Optics Letters

Light transport and localization in disordered aperiodic Mathieu lattices

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Complex optical systems such as deterministic aperiodic Mathieu lattices are known to hinder light diffraction in a manner comparable to randomized optical systems. We systematically incorporate randomness in our complex optical system, measuring its relative contribution of randomness, to understand the relationship between randomness and complexity. We introduce an experimental method for the realization of disordered aperiodic Mathieu lattices with numerically controlled disorder degree. Added disorder always enhances light transport. For lower disorder degrees, we observe diffusive-like transport, and in the range of highest light transport, we detect Anderson localization. With further increase of disorder degree, light transport is slowly decreasing and localization length decreases indicating more pronounced Anderson localization. Numerical investigation at longer propagation distances indicates that the threshold of Anderson localization detection is shifted to lower disorder degrees.

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Localization of light has drawn considerable attention in many areas of light–matter interaction owing to the evident potential for the realization in disordered media [1–4]. In contrast, Anderson localization (AL) is a well-known effect in condensed-matter physics, which predicts that electrons may become immobile in a disordered crystal. This concept of waves in disordered media has been subsequently transferred to many other areas, such as matter waves, ultracold atoms, and light or sound waves [2]. Realizing that AL is a wave phenomenon relying on interference, these concepts were extended to optics and photonics. The AL of light has been successfully demonstrated in various customized configurations, when the disorder degree (DD) is increased [5–10]. In optically induced disordered photonic quasicrystals with weak disorder, it is observed that weak disorder enhances light transport. When increasing disorder finite-time, diffusive-like transport appears, while a further increase of disorder leads first to coherent backscattering [11] and for the strong disorder to AL. Thereby, the spatial extent of the probe beam decreases and its central part of the log-plot intensity profile displays an exponential decay [9,12,13].

In nature, perfect periodicity, in contrast to disorder or aperiodicity, is not very often encountered. Deviation from periodicity results in higher complexity. In optics, the properties of various photonic quasicrystals and aperiodic systems have been studied [13–18]. Considering localization characteristics, such structures lie between periodic and random structures. Numerous aperiodic and quasiperiodic photonic structures have been realized artificially [19–21]. Non-diffracting beams, with propagation invariant transverse intensity distributions, are applicable in modern photonic research e.g. numerous two-dimensional aperiodic photonic lattices have been optically induced in photosensitive media using them [21–23]. Aperiodic lattices contain non-uniform distances between the lattice sites with non-homogeneous intensity depth distributions, and hence light propagation crucially depends on the nature of the local environment of the probe beam positions. In contrast that occurring in periodic systems, light diffraction is hampered owing to the aperiodicity [12,21,22,24]. Still, light localization in aperiodic lattices is an unexplored area of research, especially in randomized aperiodic lattices. In our previous studies, we introduced a method for the creation of various two-dimensional aperiodic photonic structures by the interference of Mathieu beams, experimentally realized in a single optical induction process in parallel [23]. We showed that such obtained aperiodic Mathieu photonic lattice (AML) hinders linear light expansion in comparison to periodic lattice and supports nonlinear light localization[24].

In this Letter, we introduce a numerical method for controllable randomization of AMLs to investigate if they support AL. We construct an experimental system for the realization of disordered lattices by a single optical induction process in parallel using a spatial light modulator (SLM) and numerically precalculated disordered patterns with adjustable DDs. This numerical method and experimental configuration, in comparison to the previous one [5,12], enable us direct control of the lattice DD and parallel optical induction of the corresponding light intensity in the whole volume of the photorefractive crystal.

Here, we investigate the light propagation in disordered AMLs numerically and experimentally. We study the conditions for light localization in such lattices as well as the effects of disorder during the propagation. For all DDs, we experimentally obtain and numerically confirm disorder-enhanced transport in

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Visualizing the Energy Flow of Tailored Light

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Exploiting the energy flow of light fields is an essential key to tailor complex optical multistate spin and orbital angular momentum (OAM) dynamics. With this work, the energy flow is identified and quantified by a novel approach that is based on the symmetry breaking induced by nonlinear light–matter interaction of OAM carrying beams at the example of Mathieu beams, showing transverse invariant intensity distributions. These complex scalar nondiffracting beams exhibit outstanding transverse energy flows on elliptic paths. Although their energy is continuously redistributed during linear propagation in homogeneous media, the beams stay nondiffracting. This approach to visualize the energy flow of light is based on the nonlinear self-action in a nonlinear crystal. By this, the sensitive equilibrium is perturbed and accumulation of rotating high-intensity spots is enabled. Intensity distributions on elliptic, chiral paths are demonstrated as a manifestation of the energy flow. Furthermore, the formation of corresponding refractive index modulations that may be implemented as chiral waveguides, is controlled via the beam power and structure size.

1. Introduction

The energy flow of light is determined by both, its spin angular momentum and its orbital angular momentum (OAM), and is generally described by the Poynting vector.[1] Controlling the spatial polarization and phase structure of light, the combination of binary spin states and multistate orbital angular momentum dynamics is an essential key to further establish modern high-dimensional singular optics. These abilities enabled breakthrough research in the areas of spatial polarization modulation,[2] classical entanglement,[3] high-density signal transmission,[4] or optical micromanipulation.[5,6]

In order to investigate two-dimensional energy flows in the transverse plane, in particular nondiffracting beams with transverse invariant intensity distributions and continuously modulated phase distributions are suited. The class of nondiffracting beams has attracted considerable interest and features not only applications in optics, but also in solid state and atom physics.[7–11] A detailed understanding of their energy flows therefore is of high importance in many communities. However, the energy flow of continuously modulated nondiffracting beams withstands a direct observation because it is hidden for the case of linear propagation in homogeneous media. The transverse intensity distribution stays invariant and the energy flow is continuously redistributed.

Four nondiffracting beam families exist as solutions of the paraxial as well as the nonparaxial Helmholtz equation in different coordinate systems:[12–17] Discrete beams in Cartesian, Bessel beams[8] in spherical, Mathieu beams in elliptic, and Weber beams in parabolic coordinates. Among these diverse families, Mathieu beams[9,10,18,19] may be interpreted as a generalized beam class, capable to interpolate between Cartesian and spherical coordinates. In contrast to parabolic Weber beams, their transverse spatial intensity distributions can form closed paths on ellipses, with spatially structured orbital angular momenta[6,20] showing periodic boundaries.

Mathieu beams are highly appealing to access fundamental physical effects in elliptical coordinates.[21] In several studies, they have been beneficially used for particle manipulation,[5] and served as lattice-writing light,[22–26] featuring the nonlinear propagation of (vortex) solitons in these previously linearly induced elliptic lattices. However, the self-action of Mathieu beams in nonlinear media was not investigated until now.

Scalar even and odd Mathieu beams exhibit only real-valued field distributions. Their transverse Poynting vector therefore vanishes. In contrast, the complex superposition of even and odd Mathieu beams leads to generalized elliptic Mathieu beams, showing outstanding continuously modulated spatial phase distributions, i.e., OAM.[5,6,20] Thus, for these beams a transverse energy flow is present. Until today, only a few works have addressed the energy flow in these complex spatially modulated beams with its unique OAM characteristics, e.g., using the OAM structure of Mathieu beams to transfer orbital angular momentum to particles that start to rotate.[5,6,20]

With this work, we present an approach to visualize the energy flow of light at the example of elliptic Mathieu beams. We demonstrate experimentally and numerically that the

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Light propagation in aperiodic photonic lattices created by synthesized Mathieu–Gauss beams

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ABSTRACT

We investigate light propagation in a two-dimensional aperiodic refractive index lattice realized using the interference of multiple Mathieu–Gauss beams. We demonstrate experimentally and numerically that such a lattice effectively hinders linear light expansion and leads to light localization, compared to periodic photonic lattices in a photorefractive crystal. Most promisingly, we show that such an aperiodic lattice supports the nonlinear confinement of light in the form of soliton-like propagation that is robust with respect to changes in a wide range of intensities.

Diffraction is a fundamental feature of wave dynamics in any branch of physics that involves waves: optics, acoustics, quantum mechanics, etc. However, in many applications, propagation-invariant transverse intensity distributions, referred to as nondiffracting beams, are needed. Nondiffracting beams are exact solutions of the Helmholtz equation, which exist in different coordinate systems:[1] superposition of plane waves in Cartesian, Bessel beams in circular cylindrical,[2] Mathieu beams in elliptic cylindrical,[3] and parabolic beams in parabolic cylindrical coordinates.[4]

The potential of nondiffracting structures is well recognized in modern photonic research.[5–9] Among them, the propagation of light through tailored refractive index modulations optically fabricated in photosensitive media by propagation-invariant intensity profiles became the subject of extensive theoretical and experimental investigations since the resulting refractive index structure represents a pure 2D material.[10–14] This field of linear and nonlinear optics in photonic lattices typically uses simple nondiffracting Cartesian beam configurations, often hexagonal light structures, to modulate the refractive index since this allows mimicking features of 2D graphene,[15] its famous bandgap structure,[16] or its nonlinear light matter interaction, leading to spatial soliton formation.[17] In a few recent studies, solitons, elliptically shaped vortex solitons, or even vortex necklaces are observed in optically induced photonic lattices by nondiffracting Mathieu beams.[12,18–20] Moreover, the superposition of this kind of elliptic nondiffracting beam allows the formation of different aperiodic photonic structures.[21]

Although the physics of periodic photonic systems is of fundamental interest, deviation from periodicity is important as it leads to higher complexity. One such deviation in optics results in the realization of photonic quasicrystals,[8] structures with a reduced degree of order between periodic and disordered ones.

The localization of waves is an intriguing research subject observed in a variety of classical and quantum systems,[22,23] including light waves,[24–27] Bose–Einstein condensates,[28] and sound waves.[29] Although the transverse expansion properties in periodic photonic lattices,[30–33] as well as in disordered ones,[34–36] have been investigated extensively, light localization and transverse expansion in photonic quasicrystals[37,38] is still an open question.

In this paper, we investigate the effects of light propagation in aperiodic photonic structures created by synthesized Mathieu–Gauss (MG) beams in a photorefractive crystal,[21] experimentally and numerically. We investigate how various input beam positions influence the diffraction and compare them with appropriate periodic waveguide arrays. We find that our approach effectively suppresses the beam expansion depending on the refractive index modulation Δn. Most importantly, in the nonlinear regime, we find localized states that are robust with respect to changes in the probing light intensities and propagation distance. Such stable solitary states are, thus, much more appealing for applications than typical spatial solitons, especially gap solitons, which react sensitively on changes in the strength of the nonlinearity.[39]

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Morphing discrete diffraction in nonlinear Mathieu lattices

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Discrete optical gratings are essential components to customize structured light waves, determined by the band structure of the periodic potential. Beyond fabricating static devices, light-driven diffraction management requires nonlinear materials. Up to now, nonlinear self-action has been limited mainly to discrete spatial solitons. Discrete solitons, however, are restricted to the eigenstates of the photonic lattice. Here, we control light formation by nonlinear discrete diffraction, allowing for versatile output diffraction states. We observe morphing of diffraction structures for discrete Mathieu beams propagating nonlinearly in photosensitive media. The self-action of a zero-order Mathieu beam in a nonlinear medium shows characteristics similar to discrete diffraction in one-dimensional waveguide arrays. Mathieu beams of higher orders show discrete diffraction along curved paths, showing the fingerprint of respective two-dimensional photonic lattices.

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Manipulating waves by customizing their interaction with functional materials enables a variety of photonic applications, e.g., tailored diffraction at gratings to discretize the waves' spectral components [1,2]. Waves in periodically structured media show dynamics that cannot be realized in homogeneous media, determined by the media's band structure. Propagation of light in dielectric media with a periodically varying refractive index can mimic the spatio–temporal characteristics that are typically encountered in discrete systems, and the underlying field evolution effectively becomes "discretized" [1]. Most importantly, the vision to control light with light is realizable only by exploiting nonlinear materials as mediators [3]. Thus, shaping the periodically varying refractive index structure allows for diffraction management to control in turn the light distribution [4].

Different types of periodic photonic structures, including arrays of evanescently coupled optical waveguides [5], optically induced lattices in photorefractive materials [6], and photonic crystals [7], have been employed to engineer and control fundamental properties of wave propagation. Arrays or lattices of evanescently coupled waveguides are prime examples of structures in which *discrete diffraction* [2,5,8] can be observed. These arrays consist of equally spaced identical waveguide elements or sites, possessing all essential characteristics of a photonic crystal structure (Brillouin zones, band structure, etc.). In such a physical setting, light couples between waveguides through tunneling, showing its diffraction characteristics. When low intensity light is injected into one or a few neighboring waveguides, it couples to more and more waveguides, broadening its spatial distribution. Fundamentally new physics occur in contrast to diffraction in homogeneous media. High-intensity light producing nonlinear responses in the refractive index is capable of forming *discrete spatial solitons* [9]. A renewed interest in nonlinear light–matter interaction goes beyond soliton formation. It is devoted to physical systems with dimensionality morphing, e.g., the continuous transformation of the lattice structure from 1D to 2D [10–12].

Nondiffracting beams, having propagation-invariant intensity distributions, allow creating 1D and 2D photonic lattices in photosensitive media. Particularly in the areas of optics and atom physics, these beams enable novel applications [13–16]. Among the variety of different nondiffracting beams, Mathieu beams [15,17] solve the Helmholtz equation in elliptic cylindrical coordinates [18]. They are used for a new type of optical lattice-writing light [19–23] allowing solitons or even elliptically shaped vortex solitons, and are beneficially used for particle manipulation [24]. However, their elliptical characteristics allow going far beyond soliton investigations and extending applications of nonlinear self-action.

In this Letter, we exploit Mathieu beams as lattice-writing light to fabricate discrete waveguide structures and investigate their nonlinear self-action in these structures, leading to morphing discrete diffraction. We investigate Mathieu beams of different orders in a photorefractive crystal, experimentally and numerically. We link linear discrete diffraction with nonlinear self-effects and demonstrate gradual transition from one to two dimensions. We use the term *morphing diffraction* to describe the nonlinear behavior similar to discrete diffraction.

Elliptical vortex necklaces in Mathieu lattices

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We demonstrate unusual kinds of discrete vortex beams, elliptical necklaces, realized by Mathieu photonic lattices. Varying the order of the Mathieu lattices and their ellipticity, we can control the shape and size of such necklaces. Besides stable vortex states, we observe oscillatory dipole states or dynamical instabilities and study their orbital angular momentum. Dynamical instabilities occur for higher beam power and higher-order vortices. Also the decay of higher-order phase singularities and their separation is observed in dependence on the ellipticity.

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I. INTRODUCTION

An optical vortex that possesses a phase singularity and a rotational flow around the singular point in a given direction can be found in physical systems of different nature and scale, ranging from water whirlpools and atmospheric tornadoes to quantized vortices in superfluids and quantized lines of magnetic flux in superconductors [1]. The study of optical vortices and associated localized vortex states is important for both fundamental and applied physics, leading to applications in many areas that include optical data storage, distribution and processing, optical interconnects between electronic chips and boards, and free-space communication links [2–4]. They also have potential uses in optical tweezers [5], optical manipulation and trapping [6,7], microscopy [8], and quantum information processing [9,10].

The evolution of nonlinear excitations in systems whose properties are modulated is especially interesting and in optics can be realized when an intense laser beam propagates in the material with a suitable transverse refractive index modulation that can be fabricated in nonlinear materials including semiconductors, liquid crystals, fused silica, polymers, and photorefractive media [11–18]. The combination of diffractive and nonlinear effects with transverse refractive index modulation in photonic lattices opens the possibility to produce spatially localized states of light [19,20]. To optically induce two-dimensional photonic lattices it is appropriate to use nondiffracting light beams that are exact solutions of the Helmholtz equation in different coordinate systems [21,22]: plane waves in Cartesian, Bessel beams in circular cylindrical [23], Mathieu beams in elliptic cylindrical [24], and parabolic beams in parabolic cylindrical coordinates [25].

In this paper we report on the existence of elliptical necklace beams in photonic lattices optically induced by Mathieu nondiffracting beams, using vortices as a probe beam. These necklace beams show discrete intensity spots on elliptical curves, associated with discrete phase vortices. We investigate the conditions for their existence as well as their properties, both experimentally and theoretically. Changing the lattice ellipticity and choosing Mathieu lattices of appropriate order, we control the shape and the size of an elliptical necklace, as well as the number of the "pearls" in the necklace. We investigate the breakup of higher-order vortices (topological charge $C_T = 2,3,4$) into $C_T = 1$ vortices and their rate of separation during propagation. Phase singularity distances increase with C_T, higher lattice ellipticity, and propagation distance. Further, we study the stability of such elliptic necklaces. Supported by the strong nonlinearity, we show the formation of oscillating dipole states in the intensity distribution for very long propagation distances and discuss our results by investigating additionally the transfer of orbital angular momentum (AM) to the lattice. Finally, a high intensity of the probe beam leads to nonlinear dynamical instabilities observable in the intensity distribution of the necklaces.

II. EXPERIMENTAL METHOD AND MODELING OF VORTEX BEAM PROPAGATION IN MATHIEU LATTICES

Figure 1 shows the experimental setup to realize elliptical necklaces. A frequency-doubled, expanded, and collimated Nd:YVO_4 laser with wavelength $\lambda = 532$ nm is split into two separate beams: an ordinary polarized writing and an extraordinary polarized probe beam. Both are spatially tailored in intensity and phase by a phase-only spatial light modulator Holoeye Pluto VIS. For this purpose, special Fourier filters (FF1 and FF2) are required [26]. The structure beam optically induces refractive index modulations in the 15-mm-long photorefractive Strontium Barium Niobate crystal doped by Cerium (SBN:Ce), thereby addressing the weaker electro-optic coefficient $r_{13} = 47$ pm/V. The birefringent crystal has refractive indices $n_o = 2.325$ and $n_e = 2.358$ and is externally biased with an electric field $E_{\text{ext}} = 1600$ V/cm aligned along the optical $c = x$ axis, perpendicular to the direction of propagation (z axis). Probing the artificial photonic structure is done with the extraordinary polarized probe beam that addresses the stronger electro-optic coefficient $r_{33} = 237$ pm/V. An imaging system consisting of a microscope objective and camera detects transverse intensity distributions at the back of the crystal.

We model our experiment by solving the nonlinear Schrödinger equation for an initial scalar electric field $A(\mathbf{r})$

Creating aperiodic photonic structures by synthesized Mathieu-Gauss beams

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We demonstrate a kind of aperiodic photonic structure realized using the interference of multiple Mathieu-Gauss beams. Depending on the beam configurations, their mutual distances, angles of rotation, or phase relations we are able to observe different classes of such aperiodic optically induced refractive index structures. Our experimental approach is based on the optical induction in a single parallel writing process.

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I. INTRODUCTION

Since nondiffracting beams have been introduced in the late 1980s [1,2] as light structures, only recently these structures have drawn considerable attention in various topics such as trapping of colloidal and *in vivo* particles in biophysics [3], atom optics [4], applications of optical lattices in quantum computing [5], as well as quantum optics [6], optical tweezing [7,8], and nonlinear optics [9–11]. Such nondiffracting structures are coming from the well-known classes of simple nondiffracting light beams that are exact solutions of the Helmholtz equation in different coordinate systems [12]: plane waves in Cartesian, Bessel beams in circular cylindrical [2], Mathieu beams in elliptic cylindrical [13], and parabolic beams in parabolic cylindrical coordinates [14].

A simple and robust implementation of optical micromanipulation technologies—optical tweezers—based on nondiffracting beams, has become a standard tool in biological, medical, and physics research laboratories [15]. Another trend in optical manipulation is the use of synthesized optical beams rather than single beams only; such beams enable a much greater freedom in object manipulation than conventional Gaussian beams [16].

The potential of nondiffracting structures is of significant importance for advances in discrete and nonlinear modern photonics [17–21]. Although the physics of periodic photonic systems are of fundamental importance, deviations from periodicity are of importance as they may result in higher complexity. One such deviation in optics results in the realization of photonic quasicrystals [20,22], the structures that lie between periodic and disordered one. They show sharp diffraction patterns that confirm the existence of wave interference resulting from their long-range order. Recently, a new serial approach for the generation of aperiodic deterministic Fibonacci and Vogel spirals as refractive index structures was presented [23,24]. In particular, the Fourier spectra of tailored aperiodic lattices can be customized to range from discrete to continuous [25], thus featuring unique light propagation as well as localization properties in aperiodic photonic lattices. Of particular interest are also flat-band lattices with a dispersionless energy band composed of entirely degenerate states, so that any excitation of these states yields nondiffracting waves. Such flat band systems have been studied in a number of lattice models including quasi-one-, two-, or three-dimensional settings, diamond ladder, Lieb, or kagome lattices [26–28].

In this paper, we demonstrate a powerful approach for the creation of two-dimensional (2D) aperiodic photonic lattices in a single writing process in parallel. It is based on synthesizing two or more nondiffracting Mathieu-Gauss (MG) beams [29]. By coherently superimposing MG beams with different orders, positions, and relative phases we realize transverse invariant propagating intensity distributions capable of optically inducing corresponding refractive index lattices in photosensitive media. Our approach features the fabrication of versatile aperiodic lattices with controllable properties as well as quasi-one-dimensional structures.

II. CHARACTERIZATION OF SYNTHESIZED MATHIEU-GAUSS BEAMS

For the experimental realization of synthesized MG beams we use the experimental setup shown in Fig. 1. We use a frequency-doubled Nd:YVO_4 laser, expand the laser beam, and illuminate as a plane wave a phase-only spatial light modulator "Holoeye Pluto VIS." The reflected light field is modulated in both amplitude and phase. This is possible by addressing a precalculated hologram to the SLM containing the information of the complex light field encoded with an additional blazed grating. By applying an appropriate Fourier filter, the tailored complex light field is realized [30,31]. Additionally, the telescope L1-L2 scales down the SLM size by a factor of 10. This extraordinary polarized "structure beam" is used to optically inscribe refractive index modulations in the 15 mm long photorefractive SBN:Ce crystal which is externally biased with an electric dc field of $E_{\text{ext}} = 2000V\,\text{cm}^{-1}$ aligned along the optical $c = x$ axis, perpendicular to the direction of propagation (z axis).

We simulate the nonlinear light propagation in a photonic structure by numerically solving the nonlinear Schrödinger equation:

$$i\partial_z A(\mathbf{r}) + \tfrac{1}{2}\Delta_\perp A(\mathbf{r}) + \tfrac{1}{2}\Gamma E(|A(\mathbf{r})|^2)A(\mathbf{r}) = 0, \qquad (1)$$

where $\Gamma = k_0^2 w_0^2 n_{o,e}^4 r_{13,33}$, $k_0 = 2\pi/\lambda$ is the wave number and defined by the wavelength $\lambda =\ 532\,\text{nm}$, $n_o = 2.325$ is the ordinary, $n_e = 2.358$ is the extraordinary bulk refractive index, $r_{13} =\ 47\,\text{pm/V}$, $r_{33} =\ 237\,\text{pm/V}$ are the corresponding

