Sound in Ring Dipolar Supersolids

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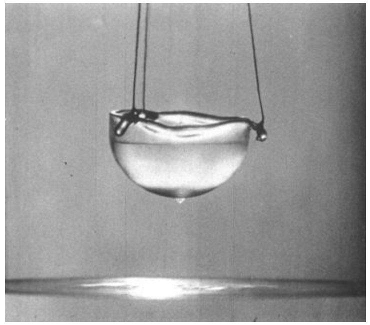
Photos by Bojan Džodan

2025

- 2019–20 Master's thesis supervisor
- 2020–21 PhD supervisor
- 2019–25 Friend and source of support

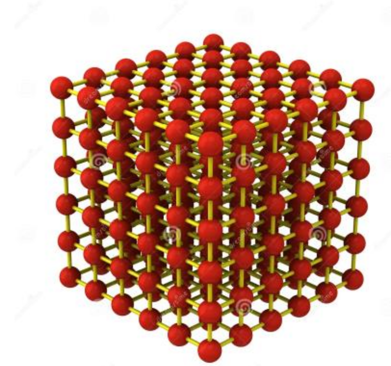
What is a supersolid?

SUPERFLUID



Spontaneous U(1) symmetry breaking

SOLID



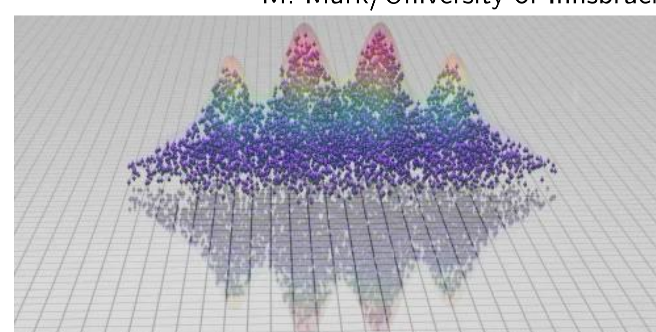
Spontaneous translational symmetry breaking

SUPERSOLID

Spontaneous and simultaneous breaking of phase and translational invariance

- Cluster supersolidity: non-commensurate crystal
- 2017: Spin-orbit coupled BECs – Ketterle
- 2017: Optical cavities – Esslinger
- 2019: Dipolar BECs – Modugno, Pfau, Ferlaino

M. Mark/University of Innsbruck



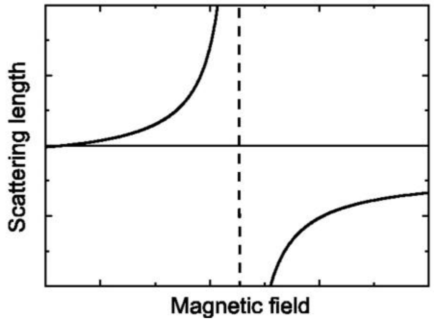
Dipolar Bose-Einstein condensates

- Permanent magnetic dipole moment
- Dilute, ultracold gas

Contact interaction

$$V_c(\boldsymbol{r}) = g\,\delta(\boldsymbol{r}), \quad g = \frac{4\pi\hbar^2 \boldsymbol{a}}{m}$$

- Particle scattering in dilute, ultracold regime
- Short-range, isotropic
- Fully characterized by the s-wave scattering length
- Tunable via Feshbach resonances

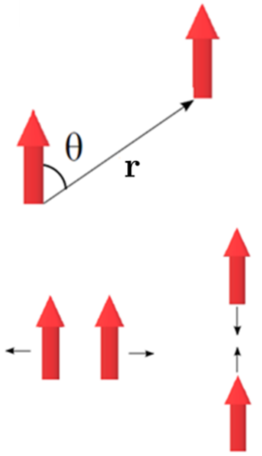


$$\epsilon_{\rm dd} = \frac{a_{\rm dd}}{a}$$

Dipole-dipole interaction

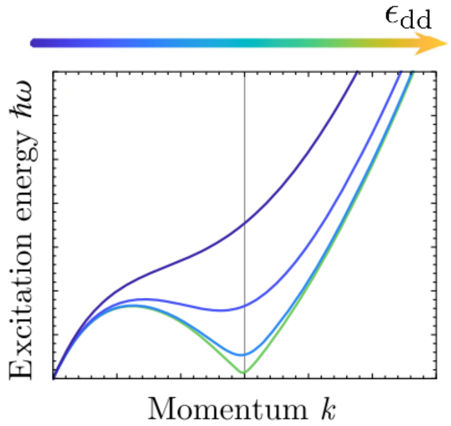
$$V_{\rm dd}(\boldsymbol{r}) = \frac{3\hbar^2 a_{\rm dd}}{m}\,\frac{1-3\cos^2\theta}{r^3}, \quad \boldsymbol{a}_{\bf dd} = \frac{\mu_0\mu^2 m}{12\pi\hbar^2}$$

- Long-ranged: decaying as r^3
- Anisotropic: attractive or repulsive
- Need for confinement along polarization direction

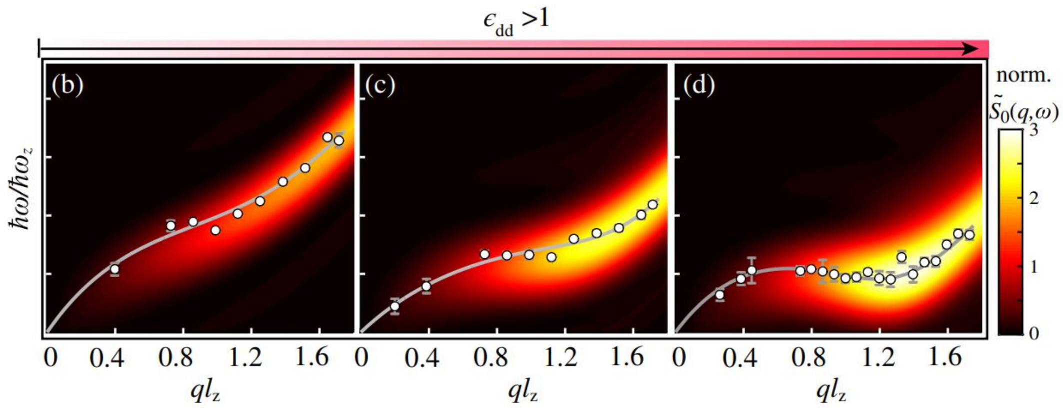


- Confinement along polarization direction leads to rotonic excitation spectrum

  Santos, Shlyapnikov, Lewenstein, PRL **90** (2003)



Phys. Rev. X **11**, 011037 (2021)



PRL **122**, 183401 (2019)

- Mean-field collapse

Dipolar Bose-Einstein condensates

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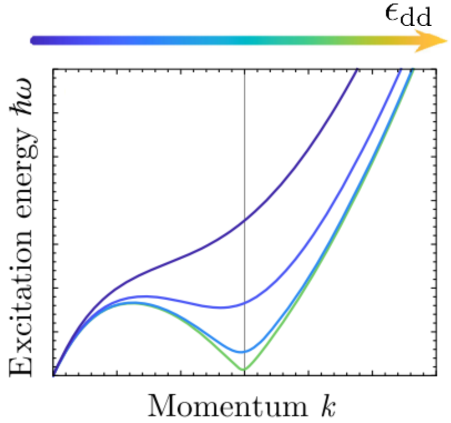
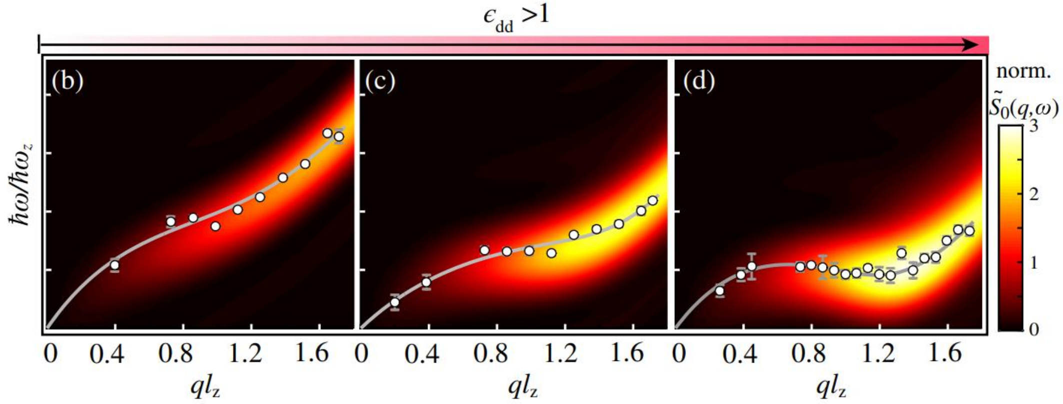
  Santos, Shlyapnikov, Lewenstein, PRL **90** (2003)

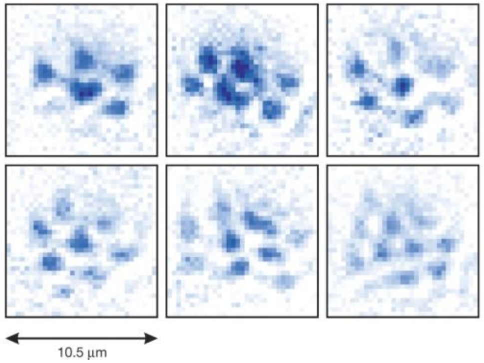


Phys. Rev. X **11**, 011037 (2021)



PRL **122**, 183401 (2019)

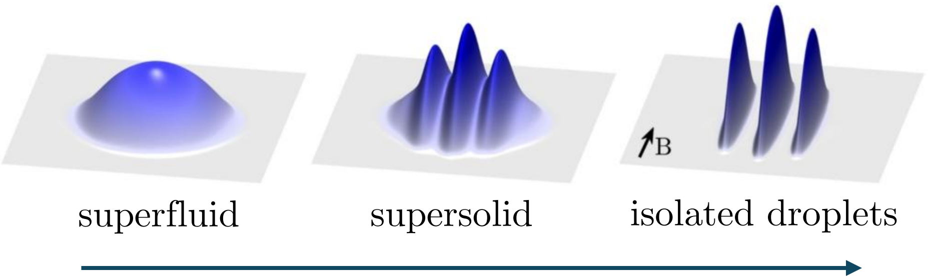
- Mean-field collapse
- Stabilization by quantum fluctuations



Nature **530**, 194 (2016)

Theory:
Petrov, PRL **115**, 155302 (2015)
Santos, PRA **93**, 061603 (2016)
Roccuzzo, PRA **99**, 041601 (2019)

Experiments:
Modugno, PRL **122**, 130405 (2019)
Pfau, PRX **9**, 011051 (2019)
Ferlaino, PRX **9**, 021012 (2019)



Rep. Prog. Phys. **84** 012403

Goldstone modes

- Goldstone theorem: spontaneous breaking of a global continuous symmetry gives rise to a gapless excitation mode

SUPERFLUID

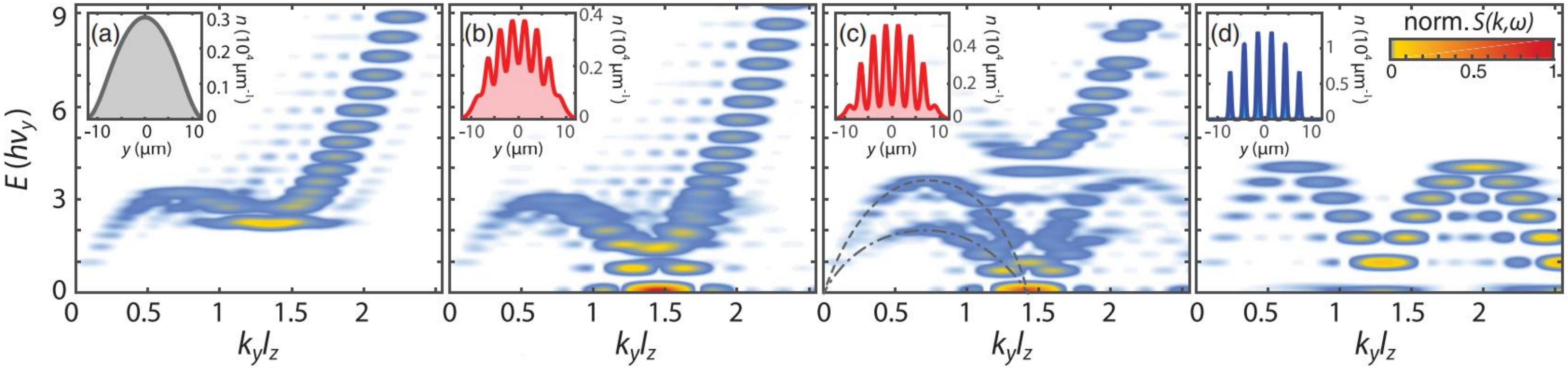
SSB of phase invariance

$\rightarrow$ 1 Goldstone mode:
superfluid phonon

SUPERSOLID

SSB of phase & translational symmetry along d directions

$\rightarrow$ $d+1$ Goldstone modes:
1 superfluid and d crystal phonons



PRL **123**, 050402 (2019)

- Tubular confinement with periodic boundary conditions $\Leftrightarrow$ ring geometry

Theoretical description

Extended Gross-Pitaevskii equation

- Microscopic theory, describes time evolution of the wavefunction (density $n(\mathbf{r},t) = |\Psi(\mathbf{r},t)|^2$)

$$i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r},t) = \left[-\frac{\hbar^2\nabla^2}{2m} + V_{\text{ext}}(\mathbf{r},t) + g|\Psi(\mathbf{r},t)|^2 + \int d\mathbf{r}' V_{\text{dd}}(\mathbf{r}-\mathbf{r}')|\Psi(\mathbf{r}',t)|^2 + \gamma(\epsilon_{\text{dd}})|\Psi(\mathbf{r},t)|^3\right]\Psi(\mathbf{r},t)$$

CONTACT INTERACTION DIPOLE-DIPOLE INTERACTION QUANTUM FLUCTUATIONS

- Solved numerically to study static and dynamic properties

Hydrodynamic theory of supersolids

Andreev&Lifshitz (1969)
Yoo&Dorsey (2010)
Hofmann&Zwerger (2019)

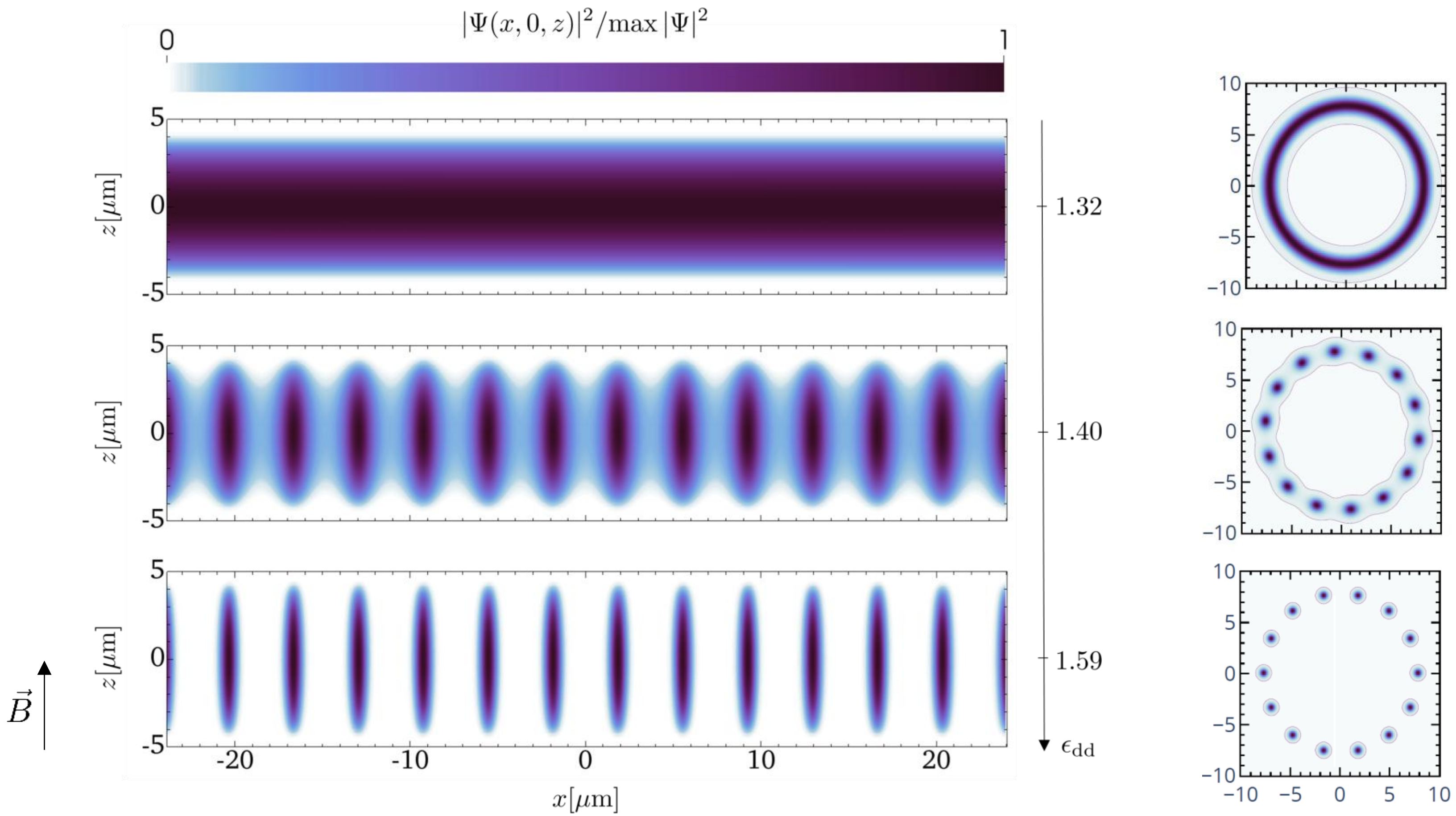
- Macroscopic theory: long-wavelength, low-energy limit
- Supersolids consist of both superfluid and normal components: $\rho = \rho_s + \rho_n$, $j = \rho_s v_s + \rho_n v_n$
- Hydrodynamic equations:

$\partial_t \rho + \partial_x j = 0$	Conservation of mass - continuity equation
$\partial_t j + \partial_x(p + \rho_n v_n^2 + \rho_s v_s^2) = 0$	Conservation of momentum
$\partial_t v_s + \partial_x(\mu + v_n v_s) = 0$	U(1) symmetry breaking - Josephson equation
$\partial_t u_x + \partial_x(v_n u_x - v_n) = 0$	Translational symmetry braking, $u_x := \partial_x u$

- Linearization around stationary values → two pairs of longitudinal sound modes: $\omega = \pm c|q|$

Ground states (numerical results)

- $80.000\ ^{164}\mathrm{Dy}$ atoms $\qquad V_{\mathrm{ext}} = \frac{1}{2} m\omega_\perp^2 (y^2 + z^2), \quad \omega_\perp = 2\pi \times 100\mathrm{Hz}$



$L = 48\mu\mathrm{m}$ $\qquad R = 7.6\mu\mathrm{m}$

The protocol for sound excitation

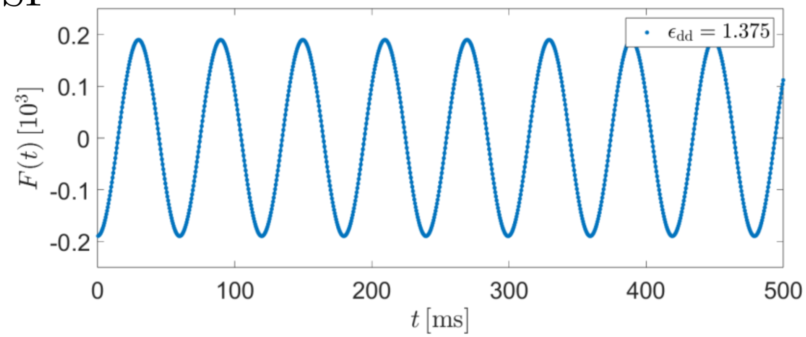
- Prepare the system in the ground state in presence of weak periodic perturbation

$$V_{\text{pert}} = V_0 \cos(qx), \quad q = \frac{2\pi}{L}$$

- At $t = 0$ set $V_0 = 0$ → excitation of longitudinal phonon modes
- Time evolution of the quantity

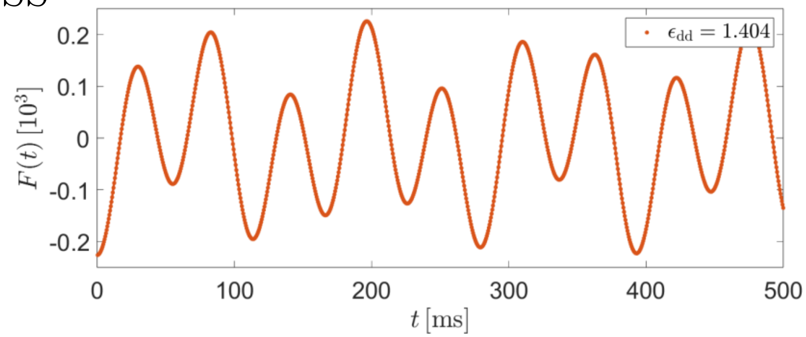
$$F(t) = \langle \cos(qx) \rangle (t)$$

SF



$$F(t) = V_0 \chi_1 \cos(\omega_1 t)$$

SS



$$F(t) = V_0 \left[\chi_1 \cos(\omega_1 t) + \chi_2 \cos(\omega_2 t) \right]$$

- Long wavelength limit, $q \to 0$: approach phononic dispersion relation

$$\omega_1(q) = c_1 q, \quad \omega_2(q) = c_2 q$$

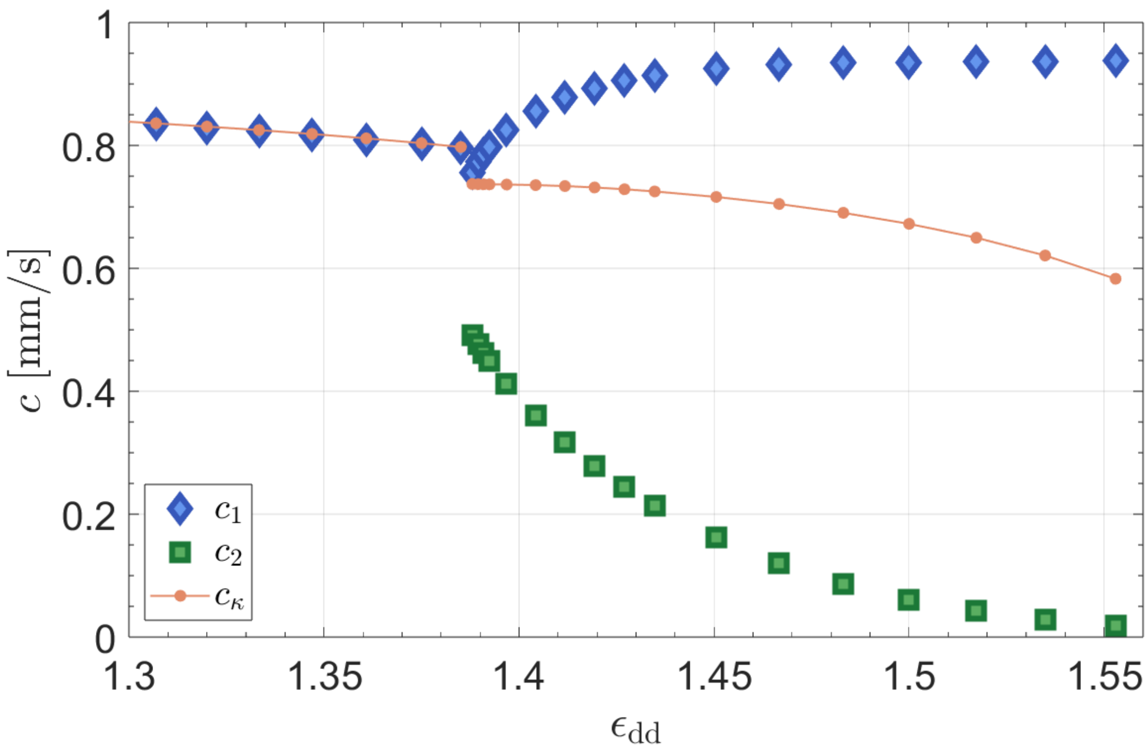
Sound speeds: eGPE simulation

- In uniform systems the speed of sound is related to compressibility:

$$c_\kappa = \sqrt{\kappa^{-1}}$$

- Determine weights to find static response function:

$$\chi(q) = \chi_1(q) + \chi_2(q) \overset{q\to 0}{=} \frac{N\kappa}{2}$$



Sound modes: HD theory

- Solving the hydrodynamic equations gives two sound modes:

$$c_{1,2}^2 = \frac{c_\kappa^2}{2}\left(1 + \beta\kappa \pm \sqrt{(1+\beta\kappa)^2 - 4 f_s \beta\kappa}\right)$$

Compressibility

$$\kappa = \left(\rho\,\frac{\partial\mu}{\partial\rho}\right)^{-1}$$

$$c_\kappa = \sqrt{\kappa^{-1}}$$

Renormalized layer compressibility modulus

$$\beta = \frac{B}{\rho_n} = \frac{1}{\rho_n}\frac{\partial^2\mathcal{E}}{\partial u_x^2}$$

Superfluid fraction

$$f_s = \frac{\rho_s}{\rho}$$

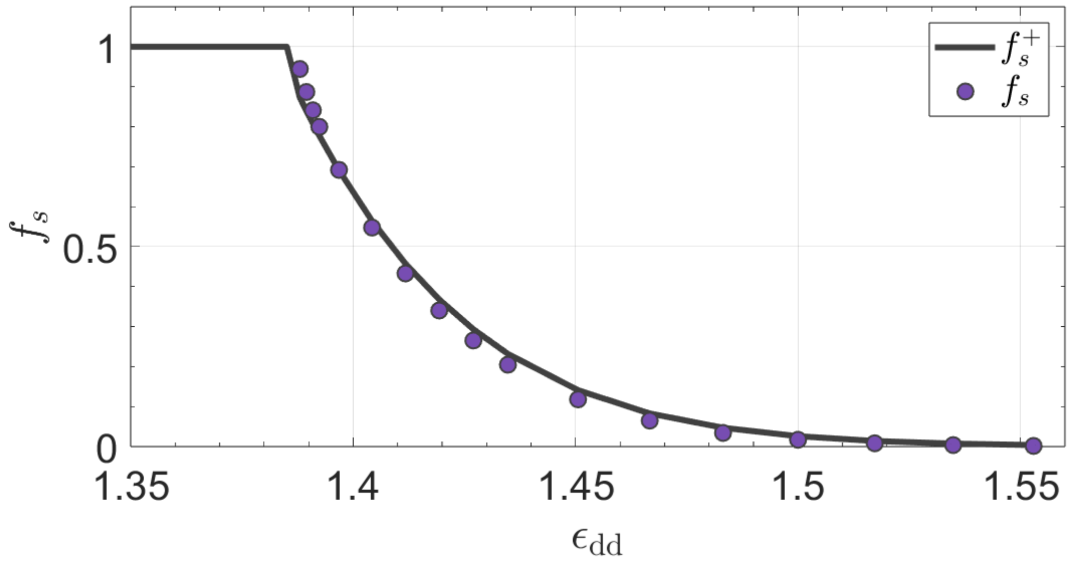
- From the "measured" sound velocities we determine

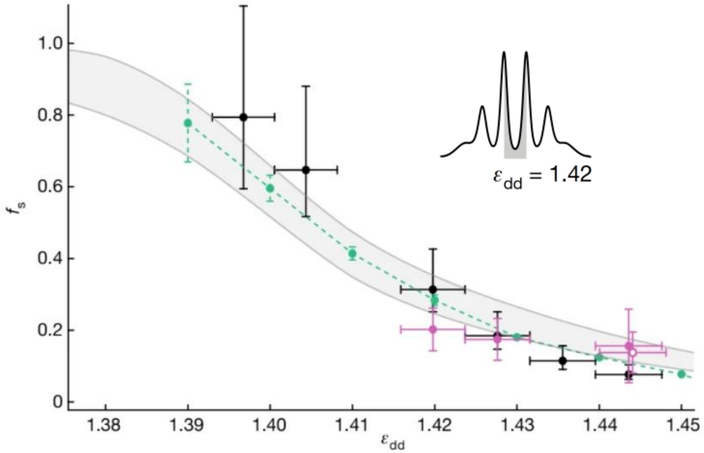
Superfluid fraction

$$f_s = \frac{c_1^2 c_2^2}{c_\kappa^2(c_1^2 + c_2^2 - c_\kappa^2)}$$

Superfluid fraction

- Leggett upper bound: $$f_s^+ = \left[\frac{1}{L}\int_0^L \frac{dx}{n(x)/n_0}\right]^{-1}$$ $$f_s = \frac{c_1^2 c_2^2}{c_\kappa^2(c_1^2 + c_2^2 - c_\kappa^2)}$$
- Consistency of the extended Gross-Pitaevskii theory with the hydrodynamic model of supersolids
- f_s very weakly influenced by finite-size effects
- Recent experimental measurement based on Josephson effect (Florence/Pisa)





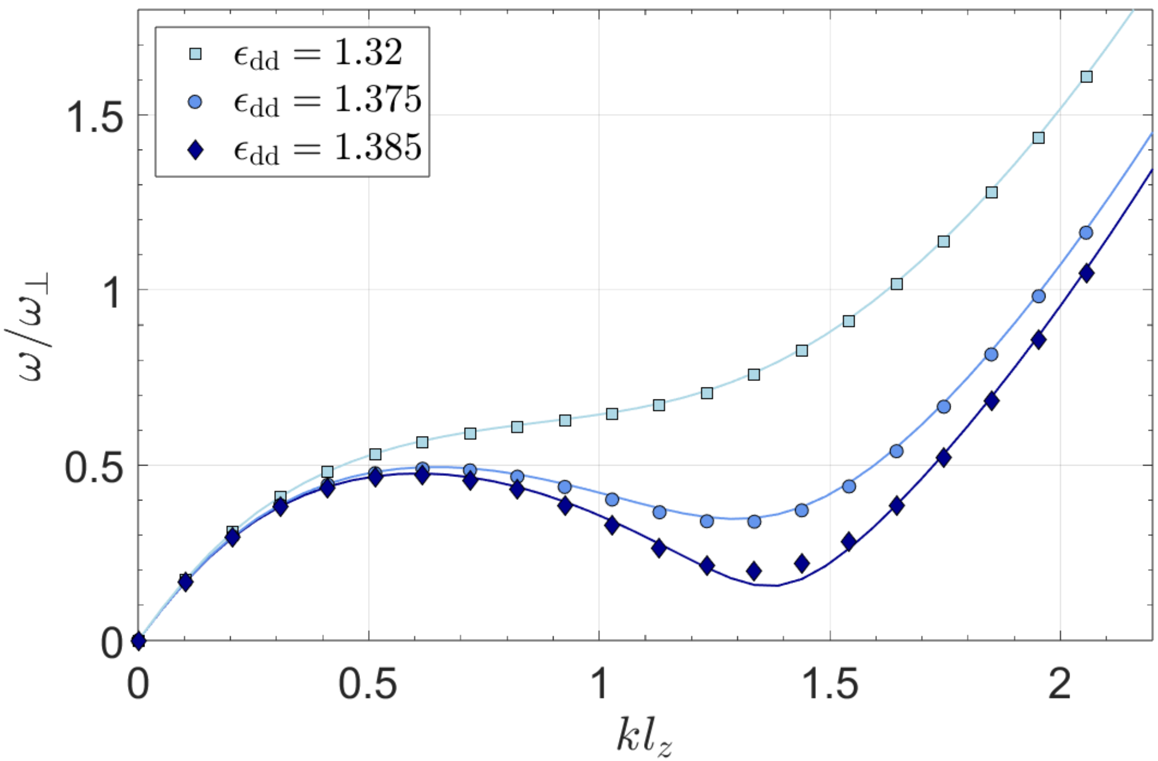
Nature **629**, 773 (2024)

Excitation spectrum

- Possibility to excite modes with larger wavevectors q

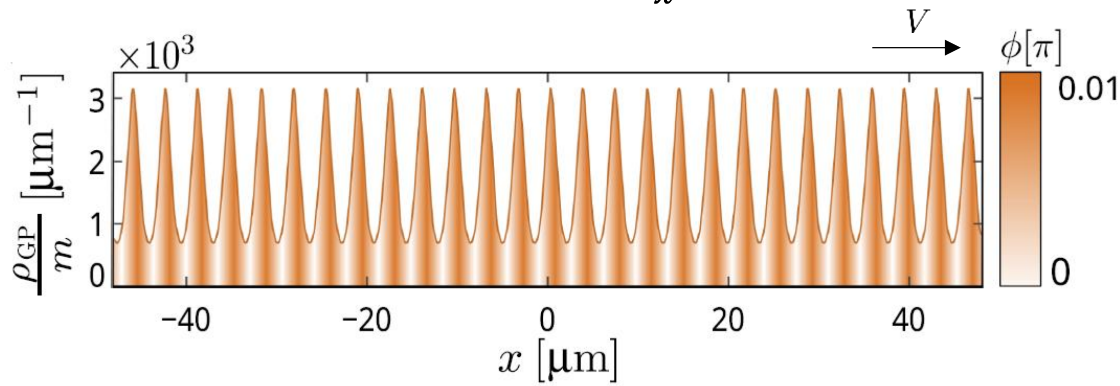
$$V_{\text{pert}} = -V_0 \cos(nqx), \quad q = \frac{2\pi}{L}, \quad n = 1, 2, 3, \ldots$$

- Excellent agreement with predictions of the Bogoliubov–de Gennes theory

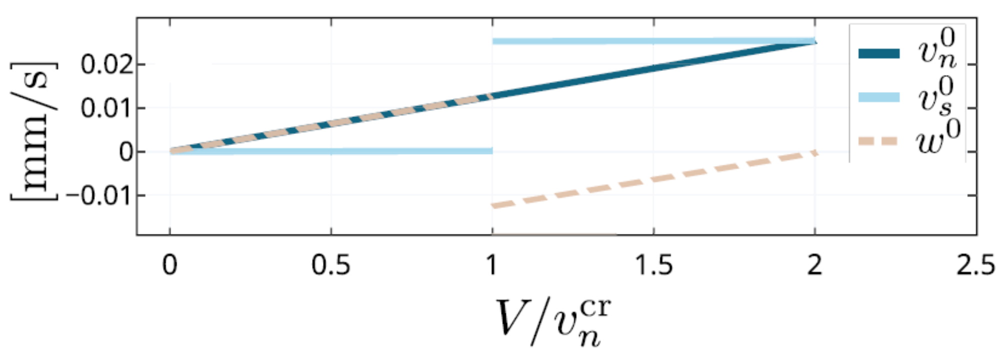


Persistent current in a supersolid

- Supersolid is set in motion: $H \to H - VP_x$



- Normal component moves with the boost velocity $v_n = V$
- Superfluid motion is quantized
  - Irrotational velocity field, $v = \frac{\hbar}{m}\nabla\varphi$
  - Single-valuedness of the wavefunction

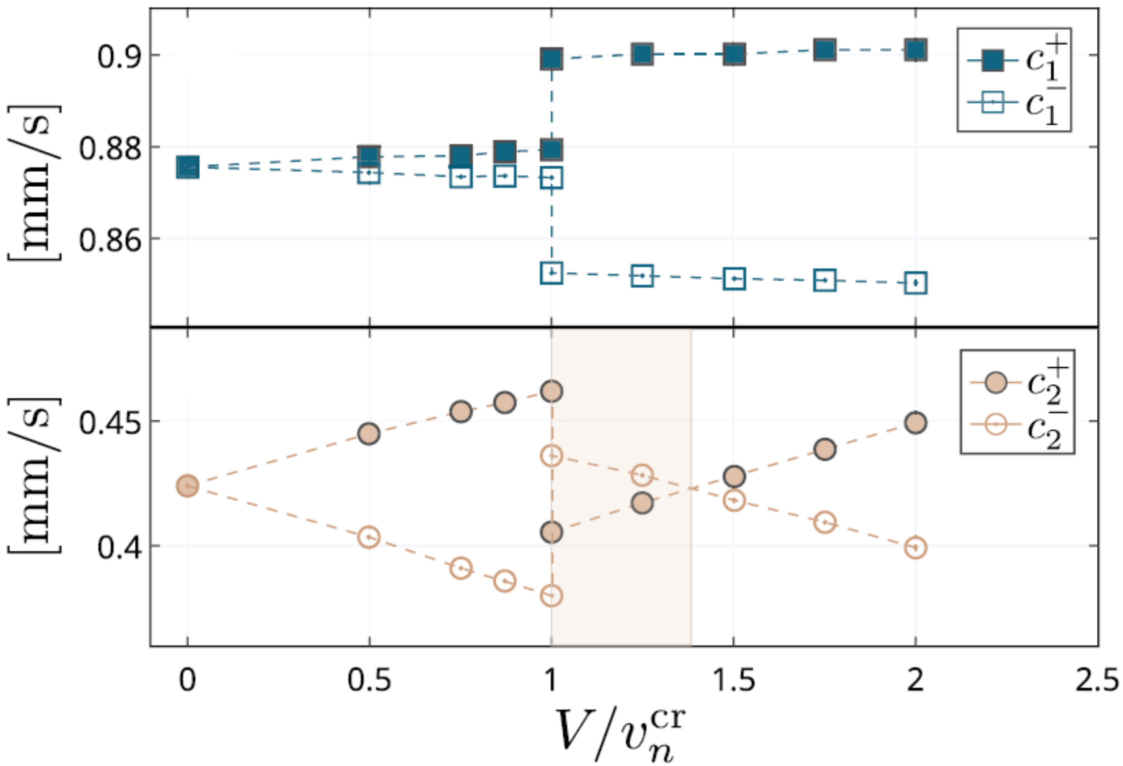


Sound velocities splitting

- Kinematic Doppler effect: $c = c_0 \pm v_f$
- $v_n \neq v_s \rightarrow$ Anomalous (nontrivial) Doppler effect

$$c_i^{\pm} = c_i^0 \pm (v_s + \delta_i\, w), \quad w = v_n - v_s, \quad i = \{1, 2\}$$

- Sound velocities obtained through excitation with periodic perturbation

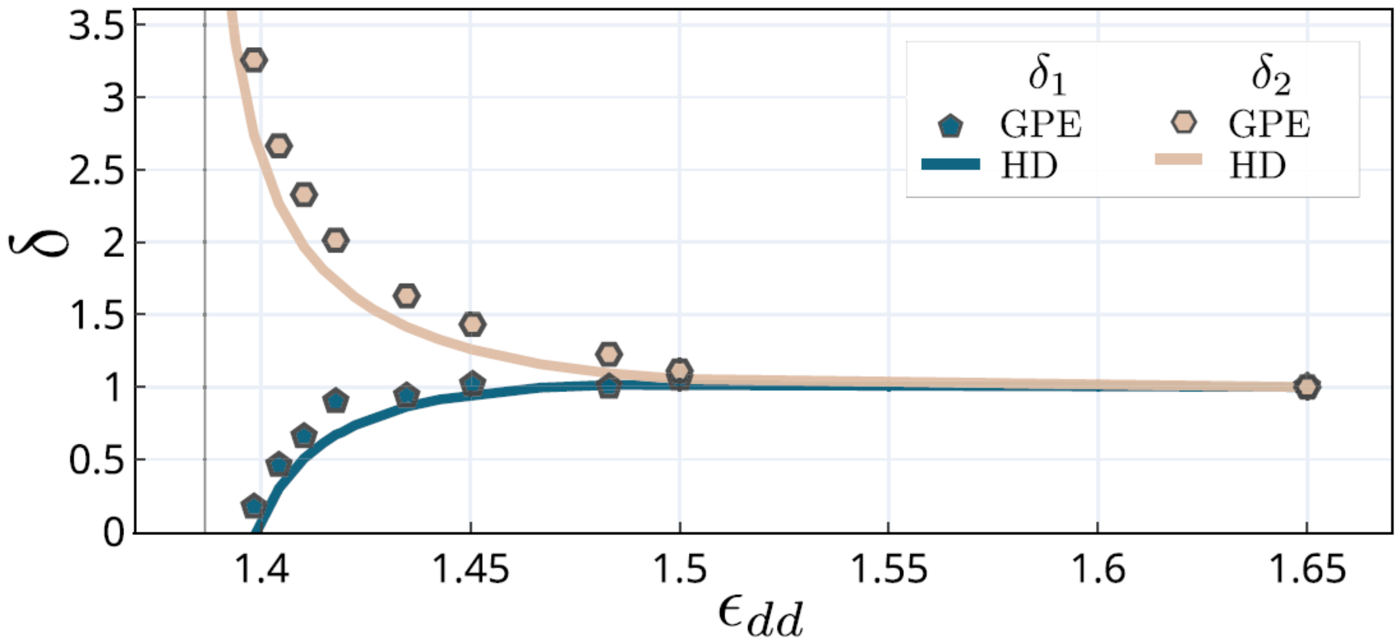


Doppler shift

- Doppler shift (HD)

$$\delta_{1,2} = \frac{\frac{\rho_n^u}{2\rho_n}\left[(c_{1,2}^0)^2 - c_\kappa^2 + \gamma\right] + 2(c_{1,2}^0)^2 - (1+f_s)c_\kappa^2 - \rho_n'\beta + \gamma\left[1 + \frac{\rho_n'}{\rho_n}(\rho - 2\rho_n)\right]}{2(c_{1,2}^0)^2 - c_\kappa^2 - \beta + 2\gamma}$$

- Parameters f_s, κ, β, γ, ρ_n', ρ_s', ρ_n^u calculated numerically from stationary (zero-current) ground state



Conclusions

- Realistic protocol to determine Goldstone modes by applying a weak periodic perturbation
- Combining extended Gross-Pitaevskii formalism with supersolid hydrodynamic theory
- Determination of superfluid fraction
- Analytic result for the anomalous Doppler shift
- Good agreement with time-dependent eGPE calculations

[1] PRL **132**, 146001 (2024)

[2] PRL **134**, 226001 (2025)

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