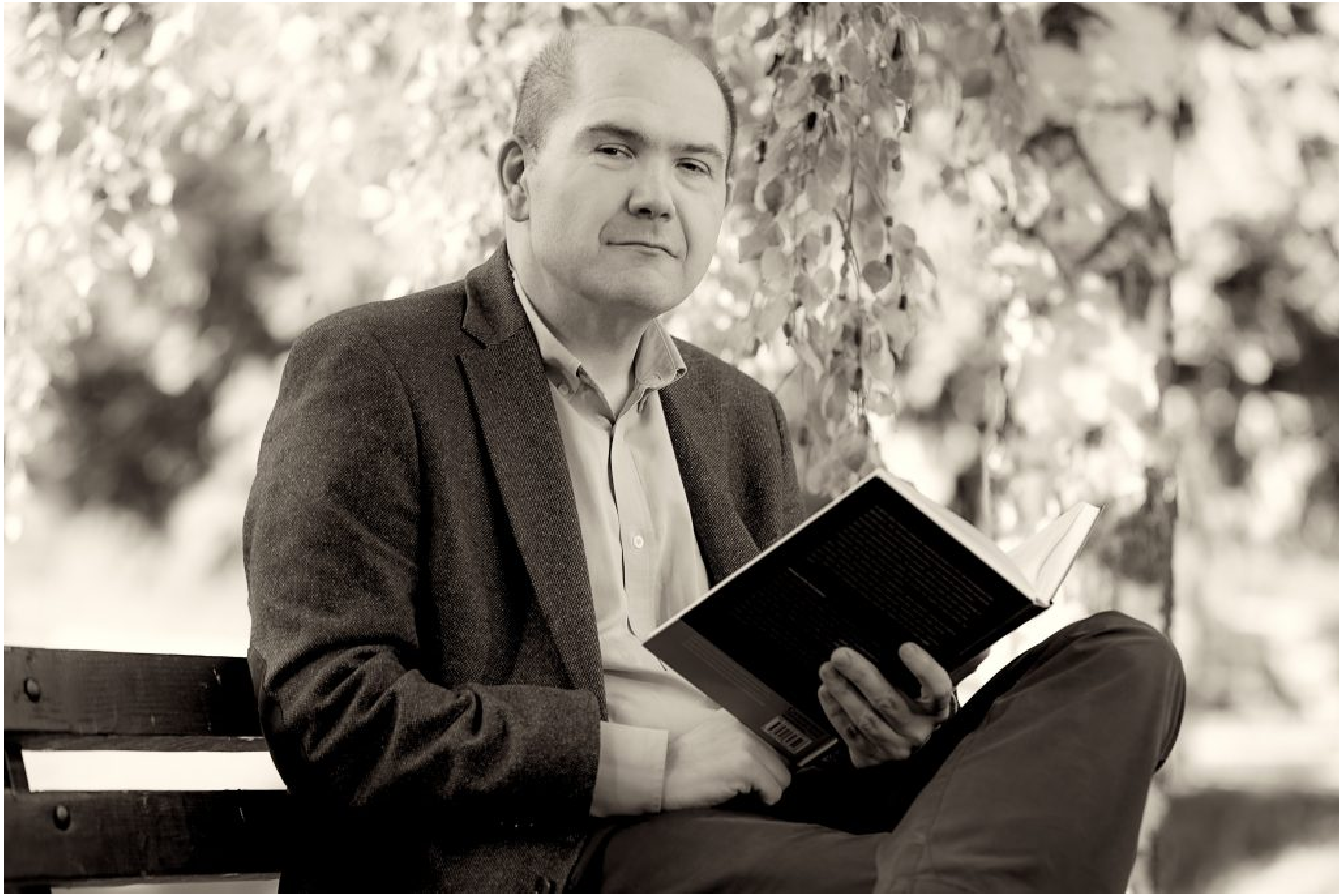
In Memoriam: Antun Balaž (1973 – 2025)



Personal Memories: Antun Balaž (1973 – 2025)

- **Original encounter**
  - Referee of his PRL 2005
  - Path integral conference at MPI-PKS Dresden (2007)
- **Talent 1: Numerics and analytics**
  - 13 common publications (2009 – 2019)
  - 6 Serbian-German projects (2009 – 2020)
- **Talent 2: Emotional intelligence**
  - Could equally well talk to junior and senior colleagues
  - Knew little personal details from my group members
- **Talent 3: Persistence**
  - Tackling problems with incredible energy
  - Preparing talks during previous nights
  - Talks at Tutzing School 2025
  - Participated at demonstrations
- **Loose ends**


METALLICA
ANALYSIS
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Welcome to BELGRADE
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ANALYSIS
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On the open-dissipative nature of photon Bose-Einstein condensates

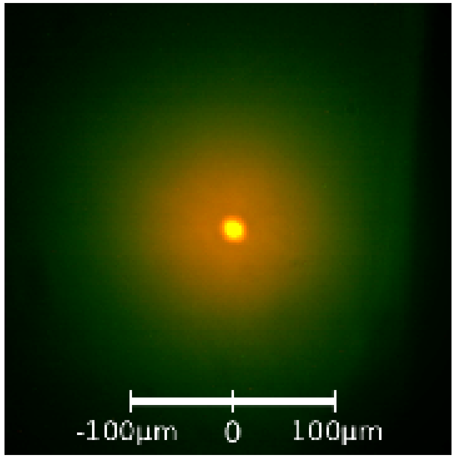
Axel Pelster

RPTU Rheinland-Pfälzische Technische Universität Kaiserslautern Landau

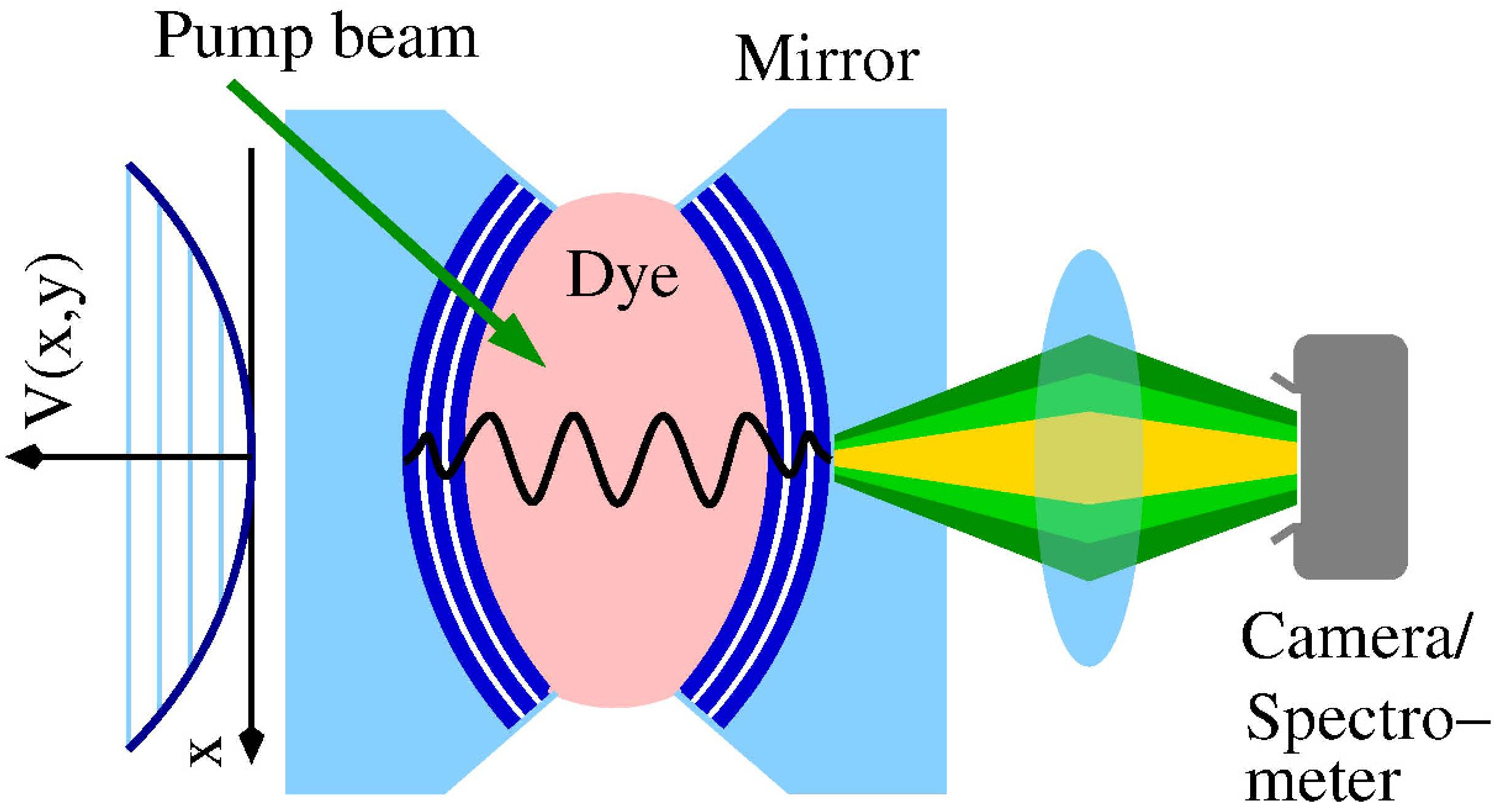
OPTIMAS TU Kaiserslautern

SFB/TR 185

1. Introduction
2. Rate Equation Model
3. Vortex
4. Outlook



1.1 Set-Up of Bonn Experiment

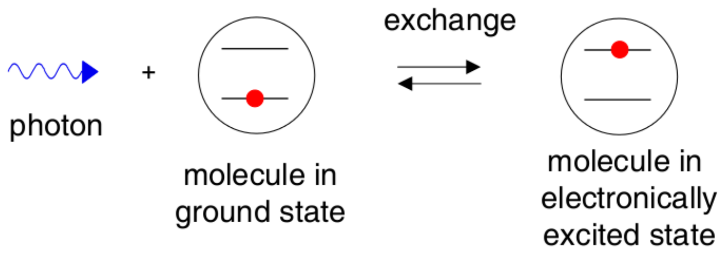


Klaers, Vewinger, and Weitz, Nature Phys. **6**, 512 (2010)

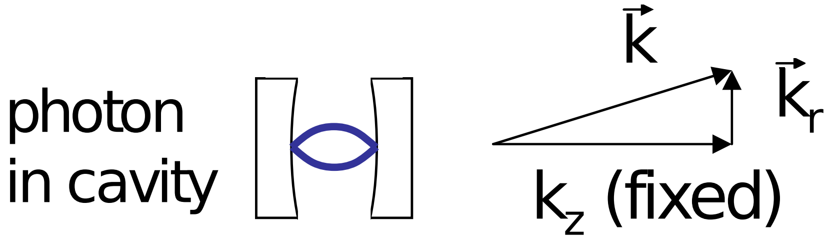
Klaers, Schmitt, Vewinger, and Weitz, Nature **468**, 545 (2010)

1.2 Basic Ingredients

- **Multiple absorption and emission of photons by dye molecules:**



- **Quadratic photon dispersion due to paraxial approximation:**

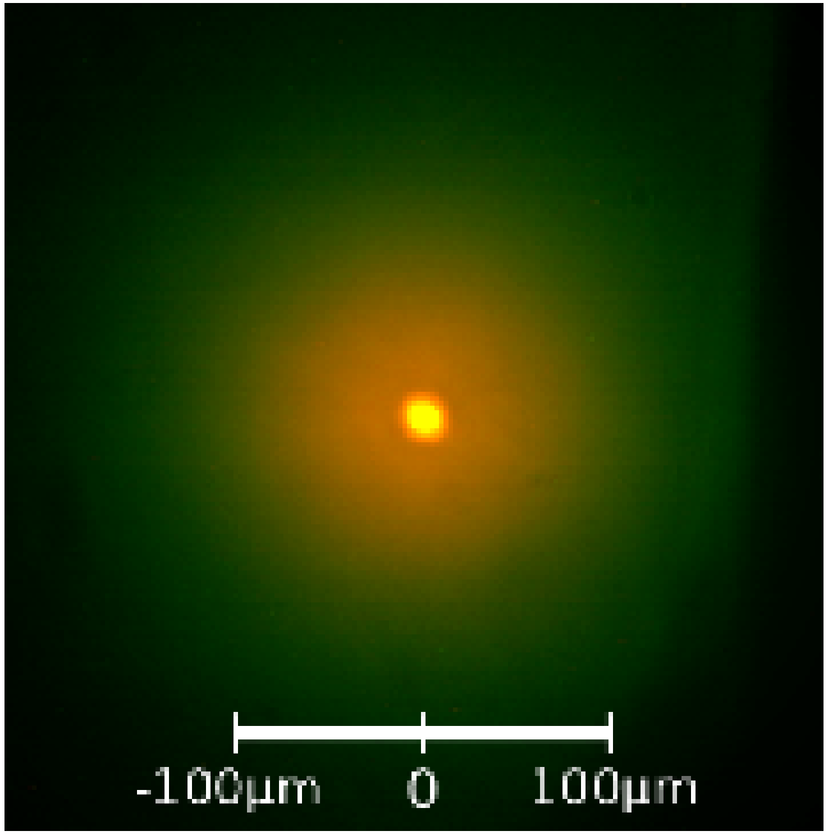


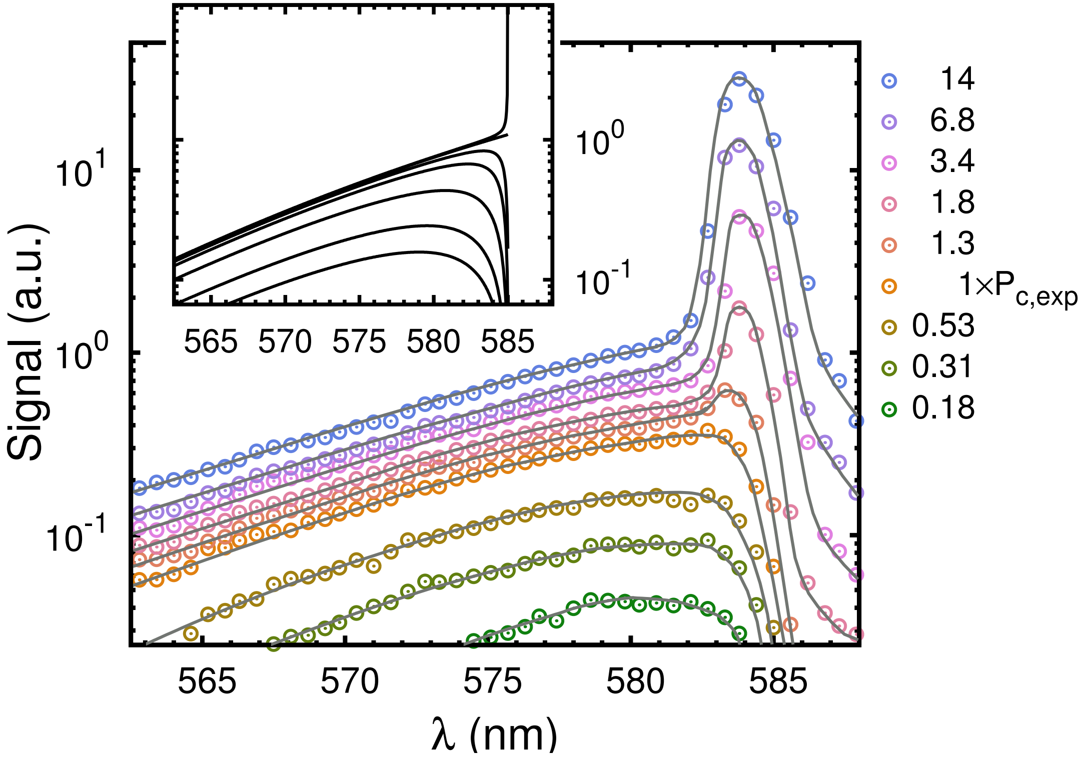
$$E_{\text{kin}} = \hbar c \sqrt{k_z^2 + k_r^2} \approx m_{\text{eff}} c^2 + \frac{\hbar^2 k_r^2}{2 m_{\text{eff}}} \,, \qquad m_{\text{eff}} = \frac{\hbar k_z}{c}$$

- **Harmonic potential from mirror curvature:**

$$E_{\text{pot}} = \frac{1}{2} m_{\text{eff}} \Omega^2 r^2 \,, \qquad \Omega = c \sqrt{\frac{2}{LR}}$$

1.3 Photon Bose-Einstein Condensate





Klärs, Schmitt, Vewinger, and Weitz, Nature **468**, 545 (2010)

Overview: Pelster, Physik-Journal **10**, Nr. 1, 20 (2011)

Pelster, Physik-Journal **13**, Nr. 3, 20 (2014)

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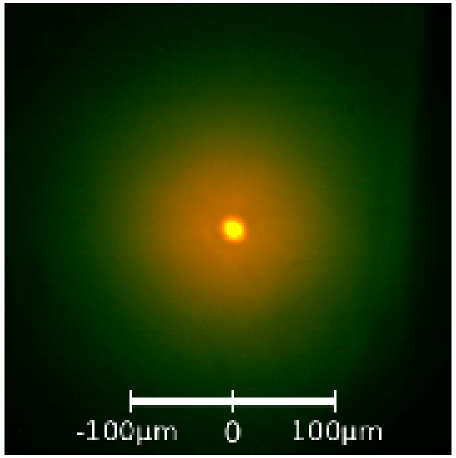
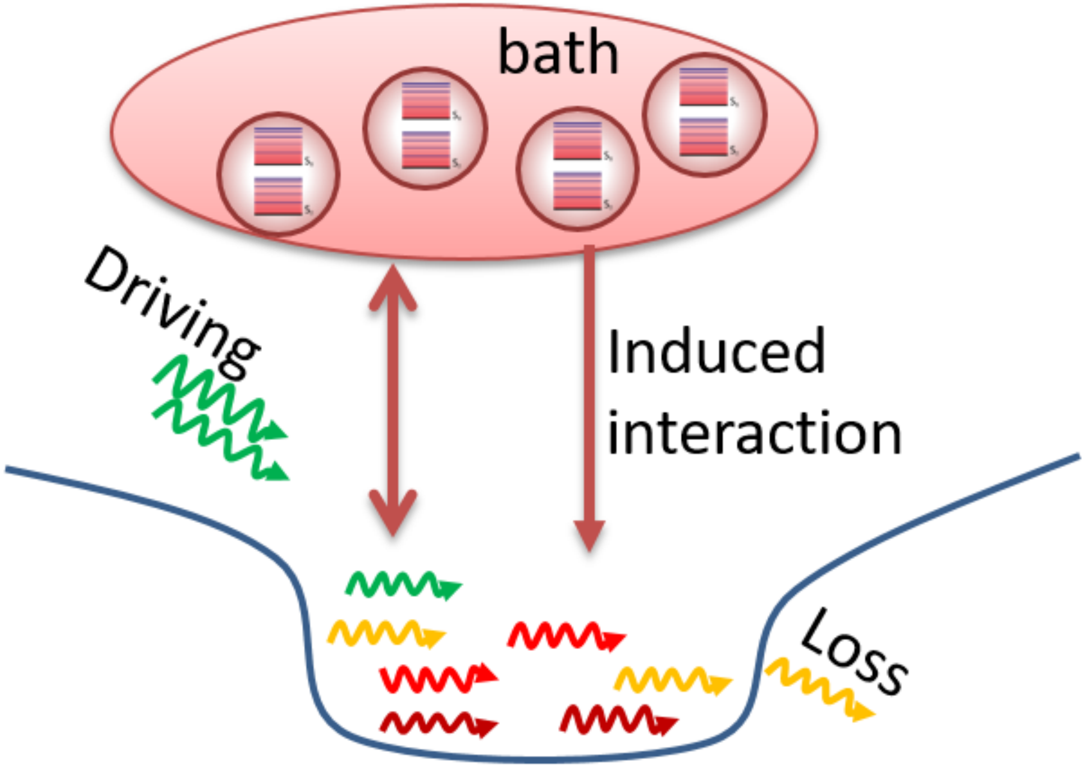


2.1 Microscopic Theory

- **Bath^2 model: Lindblad master equation**
- **Coherent and dissipative bath influences**
- **To be, or not to be BEC/LASER?**



Kirton and Keeling, Phys. Rev. Lett. **111**, 100404 (2013)

Radonjić, Kopylov, Balaž, and Pelster, New J. Phys. **20**, 055014 (2018)

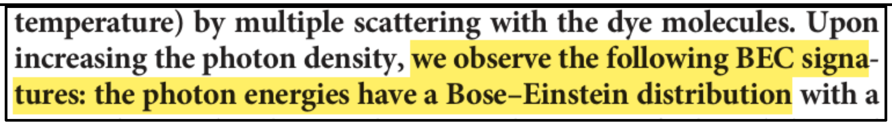
2.2 Motivation

LETTER

doi:10.1038/nature09567

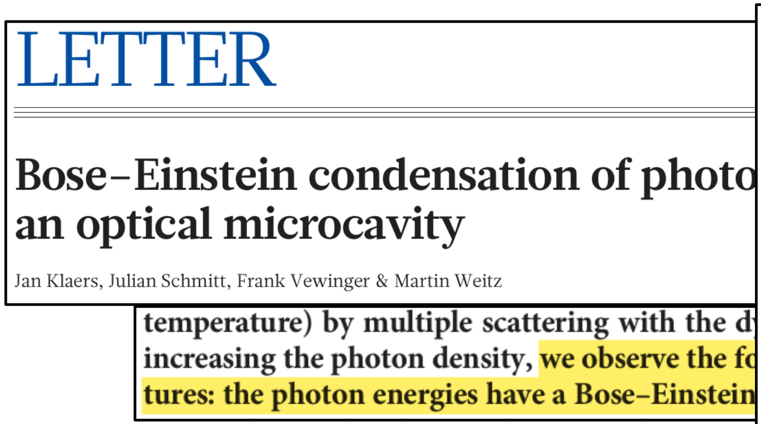
Bose–Einstein condensation of photons in an optical microcavity

Jan Klaers, Julian Schmitt, Frank Vewinger & Martin Weitz



temperature) by multiple scattering with the dye molecules. Upon increasing the photon density, we observe the following BEC signatures: the photon energies have a Bose–Einstein distribution with a

2.2 Motivation



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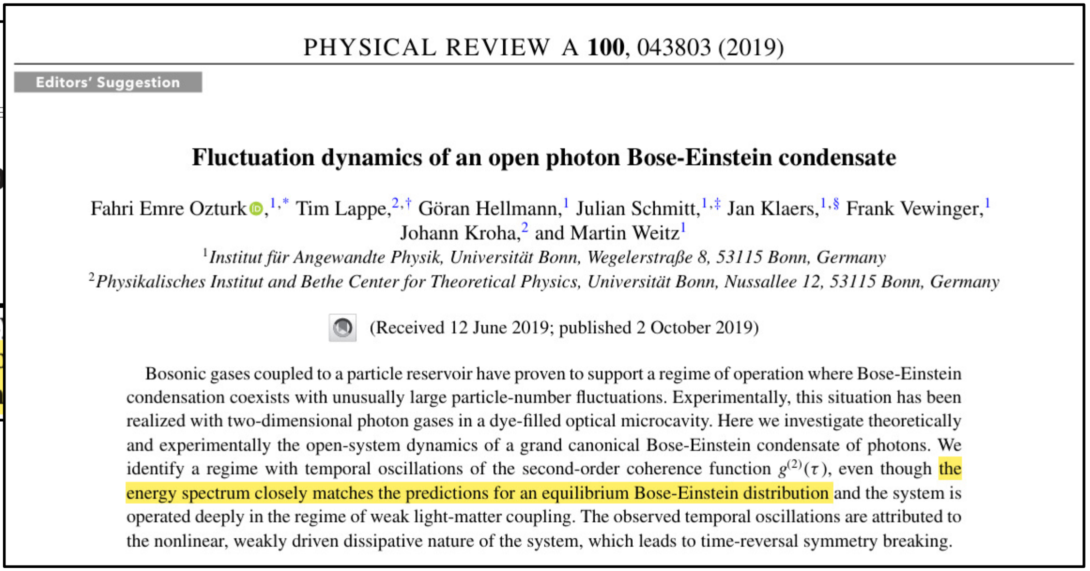


PHYSICAL REVIEW A **100**, 043803 (2019)

Editors' Suggestion

Fluctuation dynamics of an open photon Bose-Einstein condensate

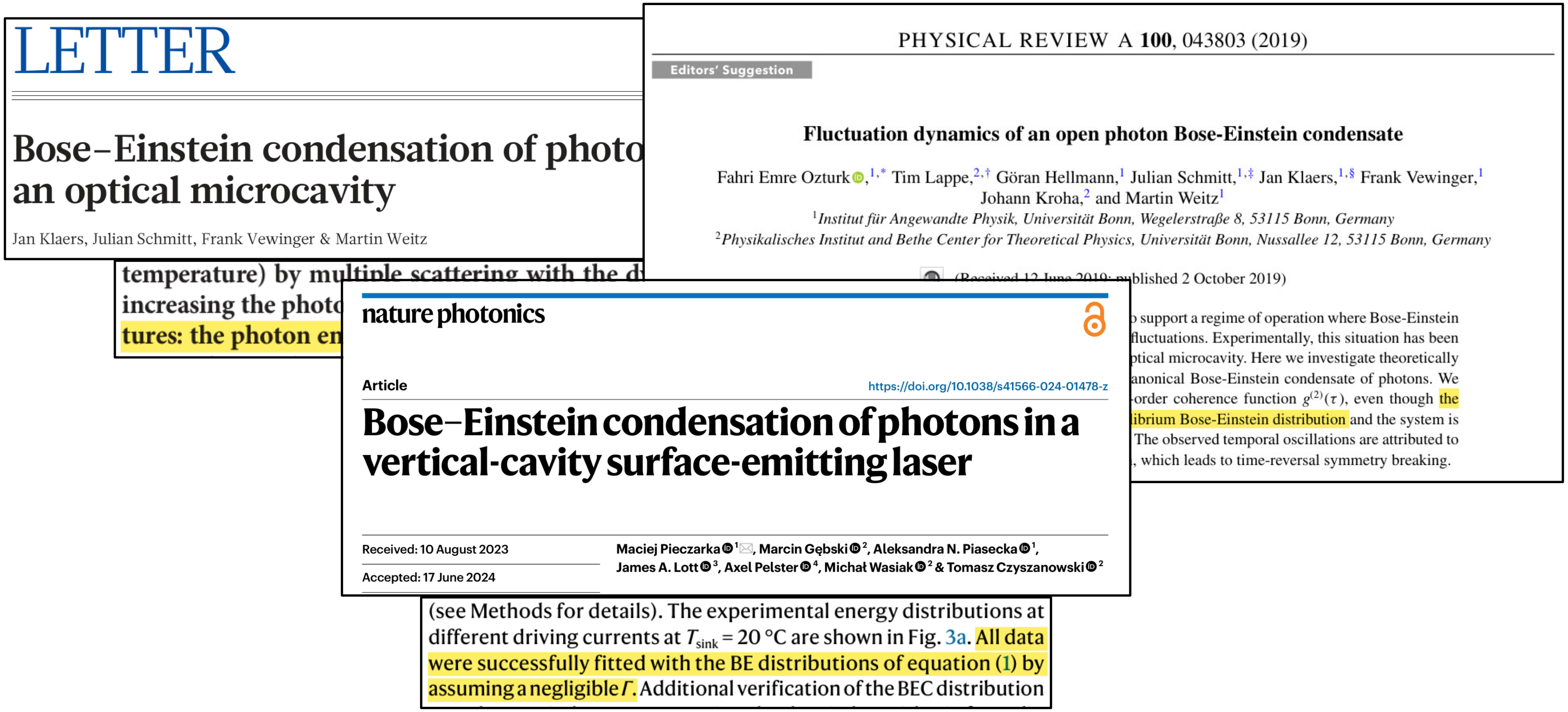
Fahri Emre Ozturk,[1,*] Tim Lappe,[2,†] Göran Hellmann,[1] Julian Schmitt,[1,‡] Jan Klaers,[1,§] Frank Vewinger,[1]
Johann Kroha,[2] and Martin Weitz[1]

[1] *Institut für Angewandte Physik, Universität Bonn, Wegelerstraße 8, 53115 Bonn, Germany*
[2] *Physikalisches Institut and Bethe Center for Theoretical Physics, Universität Bonn, Nussallee 12, 53115 Bonn, Germany*

(Received 12 June 2019; published 2 October 2019)

Bosonic gases coupled to a particle reservoir have proven to support a regime of operation where Bose-Einstein condensation coexists with unusually large particle-number fluctuations. Experimentally, this situation has been realized with two-dimensional photon gases in a dye-filled optical microcavity. Here we investigate theoretically and experimentally the open-system dynamics of a grand canonical Bose-Einstein condensate of photons. We identify a regime with temporal oscillations of the second-order coherence function $g^{(2)}(\tau)$, even though the energy spectrum closely matches the predictions for an equilibrium Bose-Einstein distribution and the system is operated deeply in the regime of weak light-matter coupling. The observed temporal oscillations are attributed to the nonlinear, weakly driven dissipative nature of the system, which leads to time-reversal symmetry breaking.

2.2 Motivation



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nature photonics

Article

https://doi.org/10.1038/s41566-024-01478-z

Bose–Einstein condensation of photons in a vertical-cavity surface-emitting laser

Received: 10 August 2023

Accepted: 17 June 2024

Maciej Pieczarka[1] ✉, Marcin Gębski[2], Aleksandra N. Piasecka[1], James A. Lott[3], Axel Pelster[4], Michał Wasiak[2] & Tomasz Czyszanowski[2]

(see Methods for details). The experimental energy distributions at different driving currents at $T_{\text{sink}} = 20$ °C are shown in Fig. 3a. All data were successfully fitted with the BE distributions of equation (1) by assuming a negligible Γ. Additional verification of the BEC distribution

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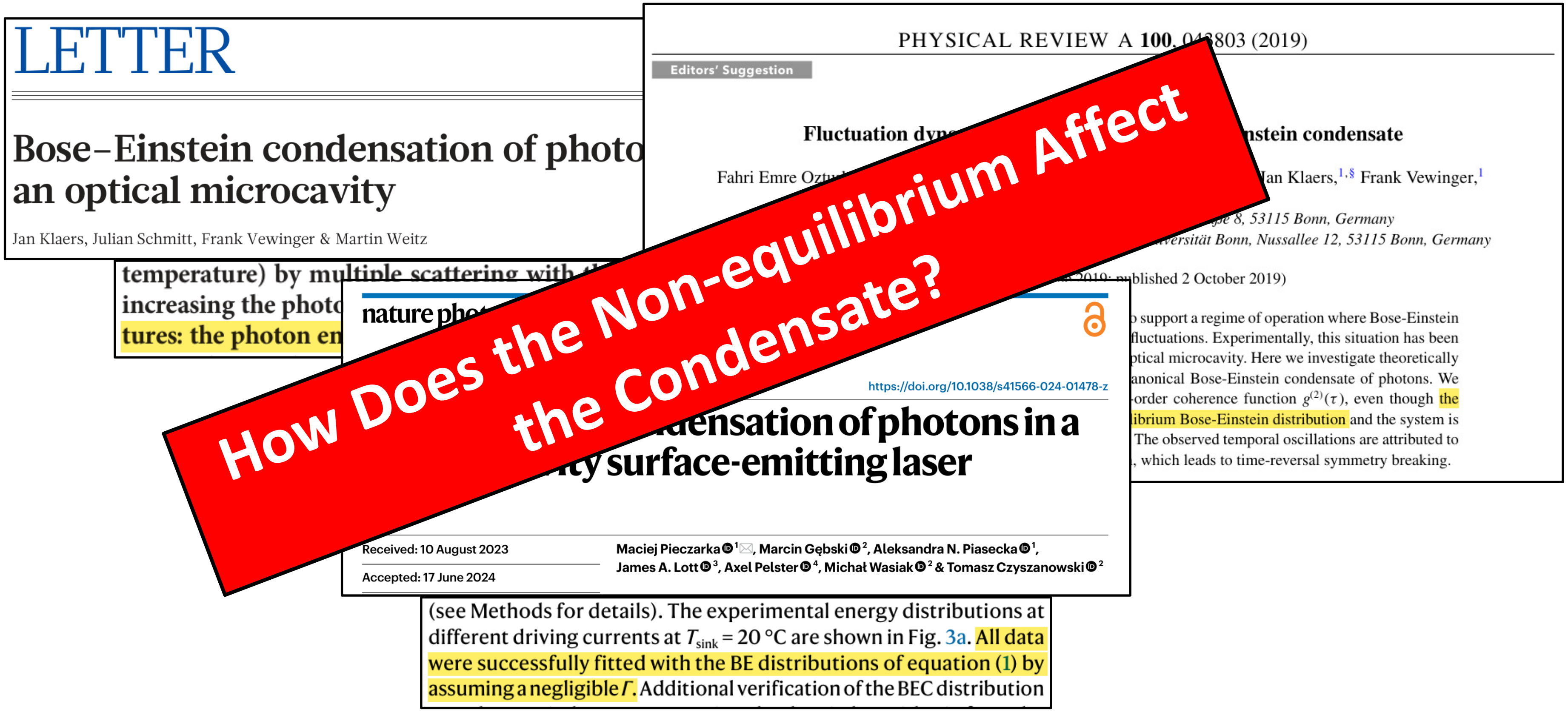

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PHYSICAL REVIEW A **100**, 0 803 (2019)

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How Does the Non-equilibrium Affect the Condensate?

2.3 Rate Equations

$$N_\ell = g_\ell n_\ell = (\ell + 1) n_\ell$$

$$\begin{aligned}
\partial_t N_\ell &= [B_{21}(\omega_\ell) M_2 - B_{12}(\omega_\ell) M_1 - \Gamma_\mathrm{c}]\, N_\ell + B_{21}(\omega_\ell) M_2 \\
\partial_t M_2 &= p M_1 - \Gamma_\mathrm{nr} M_2 - \sum_{\ell=0}^{\infty} \{[B_{21}(\omega_\ell) M_2 - B_{12}(\omega_\ell) M_1] N_\ell + B_{21}(\omega_\ell) M_2\} \\
M &= M_1 + M_2
\end{aligned}$$

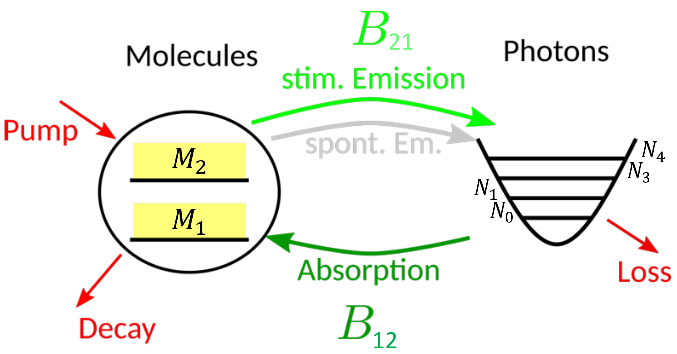
$$M = 10^{10}$$

Microscopic derivation:

P. Kirton & J. Keeling, PRA **91**, 033826 (2015)

M. Radonjić et alii, NJP **20**, 055014 (2018)

E. Stein, PhD thesis (2022)



Kennard-Stepanov (KS) relation:

$$\omega_\ell = 2\pi \cdot (500\ \mathrm{THz} + \ell \cdot 40\ \mathrm{GHz}) \qquad \omega_\mathrm{ZPL} = 2\pi \cdot 550\ \mathrm{THz}$$

$$B_{12}(\omega_\ell) = B_{21}(\omega_\ell) e^{\hbar\beta(\omega_\ell - \omega_\mathrm{ZPL})} \qquad \beta = \frac{1}{k_\mathrm{B} T} = \frac{1}{k_\mathrm{B} \cdot 300\ \mathrm{K}}$$

For $\omega_\ell = \omega_\mathrm{ZPL}$: $B_{12}(\omega_\ell) = B_{21}(\omega_\ell)$ → Laser regime!

2.4 Steady State

Photon rate equation in steady state: $(\partial_t N_\ell = 0)$

Open-dissipative BEC distribution

$$N_\ell = \frac{1}{e^{\beta(\hbar\omega_\ell - \mu)} - 1 + \frac{\Gamma_\mathrm{c}}{B_{21}(\omega_\ell)M_2}}$$

$$\mu = \hbar\omega_\mathrm{ZPL} - \frac{1}{\beta}\ln\left(\frac{M_1}{M_2}\right)$$

Chemical potential

- Emission time:
  $$t_\mathrm{EM}^{(\ell)} = \frac{1}{B_{21}(\omega_\ell)M_2} \approx 10^{-11}\ \mathrm{s}$$
- Photon life time:
  $$t_\mathrm{Ph}^{(\ell)} = \frac{1}{\Gamma_c} \leq 10^{-9}\ \mathrm{s}$$

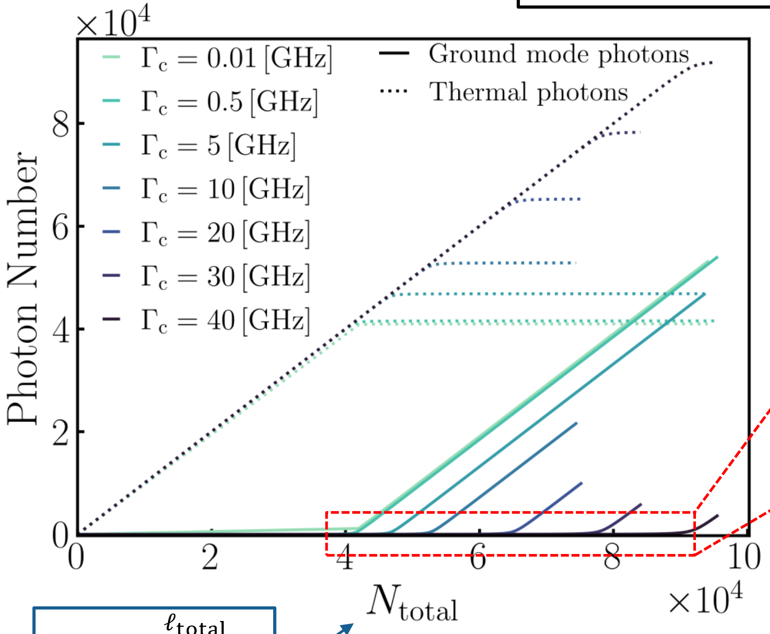
2.5 Open-Dissipative BEC Distribution

Consider:

- $\Gamma_{\mathrm{nr}} = 0\ \mathrm{MHz}$
- $\ell_{\mathrm{total}} = 2000$

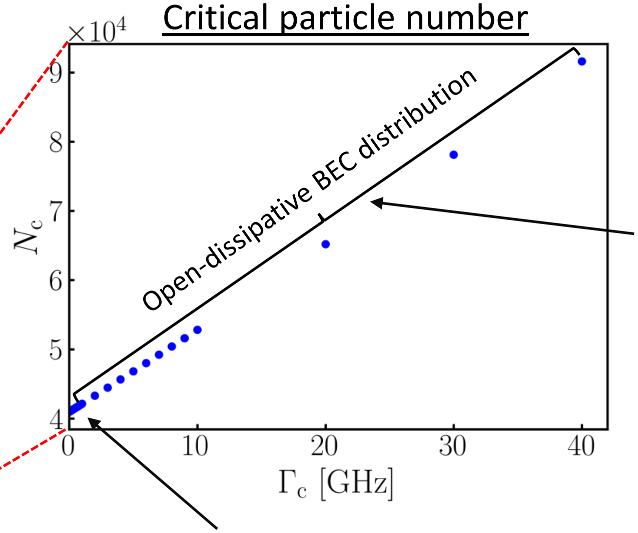
Increase of loss:

- Decrease of coherence
- Increase of decoherence



$$N_{\mathrm{total}} = \sum_{\ell=0}^{\ell_{\mathrm{total}}} N_\ell$$

Critical particle number



Can This be Measured?

Numerically determined: $N_c(\Gamma_c = 0.01\ \mathrm{GHz}) \approx 40\,970$

Expectation for BEC distribution:

B. Klünder & A. Pelster, EPJ B **68**, 457-465 (2009)

$$\boxed{N_c(\Gamma_c = 0\ \mathrm{GHz}) \approx 40\,980}$$

Krauß, Stein, and Pelster, Europ. Phys. J. Spec. Top. (2026)

In Memoriam Hermann Haken: Synergetics and Self-organisation in Complex Systems

`https://doi.org/10.1140/epjs/s11734-026-02185-2`

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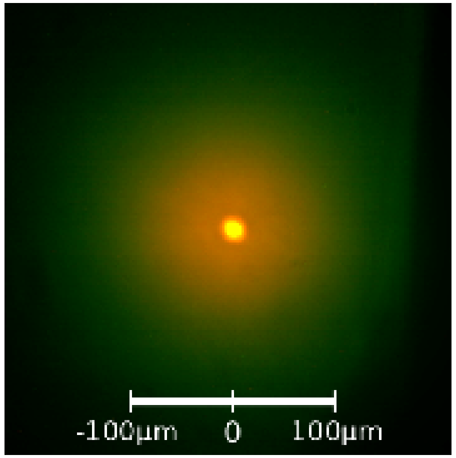
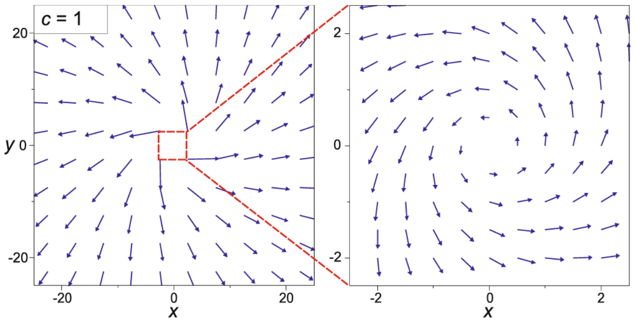


3.1 Motivation

Non-Equilibrium Quantum Fluids

V. N. Gladilin & M. Wouters, NJP **19,** 105005 (2017)

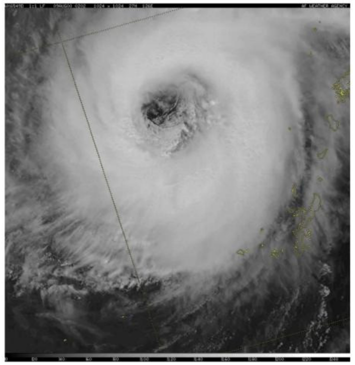


Spiral Galaxies

F. Combes, ArXiv: 2302.12913 (2023)

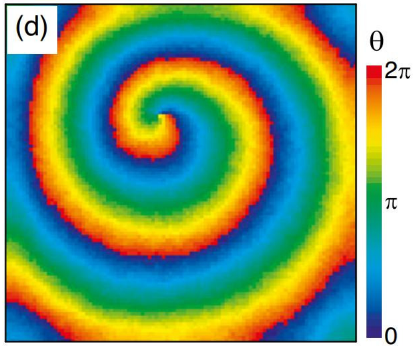
Tropical Cyclones

R. K. Smith, Lectures on Tropical Cyclones (2006)



Lattice of Photon BECs

V. N. Gladilin & M. Wouters, PRL **125,** 215301 (2020)



How can we **analytically** describe vortices in open-dissipative systems?

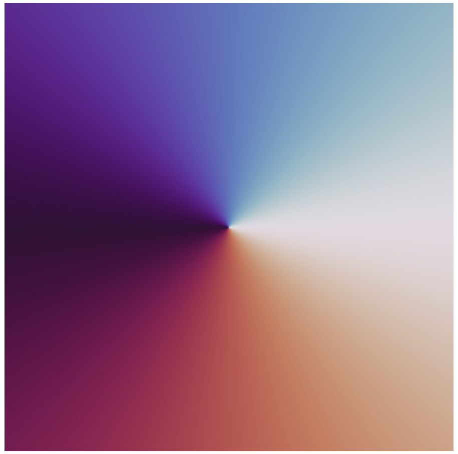
3.2 Phenomenological Model

- **Complex Gross-Pitaevskii equation:**

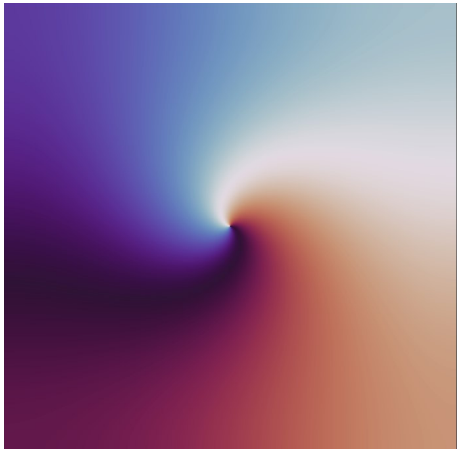
$$i\hbar \frac{\partial \psi}{\partial t} = \left[-\frac{\hbar^2}{2m} \boldsymbol{\nabla}^2 + \frac{M}{2} \omega^2 \boldsymbol{x}^2 + g\, |\psi|^2 + \frac{i}{2} \left(\gamma - \Gamma |\psi|^2 \right) \right] \psi$$

Keeling and Berloff, PRL **100**, 250401 (2008)

- **Resulting phase portrait:**



closed system



open system

3.3 Analytic Approximation Methods

- **Variational optimization method:**

  Pérez-García, Michinel, Cirac, Lewenstein, and Zoller,
  PRL **77**, 5320 (1996)

- **Cumulant optimization method:**

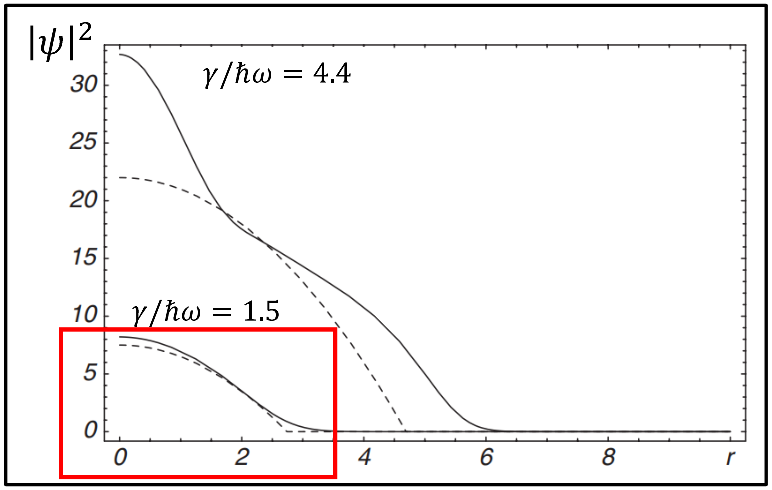
  Mann, Bakhtiari, Pelster, and Thorwart, PRL **120**, 063605 (2018)
  Stein, Vewinger, and Pelster, NJP **21**, 103044 (2019)

- **Projection optimization method:**
  - **Equation of motion:** $\mathrm{EOM}[\psi^*, \psi] = 0$
  - **Wave function ansatz:** $\psi(\boldsymbol{x}) \approx \Psi(\boldsymbol{x}, \boldsymbol{\alpha})$
  - **Projection:**

$$\left\langle \mathrm{EOM}^*[\Psi^*, \Psi], \frac{\partial \Psi^*}{\partial \alpha^i} \right\rangle + \left\langle \mathrm{EOM}[\Psi^*, \Psi], \frac{\partial \Psi}{\partial \alpha^i} \right\rangle = 0$$

Krauß, dos Santos Filho, dos Santos, and Pelster, PRR **7**, 033007 (2025)

3.4 Single Vortex



Consider ansatz for single charged ($\ell = \pm 1$) vortex:

[A. Cidrim at alii, PRA **98** (2018)]

$$\boldsymbol{v}(r,\varphi) = \frac{\hbar}{m}\boldsymbol{\nabla}\phi(r,\varphi)$$

Helmholtz decomposition

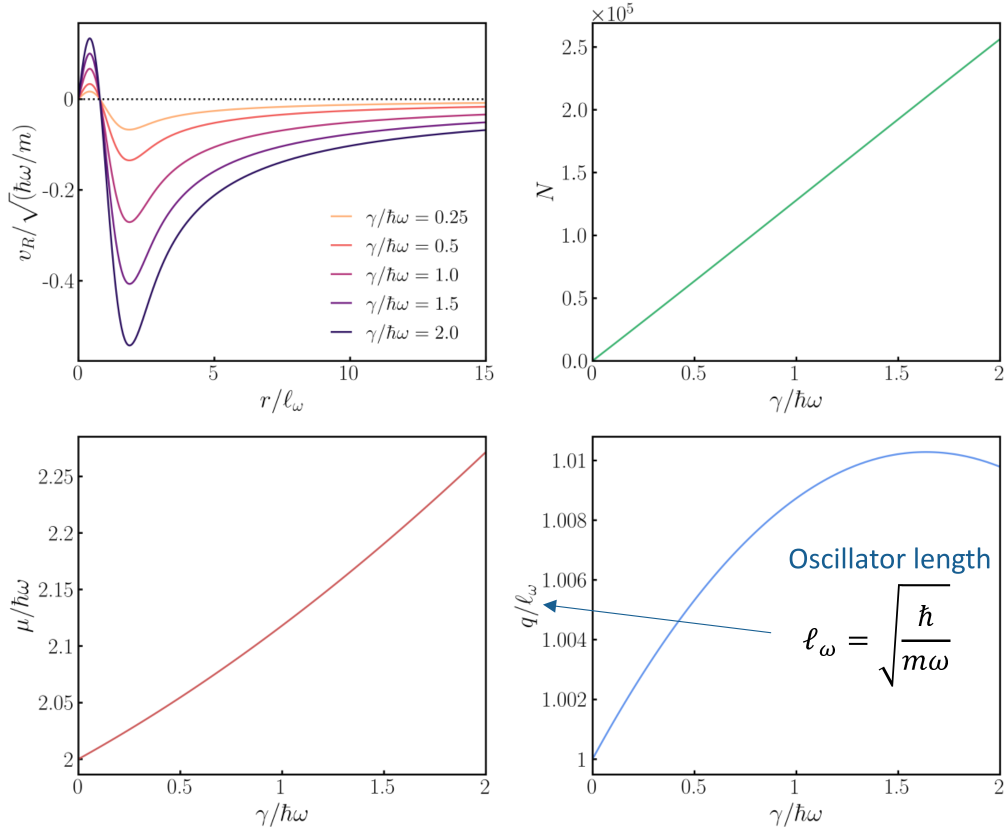
$$\psi(r,\varphi,t) = \sqrt{n(r,t)}\; e^{i\phi(r,\varphi,t)} e^{-\frac{i}{\hbar}\mu(t)} = \sqrt{\frac{N(t)}{\pi q(t)^4}}\; r\; e^{-\frac{r^2}{2q(t)^2}+i[\ell\varphi+\varphi_R(r,t)]}\; e^{-\frac{i}{\hbar}\mu(t)}$$

Madelung tranformation

To be determined:
- Particle number $N(t)$
- Chemical potential $\mu(t)$
- Condensate width $q(t)$
- Radial velocity $v_R(r,t)$

3.5 Steady State

Solution of projection optimization equations:



Oscillator length

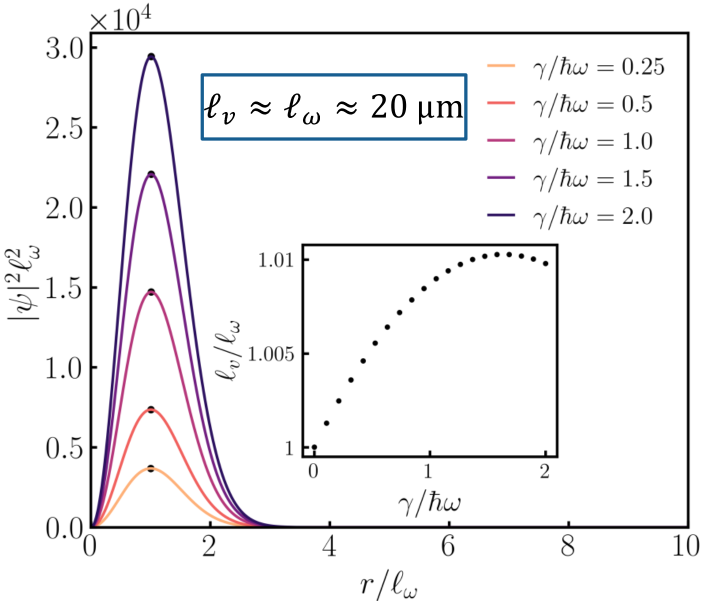
$$\ell_\omega = \sqrt{\frac{\hbar}{m\omega}}$$

Fixed:

- $\Gamma = 10^{-4} \mathrel{\widehat{=}} 10\ \text{GHz}$
- $g = 10^{-5}$

Vortex core size ℓ_v:

- Density reaches maximum value



$\ell_v \approx \ell_\omega \approx 20\ \mu\text{m}$

3.6 Stability

- Linearized dynamics:

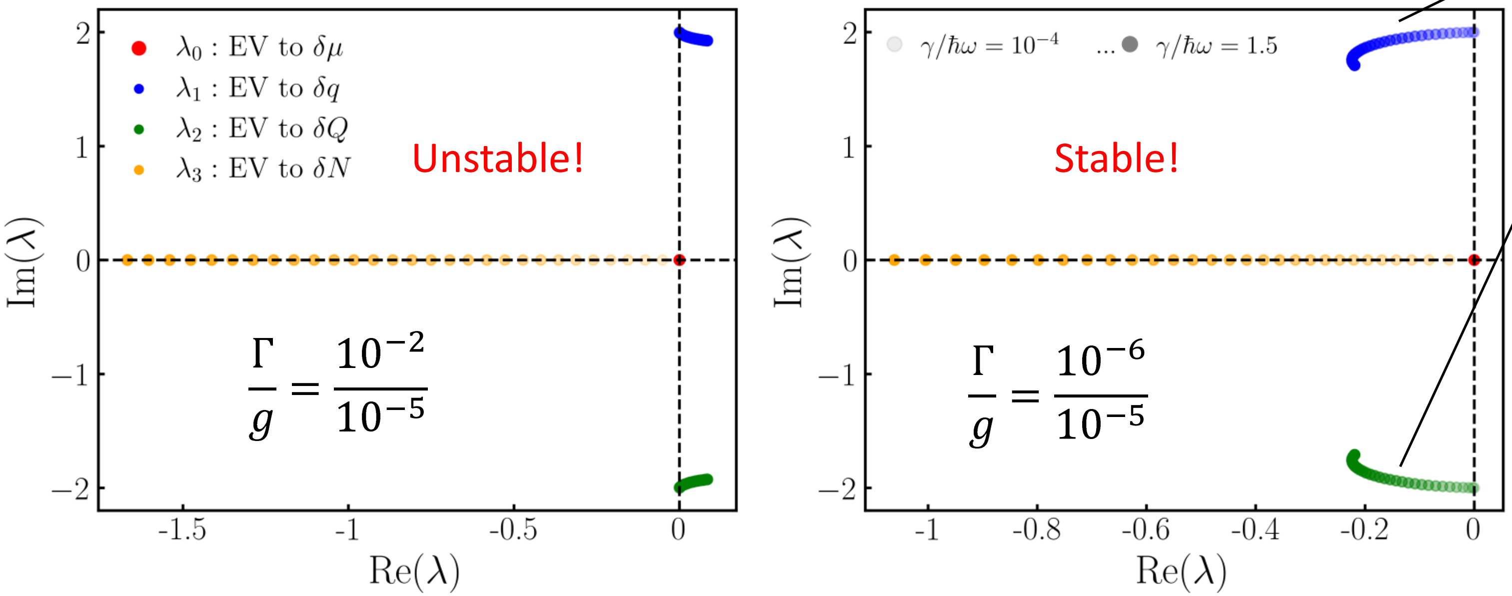
$A \in \mathrm{Mat}(4 \times 4, \mathbb{R})$

$$\left.\begin{aligned} N(t) &= N_0 + \delta N(t) \\ q(t) &= q_0 + \delta q(t) \\ \mu(t) &= \mu_0 t + \delta\mu(t) \end{aligned}\right] \longrightarrow \partial_t \begin{pmatrix} \delta N \\ \delta q \\ \delta Q \\ \delta\mu \end{pmatrix} = A \cdot \begin{pmatrix} \delta N \\ \delta q \\ \delta Q \\ \delta\mu \end{pmatrix}$$

$\delta Q = \partial_t \delta q$

- Condensate lifetime: ≈ 500 ns
- Characteristic timescale: $\mathcal{O}(1\text{ ns})$

Eigenvalues of A for $10^{-4} \leq \frac{\gamma}{\hbar\omega} \leq 1.5$:



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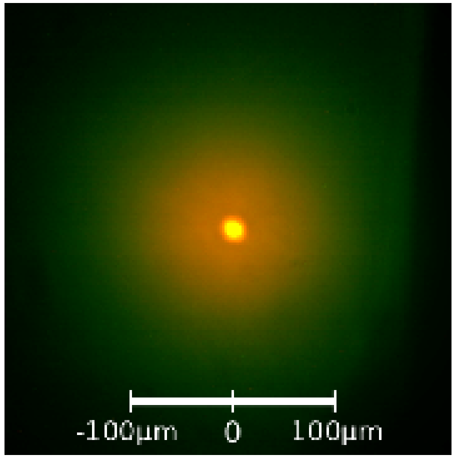
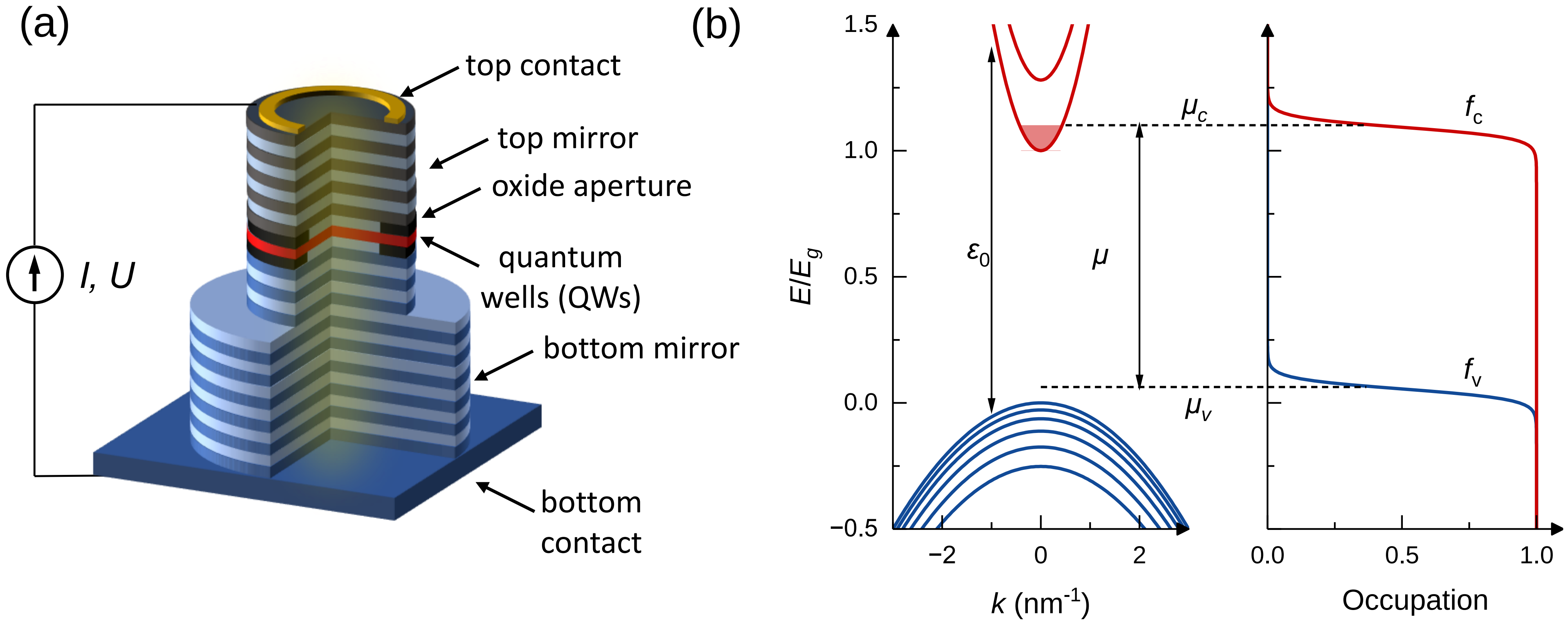


4.1 Vertical-Cavity Surface-Emitting Laser



Pieczarka, Gebski, Piasecka, Lott, Pelster, Wasiak, and Czyszanowski,
Nat. Phot. **18**, 1090 (2024)

4.2 Thermalization of photons

- **Rate equation for photon number:**

$$\frac{d}{dt}N(\varepsilon) = R_{\mathrm{em}}(\varepsilon)\big[N(\varepsilon)+1\big] - \big[R_{\mathrm{abs}}(\varepsilon)+\gamma(\varepsilon)\big]N(\varepsilon)$$

- **Detailed balance between absorption and emission:**

$$\frac{R_{\mathrm{abs}}(\varepsilon)}{R_{\mathrm{em}}(\varepsilon)} = \exp\left(\frac{\varepsilon-\mu}{k_B T}\right), \qquad \mu = \mu_c - \mu_v$$

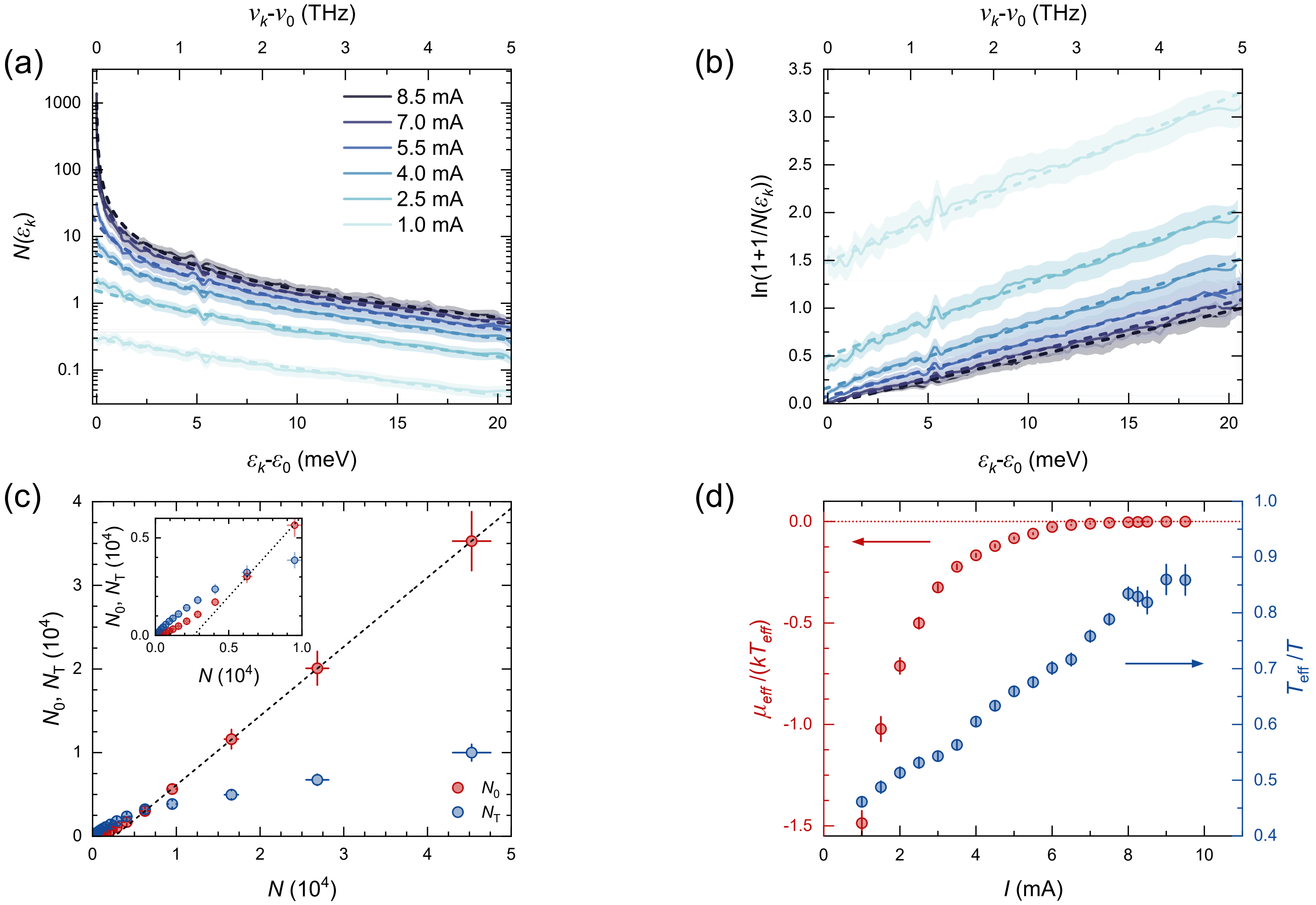
- **Steady-state photon distribution:**

$$N(\varepsilon) = \frac{1}{\exp(\frac{\varepsilon-\mu}{k_B T}) - 1 + \Gamma(\varepsilon)}$$

- **Correction parameter for fundamental mode**

$$\Gamma(\varepsilon_0) = \frac{\gamma(\varepsilon_0)}{R_{\mathrm{em}}(\varepsilon_0)} \approx \frac{24\ \mathrm{fs}}{3\ \mathrm{ps}} = 0.008$$

4.3 Energy distributions and thermodynamic quantities



28. Deutsche Physikerinnentagung

28.09. - 30.09.

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