УНИВЕРЗИТЕТ У БЕОГРАДУ
ИНСТИТУТ ЗА ФИЗИКУ | БЕОГРАД
ИНСТИТУТ ОД НАЦИОНАЛНОГ ЗНАЧАЈА ЗА РЕПУБЛИКУ СРБИЈУ
www.ipb.ac.rs
Број 0801-411/1
Датум 12 MAR 2024

Научном већу Института за физику

Предмет: Молба за покретање поступка за реизбор у звање виши научни сарадник

Молим Научно веће Института за физику у Београду да у складу са Правилником о поступку и начину вредновања и квантитативном исказивању научно-истраживачких резултата истраживача покрене поступак за мој реизбор у звање виши научни сарадник.

У прилогу достављам:

1. Мишљење руководица лабораторије са предлогом чланова комисије за избор у звање
2. Стручну биографију
3. Преглед научне активности
4. Елементе за квалитативну анализу научног доприноса
5. Елементе за квантитативну анализу научног доприноса
6. Списак објављених радова и њихове копије
7. Потрвде о цитираности радова као и списак радова са цитатима
8. Копију решења о избору у претходно звање
9. Додатке

Београд, 12.03.2024. године

Бојан Николић

др Бојан Николић
виши научни сарадник Института за физику

Научном већу Института за физику

Мишљење руководиоца лабораторије о реизбору др Бојана Николића у звање виши научни сарадник

Др Бојан Николић је члан Групе за гравитацију, честице и поља. Од 01. новембра 2003. године је запослен на Институту за физику и бави се проучавањем бозонске и суперструна у контексту некомутативности, негеометрије и Т-дуалних трансформација. Дипломирао је 2002. године на Физичком факултету Универзитета у Београду са просечном оценом 9,81. Магистрирао је децембра 2006. године, а докторирао на Физичком факултету Универзитета у Београду 2008. године. У звање виши научни сарадник изабран је 29.01.2014. године решењем Министарства просвете, науке и технолошког развоја, а 21.10.2019. године реизабран у исто звање.

Препоручујем Научном већу Института за физику да одобри **покретање поступка за реизбор др Бојана Николића у звање виши научни сарадник** из следећих разлога:

1. Испуњеност **квантитативних и квалитативних критеријума**: збир поена на основу објављених радова као и квалитет часописа у којима су радови публиковани. Радови су објављени у врхунским међународним часописима из области физике високих енергија.

2. Покретање **нове истраживачке теме** у оквиру Групе за гравитацију, честице и поља: након шестомесечног постдокторског боравка у Минхену 2012. године, у сарадњи са ментором др Браниславом Саздовићем и колегеницом др Љубицом Давидовић, покренуо је истраживачки рад у области Т-дуализације теорија са координатно зависним позадинским пољима. Један део те теме је обрађен у оквиру рада на докторској тези студента Данијела Обрића.

3. **Самосталност и квалитет** научног рада: др Николић показује висок ниво самостралности у раду који се огледа у броју објављаних

радова као и у квалитету часописа где су радови објављени. Др Бојан Николић је скоро све своје радове објавио у часописима са високим импакт факторима (категорија М21), а цитираност радова је, с обзиром на релативно малу научну заједницу која се бави сличном проблематиком, сасвим прихватљива, испуњавајући границе које је задао Матични одбор за физику.

4. Међународна сарадња: др Николић је уз помоћ стипендије Министарства за просвету, науку и технолошки развој 2012. године био на постдокторском усавршавању на Лудвиг Максимилијан Универзитету у Минхену. Боравио је и радио у групи светски познатог научника у области теорије струна др Дитера Листа. Као један од резултат тог боравка је и горе споменута нова тема - T-дуализација теорије (супер)струне са координатно зависним пољима у контексту некомутативности, нелокалности и неасоцијативности.

5. **Педагошки рад и популаризација физике**: др Бојан Николић је почев од 2015. године био ментор четири мастер рада. Са студентом Данијелом Обрићем је након одбране мастер рада 2017. године наставио сарадњу 2018. године на изради докторске дисертације. Докторска дисертација је успешно одбрањена сепетембра 2023. године на Физичком факултету Универзитета у Београду.

   У оквиру пројекта којим руководи др Марко Војиновић (QGHG-2021, Фонд за науку Републике Србије, програм ИДЕЈЕ) крајем 2023. године одржана је серија популарних предавања у просторијама Задужбине Илије М. Коларца. Др Николић је одржао једно од пет предавања.

   Др Бојан Николић је **професор** на предмету Теорија струна на **докторским студијама** Физичког факултета Универзитета у Београду. Такође др Николић је као спољни сарадник радио школске 2013/2014 у Математичкој гимназији, а од школске 2015/2016 године до данас је спољни сарадник у Земунској гимназији у оквиру посебног одељења за физику. У више наврата до 2015. године је био у комисијама за састављање и прегледање задатака на такмичењима из физике за ученике средњих школа. Од 2004. до 2006. године био је члан уредништва часописа за популаризацију физике Млади физичар.

На основу свега горе наведеног као и на основу личног увида у квалитете и посвећеност раду, препоручујем Научном већу Института за физику да др Бојану Николићу **одобри покретање поступка за реизбор у звање виши научни сарадник**. Предлажем следеће чланове комисије за избор др Бојана Николића у звање виши научни сарадник:

1. др Бранислав Цветковић, научни саветник, Институт за физику
2. др Бранислав Саздовић, научни саветник у пензији, Институт за физику
3. проф. др Воја Радовановић, редовни професор, Физички факултет.

Београд, 11.03.2024. године

Руководилац лабораторије

Бранислав Цветковић

др Бранислав Цветковић
научни саветник

Биографија са прегледом научне активности

1 Стручно-биографски подаци

Рођен сам 10. 04. 1979. године у Зајечару. У Књажевцу сам завршио основну школу и гимназију природно-математичког смера као ђак генерације.

Године 1998. уписао сам основне студије на Физичком факултету Универзитета у Београду, смер Теоријска и експериментална физика. Дипломирао сам 2002. године као први у генерацији са просечном оценом 9,81. Постдипломске студије на смеру ,,Теоријска физика елементарних честица и гравитације” уписао сам 2002., а магистарски рад са темом *Ефекат дилатонског поља на некомутативност просторно-временских координата* сам одбранио 2006. године на Физичком факултету Универзитета у Београду. Докторску дисертацију под насловом *Некомутативност и димензионалност Dp-бране* одбранио сам 2008. године такође на Физичком факултету Универзитета у Београду. Ментор магистарске тезе и докторске дисертације био је професор др Бранислав Саздовић, научни саветник Института за физику.

Од 01. 11. 2003. године, запослен сам у Центру за теоријску физику Института за физику као истраживач приправник, у оквиру пројекта ,,Градијентне теорије гравитације: симетрије и динамика” Министарства просвете, науке и технолошког развоја Републике Србије. Од 2006. до 2010. године био сам на пројекту “Алтернативне теорије гравитације“, док сам од почетка 2011. године ангажован на пројекту Министарства просвете и науке ”Физичке импликације модификованог простор-времена”. Почев од јануара 2022. године ангажован сам на пројекту из програма ИДЕЈЕ ”Quantum Gravity from Higher Gauge Theory” финансиран од стране Фонда за науку Републике Србије.

Године 2007. изабран сам у звање истраживач сарадник, у октобру 2009. у звање научни сарадник, а у садашње звање вишег научног сарадника изабран сам јануара 2014. године. У октобру 2019. године сам реизабран у звање виши научни сарадник.

Од 2004.-2006. године био сам члан редакције часописа ,,Млади физичар”. У периоду од децембра 2004. године до августа 2005. године био сам на одслужењу војног рока. Активно сам учествовао у обележавању Светске године физике 2005. године, посебно у организацији такмичења ,,Откривамо таленте за физику”. Био сам три пута члан локалног организационог комитета међународне школе и конференције из модерне математичке физике. Од априла до јула 2008. године био сам на стручном усавршавању на Институту за нуклеарна истраживања и нуклеарну енергју у Софији (Бугарска) у оквиру ОП6 Марија Кири истраживачке тренинг мреже "Forces-Universe" MRTN-CT-2004-005104. Био сам члан Државне комисије за такмичења ученика средњих школа школске 2011/2012 и 2012/2013. У периоду од 01. јула 2012. до 24. децембра 2012. био сам на стручном усавршавању у Центру за теоријску физику "Арнолд Зомерфелд" у Минхену у групи професора Дитера Листа, једног од водећих физичара у области теорије струна. Године 2015. сам одржао предавање у САНУ у оквиру једнодeневног скупа поводом сто година опште теорије релативности (ОТР). У оквиру XXXIV Републичког семинара наставника физике 2016. године одржао сам предавање по позиву о открићу гравитационих таласа "Гравитациони таласи-од открића до директне детекције". Рад је објављен у трећем броју часописа "Настава физике". Крајем 2023. године пројекат "Quantum Gravity from Higher Gauge Theory" који је финансиран од стране Фонда за науку Републике Србије уз помоћ Задужбине Илије М. Коларца је организовао серију популарних предавања из области савремене теоријске физике под називом "КВАНТНА ГРАВИТАЦИЈА - СВЕТИ ГРАЛ САВРЕМЕНЕ ФИЗИКЕ". Као члан пројекта одржао сам једно од 5 предавања са темом "Зашто квантна теорија поља?".

Школске 2013/2014 радио сам као наставник физике у одељењу трећег разреда Математичке гимназије, док од школске 2015/2016 радим као наставник Рачунског практикума 1 (2 школска часа недељно) и Рачунског практикума 2 (2 школска часа недељно) у одељењу за децу са посебним способностима за физику у Земунској гимназији. Био сам ментор више матурских радова као и четири мастер рада на Физичком факултету у Београду.

Од октобра 2018. године сам ментор докторске дисертације Данијела Обрића. Дисертација је успешно одбрањена на Физичком факултету Универзитета у Београду 18.09.2023. године. У оквиру докторских студија на Физичком факултету Универзитета у Београду, професор сам на предмету Теорија струна (ужа научна област Квантна поља, честице и гравитација).

Од 2009. године сам ожењен и отац сина Стојана (2010) и ћерки, Анастасије (2012), Савке (2015) и Данице (2019).

2 Преглед научне активности

Мој научни рад је у области **физике високих енергија**. Ангажован сам на основним истраживањима у Групи за гравитацију, честице и поља Института за физику. Основна тема мог истраживачког рада односи се на анализу бозонске струне и суперструне, некомутативност као последицу наметнутих граничних услова и T-дуализације затворене (супер)струне у присуству координатно зависних позадинских поља и некомутативности и неасоцијативности проистеклих из тога. У контексту трагања за обједињеном теоријом свих интеракција анализирају се и теорије у удвостручeним просторима где дуалне трансформације постају трансформације симетрије.

2.1 Теорија бозонске струне са дилатоном и некомутативност

Један од предмета мог интересовања је бозонска струна у присуству гравитационог поља $G_{\mu\nu}(x)$ и антисиметричног тензорског поља $B_{\mu\nu}(x)$. Ову тему сам обрађивао током рада на магистарској и делимично докторској тези.

У циљу да конформна инваријантност буде задржана и на квантном нивоу, потребно је додати још једно позадинско поље - дилатонско поље $\Phi(x)$. Наведена три позадинска поља нису произвољна већ морају задовољавати просторно-временске једначине које следе из услова конформне инваријантности, што формалним језиком значи да су β-функције које одговарају гравитационом $G_{\mu\nu}$ и антисиметричном Калб-Рамоновом пољу $B_{\mu\nu}$ једнаке нули, док трећа, која одговара дилатону, може бити нула или константа. Случај у коме је β-функција која одговара дилатонском пољу једнака константи захтева додавање Лиувиловог члана у дејство.

У теорији отворене струне поред једначина кретања од посебне важности су гранични услови. Многострукост са p просторних димензија, дефинисана скупом Дирихлеових граничних услова на крајевима отворене струне, назива се Dp-брана.

У анализи сам користио канонске методе и третирао граничне услове као канонске везе. За случај отворене струне у присуству константног гравитационог и антисиметричног тензорског поља некомутативне релације су већ биле изведене другим методама, укључујући и канонски. У новом приступу, уместо увођења Диракових заграда решене су везе које потичу од граничних услова. Добијено решење за координате, зависи не само од ефективних координата већ и од ефективних импулса. То објашњава чињеницу да је Поасонова заграда координата различита од нуле.

Мој оригинални допринос је укључивање линеарног дилатонског поља $\Phi(x) = \Phi_0 + a_i x^i$, које је одабрано тако да позадинска поља задовољавају просторно-временске једначине кретања. Поред очекиване зависности некомутативног параметра од дилатонског поља појављу се и два на први поглед неочекивана резултата. Јавља се једна комутативна координата, а у случају додатних релација између позадинских поља смањује се димензија Dp-бране. У проширеном простору који се добија додавањем конформног фактора F скупу координата x^i, могуће је објединити све случајеве и то је обрађено у раду под редним бројем 3., који је објављен у редовној свесци часописа **заједно са радом Едварда Витена**.

1. B. Nikolic and B. Sazdovic, Gauge symmetries decrease the number of Dp-brane dimensions, Phys. Rev. D 74 (2006) 045024.

2. B. Nikolic and B. Sazdovic, Gauge symmetries decrease the number of Dp-brane dimensions. II. Inclusion of the Liouville term, Phys. Rev. D 75 (2007) 085011.

3. B. Nikolic and B. Sazdovic, Noncommutativity in space-time extended by Liouville field, Adv. Theor. Math. Phys. 14 (2010) 1.

2.2 Некомутативност и Т-дуалност суперструна са константним позадинским пољима

Бозонска теорија је добар модел за анализу али у реалности постоје и **фермионски степени слободе**. На тај начин долазимо до модела **суперструне**. Бозонска и фермионска поља су повезана суперсиметријом која је потребна због конзистентности саме теорије (елиминација тахиона) па отуда и назив суперструна. Испоставља се да постоји пет конзистентних теорија суперструне: тип I, тип IIA, тип IIB и две хетеротичке теорије суперструне. Ове теорије међусобно су повезане мрежом дуалности. Теорија типа IIB описује суперструну са $N = 2$ суперсиметријом и фермионским координатама исте киралности, док су спинори у тип IIA теорији супротне киралности. Тип I је суперструна са експлицитном $N = 1$ суперсиметријом.

У мом раду посебно место заузима проучавање T-дуалних трансформација тип II теорије суперструне у формулацији чистог спинора, као и однос некомутативности и T-дуалности за случај константних позадиснких поља. Такав избор позадинских поља је у складу са великим скупом услова конзистентности.

Процедура T-дуализације је таква по конструкцији да су иницијална и T-дуална теорија физички еквивалентне, а та физичка еквивалентност има за последицу одржање броја бозонских и фермионских степени слободе. Стандардан начин T-дуализације је Бушерова процедура која представља локализацију транслационе симетрије уз увођење додатног члана у дејству који нам омогућава еквиваленцију са почетном теоријом. Фиксирањем иницијалних координата D димезионог простора добијамо теорију која зависи од градијентних поља и дуалних координата. На једначинама кретања за градијентна поља добија се дуална теорија.

У литератури је дуго времена проучавана само бозонска Т-дуалност. Међутим, недавно, у оквиру анализе симетрија амплитуда глуонских расејања, откривена је фермионска T-дуалност. Формално, она представља исти тип трансформације као и бозонска T-дуалност. Реализује се локализацијом транслационе симетрије фермионских координата. Применом Бушерове процедуре може се добити фермионски T-дуална теорија.

Метод развијен у анализи бозонске струне, применио сам на теорију суперструна типа IIB. Гранични услови су изабрани тако да је очувана $N = 1$ суперсиметрија од иницијалне $N = 2$ суперсиметрије типа IIB суперструне (гранични услови за бозонске координате су Нојманови). Решавањем граничних услова добија се некомутативност координата иницијалног простора и ефективна теорија, као иницијална теорија на решењу граничних услова. С друге стране извршена је T дуализација теорије типа IIB суперструне са константним позадинским пољима. T-дуална поља која су непарна на трансформацију парности светске површи $\Omega : \sigma \to -\sigma$ представљају некомутативне параметре док су поља која су Ω парна представљају поља ефективне теорије. Такође показано је и да Нојманови гранични услови прелазе у Дирихлеове граничне услове.

Ефективна теорија која се добија у претходно описаном случају је тип I теорија суперструне. С обзиром да је $D5$-брана стабилна и у тип IIB и тип I теорији суперструне, анализирана је некомутативност и T-дуалност тип IIB суперструне са $D5$-браном. Бозонским координатама бране се намећу Нојманови а преосталим бозонским координатама Дирихлеови гранични услови. Фермионске променљиве задовољавају идентичне граничне услове само расписане преко независних спинора у шестодимензионалном простору. Физички смисао резултата је исти као и у случају када се Нојманови гранични услови намећу свим бозонским координатама.

Такође, у контетксту некомутативности, бавио сам се и фермионским T дуалностима тип IIB суперструне. Наметањем Дирихлеових граничних услова на све координате добија се некомутативност импулса иницијалне теорије. Некомутативни параметри су (до на константу) нека од поља фермионске T дуалне теорије.

1. B. Nikolic and B. Sazdovic, Type I background fields in terms of type IIB ones, Phys. Lett. B666 (2008) 400.

2. B. Nikolic and B. Sazdovic, D5-brane type I superstring background fields in

terms of type IIB ones by canonical method and T-duality approach, Nucl. Phys. B 836 (2010) 100–126.

3. B. Nikolic and B. Sazdovic, Noncommutativity relations in type IIB theory and their supersymmetry, JHEP 08 (2010) 037.

4. B. Nikolic and B. Sazdovic, Fermionic T-duality and momenta noncommutativity, Phys. Rev. D 84 (2011) 065012.

5. B. Nikolic and B. Sazdovic, Dirichlet boundary conditions in type IIB superstring theory and fermionic T-duality, JHEP 06 (2012) 101.

2.3 Т-дуалност и удвостручени простори

Када говоримо о Т-дуалности говоримо о трансформацији која повезује физички еквивалентне теорије. Уколико бисмо удвостручили простор тако да иницијалним координатама x^μ додамо Т-дуалне координате y_μ, онда можемо говорити о Т-дуалности као симетрији теорије. Удвостручавање простора (double space) је један од праваца истраживања чији је циљ налажење јединствене теорије свих интеракција. Идеја о удвостручeним просторима је стара нешто преко две деценије. У удвостручeном простору Т-дуалност се репрезентује матрицом пермутације подскупа координата које дуализујемо и одговарајућег подскупа Т-дуалних координата. Из захтева да закон Т-дуалне трансформације буде исти за удвостручене координате Z^M и њима Т-дуалне ${}_aZ^M$ добијају се изрази за Т-дуална позадинска поља преко иницијалних позадинских поља. Показано је да је Т-дуализација у оквиру формализма удвостручених простора еквивалентна са резултатима из Бушерове процедуре која се може сматрати дефиницијом Т-дуалности, како за бозонску тако и за фермионску. У анализама је коришћен модел тип II суперструне у формулацији чистог спинора са константним позадинским пољима. Даљи рад подразумева испитивање оптшег случаја у којем је једина апроксимација да позадинска поља не зависе од праваца дуж којих се дуализује. Проучавање Т-дуализације у удвостручeним просторима представља и мали корак ка бољем разумевање M-теорије. Једна Т-дуализација преводи типIIA/B у типIIB/A, а формализам удвостручених простора обједињује те две теорије у једну, што је један изузетно вредан резултат. Такође формализам развијен у овим радовима представља мало унапређење постојећег формализма јер у себи обједињује све могуће подскупове координата по којима се дуализује.

1. B. Nikolic and B. Sazdovic, Fermionic T-duality in fermionic double space, Nucl. Phys. B917 (2017) 105-121.

2. B. Nikolic and B. Sazdovic, T-dualization of type II superstring theory in double space, Eur. Phys. J. C77 (2017) 197.

3. B. Nikolić and B. Sazdović, Advantage of the second-order formalism in double space T-dualization of type II superstring, Eur. Phys. J. C 79 (2019) 819.

2.4 Т-дуалност бозонске и суперструне типа II у присуству координатно зависних позадинских поља

Једна од врло актуелних тема у области теорије струна је проучавање Т-дуалности и њено повезивање са некомутативношћу затворене струне. разлог зашто је важно размотрити и затворену струну у контексту Т-дуалности је та што је једно од побуђења затворене струне и гравитон, преносилац гравитационе интеракције. Проучавање затворене струне у контексту Т-дуализације и координатно зависних позадинских поља је важно за неке приступе проучавања гравитационе интеракције.

У случају затворене бозонске струне која пропагира у простор-времену константне метрике и у присуству константног Калб-Рамоновог поља, координате комутирају. Задржавањем констатне метрике и увођењем слабог Калб-Рамоновог поља које зависи линеарно од координата (у складу са просторно-временским једначинама за позадинска поља) не губи се транслациона симетрија теорије али се добија, применом **уопштене Бушерове процедуре**, један наизглед неочекиван резултат - некомутативност координата затворене струне. Пошто се добија **координатно зависна некомутативност**, добија се и **релација неасоцијативности** тј. Јакобијев идентитет није једнак нули.

Слично као у теорији отворене струне, у теорији затворене струне некомутативност следи из чињенице да се применом уопштене Бушерове процедуре добија да изводи Т-дуалних координата зависе од канонских импулса и извода координата почетне теорије. Стандарно се у литератури проучавају координатно зависна позадинска поља али се Т-дуализација врши дуж изометријских праваца - праваца од којих позадинска поља не зависе уз примену нетривијалних услова намотавања. Примена уопштене Бушерове процедуре омогућава дуализацију дуж свих праваца и добијање некомутативности и неасоцијативности и уз тривијалне услове намотавања. Такође у добијеној Т-дуалној теорији нелокалност је манифестна, јер теорија зависи од величине која је дефинисана као интеграл по линији на светској површи.

Метод уопштене T-дуализације се може применити и на суперструну. Заједно са докторандом Данијелом Обрићем проучавали смо тип II суперструну у формулацији чистог спинора са координатно зависним Рамон-Рамон пољем. Рамон-Рамон поље има и константан и линеарно зависан члан уз претпоставку да је коефицијент уз координатно зависни члан инфинитезималан. Разматрали смо две могућности: општи случај и један посебан случај. Посебност тог случаја се огледа у томе да је константни део Рамон-Рамон поља симетричан тензор (на измену фермионских индекса) док је координатно зависни део антисиметричан. Услови конзистентности не забрањују такве изборе.

Посебан случај се формално потпуно своди на случај бозонске струне са слабим Калб-Рамоновим пољем. Општи случај је доста сложенији рачунски, али је резултат квалитативно исти - T-дуалне бозонске координате су некомутативне и неасоцијативне, бозонске и фермионске координате су некомутативне (рађена је и фермионска T-дуализација) док су фермионске координате остале антикомутативне. T-дуалне теорије су нелокалне из истог разлога као и бозонска теорија са слабим Калб-Рамоновим пољем. Обрађен је и случај T-дуализације поменуте теорије за посебан случај у удвострученом простору. Из рачунских разлога, потребно је претпоставити и да је константни део Рамон-Рамон поља антисиметричан тензор. Резултати су потпуно у складу са резултатима добијеним аналитичким приступом.

1. Lj. Davidovic, B. Nikolic, B. Sazdovic, Canonical approach to the closed string noncommutativity, Eur. Phys. J C74 (2014) 2734.

2. Lj. Davidovic, B. Nikolic, B. Sazdovic, T-duality diagram for a weakly curved background, Eur. Phys. J C75 (2015) 576.

3. B. Nikolic and D. Obric, Noncommutativity and nonassociativity of closed bosonic string on T-dual toroidal background, Fortsch. Phys. 66 (2018) 040009.

4. B. Nikolic and D. Obric, Directly from H-flux to the family of three nonlocal R-flux theories, JHEP 03 (2019) 136.

5. B. Nikolic, B. Sazdovic and D. Obric, Noncommutativity and nonassociativity of type II superstring with coordinate dependent RR field, Fortsch.Phys. 70 (2022) 2200048.

6. B. Nikolic and D. Obric, Combined fermionic and bosonic T-duality of type II superstring theory with coordinate dependent RR field, Fortsch.Phys. 71 (2023) 2200160.

7. B. Nikolic and D. Obric, Noncommutativity and nonassociativity of type II superstring with coordinate dependent RR field - the general case, JHEP 12 (2022) 078.

8. B. Nikolic and D. Obric, Double space T-dualization and coordinate dependent RR field, arXiv:2307.02438.

1 Елементи за квалитативну анализу рада

1.1 Квалитет научних резултата

1.1.1 Научни ниво и значај резултата, утицај научних радова

Др Бојан Николић је у свом досадашњем раду објавио 45 научних публикација, међу којима 17 радова у врхунским међународним часописима (M21), 2 рада у истакнутим међународним часописима (M22), 3 рада у међународним часописима (M23), 13 радова у категорији M33 (саопштење са међународног скупа штампано у целини), 5 радова у категорији M34 (саопштење са међународног скупа штампано у изводу), 1 рад у категорији M61 (предавање по позиву са скупа националног значаја штампано у целини) и 4 рада из категорије M63 (саопштење са скупа националног значаја штампано у целини).

Од реизбора у звање виши научни сарадник (утврђивање предлога на седници Научног већа Института за физику 23.10.2018. године), др Николић је објавио 5 радова у категорији M21, 3 рада у категорији M33 и 1 рад у M34.

Као пет најзначајнијих радова кандидата могу се узети:

[1] B. Nikolić and B. Sazdović, Noncommutativity in space-time extended by Liouville field, Adv. Theor. Math. Phys. **14** (2010) 1.
M21, DOI: 10.4310/ATMP.2010.v14.n1.a1

[2] B. Nikolić and B. Sazdović, Noncommutativity relations in type IIB theory and their supersymmetry, JHEP **08** (2010) 037.
M21, DOI: 10.1007/JHEP08(2010)037

[3] Lj. Davidović, B. Nikolić, B. Sazdović, T-duality diagram for a weakly curved background, Eur. Phys. J. C 75 (2015) 576.
M21, DOI: 10.1140/epjc/s10052-015-3808-8

[4] B. Nikolic, B. Sazdovic, Advantage of the second-order formalism in double space T-dualization of type II superstring, Eur. Phys. J. C79 (2019) 819.
M21, DOI: 10.1140/epjc/s10052-019-7338-7

[5] B. Nikolić and B. Sazdović, Fermionic T-duality and momenta noncommutativity, Phys. Rev. D 84 (2022) 065012.
M21, DOI: 10.1103/PhysRevD.84.065012

Рад [1] представља систематизацију рада на проучавању бозонске струне у присуству константне метрике $G_{\mu\nu}$, Калб-Рамоновог поља $B_{\mu\nu}$ и дилатонског поља $\Phi(x)$ које је линеарна функција координата. Ова тематика је обрађивана у оквиру израде магистарске тезе и једног дела докторске дисертације кандидата. Анализа некомутативности указује на 6 могућих случајева а тих шест случајева произилази из шест услова које задовољава градијент дилатонског поља. Три случаја се односе на ситуацију када је трећа бета функција β^Φ нула, а друга три када је једнака ненултој константи и када се уведе Лиувилов члан у дејство. Свих шест случајева се може објединити у јединственом запису при чему је простор-време x^μ проширен додатном координатом - конформним фактором метрике светске површи F ($g_{\alpha\beta} = e^{2F}\eta_{\alpha\beta}$). Случајеви са и без Лиувиловог члана су повезани трансформацијом сличности. Рад је објављен у свесци бр.1 волумена 14 часописа Advances in Theoretical and Mathematical Physics заједно са још 5 радова. Аутор једног од тих преосталих пет радова је професор **Едвард Витен**, водећи научник на светском нивоу у области теорије (супер)струна.

Други рад представља детаљну анализу некомутативности тип IIB отворене суперструне у формалацији чистог спинора. У складу са скупом услова конзистентности, изабрана су константна позадиска поља. Пошто је у питању отворена струна нужно се у анализи појављују гранични услови - Нојманови за бозонске координате, док се за фермионске промењливе бирају такви гранични услови који суперсиметрију $N = 2$ своде на суперсиметрију $N = 1$. Третирањем граничних услова као канонских веза и њиховим решавањем добијају се релације не(анти)комутативности, као и ефективна теорија - иницијална теорија на везама. Ефективна теорија је теорија тип I суперструне са позадинским пољима која су Ω парна ($\Omega : \sigma \to -\sigma$), док су параметри некомутативности Ω непарна. Показано је да су ефективна позадинска поља и параметри некомутативности позадинска поља T-дуалне теорије. Један од резултата је и повезаност релација не(анти)комутативности преко суперсиметричних трансформација.

Трећи рад представља примену процедуре уопштене T-дуализације на затворену бозонску струну у присуству тзв. слабо закривљених позадинских поља - константа метрика и координатно линеарно зависно Калб-Рамоново поље. Процедура T-дуализације се примењује дуж свих праваца, уз тривијалне услове намотавања. Извршена је симултана Т-дуализација дуж свих праваца, као и две узастопне произвољне парцијалне Т-дуализације до потпуне Т-дуализације. Главни резултат је да је група свих могућих парцијалних Т-дуализација Абелова група. Т-дуализаоване теорије имају особину нелокалности јер зависе од координате која је дефинисана као линијски интеграл.

У раду [4] разматрана је Т-дуализација тип II суперструне у формулацији чистог спинора преко удвостручених простора. Иницијално теорија је задата у формализму првог реда али, интеграцијом фермионских импулса, добија се теорија другог реда. T-дуализација је репрезентована пермутацијом једног подскупа координата иницијалног простора са одговарајућим подскупом T-дуалних координата. Из захтева да је закон трансформације иницијалне удвострчене координате и T-дуалне удвостручене координате исти, добијају се сва T-дуална позадинска поља и потпуно слагање са аналитичким приступом преко Бушерове процедуре.

Пети рад се бави фермионском T-дуализацијом тип IIB суперструне у формулацији чистог спинора. Фермионска дуализација је екстензија већ постојеће бозонске дуализације и процедура је исте форме. Разматран је случај отворене струне и за бозонске координате су изабрани Дирихлеови гранични услови. Као резултат је добијена некомутативност импулса са параметрима некомутативности који су фермионски T-дуална поља.

1.1.2 Позитивна цитираност научних радова

Према бази Scopus Хиршов индекс кандидата је 5 (искључени аутоцитати), а број укупан цитата 159, док база WoS даје Хиршов индекс 7. И једна и друга база имају мањкавости у смислу да неке публиковане радове третирају као препринте.[1]

База Google Scholar даје Хиршов индекс 9 и број цитата 219, а непосредним увидом у радове и цитате, укрштајући са базом INSPIRE HEP тачан број цитата је 213. Број цитата без аутоцитата и цитата коаутора је 28, док је број цитата **без аутоцитата 88** (минимални захтев МОФа је 30).

Прилог: листе цитираности из база Google Scholar, Scopus и WoS као и детаљан списак радова који цитирају радове кандидата.

1.1.3 Параметри квалитета часописа

Др Бојан Николић је током каријере објавио укупно 45 публикација у часописима са ИСИ листе од тога **са импакт фактором** 17 у категорији М21, 2 категорије М22 и 3 категорије М23. **Укупан импакт фактор** радова је **94,002**. Од реизбора у звање виши научни сарадник др Николић је објавио 5 радова категорије М21. Укупан импакт фактор ових радова је **29,054**.

[1] Чека се одговор аналитичара Scopus-а јер 7 радова се класификују као препринти а уствари су то радови публиковани у часописима категорије М21.

Збирно приказано др Николић је објавио [2]:

- 4 рада у Journal of High Energy Physics, ($ИФ_{2010}$ = $ИФ_{2012}$ = 4,642, $ИФ_{2019}$ = 5,875, $ИФ_{2022}$ = 6,376)
- 3 рада у Physical Review D, ($ИФ_{2006}$ =$ИФ_{2007}$ = 5,336, $ИФ_{2011}$ = 5,373)
- 4 рада у European Physical Journal C, ($ИФ_{2014}$ =ИФ$ИФ_{2015}$ = 5,436, $ИФ_{2017}$ = 5,297, $ИФ_{2019}$ = 5,172)
- 2 рада у Nuclear Physics B($ИФ_{2010}$ = 4,79, $ИФ_{2017}$ = 3,735)
- 4 рада у Fortschritte der Physik, ($ИФ_{2008}$ = 2,125, $ИФ_{2018}$ = 3,263, $ИФ_{2022}$ = 6,099, $ИФ_{2023}$ = 5,532)
- 2 рада у Romanian Journal of Physics, ($ИФ_{2012}$ = 0,526)
- 1 рад у Advances of Theoretical and Mathematical Physics, ($ИФ_{2010}$ = 2,075)
- 1 рад у Physics Letters B, ($ИФ_{2008}$ = 5,446)
- 1 рад у International Journal of Modern Physics A, ($ИФ_{2009}$0,982)

После одлуке Научног већа о утврђивању предлога за реизбор др Николића у звање виши научни сарадник (седница Научног већа Института за физику 23.10.2018. године):

- 1 рад у European Physical Journal C ($ИФ_{2019}$ = 5,172)
- 2 рада у Fortschritte der Physik ($ИФ_{2022}$ = 6,099, $ИФ_{2023}$ = 5,532)
- 2 рада у Journal of High Energy Physics ($ИФ_{2019}$ = 5,875, $ИФ_{2022}$ = 6,376)

Подаци о **додатним библиометријским параметрима** квалитета часописа у којима је кандидат објављивао радове категорије М20 у периоду од утврђивања предлога за реизбор у звање виши научни сарадник (23.10.2018. године) дате су у доњој табели.

	ИФ	**М**	**СНИП**
Укупно	29,054	40	6,71
Усредњено по чланку	5,81	7,7	1,342
Усредњено по аутору	13,511	14,667	3,14

[2] У индексима ознака ИФ стоје импакт фактори часописа са којима су израчунате укупне суме из претходног пасуса. Гледала се најповољнија вредност између године објављивања и две године уназад.

1.1.4 Степен самосталности и степен учешћа у реализацији резултата

У области истраживања којом се кандидат бави уобичајено је да се аутори потписују абецедним редом тј. не постоји појам првог аутора. Ова пракса је примењена у свим кандидатовим радовима. У том смислу кандидатов допринос у научним радовима је потпуно равноправан између свих потписаних аутора.

Узимајући у обзир све кандидатове радове из категорија М21, М22 и М23 (22 рада) само три су урађена са два коаутора, док су остали са једним коаутором (ментор или докторанд). У том смислу допринос кандидата се сматра 100%-ним како у квалитативном тако и кавантитативном смислу. Кандидат је учествовао у свим фазама израде научних радова - избору и осмишљавању тема, дискусијама, аналитичким прорачунима као и у писању самих радова. Сви радови кандидата са докторандом Данијелом Обрићем објављени су у часописима категорије М21 са високим импакт факторима.

1.2 Ангажованост у формирању научних кадрова

Под менторством др Николића урађене су **четири мастер рада**:

- мастер рад Миливоја Јојића "Т-дуалност на торусу преко комплексних параметара" урађен је и успешно одбрањен 2015. године на Физичком факултету Универзитета у Београду,
- мастер рад Данијела Обрића "Некомутативност и неасоцијативност затворене бозонске струне" успешно одбрањен на Физичком факултету Универзитета у Београду 2017. године. Резултати рада су публиковани у часопису $M21$ категорије,

  B. Nikolic, D. Obric, Fortschritte der Physik **66** (2018) 040009.
- мастер рад Немање Симовића "Некомутативност координата на Dp-брани у присуству константних позадинских поља" успешно одбрањен на Физичком факултету Универзитета у Београду 2019. године,
- мастер Јована Јањића "Тип IIB суперструна - ефективна теорија, некомутативност и Т-дуалност" успешно одбрањен на Физичком факултету Универзитета у Београду 2023. године.

Под менторством др Николића септембра 2023. године студент Данијел Обрић одбранио је **докторску дисертацију** под насловом "Т-дуализација бозонске струне и тип IIB суперструне у присуству координатно зависних позадинских поља" на Физичком факултету Универзитета у Београду.

Кандидат је више пута био члан комисије одбрану мастер и докторских радова на Физичком факултету Универзитета у Београду.

Кандидат је школске 2013/2014 радио као спољни сарадник-професор физике у Математичкој гимназији. Од школске 2015/2016 ангажован је као наставник на предметима Рачунски практикум 1 и 2 у посебном одељењу за ученике посебно надарене за физику у Земунској гимназији (4 часа недељно).

Кандидат је **професор на докторским студијама** Физичког факултета Универзитета у Београду за ужу научну област **Квантна поља, честице и гравитација** на предмету **Теорија струна**.

Прилог: уговор о ангажовању у Земунској гимназији, записници са седница Наставно-научног већа Физичког факултета у Београду на којима су одобрене мастер тезе и докторска теза поменутих кандидата, текстови мастер тезе и докторске тезе, слика web странице факултета и програм предмета Теорија струна.

1.3 Нормирање броја коауторских радова, патената и техничких решења

У реизборном периоду (почев од датума утврђивања предлога за реизбор, седница НВ Института за физику 23.10.2018. године) кандидат је укупно објавио: 5 из категорије М21, 2 из М33 и 1 из М34. Само у 1 раду (М33)

B. Nikolić, D. Obrić, T. Radenković, I. Salom, M. Vojinović, Higher category theory and n-groups as gauge symmetries for quantum gravity, J.Phys.Conf.Ser. 2667 (2023) 1 - Contribution to QTS12.

кандидат има 4 коаутора што значи да у складу са Правилником овај рад нема вредност 1 М-бода већ $\frac{1}{1+0,2(5-3)} = 0,71$. У свим осталим радовима кандидат има једног или два коаутора, што се у теоријској физици сматра за стопроцентни допринос.

Укупан нормирани број М-бодова је 42,21.

Прилог: списак радова кандидата као и табеле у делу Квантитативне анализе рада кандидата.

1.4 Руковођење пројектима, потпројектима и пројектним задацима

Др Бојан Николић је **руководио потпројектом** "Т-дуализација отворене и затворене (супер)струне" у оквиру пројекта Министарства просвете, науке и технолошког развоја ОН 171031 ”Физичке импликације модификованог простор-времена” и Групе за гравитацију, честице и поља Института за физику Београд у периоду 2011. до 2020. године. Пошто је пројекат био састављен од људи са Института за физику у Београду и Физичког факултета у Београду, кандидат је ефективно био руководилац институтског дела пројекта (писање годишњих извештаја, финансијских извештаја, као и потписивање путних налога и фактура). У прилог овоме прилаже се допис руководиоца пројекта проф. др Маје Бурић из 2018. године писан за потребе реизбора.

Такође кандидат је представљао пројекат МПНТР ОН 171031 у Научном савету Института за физику у Београду у периоду од 2011. до 2013. године (Министарство укинуло научне савете), што је и био задатак руководиоца пројекта.

Почев 01.01.2022. године кандидат је члан пројекта ”Квантна гравитација преко виших гејџ теорија” из програма ИДЕЈЕ Фонда за науку Републике Србије и у оквиру њега **руководи пројектним задатком** ”Испитивање аспеката класичне теорије гравитације” (радни пакет 1 пројекта).

Прилог: дописи руководилаца поменутих пројеката.

1.5 Активност у научно стручним друштвима

Кандидат је рецензент у часопису **Foundations of Physics**.

Др Бојан Николић је у два наврата био члан **Државне комисије** ДФС за такмичење ученика средњих школа - 2003.-2005. и 2011.-2013., и као аутор задатака и као прегледач. У периоду 2004. до 2006. био је члан редакције часописа Млади физичар, који издаје ДФС у сврху популаризације физике. Активно је учествовао у обележавању Светске године физике на Институту за физику 2005. године. Био је члан локалних организационих комитета више међународних и домаћих конференција и радионица организованих од стране Института за физику или Групе за гравитацију, честице и поља.

Прилог: E-mail у којем се уредништво часописа захваљује за обављену рецензију. Копија ”Младог физичара”, одштампане интернет странице школа и конференција, као и задаци за 2. разред средњих школа за Државно такмичење 2012. године.

1.6 Утицајност научних резултата

Цитираност као и квалитет часописа (висок ИФ) у којима др Николић публикује говоре о квалитету добијених резултата а самим тим и о њиховом (потенцијалном) утицају на научну заједницу.

С обзиром да се кандидат у изборном периоду бавио Т-дуализацијом затворене струне у присуству координатно зависних поља, а то је област зачета 2010. године, као и због релативно малог броја људи који се том темом бави, број цитата 88 (без аутоцитата) је задовољавајући јер је остварен већином у периоду од последњих 13 година.

Рад из 2010. године

B. Nikolic and B. Sazdovic, Noncommutativity in space-time extended by Liouville field, Adv. Theor. Math. Phys. **14** (2010) 1,

је објављен у свесци са још само 5 радова од којих је један рад дело Едварда Витена, водећег светског експерта у области математичке физике и теорије струна.

Рад из 2018. године

B. Nikolic, D. Obric, Noncommutativity and nonassociativity of closed bosonic string on T-dual toroidal backgrounds, Fortschritte der Physik (2018) 1800009,

је у овом часопису М21 категорије објављен по позиву главног уредника проф. др Дитера Листа (Dieter Lüst).

Прилог: листе цитираности од Google Scholar-a и Scopus-a, копија прве стране часописа као и e-mail главног уредника часописа Fortschritte der Physik, проф. др Дитера Листа.

1.7 Допринос кандидата у реализацији радова у центрима у земљи и иностранству

Кандидат је значајно допринео сваком раду који је објавио. Сви радови објављени у изборном периоду урађени су са сарадницима са Института за физику и докторандом Данијелом Обрићем. Др Николић је имао значајан допринос у свакој етапи израде публикације - како у избору теме, тако и у дискусијама, аналитичким прорачунима као и у писању самих радова и комуникацији са рецензентима часописа у току поступка објављивања.

1.8 Међународна научна сарадња

Др Николић је донео две нове теме у групу која се бави теоријом струна (др Бранислав Саздовић, др Љубица Давидовић, докторанд Илија Иванишевић, др Бојан Николић, др Данијел Обрић).

Прва је фермионска Т-дуалност (проистекла из кореспонденције са др Иљом Бахматовим[3]) а друга некомутативност затворене струне током постдокторског боравка у групи проф. Листа. У оквиру прве теме др Николић је применио фермионску Т-дуалност на случај тип II теорије суперструне у формулацији чистог спинора.

Другу споменуту тему, некомутативност затворене струне, др Николић је учио директно од њеног аутора проф. Листа током боравка у Минхену. По повратку из Минхена идеја је комбинована са уопштеном Т-дуализационом процедуром која је већ била развијена у Институту за физику од старне проф. др Бранислава Саздовића и др Љубице Давидовић. Резултат је генерализација резултата добијених у групи др. Листа као и значајно поједностављење математичког дела процедуре.

Током боравка у Минхену успостављена је сарадња са групом проф. Листа (није формализована), која се огледа у честој кореспонденцији и анализи нових радова, што доприноси вишем квалитету резултата.

1.9 Показатељи успеха у научном раду

После претходног избора у звање др Бојан Николић је одржао следећа **предавања по позиву** на скуповима од националног значаја:

M 61 Б. Николић, *Гравитациони таласи - од теорије до директне детекције*, Настава физике број 3, мај 2016, 213-221, $XXXIV$ Републички семинар о настави физике, Златибор 12.-14. мај 2016.

Члан организационог комитета неколико међународних скупова:

- IV Summer School in Modern Mathematical Physics, September 2006, Belgrade, Serbia

  http://mphys4.ipb.ac.rs/

- V Summer School in Modern Mathematical Physics, September 2008, Belgrade, Serbia

  http://mphys5.ipb.ac.rs/

- VI Summer School in Modern Mathematical Physics, September 2010, Belgrade, Serbia

  http://mphys6.ipb.ac.rs/

- VII Summer School in Modern Mathematical Physics, September 2012, Belgrade, Serbia

[3]Institute of Theoretical and Mathematical Physics, Moscow State University, Leninskie Gory 119991, Moscow, Russia.

http://mphys7.ipb.ac.rs/

- Gravity and String Theory: New ideas for unsolved problems III (In honour of Prof. Branislav Sazdović's retirement)

  http://www.gst2018.ipb.ac.rs/

Кандидат је добитник награде Института за физику у Београду 2009. године за најбоље урађену докторску тезу на Институту у току 2008. године.

Прилог: копија рада, план рада скупа, позив организатора.

ПРЕГЛЕД КВАНТИТАТИВНИХ ПОКАЗАТЕЉА
научноистраживачког рада др Бојана Николића

Остварени резултати након утврђивања предлога за реизбор у звање виши научни сарадник (редовна седница Научног већа Института за физику од 23.10.2018. године)

Категорија	М бодова по раду	Број радова	Укупно М бодова	Нормирани број М бодова
M21	8	5	40	40
M33	1	2	2	1,71
M34	0,5	1	0,5	0,5
Укупно		8	42,5	42,21

Поређење са минималним квантитативним условима за **реизбор у звање вишег научног сардника**[1]

		Неопходно	Остварено(нормирано)
Виши научни сарадник	Укупно	25	42,21
	$M_{10} + M_{20} + M_{31} + M_{32} + M_{33} + M_{41} + M_{42} + M_{90}$	20	41,71
	$M_{11} + M_{12} + M_{21} + M_{22} + M_{23}$	15	40

[1] Приликом реизбора у звање виши научни сарадник минимални квантитативни захтев је, по Правилнику, половина онога што је неопходно за избор у звање виши научни сарадник.

Spisak objavljenih radova sa citatima *

March 6, 2024

Radovi u vrhunskim medjunarodnim časopisima, M21

1. B. Nikolić and B. Sazdović, Gauge symmetries decrease the number of *Dp*-brane dimensions, *Phys. Rev.* **D 74** (2006) 045024.

2. B. Nikolić and B. Sazdović, Gauge symmetries decrease the number of *Dp*-brane dimensions. II. Inclusion of the Liouville term, *Phys. Rev.* **D 75** (2007) 085011.

3. B. Nikolić and B. Sazdović, Type I background fields in terms of type IIB ones, *Phys. Lett.* **B666** (2008) 400.

4. B. Nikolić, B. Sazdović, *D5-brane type I superstring background fields in terms of type IIB ones by canonical method and T-duality approach*, *Nucl. Phys.* **B 836** (2010) 100–126.

5. B. Nikolić and B. Sazdović, *Noncommutativity relations in type IIB theory and their supersymmetry*, *JHEP* **08** (2010) 037.

6. B. Nikolić and B. Sazdović, *Noncommutativity in space-time extended by Liouville field, Adv. Theor. Math. Phys.* **14** (2010) 1.

7. B. Nikolić and B. Sazdović, *Fermionic T-duality and momenta noncommutativity, Phys. Rev.* **D 84** (2011) 065012.

8. B. Nikolić, B. Sazdović, *Dirichlet boundary conditions in type IIB superstring theory and fermionic T-duality*, *JHEP* **06** (2012) 101.

9. * Lj. Davidović, B. Nikolić, B. Sazdović, *Canonical approach to the closed string noncommutativity*, *Eur. Phys. J* **C74** (2014) 2734.

10. * Lj. Davidović, B. Nikolić, B. Sazdović, *T-duality diagram for a weakly curved background*, *Eur. Phys. J* **C75** (2015) 576.

*Ovde je dat spisak svih radova. Radovi objavljeni u izbornom periodu su označeni zvezdicama.

11. * B. Nikolić and B. Sazdović, *T-dualization of type II superstring theory in double space*, *Eur. Phys. J.* **C77** (2017) 197.

12. * B. Nikolić and D. Obrić, *Noncommutativity and nonassociativity of closed bosonic string on T-dual toroidal background*, *Fortsch. Phys.* **66** (2018) 040009.

13. * B. Nikolić and D. Obrić, *Directly from H-flux to the family of three nonlocal R-flux theories*, *JHEP* **03** (2019) 136.

14. * B. Nikolic, B. Sazdovic, *Advantage of the second-order formalism in double space T-dualization of type II superstring*, *Eur. Phys. J.* **C79** (2019) 819.

15. * B. Nikolic, B. Sazdovic, D. Obric, *Noncommutativity and nonassociativity of type II superstring with coordinate dependent RR field*, *Fortsch. Phys.* **70** (2022) 2200048.

16. * B. Nikolic, D. Obric, *Noncommutativity and nonassociativity of type II superstring with coordinate dependent RR field - the general case*, *JHEP* **12** (2022) 078.

17. * B. Nikolic, D. Obric, *Combined fermionic and bosonic T-duality of type II superstring theory with coordinate dependent RR field*, *Fortsch. Phys.* **71** (2023) 2200160.

Rad u istaknutom medjunarodnom časopisu, M22

1. B. Nikolić and B. Sazdović, Central charge contriution to noncommutativity, *Fortschr. Phys.* **56**, 491 (2008).

2. * B. Nikolić and B. Sazdović, Fermionic T-duality in fermionic double space, *Nucl. Phys.* **B917** (2017) 105-121.

Rad u medjunarodnom časopisu, M23

1. B. Nikolić and B. Sazdović, *Type I superstring theory in the form of open type IIB with appropriate choice of boundary conditions*, *IJMP* **A24** (2009) 2857.

2. B. Nikolić and B. Sazdović, *Supersymmetry of noncommutativity relations*, Rom. Journ. Phys. Vol. 57, Nos. 5-6, P. 931-937, Bucharest, 2012.

3. B. Nikolić and B. Sazdović, *Noncommutativity and T-duality*, Rom. Journ. Phys. Vol. 57, Nos. 5-6, P. 816-829, Bucharest, 2012.

Saopštenje sa medjunarodnog skupa štampano u celini, M33 (1+1+1+1=4)

1. B. Nikolić and B. Sazdović, Noncommutativity in the presence of the dilaton field, Zbornik radova, Balkan Workshop 2005 (BW2005), II Southeastern European Workshop "Challenges Beyond the Standard Model", Facta Universitatis, Series: Physics, Chemistry and Technology, Vol. 4, N^{o} 2, (2006) 405-413, editors: G. Djordjević, Lj. Nešić i J. Wess.

2. B. Nikolić and B. Sazdović, From Neumann to Dirichlet boundary conditions, *Sixth International Conference of the Balkan Physical Union*, Istanbul, Turkey, 22.–26. august 2006.; Springer, Series: AIP Conference Proceedings, Vol.899, (2007) ISBN: 978-0-7354-0404-5 (151—152). Editors: Cetin, Serkant Ali; Hikmet, Iskender.

3. B. Nikolić and B. Sazdović, Open string boundary conditions dependence on the background fields, *4th Summer School in Modern Mathematical Physics*, Belgrade, Serbia, 3–14. September 2006, editors: B. Dragovich, Z. Rakić (SFIN, XX A1 2007) 327 — 334.

4. B. Nikolić and B. Sazdović, *Improved relations between type I and type IIB background fields*, The 5th MATHEMATICAL PHYSICS MEETING: Summer School and Conference on Modern Mathematical Physics, Belgrade, Serbia, 6–17 July 2008, editors: B. Dragovich, Z. Rakić (SFIN, XXII A1 2009) 305 — 316.

5. B. Nikolić and B. Sazdović, *Hamiltonian Approach to Dp-brane Noncommutativity*, Spring School on Strings, Cosmology and Particles, Belgrade-Niš, 31.3.-04.04.2009., Monograph issue on "Modern Trends in Strings, Cosmology and Particles", Publications of Astronomical Observatory of Belgrade No.88, July, 2010, 61; Editors: M. Ćirković, G. Djordjević, G. Senjanović, Referees: M. Hindmarsh, Dj. Šijački, D. Stojković.

6. B. Nikolić and B. Sazdović, *T-duality and noncommutativity in type IIB superstring theory*, The 6th MATHEMATICAL PHYSICS MEETING: Summer School and Conference on Modern Mathematical Physics, Belgrade, Serbia, 14–23 September 2010, editors: B. Dragovich, Z. Rakić (SFIN, XXIV A1 2011) 259 — 266.

7. $\star$ Lj. Davidović, B. Nikolić and B. Sazdović, *Complet T-dualization of a String in a Weakly Curved Background*, Springer Proc.Math.Stat. 111 (2014) 13-20, 10th International Workshop on Lie Theory and Its Applications in Physics (LT-10), 17-23 June 2013. Varna, Bulgaria.

8. $\star$ Lj. Davidović, B. Nikolić and B. Sazdović, *Weakly curved background T-duals*, 8th Mathematical Physics Meeting: Summer School and Conference on Modern Mathematical Physics, SFIN XXVIII A1 (2015) 43.

9. * Lj. Davidović, B. Nikolić and B. Sazdović, *Closed string noncomutativity in the weakly curved background*, 8th Mathematical Physics Meeting: Summer School and Conference on Modern Mathematical Physics, SFIN XXVIII A1 (2015) 51.

10. * Lj. Davidović, B. Nikolić and B. Sazdović, *T-dualization of a weakly curved background*, J.Phys.Conf.Ser. 804 (2017) no.1, 012014, 24th International Conference on Integrable Systems and Quantum Symmetries (ISQS-24), 14-18 Jun 2016. Prague, Czech Republic.

11. * B. Nikolić and B. Sazdović, *Representation of T-duality of type II pure spinor superstring in double space*, 9th Mathematical Physics Meeting: Summer School and Conference on Modern Mathematical Physics, SFIN XXXI A1 (2018) 281.

12. * B. Nikolić, D. Obrić, *From 3D torus with H-flux to torus with R-flux and back*, 10th Mathematical Physics Meeting: Summer School and Conference on Modern Mathematical Physics, SFIN XXXIII A1 (2020) 233.

13. * B. Nikolić, D. Obrić, T. Radenković, I. Salom, M. Vojinović, *Higher category theory and n-groups as gauge symmetries for quantum gravity*, J.Phys.Conf.Ser. 2667 (2023) 1 - Contribution to QTS12.

Saopštenje sa medjunarodnog skupa štampano u izvodu, M34

1. B. Nikolić and B. Sazdović, Central charge contribution to noncommutativity, *BW2007 III Southeastern European Workshop "Challenges Beyond the Standard Model"*, Kladovo, Serbia, 2–9. September 2007, editors: G. Djordjevic, M. Haack, Lj. Nesic (BW2007 Book of Abstracts, Faculty of Sciences and Mathematics, Section of Serbian Physical Society, Niš 2007) 35.

2. B. Nikolić and B. Sazdović, *Hamiltonian Approach to Dp-brane Noncommutativity*, Spring School on Strings, Cosmology and Particles, Belgrade-Niš, 31.3.-04.04.2009., Book of short contributions and extended abstracts (32-34); Editors: M. Ćirković, G. Djordjević and Lj. Nešić.

3. Lj.Davidović, B. Nikolić and B. Sazdović, *Noncommutativity and T-duality*, Balkan Summer Institute - BSI 2011, August 19 - September 1, 2011, Niš - Donji Milanovac, Serbia; Editors: G. djordjević, Lj. Nešić, G. Senjanović..

4. B. Nikolić and B. Sazdović, *Noncommutativity in type IIB superstring theory and supersymmetry*, Balkan Summer Institute - BSI 2011, August 19 - September 1, 2011, Niš - Donji Milanovac, Serbia; Editors: G. Djordjević, Lj. Nešić, G. Senjanović.

5. ⋆ B. Nikolić, *From 3D torus with H-flux to torus with R-flux and back*, Book of abstacts, 10th Mathematical Physics Meeting: School and Conference on Modern Mathematical Physics, 9.-14. Sepetember 2019, Belgrade, Serbia.

Predavanje po pozivu sa skupa nacionalnog značaja štampano u celini, M61 (1.5)

1. ⋆ B. Nikolić, *Gravitacioni talasi - od teorije do direktne detekcije*, Nastava fizike broj 3, maj 2016, 213-221, XXXIV Republički seminar o nastavi fizike, Zlatibor 12.-14. maj 2016.

Saopštenje sa skupa nacionalnog značaja štampano u celini, M63 (1)

1. B. Nikolić, i B. Sazdović, Granični uslovi za otvorenu strunu i nekomutativnost prostornih koordinata, Zbornik radova sa 11. kongresa fizičara Srbije i Crne Gore, Petrovac na moru, Srbija i Crna Gora, 3.–5. jun 2004. godine, urednici: N. Konjević, B. Vujičić i P. Miranović (2004) 5-133 — 5-136.

2. B. Nikolić and B. Sazdović, Noncommutativity in the presence of dilaton field, Zbornik radova sa naučno-stručnog skupa "Sto godina teorije relativnosti", Banja Luka, Republika Srpska, 29.–30. septembar 2005. godine, urednici: Rajko Kuzmanović, Dragoljub Mirjanić, Branko Škundrić i Zoran Rajilić (2005) 59 — 70.

3. B. Nikolić and B. Sazdović, Contribution of dilaton field to *Dp*-brane properties, FIS2007- Fundamentalne Interakcije - Srbija 2007, septembar, 26-28 2007, Irški venac, Novi Sad, Srbija. Journal of Research in Physics, Vol. 31, N^o 2, (2007), ISSN 1450-7404, (98-101). Editors: Ištvan Bikit, Ivan Aničin, Dragan Popović, Ilija Savić, Milutin Blagojević, Miroslav Vesković, Petar Adžić, Božidar Cekić, Krunoslav Subotić, Maja Burić.

4. ⋆ B. Nikolić, *Gravitacioni talasi - šta se to talasa?*, Zbornik radova "100 godina Opšte teorije relativnosti" str.43, 23. jun 2015. godine, SANU, Beograd, Srbija.

Odbranjena doktorska disertacija, M71

1. B. Nikolić, Nekomutativnost i dimenzionalnost *Dp*-brane, Fizički fakultet Univerziteta u Beogradu, 2008, mentor: prof. dr Branislav Sazdović

Odbranjena magistarska teza, M72

1. B. Nikolić, Efekat dilatonskog polja na nekomutativnost prostorno-vremenskih koordinata, Fizički fakultet Univerziteta u Beogradu, 2006, mentor: prof. dr Branislav Sazdović

Citiranost

(1) B. Nikolić and B. Sazdović, Gauge symmetries decrease the number of *Dp*-brane dimensions, *Phys. Rev.* **D 74** (2006) 045024, citiran je u:

- B. Nikolić, D. Obrić, Noncommutativity and nonassociativity of type II superstring with coordinate dependent RR field—the general case, JHEP **12** (2022) 078.
- B. Nikolić, D. Obrić, From 3D torus with H-flux to torus with R-flux and back, Proceedings of the 10th MATHEMATICAL PHYSICS MEETING: School and Conference on Modern Mathematical Physics : Belgrade, Serbia, September 9 - 14, 2019, 233-250, published 2020.
- L. Davidovic, B. Sazdovic, T-duality between effective string theories, Proceedings of the 10th MATHEMATICAL PHYSICS MEETING: School and Conference on Modern Mathematical Physics : Belgrade, Serbia, September 9 - 14, 2019, 97-109, published 2020.
- L. Davidovic, B. Sazdovic, Effective theories of two T-dual theories are also T-dual, Eur. Phys. J. C79 (2019) 770.
- B. Nikolić, D. Obrić, Directly from H-flux to the family of three nonlocal R-flux theories, JHEP **03** (2019) 136.
- B. Nikolić, D. Obrić, Noncommutativity and nonassociativity of closed bosonic string on T-dual toroidal backgrounds, Fortsch.Phys. 66 (2018) 040009.
- B. Nikolić, B. Sazdović, Hamiltonian approach to Dp-brane noncommutativity, Publ. Astron. Obs. Belgrade 88 (2010) 61-69.
- Lj. Davidović, B. Nikolić, B. Sazdović, Canonical approach to the closed string noncommutativity, Facta Universitatis, Series: Physics, Chemistry and Technology, Vol 12, No 2, Special Issue, 2014.
- Lj. Davidović, B. Nikolić, B. Sazdović, Closed string noncommutativity in the weakly curved background, 8th Mathematical Physics Meeting : Summer School and Conference on Modern Mathematical Physics. 24-31 Aug 2014. Belgrade, Serbia.
- B. Nikolić, B. Sazdović, T-duality and noncommutativity in type IIB superstring theory, 6th Summer School in Modern Mathematical Physics (MPHYS6) 14-23 Sep 2010. Belgrade, Serbia .
- Lj. Davidović, B. Sazdović, Curved Dp-brane in curved background by canonical methods, 6th Summer School in Modern Mathematical Physics (MPHYS6) 14-23 Sep 2010. Belgrade, Serbia.

- Lj. Davidović, B. Nikolić, B. Sazdović, Canonical approach to the closed string non-commutativity, Eur.Phys.J. C74 (2014) no.1, 2734.
- Lj. Davidović,B. Sazdović, *Nongeometric background arising in the solution of Neumann boundary conditions*, Eur. Phys. J. **C 72** (2012) 2199.
- B. Nikolić, B. Sazdović, *Dirichlet boundary conditions in type IIB superstring theory and fermionic T-duality*, *JHEP* **06** (2012) 101.
- Lj. Davidović, B. Sazdović, *Non-commutativity parameters depend not only on the effective coordinate but on its T-dual as well*, *JHEP* **08** (2011) 112.
- Lj. Davidović, B. Sazdović, *T-dual-coordinate dependence makes the effective Kalb-Ramond field nontrivial*, 7th Mathematical Physics Meeting: Summer School and Conference on Modern Mathematical Physics (MPHYS7) 9-19 September 2012., Belgrade, Serbia, arXiv:1105.2809.
- B. Nikolić, B. Sazdović, *Fermionic T-duality and momenta noncommutativity*, *Phys. Rev.* **D 84** (2011) 065012.
- B. Nikolić, B. Sazdović, *Noncommutativity relations in type IIB theory and their supersymmetry*, *JHEP* **08** (2010) 037.
- Lj. Davidović, B. Sazdović, *Noncommutativity in weakly curved background by canonical methods*, *Phys. Rev.* **D 83** (2011) 066014.
- B. Nikolić, B. Sazdović, *D5-brane type I superstring background fields in terms of type IIB ones by canonical method and T-duality approach*, *Nucl. Phys.* **B 836** (2010) 100–126
- D. S. Popović, B. Sazdović, *Canonical approach to noncommutative gauge theory*, *Phys.Lett.* **B683** (2010) 349-353
- B. Nikolić, B. Sazdović, *Central charge contribution to noncommutativity*, *Fortschr. Phys.* **56**, 491 (2008).
- B. Nikolić, B. Sazdović, *Type I background fields in terms of type IIB ones*, *Phys.Lett.* **B666** (2008) 400-403
- B. Nikolić and B. Sazdović, *Noncommutativity in space-time extended by Liouville field*, *Adv. Theor. Math. Phys.* **14** (2010) 1.
- B. Nikolić and B. Sazdović, *Type I superstring theory in the form of open type IIB with appropriate choice of boundary conditions*, *IJMP* **A24** (2009) 2857.

- B. Nikolić, B. Sazdović, Improved relations between type I and type IIB background fields, PROCEEDINGS. Edited by B. Dragovich, Z. Rakic. Belgrade, Inst. Phys. Belgrade, 2009. 513p. (SFIN Ser. A: Conferences; A1 (2009))

- B. Nikolić and B. Sazdović, Gauge symmetries decrease the number of *Dp*-brane dimensions. II. Inclusion of the Liouville term, *Phys. Rev.* **D 75** (2007) 085011.

- ⋆ B. Nikolić, B. Sazdović, Open string boundary conditions dependence on the background fields, SFIN A1 (2007) 327-334.

- B. Nikolić, B. Sazdović, From Neumann to Dirichlet boundary conditions, AIP Conf. Proc. 899, 151–152 (2007).

(2) B. Nikolić and B. Sazdović, Gauge symmetries decrease the number of *Dp*-brane dimensions. II. Inclusion of the Liouville term, *Phys. Rev.* **D 75** (2007) 085011, citiran je u:

- B. Nikolic, D. Obric, Directly from H-flux to the family of three nonlocal R-flux theories, JHEP **03** (2019) 136.

- B. Nikolić, D. Obrić, From 3D torus with H-flux to torus with R-flux and back, SFIN XXXIII, 233 (2020), 10th Mathematical Physics Meeting : Summer School and Conference on Modern Mathematical Physics. 09.-14. Sep 2019., Belgrade, Serbia.

- Lj. Davidović, B. Sazdović, T-duality between effective string theories, SFIN XXXIII, 97 (2020), 10th Mathematical Physics Meeting : Summer School and Conference on Modern Mathematical Physics. 09.-14. Sep 2019., Belgrade, Serbia.

- Lj. Davidović, B. Sazdović, Effective theories of two T-dual theories are also T-dual, Eur. Phys. J. **C 79** (2019) 770.

- Lj. Davidović, B. Nikolić, B. Sazdović, Canonical approach to the closed string noncommutativity, Facta Universitatis, Series: Physics, Chemistry and Technology, Vol 12, No 2, Special Issue, 2014.

- B. Nikolić, D. Obrić, Noncommutativity and nonassociativity of closed bosonic string on T-dual toroidal backgrounds, Fortsch.Phys. 66 (2018) 040009.

- B. Nikolić, B. Sazdović, Hamiltonian approach to Dp-brane noncommutativity, Publ. Astron. Obs. Belgrade 88 (2010) 61-69.

- B. Nikolić and B. Sazdović, *Type I superstring theory in the form of open type IIB with appropriate choice of boundary conditions*, *IJMP* **A24** (2009) 2857.

- Lj. Davidović, B. Nikolić, B. Sazdović, Closed string noncommutativity in the weakly curved background, 8th Mathematical Physics Meeting : Summer School and Conference on Modern Mathematical Physics. 24-31 Aug 2014. Belgrade, Serbia.

- B. Nikolić, B. Sazdović, T-duality and noncommutativity in type IIB superstring theory, 6th Summer School in Modern Mathematical Physics (MPHYS6) 14-23 Sep 2010. Belgrade, Serbia.

- Lj. Davidović, B. Sazdović, Curved Dp-brane in curved background by canonical methods, 6th Summer School in Modern Mathematical Physics (MPHYS6) 14-23 Sep 2010. Belgrade, Serbia.

- Lj. Davidović, B. Nikolić, B. Sazdović, Canonical approach to the closed string non-commutativity, Eur.Phys.J. C74 (2014) no.1, 2734.

- Lj. Davidović and B. Sazdović, Non-commutativity parametrs depend not only on the effective coordinates but on its T-dual as well, *JHEP* **08** (2011) 112.

- Lj. Davidović and B. Sazdović, T-dual-coordinate dependence makes the effective Kalb-Ramond field nontrivial, 7th Mathematical Physics Meeting: Summer School and Conference on Modern Mathematical Physics (MPHYS7) 9-19 September 2012., Belgrade, Serbia,arxiv: 1105.2809.

- B. Nikolić and B. Sazdović, Fermionic T-duality and momenta noncommutativity, *Phys. Rev.* **D 84** (2011) 065012.

- B. Nikolić and B. Sazdović, Noncommutativity relations in type IIB theory and their supersymmetry, *JHEP* **08** (2010) 037.

- Lj. Davidović and B. Sazdović, Noncommutativity in weakly curved background by canonical methods, *Phys. Rev.* **D 83** (2011) 066014.

- B. Nikolić and B. Sazdović, $D5$-brane type I superstring background fields in terms of type IIB ones by canonical method and T-duality approach, Nucl. Phys. **B 836** (2010) 100–126.

- B. Nikolić, B. Sazdović, Improved relations between type I and type IIB background fields, PROCEEDINGS. Edited by B. Dragovich, Z. Rakic. Belgrade, Inst. Phys. Belgrade, 2009. 513p. (SFIN Ser. A: Conferences; A1 (2009))

- B. Nikolić and B. Sazdović, Central charge contribution to noncommutativity, *Fortschr. Phys.* **56**, 491 (2008).

- B. Nikolić and B. Sazdović, Type I background fields in terms of type IIB ones, *Phys. Lett.* **B 666** (2008) 400.

- B. Nikolić and B. Sazdović, Noncommutativity in space-time extended by Liouville field, *Adv. Theor. Math. Phys.* **14** (2010) 1.

- B. Nikolić, B. Sazdović, *Dirichlet boundary conditions in type IIB superstring theory and fermionic T-duality*, *JHEP* **06** (2012) 101.

- Lj. Davidović,B. Sazdović, *Nongeometric background arising in the solution of Neumann boundary conditions*, Eur. Phys. J. **C 72** (2012) 2199.

- D. S. Popović, B. Sazdović, *Canonical approach to noncommutative gauge theory*, *Phys.Lett.* **B683** (2010) 349-353.

- B. Nikolić, B. Sazdović, Open string boundary conditions dependence on the background fields, SFIN A1 (2007) 327-334.

(3) B. Nikolić and B. Sazdović, Type I background fields in terms of type IIB ones, *Phys. Lett.* **B666** (2008) 400, citiran je u:

- B. Nikolić, B. Sazdović, Hamiltonian approach to Dp-brane noncommutativity, Publ. Astron. Obs. Belgrade 88 (2010) 61-69.

- B. Nikolić, B. Sazdović, Fermionic T-duality in fermionic double space, Nucl.Phys. B917 (2017) 105-121.

- B. Nikolić, B. Sazdović, T-dualization of type II superstring theory in double space, Eur.Phys.J. C77 (2017) no.3, 197.

- Lj. Davidović and B. Sazdović, Non-commutativity parametrs depend not only on the effective coordinates but on its T-dual as well, *JHEP* **08** (2011) 112.

- B. Nikolić and B. Sazdović, Fermionic T-duality and momenta noncommutativity, *Phys. Rev.* **D 84** (2011) 065012.

- B. Nikolić and B. Sazdović, Noncommutativity relations in type IIB theory and their supersymmetry, *JHEP* **08** (2010) 037.

- Lj. Davidović and B. Sazdović, Noncommutativity in weakly curved background by canonical methods, *Phys. Rev.* **D 83** (2011) 066014.

- B. Nikolić and B. Sazdović, $D5$-brane type I superstring background fieldsin terms of type IIB ones by canonical method and T-duality approach, Nucl. Phys. **B 836** (2010) 100–126.

- B. Nikolić, B. Sazdović, Dirichlet boundary conditions in type IIB superstring theory and fermionic T-duality, *JHEP* **06** (2012) 101.

- Lj. Davidović and B. Sazdović, T-dual-coordinate dependence makes the effective Kalb-Ramond field nontrivial, 7th Mathematical Physics Meeting: Summer School and Conference on Modern Mathematical Physics (MPHYS7) 9-19 September 2012., Belgrade, Serbia, arXiv:1105.2809.

- B. Nikolić, B. Sazdović, T-duality and noncommutativity in type IIB superstring theory, 6th Summer School in Modern Mathematical Physics (MPHYS6) 14-23 Sep 2010. Belgrade, Serbia.

- B. Nikolić, B. Sazdović, Improved relations between type I and type IIB background fields, PROCEEDINGS. Edited by B. Dragovich, Z. Rakic. Belgrade, Inst. Phys. Belgrade, 2009. 513p. (SFIN Ser. A: Conferences; A1 (2009)).

- B. Nikolić and B. Sazdović, *Type I superstring theory in the form of open type IIB with appropriate choice of boundary conditions*, *IJMP* **A24** (2009) 2857.

(4) B. Nikolić, B. Sazdović, D5-brane type I superstring background fields in terms of type IIB ones by canonical method and T-duality approach, *Nucl. Phys.* **B 836** (2010) 100–126. citiran je u:

- B. Nikolic, B. Sazdovic, Advantage of the second-order formalism in double space T-dualization of type II superstring, Eur. Phys. J. C79 (2019) 819.

- B Nikolic, D Obric, B Sazdovic, Noncommutativity and nonassociativity of type II superstring with coordinate dependent RR field, Fortschr. Phys. 70 (2022) 2200048.

- B. Nikolic, D. Obric, From 3D torus with H-flux to torus with R-flux and back, Proceedings of the 10th MATHEMATICAL PHYSICS MEETING: School and Conference on Modern Mathematical Physics : Belgrade, Serbia, September 9 - 14, 2019, 233-250 (2020).

- B. Nikolic, D. Obric, Representation of T-duality of type II pure spinor superstring in double space, Proceedings of the 9th MATHEMATICAL PHYSICS MEETING: School and Conference on Modern Mathematical Physics, September 18 – 23, 2017, Belgrade, Serbia, SFIN XXXI Series A: Conferences No. A1 (2018).

- B. Nikolić, D. Obrić, Noncommutativity and nonassociativity of closed bosonic string on T-dual toroidal backgrounds, Fortsch.Phys. 66 (2018) 040009.

- Lj. Davidović, Open string T-duality in a weakly curved background,Eur.Phys.J. C76 (2016) no.12, 660.

- B. Nikolić, B. Sazdović, Fermionic T-duality in fermionic double space, Nucl.Phys. B917 (2017) 105-121.
- B. Nikolić, B. Sazdović, T-dualization of type II superstring theory in double space, Eur.Phys.J. C77 (2017) no.3, 197.
- B. Sazdović, T-duality as coordinates permutation in double space for weakly curved background, JHEP 1508 (2015) 055.
- B. Nikolić, B. Sazdović, T-duality and noncommutativity in type IIB superstring theory, 6th Summer School in Modern Mathematical Physics (MPHYS6) 14-23 Sep 2010. Belgrade, Serbia
- B. Sazdović, T-duality as permutation of coordinates in double space, Chin.Phys. C41 (2017) no.5, 053101.
- Lj. Davidović, B. Nikolić, B. Sazdović, T-duality diagram for a weakly curved background, Eur.Phys.J. C75 (2015) no.12, 576.
- Lj. Davidović, B. Sazdović, T-duality in a weakly curved background, Eur.Phys.J. C74 (2014) no.1, 2683.
- Lj. Davidović and B. Sazdović, Non-commutativity parametrs depend not only on the effective coordinates but on its T-dual as well, *JHEP* **08** (2011) 112.
- B. Nikolić and B. Sazdović, Fermionic T-duality and momenta noncommutativity, *Phys. Rev.* **D 84** (2011) 065012.
- B. Nikolić and B. Sazdović, Noncommutativity relations in type IIB theory and their supersymmetry, *JHEP* **08** (2010) 037.
- Lj. Davidović and B. Sazdović, Noncommutativity in weakly curved background by canonical methods, *Phys. Rev.* **D 83** (2011) 066014.
- B. Nikolić, B. Sazdović, *Dirichlet boundary conditions in type IIB superstring theory and fermionic T-duality*, *JHEP* **06** (2012) 101.
- Lj. Davidović and B. Sazdović, T-dual-coordinate dependence makes the effective Kalb-Ramond field nontrivial, 7th Mathematical Physics Meeting: Summer School and Conference on Modern Mathematical Physics (MPHYS7) 9-19 September 2012., Belgrade, Serbia, arXiv:1105.2809.

(5) B. Nikolić and B. Sazdović, Noncommutativity relations in type IIB theory and their supersymmetry, *JHEP* **08** (2010) 037, citiran je u:

- B. Nikolić, B. Sazdović, Fermionic T-duality in fermionic double space, Nucl.Phys. B917 (2017) 105-121.

- M. Dimitrijević, Bilj. Nikolić, V. Radovanović, Non(Anti)commutative Field Theories: Model Building and Renormalizability Properties, Ann.U.Craiova Phys. 21 S18-S27.

- B. Nikolić, B. Sazdović, T-duality and noncommutativity in type IIB superstring theory, 6th Summer School in Modern Mathematical Physics (MPHYS6) 14-23 Sep 2010. Belgrade, Serbia.

- Lj. Davidović and B. Sazdović, Non-commutativity parametrs depend not only on the effective coordinates but on its T-dual as well, *JHEP* **08** (2011) 112.

- B. Nikolić and B. Sazdović, Fermionic T-duality and momenta noncommutativity, *Phys. Rev.* **D 84** (2011) 065012.

- M. Dimitrijević, Biljana Nikolić and V. Radovnović, Twisted SUSY: Twisted symmetry versus renormalizability, *Phys. Rev.* **D 83** (2011) 065010.

- Lj. Davidović and B. Sazdović, T-dual-coordinate dependence makes the effective Kalb-Ramond field nontrivial, 7th Mathematical Physics Meeting: Summer School and Conference on Modern Mathematical Physics (MPHYS7) 9-19 September 2012., Belgrade, Serbia, arXiv:1105.2809.

- B. Nikolić, B. Sazdović, Dirichlet boundary conditions in type IIB superstring theory and fermionic T-duality, *JHEP* **06** (2012) 101.

- M. Dimitrijević, Bilj. Nikolić, V. Radovanović, Non(anti)commutative field theories: model building and renormalizability, 6th Summer School in Modern Mathematical Physics (MPHYS6) 14-23 Sep 2010. Belgrade, Serbia.

(6) B. Nikolić and B. Sazdović, Noncommutativity in space-time extended by Liouville field, *Adv. Theor. Math. Phys.* **14** (2010) 1, citiran je u:

- B. Nikolic, D. Obric, Noncommutativity and nonassociativity of type II superstring with coordinate dependent RR field—the general case, JHEP **12** (2022) 078.

- Lj. Davidovic, B. Sazdovic, T-duality between effective string theories, Proceedings of the 10th MATHEMATICAL PHYSICS MEETING: School and Conference on Modern Mathematical Physics September 9 – 14, 2019, Belgrade, Serbia, SFIN XXXIII Series A: Conferences, No. A1 (2020).

- Lj. Davidovic, B. Sazdovic, Effective theories of two T-dual theories are also T-dual, Eur. Phys. J. C 79, 770 (2019).
- Lj. Davidović, B. Nikolić, B. Sazdović, Closed string noncommutativity in the weakly curved background, 8th Mathematical Physics Meeting : Summer School and Conference on Modern Mathematical Physics. 24-31 Aug 2014. Belgrade, Serbia.
- B. Nikolić, B. Sazdović, T-duality and noncommutativity in type IIB superstring theory, 6th Summer School in Modern Mathematical Physics (MPHYS6) 14-23 Sep 2010. Belgrade, Serbia.
- Lj. Davidović, B. Sazdović, Curved Dp-brane in curved background by canonical methods, 6th Summer School in Modern Mathematical Physics (MPHYS6) 14-23 Sep 2010. Belgrade, Serbia.
- Lj. Davidović, B. Nikolić, B. Sazdović, Canonical approach to the closed string noncommutativity, FACTA UNIVERSITATIS Physics, Chemistry and Technology Vol. 12, No2 , Special Issue, 2014, pp. 101–110.
- Lj. Davidović, B. Nikolić, B. Sazdović, Canonical approach to the closed string non-commutativity, Eur.Phys.J. C74 (2014) no.1, 2734.
- H. Sati, Framed M-branes, corners, and topological invariants, J. Math. Phys. 59 (2018) 6.
- D. Fiorenza, H. Sati, U. Schreiber, Extended higher cup-product Chern-Simons theories, J. Geom. Phys. 74 (2013) 130-163.
- Lj. Davidović and B. Sazdović, Non-commutativity parametrs depend not only on the effective coordinates but on its T-dual as well, *JHEP* **08** (2011) 112.
- B. Nikolić and B. Sazdović, Fermionic T-duality and momenta noncommutativity, *Phys. Rev.* **D 84** (2011) 065012.
- B. Nikolić and B. Sazdović, Noncommutativity relations in type IIB theory and their supersymmetry, *JHEP* **08** (2010) 037.
- H. Sati, M-theory with framed corners and tertiary index invariants, SIGMA 10 (2014) 024.
- Lj. Davidović and B. Sazdović, Noncommutativity in weakly curved background by canonical methods, *Phys. Rev.* **D 83** (2011) 066014.
- D. Fiorenza, H. Sati, U. Schreiber, The E8 moduli 3-stack of the C-field in M-theory, Commun. Math. Phys. 333 (2015) 1, 117-151.

- I. G. Avramidi, S. Collopy, Thermal Yang-Mills Theory In the Einstein Universe, J. Phys. A 45 (2012) 374009.
- H. Sati, Corners in M-theory, J. Phys. A 44 (2011) 255402.
- H. Sati, M-theory, the signature theorem, and geometric invariants, Phys.Rev.D 83 (2011) 126010.
- I. Avramidi, S, Collopy, Effective Action and Phase Transitions in Thermal Yang-Mills Theory on Spheres, Commun. Math. Phys. 311 (2012) 713-753.
- H. Sati, Duality and Cohomology in M-Theory with Boundary, J. Geom. Phys. 62 (2012) 1284-1297.
- B. Pozsgay, G. Takacs, Form factor expansion for thermal correlators, J. Stat. Mech. 1011 (2010) P11012.
- G. Takacs, Form factor perturbation theory from finite volume, Nucl.Phys.B 825 (2010) 466-481.
- B. Nikolić and B. Sazdović, $D5$-brane type I superstring background fieldsin terms of type IIB ones by canonical method and T-duality approach, *Nucl. Phys.* **B 836** (2010) 100–126.
- B. Nikolić and B. Sazdović, Central charge contribution to noncommutativity, *Fortschr. Phys.* **56**, 491 (2008).
- B. Nikolić and B. Sazdović, Type I background fields in terms of type IIB ones, *Phys. Lett.* **B666** (2008) 400.
- B. Nikolić, B. Sazdović, Dirichlet boundary conditions in type IIB superstring theory and fermionic T-duality, *JHEP* **06** (2012) 101.
- Lj. Davidović,B. Sazdović, Nongeometric background arising in the solution of Neumann boundary conditions, Eur. Phys. J. **C 72** (2012) 2199.
- Lj. Davidović and B. Sazdović, T-dual-coordinate dependence makes the effective Kalb-Ramond field nontrivial, 7th Mathematical Physics Meeting: Summer School and Conference on Modern Mathematical Physics (MPHYS7) 9-19 September 2012., Belgrade, Serbia, arXiv:1105.2809.
- B. Nikolić, B. Sazdović, Improved relations between type I and type IIB background fields, PROCEEDINGS. Edited by B. Dragovich, Z. Rakic. Belgrade, Inst. Phys. Belgrade, 2009. 513p. (SFIN Ser. A: Conferences; A1 (2009)).

- B. Nikolić and B. Sazdović, Type I superstring theory in the form of open type IIB with appropriate choice of boundary conditions, *IJMP* **A24** (2009) 2857.

(7) B. Nikolić and B. Sazdović, *Fermionic T-duality and momenta noncommutativity*, *Phys. Rev.* **D 84** (2011) 065012, citiran je u:

- B. Nikolic, B. Sazdovic, Representation of T-duality of type II pure spinor superstring in double space, : 9th MATHEMATICAL PHYSICS MEETING: SCHOOL AND CONFERENCE ON MODERN MATHEMATICAL PHYSICS, SFIN XXXI Series A: Conferences No. A1 (2018).

- Lj. Davidović, Open string T-duality in a weakly curved background, Eur.Phys.J. C76 (2016) no.12, 660.

- B. Nikolić, B. Sazdović, Fermionic T-duality in fermionic double space, Nucl.Phys. B917 (2017) 105-121.

- B. Nikolić, B. Sazdović, T-dualization of type II superstring theory in double space, Eur.Phys.J. C77 (2017) no.3, 197.

- Lj. Davidović, B. Nikolić, B. Sazdović, T-duality diagram for a weakly curved background, Eur.Phys.J. C75 (2015) no.12, 576.

- Ilya Bakhmatov, Fermionic T-duality and U-duality in type IIB supergravity, doktorska teza odbranjena na Queen Mary, University of London, rukovodilac prof. dr David Berman, arxiv:1112.1983.

- D. Andriot, M. Larfors, D. Lüst, P. Patalong, (Non) commutative closed string on T-dual toroidal backgrounds, JHEP 06 (2013) 021.

- I. Bakhmatov, E. O. Colgain, H. Yavartanoo, Fermionic T-duality in the pp-wave limit, *JHEP* **10** (2011) 085.

- B. Nikolić, B. Sazdović, Dirichlet boundary conditions in type IIB superstring theory and fermionic T-duality, *JHEP* **06** (2012) 101.

- Lj. Davidović, B. Sazdović, T-duality in the weakly curved background, Eur.Phys.J. C74 (2014) no.1, 2683, arxiv:1205.1991.

- E. O. Colgain, Fermionic T-duality: A snapshot review, IJMP **A27** (2012) 1230032.

(8) B. Nikolić, B. Sazdović, *Dirichlet boundary conditions in type IIB superstring theory and fermionic T-duality*, *JHEP* **06** (2012) 101, citiran je u:

- B. Nikolić, B. Sazdović, Fermionic T-duality in fermionic double space, Nucl.Phys. B917 (2017) 105-121.

- B. Nikolić, B. Sazdović, T-dualization of type II superstring theory in double space, Eur.Phys.J. C77 (2017) no.3, 197.

- Lj. Davidović, B. Nikolić, B. Sazdović, T-duality diagram for a weakly curved background, Eur.Phys.J. C75 (2015) no.12, 576.

- E. O. Colgain, *Fermionic T-duality: A snapshot review*, IJMP **A27** (2012) 1230032.

- E. O. Colgain, *Self-duality of the D1-D5 near-horizon*, *JHEP* **04** (2012) 047.

(9) Lj. Davidović, B. Nikolić, B. Sazdović, *Canonical approach to the closed string noncommutativity*, *Eur. Phys. J* **C74** (2014) 2734, citiran je u:

- B. Nikolic, D. Obric, Combined Fermionic and Bosonic T-duality of Type II Superstring Theory with Coordinate Dependent RR Field, Fortschr. Phys. 71 (2023) 2200160.

- B. Nikolic, D. Obric, B. Sazdovic, Noncommutativity and Nonassociativity of Type II Superstring with Coordinate Dependent RR Field, Fortschr. Phys. 70 (2022) 2200048.

- B. Nikolic, D. Obric, Noncommutativity and nonassociativity of type II superstring with coordinate dependent RR field—the general case, JHEP **12** (2022) 078.

- Lj. Davidovic, I. Ivanisevic, B. Sazdovic, Courant bracket as T-dual invariant extension of Lie bracket, JHEP 03 (2021) 109.

- B. Nikolic, D. Obric, Directly from H-flux to the family of three nonlocal R-flux theories, JHEP 03 (2019) 136.

- B. Nikolic, B. Sazdovic, Advantage of the second-order formalism in double space T-dualization of type II superstring, Eur. Phys. J. C 79 (2019) 819.

- B. Sazdovic, T-duality in Superstring Theory, Proceedings of the 1st CONFERENCE ON NONLINEARITY, October 11 – 12, 2019, Belgrade, Serbia.

- B. Nikolic, B. Sazdovic, Representation of T-duality of type II pure spinor superstring in double space, Proceedings of the 9th MATHEMATICAL PHYSICS MEETING: School and Conference on Modern Mathematical Physics, September 18 – 23, 2017, Belgrade, Serbia, SFIN XXXI Series A: Conferences No. A1 (2018).

- B. Nikolic, D. Obric, Noncommutativity and Nonassociativity of Closed Bosonic String on T-dual Toroidal Backgrounds, Fortschr. Phys. 66 (2018) 1800009.
- Lj. Davidović, B. Sazdović, The T-dual symmetries of a bosonic string, Eur. Phys. J. C78 (2018) 600.
- R. J. Szabo, Higher Quantum Geometry and Non-Geometric String Theory, 17th Hellenic School and Workshops on Elementary Particle Physics and Gravity (CORFU2017) 02-28 Sep 2017. Corfu, Greece.
- B. Nikolic, B. Sazdovic, Fermionic T-duality in fermionic double space, Nucl. Phys. B 917 (2017) 105–121.
- Lj. Davidović, B. Nikolić, B. Sazdović, T-dualization of a weakly curved background, J.Phys.Conf.Ser. 804 (2017) no.1, 012014.
- B. Sazdović, From geometry to non-geometry via T-duality, Chinese Phys. C42 (2018) 083106.
- Lj. Davidović, B. Nikolić, B. Sazdović, Complete T-Dualization of a String in a Weakly Curved Background, Springer Proc.Math.Stat. 111 (2014) 13-20.
- Lj. Davidović, B. Sazdović, T-dualization in a curved background in absence of a global symmetry, JHEP 1511 (2015) 119.
- B. Nikolić, B. Sazdović, T-dualization of type II superstring theory in double space, Eur.Phys.J. C77 (2017) no.3, 197.
- I. Bakas, D. Luest, T-duality, Quotients and Currents for Non-Geometric Closed Strings, Fortsch.Phys. 63 (2015) 543-570.
- B. Sazdović, T-duality as coordinates permutation in double space for weakly curved background, JHEP 1508 (2015) 055.
- B. Sazdović, T-duality as permutation of coordinates in double space, Chin.Phys. C41 (2017) no.5, 053101.
- C. D. A. Blair, E. Malek, Geometry and fluxes of SL(5) exceptional field theory, JHEP 1503 (2015) 144.
- Lj. Davidović, B. Nikolić, B. Sazdović, T-duality diagram for a weakly curved background, Eur.Phys.J. C75 (2015) no.12, 576.

- Lj. Davidović, B. Nikolić, B. Sazdović, Canonical approach to the closed string noncommutativity, FACTA UNIVERSITATIS Physics, Chemistry and Technology Vol. 12, No2 , Special Issue, 2014, pp. 101–110.

(10) Lj. Davidović, B. Nikolić, B. Sazdović, *T-duality diagram for a weakly curved background*, *Eur. Phys. J* **C75** (2015) 576, citiran je u:

- B. Nikolic, D. Obric, Noncommutativity and nonassociativity of type II superstring with coordinate dependent RR field—the general case, JHEP 12 (2022) 078.
- B. Nikolic, D. Obric, Combined Fermionic and Bosonic T-duality of Type II Superstring Theory with Coordinate Dependent RR Field, Fortschr. Phys. 71 (2023) 2200160.
- B Nikolic, D Obric, B Sazdovic, Noncommutativity and nonassociativity of type II superstring with coordinate dependent RR field, Fortschr. Phys. 70 (2022) 2200048.
- I. Ivanisević, Lj. Davidovic, B. Sazdovic, Courant bracket found out to be T-dual to Roytenberg bracket, Eur. Phys. J. C80 (2020) 571.
- B. Nikolic, D. Obric, From 3D torus with H-flux to torus with R-flux and back, Proceedings, 10th MATHEMATICAL PHYSICS MEETING: School and Conference on Modern Mathematical Physics : Belgrade, Serbia, September 9 - 14, 2019, SFIN XXXIII Series A: Conferences, No. A1 (2020), 233-250.
- B. Nikolic, D. Obric, Directly from H-flux to the family of three nonlocal R-flux theories, JHEP 03 (2019) 136.
- B. Nikolic, B. Sazdovic, Advantage of the second-order formalism in double space T-dualization of type II superstring, Eur. Phys. J. C79 (2019) 819.
- B. Nikolic, B. Sazdovic, Representation of T-duality of type II pure spinor superstring in double space, 9th MATHEMATICAL PHYSICS MEETING: SCHOOL AND CONFERENCE ON MODERN MATHEMATICAL PHYSICS, SFIN XXXI Series A: Conferences No. A1 (2018), 281-300.
- Lj. Davidovic, B. Sazdovic, Symmetries of a bosonic string, 9th MATHEMATICAL PHYSICS MEETING: SCHOOL AND CONFERENCE ON MODERN MATHEMATICAL PHYSICS, SFIN XXXI Series A: Conferences No. A1 (2018), 147-158.
- B. Sazdovic, T-duality and nongeometry, 9th MATHEMATICAL PHYSICS MEETING: SCHOOL AND CONFERENCE ON MODERN MATHEMATICAL PHYSICS, SFIN XXXI Series A: Conferences No. A1 (2018), 301-308.

- Lj. Davidović, B. Sazdović, The T-dual symmetries of a bosonic string, Eur. Phys. J. C78 (2018) 600.
- B. Nikolić, D. Obrić, Noncommutativity and nonassociativity of closed bosonic string on T-dual toroidal backgrounds, Fortsch. Phys. 66 (2018) 040009.
- Lj. Davidović, B. Nikolić, B. Sazdović, T-dualization of a weakly curved background, J.Phys.Conf.Ser. 804 (2017) no.1, 012014.
- Lj. Davidović, B. Nikolić, B. Sazdović, Weakly curved background T-duals, 8th Mathematical Physics Meeting : Summer School and Conference on Modern Mathematical Physics, 24-31 Aug 2014. Belgrade, Serbia
- B. Sazdović, From geometry to non-geometry via T-duality, Chin. Phys. C42 (2018) 083106.
- Lj. Davidović, Open string T-duality in a weakly curved background, Eur.Phys.J. C76 (2016) 660.
- Lj. Davidović, B. Sazdović, T-dualization in a curved background in absence of a global symmetry, JHEP 1511 (2015) 119.
- B. Nikolić, B. Sazdović, T-dualization of type II superstring theory in double space, Eur.Phys.J. C77 (2017) 197.
- B. Sazdović, T-duality as coordinates permutation in double space for weakly curved background, JHEP 1508 (2015) 055.
- B. Sazdović, T-duality as permutation of coordinates in double space, Chin.Phys. C41 (2017) no.5, 053101.

(11) B. Nikolić and B. Sazdović, Fermionic T-duality in fermionic double space, *Nucl. Phys.* **B917** (2017) 105-121, citiran je u:

- D. Butter, F. Hassler, C. N. Pope, H. Zhang, Generalized dualities and supergroups, JHEP 12 (2023) 052.
- B. Nikolic, D. Obric, Combined Fermionic and Bosonic T-duality of Type II Superstring Theory with Coordinate Dependent RR Field, Fort. Phys. 71 (2023) 2200160.
- B. Sazdovic, T-duality as permutation of coordinates in double space, Chin. Phys. C41 (2017) 5, 053101.
- B. Sazdovic, T-duality in Superstring Theory, Proceedings of the 1st CONFERENCE ON NONLINEARITY, October 11 – 12, 2019, Belgrade, Serbia.

- B. Sazdović, Open string T-duality in double space, Eur.Phys.J. C77 (2017) no.9, 634.

(12) B. Nikolić and B. Sazdović, *T-dualization of type II superstring theory in double space*, *Eur. Phys. J.* **C77** (2017) 197, citiran je u:

- Dmitry Kondratyev, Search for the Higgs boson decaying to a pair of muons with the CMS experiment at the Large Hadron Collider, CERN thesis-2022-260.
- B. Nikolic, B. Sazdovic, Advantage of the second-order formalism in double space T-dualization of type II superstring, Eur. Phys. J. C79 (2019) 819.
- B. Sazdovic, T-duality as permutation of coordinates in double space, Chin. Phys. C41 (2017) 5, 053101.
- B. Sazdović, Open string T-duality in double space, Eur.Phys.J. C77 (2017) no.9, 634.
- D. Florenza, H. Sati, U. Schreiber, T-Duality from super Lie n-algebra cocycles for super p-branes, Adv. Theor. Math. Phys. 22 (2018) 1209-1270.
- B. Nikolić and B. Sazdović, Fermionic T-duality in fermionic double space, *Nucl. Phys.* **B917** (2017) 105-121
- Lj. Davidović, B. Sazdović, T-dualization in a curved background in absence of a global symmetry, JHEP 1511 (2015) 119.
- B. Nikolic, B. Sazdovic, Simultaneous T-dualization of type II pure spinor superstring, Eur.Phys.J.C 78 (2018) 1030.

(13) B. Nikolić and D. Obrić, *Noncommutativity and nonassociativity of closed bosonic string on T-dual toroidal background*, *Fortsch. Phys.* **66** (2018) 040009, citiran je u:

- B. Nikolic, B. Sazdovic, D. Obric, Noncommutativity and nonassociativity of type II superstring with coordinate dependent RR field, Fortschr. Phys. 70 (2022) 2200048.
- B. Nikolic, D. Obric, Noncommutativity and nonassociativity of type II superstring with coordinate dependent RR field—the general case, JHEP 12 (2022) 078.
- R. J. Szabo, Higher Quantum Geometry and Non-Geometric String Theory, Proceedings of the 17th Hellenic School and Workshops on Elementary Particle Physics and Gravity (CORFU2017) : Corfu, Greece, September 2-28, 2017, PoS CORFU2017 (2018) 151.

- B. Nikolic, D. Obric, Directly from H-flux to the family of three nonlocal R-flux theories, JHEP 03 (2019) 136.
- B. Nikolic, B. Sazdovic, From 3D torus with H-flux to torus with R-flux and back, Proceedings, 10th MATHEMATICAL PHYSICS MEETING: School and Conference on Modern Mathematical Physics : Belgrade, Serbia, September 9 - 14, 2019, 233-250, SFIN XXXIII Series A: Conferences, No. A1 (2020).

(14) B. Nikolić, B. Sazdović, *Advantage of the second-order formalism in double space T-dualization of type II superstring*, Eur. Phys. J. C 79 (2019) 819, citiran je u:

- D. S. Berman, C. Blair, The geometry, branes and applications of exceptional field theory, Int. J. Mod. Phys. A35 (2020) 30, 2030014.
- M. Hatsuda, W. Siegel, T-dual Superstring Lagrangian with double zweibeins, JHEP 03 (2020) 058.

(15) B. Nikolić, D. Obrić, *Directly from H-flux to the family of three nonlocal R-flux theories*, JHEP 03 (2019) 136, citiran je u:

- B. Nikolic, D. Obric, Noncommutativity and nonassociativity of type II superstring with coordinate dependent RR field—the general case, JHEP 12 (2022) 078.
- B. Nikolic, D. Obric, Combined Fermionic and Bosonic T-duality of Type II Superstring Theory with Coordinate Dependent RR Field, Fort. Phys. 71 (2023) 2200160.
- B. Nikolic, B. Sazdovic, D. Obric, Noncommutativity and nonassociativity of type II superstring with coordinate dependent RR field, Fortschr. Phys. 70 (2022) 2200048.

(16) B. Nikolić, B. Sazdović, D. Obrić, *Noncommutativity and nonassociativity of type II superstring with coordinate dependent RR field*, Fortschr. Phys. 70 (2022) 2200048, citiran je u:

- B. Nikolic, D. Obric, Noncommutativity and nonassociativity of type II superstring with coordinate dependent RR field—the general case, JHEP 12 (2022) 078.
- B. Nikolic, D. Obric, Combined Fermionic and Bosonic T-duality of Type II Superstring Theory with Coordinate Dependent RR Field, Fort. Phys. 71 (2023) 2200160.

(17) B. Nikolić, D. Obrić, *Combined Fermionic and Bosonic T-duality of Type II Superstring Theory with Coordinate Dependent RR Field*, Fort. Phys. 71 (2023) 2200160, citiran je u:

- D. Obric, T-dualizacija bozonske strune i tip IIB superstrune u prisustvu koorinatno zavisnih pozadinskih polja, doktorska disertacija, Beograd, 2023.

(18) B. Nikolić, D. Obrić, *Noncommutativity and nonassociativity of type II superstring with coordinate dependent RR field—the general case*, JHEP 12 (2022) 078, citiran je u:

- D. Obric, T-dualizacija bozonske strune i tip IIB superstrune u prisustvu koorinatno zavisnih pozadinskih polja, doktorska disertacija, Beograd, 2023.

Beograd, 21. februar 2024. godine

dr Bojan Nikolić

Bojan Nikolic (Author)
Timespan:1996-2024.

Results fo	24
Sum of the	141
Average C	5.88
h-index	7

Title	Authors	Corporate	Editors	Book Edito	Source Titl	Publicatior	Publicatior	Volume	Issue
Gauge syn	Nikolic, B.; Sazdovic, B.				PHYSICAL	AUG 2006	2006	74	4
Canonical	Davidovic, Lj; Nikolic, B.; Sazdovic, B.				EUROPEA	JAN 28 20	2014	74	1
Gauge syn	Nikolic, B.; Sazdovic, B.				PHYSICAL	APR 2007	2007	75	8
<i>D</i>5-	Nikolic, B.; Sazdovic, B.				NUCLEAR	SEP 1 201	2010	836	1-2
T-duality	Davidovic, Ljubica; Nikolic, Bojan; Sazdovic				EUROPEA	DEC 7 201	2015	75	12
Noncommu	Nikolic, Bojan; Sazdovic, Branislav				ADVANCE	JAN 2010	2010	14	1
Type I bac	Nikolic, B.; Sazdovic, B.				PHYSICS	SEP 4 200	2008	666	4
Fermionic	Nikolic, B.; Sazdovic, B.				PHYSICAL	SEP 12 20	2011	84	6
Noncommu	Nikolic, B.; Sazdovic, B.				JOURNAL	AUG 2010	2010		8
Fermionic	Nikolic, B.; Sazdovic, B.				NUCLEAR	APR 2017	2017	917	
T-dualizati	Nikolic, B.; Sazdovic, B.				EUROPEA	MAR 27 20	2017	77	3
Dirichlet b	Nikolic, B.; Sazdovic, B.				JOURNAL	JUN 2012	2012		6
Directly fr	Nikolic, B.; Obric, D.				JOURNAL	MAR 22 20	2019		3
Noncommu	Nikolic, B.; Obric, D.				FORTSCH	APR 2018	2018	66	4
Noncommu	Nikolic, B.; Obric, D.; Sazdovic, B.				FORTSCH	MAY 2022	2022	70	5
Advantage	Nikolic, B.; Sazdovic, B.				EUROPEA	OCT 2019	2019	79	10
Noncommu	Obric, D.; Nikolic, B.				JOURNAL	DEC 14 20	2022		12
Combined	Nikolic, B.; Obric, D.				FORTSCH	JAN 2023	2023	71	1
Simultane	Nikolic, B.; Sazdovic, B.				EUROPEA	DEC 27 20	2018	78	12
T-dualizat	Davidovic, Lj.; Nikolic, B.; Sazdovic, B.				XXIV INT	2017	2017	804	
NONCOMI	Davidovic, L. I.; Nikolic, B.; Sazdovic, B.				ROMANIA	2012	2012	57	5-6
SUPERSY	Nikolic, B.; Sazdovic, B.				ROMANIA	2012	2012	57	5-6
Central ch	Nikolic, Bojan; Sazdovic, Branislav				FORTSCH	APR-MAY	2008	56	4-5
From neun	Nikolic, B.; Sazdovic, B.			Cetin, SA;	SIX INTE	2007	2007	899	

Part Numb	Supplemer	Special Iss	Beginning	Ending Pa	Article Nu	DOI	Conferenc	Conferenc	Total Citat
					45024	10.1103/PhysRevD.74.045024			20
					2734	10.1140/epjc/s10052-014-2734-			18
					85011	10.1103/PhysRevD.75.085011			16
			100	126		10.1016/j.nuclphysb.2010.04.01			15
					576	10.1140/epjc/s10052-015-3808-			13
			1	27					12
			400	403		10.1016/j.physletb.2008.07.067			11
					65012	10.1103/PhysRevD.84.065012			7
					37	10.1007/JHEP08(2010)037			7
			105	121		10.1016/j.nuclphysb.2017.02.00			4
					197	10.1140/epjc/s10052-017-4758-			4
					101	10.1007/JHEP06(2012)101			4
					136	10.1007/JHEP03(2019)136			3
					1800009	10.1002/prop.201800009			3
					2200048	10.1002/prop.202200048			2
					819	10.1140/epjc/s10052-019-7338-			2
					78	10.1007/JHEP12(2022)078			0
						10.1002/prop.202200160			0
					1030	10.1140/epjc/s10052-018-6504-			0
					12014	10.1088/17	24th Inter	JUN 14-18	0
		SI	816	829			4th Southe	AUG 28-SI	0
		SI	931	937			4th Southe	AUG 28-SI	0
			491	501		10.1002/pr	3rd Southe	SEP 02-09	0
			151	+			6th Interna	AUG 22-26	0

Average pe	1996	1997	1998	1999	2000	2001	2002	2003	2004
1.05	0	0	0	0	0	0	0	0	0
1.64	0	0	0	0	0	0	0	0	0
0.89	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0
1.3	0	0	0	0	0	0	0	0	0
0.8	0	0	0	0	0	0	0	0	0
0.65	0	0	0	0	0	0	0	0	0
0.5	0	0	0	0	0	0	0	0	0
0.47	0	0	0	0	0	0	0	0	0
0.5	0	0	0	0	0	0	0	0	0
0.5	0	0	0	0	0	0	0	0	0
0.31	0	0	0	0	0	0	0	0	0
0.5	0	0	0	0	0	0	0	0	0
0.43	0	0	0	0	0	0	0	0	0
0.67	0	0	0	0	0	0	0	0	0
0.33	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

2005	2006	2007	2008	2009	2010	2011	2012	2013	2014
0	0	1	2	1	4	3	4	0	1
0	0	0	0	0	0	0	0	0	0
0	0	0	2	1	4	2	4	0	1
0	0	0	0	0	1	3	2	0	1
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	2	3	4	0	1
0	0	0	0	1	2	3	3	0	0
0	0	0	0	0	0	0	2	1	1
0	0	0	0	0	0	3	3	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

2015	2016	2017	2018	2019	2020	2021	2022	2023	2024
0	0	0	1	2	0	0	1	0	0
5	0	4	3	2	0	1	3	0	0
0	0	0	0	2	0	0	0	0	0
2	0	3	1	1	0	0	1	0	0
0	1	3	3	2	1	0	3	0	0
0	0	0	0	1	0	0	1	0	0
0	0	2	0	0	0	0	0	0	0
1	0	2	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0
0	0	2	0	0	0	0	1	1	0
0	0	2	1	1	0	0	0	0	0
1	0	2	0	0	0	0	0	0	0
0	0	0	0	0	0	0	3	0	0
0	0	0	0	1	0	0	2	0	0
0	0	0	0	0	0	0	2	0	0
0	0	0	0	0	2	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

Analyze author output

< Back to author details page

Export Print Email

Nikolić, Bojan
University of Belgrade, Belgrade, Serbia
Author ID:16234167200

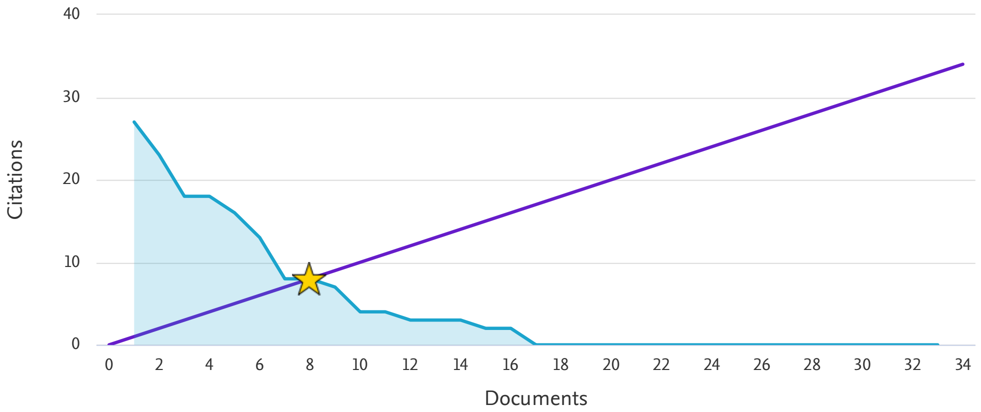
Analyze documents published between: 2006 to 2025

Exclude self citations Exclude citations from books Update Graph

Documents	Citations	Title
1	27	Gauge symmetri...
2	23	Gauge symmetri...
3	18	Canonical approa...
4	18	D5-brane type I s...
5	16	Noncommutativi...
6	13	Type I backgroun...
7	8	Fermionic T duali...
8	8	Noncommutativi...
9	7	T-duality diagra...

This author's *h*-index

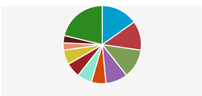
The *h*-index is based upon the number of documents and number of citations.



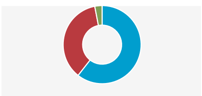
Click on cards below to see additional data.

Documents

by source

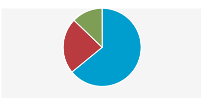


by type

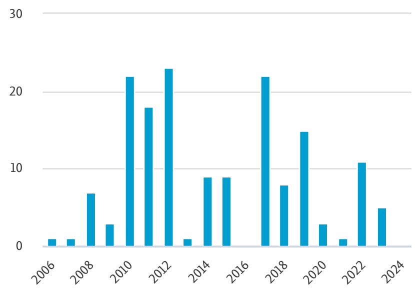


by year

by subject



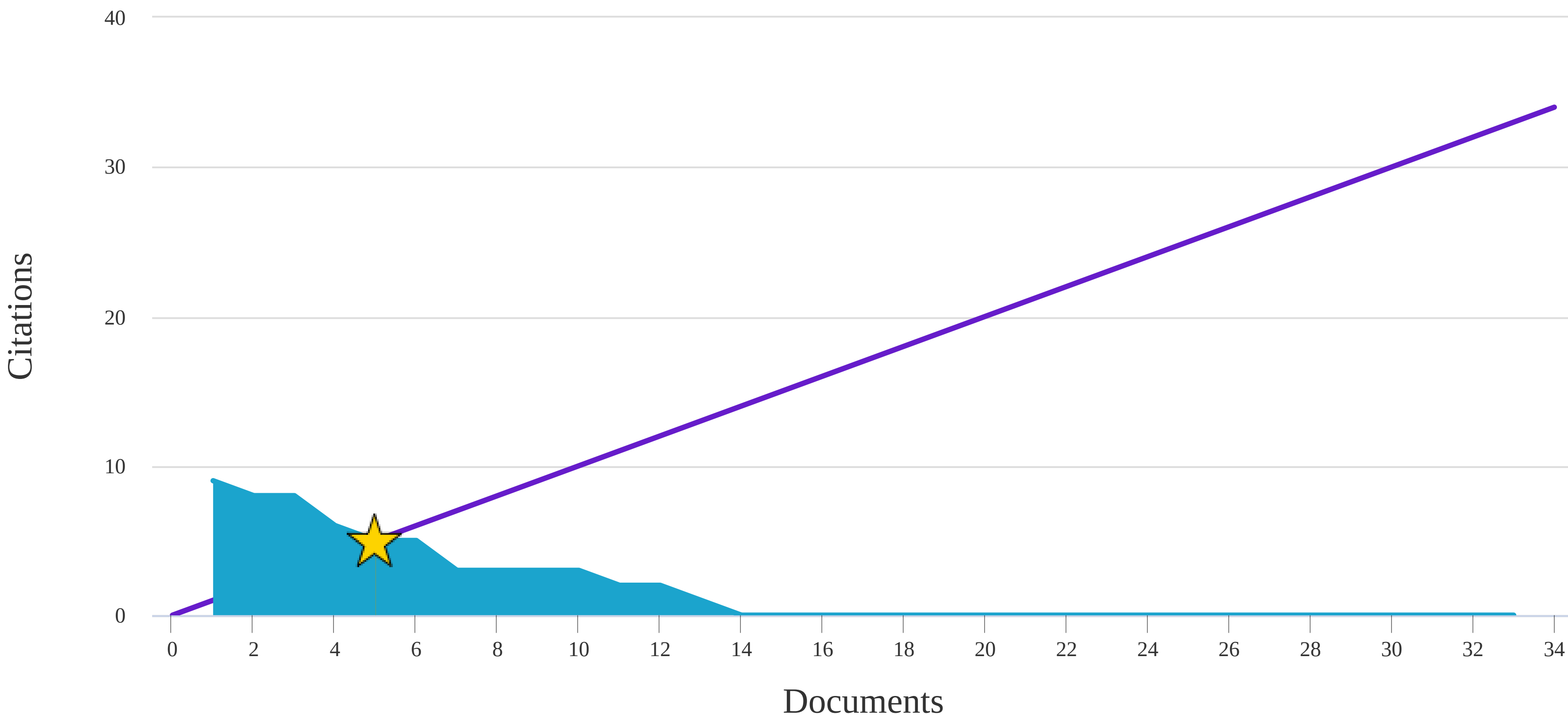
Citations 159



6 co-authors

Author Name	Co-authored Documents
Sazdović, Branislav T.	27
Obrić, Danijel	7
Davidović, Ljubica D.	7
Salom, Igor	1
Vojinović, Marko	1

The h-index is based upon the number of documents and number of citations.



Bojan Nikolic

Institute of Physics Belgrade
Верификована је имејл адреса на ipb.ac.rs
String theory

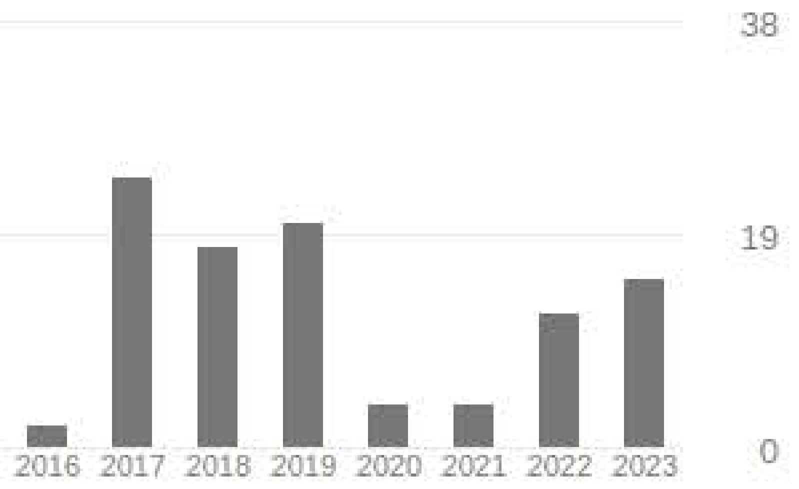
ПРАТИ

НАСЛОВ	НАВЕЛО	ГОДИНА
Gauge symmetries decrease the number of Dp-brane dimensions B Nikolić, B Sazdović Physical Review D 74 (4), 045024	31	2006
Gauge symmetries decrease the number of Dp-brane dimensions. II. Inclusion of the Liouville term B Nikolić, B Sazdović Physical Review D 75 (8), 085011	28	2007
Canonical approach to the closed string non-commutativity L Davidović, B Nikolić, B Sazdović Facta Universitatis, Series Physics, Chemistry and Technology, 101-110	~~25~~	2015
Canonical approach to the closed string non-commutativity L Davidović, B Nikolić, B Sazdović The European Physical Journal C 74 (1), 2734	25	2014
T-duality diagram for a weakly curved background L Davidović, B Nikolić, B Sazdović The European Physical Journal C 75 (12), 576	23	2015
D5-brane type I superstring background fields in terms of type IIB ones by canonical method and T-duality approach B Nikolić, B Sazdović Nuclear Physics B 836 (1-2), 100-126	20	2010
Noncommutativity in space-time extended by Liouville field B Nikolić, B Sazdović Advances in Theoretical and Mathematical Physics 14 (1), 1	20	2010
Type I background fields in terms of type IIB ones B Nikolić, B Sazdović Physics Letters B 666 (4), 400-403	13	2008
Fermionic T duality and momenta noncommutativity B Nikolić, B Sazdović	10	2011

Навело

ПРИКАЖИ СВЕ

	Све	Од 2019
Наводи	219	57
h-индекс	9	5
i10-индекс	8	0



Јавни приступ

ПРИКАЖИ СВЕ

0 чланака — 6 чланака
није доступно — доступно

На основу услова финансирања

Коаутори

ИЗМЕНИ

Branislav Sazdovic — Professor of Physics, Belgrade U…
Marko Vojinović — Institute of Physics, University of …
Tijana Radenković — Institute of physics Belgrade

Република Србија
МИНИСТАРСТВО ПРОСВЕТЕ,
НАУКЕ И ТЕХНОЛОШКОГ РАЗВОЈА
Комисија за стицање научних звања

Број:660-01-00001/695
21.10.2019. године
Б е о г р а д

ИНСТИТУТ ЗА ФИЗИКУ

ПРИМЉЕНО: 29. 11. 2019			
Рад.јед.	број	Арх.шифра	Прилог
0801	1870/1		

На основу члана 22. став 2. члана 70. став 5. и члана 86. ст. 1. и 2. Закона о научноистраживачкој делатности ("Службени гласник Републике Србије", број 110/05 и 50/06 – исправка, 18/10 и 112/15), члана 3. ст. 1. и 3., члана 32. став 1., члана 35. став 2. и члана 40. Правилника о поступку, начину вредновања и квантитативном исказивању научноистраживачких резултата истраживача ("Службени гласник Републике Србије", број 24/16, 21/17 и 38/17) и захтева који је поднео

Институт за физику у Београду

Комисија за стицање научних звања на седници одржаној 21.10.2019. године, донела је

ОДЛУКУ
О СТИЦАЊУ НАУЧНОГ ЗВАЊА

Др Бојан Николић

стиче научно звање

Виши научни сарадник

Р е и з б о р

у области природно-математичких наука - физика

О Б Р А З Л О Ж Е Њ Е

Институт за физику у Београду

утврдио је предлог број 0801/1517/1 од 23.10.2018. године на седници Научног већа Института и поднео захтев Комисији за стицање научних звања број 0801-1579/1 од 29.10.2018. године за доношење одлуке о испуњености услова за реизбор у научно звање ***Виши научни сарадник***.

Комисија за стицање научних звања је по претходно прибављеном позитивном мишљењу Матичног научног одбора за физику на седници одржаној 21.10.2019. године разматрала захтев и утврдила да именовани испуњава услове из члана 70. став 5. и члана 86. ст. 1. и 2. Закона о научноистраживачкој делатности ("Службени гласник Републике Србије", број 110/05 и 50/06 – исправка, 18/10 и 112/15), члана 3. ст. 1. и 3., члана 32. став 1., члана 35. став 2. и члана 40. Правилника о поступку, начину вредновања и квантитативном исказивању научноистраживачких резултата истраживача ("Службени гласник Републике Србије", број 24/16, 21/17 и 38/17) за реизбор у научно звање ***Виши научни сарадник***, па је одлучила као у изреци ове одлуке.

Доношењем ове одлуке именовани стиче сва права која му на основу ње по закону припадају.

Одлуку доставити подносиоцу захтева, именованом и архиви Министарства просвете, науке и технолошког развоја у Београду.

ПРЕДСЕДНИК КОМИСИЈЕ

Др Ђурђица Јововић,
научни саветник

МИНИСТАР

Младен Шарчевић

Eur. Phys. J. C (2017) 77:197
DOI 10.1140/epjc/s10052-017-4758-0

THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

T-dualization of type II superstring theory in double space

B. Nikolić[a], **B. Sazdović**[b]

Institute of Physics Belgrade, University of Belgrade, Pregrevica 118, Belgrade, Serbia

Received: 12 October 2016 / Accepted: 14 March 2017 / Published online: 27 March 2017

Abstract In this article we offer a new interpretation of the T-dualization procedure of type II superstring theory in the double space framework. We use the ghost free action of type II superstring in pure spinor formulation in approximation of constant background fields up to the quadratic terms. T-dualization along any subset of the initial coordinates, x^a, is equivalent to the permutation of this subset with subset of the corresponding T-dual coordinates, y_a, in double space coordinate $Z^M = (x^\mu, y_\mu)$. Requiring that the T-dual transformation law after the exchange $x^a \leftrightarrow y_a$ has the same form as the initial one, we obtain the T-dual NS–NS and NS–R background fields. The T-dual R–R field strength is determined up to one arbitrary constant under some assumptions. The compatibility between supersymmetry and T-duality produces a change of bar spinors and R–R field strength. If we dualize an odd number of dimensions x^a, such a change flips type IIA/B to type II B/A. If we T-dualize the time-like direction, one imaginary unit i maps type II superstring theories to type II* ones.

1 Introduction

T-duality is a fundamental feature of string theory [1–8]. As a consequence of T-duality there is no physical difference between string theory compactified on a circle of radius R and circle of radius $1/R$. This conclusion can be generalized to tori of various dimensions.

The mathematical realization of T-duality is given by Buscher T-dualization procedure [4,5]. If the background fields have global isometries along some directions then we can localize that symmetry introducing gauge fields. The next step is to add the new term in the action with Lagrange multipliers which forces these gauge fields to be unphysical. Finally, we can use gauge freedom to fix initial coordinates. Varying this gauge fixed action with respect to the Lagrange multipliers one gets the initial action and varying with respect to the gauge fields one gets the T-dual action.

Buscher T-dualization can be applied along directions on which background fields do not depend [4–10]. Such a procedure was used in Refs. [11–18] in the context of closed string noncommutativity. There is a generalized Buscher procedure which deals with background fields depending on all coordinates. The generalized procedure was applied to the case of bosonic string moving in the weakly curved background [19,20]. It leads directly to closed string noncommutativity [21].

The Buscher procedure can be considered as the definition of T-dualization. But there are also other frameworks in which we can represent T-dualization which must be in accordance with the Buscher procedure. Here we talk about the double space formalism which was the subject of the articles about 20 years ago [22–26]. Double space is spanned by coordinates $Z^M = (x^\mu, y_\mu)$ $(\mu = 0, 1, 2, \ldots, D-1)$, where x^μ and y_μ are the coordinates of the D-dimensional initial and T-dual space-time, respectively. Interest for this subject emerged again with Refs. [27–34], where T-duality is related with $O(d, d)$ transformations. The approach of Ref. [22] has been recently improved when the T-dualization along some subset of the initial and corresponding subset of the T-dual coordinates has been interpreted as permutation of these subsets in the double space coordinates [35,36].

Let us motivate our interest in this subject. It is well known that T-duality is important feature in understanding M-theory. In fact, five consistent superstring theories are connected by a web of T and S dualities. In the beginning we are going to pay attention to the T-duality. To obtain formulation of M-theory it is not enough to find all corresponding T-dual theories. We must construct one theory which contains the initial theory and all corresponding T-dual ones.

This work is supported in part by the Serbian Ministry of Education, Science and Technological Development, under contract No. 171031.

[a] e-mail: bnikolic@ipb.ac.rs
[b] e-mail: sazdovic@ipb.ac.rs

We have succeeded to realize such program in the bosonic case, for both constant and weakly curved background. In Refs. [35,36] we doubled all bosonic coordinates and obtain the theory which contains the initial and all corresponding T-dual theories. In such theory T-dualization along an arbitrary set of coordinates x^a is equivalent to replacement of these coordinates with the corresponding T-dual ones, y_a. Therefore, T-duality in double space becomes symmetry transformation with respect to permutation group.

Performing T-duality in supersymmetric case generates new problems. In the present paper we are going to extend such an approach to the type II theories. In fact, doubling all bosonic coordinates we have unified types IIA, IIB as well as type II* [37] (obtained by T-dualization along time-like direction) theories. We expect that such a program could be a step toward better understanding M-theory.

In the present article we apply the approach of Refs. [35, 36] in the cases of complete (along all bosonic coordinates) and partial (subset of the bosonic coordinates) T-dualization of the type II superstring theory [1–3]. We use ghost free type II superstring theory in pure spinor formulation [33,38–44] in the approximation of constant background fields and up to the quadratic terms. This action is obtained from the general type II superstring action [45] which is given in the form of an expansion in powers of fermionic coordinates θ^α and $\bar\theta^\alpha$. In the first step of our consideration we will limit our analysis to the basic term of the action neglecting θ^α and $\bar\theta^\alpha$ dependent terms. Later, in the discussion of proper fermionic variables, using an iterative procedure [45], we take into consideration higher power terms and restore the supersymmetric invariants $\Pi^\mu_\pm$, d_α and $\bar d_\alpha$ as variables in the theory.

Rewriting the T-dual transformation laws in terms of the double space coordinates Z^M we introduce the generalized metric $\mathcal{H}_{MN}$ and the generalized current $J_{\pm M}$. The permutation matrix $(\mathcal{T}^a)^M{}_N$ exchanges the places of x^a and y_a, where the index a marks the directions along which we make T-dualization. The basic request is that T-dual double space coordinates, ${}_aZ^M = (\mathcal{T}^a)^M{}_N Z^N$, satisfy the transformation law of the same form as initial coordinates, Z^M. It produces the expressions for the T-dual generalized metric, ${}_a\mathcal{H}_{MN} = (\mathcal{T}^a\mathcal{H}\mathcal{T}^a)_{MN}$, and the T-dual current, ${}_aJ_{\pm M} = (\mathcal{T}^a J_\pm)_M$. This is equivalent to the requirement that transformations of the coordinates and background fields, $Z^M \to {}_aZ^M$, $\mathcal{H}_{MN} \to {}_a\mathcal{H}_{MN}$ and $J_{\pm M} \to {}_aJ_{\pm M}$, are symmetry transformations of the double space action. From transformation of the generalized metric we obtain T-dual NS–NS background fields and from transformation of the current we obtain T-dual NS–R fields.

The supersymmetry case includes the new features in both the Buscher and the double space T-duality approaches. In the bosonic case the left and right world-sheet chiralities have different T-duality transformations. It implies that in T-dual theory two fermionic coordinates, θ^α and $\bar\theta^\alpha$, and corresponding canonically conjugated momenta, π_α and $\bar\pi_\alpha$ (with different world-sheet chiralities), have different supersymmetry transformations. As shown in [46,47] it is possible to make a supersymmetry transformation in T-dual theory unique if we change one world-sheet chirality sector. Therefore, compatibility between supersymmetry and T-duality can be achieved by action on the bar variables with the operator ${}_a\Omega$, ${}^\bullet\bar\pi_\alpha = {}_a\Omega_\alpha{}^\beta\, {}_a\bar\pi_\beta$. As a consequence of the relation $\Gamma^{11}\, {}_a\Omega = (-1)^d\, {}_a\Omega\Gamma^{11}$ it follows that such transformations for odd d change space-time chiralities of the bar spinors. In such a way the operator ${}_a\Omega$ for odd d maps type IIA/B to type IIB/A theory. Here d denotes the number of T-dualized directions.

There is one difference compared with the bosonic string case [35,36] where all results from the Buscher procedure were reproduced. In the T-dual transformation laws of type II superstring theory the R–R field strength $F^{\alpha\beta}$ does not appear. The reason is that R–R field strength couples only with the fermionic degrees of freedom, which are not dualized. This is in analogy with the term $\partial_+ x^i \Pi_{+ij}\partial_- x^j$ in the bosonic case, where background field Π_{+ij} couples only with coordinates x^i, which are undualized [27–29]. To reproduce the Buscher form of the T-dual R–R field strength we should make some additional assumptions.

There is an appendix, which contains the block-wise expressions for the tensors used in this article and useful relations.

2 Buscher T-dualization of type II superstring theory

In this section we will consider type II superstring action in pure spinor formulation [38,43,44] in the approximation of constant background fields and up to the quadratic terms. Then we will give the overview of the results obtained by Buscher T-dualization procedure [9,10,46,47].

2.1 Type II superstring in pure spinor formulation

The sigma model action for the type II superstring of Ref. [45] is of the form

$$S = \int_\Sigma d^2\xi (X^T)^M A_{MN} \bar X^N + S_\lambda + S_{\bar\lambda}, \tag{2.1}$$

where the vectors X^M and $\bar X^N$ are left and right chiral supersymmetric variables,

$$X^M = \begin{pmatrix} \partial_+\theta^\alpha \\ \Pi^\mu_+ \\ d_\alpha \\ \frac{1}{2}N^{\mu\nu}_+ \end{pmatrix}, \quad \bar X^N = \begin{pmatrix} \partial_-\bar\theta^\alpha \\ \Pi^\mu_- \\ \bar d_\alpha \\ \frac{1}{2}\bar N^{\mu\nu}_- \end{pmatrix}, \tag{2.2}$$

of which the components are defined as

$$\Pi_+^\mu = \partial_+ x^\mu + \frac{1}{2}\theta^\alpha (\Gamma^\mu)_{\alpha\beta}\partial_+\theta^\beta,$$
$$\Pi_-^\mu = \partial_- x^\mu + \frac{1}{2}\bar\theta^\alpha (\Gamma^\mu)_{\alpha\beta}\partial_-\bar\theta^\beta, \tag{2.3}$$

$$d_\alpha = \pi_\alpha - \frac{1}{2}(\Gamma_\mu\theta)_\alpha\left[\partial_+ x^\mu + \frac{1}{4}(\theta\Gamma_\mu\partial_+\theta)\right],$$
$$\bar d_\alpha = \bar\pi_\alpha - \frac{1}{2}(\Gamma_\mu\bar\theta)_\alpha\left[\partial_- x^\mu + \frac{1}{4}(\bar\theta\Gamma_\mu\partial_-\bar\theta)\right], \tag{2.4}$$

$$N_+^{\mu\nu} = \frac{1}{2}w_\alpha(\Gamma^{[\mu\nu]})^\alpha{}_\beta\lambda^\beta, \quad \bar N_-^{\mu\nu} = \frac{1}{2}\bar w_\alpha(\Gamma^{[\mu\nu]})^\alpha{}_\beta\bar\lambda^\beta. \tag{2.5}$$

Inserting the supermatrix A_{MN}

$$A_{MN} = \begin{pmatrix} A_{\alpha\beta} & A_{\alpha\nu} & E_\alpha{}^\beta & \Omega_{\alpha,\mu\nu} \\ A_{\mu\beta} & A_{\mu\nu} & \bar E_\mu^\beta & \Omega_{\mu,\nu\rho} \\ E^\alpha{}_\beta & E^\alpha_\nu & \mathrm{P}^{\alpha\beta} & C^\alpha{}_{\mu\nu} \\ \Omega_{\mu\nu,\beta} & \Omega_{\mu\nu,\rho} & \bar C_{\mu\nu}{}^\beta & S_{\mu\nu,\rho\sigma} \end{pmatrix}, \tag{2.6}$$

in (2.1), the action gets the expanded form [45]

$$\begin{aligned} S = \int d^2\xi \Big[& \partial_+\theta^\alpha A_{\alpha\beta}\partial_-\bar\theta^\beta + \partial_+\theta^\alpha A_{\alpha\mu}\Pi_-^\mu + \Pi_+^\mu A_{\mu\alpha}\partial_-\bar\theta^\alpha \\ & + \Pi_+^\mu A_{\mu\nu}\Pi_-^\nu + d_\alpha E^\alpha{}_\beta\partial_-\bar\theta^\beta + d_\alpha E^\alpha{}_\mu\Pi_-^\mu \\ & + \partial_+\theta^\alpha E_\alpha{}^\beta\bar d_\beta + \Pi_+^\mu E_\mu{}^\beta\bar d_\beta + d_\alpha \mathrm{P}^{\alpha\beta}\bar d_\beta \\ & + \frac{1}{2}N_+^{\mu\nu}\Omega_{\mu\nu,\beta}\partial_-\bar\theta^\beta + \frac{1}{2}N_+^{\mu\nu}\Omega_{\mu\nu,\rho}\Pi_-^\rho \\ & + \frac{1}{2}\partial_+\theta^\alpha\Omega_{\alpha,\mu\nu}\bar N_-^{\mu\nu} + \frac{1}{2}\Pi_+^\mu\Omega_{\mu,\nu\rho}\bar N_-^{\nu\rho} \\ & + \frac{1}{2}N_+^{\mu\nu}\bar C_{\mu\nu}{}^\beta\bar d_\beta + \frac{1}{2}d_\alpha C^\alpha{}_{\mu\nu}\bar N_-^{\mu\nu} \\ & + \frac{1}{4}N_+^{\mu\nu}S_{\mu\nu,\rho\sigma}\bar N_-^{\rho\sigma}\Big] + S_\lambda + S_{\bar\lambda}. \end{aligned} \tag{2.7}$$

The world sheet Σ is parameterized by $\xi^m = (\xi^0 = \tau, \xi^1 = \sigma)$ and $\partial_\pm = \partial_\tau \pm \partial_\sigma$. Superspace is spanned by bosonic coordinates, x^μ ($\mu = 0, 1, 2, \ldots, 9$), and fermionic ones, θ^α and $\bar\theta^\alpha$ ($\alpha = 1, 2, \ldots, 16$). The variables π_α and $\bar\pi_\alpha$ are canonically conjugated momenta to θ^α and $\bar\theta^\alpha$, respectively. The actions for pure spinors, S_λ and $S_{\bar\lambda}$, are free field actions

$$S_\lambda = \int d^2\xi\, w_\alpha\partial_-\lambda^\alpha, \quad S_{\bar\lambda} = \int d^2\xi\, \bar w_\alpha\partial_+\bar\lambda^\alpha, \tag{2.8}$$

where λ^α and $\bar\lambda^\alpha$ are pure spinors and w_α and $\bar w_\alpha$ are their canonically conjugated momenta, respectively. The pure spinors satisfy the so-called pure spinor constraints,

$$\lambda^\alpha(\Gamma^\mu)_{\alpha\beta}\lambda^\beta = \bar\lambda^\alpha(\Gamma^\mu)_{\alpha\beta}\bar\lambda^\beta = 0. \tag{2.9}$$

The matrix A_{MN} containing type II superfields generally depends on x^μ, θ^α and $\bar\theta^\alpha$. The superfields $A_{\mu\nu}$, $\bar E_\mu{}^\alpha$, $E^\alpha{}_\mu$ and $\mathrm{P}^{\alpha\beta}$ are physical superfields, because their first components are supergravity fields. The fields in the first column and first row are auxiliary superfields because they can be expressed in terms of the physical ones [45]. The remaining ones, $\Omega_{\mu,\nu\rho}(\Omega_{\mu\nu,\rho})$, $C^\alpha{}_{\mu\nu}(\bar C_{\mu\nu}{}^\alpha)$ and $S_{\mu\nu,\rho\sigma}$, are the curvatures (field strengths) for the physical superfields.

The action from which we start (2.7) could be considered as an expansion in powers of θ^α and $\bar\theta^\alpha$. In the iterative procedure presented in [45] it has been shown that each component in the expansion can be obtained from the previous one. Therefore, for practical reasons (computational simplicity), in the first step we limit our considerations to the basic component i.e. we neglect all terms in the action containing θ^α and $\bar\theta^\alpha$. As a consequence the θ^α and $\bar\theta^\alpha$ terms disappear from $\Pi_\pm^\mu$, d_α and $\bar d_\alpha$ and in the solutions for the physical superfields just x-dependent supergravity fields survive. Therefore we lose explicit supersymmetry in such approximation. Later, when we discuss proper fermionic variables, we would go further in the expansion and take higher power terms, which means that supersymmetric invariants, $\Pi_\pm^\mu$, d_α and $\bar d_\alpha$, would play the roles of $\partial_\pm x^\mu$, π_α and $\bar\pi_\alpha$, respectively.

We are going to perform T-dualization along some subset of bosonic coordinates x^a. Therefore, we will assume that these directions are Killing vectors. Since $\partial_\pm x^a$ appears in $\Pi_\pm^\mu$, d_α and $\bar d_\alpha$, it essentially means that corresponding superfields (A_{ab}, $\bar E_a{}^\alpha$, $E^\alpha{}_a$, $\mathrm{P}^{\alpha\beta}$) should not depend on x^a. This assumption regarding Killing spinors could be extended on all space-time directions x^μ, which effectively means, in the first step, that physical superfields are constant. All auxiliary superfields can be expressed in terms of space-time derivatives of physical supergravity fields [45]. Then, in the first step, the auxiliary superfields are zero, because all physical superfields are constant. On the other hand, having constant physical superfields means that their field strengths, $\Omega_{\mu,\nu\rho}(\Omega_{\mu\nu,\rho})$, $C^\alpha{}_{\mu\nu}(\bar C_{\mu\nu}{}^\alpha)$ and $S_{\mu\nu,\rho\sigma}$, are zero. In this way, in the first step, we eliminated from the action terms containing variables $N_+^{\mu\nu}$ and $\bar N_-^{\mu\nu}$ (2.5).

This choice of background fields should be discussed from the viewpoint of space-time field equations of type II superstring action [48]. Let us pay attention on the space-time field equations for type II superstring given in Appendix B of [48]. Equation (B.7) from this set of equations represents the back-reaction of $\mathrm{P}^{\alpha\beta}$ on the metric $G_{\mu\nu}$. If we take a constant dilaton Φ and a constant antisymmetric NS–NS field $B_{\mu\nu}$ we obtain

$$R_{\mu\nu} - \frac{1}{2}G_{\mu\nu}R \sim (\mathrm{P}^{\alpha\beta})^2_{\mu\nu}. \tag{2.10}$$

If we choose the background field $\mathrm{P}^{\alpha\beta}$ to be constant, in general, we will have a constant Ricci tensor, which means that the metric tensor is a quadratic function of the space-time coordinates i.e. there is back-reaction of R–R field strength on the metric tensor. If one wants to cancel non-quadratic terms originating from back-reaction, additional conditions must be imposed on the R–R field strength—the $AdS_5 \times S_5$ coset geometry or self-duality condition (see Ref. [38]).

Taking into account the above analysis and arguments, our approximation can be realized in the following way:

$$\Pi_{\pm}^{\mu} \to \partial_{\pm}x^{\mu}, \quad d_{\alpha} \to \pi_{\alpha}, \quad \bar{d}_{\alpha} \to \bar{\pi}_{\alpha}, \tag{2.11}$$

and the physical superfields take the form

$$A_{\mu\nu} = \kappa\left(\frac{1}{2}G_{\mu\nu} + B_{\mu\nu}\right), \quad E_{\nu}^{\alpha} = -\Psi_{\nu}^{\alpha}, \quad \bar{E}_{\mu}^{\alpha} = \bar{\Psi}_{\mu}^{\alpha},$$
$$\mathrm{P}^{\alpha\beta} = \frac{2}{\kappa}P^{\alpha\beta} = \frac{2}{\kappa}e^{\frac{\Phi}{2}}F^{\alpha\beta}, \tag{2.12}$$

where $G_{\mu\nu}$ is the metric tensor and $B_{\mu\nu}$ is the antisymmetric NS–NS background field. Consequently, the full action S is

$$S = \kappa\int_{\Sigma} d^2\xi\left[\partial_{+}x^{\mu}\Pi_{+\mu\nu}\partial_{-}x^{\nu} + \frac{1}{4\pi\kappa}\Phi R^{(2)}\right]$$
$$+\int_{\Sigma} d^2\xi\left[-\pi_{\alpha}\partial_{-}(\theta^{\alpha} + \Psi_{\mu}^{\alpha}x^{\mu}) + \partial_{+}(\bar{\theta}^{\alpha} + \bar{\Psi}_{\mu}^{\alpha}x^{\mu})\bar{\pi}_{\alpha}\right.$$
$$\left.+\frac{2}{\kappa}\pi_{\alpha}P^{\alpha\beta}\bar{\pi}_{\beta}\right], \tag{2.13}$$

where

$$\Pi_{\pm\mu\nu} = B_{\mu\nu} \pm \frac{1}{2}G_{\mu\nu}. \tag{2.14}$$

The actions S_{λ} and $S_{\bar{\lambda}}$ are decoupled from the rest and can be neglected in the further analysis. The action, in its final form, is ghost independent.

The NS–NS sector of the theory described by (2.13) contains the gravitational $G_{\mu\nu}$, the antisymmetric Kalb–Ramond field $B_{\mu\nu}$ and the dilaton field Φ. In the NS–R sector there are two gravitino fields, Ψ_{μ}^{α} and $\bar{\Psi}_{\mu}^{\alpha}$, which are Majorana–Weyl spinors of the opposite chirality in type IIA and of the same chirality in type IIB theory. The field $F^{\alpha\beta}$ is the R–R field strength and can be expressed in terms of the antisymmetric tensors $F_{(k)}$ [9,49–51]

$$F^{\alpha\beta} = \sum_{k=0}^{D}\frac{1}{k!}F_{(k)}\Gamma_{(k)}^{\alpha\beta}, \quad \left[\Gamma_{(k)}^{\alpha\beta} = (\Gamma^{[\mu_1\cdots\mu_k]})^{\alpha\beta}\right] \tag{2.15}$$

where

$$\Gamma^{[\mu_1\mu_2\cdots\mu_k]} \equiv \Gamma^{[\mu_1}\Gamma^{\mu_2}\ldots\Gamma^{\mu_k]} \tag{2.16}$$

is the completely antisymmetrized product of gamma matrices. The bispinor $F^{\alpha\beta}$ satisfies the chirality condition, $\Gamma^{11}F = \pm F\Gamma^{11}$, where Γ^{11} is a product of gamma matrices in $D = 10$ dimensional space-time and the sign + corresponds to type IIA, while the sign − corresponds to type IIB superstring theory. Consequently, type IIA theory contains only even rank tensors $F_{(k)}$, while type IIB contains only odd rank tensors. Because of the duality relation, the independent tensors are $F_{(0)}$, $F_{(2)}$ and $F_{(4)}$ for type IIA, while $F_{(1)}$, $F_{(3)}$ and the self-dual part of $F_{(5)}$ for type IIB superstring theory. Using the mass-shell condition (massless Dirac equation for $F^{\alpha\beta}$) these tensors can be solved in terms of the potentials $F_{(k)} = dA_{(k-1)}$. The factor $e^{\frac{\Phi}{2}}$ is in accordance with the conventions adopted from [52].

2.2 T-dualization along arbitrary number of coordinates

Let us start with the action (2.13) and apply the standard T-dualization procedure [4,5,19,20]. It means that we localize the shift symmetry for some coordinates x^a. We substitute the ordinary derivatives with covariant ones, introducing gauge fields v_{α}^{a}. Then we add the term $\frac{1}{2}y_a F_{+-}^{a}$ to the Lagrangian in order to force the field strength F_{+-}^{a} to vanish and preserve equivalence between original and T-dual theories. Finally, we fix the gauge $x^a = 0$ and obtain

$$S_{\text{fix}}(v_{\pm}^{a}, x^i, \theta^{\alpha}, \bar{\theta}^{\alpha}, \pi_{\alpha}, \bar{\pi}_{\alpha})$$
$$= \int_{\Sigma} d^2\xi\left[\kappa v_{+}^{a}\Pi_{+ab}v_{-}^{b} + \kappa v_{+}^{a}\Pi_{+aj}\partial_{-}x^{j} + \kappa\partial_{+}x^{i}\Pi_{+ib}v_{-}^{b}\right.$$
$$+\kappa\partial_{+}x^{i}\Pi_{+ij}\partial_{-}x^{j} + \frac{1}{4\pi}\Phi R^{(2)} - \pi_{\alpha}\Psi_{b}^{\alpha}v_{-}^{b}$$
$$+v_{+}^{a}\bar{\Psi}_{a}^{\alpha}\bar{\pi}_{\alpha} - \pi_{\alpha}\partial_{-}(\theta^{\alpha} + \Psi_{i}^{\alpha}x^{i}) + \partial_{+}(\bar{\theta}^{\alpha} + \bar{\Psi}_{i}^{\alpha}x^{i})\bar{\pi}_{\alpha}$$
$$\left.+\frac{1}{2\kappa}e^{\frac{\Phi}{2}}\pi_{\alpha}F^{\alpha\beta}\bar{\pi}_{\beta} + \frac{\kappa}{2}(v_{+}^{a}\partial_{-}y_a - v_{-}^{a}\partial_{+}y_a)\right]. \tag{2.17}$$

Varying the gauge fixed action with respect to the Lagrange multipliers y_a we get the solution for the gauge fields in the form

$$v_{\pm}^{a} = \partial_{\pm}x^{a}, \tag{2.18}$$

while varying with respect to the gauge fields $v_{\pm}^{a}$ we have

$$v_{\pm}^{a} = -2\kappa\hat{\theta}_{\pm}^{ab}\Pi_{\mp bi}\partial_{\pm}x^{i} - \kappa\hat{\theta}_{\pm}^{ab}\partial_{\pm}y_b \pm 2\hat{\theta}_{\pm}^{ab}\Psi_{\pm b}^{\alpha}\pi_{\pm\alpha}. \tag{2.19}$$

Substituting $v_{\pm}^{a}$ in (2.17) we find

$$S_{\text{fix}}(y_a, x^i, \theta^{\alpha}, \bar{\theta}^{\alpha}, \pi_{\alpha}, \bar{\pi}_{\alpha})$$
$$= \int_{\Sigma} d^2\xi\left[\frac{\kappa^2}{2}\partial_{+}y_a\hat{\theta}_{-}^{ab}\partial_{-}y_b + \kappa^2\partial_{+}y_a\hat{\theta}_{-}^{ab}\Pi_{+bj}\partial_{-}x^{j}\right.$$
$$-\kappa^2\partial_{+}x^{i}\Pi_{+ia}\hat{\theta}_{-}^{ab}\partial_{-}y_b + \frac{1}{4\pi}\Phi R^{(2)}$$
$$+\kappa\partial_{+}x^{i}(\Pi_{+ij} - 2\kappa\Pi_{+ia}\hat{\theta}_{-}^{ab}\Pi_{+bj})\partial_{-}x^{j}$$
$$-\pi_{\alpha}\partial_{-}(\theta^{\alpha} + \Psi_{i}^{\alpha}x^{i} - 2\Psi_{a}^{\alpha}\hat{\theta}_{-}^{ab}\Pi_{+bj}x^{j} - \Psi_{a}^{\alpha}\hat{\theta}_{-}^{ab}y_b)$$
$$+\partial_{+}(\bar{\theta}^{\alpha} + \bar{\Psi}_{i}^{\alpha}x^{i} + 2\bar{\Psi}_{a}^{\alpha}\hat{\theta}_{+}^{ab}\Pi_{-bj}x^{j} + \bar{\Psi}_{a}^{\alpha}\hat{\theta}_{+}^{ab}y_b)\bar{\pi}_{\alpha}$$
$$\left.+2\pi_{\alpha}\Psi_{a}^{\alpha}\hat{\theta}_{-}^{ab}\bar{\Psi}_{b}^{\beta}\bar{\pi}_{\beta} + \frac{1}{2\kappa}e^{\frac{\Phi}{2}}\pi_{\alpha}F^{\alpha\beta}\bar{\pi}_{\beta}\right]. \tag{2.20}$$

Before we read the T-dual background fields, we must express this action in terms of the appropriate spinor coordinates, which we will discuss in the next subsections.

Combining two solutions for the gauge fields (2.18) and (2.19) we obtain the transformation law between initial x^a

and T-dual coordinates y_a,

$$\partial_\pm x^a \cong -2\kappa\hat{\theta}^{ab}_\pm \Pi_{\mp bi}\partial_\pm x^i - \kappa\hat{\theta}^{ab}_\pm(\partial_\pm y_b - J_{\pm b}). \tag{2.21}$$

Its inverse is the solution of the last equation in terms of y_a

$$\partial_\pm y_a \cong -2\Pi_{\mp ab}\partial_\pm x^b - 2\Pi_{\mp ai}\partial_\pm x^i + J_{\pm a}, \tag{2.22}$$

where we use $\cong$ to emphasize that these are T-duality relations. Here we introduced the current $J_{\pm\mu}$ in the form

$$J_{\pm\mu} = \pm\frac{2}{\kappa}\Psi^\alpha_{\pm\mu}\pi_{\pm\alpha}, \tag{2.23}$$

where

$$\Psi^\alpha_{+\mu} \equiv \Psi^\alpha_\mu, \quad \Psi^\alpha_{-\mu} \equiv \bar{\Psi}^\alpha_\mu, \quad \pi_{+\alpha} \equiv \pi_\alpha, \quad \pi_{-\alpha} \equiv \bar{\pi}_\alpha, \tag{2.24}$$

and the expression $\hat{\theta}^{ab}_\pm$ is defined in (A.9).

2.3 Relation between left and right chirality in T-dual theory

One can see from (2.21) and (2.22) that the left and right chiralities transform differently in T-dual theory. As a consequence, in T-dual theory we will have two types of vielbeins, two types of Γ-matrices, two types of spin connections and two types of supersymmetry transformations. We want to have a single geometry in T-dual theory. Therefore, we will show that all these different representations of the same variables can be connected by Lorentz transformations [46,47].

2.3.1 Two sets of vielbeins in T-dual theory

The T-dual transformations of the coordinates (2.22) can be put in the form

$$\begin{pmatrix}\partial_\pm y_a\\ \partial_\pm x^i\end{pmatrix} = \begin{pmatrix}-2\Pi_{\mp ab} & -2\Pi_{\mp aj}\\ 0 & \delta^i_j\end{pmatrix}\begin{pmatrix}\partial_\pm x^b\\ \partial_\pm x^j\end{pmatrix} + \begin{pmatrix}J_{\pm a}\\ 0\end{pmatrix}, \tag{2.25}$$

which can be rewritten as

$$\begin{aligned}\partial_+ ({}_aX)_{\hat\mu} &= (\bar{Q}^{-1T})_{\hat\mu\nu}\partial_+ x^\nu + J_{+\hat\mu},\\ \partial_- ({}_aX)_{\hat\mu} &= (Q^{-1T})_{\hat\mu\nu}\partial_- x^\nu + J_{-\hat\mu},\end{aligned} \tag{2.26}$$

where we introduced the T-dual variables ${}_aX_{\hat\mu} = \{y_a, x^i\}$. Here and further on the left subscript a denotes the T-dualization along x^a directions. For coordinates which contain both x^i and y_a we will use "hat" indices $\hat\mu$, $\hat\nu$. The matrices

$$Q^{\hat\mu\nu} = \begin{pmatrix}\kappa\hat{\theta}^{ab}_+ & 0\\ -2\kappa\Pi_{-ic}\hat{\theta}^{cb}_+ & \delta^i_j\end{pmatrix}, \quad \bar{Q}^{\hat\mu\nu} = \begin{pmatrix}\kappa\hat{\theta}^{ab}_- & 0\\ -2\kappa\Pi_{+ic}\hat{\theta}^{cb}_- & \delta^i_j\end{pmatrix}, \tag{2.27}$$

and theirs inverse

$$Q^{-1}_{\mu\hat\nu} = \begin{pmatrix}2\Pi_{-ab} & 0\\ 2\Pi_{-ib} & \delta^j_i\end{pmatrix}, \quad \bar{Q}^{-1}_{\mu\hat\nu} = \begin{pmatrix}2\Pi_{+ab} & 0\\ 2\Pi_{+ib} & \delta^j_i\end{pmatrix}, \tag{2.28}$$

perform T-dualization for the vector indices.

Note that different chiralities transform with different matrices $Q^{\hat\mu\nu}$ and $\bar{Q}^{\hat\mu\nu}$. Therefore, there are two types of T-dual vielbeins

$${}_ae^{\underline{a}\hat\mu} = e^{\underline{a}}{}_\nu(Q^T)^{\nu\hat\mu}, \quad {}_a\bar{e}^{\underline{a}\hat\mu} = e^{\underline{a}}{}_\nu(\bar{Q}^T)^{\nu\hat\mu}, \tag{2.29}$$

with the same T-dual metric

$$\begin{aligned}{}_aG^{\hat\mu\hat\nu} &\equiv ({}_ae^T\eta\,{}_ae)^{\hat\mu\hat\nu} = (QGQ^T)^{\hat\mu\hat\nu} = {}_a\bar{G}^{\hat\mu\hat\nu}\\ &\equiv ({}_a\bar{e}^T\eta\,{}_a\bar{e})^{\hat\mu\hat\nu} = (\bar{Q}G\bar{Q}^T)^{\hat\mu\hat\nu}.\end{aligned} \tag{2.30}$$

The Lorentz indices are underlined (denoted by $\underline{a}$, $\underline{b}$).

The two T-dual vielbeins are equivalent because they are related by the particular local Lorentz transformation

$${}_a\bar{e}^{\underline{a}\hat\mu} = \Lambda^{\underline{a}}{}_{\underline{b}}\,{}_ae^{\underline{b}\hat\mu}, \quad \Lambda^{\underline{a}}{}_{\underline{b}} = e^{\underline{a}}{}_\mu(Q^{-1}\bar{Q})^{T\mu}{}_\nu(e^{-1})^\nu{}_{\underline{b}}. \tag{2.31}$$

From (2.27) and (2.28) we have

$$(Q^{-1}\bar{Q})^{T\mu}{}_\nu = \begin{pmatrix}\delta^a{}_b + 2\kappa\hat{\theta}^{ac}_+G_{cb} & 2\kappa\hat{\theta}^{ac}_+G_{cj}\\ 0 & \delta^i_j\end{pmatrix}, \tag{2.32}$$

which produces

$$\Lambda^{\underline{a}}{}_{\underline{b}} = \delta^{\underline{a}}{}_{\underline{b}} - 2\omega^{\underline{a}}{}_{\underline{b}}, \qquad \omega^{\underline{a}}{}_{\underline{b}} = -\kappa e^{\underline{a}}{}_a\hat{\theta}^{ab}_+(e^T)_b{}^{\underline{c}}\eta_{\underline{cb}}. \tag{2.33}$$

It satisfies definition of Lorentz transformations

$$\Lambda^T\eta\Lambda = \eta \quad\Longrightarrow\quad \det\Lambda^{\underline{a}}{}_{\underline{b}} = \pm 1. \tag{2.34}$$

After careful calculations we have $\det\Lambda^{\underline{a}}{}_{\underline{b}} = (-1)^d$, where d is the number of dimensions along which we perform T-duality.

2.3.2 Two sets of Γ-matrices in T-dual theory

Because in T-dual theory there are two vielbeins, there must also be two sets of Γ-matrices in curved space

$$\begin{aligned}{}_a\Gamma_{\hat\mu} &= ({}_ae^{-1})_{\hat\mu\underline{a}}\,\Gamma^{\underline{a}} = ({}_ae^{-1}\Gamma)_{\hat\mu},\\ {}_a\bar{\Gamma}_{\hat\mu} &= ({}_a\bar{e}^{-1})_{\hat\mu\underline{a}}\,\Gamma^{\underline{a}} = ({}_a\bar{e}^{-1}\Gamma)_{\hat\mu}.\end{aligned} \tag{2.35}$$

They are related by the expression

$${}_a\bar{\Gamma}_{\hat\mu} = {}_a\Omega^{-1}\,{}_a\Gamma_{\hat\mu}\,{}_a\Omega, \tag{2.36}$$

where ${}_a\Omega$ is a spinorial representation of the Lorentz transformation

$${}_a\Omega^{-1}\,\Gamma^{\underline{a}}\,{}_a\Omega = (\Lambda^{-1})^{\underline{a}}{}_{\underline{b}}\,\Gamma^{\underline{b}}. \tag{2.37}$$

2.3.3 Two sets of spin connections in T-dual theory

The spin connection can be expressed in terms of vielbeins as

$$\omega_\mu{}^{\underline{ab}} = \frac{1}{2}(e^{\nu\underline{a}}c^{\underline{b}}_{\mu\nu} - e^{\nu\underline{b}}c^{\underline{a}}_{\mu\nu}) - \frac{1}{2}e^{\rho\underline{a}}e^{\sigma\underline{b}}c_{\underline{c}\rho\sigma}e^{\underline{c}}{}_\mu, \qquad (2.38)$$

where

$$c^{\underline{a}}_{\mu\nu} = \partial_\mu e^{\underline{a}}{}_\nu - \partial_\nu e^{\underline{a}}{}_\mu. \qquad (2.39)$$

Therefore, in T-dual theory there are two spin connections, defined in terms of two vielbeins. As a consequence of (2.31) they are related as

$$_a\bar\omega^{\hat\mu\underline{a}}{}_{\underline{b}} = \Lambda^{\underline{a}}{}_{\underline{c}}\; {}_a\omega^{\hat\mu\underline{c}}{}_{\underline{d}}\,(\Lambda^{-1})^{\underline{d}}{}_{\underline{b}} + \Lambda^{\underline{a}}{}_{\underline{c}}\,\partial^{\hat\mu}\,(\Lambda^{-1})^{\underline{c}}{}_{\underline{b}}. \qquad (2.40)$$

It is useful to introduce the spin connection in the form

$$\omega_\mu = \omega_{\mu\underline{ab}}\Gamma^{\underline{ab}}, \qquad (2.41)$$

where

$$\Gamma^{\underline{ab}} = \Gamma^{\underline{a}}\Gamma^{\underline{b}} - \Gamma^{\underline{b}}\Gamma^{\underline{a}}. \qquad (2.42)$$

Then from (2.37) for $_a\Omega = \text{const}$ we obtain

$$_a\bar\omega^{\hat\mu} = {}_a\Omega^{-1}\; {}_a\omega^{\hat\mu}\; {}_a\Omega. \qquad (2.43)$$

2.3.4 Single form of supersymmetry invariants in T-dual theory and new spinor coordinates

So far we used the action from Ref. [45] which is an expansion in powers of θ^α and $\bar\theta^\alpha$. We performed the procedure of bosonic T-dualization using the first term in the expansion i.e. θ^α and $\bar\theta^\alpha$ independent part of the action. Consequently, the supersymmetric invariants, $\Pi^\mu_\pm$, d_α and $\bar d_\alpha$, in that approximation became $\partial_\pm x^\mu$, π_α and $\bar\pi_\alpha$. But if we would take higher power terms into consideration, then these invariants would appear again in the theory. Consequently, we can use these invariants to find proper spinor variables.

From the compatibility between supersymmetry and T-duality we will find appropriate spinor variables changing the bar ones. We are not going to apply such a procedure to background fields which transformation we will find from T-dualization. In Sect. 2.5 we will check that both T-dual gravitinos satisfy a single supersymmetry transformation rule.

Note that according to [38,53–59] fermionic coordinates, θ^α and $\bar\theta^\alpha$, and their canonically conjugated momenta, π_α and $\bar\pi_\alpha$, are parts of the supersymmetry invariant variables,

$$d_\alpha = \pi_\alpha - \frac{1}{2}(\Gamma_\mu\theta)_\alpha\left(\partial_+x^\mu + \frac{1}{4}\theta\Gamma^\mu\partial_+\theta\right)$$
$$\bar d_\alpha = \bar\pi_\alpha - \frac{1}{2}(\Gamma_\mu\bar\theta)_\alpha\left(\partial_-x^\mu + \frac{1}{4}\bar\theta\Gamma^\mu\partial_-\bar\theta\right). \qquad (2.44)$$

In T-dual theory, as a consequence of two types of Γ matrices, there are two types supersymmetry invariant variables,

$$_ad_\alpha = {}_a\pi_\alpha - \frac{1}{2}({}_a\Gamma^{\hat\mu}\;{}_a\theta)_\alpha\left(\partial_+\;{}_aX_{\hat\mu} + \frac{1}{4}{}_a\theta_a\Gamma_{\hat\mu}\partial_+\;{}_a\theta\right), \qquad (2.45)$$

$$_a\bar d_\alpha = {}_a\bar\pi_\alpha - \frac{1}{2}({}_a\bar\Gamma^{\hat\mu}\;{}_a\bar\theta)_\alpha\left(\partial_-\;{}_aX_{\hat\mu} + \frac{1}{4}{}_a\bar\theta_a\bar\Gamma_{\hat\mu}\partial_-\;{}_a\bar\theta\right). \qquad (2.46)$$

We want the two expressions to have the same Γ matrices. Using Eq. (2.36) we can rewrite the bar expressions as

$$({}_a\Omega\;{}_a\bar d)_\alpha = ({}_a\Omega\;{}_a\bar\pi)_\alpha - \frac{1}{2}({}_a\Gamma^{\hat\mu}{}_a\Omega\;{}_a\bar\theta)_\alpha\left(\partial_-\;{}_aX_{\hat\mu} + \frac{1}{4}{}_a\bar\theta\;{}_a\Omega^{-1}{}_a\Gamma_{\hat\mu}\;{}_a\Omega\;\partial_-{}_a\bar\theta\right). \qquad (2.47)$$

Therefore, if we preserve expressions for $_a\theta^\alpha = \theta^\alpha$ and $_a\pi_\alpha = \pi_\alpha$, change bar variables

$$^\bullet\bar\theta^\alpha \equiv {}_a\Omega^\alpha{}_\beta\;{}_a\bar\theta^\beta, \quad {}^\bullet\bar\pi_\alpha \equiv {}_a\Omega_\alpha{}^\beta\;{}_a\bar\pi_\beta, \qquad (2.48)$$

and take

$$\Omega^2 = 1, \qquad (2.49)$$

the transformation with bar variables will get the same form as those without bar in $_ad_\alpha$. Consequently, the T-dual supersymmetric invariant variables $_ad_\alpha$ and $_a\Omega_\alpha{}^\beta\;{}_a\bar d_\beta$ are expressed in a unique form in terms of the true T-dual spinor variables θ^α, π_α, $^\bullet\bar\theta^\alpha$ and $^\bullet\bar\pi_\alpha$,

$$_ad_\alpha = d_\alpha, \quad {}^\bullet\bar d_\alpha = {}_a\Omega_\alpha{}^\beta\bar d_\beta = {}^\bullet\bar\pi_\alpha - \frac{1}{2}({}_a\Gamma^{\hat\mu}\;{}^\bullet\bar\theta)_\alpha \times(\partial_-\;{}_aX_{\hat\mu} + \frac{1}{4}{}^\bullet\bar\theta\;{}_a\Gamma_{\hat\mu}\partial_-\;{}^\bullet\bar\theta), \qquad (2.50)$$

if condition (2.49) is satisfied.

2.3.5 Spinorial representation of the Lorentz transformation

In order to find expressions for the bar spinors in a T-dual background we should first solve Eq. (2.37) and find the expression for $_a\Omega$. We will do it for $B_{\mu\nu} \to 0$, so that $\hat\theta^{ab}_+ \to -\frac{1}{\kappa}(\tilde G^{-1})^{ab}$, where $\tilde G_{ab}$ is the ab component of $G_{\mu\nu}$. Then from (2.33) it follows that

$$_a\omega^{\underline{ab}} \to e^{\underline{a}}{}_a(\tilde G^{-1})^{ab}(e^T)_b{}^{\underline{b}} \equiv {}_aP^{\underline{ab}}, \qquad (2.51)$$

where $_aP^{\underline{a}}{}_{\underline{b}}$ is some a dependent projector on the $\underline{ab}$ subspace $_aP^{\underline{a}}{}_{\underline{c}}\;{}_aP^{\underline{c}}{}_{\underline{b}} = {}_aP^{\underline{a}}{}_{\underline{b}}$. If we introduce the Γ-matrices in curved space

$$\Gamma^\mu = (e^{-1})^\mu{}_{\underline{a}}\Gamma^{\underline{a}}, \qquad (2.52)$$

we can rewrite Eq. (2.37) in the form

$$_a\Omega\,\Gamma^\mu = \left[\Gamma^\mu - 2\,((e^{-1})^\mu{}_{\underline{a}}\;{}_aP^{\underline{a}}{}_{\underline{b}}\,\Gamma^{\underline{b}}\right]{}_a\Omega. \qquad (2.53)$$

To simplify the derivation from now on we will suppose that the metric tensor is diagonal. Then $(e^{-1})^{\mu}{}_{\underline{a}}\, {}_aP^{\underline{a}}{}_{\underline{b}} = \delta^{\mu}_{a}(e^{-1})^{a}{}_{\underline{a}}$ and we have

$$ {}_a\Omega\, \Gamma^{\mu} = \left[\Gamma^{\mu} - 2\,\delta^{\mu}_{a}\,\Gamma^{a}\right] {}_a\Omega. \tag{2.54}$$

For $\mu = a$ and $\mu = i$ we obtain

$$ {}_a\Omega\ \Gamma^{a} = -\Gamma^{a}\ {}_a\Omega, \quad {}_a\Omega\ \Gamma^{i} = \Gamma^{i}\ {}_a\Omega. \tag{2.55}$$

The Γ-matrices in curved space for a diagonal metric satisfy the algebra

$$\{\Gamma^{a},\Gamma^{b}\} = 2(G^{-1})^{ab}, \quad \{\Gamma^{a},\Gamma^{i}\} = 0, \quad \{\Gamma^{i},\Gamma^{j}\} = 2(G^{-1})^{ij}. \tag{2.56}$$

We should find such an ${}_a\Omega$ as anticommutes with all matrices Γ^{a} and commutes with all matrices Γ^{i}. Let us first introduce the Γ^{11} matrix,

$$\Gamma^{11} = (i)^{\frac{D(D-1)}{2}} \frac{1}{\prod_{\mu=0}^{D-1} G_{\mu\mu}} \varepsilon_{\mu_1\mu_2\cdots\mu_D}\Gamma^{\mu_1}\Gamma^{\mu_2}\cdots\Gamma^{\mu_D}, \tag{2.57}$$

where the normalization constant is chosen so that Γ^{11} satisfies the condition $(\Gamma^{11})^2 = 1$.

Then we define an analogy of the Γ^{11} matrix in the subspace spanned by the T-dualized directions

$$ {}_a\Gamma = (i)^{\frac{d(d-1)}{2}} \prod_{i=1}^{d} \Gamma^{a_i} = (i)^{\frac{d(d-1)}{2}} \Gamma^{a_1}\Gamma^{a_2}\cdots\Gamma^{a_d}, \tag{2.58}$$

so that

$$({}_a\Gamma)^2 = \prod_{i=1}^{d} G^{a_i a_i} = \frac{1}{\prod_{i=1}^{d} G_{a_i a_i}}. \tag{2.59}$$

Their commutation (anticommutation) relations with one Γ matrix depend on the number of coordinates d, along which we perform T-dualizations. Therefore we have

$$ {}_a\Gamma\,\Gamma^{a} = (-1)^{d+1}\,\Gamma^{a}\ {}_a\Gamma, \quad {}_a\Gamma\,\Gamma^{i} = (-1)^{d}\,\Gamma^{i}\ {}_a\Gamma, \tag{2.60}$$

which means that the solution of Eq. (2.55) is proportional to

$$ {}_a\Omega \ \sim\ {}_a\Gamma\,(\Gamma^{11})^{d}. \tag{2.61}$$

Taking into account (2.49), ${}_a\Omega^2 = 1$, we obtain

$$ {}_a\Omega = \sqrt{\prod_{i=1}^{d} G_{a_i a_i}}\ {}_a\Gamma\,(i\,\Gamma^{11})^{d}. \tag{2.62}$$

This is a general solution. Note that for $a_1 \bigcap a_2 = 0$ we have ${}_{a_1}\Omega\ {}_{a_2}\Omega = (-1)^{d_1 d_2}{}_a\Omega$, where $a = a_1 \bigcup a_2$.

When the number of coordinates along which we perform T-duality is even ($d = 2k$), we have ${}_a\Omega = (-1)^{\frac{d}{2}}\sqrt{\prod_{i=1}^{d} G_{a_i a_i}}\ {}_a\Gamma$. As a consequence of the relation $\Gamma^{11}\,{}_a\Omega = (-1)^{d}\ {}_a\Omega\ \Gamma^{11}$ we can conclude that in that case bar spinors preserve chirality. When the number of coordinates along which we perform T-duality is odd ($d = 2k+1$), we have ${}_a\Omega = (-1)^{\frac{d-1}{2}}\sqrt{\prod_{i=1}^{d} G_{a_i a_i}}\ i\ {}_a\Gamma\,\Gamma^{11}$. As a consequence of the above relation such a transformation changes the chirality of the bar spinors.

In the particular case that we perform T-dualization along only one direction, x^{a_1}, ${}_a\Gamma \to \Gamma^{a_1}$, $d \to 1$ and we obtain the result, well known in the literature [1–3,46,47],

$$ {}_{a_1}\Omega = i\sqrt{G_{a_1a_1}}\,\Gamma^{a_1}\,\Gamma^{11}. \tag{2.63}$$

This is the case of the transition between IIA and IIB theory, when T-duality changes the chirality of the bar spinors.

When we perform T-dualization along all coordinates, $d \to D = 10$, ${}_a\Gamma \to \frac{\Gamma^{11}}{\sqrt{\prod_{\mu=0}^{D-1} G_{\mu\mu}}}$ and from (2.62) we obtain

$$ {}^{\star}\Omega = (-1)^{\frac{D}{2}}\ \Gamma^{11} = -\Gamma^{11}. \tag{2.64}$$

2.4 Choice of the proper fermionic coordinates and T-dual background fields

We have already learned that in order to have compatibility between supersymmetry and T-duality, we should choose the dual bar variables with a bullet in accordance with (2.48). Therefore, before we read the T-dual background fields, we will reexpress the action (2.20) in terms of the appropriate spinor coordinates (2.48) which, with the help of the relation ${}_a\Omega^2 = 1$, produces

$$\begin{aligned} {}_aS(y_a, x^i, \theta^{\alpha}, {}^{\bullet}\bar\theta^{\alpha}, \pi_{\alpha}, {}^{\bullet}\bar\pi_{\alpha}) &= \int_{\Sigma} d^2\xi\, \Big\{ \frac{\kappa^2}{2}\partial_+ y_a \hat\theta^{ab}_{-}\partial_- y_b + \kappa^2\partial_+ y_a \hat\theta^{ab}_{-}\Pi_{+bj}\partial_- x^j \\ &- \kappa^2\partial_+ x^i\,\Pi_{+ia}\hat\theta^{ab}_{-}\partial_- y_b + \frac{1}{4\pi}\Phi R^{(2)} \\ &+ \kappa\partial_+ x^i(\Pi_{+ij} - 2\kappa\,\Pi_{+ia}\hat\theta^{ab}_{-}\Pi_{+bj})\partial_- x^j \\ &- \pi_{\alpha}\partial_-(\theta^{\alpha} + \Psi^{\alpha}_{i}x^i - 2\Psi^{\alpha}_{a}\hat\theta^{ab}_{-}\Pi_{+bj}x^j - \Psi^{\alpha}_{a}\hat\theta^{ab}_{-}y_b) \\ &+ \partial_+[{}^{\bullet}\bar\theta^{\gamma}\,{}_a\Omega_{\gamma}{}^{\alpha} + \bar\Psi^{\alpha}_{i}x^i + 2\bar\Psi^{\alpha}_{a}\hat\theta^{ab}_{+}\Pi_{-bj}x^j \\ &+ \bar\Psi^{\alpha}_{a}\hat\theta^{ab}_{+}y_b]{}_a\Omega_{\alpha}{}^{\beta}\,{}^{\bullet}\bar\pi_{\beta} + 2\pi_{\alpha}\Psi^{\alpha}_{a}\hat\theta^{ab}_{-}\bar\Psi^{\beta}_{b}\,{}_a\Omega_{\beta}{}^{\gamma}\,{}^{\bullet}\bar\pi_{\gamma} \\ &+ \frac{1}{2\kappa}\,e^{\frac{\Phi}{2}}\,\pi_{\alpha}F^{\alpha\beta}\,{}_a\Omega_{\beta}{}^{\gamma}\,{}^{\bullet}\bar\pi_{\gamma}\Big\}. \end{aligned} \tag{2.65}$$

Consequently, applying the Buscher T-dualization procedure [4,5] along the bosonic coordinates x^a of the action (2.13) the T-dual action gets the form

$$\begin{aligned} {}_aS = \int_{\Sigma} d^2\xi\, \Big[&\kappa\,\partial_+({}_aX)_{\hat\mu}\,{}_a\Pi_{+}^{\hat\mu\hat\nu}\partial_-({}_aX)_{\hat\nu} + \frac{1}{4\pi}{}_a\Phi R^{(2)} \\ &- \pi_{\alpha}\partial_-[\theta^{\alpha} + {}_a\Psi^{\alpha\hat\mu}({}_aX)_{\hat\mu}] + \partial_+[{}^{\bullet}\bar\theta^{\alpha} + {}_a\bar\Psi^{\alpha\hat\mu}({}_aX)_{\hat\mu}]\,{}^{\bullet}\bar\pi_{\alpha} \\ &+ \frac{1}{2\kappa}e^{\frac{{}_a\Phi}{2}}\pi_{\alpha}\ {}_aF^{\alpha\beta}\,{}^{\bullet}\bar\pi_{\beta}\Big], \end{aligned} \tag{2.66}$$

where $({}_aX)_{\hat\mu} = (y_a, x^i)$, ${}_a\Psi^{\alpha\hat\mu} = ({}_a\Psi^{\alpha a}, {}_a\Psi^\alpha_i)$ and ${}_a\bar\Psi^{\alpha\hat\mu} = ({}_a\bar\Psi^{\alpha a}, {}_a\bar\Psi^\alpha_i)$.

Now, we are ready to read the T-dual background fields

$$ {}_a\Pi^{ab}_\pm = \frac{\kappa}{2}\hat\theta^{ab}_\mp, \tag{2.67}$$

$$ {}_a\Pi_{\pm i}{}^a = -\kappa\Pi_{\pm ib}\hat\theta^{ba}_\mp, \quad {}_a(\Pi_\pm)^a{}_i = \kappa\hat\theta^{ab}_\mp\Pi_{\pm bi}, \tag{2.68}$$

$$ {}_a\Pi_{\pm ij} = \Pi_{\pm ij} - 2\kappa\Pi_{\pm ia}\hat\theta^{ab}_\mp\Pi_{\pm bj}, \tag{2.69}$$

$$ {}_a\Psi^{\alpha a} = \kappa\hat\theta^{ab}_+\Psi^\alpha_b, \quad {}_a\bar\Psi^{\alpha a} = \kappa\, {}_a\Omega^\alpha{}_\beta\,\hat\theta^{ab}_-\bar\Psi^\beta_b, \tag{2.70}$$

$$ {}_a\Psi^\alpha_i = \Psi^\alpha_i - 2\kappa\Pi_{-ib}\hat\theta^{ba}_+\Psi^\alpha_a,$$

$$ {}_a\bar\Psi^\alpha_i = {}_a\Omega^\alpha{}_\beta(\bar\Psi^\beta_i - 2\kappa\Pi_{+ib}\hat\theta^{ba}_-\bar\Psi^\beta), \tag{2.71}$$

$$ e^{\frac{{}_a\Phi}{2}}\,{}_aF^{\alpha\beta} = \left(e^{\frac{\Phi}{2}}F^{\alpha\gamma} + 4\kappa\Psi^\alpha_a\hat\theta^{ab}_-\bar\Psi^\gamma_b\right){}_a\Omega_\gamma{}^\beta \tag{2.72}$$

when ${}_a\Omega$ is defined in (2.62).

The dilaton transformation in the term $\Phi R^{(2)}$ originates from quantum theory and will be discussed in Sect. 2.6.

2.5 Supersymmetry transformations of T-dual gravitinos

Note that in the expressions for the T-dual fields ${}_a\bar\Psi^{\alpha a}$, ${}_a\bar\Psi^\alpha_i$ and ${}_aF^{\alpha\beta}$ the matrix ${}_a\Omega$ appears as a consequence of the T-dualization procedure and adoption of the bullet spinor coordinates. In Refs. [46,47] it appears as a consequence of the compatibility between supersymmetry and T-duality.

A supersymmetry transformation of the gravitino is expressed in terms of covariant derivatives,

$$ \delta_\varepsilon\Psi^\alpha_\mu = D_\mu\,\varepsilon^\alpha + \cdots, \quad \delta_{\bar\varepsilon}\bar\Psi^\alpha_\mu = D_\mu\,\bar\varepsilon^\alpha + \cdots, \tag{2.73}$$

with the same covariant derivative on both left and right spinors,

$$ D_\mu = \partial_\mu + \omega_\mu. \tag{2.74}$$

In the T-dual theory, as a consequence of the two kinds of spin connections, there are two kinds of covariant derivatives,

$$ {}_aD^{\hat\mu} = \partial^{\hat\mu} + {}_a\omega^{\hat\mu}, \quad {}_a\bar D^{\hat\mu} = \partial^{\hat\mu} + {}_a\bar\omega^{\hat\mu}, \tag{2.75}$$

such that

$$ {}_a\delta_\varepsilon\,{}_a\Psi^{\alpha\hat\mu} = {}_aD^{\hat\mu}\varepsilon^\alpha, \quad {}_a\bar\delta_{\bar\varepsilon}\,{}_a\bar\Psi^{\alpha\hat\mu} = {}_a\bar D^{\hat\mu}\bar\varepsilon^\alpha. \tag{2.76}$$

Let us show that improvement with ${}_a\Omega$ in the transformation of the bar gravitinos just turns ${}_a\bar D^{\hat\mu}$ to ${}_aD^{\hat\mu}$. In fact, from

$$ {}_a\bar\delta_{\bar\varepsilon}\,{}_a\bar\Psi^{\alpha\hat\mu} = {}_a\Omega^\alpha{}_\beta\left(\partial^{\hat\mu}\bar\varepsilon^\beta + {}_a\bar\omega^{\hat\mu\beta}{}_\gamma\bar\varepsilon^\gamma\right), \tag{2.77}$$

with the help of (2.43), for constant ${}_a\Omega$, we have

$$ {}_a\bar\delta_{\bar\varepsilon}\,{}_a\bar\Psi^{\alpha\hat\mu} = \partial^{\hat\mu}({}_a\Omega^\alpha{}_\beta\bar\varepsilon^\beta) + {}_a\omega^{\hat\mu\alpha}{}_\beta\,{}_a\Omega^\beta{}_\gamma\bar\varepsilon^\gamma$$
$$ = {}_aD^{\hat\mu}({}_a\Omega^\alpha{}_\beta\bar\varepsilon^\beta) = {}_a\delta_{{}_a\Omega\bar\varepsilon}\,{}_a\bar\Psi^{\alpha\hat\mu}. \tag{2.78}$$

Therefore, it is clear that in order to preserve the same spin connection for the two chiralities we should additionally change the bar supersymmetry parameter

$$ {}^\bullet\bar\varepsilon^\alpha \equiv ({}_a\Omega)^\alpha{}_\beta\,{}_a\bar\varepsilon^\beta. \tag{2.79}$$

2.6 Transformation of pure spinors

In this subsection we will find transformation laws for pure spinors, λ^α and $\bar\lambda^\alpha$, which are the main ingredient of the pure spinor formalism.

It is well known that pure spinors satisfy the so-called pure spinor constraints,

$$ \lambda^\alpha(\Gamma^\mu)_{\alpha\beta}\lambda^\beta = 0, \quad \bar\lambda^\alpha(\Gamma^\mu)_{\alpha\beta}\bar\lambda^\beta = 0. \tag{2.80}$$

After T-dualization they turn into

$$ {}_a\lambda^\alpha({}_a\Gamma_{\hat\mu})_{\alpha\beta}\,{}_a\lambda^\beta = 0, \quad {}_a\bar\lambda^\alpha({}_a\bar\Gamma_{\hat\mu})_{\alpha\beta}\,{}_a\bar\lambda^\beta = 0. \tag{2.81}$$

The relation between matrices ${}_a\Gamma_{\hat\mu}$ and ${}_a\bar\Gamma_{\hat\mu}$ is given in (2.36). In order to have the two cases expressed with the same gamma matrices, as before, we preserve the expression for the unbar variables,

$$ {}_a\lambda^\alpha = \lambda^\alpha. \tag{2.82}$$

and change bar variables

$$ {}^\bullet\bar\lambda^\alpha = {}_a\Omega^\alpha{}_\beta\,{}_a\bar\lambda^\beta. \tag{2.83}$$

The variables w_α and $\bar w_\alpha$ are canonically conjugated momenta to the pure spinors λ^α and $\bar\lambda^\alpha$, respectively. The transformation laws for pure spinor momenta can be found from the expressions for $N^{\mu\nu}_+$ and $\bar N^{\mu\nu}_-$ (2.5) which would appear in the action if we would take higher power terms in θ^α and $\bar\theta^\alpha$. After T-dualization these expressions become

$$ {}_aN_{+\hat\mu\hat\nu} = \frac{1}{2}{}_aw_\alpha({}_a\Gamma_{[\hat\mu\hat\nu]})^\alpha{}_\beta\,{}_a\lambda^\beta,$$
$$ {}_a\bar N_{-\hat\mu\hat\nu} = \frac{1}{2}{}_a\bar w_\alpha({}_a\bar\Gamma_{[\hat\mu\hat\nu]})^\alpha{}_\beta\,{}_a\bar\lambda^\beta. \tag{2.84}$$

Using Eq. (2.36) and the definition of $\Gamma^{[\mu\nu]}$ (2.16) we see that the relation between ${}_a\Gamma_{[\hat\mu\hat\nu]}$ and ${}_a\bar\Gamma_{[\hat\mu\hat\nu]}$ is the same as between the gamma matrices (2.36). As in the previous case, in order to have unique set of gamma matrices, we do not change the unbar variables,

$$ {}_aw_\alpha = w_\alpha, \tag{2.85}$$

while we choose bar variables in the form

$$ {}^\bullet\bar w_\alpha = {}_a\Omega_\alpha{}^\beta\,{}_a\bar w_\beta. \tag{2.86}$$

Let us note that free field actions S_λ and $S_{\bar\lambda}$ are invariant under T-dualization because ${}_a\Omega^2 = 1$.

2.7 T-dual transformation of antisymmetric fields: from IIB to IIA theory

To find the T-dual transformation laws for antisymmetric fields we will start with Eq. (2.72). First, as explained in Refs. [4,5,60] the quantization procedure produces the well-known shift in the dilaton transformation

$$
{}_a\Phi = \Phi - \ln\det(2\Pi_{+ab}) = \Phi - \ln\sqrt{\frac{\det G_{ab}}{\det {}_aG^{ab}}}. \tag{2.87}
$$

Together with (2.72) it gives a relation between the initial and T-dual background fields,

$$
{}_aF^{\alpha\beta} = \sqrt[4]{\frac{\det G_{ab}}{\det {}_aG^{ab}}}\,(F^{\alpha\gamma} + 4e^{-\frac{\Phi}{2}}\kappa\Psi^\alpha_a\hat\theta^{ab}_-\bar\Psi^\gamma_b)_a\Omega_\gamma{}^\beta. \tag{2.88}
$$

For $B_{\mu\nu}=0$ we have ${}_aG^{ab}=(G_E^{-1})^{ab}=(G^{-1})^{ab}$, and consequently $\sqrt[4]{\frac{\det G_{ab}}{\det {}_aG^{ab}}}=\sqrt[4]{(\det G_{ab})^2}=\sqrt{|\det G_{ab}|}$. It is important to stress that unlike in Eq. (2.62) for ${}_a\Omega$ here we have the absolute value under the square root. For a diagonal metric $G_{\mu\nu}$ we have $\det G_{ab}=\prod_{i=1}^d G_{a_ia_i}$ and taking into account Eq. (2.62) we find

$$
\begin{aligned}
{}_aF^{\alpha\beta} &= i^d\sqrt{\text{sign}\left(\prod_{i=1}^d G_{a_ia_i}\right)\prod_{i=1}^d G_{a_ia_i}} \\
&\times\left(F^{\alpha\gamma}+4e^{-\frac{\Phi}{2}}\kappa\Psi^\alpha_a\hat\theta^{ab}_-\bar\Psi^\gamma_b\right)({}_a\Gamma\,\Gamma^d_{11})_\gamma{}^\beta.
\end{aligned} \tag{2.89}
$$

Note that we are going to T-dualize all D-directions. Then it is necessary to perform T-dualization along the time-like direction. Here the above square root has important consequences. For our signature $(+,-,-,\dots,-)$, the square of the field strength $({}_aF^{\alpha\beta})^2$ and, consequently, the square of all antisymmetric fields will change the sign when we perform T-dualization along the time-like direction. This is just what we need to obtain type $II^\star$ theories in accordance with Ref. [37].

In a simple case when gravitino fields and Kalb–Ramond field are zero and metric is diagonal we will express the transition from type IIB to type IIA theory. Taking $d=1$ we have

$$
{}_aF^{\alpha\beta} = i\sqrt{\text{sign}(G_{aa})}\;G_{aa}\;F^{\alpha\gamma}\,(\Gamma^{11}\Gamma^a)_\gamma{}^\beta. \tag{2.90}
$$

Let us choose type IIB as a starting theory. The matrix Γ^{11} turns $F^{(n)}$ to $F^{(10-n)}$ where

$$
(\mathrm{F}^{(n)})^{\alpha\beta} = \frac{1}{n!}F_{\mu_1\mu_2\cdots\mu_n}(\Gamma^{[\mu_1\mu_2\cdots\mu_n}])^{\alpha\beta}. \tag{2.91}
$$

As a consequence of the chirality condition $F\Gamma^{11}=-\Gamma^{11}F$ the independent tensors are $F^{(1)}$, $F^{(3)}$ and self-dual part of $F^{(5)}$. So we can write

$$
F^{\alpha\gamma}(\Gamma^{11})_\gamma{}^\beta = \left(\mathrm{F}^{(1)}+\mathrm{F}^{(3)}+\frac{1}{2}\mathrm{F}^{(5)}\right)^{\alpha\beta}. \tag{2.92}
$$

Similarly, in T-dual theory (here it is IIA) we have

$$
{}_aF^{\alpha\beta} = ({}_a\mathrm{F}^{(2)}+{}_a\mathrm{F}^{(4)})^{\alpha\beta}, \tag{2.93}
$$

where now

$$
({}_a\mathrm{F}^{(n)})^{\alpha\beta} = \frac{1}{n!}{}_aF^{\hat\mu_1\hat\mu_2\cdots\hat\mu_n}({}_a\Gamma_{[\hat\mu_1\hat\mu_2\cdots\hat\mu_n}])^{\alpha\beta}. \tag{2.94}
$$

The Γ-matrices on both sides are defined in curved space. For the initial theory it is just (2.52), while for T-dual theory it is defined in the first relation in Eq. (2.35) as ${}_a\Gamma_{\hat\mu}=({}_ae^{-1})_{\hat\mu\underline{a}}\Gamma^{\underline{a}}$. As a consequence of the first relation (2.29) between the vielbeins ${}_ae^{\underline{a}\hat\mu}=e^{\underline{a}}{}_\nu(Q^T)^{\nu\hat\mu}$ we can find the relation between the Γ-matrices,

$$
{}_a\Gamma_{\hat\mu} = (Q^{-1T})_{\hat\mu\nu}\Gamma^\nu, \tag{2.95}
$$

which produces

$$
({}_a\mathrm{F}^{(n)})^{\alpha\beta} = \frac{1}{n!}({}^Q_aF)_{\mu_1\mu_2\cdots\mu_n}(\Gamma^{[\mu_1\mu_2\cdots\mu_n]})^{\alpha\beta}, \tag{2.96}
$$

where

$$
\begin{aligned}
({}^Q_aF)_{\mu_1\mu_2\cdots\mu_n} &= {}_aF^{\hat\mu_1\hat\mu_2\cdots\hat\mu_n}(Q^{-1T})_{\hat\mu_1\mu_1} \\
&\times(Q^{-1T})_{\hat\mu_2\mu_2}\cdots(Q^{-1T})_{\hat\mu_n\mu_n}.
\end{aligned} \tag{2.97}
$$

Using the standard relation between the Γ-matrices,

$$
\Gamma^{[\mu_1\mu_2\cdots\mu_n]}\Gamma^a=\Gamma^{\mu\mu_1\mu_2\cdots\mu_na}-\frac{1}{(n-1)!}G^{a[\mu_n}\Gamma^{\mu_1\mu_2\cdots\mu_{n-1}]}, \tag{2.98}
$$

we obtain

$$
\begin{aligned}
F^{(n)}\Gamma^a &= \frac{1}{n!}F_{\mu_1\mu_2\cdots\mu_n}\Gamma^{[\mu_1\mu_2\cdots\mu_na]} \\
&-\frac{1}{(n-1)!}F_{\mu_1\mu_2\cdots\mu_{n-1}}{}^a\,\Gamma^{[\mu_1\mu_2\cdots\mu_{n-1}]}.
\end{aligned} \tag{2.99}
$$

Therefore, from (2.90), (2.92), (2.93), (2.96), (2.97) and (2.99) we can find a general relation connecting antisymmetric fields of Type IIA and type IIB theories,

$$
\begin{aligned}
{}_a\mathrm{F}^{\hat\mu_1\hat\mu_2\cdots\hat\mu_n} &= \sqrt{\text{sign}G_{aa}}G_{aa}(nF_{\mu_1\mu_2\cdots\mu_{n-1}}\delta^a{}_{\mu_n} \\
&-F_{\mu_1\mu_2\cdots\mu_n}{}^a)(Q^T)^{\mu_1\hat\mu_1}(Q^T)^{\mu_2\hat\mu_2}\cdots(Q^T)^{\mu_n\hat\mu_n}.
\end{aligned} \tag{2.100}
$$

Under our assumptions we have

$$
(Q^T)^{\mu\hat\mu} = \begin{pmatrix} -G^{aa} & 0 \\ 0 & \delta_i{}^j\end{pmatrix}, \tag{2.101}
$$

and consequently

$$ {}_aF_{ij} = -i\sqrt{\text{sign}G_{aa}}G_{aa}F_{ij}{}^a, \quad {}_aF_i{}^a = -2i\sqrt{\text{sign}G_{aa}}F_i, \tag{2.102} $$

$$ {}_aF_{ijkq} = -\frac{i}{2}\sqrt{\text{sign}G_{aa}}G_{aa}F_{ijkq}{}^a, \\ {}_aF_{ijk}{}^a = -4i\sqrt{\text{sign}G_{aa}}F_{ijk}. \tag{2.103} $$

For the space-like directions $G_{aa} < 0$ and $i\sqrt{\text{sign}G_{aa}}$ is real. For time-like direction $\sqrt{\text{sign}G_{aa}} \to \sqrt{\text{sign}G_{00}} = 1$ and the remaining imaginary unit causes squares of the antisymmetric fields to get an additional minus sign and type II theories to swap to type II* ones [37].

3 Double space formulation

In this section we will introduce double space, doubling all bosonic coordinates x^μ by corresponding T-dual ones y_μ. We will rewrite the transformation laws in double space and show that both the equations of motion and the Bianchi identities can be written by that single equation.

3.1 T-dualization along all bosonic directions

Applying the Buscher T-dualization procedure [4,5] along all bosonic coordinates of the action (2.13) the T-dual action has been obtained in Ref. [9]. This is a particular case of our relations (2.67)–(2.72) where the T-dual background fields are of the form

$$ {}^\star\Pi_\pm^{\mu\nu} \equiv {}^\star B^{\mu\nu} \pm \frac{1}{2}{}^\star G^{\mu\nu} = \frac{\kappa}{2}\Theta_\mp^{\mu\nu}, \tag{3.1} $$

$$ {}^\star\Psi^{\alpha\mu} = \kappa\Theta_+^{\mu\nu}\Psi_\nu^\alpha, \quad {}^\star\bar{\Psi}^{\alpha\mu} = \kappa\, {}^\star\Omega^\alpha{}_\beta\,\Theta_-^{\mu\nu}\bar{\Psi}_\nu^\beta, \tag{3.2} $$

$$ e^{\frac{{}^\star\Phi}{2}}\, {}^\star F^{\alpha\beta} = \left(e^{\frac{\Phi}{2}}F^{\alpha\gamma} + 4\kappa\Psi_\mu^\alpha\Theta_-^{\mu\nu}\bar{\Psi}_\nu^\gamma\right){}^\star\Omega_\gamma{}^\beta. \tag{3.3} $$

Here we use the notation

$$ G^E_{\mu\nu} = G_{\mu\nu} - 4(BG^{-1}B)_{\mu\nu},\ \Theta^{\mu\nu} = -\frac{2}{\kappa}(G_E^{-1}BG^{-1})^{\mu\nu}, \\ {}^\star\Omega = -\Gamma^{11}, \tag{3.4} $$

and

$$ \Theta_\pm^{\mu\nu} = -\frac{2}{\kappa}(G_E^{-1}\Pi_\pm G^{-1})^{\mu\nu} = \Theta^{\mu\nu} \mp \frac{1}{\kappa}(G_E^{-1})^{\mu\nu}, \tag{3.5} $$

so that

$$ (\Pi_\pm\Theta_\mp)_\mu{}^\nu = \frac{1}{2\kappa}\delta_\mu{}^\nu. \tag{3.6} $$

From (3.1) and (3.5) it follows that

$$ {}^\star G^{\mu\nu} = (G_E^{-1})^{\mu\nu}, \quad {}^\star B^{\mu\nu} = \frac{\kappa}{2}\Theta^{\mu\nu}. \tag{3.7} $$

In this case the transformation laws (2.21) and (2.22) (the relations between the initial x^μ and T-dual coordinates y_μ) get the form

$$ \partial_\pm x^\mu \cong -\kappa\Theta_\pm^{\mu\nu}\partial_\pm y_\nu + \kappa\Theta_\pm^{\mu\nu}J_{\pm\nu}, \\ \partial_\pm y_\mu \cong -2\Pi_{\mp\mu\nu}\partial_\pm x^\nu + J_{\pm\mu}. \tag{3.8} $$

3.2 Transformation laws in double space

Rewriting Eq. (3.8) in the form where terms multiplied by $\varepsilon_\pm{}^\pm = \pm 1$ are on the left-hand side of the equation, we obtain

$$ \pm\partial_\pm y_\mu \cong G_{E\mu\nu}\partial_\pm x^\nu - 2(BG^{-1})_\mu{}^\nu\partial_\pm y_\nu \\ +2(\Pi_\pm G^{-1})_\mu{}^\nu J_{\pm\nu}, \tag{3.9} $$

$$ \pm\partial_\pm x^\mu \cong (G^{-1})^{\mu\nu}\partial_\pm y_\nu \\ +2(G^{-1}B)^\mu{}_\nu\partial_\pm x^\nu - (G^{-1})^{\mu\nu}J_{\pm\nu}. \tag{3.10} $$

Let us introduce double space coordinates

$$ Z^M = \begin{pmatrix} x^\mu \\ y_\mu \end{pmatrix}, \tag{3.11} $$

which contain all initial and T-dual coordinates. In terms of double coordinates Eqs. (3.9) and (3.10) are replaced by one equation:

$$ \partial_\pm Z^M \cong \pm\Omega^{MN}(\mathcal{H}_{NP}\partial_\pm Z^P + J_{\pm N}), \tag{3.12} $$

where the matrix $\mathcal{H}_{MN}$ is known in the literature as the generalized metric and has the form

$$ \mathcal{H}_{MN} = \begin{pmatrix} G^E_{\mu\nu} & -2B_{\mu\rho}(G^{-1})^{\rho\nu} \\ 2(G^{-1})^{\mu\rho}B_{\rho\nu} & (G^{-1})^{\mu\nu} \end{pmatrix}. \tag{3.13} $$

The double current $J_{\pm M}$ is defined as

$$ J_{\pm M} = \begin{pmatrix} 2(\Pi_\pm G^{-1})_\mu{}^\nu J_{\pm\nu} \\ -(G^{-1})^{\mu\nu}J_{\pm\nu} \end{pmatrix}, \tag{3.14} $$

and

$$ \Omega^{MN} = \begin{pmatrix} 0 & 1_D \\ 1_D & 0 \end{pmatrix}, \tag{3.15} $$

is a constant symmetric matrix. Here 1_D denotes the identity operator in D dimensions. Let us stress that the matrix ${}_a\Omega$ and Ω^{MN} are different quantities.

By straightforward calculation we can prove the relations

$$ \mathcal{H}^T\Omega\mathcal{H} = \Omega, \quad \Omega^2 = 1, \quad \det\mathcal{H}_{MN} = 1, \tag{3.16} $$

which means that $\mathcal{H} \in SO(D, D)$. In calculation of determinant we use the rule for block matrices

$$ \det\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \det D\det(A - BD^{-1}C). \tag{3.17} $$

In double field theory Ω^{MN} is the $SO(D, D)$ invariant metric and denoted by η^{MN}.

3.3 Equations of motion and double space action

It is well known that the equations of motion of the initial theory are the Bianchi identities in T-dual picture and vice versa [12,19,22,60]. As a consequence of the identity

$$\partial_+\partial_- Z^M - \partial_-\partial_+ Z^M = 0, \tag{3.18}$$

known as the Bianchi identity, and Eq. (3.12), we obtain the consistency condition

$$\partial_+[\mathcal{H}_{MN}\partial_- Z^N + J_{-M}] + \partial_-[\mathcal{H}_{MN}\partial_+ Z^N + J_{+M}] = 0. \tag{3.19}$$

In components it takes the form

$$\partial_+\partial_- x^\mu = -\frac{1}{\kappa}(G^{-1})^{\mu\nu}\left(\bar\Psi^\alpha_\nu\partial_+\bar\pi_\alpha + \Psi^\alpha_\mu\partial_-\pi_\alpha\right),$$
$$\partial_+\partial_- y_\nu = -\frac{1}{\kappa}G^E_{\mu\nu}\left({}^\star\bar\Psi^{\alpha\mu}\partial_+\bar\pi_\alpha + {}^\star\Psi^{\alpha\mu}\partial_-\pi_\alpha\right). \tag{3.20}$$

These equations are equations of motion of the initial and T-dual theory. Double space formalism enables us to write both equations of motion and Bianchi identities by the single relation (3.12).

Equation (3.19) is the equation of motion of the following action:

$$S = \frac{\kappa}{4}\int d^2\xi[\partial_+ Z^M\mathcal{H}_{MN}\partial_- Z^N + \partial_+ Z^M J_{-M} + J_{+M}\partial_- Z^M + L(\pi_\alpha,\bar\pi_\alpha)], \tag{3.21}$$

where $L(\pi_\alpha,\bar\pi_\alpha)$ is an arbitrary functional of the fermionic momenta.

4 T-dualization of type II superstring theory as a permutation of coordinates in double space

In this section we will derive the transformations of the generalized metric and current, which are a consequence of the permutation of some subset of the bosonic coordinates with the corresponding T-dual ones. First we will present the method in the case of the complete T-dualization (along all bosonic coordinates) and find the expressions for T-dual background fields. Then we will apply the results to the case of partial T-dualization.

4.1 The case of complete T-dualization

In order to exchange all initial and T-dual coordinates let us introduce the permutation matrix

$$\mathcal{T}^M{}_N = \begin{pmatrix} 0 & 1_D \\ 1_D & 0 \end{pmatrix}, \tag{4.1}$$

so that the double T-dual coordinate ${}^\star Z^M$ is obtained:

$${}^\star Z^M = \mathcal{T}^M{}_N Z^N = \begin{pmatrix} y_\mu \\ x^\mu \end{pmatrix}. \tag{4.2}$$

We require that the T-dual transformation law for the double T-dual coordinate ${}^\star Z^M$ has the same form as for the initial coordinate Z^M (3.12)

$$\partial_\pm {}^\star Z^M \cong \pm\Omega^{MN}({}^\star\mathcal{H}_{NP}\partial_\pm {}^\star Z^P + {}^\star J_{\pm N}). \tag{4.3}$$

Then the T-dual generalized metric ${}^\star\mathcal{H}_{MN}$ and T-dual current ${}^\star J_{\pm M}$ are

$${}^\star\mathcal{H}_{MN} = \mathcal{T}_M{}^K\mathcal{H}_{KL}\mathcal{T}^L{}_N, \quad {}^\star J_{\pm M} = \mathcal{T}_M{}^N J_{\pm N}. \tag{4.4}$$

Permutation of the coordinates (4.2) together with transformations of the background fields (4.4) represents the symmetry transformations of the action (3.21).

Using the corresponding expressions for $\mathcal{T}^M{}_N$, $\mathcal{H}_{MN}$ and $J_{\pm M}$, we obtain from the generalized metric transformation

$${}^\star G^{\mu\nu} = (G_E^{-1})^{\mu\nu}, \quad {}^\star B^{\mu\nu} = \frac{\kappa}{2}\Theta^{\mu\nu}. \tag{4.5}$$

Taking into account that as a consequence of (2.48) the bar dual variable is ${}^{\bullet\star}\bar\pi_\alpha = ({}^\star\Omega^T)_\alpha{}^\beta\bar\pi_\beta$, from the current transformations we have

$${}^\star\Psi^{\alpha\mu} = \kappa\Theta_+^{\mu\nu}\Psi^\alpha_\nu, \quad {}^\star\bar\Psi^{\alpha\mu} = \kappa\ {}^\star\Omega^\alpha{}_\beta\Theta_-^{\mu\nu}\bar\Psi^\beta_\nu, \tag{4.6}$$

where ${}^\star\Omega = -\Gamma^{11}$.

Consequently, using double space we can easily reproduce the results of T-dualization, Eqs. (3.7) and (3.2). The problem with T-dualization of the R–R field strength $F^{\alpha\beta}$ will be discussed in Sect. 5.3.

4.2 The case of partial T-dualization

Applying the procedure presented in the previous subsection to the arbitrary subset of bosonic coordinates we will, in fact, describe all possible bosonic T-dualizations. Let us split the coordinate index μ into a and i ($a = 0,\dots,d-1$, $i = d,\dots,D-1$) and denote T-dualization along direction x^a and y_a by

$$\mathcal{T}^a = T^a\circ T_a, \quad T^a \equiv T^0\circ T^1\circ\cdots\circ T^{d-1},$$
$$T_a \equiv T_0\circ T_1\circ\cdots\circ T_{d-1}, \tag{4.7}$$

where $\circ$ marks the operation of composition of T-dualizations. Permutation of the initial coordinates x^a with its T-dual y_a is realized by multiplying the double space coordinate by the constant symmetric matrix $(\mathcal{T}^a)^M{}_N$,

$${}_aZ^M \equiv \begin{pmatrix} y_a \\ x^i \\ x^a \\ y_i \end{pmatrix} = (\mathcal{T}^a)^M{}_N Z^N \equiv \begin{pmatrix} 0 & 0 & 1_a & 0 \\ 0 & 1_i & 0 & 0 \\ 1_a & 0 & 0 & 0 \\ 0 & 0 & 0 & 1_i \end{pmatrix}\begin{pmatrix} x^a \\ x^i \\ y_a \\ y_i \end{pmatrix}, \tag{4.8}$$

where 1_a and 1_i are identity operators in the subspaces spanned by x^a and x^i, respectively. It is easily to check the following relations:

$$(\mathcal{T}^a\mathcal{T}^a)^M{}_N = \delta^M{}_N, \qquad (\mathcal{T}^a\Omega\mathcal{T}^a)^M{}_N = \Omega^M{}_N. \tag{4.9}$$

The first relation means that after two T-dualizations we get the initial theory, while the second relation means that $\mathcal{T}^a \in O(D, D)$.

Let us apply the same approach as in the case of the full T-dualization presented in the previous subsection. We require that the double T-dual coordinate ${}_aZ^M$ satisfy the T-duality transformations of the form like the initial one Z^M (3.12),

$$\partial_\pm\, {}_aZ^M \cong \pm\Omega^{MN}({}_a\mathcal{H}_{NK}\,\partial_\pm\, {}_aZ^K + {}_aJ_{\pm N}). \tag{4.10}$$

Consequently, we find the T-dual generalized metric

$${}_a\mathcal{H}_{MN} = (\mathcal{T}^a)_M{}^K\mathcal{H}_{KL}(\mathcal{T}^a)^L{}_N, \tag{4.11}$$

and the T-dual current

$${}_aJ_{\pm M} = (\mathcal{T}^a)_M{}^N J_{\pm N}. \tag{4.12}$$

Note that Eqs. (4.8), (4.11) and (4.12) are symmetry transformations of the action (3.21). The left subscript a means dualization along the x^a directions.

5 T-dual background fields

In this section we will show that permutation of some bosonic coordinates leads to the same T-dual background fields as standard Buscher procedure [9]. The transformation of the generalized metric (4.11) produces expressions for NS–NS T-dual background fields ($G_{\mu\nu}$ and $B_{\mu\nu}$). They are the same as in bosonic string case obtained in Ref. [35]. Therefore, we will just shortly repeat these results. From the transformation of the current $J_{\pm M}$ (4.12) we will find T-dual background fields of the NS–R sector (Ψ^α_μ and $\bar\Psi^\alpha_\mu$). Because R–R field strength $F^{\alpha\beta}$ does not appear in T-dual transformations, we will find its T-dual under some assumptions.

5.1 T-dual NS–NS background fields $G_{\mu\nu}$, $B_{\mu\nu}$

Requiring that the T-dual generalized metric ${}_a\mathcal{H}_{MN}$ has the same form as the initial one $\mathcal{H}_{MN}$ (3.13) but in terms of the T-dual fields

$${}_a\mathcal{H}_{MN} = \begin{pmatrix} {}_aG_E^{\mu\nu} & -2({}_aB\,{}_aG^{-1})^\mu{}_\nu \\ 2({}_aG^{-1}\,{}_aB)_\mu{}^\nu & ({}_aG^{-1})_{\mu\nu}\end{pmatrix}, \tag{5.1}$$

and using Eq. (4.11), one finds expressions for the NS–NS T-dual background fields ${}_a\Pi^{\mu\nu}_\pm$ in terms of the initial ones,

$${}_a\Pi^{\mu\nu}_\pm = \begin{pmatrix} \tilde g^{-1}\beta_1 D^{-1}\gamma - A^{-1}(\tilde\beta \mp \frac12) & \frac12 A^{-1}g^T - 2\tilde g^{-1}\beta_1 D^{-1}(\bar\beta^T \mp \frac12) \\ \frac12 D^{-1}\gamma - 2\bar\gamma^{-1}\beta_1^T A^{-1}(\tilde\beta \mp \frac12) & \bar\gamma^{-1}\beta_1^T A^{-1}g^T - D^{-1}(\bar\beta^T \mp \frac12)\end{pmatrix}, \tag{5.2}$$

where γ and $\bar\gamma$ are defined in (A.4), g and $\tilde g$ in (A.5), while β_1, $\tilde\beta$ and $\bar\beta$ are defined in (A.7). The quantities A and D are given in (A.11) and (A.13), respectively. In more compact form we have

$${}_a\Pi^{\mu\nu}_\pm = \begin{pmatrix} \frac{\kappa}{2}\hat\theta^{ab}_\mp & \kappa\hat\theta^{ab}_\mp\Pi_{\pm bi} \\ -\kappa\Pi_{\pm ib}\hat\theta^{ba}_\mp & \Pi_{\pm ij} - 2\kappa\Pi_{\pm ia}\hat\theta^{ab}_\mp\Pi_{\pm bj}\end{pmatrix}, \tag{5.3}$$

where $\hat\theta^{ab}_\pm$ has been defined in (A.9). Details regarding the derivation of the Eqs. (5.2) and (5.3) are given in Ref. [35]. Reading the block components we obtained the NS–NS T-dual background fields in the flat background after dualization along the directions x^a, $(a = 0, 1, \ldots, d-1)$

$${}_a\Pi^{ab}_\pm = \frac{\kappa}{2}\hat\theta^{ab}_\mp, \quad {}_a\Pi^a_{\pm i} = \kappa\hat\theta^{ab}_\mp\Pi_{\pm bi}, \tag{5.4}$$

$${}_a\Pi_{\pm i}{}^a = -\kappa\Pi_{\pm ib}\hat\theta^{ba}_\mp,$$
$${}_a\Pi_{\pm ij} = \Pi_{\pm ij} - 2\kappa\Pi_{\pm ia}\hat\theta^{ab}_\mp\Pi_{\pm bj}. \tag{5.5}$$

These are just the Eqs. (2.67)–(2.69). The symmetric and antisymmetric parts of these expressions are the T-dual metric and T-dual Kalb–Ramond field, which are in full agreement with the Refs. [9,20].

5.2 T-dual NS–R background fields Ψ^α_μ, $\bar\Psi^\alpha_\mu$

Let us find the form of T-dual NS–R background fields, ${}_a\Psi^{\alpha a}$, ${}_a\Psi^\alpha_i$, ${}_a\bar\Psi^{\alpha a}$ and ${}_a\bar\Psi^\alpha_i$. The T-dual current ${}_aJ_{\pm M}$ (4.12) should have the same form as the initial one, Eq. (3.14), but in terms of the T-dual background fields

$$\begin{pmatrix} 2({}_a\Pi_\pm\,{}_aG^{-1})^a{}_b\;({}_aJ)^b_\pm + 2({}_a\Pi_\pm\,{}_aG^{-1})^{ai}\;({}_aJ)_{\pm i} \\ 2({}_a\Pi_\pm\,{}_aG^{-1})_{ia}\;({}_aJ)^a_\pm + 2({}_a\Pi_\pm\,{}_aG^{-1})_i{}^j\;({}_aJ)_{\pm j} \\ -({}_aG^{-1})_{ab}\;({}_aJ)^b_\pm - ({}_aG^{-1})_a{}^i\;({}_aJ)_{\pm i} \\ -({}_aG^{-1})^i{}_a\;({}_aJ)^a_\pm - ({}_aG^{-1})^{ij}\;({}_aJ)_{\pm j}\end{pmatrix}$$
$$= \begin{pmatrix} -(G^{-1})^{a\mu}J_{\pm\mu} \\ 2(\Pi_\pm G^{-1})_i{}^\mu J_{\pm\mu} \\ 2(\Pi_\pm G^{-1})_a{}^\mu J_{\pm\mu} \\ -(G^{-1})^{i\mu}J_{\pm\mu}\end{pmatrix}. \tag{5.6}$$

On the left-hand side of this equation we split the index μ in a and i components because in the T-dual picture the index a has a different position, it is now up. T-dual currents are written between the brackets to make a distinction between a left subscript a denoting partial T-dualization and summation indices in the subspace spanned by x^a.

We can obtain the information about T-dual NS–R background fields from the lower D components of the above equation. In order to find the solution of these equations it is more practical to rewrite them using the block-wise form of matrices given in the appendix and Ref. [35],

$$-\tilde{g}_{ab}\ ({}_aJ)^b_\pm + 2(\beta_1)_a{}^i\ ({}_aJ)_{\pm i}$$
$$= 2\left(\tilde{\beta} \pm \frac{1}{2}\right)_a{}^b J_{\pm b} + 2(\beta_1)_a{}^i J_{\pm i},$$
$$-2(\beta_1^T)^i{}_b\ ({}_aJ)^b_\pm + \bar{\gamma}^{ij}\ ({}_aJ)_{\pm j} = \gamma^{ia}\ J_{\pm a} + \bar{\gamma}^{ij} J_{\pm j}. \tag{5.7}$$

From Eq. (3.18) of [35]

$$({}_aG^{-1})_{\mu\nu} = \begin{pmatrix} g_{ab} & -2(BG^{-1})_a{}^j \\ 2(G^{-1}B)^i{}_b & (G^{-1})^{ij} \end{pmatrix} = \begin{pmatrix} \tilde{g} & -2\beta_1 \\ -2\beta_1^T & \bar{\gamma} \end{pmatrix}, \tag{5.8}$$

(A.4) and (A.7), we find the components of ${}_aG^{-1}$, G^{-1} and BG^{-1}, respectively. In the first equation on right-hand side for $(\Pi_\pm G^{-1})_a{}^i$ just stands for $(\beta_1)_a{}^i$ because $\delta_a{}^i = 0$.

The difference

$$({}_aJ)_{\pm i} - J_{\pm i} = (\bar{\gamma}^{-1})_{ij}\left[\gamma^{ja} J_{\pm a} + 2(\beta_1^T)^j{}_b\ ({}_aJ)^b_\pm\right], \tag{5.9}$$

obtained from the second equation, we put in the first equation, which produces

$$2\left[\left(\tilde{\beta} \pm \frac{1}{2}\right) - \beta_1\bar{\gamma}^{-1}\gamma\right]_a{}^b J_{\pm b} = -\left(\tilde{g} - 4\beta_1\bar{\gamma}^{-1}\beta_1^T\right)_{ab}\ {}_aJ^b_\pm. \tag{5.10}$$

From the definition of the quantity A_{ab} (A.11) we get

$$({}_aJ)^b_\pm = 2\left[-A^{-1}(\tilde{\beta} \pm \frac{1}{2}) + A^{-1}\beta_1\bar{\gamma}^{-1}\gamma\right]^{bc} J_{\pm c}. \tag{5.11}$$

Using the expression $A_{ab} = \hat{g}_{ab}$ (proved in [35]) and Eq. (A.12), we recognize the ab block component of Eq. (5.2). Therefore, with the help of (5.3) it is easy to see that

$$({}_aJ)^b_\pm = 2\ {}_a\Pi^{bc}_\mp J_{\pm c} = \kappa\ \hat{\theta}^{bc}_\pm J_{\pm c}. \tag{5.12}$$

Note that now the T-dual current ${}_aJ^{\hat{\mu}}_\pm$ is of the form

$${}_aJ^{\hat{\mu}}_\pm = \pm\frac{2}{\kappa}\ {}_a\Psi^{\alpha\hat{\mu}}_\pm\ {}_a\pi_{\pm\alpha}, \tag{5.13}$$

where

$${}_a\Psi^{\alpha\hat{\mu}}_+ \equiv {}_a\Psi^{\alpha\hat{\mu}}, \quad {}_a\Psi^{\alpha\hat{\mu}}_- \equiv {}_a\bar{\Psi}^{\alpha\hat{\mu}}, \quad {}_a\pi_{+\alpha} \equiv \pi_\alpha,$$
$${}_a\pi_{-\alpha} \equiv {}^\bullet\bar{\pi}_\alpha, \tag{5.14}$$

and as before

$$J_{\pm\mu} = \pm\frac{2}{\kappa}\Psi^\alpha_{\pm\mu}\pi_{\pm\alpha}. \tag{5.15}$$

Therefore, the a components of the T-dual NS–R fields are of the form

$${}_a\Psi^{\alpha a} = \kappa\hat{\theta}^{ab}_+\Psi^\alpha_b, \quad {}_a\bar{\Psi}^{\alpha a} = \kappa\ {}_a\Omega^\alpha{}_\beta\hat{\theta}^{ab}_-\bar{\Psi}^\beta_b. \tag{5.16}$$

Substituting (5.11) into (5.9) we obtain

$$({}_aJ)_{\pm i} - J_{\pm i} = \left(\bar{\gamma}^{-1} + 4\bar{\gamma}^{-1}\beta_1^T A^{-1}\beta_1\bar{\gamma}^{-1}\right)_{ij}\gamma^{jb}J_{\pm b} - 4\left[\bar{\gamma}^{-1}\beta_1^T A^{-1}(\tilde{\beta} \pm \frac{1}{2})\right]_i{}^a J_{\pm a}. \tag{5.17}$$

With the help of (A.13) Eq. (5.17) transforms into

$$({}_aJ)_{\pm i} - J_{\pm i} = 2\left[\frac{1}{2}D^{-1}\gamma - 2\bar{\gamma}^{-1}\beta_1^T A^{-1}\left(\tilde{\beta} \pm \frac{1}{2}\right)\right]_i{}^a J_{\pm a}. \tag{5.18}$$

From the ${}_i{}^a$ component of (5.2) and (5.3) we finally have

$$({}_aJ)_{\pm i} = J_{\pm i} - 2\kappa\Pi_{\mp ib}\hat{\theta}^{ba}_\pm J_{\pm a}. \tag{5.19}$$

As in the previous case, using the expressions for the currents (5.13) and (5.15), the final form of the T-dual fields is

$${}_a\Psi^\alpha_i = \Psi^\alpha_i - 2\kappa\Pi_{-ib}\hat{\theta}^{ba}_+\Psi^\alpha_a,$$
$${}_a\bar{\Psi}^\alpha_i = {}_a\Omega^\alpha{}_\beta(\bar{\Psi}^\beta_i - 2\kappa\Pi_{+ib}\hat{\theta}^{ba}_-\bar{\Psi}^\beta_a). \tag{5.20}$$

Equations (5.16) and (5.20) are in full agreement with the results from Ref. [9] given by Eqs. (2.70) and (2.71).

The upper D components of Eq. (5.6) produce the same result for T-dual background fields.

5.3 T-dual R–R field strength $F^{\alpha\beta}$

Using the relations ${}_a\mathcal{H} = \mathcal{T}^a\mathcal{H}\mathcal{T}^a$ and ${}_aJ_\pm = \mathcal{T}^aJ_\pm$ we obtained the form of the NS–NS and NS–R T-dual background fields of type II superstring theory. But we know from the Buscher T-dualization procedure that the T-dual R–R field strength ${}_aF^{\alpha\beta}$ has the form given in Eq. (2.72). In this subsection we will derive this relation within the double space framework.

The R–R field strength $F^{\alpha\beta}$ appears in the action (2.13) coupled with the fermionic momenta π_α and $\bar{\pi}_\alpha$ along which we do not perform T-dualization. Therefore, we did not double these variables. It is an analog of the ij-term in approach of Refs. [27–29] where x^i coordinates are not doubled. Consequently, as in [27–29] we should make some assumptions. Let us suppose that the fermionic term $L(\pi_\alpha, \bar{\pi}_\alpha)$ is symmetric under exchange of the R–R field strength $F^{\alpha\beta}$ with its T-dual ${}_aF^{\alpha\beta}$

$$L = e^{\frac{\Phi}{2}}\pi_\alpha F^{\alpha\beta}\bar{\pi}_\beta + e^{\frac{{}_a\Phi}{2}}\ {}_a\pi_\alpha\ {}_aF^{\alpha\beta}\ {}_a\bar{\pi}_\beta \equiv \mathcal{L} + {}_a\mathcal{L}, \tag{5.21}$$

for some $F^{\alpha\beta}$ and ${}_aF^{\alpha\beta}$. This term should be invariant under the T-dual transformation

$${}_a\mathcal{L} = \mathcal{L} + \Delta\mathcal{L}. \tag{5.22}$$

Taking into account the fact that two successive T-dualizations are the identity transformation, we obtain from (5.22)

$$\mathcal{L} = {}_a\mathcal{L} + {}_a\Delta\mathcal{L}. \tag{5.23}$$

Combining the last two relations we get

$${}_a\Delta\mathcal{L} = -\Delta\mathcal{L}. \tag{5.24}$$

If $\Delta\mathcal{L}$ has the form $\Delta\mathcal{L} = \pi_\alpha \Delta^{\alpha\beta}\bar{\pi}_\beta$ and consequently ${}_a\Delta\mathcal{L} = {}_a\pi_\alpha\, {}_a\Delta^{\alpha\beta}\, {}_a\bar{\pi}_\beta$, then with the help of the first relation of Eq. (2.48) we obtain the condition for $\Delta^{\alpha\beta}$

$${}_a\Delta^{\alpha\beta} = -\,\Delta^{\alpha\gamma}\, {}_a\Omega_\gamma{}^\beta. \tag{5.25}$$

Therefore, we should find the combination of background fields with two upper spinor indices which under T-dualization transforms as in (5.25). Using the expression for the NS–R fields (2.70) and the equation $({}_a\hat{\theta}_\pm)_{ab} = \frac{2}{\kappa}\Pi_{\mp ab} = \frac{1}{\kappa^2}(\hat{\theta}_\pm^{-1})_{ab}$ [see the T-dual of (5.4) and (A.10)], it is easy to check that there are D different solutions,

$$\Delta_d^{\alpha\beta} = c\Psi_a^\alpha \hat{\theta}_-^{ab}\bar{\Psi}_b^\beta, \tag{5.26}$$

where $d = 1, 2, \ldots D$ and c is an arbitrary constant. Consequently, when we T-dualize d dimensions x^a ($a = 0, 1, \ldots d-1$), from (5.22) we can conclude that the T-dual R–R field strength has the form

$$e^{\frac{a\Phi}{2}}\, {}_aF^{\alpha\beta} = \left(e^{\frac{\Phi}{2}} F^{\alpha\gamma} + c\Psi_a^\alpha \hat{\theta}_-^{ab}\bar{\Psi}_b^\gamma\right) {}_a\Omega_\gamma{}^\beta. \tag{5.27}$$

For $c = 4\kappa$ we obtain the agreement with Eq. (2.72). Note that the fermionic term $L_d(\pi_\alpha, \bar{\pi}_\alpha)$ depends on d, the number of directions along which we perform T-duality, just in Refs. [27–29].

6 Conclusion

In this article we showed that the new interpretation of the bosonic T-dualization procedure in the double space formalism offered in [35,36] is also valid in the case of type II superstring theory. We used the ghost free action of type II superstring theory in a pure spinor formulation in the approximation of quadratic terms and constant background fields. One can obtain this action from the action (2.7), which could be considered as an expansion in powers of fermionic coordinates. In the first part of the analysis we neglect all terms in the action containing powers of θ^α and $\bar{\theta}^\alpha$. This approximation is justified by the fact that the action is a result of an iterative procedure in which every step results from the previous one. Later, when we discuss proper fermionic variables, taking higher power terms we restore supersymmetric invariants $(\Pi_\pm^\mu, d_\alpha, \bar{d}_\alpha)$ as variables instead of $\partial_\pm x^\mu$, π_α and $\bar{\pi}_\alpha$.

We introduced the double space coordinate $Z^M = (x^\mu, y_\mu)$ adding to all bosonic initial coordinates, x^μ, the T-dual ones, y_μ. Then we rewrote the T-dual transformation laws (3.8) in terms of double space variables (3.12) introducing the generalized metric $\mathcal{H}_{MN}$ and the current $J_{\pm M}$. The generalized metric depends only on the NS–NS background fields of the initial theory. The current $J_{\pm M}$ contains fermionic momenta π_α and $\bar{\pi}_\alpha$, along which we do not make a T-dualization, and it depends also on NS–R background fields. The R–R background fields do not appear in T-dual transformation laws.

The coordinate index μ is split in $a = (0, 1, \ldots d-1)$ and $i = (d, d+1, \ldots D-1)$, where index a marks subsets of the initial and T-dual coordinates, x^a and y_a, along which we make T-dualization. T-dualization is realized as permutation of the subsets x^a and y_a in the double space coordinate Z^M. The main require is that T-dual double space coordinates ${}_aZ^M = (\mathcal{T}^a)^M{}_N Z^N$ satisfy the transformation law of the same form as the initial coordinates Z^M. From this condition we found the T-dual generalized metric ${}_a\mathcal{H}_{MN}$ and the T-dual current ${}_aJ_{\pm M}$. Because the initial and T-dual theory are physically equivalent, ${}_a\mathcal{H}_{MN}$ and ${}_aJ_{\pm M}$ should have the same form as the initial ones, $\mathcal{H}$ and $J_{\pm M}$, but in terms of the T-dual background fields. It produces the form of the NS–NS and NS–R T-dual background fields in terms of the initial ones which are in full accordance with the results obtained by the Buscher T-dualization procedure [9,10].

The supersymmetry case is not a simple generalization of the bosonic one, but it requires some new interesting steps. The origin of the problem is the different T-duality transformations of the world-sheet chirality sectors. It produces two possible sets of vielbeins in the T-dual theory with the same T-dual metric. These vielbeins are related by a particular local Lorentz transformation which depends on T-duality transformation and of which the determinant is $(-1)^d$, where d is the number of T-dualized coordinates. Therefore, when we perform T-dualization along an odd number of coordinates then such transformation contains a parity transformation. Consistency of T-duality with supersymmetry requires changing one of two spinor sectors. We redefine the bar spinor coordinates, ${}_a\bar{\theta} \to {}_a^\bullet\bar{\theta}^\alpha = {}_a\Omega^\alpha{}_\beta\bar{\theta}^\beta$, and the variable ${}_a\bar{\pi}_\alpha$, ${}_a\bar{\pi}_\alpha \to {}_a^\bullet\bar{\pi}_\alpha = {}_a\Omega_\alpha{}^\beta\bar{\pi}_\beta$. As a consequence the bar NS–R and R–R background fields include ${}_a\Omega$ in their T-duality transformations. For an odd number of coordinates d along which T-dualization is performed, ${}_a\Omega$ changes the chirality of the bar gravitino $\bar{\Psi}_\mu^\alpha$ and the chirality condition for $F^{\alpha\beta}$. We need it to relate type IIA and type IIB theories.

The transformation law (3.12) induces the consistency condition which can be considered as equation of motion of the double space action (3.21). It contains an arbitrary term depending on the undualized variables $L(\pi_\alpha, \bar{\pi}_\alpha)$. This is in analogy with the term $\partial_+ x^i \Pi_{+ij}\partial_- x^j$ in the approach presented in Refs. [27–29]. Therefore, to obtain the T-dual transformation of the R–R field strength $F^{\alpha\beta}$ we should make some additional assumptions. Supposing that the term

$L(\pi_\alpha, \bar{\pi}_\alpha)$ is T-dual invariant and taking into account that two successive T-dualizations act as the identity operator, we found the form of the T-dual R–R field strength up to one arbitrary constant c. For $c = 4\kappa$ we get the T-dual R–R field strength ${}_aF^{\alpha\beta}$ as in the Buscher procedure [9].

A T-duality transformation of the R–R field strength $F^{\alpha\beta}$ has two contributions in the form of square roots. The contribution of the dilaton produces the term $\sqrt{|\prod_{i=1}^d G_{a_ia_i}|}$. On the other hand the contribution of the spinorial representation of a Lorentz transformation ${}_a\Omega$ contains the same expression without the absolute value $i^d\sqrt{\prod_{i=1}^d G_{a_ia_i}}$. Therefore, the T-dual R–R field strength ${}_aF^{\alpha\beta}$, besides a rational expression, contains the expression $i^d\sqrt{\text{sign}(\prod_{i=1}^d G_{a_ia_i})}$ (2.89). If we T-dualize along the time-like direction ($G_{00} > 0$), the square root does not produce an imaginary unit i, not canceling the one in front of the square root. Therefore, T-dualization along the time-like direction maps type II superstring theories to type II* ones [37].

The successive T-dualizations make a group called the T-duality group. In the case of type II superstring T-duality, transformations are performed by the same matrices $\mathcal{T}^a$ as in the bosonic string case [35,36]. Consequently, the corresponding T-duality group is the same.

If we want to find a T-dual transformation of $F^{\alpha\beta}$ without any assumptions, we should follow the approach of [35,36] and, besides all bosonic coordinates x^μ, double also all fermionic variables π_α and $\bar{\pi}_\alpha$. In other words, besides bosonic T-duality we should also consider fermionic T-duality [53–59].

Funded by SCOAP[3].

Appendix A: Block-wise expressions for background fields

In order to simplify notation we will introduce notations for the component fields following Ref. [35].

For block-wise matrices there is a rule for inversion,

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix}^{-1} = \begin{pmatrix} (A - BD^{-1}C)^{-1} & -A^{-1}B(D - CA^{-1}B)^{-1} \\ -D^{-1}C(A - BD^{-1}C)^{-1} & (D - CA^{-1}B)^{-1} \end{pmatrix}. \tag{A.1}$$

For the metric tensor and the Kalb–Ramond background fields we define

$$G_{\mu\nu} = \begin{pmatrix} \tilde{G}_{ab} & G_{aj} \\ G_{ib} & \bar{G}_{ij} \end{pmatrix} \equiv \begin{pmatrix} \tilde{G} & G^T \\ G & \bar{G} \end{pmatrix} \tag{A.2}$$

and

$$B_{\mu\nu} = \begin{pmatrix} \tilde{b}_{ab} & b_{aj} \\ b_{ib} & \bar{b}_{ij} \end{pmatrix} \equiv \begin{pmatrix} \tilde{b} & -b^T \\ b & \bar{b} \end{pmatrix}. \tag{A.3}$$

We also define the notation for inverse of the metric,

$$(G^{-1})^{\mu\nu} = \begin{pmatrix} \tilde{\gamma}^{ab} & \gamma^{aj} \\ \gamma^{ib} & \bar{\gamma}^{ij} \end{pmatrix} \equiv \begin{pmatrix} \tilde{\gamma} & \gamma^T \\ \gamma & \bar{\gamma} \end{pmatrix}, \tag{A.4}$$

and for the effective metric

$$G^E_{\mu\nu} = G_{\mu\nu} - 4B_{\mu\rho}(G^{-1})^{\rho\sigma} \quad B_{\mu\nu} = \begin{pmatrix} \tilde{g}_{ab} & g_{aj} \\ g_{ib} & \bar{g}_{ij} \end{pmatrix} \equiv \begin{pmatrix} \tilde{g} & g^T \\ g & \bar{g} \end{pmatrix}. \tag{A.5}$$

Note that because $G^{\mu\nu}$ is the inverse of $G_{\mu\nu}$ we have

$$\begin{aligned} &\gamma = -\tilde{G}^{-1}G\tilde{\gamma} = -\bar{\gamma}G\tilde{G}^{-1}, \\ &\gamma^T = -\tilde{G}^{-1}G^T\bar{\gamma} = -\tilde{\gamma}G^T\bar{G}^{-1}, \\ &\tilde{\gamma} = (\tilde{G} - G^T\bar{G}^{-1}G)^{-1}, \quad \bar{\gamma} = (\bar{G} - G\tilde{G}^{-1}G^T)^{-1}, \\ &\tilde{G}^{-1} = \tilde{\gamma} - \gamma^T\bar{\gamma}^{-1}\gamma, \quad \bar{G}^{-1} = \bar{\gamma} - \gamma\tilde{\gamma}^{-1}\gamma^T. \end{aligned} \tag{A.6}$$

It is also useful to introduce a new notation for the expression

$$(BG^{-1})_\mu{}^\nu = \begin{pmatrix} \tilde{b}\tilde{\gamma} - b^T\gamma & \tilde{b}\gamma^T - b^T\bar{\gamma} \\ b\tilde{\gamma} + \bar{b}\gamma & b\gamma^T + \bar{b}\bar{\gamma} \end{pmatrix} \equiv \begin{pmatrix} \tilde{\beta} & \beta_1 \\ \beta_2 & \bar{\beta} \end{pmatrix}. \tag{A.7}$$

We denote by a hat expressions similar to the effective metric (A.5) and non-commutativity parameters but with all contributions from the ab subspace,

$$\hat{g}_{ab} = (\tilde{G} - 4\tilde{b}\tilde{G}^{-1}\tilde{b})_{ab}, \quad \hat{\theta}^{ab} = -\frac{2}{\kappa}(\hat{g}^{-1}\tilde{b}\tilde{G}^{-1})^{ab}. \tag{A.8}$$

Note that $\hat{g}_{ab} \neq \tilde{g}_{ab}$ because $\tilde{g}_{ab}$ is a projection of $g_{\mu\nu}$ on the subspace ab. It is extremely useful to introduce background field combinations,

$$\begin{aligned} &\Pi_{\pm ab} = B_{ab} \pm \frac{1}{2}G_{ab} \\ &\hat{\theta}^{ab}_\pm = -\frac{2}{\kappa}(\hat{g}^{-1}\tilde{\Pi}_\pm\tilde{G}^{-1})^{ab} = \hat{\theta}^{ab} \mp \frac{1}{\kappa}(\hat{g}^{-1})^{ab}, \end{aligned} \tag{A.9}$$

which are inverses to each other

$$\hat{\theta}^{ac}_\pm\Pi_{\mp cb} = \frac{1}{2\kappa}\delta^a_b. \tag{A.10}$$

The quantity A_{ab} is defined as

$$A_{ab} = (\tilde{g} - 4\beta_1\bar{\gamma}^{-1}\beta_1^T)_{ab}. \tag{A.11}$$

One can prove the relation [35]

$$(\tilde{g}^{-1}\beta_1 D^{-1})^a{}_i = (\hat{g}^{-1}\beta_1\bar{\gamma}^{-1})^a{}_i, \tag{A.12}$$

where D^{ij} is defined in Eq. (3.21) of [35],

$$D^{ij} = (\bar{\gamma} - 4\beta_1^T \tilde{g}^{-1}\beta_1)^{ij},$$
$$(D^{-1})_{ij} = \left(\bar{\gamma}^{-1} + 4\bar{\gamma}^{-1}\beta_1^T A^{-1}\beta_1\bar{\gamma}^{-1}\right)_{ij}. \quad \text{(A.13)}$$

References

1. K. Becker, M. Becker, J. Schwarz, *String Theory and M-Theory: A Modern Introduction* (Cambridge University Press, Cambridge, 2007)
2. B. Zwiebach, *A First Course in String Theory* (Cambridge University Press, Cambridge, 2004)
3. J. Polchinski, *String Theory*, vol. II (Cambridge University Press, Cambridge, 1998)
4. T. Buscher, Phys. Lett. B **194**, 59 (1987)
5. T. Buscher, Phys. Lett. **201**, 466 (1988)
6. M. Roček, E. Verlinde, Nucl. Phys. B **373**, 630 (1992)
7. A. Giveon, M. Porrati, E. Rabinovici, Phys. Rep. **244**, 77 (1994)
8. E. Alvarez, L. Alvarez-Gaume, J. Barbon, Y. Lozano, Nucl. Phys. B **415**, 71 (1994)
9. B. Nikolić, B. Sazdović, Nucl. Phys. B **836**, 100 (2010)
10. D.S. Berman, D.C. Thompson, Phys. Rep. **566**, 1–60 (2014)
11. D. Lust, JHEP **12**, 084 (2010)
12. D. Andriot, M. Larfors, D. Luest, P. Patalong, JHEP **06**, 021 (2013)
13. D. Andriot, O. Hohm, M. Larfors, D. Lust, P. Patalong, Phys. Rev. Lett. **108**, 261602 (2012)
14. D. Lüst, in *Proceedings of the Corfu Summer Institute 2011 School and Workshops on Elementary Particle Physics and Gravity*. September 4–18, 2011. Corfu, Greece. arXiv:1205.0100 [hep-th]
15. R. Blumenhagen, A. Deser, D. Luest, E. Plauschinn, F. Rennecke, J. Phys. A **44**, 385401 (2011)
16. C. Condeescu, I. Florakis, D. Luest, JHEP **04**, 121 (2012)
17. J. Shelton, W. Taylor, B. Wecht, JHEP **10**, 085 (2005)
18. A. Dabholkar, C. Hull, JHEP **05**, 009 (2006)
19. Lj. Davidović, B. Sazdović, EPJ C **74**, 2683 (2014)
20. Lj. Davidović, B. Nikolić, B. Sazdović, EPJ C **75**, 576 (2015)
21. Lj. Davidović, B. Nikolić, B. Sazdović, EPJ C **74**, 2734 (2014)
22. M. Duff, Nucl. Phys. B **335**, 610 (1990)
23. A.A. Tseytlin, Phys. Lett. B **242**, 163 (1990)
24. A.A. Tseytlin, Nucl. Phys. B **350**, 395 (1991)
25. W. Siegel, Phys. Rev. D **48**, 2826 (1993)
26. W. Siegel, Phys. Rev. D **47**, 5453 (1993)
27. C.M. Hull, JHEP **10**, 065 (2005)
28. C.M. Hull, JHEP **10**, 057 (2007)
29. C.M. Hull, JHEP **07**, 080 (2007)
30. D.S. Berman, M. Cederwall, M.J. Perry, JHEP **09**, 066 (2014)
31. D.S. Berman, C.D.A. Blair, E. Malek, M.J. Perry, Int. J. Mod. Phys. A **29**(15), 1450080 (2014)
32. C.D.A. Blair, E. Malek, A.J. Routh, Class. Quantum Gravity **31**(20), 205011 (2014)
33. C.M. Hull, R.A. Reid-Edwards, JHEP **09**, 014 (2009)
34. O. Hohm, B. Zwiebach, JHEP **11**, 075 (2014)
35. B. Sazdović, T-duality as coordinates permutation in double space. arXiv:1501.01024. doi:10.1088/1674-1137/41/5/053101
36. B. Sazdović, JHEP **08**, 055 (2015)
37. C.M. Hull, JHEP **07**, 021 (1998)
38. N. Berkovits, ICTP Lectures on Covariant Quantisation of Superstrings, arXiv:hep-th/0209059
39. P.A. Grassi, G. Policastro, P. van Nieuwenhuizen, JHEP **10**, 054 (2002)
40. P.A. Grassi, G. Policastro, P. van Nieuwenhuizen, JHEP **11**, 004 (2002)
41. P.A. Grassi, G. Policastro, P. van Nieuwenhuizen, Adv. Theor. Math. Phys. **7**, 499 (2003)
42. P.A. Grassi, G. Policastro, P. van Nieuwenhuizen, Phys. Lett. B **553**, 96 (2003)
43. J. de Boer, P.A. Grassi, P. van Nieuwenhuizen, Phys. Lett. B **574**, 98 (2003)
44. N. Berkovits, P. Howe, Nucl. Phys. B **635**, 75 (2002)
45. P.A. Grassi, L. Tamassia, JHEP **07**, 071 (2004)
46. S.F. Hassan, Nucl. Phys. B **568**, 145 (2000)
47. R. Benichou, G. Policastro, J. Troost, Phys. Lett. B **661**, 129 (2008)
48. M.J. Duff, TASI Lectures on Branes, Black Holes and Anti-de Sitter Space, arXiv:hep-th/9912164v2
49. M.J. Duff, R.R. Khuri, J.X. Lu, Phys. Rep. **259**, 213 (1995)
50. E. Kiritsis, *Introduction to Superstring Theory* (Leuven University Press, Leuven, 1998). arXiv:hep-th/9709062
51. B. Nikolić, B. Sazdović, Phys. Lett. B **666**, 400 (2008)
52. D.C. Thompson, *T-Duality Invariant Approaches to String Theory*, PhD thesis. arXiv:1012.4393
53. N. Berkovits, J. Maldacena, JHEP **09**, 062 (2008)
54. K. Sfetsos, K. Siampos, D.C. Thompson, Class. Quantum Gravity **28**, 055010 (2011)
55. I. Bakhmatov, D.S. Berman, Nucl. Phys. B **832**, 89–108 (2010)
56. N. Beisert, R. Ricci, A.A. Tseytlin, M. Wolf, Phys. Rev. D **78**, 126004 (2008)
57. R. Ricci, A.A. Tseytlin, M. Wolf, JHEP **12**, 082 (2007)
58. B. Nikolić, B. Sazdović, Phys. Rev. D **84**, 065012 (2011)
59. B. Nikolić, B. Sazdović, JHEP **06**, 101 (2012)
60. A. Giveon, M. Roček, Nucl. Phys. B **421**, 173 (1994)

Fortschritte der Physik
Progress of Physics
www.fp-journal.org

Noncommutativity and Nonassociativity of Closed Bosonic String on T-dual Toroidal Backgrounds

B. Nikolić and D. Obrić*

In this article we consider closed bosonic string in the presence of constant metric and Kalb-Ramond field with one non-zero component, $B_{xy} = Hz$, where field strength H is infinitesimal. Using Buscher T-duality procedure we dualize along x and y directions and using generalized T-duality procedure along z direction imposing trivial winding conditions. After first two T-dualizations we obtain Q flux theory which is just locally well defined, while after all three T-dualizations we obtain nonlocal R flux theory. Origin of non-locality is variable ΔV defined as line integral, which appears as an argument of the background fields. Rewriting T-dual transformation laws in the canonical form and using standard Poisson algebra, we obtained that Q flux theory is commutative one and the R flux theory is noncommutative and nonassociative one. Consequently, there is a correlation between non-locality and closed string noncommutativity and nonassociativity.

1. Introduction

Coordinate noncommutativity means that there exists minimal possible length, which imposes natural UV cutoff. Idea of coordinate noncommutativity is very old. Heisenberg suggested coordinate noncommutativity to solve the problem of the occurrence of infinite quantities before renormalization procedure was developed and accepted. The first scientific paper considering this subject appeared 1947[1] where construction of discrete Lorentz invariant space-time is presented. Later in the period of 1980s A. Connes developed noncommutative geometry as a generalization of the standard commutative geometry.[2]

Noncommutativity became again interesting for particle physicists when the paper[3] appeared. In this article it is shown using propagators that open string endpoints in the presence of the constant metric and Kalb-Ramond field become noncommutative. D-brane on which the string endpoints are forced to move becomes noncommutative manifold. After this article many articles[4] appeared addressing the same subject but using different approaches - Fourier expansion, canonical methods, solving of boundary conditions etc.

B. Nikolić
Institute of Physics Belgrade
University of Belgrade
Pregrevica 118, Belgrade, Serbia
E-mail: bnikolic@ipb.ac.rs

D. Obrić
Faculty of Physics
University of Belgrade
Studentski trg 12, Belgrade, Serbia

DOI: 10.1002/prop.201800009

In the last two articles of[4] the method of solving of boundary conditions is presented. The basic idea is that open string boundary condition is treated as canonical constraint. Investigating the consistency of the canonical constraint we obtained the σ dependent form of the boundary condition. Further, we can proceed twofold: to introduce Dirac brackets or solve the constraint. Solving the constraint, we obtained the initial coordinate as a linear combination of the effective coordinate and momenta. Consequently, initial coordinates are noncommutative and the main contribution to noncomutativity parameter comes from Kalb-Ramond field as it was expected.

Following the result of the article[5] it can be proven that gauge fields "live" at the open string endpoints. Consequently, many interesting papers concerning non-commutative Yang-Mills theories and their renormalisability appeared.[6] In the papers[7] cross sections for some decays, allowed in noncommutative Yang-Mills theories and forbidden in commutative ones, are calculated, which offers a possibility of the experimental check of the noncommutativity idea and further, indirectly, idea of strings.

It is obvious that closed bosonic string in the presence of constant background fields remains commutative. There are no boundaries and, consequently, boundary conditions constraining string dynamics. In the case of open string we obtained initial coordinate in the form of linear combination of effective coordinates and momenta using boundary condition. That is achieved in the closed string case[8] using T-duality procedure and coordinate dependent background.

T-duality as a fundamental feature of string theory,[9–15] unexperienced by point particle, makes that there is no physical difference between string theory compactified on a circle of radius R and circle of radius $1/R$. Buscher T-dualization procedure[10] represents a mathematical frame in which T-dualization is realized. If the background fields do not depend on some coordinates then those coordinates are isometry directions. Consequently, that symmetry can be localized replacing ordinary world-sheet derivatives $\partial_\pm$ by covariant ones $D_\pm x^\mu = \partial_\pm x^\mu + v_\pm^\mu$, where $v_\pm^\mu$ are gauge fields. In order to make T-dual theory has the same number of degrees of freedom, the new term with Lagrange multipliers is added to the action which forces the gauge fields to be unphysical degrees of freedom. Because of the shift symmetry, using gauge freedom we fix initial coordinates. Variation of this gauge fixed action with respect to the Lagrange multipliers produces initial action and with respect to the gauge fields produces T-dual action.

Standard Buscher T-dualization was applied in closed string case in the papers.[8,16–19] In Ref. [16] authors consider 3-torus in the presence of constant metric and Kalb-Ramond field with one nonzero component $B_{xy} = Hz$, where field strength H is infinitesimal. They systematically apply Buscher procedure and, after two T-dualizations along isometry directions, obtain theory with Q flux which is noncommutative. In the calculations they used nontrivial boundary conditions (winding conditions). The result is that T-dual closed string coordinates are noncommutative for the same values of parameters $\sigma = \bar{\sigma}$ with noncommutativity parameter proportional to field strength H and N_3, winding number for z coordinate.

But, except this standard Buscher procedure, there is a generalized Buscher procedure dealing with background fields depending on all coordinates. The generalized procedure was applied to the case of bosonic string moving in the weakly curved background[20–22] and in the case where metric is quadratic in coordinates and Kalb-Ramond field is linear function of coordinates.[23] The generalized procedure enables us to make T-dualization in mentioned cases along arbitrary subset of coordinates.

Double space is one picturesque framework for representation of T-duality. Double space is introduced two to three decades ago.[24–28] It is spanned by double coordinates $Z^M = (x^\mu, y_\mu)$ $(\mu = 0, 1, 2, \dots, D-1)$, where x^μ are the coordinates of the initial theory and y_μ are T-dual coordinates. In this space T-dualization is represented as $O(d, d)$ transformation.[29–33] Permutation of the appropriate subsets of the initial and T-dual coordinates is interpreted as partial T-dualization[34,35] expanding Duff's idea.[24] The newly invented intrinsic noncommutativity[36] is related to double space. Intrinsic noncommutativity exists in the constant background case because it is considered within double space framework.

In this article we will deal with closed bosonic string propagating in the constant metric and linear dependent Kalb-Ramond field with $B_{xy} = Hz$, the same background as in [16]. This configuration is known in literature as torus with H-flux. As in the Ref. [16] we will use approximation of diluted flux, which means that in all calculations we keep constant and linear terms in infinitesimal field strength H. Transformation laws, relations which connect initial and T-dual variables, we will write in canonical form expressing initial momenta in terms of the T-dual coordinates. Unlike Ref. [16], except T-dualization along two isometry directions, we will make one step more and T-dualize along z coordinate using generalized T-dualization procedure. During dualization procedure we will use trivial boundary (winding) conditions.

Transformation laws in canonical form enable us to express sigma derivative of the T-dual coordinate as a linear combination of the initial momenta and coordinates. Because initial theory is geometrical locally and globally, its coordinates and canonically conjugated momenta satisfy standard Poisson algebra. This fact means that we can calculate the Poisson brackets of the T-dual coordinates using technical instruction given in subsection 4.1.

After T-dualizations along isometry directions (along x and y) we obtain the same background as in Ref. [16] but, obtained Q flux theory, which is still locally well defined, is commutative. This is a consequence of the imposed trivial winding conditions. Having in mind the generalized T-duality procedure,[20,21,23] T-dualization along z coordinate produces R flux nonlocal theory because it depends on the variable ΔV which is defined as line integral. Calculating Poisson brackets of the T-dual coordinates we obtain two nonzero Poisson brackets and show that there is a correlation between non-locality and closed string noncommutativity.

The form of noncommutativity is such that it exists when arguments of the coordinates are different, $\sigma \neq \bar{\sigma}$. That is another difference with respect to the result of Ref. [16] but there is no contradiction because the origins of noncommutativity are different. In this article non-locality is related with noncommutativity of R flux theory under trivial winding conditions while in Ref. [16] it is about noncommutativity of Q flux theory under nontrivial winding conditions.

From the noncommutativity relations it follows that Jacobi identity is broken i.e. nonassociativity occurs. Nonassociativity parameter, R flux, is proportional to the field strength H. Using generalized T-duality[20,21,23] we obtain the concrete form of nonassociativity from string dynamics. Similar as noncommutativity, discovery of nonassociativity pushes the scientists to explore the effects of nonassociativity in the field of renormalisability of ϕ^4 theory[37] as well as formulation of nonassociative gravity.[38]

At the end we add an appendix containing some conventions used in the paper.

2. Bosonic String Action and Choice of Background Fields

The action of the closed bosonic string in the presence of the space-time metric $G_{\mu\nu}(x)$, Kalb-Ramond antisymmetric field $B_{\mu\nu}(x)$, and dilaton scalar field $\Phi(x)$ is given by the following expression[9]

$$S = \kappa \int_\Sigma d^2\xi \sqrt{-g} \times \left\{ \left[\frac{1}{2} g^{\alpha\beta} G_{\mu\nu}(x) + \frac{\varepsilon^{\alpha\beta}}{\sqrt{-g}} B_{\mu\nu}(x) \right] \partial_\alpha x^\mu \partial_\beta x^\nu + \Phi(x) R^{(2)} \right\}, \tag{2.1}$$

where Σ is the world-sheet surface parameterized by $\xi^\alpha = (\tau, \sigma)$ $[(\alpha = 0, 1),\ \sigma \in (0, \pi)]$, while the D-dimensional space-time is spanned by the coordinates x^μ $(\mu = 0, 1, 2, \dots, D-1)$. We denote intrinsic world sheet metric with $g_{\alpha\beta}$, and the corresponding scalar curvature with $R^{(2)}$.

In order to keep conformal symmetry on the quantum level background fields must obey space-time field equations[39]

$$\beta^G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{4} B_{\mu\rho\sigma} B_\nu{}^{\rho\sigma} + 2 D_\mu a_\nu = 0, \tag{2.2}$$

$$\beta^B_{\mu\nu} \equiv D_\rho B^\rho{}_{\mu\nu} - 2 a_\rho B^\rho{}_{\mu\nu} = 0, \tag{2.3}$$

$$\beta^\Phi \equiv 2\pi\kappa \frac{D-26}{6} - R - \frac{1}{24} B_{\mu\rho\sigma} B^{\mu\rho\sigma} - D_\mu a^\mu + 4a^2 = c, \tag{2.4}$$

where c is an arbitrary constant. The function β^{Φ} could be a constant because of the relation

$$D^{\nu}\beta^{G}_{\nu\mu} + \partial_\mu \beta^{\Phi} = 0. \tag{2.5}$$

Further, $R_{\mu\nu}$ and D_μ are Ricci tensor and covariant derivative with respect to the space-time metric $G_{\mu\nu}$, while

$$B_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} + \partial_\nu B_{\rho\mu} + \partial_\rho B_{\mu\nu}, \quad a_\mu = \partial_\mu \Phi, \tag{2.6}$$

are field strength for Kalb-Ramond field $B_{\mu\nu}$ and dilaton gradient, respectively. Trivial solution of these equations is that all three background fields are constant. This case was pretty exploited in the analysis of the open string noncommutativity.

The less trivial case would be a case where some background fields are coordinate dependent. If we choose Kalb-Ramond field to be linearly coordinate dependent and dilaton field to be constant then the first equation (2.2) becomes

$$R_{\mu\nu} - \frac{1}{4} B_{\mu\rho\sigma} B_{\nu}{}^{\rho\sigma} = 0. \tag{2.7}$$

The field strength $B_{\mu\nu\rho}$ is constant and, if we assume that it is infinitesimal, then we can take $G_{\mu\nu}$ to be constant in approximation linear in $B_{\mu\nu\rho}$. Consequently, all three space-time field equations are satisfied. Especially, the third one is of the form

$$2\pi\kappa \frac{D-26}{6} = c, \tag{2.8}$$

which enables us to work in arbitrary number of space-time dimensions.

In this article we will work in $D = 3$ dimensions with the following choice of background fields

$$G_{\mu\nu} = \begin{pmatrix} R_1^2 & 0 & 0 \\ 0 & R_2^2 & 0 \\ 0 & 0 & R_3^2 \end{pmatrix}, \quad B_{\mu\nu} = \begin{pmatrix} 0 & Hz & 0 \\ -Hz & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{2.9}$$

where $R_\mu (\mu = 1, 2, 3)$ are radii of the compact dimensions. This choice of background fields is known in geometry as torus with flux (field strength) H.[16] Our choice of infinitesimal H can be understood in terms of the radii as that

$$\left(\frac{H}{R_1 R_2 R_3}\right)^2 = 0. \tag{2.10}$$

This approximation is known in literature as the approximation of diluted flux. Physically, this means that we work with the torus which is sufficiently large. Consequently, we can rescale the coordinates

$$x^\mu \longmapsto \frac{x^\mu}{R_\mu}, \tag{2.11}$$

which simplifies the form of the metric

$$G_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{2.12}$$

The final form of the closed bosonic string action is

$$\begin{aligned} S &= \kappa \int_\Sigma d^2\xi\, \partial_+ x^\mu \Pi_{+\mu\nu} \partial_- x^\nu \\ &= \kappa \int_\Sigma d^2\xi \left[\frac{1}{2}(\partial_+ x \partial_- x + \partial_+ y \partial_- y + \partial_+ z \partial_- z) \right. \\ &\quad \left. + \partial_+ x Hz \partial_- y - \partial_+ y Hz \partial_- x\right], \end{aligned} \tag{2.13}$$

where $\partial_\pm = \partial_\tau \pm \partial_\sigma$ is world-sheet derivative with respect to the light-cone coordinates $\xi^\pm = \frac{1}{2}(\tau \pm \sigma)$, $\Pi_{\pm\mu\nu} = B_{\mu\nu} \pm \frac{1}{2} G_{\mu\nu}$ and

$$x^\mu = \begin{pmatrix} x \\ y \\ z \end{pmatrix}. \tag{2.14}$$

Let us note that we do not write dilaton term because its T-dualization is performed separately within quantum formalism and here will be skipped.

3. T-dualization of the Bosonic Closed String Action

In this section we will perform T-dualization along three directions, one direction at time. Our goal is to find the relations connecting initial variables with T-dual ones called transformation laws. Using transformation laws we will find noncommutativity and nonassociativity relations.

3.1. T-dualization Along x Direction – from Torus with H Flux to the Twisted Torus

Let us perform standard Buscher T-dualization[10] of action (2.13) along x direction. Note that x direction is an isometry direction which means that action has a global shift symmetry, $x \longrightarrow x + a$. In order to perform Buscher procedure, we have to localize this symmetry introducing covariant world-sheet derivatives instead of the ordinary ones

$$\partial_\pm x \longrightarrow D_\pm x = \partial_\pm x + v_\pm, \tag{3.1}$$

where $v_\pm$ are gauge fields which transform as $\delta v_\pm = -\partial_\pm a$. Because T-dual action must have the same number of degrees of freedom as initial one, we have to make these fields $v_\pm$ be unphysical degrees of freedom. This is accomplished by adding following term to the action

$$S_{add} = \frac{\kappa}{2} \int_\Sigma d^2\xi\, y_1(\partial_+ v_- - \partial_- v_+), \tag{3.2}$$

where y_1 is a Lagrange multiplier. After gauge fixing, $x = const.$, the action gets the form

$$\begin{aligned} S_{fix} = \kappa \int d^2\xi \left[\frac{1}{2}(v_+ v_- + \partial_+ y \partial_- y + \partial_+ z \partial_- z) + v_+ Hz \partial_- y \right. \\ \left. - \partial_+ y Hz v_- + \frac{1}{2} y_1 (\partial_+ v_- - \partial_- v_+)\right]. \end{aligned} \tag{3.3}$$

From the equations of motion for y_1 we obtain that field strength for the gauge field $v_\pm$ is equal to zero

$$F_{+-} = \partial_+ v_- - \partial_- v_+ = 0, \tag{3.4}$$

which gives us the solution for gauge field

$$v_\pm = \partial_\pm x. \tag{3.5}$$

Inserting this solution for gauge field into gauge fixed action (3.3) we obtain initial action given by Eq. (2.13). Equations of motion for $v_\pm$ will lead to the T-dual action. Varying the gauge fixed action (3.3) with respect to the gauge field v_+ we get

$$v_- = -\partial_- y_1 - 2Hz\partial_- y, \tag{3.6}$$

while on the equation of motion for v_- it holds

$$v_+ = \partial_+ y_1 + 2Hz\partial_+ y. \tag{3.7}$$

Inserting relations (3.6) and (3.7) into expression for gauge fixed action (3.3), keeping terms linear in H, we obtain the T-dual action

$${}_xS = \kappa \int_\Sigma d^2\xi\, \partial_+ ({}_xX)^\mu\, {}_x\Pi_{+\mu\nu} \partial_- ({}_xX)^\nu, \tag{3.8}$$

where subscript $_x$ denotes quantity obtained after T-dualization along x direction and

$${}_xX^\mu = \begin{pmatrix} y_1 \\ y \\ z \end{pmatrix}. \tag{3.9}$$

Further we have the T-dual background fields

$${}_x\Pi_{+\mu\nu} = {}_xB_{\mu\nu} + \frac{1}{2}\,{}_xG_{\mu\nu}, \quad {}_xB_{\mu\nu} = 0,$$

$${}_xG_{\mu\nu} = \begin{pmatrix} 1 & 2Hz & 0 \\ 2Hz & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{3.10}$$

Obtained background fields (3.10) define that what is known in literature as *twisted torus geometry*. String theory after one T-dualization is geometrically well defined globally and locally or, simply, theory is geometrical (flux H takes the role of connection).

Combining the solutions of equations of motion for Lagrange multiplier (3.5) and for gauge fields, (3.6) and (3.7), we get the transformation laws connecting initial, x^μ, and T-dual, ${}_xX^\mu$, coordinates

$$\partial_\pm x \cong \pm\partial_\pm y_1 \pm 2Hz\partial_\pm y, \tag{3.11}$$

where $\cong$ denotes T-duality relation. The momentum π_x is canonically conjugated to the initial coordinate x. Using the initial action (2.13) we get

$$\pi_x = \frac{\delta S}{\delta \dot{x}} = \kappa(\dot{x} - 2Hzy'), \tag{3.12}$$

where $\dot{A} \equiv \partial_\tau A$ and $A' \equiv \partial_\sigma A$. From transformation law (3.11) it is straightforward to obtain

$$\dot{x} \cong y_1' + 2Hzy', \tag{3.13}$$

which, inserted in the expression for momentum π_x, gives transformation law in canonical form

$$\pi_x \cong \kappa y_1'. \tag{3.14}$$

3.2. From Twisted Torus to Non-geometrical Q Flux

In this subsection we will continue the T-dualization of action (3.8) along y direction. After x and y T-dualization we obtain the structure which has local geometrical interpretation but global omissions. Such structure is known in literature as non-geometry.

We repeat the procedure from the previous subsection and form the gauge fixed action

$$S_{fix} = \kappa \int_\Sigma d^2\xi \left[\frac{1}{2}(\partial_+ y_1 \partial_- y_1 + v_+ v_- + \partial_+ z \partial_- z) + \partial_+ y_1 Hz v_- \right.$$
$$\left. + v_+ Hz \partial_- y_1 + \frac{1}{2} y_2 (\partial_+ v_- - \partial_- v_+)\right]. \tag{3.15}$$

From the equation of motion for Lagrange multiplier y_2

$$\partial_+ v_- - \partial_- v_+ = 0 \longrightarrow v_\pm = \partial_\pm y, \tag{3.16}$$

gauge fixed action becomes initial one (3.8). Varying the gauge fixed action (3.15) with respect to the gauge fields we get

$$v_\pm = \pm\partial_\pm y_2 - 2Hz\partial_\pm y_1. \tag{3.17}$$

Inserting these expressions for gauge fields into gauge fixed action, keeping the terms linear in H, gauge fixed action is driven into T-dual action

$${}_{xy}S = \kappa \int d^2\xi\, \partial_+ ({}_{xy}X)^\mu\, {}_{xy}\Pi_{+\mu\nu} \partial_- ({}_{xy}X)^\nu, \tag{3.18}$$

where

$$({}_{xy}X)^\mu = \begin{pmatrix} y_1 \\ y_2 \\ z \end{pmatrix},$$

$${}_{xy}\Pi_{+\mu\nu} = {}_{xy}B_{\mu\nu} + \frac{1}{2}\,{}_{xy}G_{\mu\nu} = \begin{pmatrix} \frac{1}{2} & -Hz & 0 \\ Hz & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} \end{pmatrix}. \tag{3.19}$$

Explicit expressions for background fields are

$${}_{xy}B_{\mu\nu} = \begin{pmatrix} 0 & -Hz & 0 \\ Hz & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = -B_{\mu\nu}, \quad {}_{xy}G_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{3.20}$$

Let us note that background fields obtained after two T-dualizations are similar to the geometric background of torus with H flux, but they should be considered only locally. Their global properties are non-trivial and because of that the term "non-geometry" is introduced.

Combining the equations of motion for Lagrange multiplier y_2 and for gauge fields $v_\pm$, we obtain T-dual transformation laws

$$\partial_\pm y \cong \pm\partial_\pm y_2 - 2Hz\partial_\pm y_1. \tag{3.21}$$

The y component of the initial canonical momentum π_y is a variation of the initial action with respect to the $\dot{y}$

$$\pi_y = \frac{\delta S}{\delta \dot{y}} = \kappa(\dot{y} + 2Hzx'). \tag{3.22}$$

Using T-dual transformation laws (3.21) we easily get

$$\dot{y} \cong y_2' - 2Hz\dot{y}_1, \tag{3.23}$$

while from the transformation law (3.11), at zeroth order in H, it holds $x' \cong \dot{y}_1$. Inserting last two expression into π_y we obtain transformation law in canonical form

$$\pi_y \cong \kappa y_2'. \tag{3.24}$$

After two T-dualizations along isometry directions, in the approximation of the diluted flux (keeping just terms linear in H), according to the canonical forms of the transformation laws (3.14) and (3.24), we see that T-dual coordinates y_1 and y_2 are still commutative. This is a consequence of the simple fact that variables of the initial theory, which is geometrical one, satisfy standard Poisson algebra

$$\left\{x^\mu(\sigma), \pi_\nu(\bar{\sigma})\right\} = \delta^\mu{}_\nu\delta(\sigma - \bar{\sigma}), \quad \{x^\mu, x^\nu\} = \left\{\pi_\mu, \pi_\nu\right\} = 0, \tag{3.25}$$

where

$$\pi_\mu = \begin{pmatrix} \pi_x \\ \pi_y \\ \pi_z \end{pmatrix}. \tag{3.26}$$

3.3. From *Q* to *R* Flux – T-dualization Along *z* Coordinate

In this subsection we will finalize the process of T-dualization dualizing along remaining z direction. For this purpose we will use generalized T-dualization procedure.[20,21,23] The result is a theory which is not well defined even locally and is known in literature as theory with R-flux.

We start with the action obtained after T-dualizations along x and y directions (3.18). The Kalb-Ramond field (3.20) depends on z and it seems that it is not possible to perform T-dualization. Let us assume that Kalb-Ramond field linearly depends on all coordinates, $B_{\mu\nu} = b_{\mu\nu} + \frac{1}{3}B_{\mu\nu\rho}x^\rho$ and check if some global transformation can be treated as isometry one. We start with global shift transformation

$$\delta x^\mu = \lambda^\mu, \tag{3.27}$$

and make a variation of action

$$\begin{aligned}\delta S &= \frac{\kappa}{3}B_{\mu\nu\rho}\lambda^\rho\int_\Sigma d^2\xi\,\partial_+x^\mu\partial_-x^\nu \\ &= \frac{2k}{3}B_{\mu\nu\rho}\lambda^\rho\epsilon^{\alpha\beta}\int_\Sigma d^2\xi[\partial_\alpha(x^\mu\partial_\beta x^\nu) - x^\mu(\partial_\alpha\partial_\beta x^\nu)].\end{aligned} \tag{3.28}$$

The second term vanishes explicitly, while the first term is surface one. Consequently, in the case of constant metric and linearly dependent Kalb-Ramond field, global shift transformation is an isometry transformation. This means that we can make T-dualization along z coordinate using generalized T-dualization procedure.

The generalized T-dualization procedure is presented in detail in Ref. [20]. In order to localize shift symmetry of the action (3.18) along z direction we introduce covariant derivative

$$\partial_\pm z \longrightarrow D_\pm z = \partial_\pm z + v_\pm, \tag{3.29}$$

which is a part of the standard Buscher procedure. The novelty is introduction of the invariant coordinate as line integral

$$\begin{aligned}z^{inv} &= \int_P d\xi^\alpha D_\alpha z \\ &= \int_P d\xi^+ D_+z + \int_P d\xi^- D_-z = z(\xi) - z(\xi_0) + \Delta V,\end{aligned} \tag{3.30}$$

where

$$\Delta V = \int_P d\xi^\alpha v_\alpha = \int_P (d\xi^+ v_+ + d\xi^- v_-). \tag{3.31}$$

Here ξ and ξ_0 are the current and initial point of the world-sheet line P. At the end, as in the standard Buscher procedure, in order to make $v_\pm$ to be unphysical degrees of freedom we add to the action term with Lagrange multiplier

$$S_{add} = \frac{\kappa}{2}\int_\Sigma d^2\xi\; y_3(\partial_+v_- - \partial_+v_-). \tag{3.32}$$

The final form of the action is

$$\begin{aligned}\bar{S} = \kappa\int_\Sigma d^2\xi\Big[&-Hz^{inv}(\partial_+y_1\partial_-y_2 - \partial_+y_2\partial_-y_1) \\ &+ \frac{1}{2}(\partial_+y_1\partial_-y_1 + \partial_+y_2\partial_-y_2 + D_+zD_-z) \\ &+ \frac{1}{2}\,y_3(\partial_+v_- - \partial_-v_+)\Big].\end{aligned} \tag{3.33}$$

Because of existing shift symmetry we fix the gauge, $z(\xi) = z(\xi_0)$, and then the gauge fixed action takes the form

$$S_{fix} = \kappa \int_\Sigma d^2\xi \Big[- H\Delta V(\partial_+ y_1 \partial_- y_2 - \partial_+ y_2 \partial_- y_1) + \frac{1}{2}(\partial_+ y_1 \partial_- y_1 + \partial_+ y_2 \partial_- y_2 + v_+ v_-) + \frac{1}{2} y_3(\partial_+ v_- - \partial_- v_+)\Big]. \tag{3.34}$$

From the equation of motion for Lagrange multiplier y_3 we obtain

$$\partial_+ v_- - \partial_- v_+ = 0 \Longrightarrow v_\pm = \partial_\pm z, \quad \Delta V = \Delta z, \tag{3.35}$$

which drives back the gauge fixed action to the initial action (3.18). Varying the gauge fixed action (3.34) with respect to the gauge fields $v_\pm$ we get the following equations of motion

$$v_\pm = \pm\partial_\pm y_3 - 2\beta^\mp, \tag{3.36}$$

where $\beta^\pm$ functions are defined as

$$\beta^\pm = \pm\frac{1}{2} H(y_1 \partial_\mp y_2 - y_2 \partial_\mp y_1). \tag{3.37}$$

The $\beta^\pm$ functions are obtained as a result of the variation of the term containing ΔV

$$\delta_v \left(-2\kappa \int d^2\xi \varepsilon^{\alpha\beta} H \partial_\alpha y_1 \partial_\beta y_2 \Delta V\right) = \kappa \int d^2\xi \left(\beta^+ \delta v_+ + \beta^- \delta v_-\right), \tag{3.38}$$

using partial integration and the fact that $\partial_\pm V = v_\pm$. Inserting the relations (3.36) into the gauge fixed action, keeping linear terms in H, we obtain the T-dual action

$$_{xyz}S = \kappa \int_\Sigma d^2\xi\, \partial_{+\,xyz} X^\mu\, {}_{xyz}\Pi_{+\mu\nu}\, \partial_{-\,xyz} X^\nu, \tag{3.39}$$

where

$$_{xyz}X^\mu = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}, \quad {}_{xyz}\Pi_{+\mu\nu} = {}_{xyz}B_{\mu\nu} + \frac{1}{2}\, {}_{xyz}G_{\mu\nu}, \tag{3.40}$$

$$_{xyz}B_{\mu\nu} = \begin{pmatrix} 0 & -H\Delta\tilde{y}_3 & 0 \\ H\Delta\tilde{y}_3 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad {}_{xyz}G_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{3.41}$$

Here we introduced double coordinate $\tilde{y}_3$ defined as

$$\partial_\pm y_3 \equiv \pm\partial_\pm \tilde{y}_3. \tag{3.42}$$

Let us note that ΔV stands beside field strength H, which implicates that, according to the diluted flux approximation, we calculate ΔV in the zeroth order in H

$$\Delta V = \int d\xi^+ \partial_+ y_3 - \int d\xi^- \partial_- y_3. \tag{3.43}$$

Having this into account it is clear why we defined double coordinate $\tilde{y}_3$ as in Eq. (3.42). Also it is useful to note that presence of ΔV, which is defined as line integral, represents the source of non-locality of the T-dual theory. the result of the three T-dualization is a theory with R flux as it is known in the literature.

Combining the equations of motion for Lagrange multiplier (3.35), $v_\pm = \partial_\pm z$, and equations of motion for gauge fields (3.36), we obtain the T-dual transformation law

$$\partial_\pm z \cong \pm\partial_\pm y_3 - 2\beta^\mp. \tag{3.44}$$

Adding transformation laws for $\partial_+ z$ and $\partial_- z$ we get the transformation law for $\dot{z}$

$$\dot{z} \cong y_3' + H(y_1 y_2' - y_2 y_1'), \tag{3.45}$$

which enables us to write down the transformation law in the canonical form

$$y_3' \cong \frac{1}{\kappa}\pi_z - H(xy' - yx'). \tag{3.46}$$

Here we used the expression for the canonical momentum of the initial theory (2.13)

$$\pi_z = \frac{\delta S}{\delta \dot{z}} = \kappa \dot{z}. \tag{3.47}$$

4. Noncommutativity and Nonassociativity Using T-duality

In the open string case noncommutativity comes from the boundary conditions which makes that coordinates x^μ depend both on the effective coordinates and on the effective momenta.[4] Effective coordinates and momenta do not commute and, consequently, coordinates x^μ do not commute. In the closed bosonic string case the logic is the same but the execution is different. Using T-duality we obtained transformation laws, (3.11), (3.21) and (3.44), which relate T-dual coordinates with the initial coordinates and their canonically conjugated momenta. In this section we will use these relations to get noncommutativity and nonassociativity relations.

4.1. Noncommutativity Relations

Let us start with the Poisson bracket of the σ derivatives of two arbitrary coordinates in the form

$$\{A'(\sigma), B'(\bar{\sigma})\} = U'(\sigma)\delta(\sigma - \bar{\sigma}) + V(\sigma)\delta'(\sigma - \bar{\sigma}), \tag{4.1}$$

where $\delta'(\sigma-\bar\sigma)\equiv\partial_\sigma\delta(\sigma-\bar\sigma)$. In order to find the form of the Poisson bracket

$$\{A(\sigma),\,B(\bar\sigma)\},$$

we have to find the form of the Poisson bracket

$$\{\Delta A(\sigma,\sigma_0),\,\Delta B(\bar\sigma,\bar\sigma_0)\},$$

where

$$\Delta A(\sigma,\sigma_0)=\int_{\sigma_0}^{\sigma}dx A'(x)=A(\sigma)-A(\sigma_0),$$

$$\Delta B(\bar\sigma,\bar\sigma_0)=\int_{\bar\sigma_0}^{\bar\sigma}dx B'(x)=B(\bar\sigma)-B(\bar\sigma_0). \tag{4.2}$$

Now we have

$$\{\Delta A(\sigma,\sigma_0),\,\Delta B(\bar\sigma,\bar\sigma_0)\}=\int_{\sigma_0}^{\sigma}dx\int_{\bar\sigma_0}^{\bar\sigma}dy\,\left[U'(x)\delta(x-y)+V(x)\delta'(x-y)\right]. \tag{4.3}$$

After integration over y we get

$$\{\Delta A(\sigma,\sigma_0),\,\Delta B(\bar\sigma,\bar\sigma_0)\}=\int_{\sigma_0}^{\sigma}dx\,\Big\{U'(x)\left[\theta(x-\bar\sigma_0)-\theta(x-\bar\sigma)\right]+V(x)\left[\delta(x-\bar\sigma_0)-\delta(x-\bar\sigma)\right]\Big\}, \tag{4.4}$$

where function $\theta(x)$ is defined as

$$\theta(x)=\int_0^x d\eta\delta(\eta)=\frac{1}{2\pi}\left[x+2\sum_{n\geq1}\frac{1}{n}\sin(nx)\right]=\begin{cases}0 & \text{if } x=0\\ 1/2 & \text{if } 0<x<2\pi.\\ 1 & \text{if } x=2\pi\end{cases} \tag{4.5}$$

Integrating over x using partial integration finally we obtain

$$\begin{aligned}\{\Delta A(\sigma,\sigma_0),\,\Delta B(\bar\sigma,\bar\sigma_0)\}&=U(\sigma)[\theta(\sigma-\bar\sigma_0)-\theta(\sigma-\bar\sigma)]\\ &-U(\sigma_0)[\theta(\sigma_0-\bar\sigma_0)-\theta(\sigma_0-\bar\sigma)]-U(\bar\sigma_0)[\theta(\sigma-\bar\sigma_0)-\theta(\sigma_0-\bar\sigma_0)]\\ &+U(\bar\sigma)[\theta(\sigma-\bar\sigma)-\theta(\sigma_0-\bar\sigma)]+V(\bar\sigma_0)[\theta(\sigma-\bar\sigma_0)-\theta(\sigma_0-\bar\sigma_0]\\ &-V(\bar\sigma)[\theta(\sigma-\bar\sigma)-\theta(\sigma_0-\bar\sigma)].\end{aligned} \tag{4.6}$$

From the last expression, using the right-hand sides of the expressions in Eq. (4.2), we extract the desired Poisson bracket

$$\{A(\sigma),\,B(\bar\sigma)\}=-[U(\sigma)-U(\bar\sigma)+V(\bar\sigma)]\theta(\sigma-\bar\sigma). \tag{4.7}$$

Let us rewrite the canonical forms of the transformation laws, (3.14), (3.24) and (3.46), in the following way

$$y_1'\cong\frac{1}{\kappa}\pi_x,\quad y_2'\cong\frac{1}{\kappa}\pi_y,\quad y_3'\cong\frac{1}{\kappa}\pi_z-H(xy'-yx'). \tag{4.8}$$

In order to find the Poisson brackets between T-dual coordinates y_μ we will use the algebra of the coordinates and momenta of the initial theory (3.25). It is obvious that only nontrivial Poisson brackets will be $\{y_1(\sigma),\,y_3(\bar\sigma)\}$ and $\{y_2(\sigma),\,y_3(\bar\sigma)\}$.

Let us first write the corresponding Poisson brackets of the sigma derivatives of T-dual coordinates y_μ using (4.8)

$$\{y_1'(\sigma),\,y_3'(\bar\sigma)\}\cong\frac{2}{\kappa}Hy'(\sigma)\delta(\sigma-\bar\sigma)+\frac{1}{\kappa}Hy(\sigma)\delta'(\sigma-\bar\sigma), \tag{4.9}$$

$$\{y_2'(\sigma),\,y_3'(\bar\sigma)\}\cong-\frac{2}{\kappa}Hx'(\sigma)\delta(\sigma-\bar\sigma)-\frac{1}{\kappa}Hx(\sigma)\delta'(\sigma-\bar\sigma), \tag{4.10}$$

while all other Poisson brackets are zero. We see that these Poisson brackets are of the form (4.1), so, we can apply the result (4.7). Consequently, we get

$$\{y_1(\sigma),\,y_3(\bar\sigma)\}\cong-\frac{H}{\kappa}\left[2y(\sigma)-y(\bar\sigma)\right]\theta(\sigma-\bar\sigma), \tag{4.11}$$

$$\{y_2(\sigma),\,y_3(\bar\sigma)\}\cong\frac{H}{\kappa}\left[2x(\sigma)-x(\bar\sigma)\right]\theta(\sigma-\bar\sigma), \tag{4.12}$$

where function $\theta(x)$ is defined in (4.5). Let us note that these two Poisson brackets are zero when $\sigma=\bar\sigma$ and/or field strength H is equal to zero. But if we take that $\sigma-\bar\sigma=2\pi$ then we have $\theta(2\pi)=1$ and it follows

$$\{y_1(\sigma+2\pi),\,y_3(\sigma)\}\cong-\frac{H}{\kappa}\left[4\pi N_y+y(\sigma)\right], \tag{4.13}$$

$$\{y_2(\sigma+2\pi),\,y_3(\sigma)\}\cong\frac{H}{\kappa}\left[4\pi N_x+x(\sigma)\right], \tag{4.14}$$

where N_x and N_y are winding numbers defined as

$$x(\sigma+2\pi)-x(\sigma)=2\pi N_x,\quad y(\sigma+2\pi)-y(\sigma)=2\pi N_y. \tag{4.15}$$

From these relations we can see that if we choose such σ for which $x(\sigma)=0$ and $y(\sigma)=0$ then noncommutativity relations are proportional to winding numbers. On the other side, for winding numbers which are equal to zero there is still noncommutativity between T-dual coordinates.

4.2. Nonassociativity

In order to calculate Jacobi identity of the T-dual coordinates we first have to find Poisson brackets $\{y_1(\sigma),\,x(\bar\sigma)\}$ as well as $\{y_2(\sigma),\,y(\bar\sigma)\}$. We start with

$$\{\Delta y_1(\sigma,\sigma_0),\,x(\bar\sigma)\}=\left\{\int_{\sigma_0}^{\sigma}d\eta y_1'(\eta),\,x(\bar\sigma)\right\}, \tag{4.16}$$

and then use the T-dual transformation for x-direction in canonical form

$$\pi_x\cong\kappa y_1'. \tag{4.17}$$

From these two equations it follows

$$\{\Delta y_1(\sigma,\sigma_0), x(\bar\sigma)\} \cong \frac{1}{\kappa}\left\{\int_{\sigma_0}^{\sigma} d\eta \pi_x(\eta), x(\bar\sigma)\right\}, \tag{4.18}$$

which, using the standard Poisson algebra, produces

$$\{\Delta y_1(\sigma,\sigma_0), x(\bar\sigma)\} \cong -\frac{1}{\kappa}\left[\theta(\sigma-\bar\sigma)-\theta(\sigma_0-\bar\sigma)\right]$$
$$\Longrightarrow \{y_1(\sigma), x(\bar\sigma)\} \cong -\frac{1}{\kappa}\theta(\sigma-\bar\sigma). \tag{4.19}$$

The relation $\{y_2(\sigma), y(\bar\sigma)\}$ can be obtained in the same way. Because the transformation law for y-direction is of the same form as for x-direction, the Poisson bracket is of the same form

$$\{y_2(\sigma), y(\bar\sigma)\} \cong -\frac{1}{\kappa}\theta(\sigma-\bar\sigma). \tag{4.20}$$

Now we can calculate Jacobi identity using noncommutativity relations (4.11) and (4.12) and above two Poisson brackets

$$\{y_1(\sigma_1), y_2(\sigma_2), y_3(\sigma_3)\} \equiv \{y_1(\sigma_1), \{y_2(\sigma_2), y_3(\sigma_3)\}\}$$
$$+\{y_2(\sigma_2), \{y_3(\sigma_3), y_1(\sigma_1)\}\} + \{y_3(\sigma_3), \{y_1(\sigma_1), y_2(\sigma_2)\}\}$$
$$\cong -\frac{2H}{\kappa^2}\big[\theta(\sigma_1-\sigma_2)\theta(\sigma_2-\sigma_3)$$
$$+\theta(\sigma_2-\sigma_1)\theta(\sigma_1-\sigma_3)+\theta(\sigma_1-\sigma_3)\theta(\sigma_3-\sigma_2)\big]. \tag{4.21}$$

Jacobi identity is nonzero which means that theory with R-flux is nonassociative. For $\sigma_2=\sigma_3=\sigma$ and $\sigma_1=\sigma+2\pi$ we get

$$\{y_1(\sigma+2\pi), y_2(\sigma), y_3(\sigma)\} \cong \frac{2H}{\kappa^2}. \tag{4.22}$$

From the last two equations, general form of Jacobi identity and Jacobi identity for special choice of σ's, we see that presence of the coordinate dependent Kalb-Ramond field is a source of noncommutativity and nonassociativity.

5. Conclusion

In this article we have considered the closed bosonic string propagating in the three-dimensional constant metric and Kalb-Ramond field with just one nonzero component $B_{xy}=Hz$. This choice of background is in accordance with consistency conditions in the sense that all calculations were made in approximation linear in Kalb-Ramond field strength H. Geometrically, this settings corresponds to the torus with H flux. Then we performed standard Buscher T-dualization procedure along isometry directions, first along x and then along y direction. At the end we performed generalized T-dualization procedure along z direction and obtained nonlocal theory with R flux. Using the relations between initial and T-dual variables, called T-dual transformation laws, in canonical form we find the noncommutativity and nonassociativity relations between T-dual coordinates.

After T-dualization along x direction we obtained theory embedded in geometry known in literature as twisted torus geometry. The relation between initial and T-dual variables is trivial, $\pi_x \cong \kappa y_1'$, where π_x is x component of the canonical momentum of the initial theory and y_1 is coordinate T-dual to x. Consequently, flux H takes a role of connection, obtained theory is globally and locally well defined and commutative, because the coordinates and their canonically conjugated momenta satisfy the standard Poisson algebra (3.25).

The second T-dualization, along y direction, produces nongeometrical theory, in literature known as Q flux theory. The metric is the same as initial one and Kalb-Ramond field have the same form as initial up to minus sign. But, this theory has just local geometrical interpretation. We obtained that, in approximation linear in H, the transformation law in canonical form is again trivial, $\pi_y \cong \kappa y_2'$, where π_y is y component od the canonical momentum of the initial theory and y_2 is coordinate T-dual to y. As a consequence of the standard Poisson algebra (3.25), we conclude that Q flux theory is still commutative. This result seems to be opposite from the result of the reference [16] where in detailed calculation it is shown that Q flux theory is noncommutative. The difference is in the so called boundary condition i.e. winding condition. In the Ref. [16] they imposed nontrivial winding condition which mixes the coordinates and their T-dual partners (condition given in Eq. (C.18) of Ref. [16]) and the result is noncommutativity. In this article the trivial winding condition is imposed on x and y coordinates. The consequence is that Q flux theory is commutative. But as it is written in Ref. [16] on page 42, "a priori other reasonings could as well be pursued".

T-dualizing along coordinate z using the machinery of the generalized T-dualization procedure[20,21,23] we obtain the nonlocal theory (theory with R flux) and nontrivial transformation law in canonical form. Non-locality stems from the fact that background fields are expressed in terms of the variable ΔV which is defined as line integral. On the other side, dependence of the Kalb-Ramond field on z coordinate produces the $\beta^{\pm}(x, y)$ functions and nontrivial transformation law for π_z. Consequently, coordinate dependent background gives non-locality and, further, nonzero Poisson brackets of the T-dual coordinates. We can claim that there is a correlation between non-locality (R-flux theory) and closed string noncommutativity and nonassociativity. In addition, nonzero Poisson bracket implies nonzero Jacobi identity which is a signal of nonassociativity.

From the expressions (4.11), (4.12) and (4.21) it follows that parameters of noncommutativity and nonassociativity are proportional to the field strength H. That means that closed string noncommuatativity and nonassociativity are consequence of the fact that Kalb-Ramond field is coordinate dependent, $B_{xy}=Hz$, where H is an infinitesimal parameter according to the approximation of diluted flux. Using T-duality and trivial winding conditions we obtained noncommutativity relations. The noncommutativity relations are zero if $\sigma=\bar\sigma$ because in noncommuatativity relations function $\theta(\sigma-\bar\sigma)$ is present, which is zero if its argument is zero. This is also at the first glance opposite to the result of Ref. [16], but, having in mind that origin of noncommutativity is not same, this difference is not surprising. If we made a round in sigma choosing $\sigma\to\sigma+2\pi$ and $\bar\sigma\to\sigma$, because of $\theta(2\pi)=1$, we obtained nonzero Poisson brackets. From the relations (4.13) and (4.14) we see that noncommutativity exists even in the case when winding numbers are zero, noncommutativity relations still stand unlike the result in [16]. Consequently, we can speak about some essential noncommutativity originating from non-locality.

We showed that in *ordinary* space coordinate dependent background is a sufficient condition for closed string noncommutativity. Some papers[36] show that noncommutativity is possible even in the constant background case. But that could be realized using the *double space formalism*. At the zeroth order the explanation follows from the fact that transformation law in canonical form is of the form $\pi_\mu \cong \kappa y'_\mu$, where y_μ is T-dual coordinate. Forming double space spanned by $Z^M = (x^\mu, y_\mu)$, we obtained noncommuative (double) space. In literature this kind of noncommutativity is called intrinsic one.

Appendix: Light-Cone Coordinates

In the paper we often use light-cone coordinates defined as

$$\xi^\pm = \frac{1}{2}(\tau \pm \sigma). \quad \text{(A.1)}$$

The corresponding partial derivatives are

$$\partial_\pm \equiv \frac{\partial}{\partial \xi^\pm} = \partial_\tau \pm \partial_\sigma. \quad \text{(A.2)}$$

Two dimensional Levi-Civita $\varepsilon^{\alpha\beta}$ is chosen in (τ, σ) basis as $\varepsilon^{\tau\sigma} = -1$. Consequently, in the light-cone basis the form of tensor is

$$\varepsilon_{lc} = \begin{pmatrix} 0 & \frac{1}{2} \\ -\frac{1}{2} & 0 \end{pmatrix}. \quad \text{(A.3)}$$

The flat world-sheet metric is of the form in (τ, σ) and light-cone basis, respectively

$$\eta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \eta_{lc} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}. \quad \text{(A.4)}$$

Acknowledgements

Work supported in part by the Serbian Ministry of Education, Science and Technological Development, under contract No. 171031. I also want to thank to Prof. Dr. Branislav Sazdović and Dr. Ljubica Davidović from Institute of Physics Belgrade for useful discussions.

Conflict of Interest

The authors have declared no conflict of interest.

Keywords

closed string, nonassociativity, noncommutativity, non-locality

Received: January 29, 2018

[1] H. S. Snyder, *Phys. Rev.* **1947**, *71*, 38.
[2] A. Connes, *Inst. Hautes tudes Sci. Publ. Math.* **1985**, *62*, 257.
[3] N. Seiberg, E. Witten, *JHEP* **1999**, *09*, 032.
[4] A. Connes, M. R. Douglas, A. Schwarz, *JHEP* **1998**, *02*, 003; M. R. Douglas, C. Hull, *JHEP* **1998**, *02*, 008; V. Schomerus, *JHEP* **1999**, *06*, 030; F. Ardalan, H. Arfaei, M. M. Sheikh-Jabbari, *JHEP* **1999**, *02*, 016; C. S. Chu, P. M. Ho, *Nucl. Phys.* **1999**, *B550*, 151; F. Ardalan, H. Arfaei, M. M. Sheikh-Jabbari, *Nucl. Phys.* **2000**, *B576*, 578; C. S. Chu, P. M. Ho, *Nucl. Phys.* **2000**, *B568*, 447; T. Lee, *Phys. Rev.* **2000**, *D62*, 024022; B. Nikolić, B. Sazdović, *Phys. Rev.* **2006**, *D74*, 045024; *Phys. Rev.* 2007, **D75**, 085011.
[5] M. Evans, B. Ovrut, *Phys. Rev.* **1989**, *D39*, 3016.
[6] D. Latas, V. Radovanovic, J. Trampetic, *Phys. Rev.* **2007**, *D76*,085006; M. Buric, V. Radovanovic, J. Trampetic, *JHEP* **2007**, *03*, 030; D. N. Blaschke, H. Grosse, J.-C. Wallet, *JHEP* **2013**, *06*, 038; C. P. Martin, C. Tamarit, *JHEP* **2009**, *11*, 092.
[7] B. Melic, K. Passek-Kumericki, J. Trampetic, *Phys. Rev.* **2005**, *D72*, 057502; M. Buric, D. Latas, V. Radovanovic, J. Trampetic, *Phys. Rev.* **2007**, *D 75*, 097701.
[8] D. Lust, *JHEP* **2010**, *12*, 084.
[9] K. Becker, M. Becker, J. Schwarz *String Theory and M-Theory: A Modern Introduction*; B. Zwiebach, *A First Course in String Theory*, Cambridge University Press, **2004**; J. Polchinski, *String theory - Volume II*, Cambridge University Press, **1998**.
[10] T. Buscher, *Phys. Lett.* **1987**, *B 194*, 59; 1988, **201**, 466.
[11] M. Roček, E. Verlinde, *Nucl. Phys.* **1992**, *B 373*, 630.
[12] A. Giveon, M. Porrati, E. Rabinovici, *Phys. Rep.* **1994**, *244*, 77.
[13] E. Alvarez, L. Alvarez-Gaume, J. Barbon, Y. Lozano, *Nucl. Phys.* **1994**, *B 415*, 71.
[14] B. Nikolić, B. Sazdović, *Nucl. Phys.* **2010**, *B 836*, 100.
[15] D. S. Berman, D. C. Thompson, *Phys. Rept.* **2014**, *566*, 1.
[16] D. Andriot, M. Larfors, D. Luest, P. Patalong, *JHEP* **2013**, *06*, 021.
[17] D. Andriot, O. Hohm, M. Larfors, D. Lust, P. Patalong, *Phys. Rev. Lett.* **2012**, *108*, 261602.
[18] D. Luest, arxiv:1205.0100 [hep-th]; R. Blumenhagen, A. Deser, D. Luest, E. Plauschinn, F. Rennecke, *J. Phys.* **2011**, *A44*, 385401; C. Condeescu, I. Florakis, D. Luest, *JHEP* **2012**, *04*, 121.
[19] J. Shelton, W. Taylor, B. Wecht, *JHEP* **2005**, *10*, 085; A. Dabholkar, C. Hull, *JHEP* **2006**, *05*, 009.
[20] Lj. Davidović, B. Sazdović, *EPJ* **2014**, *C 74*, 2683.
[21] Lj. Davidović, B. Nikolić, B. Sazdović, *EPJ* **2015**, *C 75*, 576.
[22] Lj. Davidović, B. Nikolić, B. Sazdović, *EPJ* **2014**, *C 74*, 2734.
[23] Lj. Davidović, B. Sazdović, *JHEP* **2015**, *11*, 119.
[24] M. Duff, *Nucl. Phys.* **1990**, *B 335*, 610.
[25] A. A. Tseytlin, *Phys. Lett.* **1990**, *B 242*, 163.
[26] A. A. Tseytlin, *Nucl. Phys.* **1991**, *B 350*, 395.
[27] W. Siegel, *Phys. Rev.* **1993**, *D 48*, 2826.
[28] W. Siegel, *Phys. Rev.* **1993**, *D 47*, 5453.
[29] C. M. Hull, *JHEP* **2005**, *10*, 065.
[30] C. M. Hull, *JHEP* **2007**, *10*, 057; 2007, **07**, 080.
[31] D. S. Berman, M. Cederwall, M. J. Perry, *JHEP* **2014**, *09*, 066; D. S. Berman, C. D. A. Blair, E. Malek, M. J. Perry, *Int. J. Mod. Phys.* **2014**, *A29*, 15, 1450080; C. D. A. Blair, E. Malek, A. J. Routh, *Class. Quant. Grav.* **2014**, *31*, 20, 205011.
[32] C. M. Hull, R. A. Reid-Edwards, *JHEP* **2009**, *09*, 014.
[33] O. Hohm, B. Zwiebach, *JHEP* **2014**, *11*, 075.
[34] B. Sazdović, *Chin. Phys.* **2017**, *C 41*, 053101.
[35] B. Sazdović, *JHEP* **2015**, *08*, 055.
[36] L. Freidel, R. G. Leigh, Dj. Minic, *JHEP* **2017**, *09*, 060.
[37] S. Meljanac, S. Mignemi, J. Trampetic, J. You, *Phys. Rev.* **2017**, *D 96*, 045021; arXiv: 1711.09639.
[38] P. Aschieri, M. D. Ciric, R. J. Szabo, arXiv:1710.11467; R. Blumenhagen, M. Fuchs, *JHEP* **2016**, *07*, 019.
[39] C. G. Callan, D. Friedan, E. J. Martinec, M. J. Perry, *Nucl. Phys.* **1985**, *B262*, 593.

Available online at www.sciencedirect.com

ScienceDirect

Nuclear Physics B 917 (2017) 105–121

NUCLEAR PHYSICS B

www.elsevier.com/locate/nuclphysb

Fermionic T-duality in fermionic double space ☆

B. Nikolić *, B. Sazdović

Institute of Physics Belgrade, University of Belgrade, Pregrevica 118, 11080 Belgrade, Serbia

Received 9 September 2016; received in revised form 26 January 2017; accepted 9 February 2017
Available online 13 February 2017
Editor: Stephan Stieberger

Abstract

In this article we offer the interpretation of the fermionic T-duality of the type II superstring theory in double space. We generalize the idea of double space doubling the fermionic sector of the superspace. In such doubled space fermionic T-duality is represented as permutation of the fermionic coordinates θ^α and $\bar{\theta}^\alpha$ with the corresponding fermionic T-dual ones, ϑ_α and $\bar{\vartheta}_\alpha$, respectively. Demanding that T-dual transformation law has the same form as initial one, we obtain the known form of the fermionic T-dual NS–R and R–R background fields. Fermionic T-dual NS–NS background fields are obtained under some assumptions. We conclude that only symmetric part of R–R field strength and symmetric part of its fermionic T-dual contribute to the fermionic T-duality transformation of dilaton field and analyze the dilaton field in fermionic double space. As a model we use the ghost free action of type II superstring in pure spinor formulation in approximation of constant background fields up to the quadratic terms.

1. Introduction

Two theories T-dual to one another can be viewed as being physically identical [1,2]. T-duality presents an important tool which shows the equivalence of different geometries and topologies. The useful T-duality procedure was first introduced by Buscher [3].

☆ Work supported in part by the Serbian Ministry of Education, Science and Technological Development, under contract No. 171031.

* Corresponding author.
E-mail addresses: bnikolic@ipb.ac.rs (B. Nikolić), sazdovic@ipb.ac.rs (B. Sazdović).

http://dx.doi.org/10.1016/j.nuclphysb.2017.02.003

Mathematical realization of T-duality is given by Buscher T-dualization procedure [3], which is considered as standard one. There are also other frameworks in which we can represent T-dualization which should agree with the Buscher procedure. It is double space formalism which was the subject of the articles about twenty years ago [4–8]. Double space is spanned by coordinates $Z^M = (x^\mu \quad y_\mu)^T$ ($\mu = 0, 1, 2, \ldots, D-1$), where x^μ and y_μ are the coordinates of the D-dimensional initial and T-dual space–time, respectively. Interest for this subject emerged recently with papers [9–13], where T-duality along some subset of d coordinates is considered as $O(d, d)$ symmetry transformation and [14,15], where it is considered as permutation of d initial with corresponding d T-dual coordinates.

Until recently only T-duality along bosonic coordinates has been considered. Analyzing the gluon scattering amplitudes in $N = 4$ super Yang–Mills theory, a new kind of T-dual symmetry, fermionic T-duality, was discovered [16,17]. It is a part of the dual superconformal symmetry which should be connected to integrability and it is valid just at string tree level. Mathematically, fermionic T-duality is realized within the same procedure as bosonic one, except that dualization is performed along fermionic variables. So, it can be considered as a generalization of Buscher T-duality. Fermionic T-duality consists in certain non-local redefinitions of the fermionic variables of the superstring mapping a supersymmetric background to another supersymmetric background. In Refs. [16,17] it was shown that fermionic T-duality maps gluon scattering amplitudes in the original theory to an object very close to Wilson loops in the dual one. Calculation of gluon scattering amplitudes in the initial theory is equivalent to the calculation of Wilson loops in fermionic T-dual theory. Generalizing the idea of double space to the fermionic case we would get fermionic double space in which fermionic T-duality is a symmetry [18] which exchanges scattering amplitudes and Wilson loops. Fermionic double space can be also successfully applied in random lattice [19], where doubling of the supercoordinate was done. Relation between fermionic T-duality and open string noncommutativity was considered in Ref. [20].

Let us explain our motivation for fermionic T-duality. It is well known that T-duality is important feature in understanding the M-theory. In fact, five consistent superstring theories are connected by web of T and S dualities. We are going to pay attention to the T-duality, hoping that S-duality (which can be understood as transformation of dilaton background field also) can be later successfully incorporated into our procedure. If we start with arbitrary (of five consistent superstring) theory and find all corresponding T-dual theories we can achieve any of other four consistent superstring theories. But to obtain formulation of M-theory it is not enough. We must construct one theory which contains the initial theory and all corresponding T-dual ones.

In the bosonic case (which is substantially simpler that supersymmetric one) we have succeeded to realize such program. In Refs. [14,15] we doubled all bosonic coordinates and showed that such theory contained the initial and all corresponding T-dual theories. We can connect arbitrary two of these theories just replacing some initial coordinates x^a with corresponding T-dual ones y_a. This is equivalent with T-dualization along coordinates x^a. So, introducing double space T-duality ceases to be transformation which connects two physically equivalent theories but it becomes symmetry transformation in extended space with respect to permutation group. We proved this in the bosonic string case both for constant and for weakly curved background with linear dependence on coordinates.

Unfortunately, this is not enough for construction of M-theory, because the T-duality for superstrings is much more complicated then in the bosonic case [21]. In Ref. [22] we have tried to extend such approach to the type II theories. In fact, doubling all bosonic coordinates we have unified types IIA, IIB as well as type $II^\star$ [23] (obtained by T-dualization along time-like direction) theories. There is an incompleteness in such approach. Doubling all bosonic coordinates, by simple permutations of initial with corresponding T-dual coordinates, we obtained all T-dual background fields except T-dual R–R field strength $F^{\alpha\beta}$. To obtain ${}_aF^{\alpha\beta}$ (the field strength after T-dualization along coordinates x^a) we need to introduce some additional assumptions. The explanation is that R–R field strength $F^{\alpha\beta}$ appears coupled with fermionic momenta π_α and $\bar{\pi}_\alpha$ along which we did not performed T-dualization and consequently we did not double these variables. It is an analogue of ij-term in approach of Refs. [9,10] where x^i coordinates are not doubled.

Therefore, in the first step of our approach to the formulation of M-theory (unification of types II theories) we must include T-dualization along fermionic variables (π_α and $\bar{\pi}_\alpha$ in particular case). It means that we should doubled these fermionic variables, also. The present article represents a necessary step for understanding T-dualization along all fermionic coordinates in fermionic double space. We expect that final step in construction of M-theory will be unification of all theories obtained after T-dualization along all bosonic and all fermionic variables [18,19]. In that case we should double all coordinates in superspace, anticipating that some superpermutation will connect arbitrary two of our five consistent supersymmetric string theories.

In this article we are going to double fermionic sector of type II theories adding to the coordinates θ^α and $\bar{\theta}^\alpha$ their fermionic T-duals, ϑ_α and $\bar{\vartheta}_\alpha$, where index α counts independent real components of the spinors, $\alpha = 1, 2, \ldots, 16$. Rewriting T-dual transformation laws in terms of the double coordinates, $\Theta^A = (\theta^\alpha, \vartheta_\alpha)$ and $\bar{\Theta}^A = (\bar{\theta}^\alpha, \bar{\vartheta}_\alpha)$, we define the "fermionic generalized metric" $\mathcal{F}_{AB}$ and the generalized currents $\bar{\mathcal{J}}_{+A}$ and $\mathcal{J}_{-A}$. The permutation matrix $\mathcal{T}^A{}_B$ exchanges $\bar{\theta}^\alpha$ and θ^α with their T-dual partners, $\bar{\vartheta}_\alpha$ and ϑ_α, respectively. From the requirement that fermionic T-dual coordinates, ${}^\star\Theta^A = \mathcal{T}^A{}_B\Theta^B$ and ${}^\star\bar{\Theta}^A = \mathcal{T}^A{}_B\bar{\Theta}^B$, have the same transformation law as initial ones, Θ^A and $\bar{\Theta}^A$, we obtain the expressions for fermionic T-dual generalized metric, ${}^\star\mathcal{F}_{AB} = (\mathcal{T}\mathcal{F}\mathcal{T})_{AB}$, and T-dual currents, ${}^\star\bar{\mathcal{J}}_{+A} = \mathcal{T}_A{}^B\bar{\mathcal{J}}_{+B}$ and ${}^\star\mathcal{J}_{-A} = \mathcal{T}_A{}^B\mathcal{J}_{-B}$, in terms of the initial ones. These expressions produce the expression for fermionic T-dual NS–R fields and R–R field strength. Expressions for fermionic T-dual metric and Kalb–Ramond field are obtained separately under some assumptions. We conclude that only symmetric part of R–R field strength, $F_s^{\alpha\beta} = \frac{1}{2}(F^{\alpha\beta} + F^{\beta\alpha})$, and symmetric part of its fermionic T-dual, ${}^\star F^s_{\alpha\beta} = \frac{1}{2}({}^\star F_{\alpha\beta} + {}^\star F_{\beta\alpha})$, give contribution to the dilaton field transformation under fermionic T-duality. We also investigate the dilaton field in double space.

2. Type II superstring and fermionic T-duality

In this section we will introduce the action of type II superstring theory in pure spinor formulation and perform fermionic T-duality [16,17,20] using fermionic analogue of Buscher rules [3].

2.1. Action and supergravity constraints

In this manuscript we use the action of type II superstring theory in pure spinor formulation [24] up to the quadratic terms with constant background fields. Here we will derive the final form of the action which will be exploited in the further analysis. It corresponds to the actions used in Refs. [25–28].

The sigma model action for type II superstring of Ref. [29] is of the form

$$S = S_0 + V_{SG}\,, \tag{2.1}$$

where S_0 is the action in the flat background

$$S_0 = \int_\Sigma d^2\xi \left(\frac{\kappa}{2}\eta^{mn}\eta_{\mu\nu}\partial_m x^\mu \partial_n x^\nu - \pi_\alpha \partial_- \theta^\alpha + \partial_+ \bar{\theta}^\alpha \bar{\pi}_\alpha\right) + S_\lambda + S_{\bar{\lambda}}\,, \tag{2.2}$$

and it is deformed by integrated form of the massless type II supergravity vertex operator

$$V_{SG} = \int_\Sigma d^2\xi (X^T)^M A_{MN} \bar{X}^N\,. \tag{2.3}$$

The vectors X^M and $\bar{X}^N$ are defined as

$$X^M = \begin{pmatrix} \partial_+\theta^\alpha \\ \Pi_+^\mu \\ d_\alpha \\ \frac{1}{2}N_+^{\mu\nu} \end{pmatrix}, \quad \bar{X}^N = \begin{pmatrix} \partial_-\bar{\theta}^\alpha \\ \Pi_-^\mu \\ \bar{d}_\alpha \\ \frac{1}{2}\bar{N}_-^{\mu\nu} \end{pmatrix}, \tag{2.4}$$

and supermatrix A_{MN} is of the form

$$A_{MN} = \begin{pmatrix} A_{\alpha\beta} & A_{\alpha\nu} & E_\alpha{}^\beta & \Omega_{\alpha,\mu\nu} \\ A_{\mu\beta} & A_{\mu\nu} & \bar{E}_\mu^\beta & \Omega_{\mu,\nu\rho} \\ E^\alpha{}_\beta & E^\alpha_\nu & \mathrm{P}^{\alpha\beta} & C^\alpha{}_{\mu\nu} \\ \Omega_{\mu\nu,\beta} & \Omega_{\mu\nu,\rho} & \bar{C}_{\mu\nu}{}^\beta & S_{\mu\nu,\rho\sigma} \end{pmatrix}, \tag{2.5}$$

where notation and definitions are taken from Ref. [29]. The actions for pure spinors, S_λ and $S_{\bar{\lambda}}$, are free field actions and fully decoupled from the rest of action S_0. The world sheet Σ is parameterized by $\xi^m = (\xi^0 = \tau\,, \xi^1 = \sigma)$ and $\partial_\pm = \partial_\tau \pm \partial_\sigma$. Bosonic part of superspace is spanned by coordinates x^μ ($\mu = 0, 1, 2, \dots, 9$), while the fermionic one is spanned by θ^α and $\bar{\theta}^\alpha$ ($\alpha = 1, 2, \dots, 16$). The variables π_α and $\bar{\pi}_\alpha$ are canonically conjugated momenta to θ^α and $\bar{\theta}^\alpha$, respectively. All spinors are Majorana–Weyl ones, which means that each of them has 16 independent real components. Matrix with superfields generally depends on x^μ, θ^α and $\bar{\theta}^\alpha$.

The superfields $A_{\mu\nu}$, $\bar{E}_\mu{}^\alpha$, $E^\alpha{}_\mu$ and $\mathrm{P}^{\alpha\beta}$ are known as physical superfields, while the fields given in the first column and first row are auxiliary superfields because they can be expressed in terms of the physical ones [29]. The rest ones, $\Omega_{\mu,\nu\rho}(\Omega_{\mu\nu,\rho})$, $C^\alpha{}_{\mu\nu}(\bar{C}_{\mu\nu}{}^\alpha)$ and $S_{\mu\nu,\rho\sigma}$, are curvatures (field strengths) for physical superfields.

The expanded form of the vertex operator (2.3) is [29]

$$\begin{aligned} V_{SG} = \int d^2\xi \big[&\partial_+\theta^\alpha A_{\alpha\beta}\partial_-\bar{\theta}^\beta + \partial_+\theta^\alpha A_{\alpha\mu}\Pi_-^\mu + \Pi_+^\mu A_{\mu\alpha}\partial_-\bar{\theta}^\alpha + \Pi_+^\mu A_{\mu\nu}\Pi_-^\nu \\ &+ d_\alpha E^\alpha{}_\beta \partial_-\bar{\theta}^\beta + d_\alpha E^\alpha{}_\mu \Pi_-^\mu + \partial_+\theta^\alpha E_\alpha{}^\beta \bar{d}_\beta + \Pi_+^\mu E_\mu{}^\beta \bar{d}_\beta + d_\alpha \mathrm{P}^{\alpha\beta}\bar{d}_\beta \\ &+ \frac{1}{2}N_+^{\mu\nu}\Omega_{\mu\nu,\beta}\partial_-\bar{\theta}^\beta + \frac{1}{2}N_+^{\mu\nu}\Omega_{\mu\nu,\rho}\Pi_-^\rho + \frac{1}{2}\partial_+\theta^\alpha\Omega_{\alpha,\mu\nu}\bar{N}_-^{\mu\nu} + \frac{1}{2}\Pi_+^\mu\Omega_{\mu,\nu\rho}\bar{N}_-^{\nu\rho} \\ &+ \frac{1}{2}N_+^{\mu\nu}\bar{C}_{\mu\nu}{}^\beta\bar{d}_\beta + \frac{1}{2}d_\alpha C^\alpha{}_{\mu\nu}\bar{N}_-^{\mu\nu} + \frac{1}{4}N_+^{\mu\nu}S_{\mu\nu,\rho\sigma}\bar{N}_-^{\rho\sigma}\big]\,. \end{aligned} \tag{2.6}$$

The supergravity constraints are the conditions obtained as a consequence of nilpotency and (anti)holomorphicity of BRST operators $Q = \int \lambda^\alpha d_\alpha$ and $\bar{Q} = \int \bar{\lambda}^\alpha \bar{d}_\alpha$, where λ^α and $\bar{\lambda}^\alpha$ are pure spinors and d_α and $\bar{d}_\alpha$ are independent variables. Let us discuss the choice of background fields satisfying superspace equations of motion in the context of supergravity constraints which are explained in details for pure spinor formalism in Refs. [32,29].

In order to implement T-duality many restrictions should be imposed. For example, in bosonic case one should assume the existence of Killing vectors, which in fact means background fields independence on corresponding suitably selected coordinates. The idea is to avoid dependence on the coordinate x^μ and allow only dependence on the σ and τ derivatives of the coordinates, $\dot{x}^\mu$ and x'^μ. The case with explicit dependence on the coordinate requires particular attention and has been considered in Ref. [30]. Similar simplifications must be imposed in consideration of the non-commutativity of the coordinates [31,30].

A similar situation occurs in the supersymmetric case. In order to perform fermionic T-duality we must avoid explicit dependence of background fields on the fermionic coordinates θ^α and $\bar\theta^\alpha$ (fermionic coordinates are Killing spinors) and allow only dependence on the σ and τ derivatives of these coordinates. Assumption of existence of Killing spinors produces that the auxiliary superfields should be taken to be zero what can be seen from Eq. (5.5) of Ref. [29].

The right-hand side of the equations of motion for background fields (see for example [33]) is energy-momentum tensor which is generally square of field strengths. In our case physical superfields $G_{\mu\nu}$, $B_{\mu\nu}$, Φ, Ψ^α_μ and $\bar\Psi^\alpha_\mu$ are constant (do not depend on x^μ, θ^α, $\bar\theta^\alpha$) and corresponding field strengths, $\Omega_{\mu,\nu\rho}(\Omega_{\mu\nu,\rho})$, $C^\alpha{}_{\mu\nu}(\bar C_{\mu\nu}{}^\alpha)$ and $S_{\mu\nu,\rho\sigma}$, are zero. The only nontrivial contribution of the quadratic terms in equations of motion comes from constant field strength $\mathrm{P}^{\alpha\beta}$. It can induce back-reaction to the background fields. In order to analyze this issue we will use relations from Eq. (3.6) of Ref. [29] labeled by $(\frac{1}{2}, \frac{3}{2}, \frac{3}{2})$

$$D_\alpha \mathrm{P}^{\beta\gamma} - \frac{1}{4}(\Gamma^{\mu\nu})_\alpha{}^\beta \bar C_{\mu\nu}{}^\gamma = 0\,, \quad \bar D_\alpha \mathrm{P}^{\beta\gamma} - \frac{1}{4}(\Gamma^{\mu\nu})_\alpha{}^\gamma C^\beta{}_{\mu\nu} = 0\,, \tag{2.7}$$

in which derivative of $\mathrm{P}^{\alpha\beta}$ appears. Here

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + \frac{1}{2}(\Gamma^\mu\theta)_\alpha \frac{\partial}{\partial x^\mu}\,, \quad \bar D_\alpha = \frac{\partial}{\partial\bar\theta^\alpha} + \frac{1}{2}(\Gamma^\mu\bar\theta)_\alpha \frac{\partial}{\partial x^\mu}\,, \tag{2.8}$$

are superspace covariant derivatives and $C^\alpha{}_{\mu\nu}$ and $\bar C_{\mu\nu}{}^\alpha$ are field strengths for gravitino fields Ψ^α_μ and $\bar\Psi^\alpha_\mu$, respectively. In order to perform fermionic T-dualization along all fermionic directions, θ^α and $\bar\theta^\alpha$, we assume that they are Killing spinors which means

$$\frac{\partial \mathrm{P}^{\beta\gamma}}{\partial\theta^\alpha} = \frac{\partial \mathrm{P}^{\beta\gamma}}{\partial\bar\theta^\alpha} = 0\,. \tag{2.9}$$

Taking into account that gravitino fields, Ψ^α_μ and $\bar\Psi^\alpha_\mu$, are constant (corresponding field strengths are zero), from the equations (2.7) it follows

$$(\Gamma^\mu)_{\alpha\delta}\partial_\mu \mathrm{P}^{\beta\gamma} = 0\,. \tag{2.10}$$

Note that this is more general case than equation of motion for R–R field strength,

$$(\Gamma^\mu)_{\alpha\beta}\partial_\mu \mathrm{P}^{\beta\gamma} = 0,$$

given in Eq. (3.11) of Ref. [29] where there is summation over spinor indices. Our choice of constant $\mathrm{P}^{\alpha\beta}$ is consistent with this condition. It is known fact that even constant R–R field strength produces back-reaction on background fields. In order to cancel non-quadratic terms originating from back-reaction, the constant R–R field strength must satisfy additional conditions – $AdS_5 \times S_5$ coset geometry or self-duality condition.

Taking into account these assumptions there exists solution

$$\Pi^\mu_\pm \to \partial_\pm x^\mu\,, \quad d_\alpha \to \pi_\alpha\,, \quad \bar d_\alpha \to \bar\pi_\alpha\,, \tag{2.11}$$

and only nontrivial superfields take the form

$$A_{\mu\nu}=\kappa(\frac{1}{2}g_{\mu\nu}+B_{\mu\nu}), \quad E^\alpha_\nu=-\Psi^\alpha_\nu, \quad \bar{E}^\alpha_\mu=\bar{\Psi}^\alpha_\mu, \quad \mathrm{P}^{\alpha\beta}=\frac{2}{\kappa}P^{\alpha\beta}=\frac{2}{\kappa}e^{\frac{\Phi}{2}}F^{\alpha\beta}, \tag{2.12}$$

where $g_{\mu\nu}$ is symmetric and $B_{\mu\nu}$ is antisymmetric tensor.

The final form of the vertex operator under these assumptions is

$$V_{SG}=\int_\Sigma d^2\xi\left[\kappa(\frac{1}{2}g_{\mu\nu}+B_{\mu\nu})\partial_+x^\mu\partial_-x^\nu-\pi_\alpha\Psi^\alpha_\mu\partial_-x^\mu+\partial_+x^\mu\bar{\Psi}^\alpha_\mu\bar{\pi}_\alpha+\frac{2}{\kappa}\pi_\alpha P^{\alpha\beta}\pi_\beta\right]. \tag{2.13}$$

Consequently, the action S is of the form

$$S=\kappa\int_\Sigma d^2\xi\left[\partial_+x^\mu\Pi_{+\mu\nu}\partial_-x^\nu+\frac{1}{4\pi\kappa}\Phi R^{(2)}\right] \tag{2.14}$$
$$+\int_\Sigma d^2\xi\left[-\pi_\alpha\partial_-(\theta^\alpha+\Psi^\alpha_\mu x^\mu)+\partial_+(\bar{\theta}^\alpha+\bar{\Psi}^\alpha_\mu x^\mu)\bar{\pi}_\alpha+\frac{2}{\kappa}\pi_\alpha P^{\alpha\beta}\bar{\pi}_\beta\right],$$

where $G_{\mu\nu}=\eta_{\mu\nu}+g_{\mu\nu}$ and

$$\Pi_{\pm\mu\nu}=B_{\mu\nu}\pm\frac{1}{2}G_{\mu\nu}. \tag{2.15}$$

All terms containing pure spinors vanished because curvatures are zero under our assumption that physical superfields are constant. Actions S_λ and $S_{\bar{\lambda}}$ are fully decoupled from the rest action and can be neglected in the further analysis. The action, in its final form, is ghost independent.

Here we work both with type IIA and type IIB superstring theory. The difference is in the chirality of NS–R background fields and content of R–R sector. In NS–R sector there are two gravitino fields Ψ^α_μ and $\bar{\Psi}^\alpha_\mu$ which are Majorana–Weyl spinors of the opposite chirality in type IIA and same chirality in type IIB theory. The same feature stands for the pairs of spinors $(\theta^\alpha,\bar{\theta}^\alpha)$ and $(\pi_\alpha,\bar{\pi}_\alpha)$. The R–R field strength $F^{\alpha\beta}$ is expressed in terms of the antisymmetric tensors $F_{(k)}=F_{\mu_1\mu_2\dots\mu_k}$ [1]

$$F^{\alpha\beta}=\sum_{k=0}^{D}\frac{1}{k!}F_{(k)}(C\Gamma_{(k)})^{\alpha\beta}, \quad \left[\Gamma^{\alpha\beta}_{(k)}=(\Gamma^{[\mu_1\dots\mu_k]})^{\alpha\beta}\right] \tag{2.16}$$

where

$$\Gamma^{[\mu_1\mu_2\dots\mu_k]}\equiv\Gamma^{[\mu_1}\Gamma^{\mu_2}\dots\Gamma^{\mu_k]}, \tag{2.17}$$

is basis of completely antisymmetrized product of gamma matrices and C is charge conjugation operator. For more technical details regarding gamma matrices see the first reference in [1].

R–R field strength satisfies the chirality condition $\Gamma^{11}F=\pm F\Gamma^{11}$, where Γ^{11} is a product of gamma matrices in $D=10$ dimensional space–time. The sign $+$ corresponds to type IIA while sign $-$ corresponds to type IIB superstring theory. Consequently, type IIA theory contains only even rank tensors $F_{(k)}$, while type IIB contains only odd rank tensors. For type IIA the independent tensors are $F_{(0)}$, $F_{(2)}$ and $F_{(4)}$, while independent tensors for type IIB are $F_{(1)}$, $F_{(3)}$ and self-dual part of $F_{(5)}$.

2.2. Fixing the chiral gauge invariance

The fermionic part of the action (2.14) has the form of the first order theory. We want to eliminate the fermionic momenta and work with the action expressed in terms of coordinates and their derivatives. So, on the equations of motion for fermionic momenta π_α and $\bar\pi_\alpha$,

$$\pi_\alpha = -\frac{\kappa}{2}\partial_+\left(\bar\theta^\beta + \bar\Psi^\beta_\mu x^\mu\right)(P^{-1})_{\beta\alpha}\,,\quad \bar\pi_\alpha = \frac{\kappa}{2}(P^{-1})_{\alpha\beta}\partial_-\left(\theta^\beta + \Psi^\beta_\mu x^\mu\right)\,, \tag{2.18}$$

the action gets the form

$$\begin{aligned} S(\partial_\pm x, \partial_-\theta, \partial_+\bar\theta) &= \kappa\int_\Sigma d^2\xi\,\partial_+ x^\mu \Pi_{+\mu\nu}\partial_- x^\nu + \frac{1}{4\pi}\int_\Sigma d^2\xi\,\Phi R^{(2)} \\ &+ \frac{\kappa}{2}\int_\Sigma d^2\xi\,\partial_+\left(\bar\theta^\alpha + \bar\Psi^\alpha_\mu x^\mu\right)(P^{-1})_{\alpha\beta}\partial_-\left(\theta^\beta + \Psi^\beta_\nu x^\nu\right) \\ &= \kappa\int_\Sigma d^2\xi\,\partial_+ x^\mu\left[\Pi_{+\mu\nu} + \frac{1}{2}\bar\Psi^\alpha_\mu (P^{-1})_{\alpha\beta}\Psi^\beta_\nu\right]\partial_- x^\nu + \frac{1}{4\pi}\int_\Sigma d^2\xi\,\Phi R^{(2)} \\ &+ \frac{\kappa}{2}\int_\Sigma d^2\xi\left[\partial_+\bar\theta^\alpha (P^{-1})_{\alpha\beta}\partial_-\theta^\beta + \partial_+\bar\theta^\alpha (P^{-1}\Psi)_{\alpha\mu}\partial_- x^\mu + \partial_+ x^\mu (\bar\Psi P^{-1})_{\mu\alpha}\partial_-\theta^\alpha\right]. \end{aligned} \tag{2.19}$$

In the above action θ^α appears only in the form $\partial_-\theta^\alpha$ and $\bar\theta^\alpha$ in the form $\partial_+\bar\theta^\alpha$. This means that the theory has a local symmetry

$$\delta\theta^\alpha = \varepsilon^\alpha(\sigma^+)\,,\quad \delta\bar\theta^\alpha = \bar\varepsilon^\alpha(\sigma^-)\,,\quad (\sigma^\pm = \tau \pm \sigma)\,. \tag{2.20}$$

We will treat this symmetry within BRST formalism. The corresponding BRST transformations are

$$s\theta^\alpha = c^\alpha(\sigma^+)\,,\quad s\bar\theta^\alpha = \bar c^\alpha(\sigma^-)\,, \tag{2.21}$$

where for each gauge parameter $\varepsilon^\alpha(\sigma^+)$ and $\bar\varepsilon^\alpha(\sigma^-)$ we introduced the ghost fields $c^\alpha(\sigma^+)$ and $\bar c^\alpha(\sigma^-)$, respectively. Here s is BRST nilpotent operator.

To fix gauge freedom we introduce gauge fermion with ghost number -1

$$\Psi = \frac{\kappa}{2}\int d^2\xi\left[\bar C_\alpha\left(\partial_+\theta^\alpha + \frac{\alpha^{\alpha\beta}}{2}b_{+\beta}\right) + \left(\partial_-\bar\theta^\alpha + \frac{1}{2}\bar b_{-\beta}\alpha^{\beta\alpha}\right)C_\alpha\right], \tag{2.22}$$

where $\alpha^{\alpha\beta}$ is arbitrary non-singular matrix, $\bar C_\alpha$ and C_α are antighost fields, while $b_{+\alpha}$ and $\bar b_{-\alpha}$ are Nakanishi–Lautrup auxiliary fields which satisfy

$$sC_\alpha = b_{+\alpha}\,,\quad s\bar C_\alpha = \bar b_{-\alpha}\,,\quad sb_{+\alpha} = 0\quad s\bar b_{-\alpha} = 0\,. \tag{2.23}$$

BRST transformation of gauge fermion Ψ produces the gauge fixed and Fadeev–Popov action

$$\begin{aligned} s\Psi &= S_{gf} + S_{FP}\,, \\ S_{gf} &= \frac{\kappa}{2}\int d^2\xi\left[\bar b_{-\alpha}\partial_+\theta^\alpha + \partial_-\bar\theta^\alpha b_{+\alpha} + \bar b_{-\alpha}\alpha^{\alpha\beta}b_{+\beta}\right], \\ S_{FD} &= \frac{\kappa}{2}\int d^2\xi\left[\bar C_\alpha\partial_+ c^\alpha + (\partial_-\bar c^\alpha)C_\alpha\right]. \end{aligned} \tag{2.24}$$

The Fadeev–Popov action is decoupled from the rest and, consequently, it can be omitted in further analysis. On the equations of motion for b-fields

$$b_{+\alpha} = -(\alpha^{-1})_{\alpha\beta}\partial_+\theta^\beta\,, \quad \bar{b}_{-\alpha} = -\partial_-\bar{\theta}^\beta(\alpha^{-1})_{\beta\alpha}\,, \tag{2.25}$$

we obtain the final form of the BRST gauge fixed action

$$S_{gf} = -\frac{\kappa}{2}\int d^2\xi\,\partial_-\bar{\theta}^\alpha(\alpha^{-1})_{\alpha\beta}\partial_+\theta^\beta\,. \tag{2.26}$$

2.3. Fermionic T-duality

We will perform fermionic T-duality using fermionic version of Buscher procedure similarly to Refs. [20] where we worked without chiral gauge fixing. After introducing S_{gf} the action still has a global shift symmetry in θ^α and $\bar{\theta}^\alpha$ directions. We introduce gauge fields $v_\pm^\alpha$ and $\bar{v}_\pm^\alpha$ and replace ordinary world-sheet derivatives with covariant ones

$$\partial_\pm\theta^\alpha \to D_\pm\theta^\alpha \equiv \partial_\pm\theta^\alpha + v_\pm^\alpha\,, \quad \partial_\pm\bar{\theta}^\alpha \to D_\pm\bar{\theta}^\alpha \equiv \partial_\pm\bar{\theta}^\alpha + \bar{v}_\pm^\alpha\,. \tag{2.27}$$

In order to make the fields $v_\pm^\alpha$ and $\bar{v}_\pm^\alpha$ to be unphysical we add the following terms in the action

$$\begin{aligned} S_{gauge}(\vartheta, v_\pm, \bar{\vartheta}, \bar{v}_\pm) = {} & \frac{1}{2}\kappa\int_\Sigma d^2\xi\,\bar{\vartheta}_\alpha(\partial_+ v_-^\alpha - \partial_- v_+^\alpha) \\ & + \frac{1}{2}\kappa\int_\Sigma d^2\xi(\partial_+\bar{v}_-^\alpha - \partial_-\bar{v}_+^\alpha)\vartheta_\alpha\,, \end{aligned} \tag{2.28}$$

where ϑ_α and $\bar{\vartheta}_\alpha$ are Lagrange multipliers. The full gauge invariant action is of the form

$$\begin{aligned} S_{inv}(x, \theta, \bar{\theta}, \vartheta, \bar{\vartheta}, v_\pm, \bar{v}_\pm) = {} & S(\partial_\pm x, D_-\theta, D_+\bar{\theta}) \\ & + S_{gf}(D_-\theta, D_+\bar{\theta}) + S_{gauge}(\vartheta, \bar{\vartheta}, v_\pm, \bar{v}_\pm)\,. \end{aligned} \tag{2.29}$$

Fixing θ^α and $\bar{\theta}^\alpha$ to zero we obtain the gauge fixed action

$$\begin{aligned} S_{fix} = {} & \kappa\int_\Sigma d^2\xi\,\partial_+x^\mu\left[\Pi_{+\mu\nu} + \frac{1}{2}\bar{\Psi}_\mu^\alpha(P^{-1})_{\alpha\beta}\Psi_\nu^\beta\right]\partial_-x^\nu + \frac{1}{4\pi}\int_\Sigma d^2\xi\,\Phi R^{(2)} \\ & + \frac{\kappa}{2}\int_\Sigma\left[\bar{v}_+^\alpha(P^{-1})_{\alpha\beta}v_-^\beta + \bar{v}_+^\alpha(P^{-1})_{\alpha\beta}\Psi_\nu^\beta\partial_-x^\nu + \partial_+x^\mu\bar{\Psi}_\mu^\alpha(P^{-1})_{\alpha\beta}v_-^\beta - \bar{v}_-^\alpha(\alpha^{-1})_{\alpha\beta}v_+^\beta\right] \\ & + \frac{\kappa}{2}\int_\Sigma d^2\xi\,\bar{\vartheta}_\alpha(\partial_+v_-^\alpha - \partial_-v_+^\alpha) + \frac{\kappa}{2}\int_\Sigma d^2\xi(\partial_+\bar{v}_-^\alpha - \partial_-\bar{v}_+^\alpha)\vartheta_\alpha\,. \end{aligned} \tag{2.30}$$

Varying the above action with respect to the Lagrange multipliers ϑ_α and $\bar{\vartheta}_\alpha$ we obtain the initial action (2.19) because

$$\partial_+v_-^\alpha - \partial_-v_+^\alpha = 0 \Longrightarrow v_\pm^\alpha = \partial_\pm\theta^\alpha\,, \quad \partial_+\bar{v}_-^\alpha - \partial_-\bar{v}_+^\alpha = 0 \Longrightarrow \bar{v}_\pm^\alpha = \partial_\pm\bar{\theta}^\alpha\,. \tag{2.31}$$

The equations of motion for $v_\pm^\alpha$ and $\bar{v}_\pm^\alpha$ give

$$\bar{v}_-^\alpha = \partial_-\bar{\vartheta}_\beta\alpha^{\beta\alpha}\,, \quad \bar{v}_+^\alpha = \partial_+\bar{\vartheta}_\beta P^{\beta\alpha} - \partial_+x^\mu\bar{\Psi}_\mu^\alpha\,, \tag{2.32}$$

$$v_+^\alpha = -\alpha^{\alpha\beta}\partial_+\vartheta_\beta\,, \quad v_-^\alpha = -P^{\alpha\beta}\partial_-\vartheta_\beta - \Psi_\mu^\alpha\partial_- x^\mu\,. \tag{2.33}$$

Substituting these expressions in the action S_{fix} we obtain the fermionic T-dual action

$$^\star S(\partial_\pm x, \partial_-\vartheta, \partial_+\bar\vartheta) = \kappa\int_\Sigma d^2\xi\,\partial_+ x^\mu\Pi_{+\mu\nu}\partial_- x^\nu + \frac{1}{4\pi}\int_\Sigma d^2\xi\ {}^\star\Phi R^{(2)}\,, \tag{2.34}$$

$$+\frac{\kappa}{2}\int_\Sigma d^2\xi\left[\partial_+\bar\vartheta_\alpha P^{\alpha\beta}\partial_-\vartheta_\beta - \partial_+ x^\mu\bar\Psi_\mu^\alpha\partial_-\vartheta_\alpha + \partial_+\bar\vartheta_\alpha\Psi_\mu^\alpha\partial_- x^\mu - \partial_-\bar\vartheta_\alpha\alpha^{\alpha\beta}\partial_+\vartheta_\beta\right].$$

It should be in the form of the initial action (2.19)

$$^\star S = \kappa\int_\Sigma d^2\xi\,\partial_+ x^\mu\left[{}^\star\Pi_{+\mu\nu} + \frac{1}{2}{}^\star\Psi^{\alpha\mu}({}^\star P^{-1})^{\alpha\beta}\,{}^\star\Psi_{\beta\nu}\right]\partial_- x^\nu + \frac{1}{4\pi}\int_\Sigma d^2\xi\,{}^\star\Phi R^{(2)} \tag{2.35}$$

$$+\frac{\kappa}{2}\int_\Sigma d^2\xi\left[\partial_+\bar\vartheta_\alpha({}^\star P^{-1})^{\alpha\beta}\partial_-\vartheta_\beta + \partial_+ x^\mu({}^\star\bar\Psi\,{}^\star P^{-1})_\mu^\alpha\partial_-\vartheta^\alpha + \partial_+\bar\vartheta_\alpha({}^\star P^{-1}\,{}^\star\Psi)_\mu^\alpha\partial_- x^\nu\right]$$

$$-\frac{\kappa}{2}\int d^2\xi\,\partial_-\bar\vartheta_\alpha({}^\star\alpha^{-1})^{\alpha\beta}\partial_+\vartheta_\beta\,, \tag{2.36}$$

and so we get

$$^\star\Psi_{\alpha\mu} = (P^{-1}\Psi)_{\alpha\mu}\,,\ {}^\star\bar\Psi_{\mu\alpha} = -(\bar\Psi P^{-1})_{\mu\alpha}\,, \tag{2.37}$$

$$^\star P_{\alpha\beta} = (P^{-1})_{\alpha\beta}\,, \quad {}^\star\alpha_{\alpha\beta} = (\alpha^{-1})_{\alpha\beta}\,. \tag{2.38}$$

From the condition

$$^\star\Pi_{+\mu\nu} + \frac{1}{2}{}^\star\bar\Psi_{\alpha\mu}\,({}^\star P^{-1})^{\alpha\beta}\,{}^\star\Psi_{\beta\nu} = \Pi_{+\mu\nu}\,, \tag{2.39}$$

we read the fermionic T-dual metric and Kalb–Ramond field

$$^\star G_{\mu\nu} = G_{\mu\nu} + \frac{1}{2}\left[(\bar\Psi P^{-1}\Psi)_{\mu\nu} + (\bar\Psi P^{-1}\Psi)_{\nu\mu}\right],$$

$$^\star B_{\mu\nu} = B_{\mu\nu} + \frac{1}{4}\left[(\bar\Psi P^{-1}\Psi)_{\mu\nu} - (\bar\Psi P^{-1}\Psi)_{\nu\mu}\right]. \tag{2.40}$$

Dilaton transformation under fermionic T-duality will be presented in the section 4. Let us note that two successive dualizations give the initial background fields.

The T-dual transformation laws are connection between initial and T-dual coordinates. We can obtain them combining the different solutions of equations of motion for $v_\pm^\alpha$ and $\bar v_\pm^\alpha$ (2.31) and (2.32)–(2.33)

$$\partial_-\theta^\alpha \cong -P^{\alpha\beta}\partial_-\vartheta_\beta - \Psi_\mu^\alpha\partial_- x^\mu\,, \quad \partial_+\bar\theta^\alpha \cong \partial_+\bar\vartheta_\beta P^{\beta\alpha} - \partial_+ x^\mu\bar\Psi_\mu^\alpha\,, \tag{2.41}$$

$$\partial_+\theta^\alpha \cong -\alpha^{\alpha\beta}\partial_+\vartheta_\beta\,, \quad \partial_-\bar\theta^\alpha \cong \partial_-\bar\vartheta_\beta\alpha^{\beta\alpha}\,. \tag{2.42}$$

Here the symbol $\cong$ denotes the T-duality relation. From these relations we can obtain inverse transformation rules

$$\partial_-\vartheta_\alpha \cong -(P^{-1})_{\alpha\beta}\partial_-\theta^\beta - (P^{-1})_{\alpha\beta}\Psi_\mu^\beta\partial_- x^\mu\,,$$

$$\partial_+\bar\vartheta_\alpha \cong \partial_+\bar\theta^\beta(P^{-1})_{\beta\alpha} + \partial_+ x^\mu\bar\Psi_\mu^\beta(P^{-1})_{\beta\alpha}\,, \tag{2.43}$$

$$\partial_+\vartheta_\alpha \cong -(\alpha^{-1})_{\alpha\beta}\partial_+\theta^\beta\,, \qquad \partial_-\bar\vartheta_\alpha \cong \partial_-\bar\theta^\beta(\alpha^{-1})_{\beta\alpha}\,. \tag{2.44}$$

Note that without gauge fixing in subsection 2.2, instead expressions for $\bar{v}^\alpha_-$ and v^α_+ (first relations of (2.32) and (2.33)), we would have only constraints on the T-dual variables, $\partial_-\bar{\vartheta}_\alpha = 0$ and $\partial_+\vartheta_\alpha = 0$. Consequently, integration over $v^\alpha_\pm$ and $\bar{v}^\alpha_\pm$ would be singular and we would lose the part of T-dual transformations (2.42) and (2.44).

3. Fermionic T-dualization in fermionic double space

In this section we will extend the meaning of the double space. We will introduce double fermionic space adding to the fermionic coordinates, θ^α and $\bar{\theta}^\alpha$, the fermionic T-dual ones, ϑ_α and $\bar{\vartheta}_\alpha$. Then we will show that fermionic T-dualization can be represented as permutation of the appropriate fermionic coordinates and their T-dual partners.

3.1. Transformation laws in fermionic double space

In the same way as the double bosonic coordinates were introduced [4,14,15], we double both fermionic coordinate as

$$\Theta^A = \begin{pmatrix} \theta^\alpha \\ \vartheta_\alpha \end{pmatrix}, \quad \bar{\Theta}^A = \begin{pmatrix} \bar{\theta}^\alpha \\ \bar{\vartheta}_\alpha \end{pmatrix}. \tag{3.1}$$

Each double coordinate has 32 real components. In terms of the double fermionic coordinates the transformation laws, (2.41)–(2.44), can be rewritten in the form

$$\partial_-\Theta^A \cong -\Omega^{AB}\left[\mathcal{F}_{BC}\partial_-\Theta^C + \mathcal{J}_{-B}\right], \quad \partial_+\bar{\Theta}^A \cong \left[\partial_+\bar{\Theta}^C\mathcal{F}_{CB} + \bar{\mathcal{J}}_{+B}\right]\Omega^{BA}, \tag{3.2}$$

$$\partial_+\Theta^A \cong -\Omega^{AB}\mathcal{A}_{BC}\partial_+\Theta^C, \quad \partial_-\bar{\Theta}^A \cong \partial_-\bar{\Theta}^C\mathcal{A}_{CB}\Omega^{BA}, \tag{3.3}$$

where “fermionic generalized metric” $\mathcal{F}_{AB}$ has the form

$$\mathcal{F}_{AB} = \begin{pmatrix} (P^{-1})_{\alpha\beta} & 0 \\ 0 & P^{\gamma\delta} \end{pmatrix}, \tag{3.4}$$

and

$$\mathcal{A}_{AB} = \begin{pmatrix} (\alpha^{-1})_{\alpha\beta} & 0 \\ 0 & \alpha^{\gamma\delta} \end{pmatrix}. \tag{3.5}$$

$\mathcal{F}_{AB}$ is bosonic variable but we put the name fermionic because it appears in the case of fermionic T-duality.

The double currents, $\bar{\mathcal{J}}_{+A}$ and $\mathcal{J}_{-A}$, are fermionic variables of the form

$$\bar{\mathcal{J}}_{+A} = \begin{pmatrix} (\bar{\Psi}P^{-1})_{\mu\alpha}\partial_+x^\mu \\ -\bar{\Psi}^\alpha_\mu\partial_+x^\mu \end{pmatrix}, \quad \mathcal{J}_{-A} = \begin{pmatrix} (P^{-1}\Psi)_{\alpha\mu}\partial_-x^\mu \\ \Psi^\alpha_\mu\partial_-x^\mu \end{pmatrix}, \tag{3.6}$$

while the matrix Ω^{AB} is constant

$$\Omega^{AB} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \tag{3.7}$$

where identity matrix is 16×16. By straightforward calculation we can prove the relations

$$\Omega^2 = 1, \quad \det\mathcal{F}_{AB} = 1. \tag{3.8}$$

Consistency of the transformation laws (3.2) produces

$$(\Omega\mathcal{F})^2 = 1, \quad \mathcal{J}_- = \mathcal{F}\Omega\mathcal{J}_-, \quad \bar{\mathcal{J}}_+ = -\bar{\mathcal{J}}_+\Omega\mathcal{F}. \tag{3.9}$$

3.2. Double action

It is well known that equations of motion of initial theory are Bianchi identities in T-dual picture and vice versa. As a consequence of the identities

$$\partial_+\partial_-\Theta^A - \partial_-\partial_+\Theta^A = 0\,, \quad \partial_+\partial_-\bar{\Theta}^A - \partial_-\partial_+\bar{\Theta}^A = 0\,, \tag{3.10}$$

known as Bianchi identities, and relations (3.2) and (3.3), we obtain the consistency conditions

$$\partial_+(\mathcal{F}_{AB}\partial_-\Theta^B + J_{-A}) - \partial_-(\mathcal{A}_{AB}\partial_+\Theta^B) = 0\,, \tag{3.11}$$

$$\partial_-(\partial_+\bar{\Theta}^B\mathcal{F}_{BA} + \bar{J}_{+A}) - \partial_+(\partial_-\bar{\Theta}^B\mathcal{A}_{BA}) = 0\,. \tag{3.12}$$

The equations (3.11) and (3.12) are equations of motion of the following action

$$S_{double}(\Theta,\bar{\Theta}) = \tag{3.13}$$

$$= \frac{\kappa}{2}\int d^2\xi \left[\partial_+\bar{\Theta}^A\mathcal{F}_{AB}\partial_-\Theta^B + \bar{\mathcal{J}}_{+A}\partial_-\Theta^A + \partial_+\bar{\Theta}^A\mathcal{J}_{-A} - \partial_-\bar{\Theta}^A\mathcal{A}_{AB}\partial_+\Theta^B + L(x)\right],$$

where $L(x)$ is arbitrary functional of the bosonic coordinates.

3.3. Fermionic T-dualization of type II superstring theory as permutation of fermionic coordinates in double space

In order to exchange θ^α with ϑ_α and $\bar{\theta}$ with $\bar{\vartheta}_\alpha$, let us introduce the permutation matrix

$$\mathcal{T}^A{}_B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \tag{3.14}$$

so that double T-dual coordinates are

$${}^\star\Theta^A = \mathcal{T}^A{}_B\Theta^B\,, \quad {}^\star\bar{\Theta}^A = \mathcal{T}^A{}_B\bar{\Theta}^B\,. \tag{3.15}$$

We demand that T-dual transformation laws for double T-dual coordinates ${}^\star\Theta^A$ and ${}^\star\bar{\Theta}^A$ have the same form as for initial ones Θ^A and $\bar{\Theta}^A$ (3.2) and (3.3)

$$\partial_-{}^\star\Theta^A \cong -\Omega^{AB}\left[{}^\star\mathcal{F}_{BC}\partial_-{}^\star\Theta^C + {}^\star\mathcal{J}_{-B}\right], \quad \partial_+{}^\star\bar{\Theta}^A \cong \left[\partial_+{}^\star\bar{\Theta}^C\,{}^\star\mathcal{F}_{CB} + {}^\star\bar{\mathcal{J}}_{+B}\right]\Omega^{BA}\,, \tag{3.16}$$

$$\partial_+{}^\star\Theta^A \cong -\Omega^{AB}\,{}^\star\mathcal{A}_{BC}\partial_+{}^\star\Theta^C\,, \quad \partial_-{}^\star\bar{\Theta}^A \cong \partial_-{}^\star\bar{\Theta}^C\,{}^\star\mathcal{A}_{CB}\Omega^{BA}\,. \tag{3.17}$$

Then the fermionic T-dual "generalized metric" ${}^\star\mathcal{F}_{AB}$ and T-dual currents, ${}^\star\bar{\mathcal{J}}_{+A}$ and ${}^\star\mathcal{J}_{-A}$, with the help of (3.15) and (3.2), can be expressed in terms of initial ones

$${}^\star\mathcal{F}_{AB} = \mathcal{T}_A{}^C\mathcal{F}_{CD}\mathcal{T}^D{}_B\,, \quad {}^\star\bar{\mathcal{J}}_{+A} = \mathcal{T}_A{}^B\bar{\mathcal{J}}_{+B}\,, \quad {}^\star\mathcal{J}_{-A} = \mathcal{T}_A{}^B\mathcal{J}_{-B}\,. \tag{3.18}$$

The matrix $\mathcal{A}_{AB}$ transforms as

$${}^\star\mathcal{A}_{AB} = \mathcal{T}_A{}^C\mathcal{A}_{CD}\mathcal{T}^D{}_B = (\mathcal{A}^{-1})_{AB}\,. \tag{3.19}$$

Note that, as well as bosonic case, double space action (3.13) has global symmetry under transformations (3.15) if the conditions (3.18) are satisfied.

From the first relation in (3.18) we obtain the form of the fermionic T-dual R–R background field

$$^\star P_{\alpha\beta} = (P^{-1})_{\alpha\beta}\,, \tag{3.20}$$

while from the second and third equation we obtain the form of the fermionic T-dual NS–R background fields

$$^\star\Psi_{\alpha\mu} = (P^{-1})_{\alpha\beta}\Psi^\beta_\mu\,, \quad {}^\star\bar\Psi_{\alpha\mu} = -\bar\Psi^\beta_\mu (P^{-1})_{\beta\alpha}\,. \tag{3.21}$$

The non-singular matrix $\alpha^{\alpha\beta}$ transforms as

$$(^\star\alpha)_{\alpha\beta} = (\alpha^{-1})_{\alpha\beta}\,. \tag{3.22}$$

The expressions (3.20)–(3.22) are in full agreement with the relations (2.37) and (2.38) obtained by the standard fermionic Buscher procedure. Consequently, we showed that permutation of fermionic coordinates defined in (3.14) and (3.15) completely reproduces fermionic T-dual R–R and NS–R background fields.

3.4. *Fermionic T-dual metric $^\star G_{\mu\nu}$ and Kalb–Ramond field $^\star B_{\mu\nu}$*

The expression $\Pi_{+\mu\nu} + \frac{1}{2}\Psi^\alpha_\mu (P^{-1})_{\alpha\beta}\Psi^\beta_\nu$ appears in the action (2.19) coupled with $\partial_\pm x^\mu$, along which we do not T-dualize. It is an analogue of ij-term of Refs. [9,10] where x^i coordinates are not T-dualized, and $\alpha\beta$-term in [22] where fermionic directions are undualized.

Taking into account the form of the doubled action (3.13) we suppose that term $L(x)$ has the form

$$L(x) = 2\partial_+ x^\mu \left(\Pi_{+\mu\nu} + {}^\star\Pi_{+\mu\nu}\right)\partial_- x^\nu \equiv \mathcal{L} + {}^\star\mathcal{L}\,, \tag{3.23}$$

where $\Pi_{+\mu\nu}$ is defined in (2.15) and $^\star\Pi_{+\mu\nu}$ is fermionic T-dual which we are going to find. This term should be invariant under T-dual transformation

$$^\star\mathcal{L} = \mathcal{L} + \Delta\mathcal{L}\,. \tag{3.24}$$

Using the fact that two successive T-dualizations are identity transformation, we obtain

$$\mathcal{L} = {}^\star\mathcal{L} + {}^\star\Delta\mathcal{L}\,. \tag{3.25}$$

Combining last two relations we get

$$^\star\Delta\mathcal{L} = -\Delta\mathcal{L}\,. \tag{3.26}$$

If $\Delta\mathcal{L} = 2\partial_+ x^\mu \Delta_{\mu\nu}\partial_- x^\nu$, we obtain the condition for $\Delta_{\mu\nu}$

$$^\star\Delta_{\mu\nu} = -\,\Delta_{\mu\nu}\,. \tag{3.27}$$

Using the relations (2.37) and (2.38) we realize that, up to multiplication constant, combination

$$\Delta_{\mu\nu} = \bar\Psi^\alpha_\mu (P^{-1})_{\alpha\beta}\Psi^\beta_\nu\,, \tag{3.28}$$

satisfies the condition (3.27). So, we conclude that

$$^\star\Pi_{+\mu\nu} = \Pi_{+\mu\nu} + c\bar\Psi^\alpha_\mu (P^{-1})_{\alpha\beta}\Psi^\beta_\nu\,, \tag{3.29}$$

where c is an arbitrary constant. For $c = \frac{1}{2}$ we obtain the relations (2.40). So, in double space formulation the fermionic T-dual NS–NS background fields can be obtained up to an arbitrary constant under assumption that two successive T-dualizations produce initial action.

4. Dilaton field in double fermionic space

Dilaton field transformation under fermionic T-duality is considered [16]. Here we will discuss some new features of dilaton transformation under fermionic T-duality as well as the dilaton field in fermionic double space.

Because the dilaton transformation has quantum origin we start with the path integral for the gauge fixed action given in Eq. (2.30)

$$Z = \int d\bar{v}_+^\alpha d\bar{v}_-^\alpha dv_+^\alpha dv_-^\alpha d\bar{\vartheta}_\alpha d\vartheta_\alpha e^{i\, S_{fix}(v_\pm,\bar{v}_\pm,\partial_\pm\vartheta,\partial_\pm\bar{\vartheta})} . \tag{4.1}$$

For constant background case, after integration over the fermionic gauge fields $\bar{v}_\pm^\alpha$ and $v_\pm^\alpha$, we obtain the generating functional Z in the form

$$Z = \int d\bar{\vartheta}_\alpha d\vartheta_\alpha \det\Big[(P^{-1}\alpha^{-1})_{\alpha\beta}\Big] e^{i\, {}^\star S(\vartheta,\bar{\vartheta})} , \tag{4.2}$$

where ${}^\star S(\vartheta,\bar{\vartheta})$ is T-dual action given in Eq. (2.34). We are able to perform such integration thank to the facts that we fixed the gauge in subsection 2.2.

Note that here we multiplied with determinants of P^{-1} and α^{-1} because we integrate over Grassman fields $v_\pm^\alpha$ and $\bar{v}_\pm^\alpha$. We can choose that $\det\alpha = 1$, and the generating functional gets the form

$$Z = \int d\bar{\vartheta}_\alpha d\vartheta_\alpha \det\Big[(P^{-1})_{\alpha\beta}\Big] e^{i\, {}^\star S(\vartheta,\bar{\vartheta})} . \tag{4.3}$$

This produces the fermionic T-dual transformation of dilaton field

$${}^\star\Phi = \Phi + \ln\det\Big[(P^{-1})_{\alpha\beta}\Big] = \Phi - \ln\det P^{\alpha\beta} . \tag{4.4}$$

Let us calculate $\det P^{\alpha\beta}$ using the expression

$$(P P_s^{-1} P^T)^{\alpha\beta} = P_s^{\alpha\beta} - P_a^{\alpha\gamma}(P_s^{-1})_{\gamma\delta} P_a^{\delta\beta} , \tag{4.5}$$

where we introduce the symmetric and antisymmetric parts for initial background fields

$$P_s^{\alpha\beta} = \frac{1}{2}\left(P^{\alpha\beta} + P^{\beta\alpha}\right) , \quad P_a^{\alpha\beta} = \frac{1}{2}\left(P^{\alpha\beta} - P^{\beta\alpha}\right) , \tag{4.6}$$

and similar expressions for T-dual background fields, ${}^\star P^s_{\alpha\beta}$ and ${}^\star P^a_{\alpha\beta}$. Taking into account that

$$(P \cdot {}^\star P)^\alpha{}_\beta = \delta^\alpha{}_\beta , \tag{4.7}$$

we obtain

$$P_s^{\alpha\gamma}\; {}^\star P^s_{\gamma\beta} + P_a^{\alpha\gamma}\; {}^\star P^a_{\gamma\beta} = \delta^\alpha{}_\beta , \quad P_s^{\alpha\gamma}\, {}^\star P^a_{\gamma\beta} + P_a^{\alpha\gamma}\, {}^\star P^s_{\gamma\beta} = 0 . \tag{4.8}$$

From these two equations we obtain

$${}^\star P^s_{\alpha\beta} = \Big[(P_s - P_a P_s^{-1} P_a)^{-1}\Big]_{\alpha\beta} , \tag{4.9}$$

and, consequently, we have

$$(P P_s^{-1} P^T)^{\alpha\beta} = \Big[({}^\star P_s)^{-1}\Big]^{\alpha\beta} . \tag{4.10}$$

Taking determinant of the left and right-hand side of the above equation we get

$$\det P^{\alpha\beta} = \sqrt{\frac{\det P_s}{\det {}^\star P_s}}\,, \tag{4.11}$$

which produces

$$^\star\Phi = \Phi - \ln\sqrt{\frac{\det P_s}{\det {}^\star P_s}}\,. \tag{4.12}$$

Using the fact that $P^{\alpha\beta} = e^{\frac{\Phi}{2}} F^{\alpha\beta}$ and $^\star P^{\alpha\beta} = e^{\frac{^\star\Phi}{2}}\, {}^\star F^{\alpha\beta}$, fermionic T-dual transformation law for dilaton takes the form

$$^\star\Phi = \Phi - \ln\sqrt{e^{8(\Phi - {}^\star\Phi)}\frac{\det F_s}{\det {}^\star F_s}}\,, \tag{4.13}$$

and finally we have

$$^\star\Phi = \Phi + \frac{1}{6}\ln\frac{\det F_s}{\det {}^\star F_s}\,. \tag{4.14}$$

It is obvious that two successive T-dualizations act as identity transformation

$$^{\star\star}\Phi = \Phi\,. \tag{4.15}$$

We can conclude that only symmetric parts of the R–R field strengths give contribution to the transformation of dilaton field under fermionic T-duality. In type IIA superstring theory R–R field strength $F^{\alpha\beta}$ contains tensors F^A_0, $F^A_{\mu\nu}$ and $F^A_{\mu\nu\rho\lambda}$, while in type IIB $F^{\alpha\beta}$ contains F^B_μ, $F^B_{\mu\nu\rho}$ and self dual part of $F^B_{\mu\nu\rho\lambda\omega}$. Using the conventions for gamma matrices from the appendix of the first reference in [1] (see Appendix A), we conclude that symmetric part of $F^{\alpha\beta}$ in type IIA contains scalar F^A_0 and 2-rank tensor $F^A_{\mu\nu}$, while in type IIB superstring theory it contains 1-rank F^B_μ and self dual part of 5-rank tensor $F^B_{\mu\nu\rho\lambda\omega}$.

Let us write the path integral for double action (3.13)

$$Z_{double} = \int d\Theta^A d\bar{\Theta}^A e^{i S_{double}(\Theta,\bar{\Theta})}\,. \tag{4.16}$$

Because $\det\mathcal{F} = 1$ and $\det\mathcal{A} = 1$ we obtain that dilaton field in double space is invariant under fermionic T-duality. Consequently, a new dilaton should be introduced (see [14,15]), invariant under T-duality transformations. Because of the relation (4.15) we define the T-duality invariant dilaton as

$$\Phi_{inv} = \frac{1}{2}\left({}^\star\Phi + \Phi\right) = \Phi + \frac{1}{12}\ln\frac{\det F_s}{\det {}^\star F_s}\,, \qquad {}^\star\Phi_{inv} = \Phi_{inv}\,. \tag{4.17}$$

5. Concluding remarks

In this article we considered the fermionic T-duality of the type II superstring theory using the double space approach. We used the action of the type II superstring theory in pure spinor formulation neglecting ghost terms and keeping all terms up to the quadratic ones which means that all background fields are constant.

Using equations of motion with respect to the fermionic momenta π_α and $\bar{\pi}_\alpha$ we eliminated them from the action. We obtained the action expressed in terms of the derivatives $\partial_\pm x^\mu$, $\partial_-\theta^\alpha$ and $\partial_+\bar{\theta}^\alpha$, where θ^α and $\bar{\theta}^\alpha$ are fermionic coordinates. Because θ^α appears in the action in the form $\partial_-\theta^\alpha$ and $\bar{\theta}^\alpha$ in the form $\partial_+\bar{\theta}^\alpha$, there is a local chiral gauge symmetry with parameters depending on $\sigma^\pm = \tau \pm \sigma$. We fixed this gauge invariance using BRST approach.

Using the Buscher approach we performed fermionic T-duality procedure and obtained the form of the fermionic T-dual background fields. It is obvious that two successive fermionic T-dualizations produce initial theory i.e. they are equivalent to the identity transformation.

In the central point of the article we generalize the idea of double space and show that fermionic T-duality can be represented as permutation in fermionic double space. In the bosonic case double space spanned by coordinates $Z^M = (x^\mu, y_\mu)$ can be obtained adding T-dual coordinates y_μ to the initial ones x^μ. Using analogy with the bosonic case we introduced double fermionic space doubling the initial coordinates θ^α and $\bar{\theta}^\alpha$ with their fermionic T-duals, ϑ_α and $\bar{\vartheta}_\alpha$. Double fermionic space is spanned by the coordinates $\Theta^A = (\theta^\alpha, \vartheta_\alpha)$ and $\bar{\Theta}^A = (\bar{\theta}^\alpha, \bar{\vartheta}_\alpha)$.

T-dual transformation laws and their inverse ones are rewritten in fermionic double space by single relation introducing the fermionic generalized metric $\mathcal{F}_{AB}$ and currents $\mathcal{J}_{-A}$ and $\bar{\mathcal{J}}_{+A}$. Demanding that transformation laws for fermionic T-dual double coordinates, ${}^\star\Theta^A = \mathcal{T}^A{}_B\Theta^B$ and ${}^\star\bar{\Theta}^A = \mathcal{T}^A{}_B\bar{\Theta}^B$, are of the same form as those for Θ^A and $\bar{\Theta}^A$, we obtained fermionic T-dual generalized metric ${}^\star\mathcal{F}_{AB}$ and currents ${}^\star\mathcal{J}_{-A}$ and ${}^\star\bar{\mathcal{J}}_{+A}$. These transformations act as symmetry transformations of the double action (3.13). They produce the form of the fermionic T-dual NS–R and R–R background fields which are in full accordance with the results obtained by standard Buscher procedure.

The expressions for T-dual metric ${}^\star G_{\mu\nu}$ and Kalb–Ramond field ${}^\star B_{\mu\nu}$ cannot be found from double space formalism because they do not appear in the T-dual transformation laws. These expressions, up to arbitrary constant, are obtained assuming that two successive T-dualizations act as identity transformation.

We considered transformation of dilaton field under fermionic T-duality. We derived the transformation law for dilaton field and concluded that just symmetric parts of R–R field strengths, $F_s^{\alpha\beta}$ and ${}^\star F^s_{\alpha\beta}$, affected the dilaton transformation law. This means that in the case of type IIA scalar and 2-rank tensor have influence on the dilaton transformation, while in the case of type IIB 1-rank tensor and self-dual part of 5-rank tensor take that role.

Therefore, we extended T-dualization in double space to the fermionic case. We proved that permutation of fermionic coordinates with corresponding T-dual ones in double space is equivalent to the fermionic T-duality along initial coordinates θ^α and $\bar{\theta}^\alpha$.

Appendix A. Gamma matrices

In the appendix of the first reference of [1] one specific representation of gamma matrices is given. Here we will calculate the transpositions of basis matrices $(C\Gamma_{(k)})^{\alpha\beta}$ for $k = 1, 2, 3, 4, 5$, where C is charge conjugation operator.

The charge conjugation operator is antisymmetric matrix, $C^T = -C$, and it acts on gamma matrices as

$$C\Gamma^\mu C^{-1} = -(\Gamma^\mu)^T \,. \tag{A.1}$$

Now we have

$$(C\Gamma^\mu)^T = (\Gamma^\mu)^T C^T = -(\Gamma^\mu)^T C = C\Gamma^\mu C^{-1} C = C\Gamma^\mu \,, \tag{A.2}$$

$$(C\Gamma^\mu\Gamma^\nu)^T = C\Gamma^\mu\Gamma^\nu \Longrightarrow (C\Gamma^{[\mu\nu]})^T = C\Gamma^{[\mu\nu]}, \tag{A.3}$$

$$(C\Gamma^\mu\Gamma^\nu\Gamma^\rho)^T = -C\Gamma^\mu\Gamma^\nu\Gamma^\rho \Longrightarrow (C\Gamma^{[\mu\nu\rho]})^T = -C\Gamma^{[\mu\nu\rho]}, \tag{A.4}$$

$$(C\Gamma^\mu\Gamma^\nu\Gamma^\rho\Gamma^\lambda)^T = -C\Gamma^\mu\Gamma^\nu\Gamma^\rho\Gamma^\lambda \Longrightarrow (C\Gamma^{[\mu\nu\rho\lambda]})^T = -C\Gamma^{[\mu\nu\rho\lambda]}, \tag{A.5}$$

$$(C\Gamma^\mu\Gamma^\nu\Gamma^\rho\Gamma^\lambda\Gamma^\omega)^T = C\Gamma^\mu\Gamma^\nu\Gamma^\rho\Gamma^\lambda\Gamma^\omega \Longrightarrow (C\Gamma^{[\mu\nu\rho\lambda\omega]})^T = C\Gamma^{[\mu\nu\rho\lambda\omega]}. \tag{A.6}$$

References

[1] J. Polchinski, String Theory – Volume II, Cambridge University Press, 1998;
K. Becker, M. Becker, J.H. Schwarz, String Theory and M-Theory – A Modern Introduction, Cambridge University Press, 2007.

[2] E. Alvarez, L. Alvarez-Gaume, Y. Lozano, An introduction to T-duality in string theory, arXiv:hep-th/9410237;
A. Giveon, M. Porrati, E. Rabinovici, Target space duality in string theory, Phys. Rep. 244 (1994) 77–202, arXiv:hep-th/9401139;
I. Bandos, B. Julia, J. High Energy Phys. 08 (2003) 032;
D. Luest, J. High Energy Phys. 12 (2010) 084.

[3] T.H. Buscher, Phys. Lett. B 194 (1987) 59;
T.H. Buscher, Phys. Lett. B 201 (1988) 466.

[4] M. Duff, Nucl. Phys. B 335 (1990) 610.

[5] A.A. Tseytlin, Phys. Lett. B 242 (1990) 163.

[6] A.A. Tseytlin, Nucl. Phys. B 350 (1991) 395.

[7] W. Siegel, Phys. Rev. D 48 (1993) 2826.

[8] W. Siegel, Phys. Rev. D 47 (1993) 5453.

[9] C.M. Hull, J. High Energy Phys. 10 (2005) 065.

[10] C.M. Hull, J. High Energy Phys. 10 (2007) 057;
C.M. Hull, J. High Energy Phys. 07 (2007) 080.

[11] D.S. Berman, M. Cederwall, M.J. Perry, J. High Energy Phys. 09 (2014) 066;
D.S. Berman, C.D.A. Blair, E. Malek, M.J. Perry, Int. J. Mod. Phys. A 29 (15) (2014) 1450080;
C.D.A. Blair, E. Malek, A.J. Routh, Class. Quantum Gravity 31 (20) (2014) 205011.

[12] C.M. Hull, R.A. Reid-Edwards, J. High Energy Phys. 09 (2009) 014.

[13] O. Hohm, B. Zwiebach, J. High Energy Phys. 11 (2014) 075.

[14] B. Sazdović, T-duality as coordinates permutation in double space, arXiv:1501.01024.

[15] B. Sazdović, J. High Energy Phys. 08 (2015) 055.

[16] N. Berkovits, J. Maldacena, J. High Energy Phys. 09 (2008) 062;
I. Bakhmatov, D.S. Berman, Nucl. Phys. B 832 (2010) 89–108;
K. Sfetsos, K. Siampos, D.C. Thompson, QMUL-PH-10-08, arXiv:1007.5142;
I. Bakhmatov, Fermionic T-duality and U-duality in type II supergravity, arXiv:1112.1983.

[17] N. Beisert, R. Ricci, A.A. Tseytlin, M. Wolf, Phys. Rev. D 78 (2008) 126004;
R. Ricci, A.A. Tseytlin, M. Wolf, J. High Energy Phys. 12 (2007) 082.

[18] M. Hatsuda, K. Kamimura, W. Siegel, J. High Energy Phys. 06 (2014) 039.

[19] W. Siegel, Phys. Rev. D 50 (1994) 2799–2805.

[20] B. Nikolić, B. Sazdović, Phys. Rev. D 84 (2011) 065012;
B. Nikolić, B. Sazdović, J. High Energy Phys. 06 (2012) 101.

[21] R. Benichou, G. Policastro, J. Troost, Phys. Lett. B 661 (2008) 192–195.

[22] B. Nikolić, B. Sazdović, arXiv:1505.06044.

[23] C.M. Hull, J. High Energy Phys. 07 (1998) 021.

[24] N. Berkovits, arXiv:hep-th/0209059;
P.A. Grassi, G. Policastro, P. van Nieuwenhuizen, J. High Energy Phys. 10 (2002) 054;
P.A. Grassi, G. Policastro, P. van Nieuwenhuizen, J. High Energy Phys. 11 (2002) 004;
P.A. Grassi, G. Policastro, P. van Nieuwenhuizen, Adv. Theor. Math. Phys. 7 (2003) 499;
P.A. Grassi, G. Policastro, P. van Nieuwenhuizen, Phys. Lett. B 553 (2003) 96.

[25] J. de Boer, P.A. Grassi, P. van Nieuwenhuizen, Phys. Lett. B 574 (2003) 98.

[26] B. Nikolić, B. Sazdović, Phys. Lett. B 666 (2008) 400.

[27] B. Nikolić, B. Sazdović, J. High Energy Phys. 08 (2010) 037.

[28] B. Nikolić, B. Sazdović, Nucl. Phys. B 836 (2010) 100–126.

[29] P.A. Grassi, L. Tamassia, J. High Energy Phys. 07 (2004) 071.
[30] Lj. Davidović, B. Sazdović, Eur. Phys. J. C 74 (2014) 2683.
[31] Lj. Davidović, B. Nikolić, B. Sazdović, Eur. Phys. J. C 74 (2014) 2734.
[32] N. Berkovits, P. Howe, Nucl. Phys. B 635 (2002) 75–105.
[33] M.J. Duff, arXiv:hep-th/9912164v2.

Published for SISSA by Springer

Received: *November 5, 2022*
Accepted: *December 5, 2022*
Published: *December 14, 2022*

Noncommutativity and nonassociativity of type II superstring with coordinate dependent RR field — the general case

D. Obrić and B. Nikolić

Institute of Physics Belgrade, University of Belgrade,
Pregrevica 118, Belgrade, Serbia

E-mail: dobric@ipb.ac.rs, bnikolic@ipb.ac.rs

Abstract: In this paper we consider non-commutativity that arises from T-duality of bosonic coordinates of type II superstring in presence of coordinate dependent Ramond-Ramond field. Action with such choice of the background fields is not translational invariant. Consequently, we will employ generalization of Buscher procedure that can be applied to cases that have coordinate dependent fields and that do not possess translational isometry. Bosonic part of newly obtained T-dual theory is non-local and defined in non-geometric double space spanned by Lagrange multipliers y_μ and double coordinate ΔV^μ. We will apply Buscher procedure once more on T-dual theory to check if original theory can be salvaged. Finally, we will use T-dual transformation laws along with Poisson brackets of original theory to derive Poisson bracket structure of T-dual theory.

Keywords: String Duality, Superstrings and Heterotic Strings

ArXiv ePrint: 2211.00455

https://doi.org/10.1007/JHEP12(2022)078

JHEP12(2022)078

Contents

JHEP12(2022)078

1 Introduction

String theory as a possible candidate for unification of all known interactions offers a framework for description of both gauge interactions and gravity. Analyzing the relation between world-sheet diffeomorphisms and transformations of the background fields for open bosonic string [1–3] it is concluded that Kalb-Ramond field gets one additional term that is in fact a field strength of some gauge field. In sigma-model action it looks like that gauge fields are attached at the string endpoints moving along Dp-brane. Finiteness of the gauge theories (UV cutoff) demands existence of some minimal length. Consequently, noncommutativity naturally arose in open bosonic string theory in the presence of the constant background fields [4–7].

The fact that noncommutativity appears together with gauge theory produces a new line of investigation in quantum field theory — noncommutative gauge theories [8–10], but not noncommutative gravity.

The open bosonic string in the presence of the constant background fields gives constant noncommutativity [4–7], but, consequently, Jacobi identity is zero and associativity is not broken. Closed string in the presence of the constant background fields remains commutative.

Noncommutativity in open string theory comes from the boundary conditions (see [5–7]). Coordinates and their canonically conjugated momenta are mixed in the boundary conditions, and because they obey standard Poisson algebra, at the end we get noncommutativity of the initial coordinates. All these facts tell us about the way how we can reach noncommutativity in the bosonic closed string case. Kalb-Ramond field must be, at least, linearly coordinate dependent, and the generalized T-dualization procedure is a machinery [11–19]. The noncommutativity relations are coordinate dependent and produce nonzero Jacobi identity — nonassociativity appears in the closed string theory [13, 18, 20–22].

The noncommutativity and nonassociativity can be considered also within superstring theory [23, 24]. In ref. [25] we considered one special case of the type II superstring theory in pure spinor formulation [26–30] — all physical background fields are constant except Ramond-Ramond (RR) field strength. The RR field strength consists of the constant part and linearly coordinate dependent one, which is infinitesimal. In accordance with consistency conditions, we have chosen constant part of RR field strength to be symmetric and cooordinate dependent part to be antisymmetric tensor.

The motivation for this choice of background fields is the quest for the anticommutation relation between fermionic coordinates suggested in [30, 31]. Formally, this case is similar to the bosonic string case with coordinate dependent Kalb-Ramond field (weakly curved background). The difference is that in the superstring case noncommutativity parameter depends both on the bosonic and fermionic coordinates. Also we obtained that Jacobiator is nonzero. Both noncommutativity and nonassociativity parameters are proportional to the infinitesimal tensor from RR field strength.

In this article we consider the same action as in [25], but we will not imply the additional restrictions on the constant and coordinate dependent part of RR field strength as in [25]. The fundamental difference in relation to the choice of background field in [25] is in the fact that action with RR field strength without restrictions does not possess translational isometry. In that sense this case can be considered as general one comparing with [25].

We will use the generalized T-dualization procedure [16, 19] along bosonic directions. Because this general case cannot be deduced to the form of the bosonic string with linearly dependent Kalb-Ramond field, as it could in the case [25], we obtained more complicated form of T-dual transformation laws and, consequently, the generalization of β_μ functions in the form of $N(\xi)$ functions. Besides the complexity of the T-dual transformation laws we succeeded to find expressions for noncommutativity and nonassociativity as well as the form of the T-dual theory.

At the end we give some concluding remarks. In the appendices we present the derivation of $N(\xi)$ functions and show their properties.

2 General type II superstring action and choice of background fields

In this section we will shortly present how we derive the action of type II superstring in pure spinor formulation with all constant background fields except RR field strength from the general form of that action given in [26–30].

2.1 General form of the pure spinor type II superstring action

The general form of the type II superstring action in pure spinor formalism is derived and given in [30]. It consists of two parts and can be represent as their sum

$$S = S_0 + V_{SG}\,, \tag{2.1}$$

where S_0 describes the motion of string in flat background

$$S_0 = \int_\Sigma d^2\xi \left(\frac{k}{2}\eta_{\mu\nu}\partial_m x^\mu \partial_n x^\nu \eta^{mn} - \pi_\alpha \partial_- \theta^\alpha + \partial_+ \bar\theta^\alpha \bar\pi_\alpha \right) + S_\lambda + S_{\bar\lambda}\,, \tag{2.2}$$

while the second one contains all possible interactions

$$V_{SG} = \int_\Sigma d^2\xi (X^T)^M A_{MN} \bar X^N. \tag{2.3}$$

The second part of the action is expressed in terms of the integrated form of massless type II supergravity vertex operator V_{SG}. The actions S_λ and $S_{\bar\lambda}$ in (2.2) are free-field actions for pure spinors

$$S_\lambda = \int d^2\xi \omega_\alpha \partial_- \lambda^\alpha, \quad S_{\bar\lambda} = \int d^2\xi \bar\omega_\alpha \partial_+ \bar\lambda^\alpha\,. \tag{2.4}$$

Here, λ^α and $\bar\lambda^\alpha$ are pure spinors whose canonically conjugated momenta are ω_α and $\bar\omega_\alpha$, respectively.

The vectors X^M and X^N and matrix A_{MN} are of the form

$$X^M = \begin{pmatrix} \partial_+\theta^\alpha \\ \Pi_+^\mu \\ d_\alpha \\ \frac{1}{2}N_+^{\mu\nu} \end{pmatrix}, \quad \bar X^M = \begin{pmatrix} \partial_-\bar\theta^\lambda \\ \Pi_-^\mu \\ \bar d_\lambda \\ \frac{1}{2}\bar N_-^{\mu\nu} \end{pmatrix}, \quad A_{MN} = \begin{bmatrix} A_{\alpha\beta} & A_{\alpha\nu} & E_\alpha{}^\beta & \Omega_{\alpha,\mu\nu} \\ A_{\mu\beta} & A_{\mu\nu} & \bar E_\mu^\beta & \Omega_{\mu,\nu\rho} \\ E^\alpha{}_\beta & E_\nu^\alpha & P^{\alpha\beta} & C^\alpha{}_{\mu\nu} \\ \Omega_{\mu\nu,\beta} & \Omega_{\mu\nu,\rho} & \bar C^\beta{}_{\mu\nu} & S_{\mu\nu,\rho\sigma} \end{bmatrix}, \tag{2.5}$$

where notation is taken from refs. [25, 30]. Every component of the matrix A_{MN} is function of bosonic, x^μ, and fermionic, θ^α and $\bar\theta^\alpha$, coordinates. For more details about derivation of the components consult [30]. The superfields $A_{\mu\nu}$, $\bar E_\mu^\alpha$, E_μ^α and $P^{\alpha\beta}$ are known as physical superfields, superfields that are in the first row and the first column are known as auxiliary because they can be expressed in terms of physical ones [30]. Remaining superfields $\Omega_{\mu,\nu\rho}$ $(\Omega_{\mu\nu,\rho})$, $C^\alpha{}_{\mu\nu}$ $(\bar C^\beta{}_{\mu\nu})$ and $S_{\mu\nu,\rho\sigma}$, are curvatures (field strengths) for physical fields. Components of X^M and $\bar X^N$ are of the form

$$\Pi_+^\mu = \partial_+ x^\mu + \frac{1}{2}\theta^\alpha (\Gamma^\mu)_{\alpha\beta}\partial_+\theta^\beta, \qquad \Pi_-^\mu = \partial_- x^\mu + \frac{1}{2}\bar\theta^\alpha (\Gamma^\mu)_{\alpha\beta}\partial_-\bar\theta^\beta, \tag{2.6}$$

$$d_\alpha = \pi_\alpha - \frac{1}{2}(\Gamma_\mu\theta)_\alpha \left[\partial_+ x^\mu + \frac{1}{4}(\theta\Gamma^\mu\partial_+\theta)\right],$$

$$\bar d_\alpha = \bar\pi_\alpha - \frac{1}{2}(\Gamma_\mu\bar\theta)_\alpha \left[\partial_- x^\mu + \frac{1}{4}(\bar\theta\Gamma^\mu\partial_-\bar\theta)\right], \tag{2.7}$$

$$N_+^{\mu\nu} = \frac{1}{2}\omega_\alpha (\Gamma^{[\mu\nu]})^\alpha{}_\beta \lambda^\beta, \qquad \bar N_-^{\mu\nu} = \frac{1}{2}\bar\omega_\alpha (\Gamma^{[\mu\nu]})^\alpha{}_\beta \bar\lambda^\beta. \tag{2.8}$$

The world-sheet is spanned by $\xi^m = (\xi^0 = \tau, \xi^1 = \sigma)$, while world-sheet light-cone partial derivatives are defined as $\partial_\pm = \partial_\tau \pm \partial_\sigma$. Superspace contains bosonic x^μ $(\mu = 0, 1, \dots, 9)$ and fermionic θ^α, $\bar\theta^\alpha$ $(\alpha = 1, 2, \dots, 16)$ coordinates. Variables π_α and $\bar\pi_\alpha$ are canonically conjugated momenta to the fermionic coordinates θ^α and $\bar\theta^\alpha$, respectively.

JHEP12(2022)078

2.2 Choice of the background fields

In this particular case we will use the supermatrix A_{MN} where all physical background fields, except RR field strength $P^{\alpha\beta}$, are constant. RR fields strength will have linear coordinate dependence on bosonic coordinate x^μ. Consequently, supermatrix A_{MN} is of the following form

$$A_{MN} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & k(\frac{1}{2}g_{\mu\nu} + B_{\mu\nu}) & \bar{\Psi}^\beta_\mu & 0 \\ 0 & -\Psi^\alpha_\nu & \frac{2}{k}(f^{\alpha\beta} + C^{\alpha\beta}_\rho x^\rho) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \tag{2.9}$$

where $g_{\mu\nu}$ is symmetric tensor, $B_{\mu\nu}$ is Kalb-Ramond antisymmetric field, Ψ^α_μ and $\bar{\Psi}^\alpha_\mu$ are Mayorana-Weyl gravitino fields and $f^{\alpha\beta}$ and $C^{\alpha\beta}_\rho$ are constant tensors. Let us stress this will be a classical analysis and we will not calculate the dilaton shift under T-duality transformation.

From the consistency conditions given in ref. [30], following this choice of background fields, it follows

$$\gamma^\mu_{\alpha\beta} C^{\beta\gamma}_\mu = 0, \quad \gamma^\mu_{\alpha\beta} C^{\gamma\beta}_\mu = 0. \tag{2.10}$$

Because all background fields are expanded in powers of θ^α and $\bar{\theta}^\alpha$, θ^α and $\bar{\theta}^\alpha$ terms in X^M and $\bar{X}^N$ will be neglected. Taking into account all imposed assumptions and approximations, the full action S is getting the form

$$S = \int_\Sigma d^2\xi \left[\frac{k}{2}\Pi_{+\mu\nu}\partial_+ x^\mu \partial_- x^\nu - \pi_\alpha(\partial_-\theta^\alpha + \Psi^\alpha_\nu \partial_- x^\nu) + (\partial_+\bar{\theta}^\alpha + \partial_+ x^\mu \bar{\Psi}^\alpha_\mu)\bar{\pi}_\alpha \right. \\ \left. + \frac{2}{k}\pi_\alpha (f^{\alpha\beta} + C^{\alpha\beta}_\rho x^\rho)\bar{\pi}_\beta \right], \tag{2.11}$$

where $\Pi_{\pm\mu\nu} = B_{\mu\nu} \pm \frac{1}{2}G_{\mu\nu}$, and $G_{\mu\nu} = \eta_{\mu\nu} + g_{\mu\nu}$ is metric tensor. The actions S_λ and $S_{\bar{\lambda}}$ are fully decoupled from the rest and they will not be analyzed from now on.

It is easy to notice that fermionic momenta play the roles of the auxiliary fields in full action. They can be integrated out finding equations for motion for both π_α and $\bar{\pi}_\alpha$

$$\bar{\pi}_\beta = \frac{k}{2}\left(F^{-1}(x)\right)_{\beta\alpha}(\partial_-\theta^\alpha + \Psi^\alpha_\nu \partial_- x^\nu), \tag{2.12}$$

$$\pi_\alpha = -\frac{k}{2}\left(\partial_+\bar{\theta}^\beta + \partial_+ x^\mu \bar{\Psi}^\beta_\mu\right)\left(F^{-1}(x)\right)_{\beta\alpha}, \tag{2.13}$$

where $F^{\alpha\beta}(x)$ and $(F^{-1}(x))_{\alpha\beta}$ are of the form

$$F^{\alpha\beta}(x) = f^{\alpha\beta} + C^{\alpha\beta}_\mu x^\mu, \quad (F^{-1}(x))_{\alpha\beta} = (f^{-1})_{\alpha\beta} - (f^{-1})_{\alpha\alpha_1} C^{\alpha_1\beta_1}_\rho x^\rho (f^{-1})_{\beta_1\beta}. \tag{2.14}$$

For practical reasons, we assume that $C^{\alpha\beta}{}_\mu$ is infinitesimal. This assumption is in accordance with constraints (2.10). Substituting equations (2.12) and (2.13) into (2.11) the final form of action is

$$S = k\int_\Sigma d^2\xi \left[\Pi_{+\mu\nu}\partial_+ x^\mu \partial_- x^\nu + \frac{1}{2}(\partial_+\bar{\theta}^\alpha + \partial_+ x^\mu \bar{\Psi}^\alpha_\mu)\left(F^{-1}(x)\right)_{\alpha\beta}(\partial_-\theta^\beta + \Psi^\beta_\nu \partial_- x^\nu)\right]. \tag{2.15}$$

JHEP12(2022)078

Let us note that we did not impose any conditions on tensors $f^{\alpha\beta}$ and $C_\mu^{\alpha\beta}$ as we did in ref. [25]. The case considered in this article is more general than the case studied in [25], because action does not possess translational symmetry.

3 T-dualization

Here we will make T-dualization of all bosonic directions aiming to find T-dual transformation laws — relations between T-dual coordinates and canonical variables of the original theory. The T-dual transformation laws will be used to calculate Poisson brackets of the T-dual coordinates.

3.1 Implementation of the generalized T-dualization procedure

In implementing of T-dualization procedure we will use *generalized Buscher T-dualization procedure* [16]. The standard Buscher procedure [14, 15] is made to be used along directions on which background fields do not depend (isometry directions), while generalized Buscher procedure can be applied to theories with coordinate dependent background fields along all directions. The generalized T-dualization procedure follows three steps — introduction of covariant derivatives, invariant coordinates and additional gauge fields, which produces additional degrees of freedom. The starting and T-dual theory must have the same number of degrees of freedom. In order to achieve that we eliminate all excessive degrees of freedom demanding that field strength of gauge fields ($F_{+-} = \partial_+ v_- - \partial_- v_+$) vanishes by addition of Lagrange multipliers. Then we fix the gauge symmetry (shift symmetry) and action is left with gauge fields and their derivatives. Finding equations of motion for gauge fields, expressing in terms of the Lagrange multipliers and inserting those equations into action we obtain T-dual action, where Lagrange multipliers have roles of T-dual coordinates.

T-duality can be performed also in the cases of the absence of shift symmetry [19]. Then we replace original action with translation invariant auxiliary action. Form of the auxiliary action is exactly the same as the form of action where translation symmetry was localized and gauge fixed. It produces correct T-dual theory only if original action can be salvaged from it.

Action (2.15) is not translational invariant. Consequently, we make the following substitutions

$$\partial_\pm x^\mu \to v_\pm^\mu, \tag{3.1}$$

$$x^\rho \to \Delta V^\rho = \int_P d\xi'^m v_m^\rho(\xi'), \tag{3.2}$$

$$S \to S + \frac{k}{2}\int_\Sigma d^2\xi \left[v_+^\mu \partial_- y_\mu - v_-^\mu \partial_+ y_\mu\right], \tag{3.3}$$

and insert them in action (2.15). The result is auxiliary action convenient for T-dualization procedure

$$\begin{aligned} S_{aux} = k\int_\Sigma d^2\xi \Big[&\Pi_{+\mu\nu} v_+^\mu v_-^\nu + \frac{1}{2}(\partial_+\bar\theta^\alpha + v_+^\mu \bar\Psi_\mu^\alpha)\left(F^{-1}(\Delta V)\right)_{\alpha\beta}(\partial_-\theta^\beta + \Psi_\nu^\beta v_-^\nu) \\ &+\frac{1}{2}(v_+^\mu \partial_- y_\mu - v_-^\mu \partial_+ y_\mu)\Big]. \end{aligned} \tag{3.4}$$

Let us note that path P starts from ξ_0 and ends in ξ. In this way action becomes non-local.

Finding equations of motion for Lagrange multipliers

$$\partial_- v_+^\mu - \partial_+ v_-^\mu = 0 \quad v_\pm^\mu = \partial_\pm x^\mu, \tag{3.5}$$

and inserting them into (3.2) we have

$$\Delta V^\rho = \int_P d\xi'^m \partial_m x^\rho(\xi') = x^\rho(\xi) - x^\rho(\xi_0) = \Delta x^\rho. \tag{3.6}$$

In absence of translational symmetry, in order to extract starting action from auxiliary one, we impose $x^\rho(\xi_0) = 0$ as a constraint. Taking all this into account, we get the starting action (2.15).

Euler-Lagrange equations of motion for gauge fields $v_\pm(\kappa)$ give the following ones

$$\begin{aligned}
&-\frac{1}{2}\partial_- y_\mu(\kappa) = \Pi_{+\mu\nu} v_-^\nu(\kappa) + \frac{1}{2}\bar\Psi_\mu^\alpha \left(F^{-1}(\Delta V)\right)_{\alpha\beta} (\partial_-\theta^\beta(\kappa) + \Psi_\nu^\beta v_-^\nu(\kappa)) && (3.7)\\
&-\frac{1}{2}\int_\Sigma d^2\xi [\partial_+\bar\theta^\alpha(\xi) + v_+^{\nu_1}(\xi)\bar\Psi_{\nu_1}^\alpha](f^{-1})_{\alpha\alpha_1} C_\mu^{\alpha_1\beta_1}(f^{-1})_{\beta_1\beta} N(\kappa^+)[\partial_-\theta^\beta(\xi) + \Psi_{\nu_2}^\beta v_-^{\nu_2}(\xi)],\\
&\frac{1}{2}\partial_+ y_\mu(\kappa) = \Pi_{+\nu\mu} v_+^\nu(\kappa) + \frac{1}{2}(\partial_+\bar\theta^\alpha(\kappa) + v_+^\nu(\kappa)\bar\Psi_\nu^\alpha) \left(F^{-1}(\Delta V)\right)_{\alpha\beta} \Psi_\mu^\beta && (3.8)\\
&-\frac{1}{2}\int_\Sigma d^2\xi [\partial_+\bar\theta^\alpha(\xi) + v_+^{\nu_1}(\xi)\bar\Psi_{\nu_1}^\alpha](f^{-1})_{\alpha\alpha_1} C_\mu^{\alpha_1\beta_1}(f^{-1})_{\beta_1\beta} N(\kappa^-)[\partial_-\theta^\beta(\xi) + \Psi_{\nu_2}^\beta v_-^{\nu_2}(\xi)].
\end{aligned}$$

Here, function $N(\kappa^\pm)$ is obtained from variation of term containing ΔV^ρ in expression for $F^{-1}(\Delta V)$ (details are presented in appendix A). They represent the generalization of beta functions introduced in ref. [25]

$$N(\kappa^+) = \delta\Big(\xi'^-((\xi'^+)^{-1}(\kappa^+)) - \kappa^-\Big)\left[H(\xi^+ - \kappa^+) - H(\xi_0^+ - \kappa^+)\right], \tag{3.9}$$

$$N(\kappa^-) = \delta\Big(\xi'^+((\xi'^-)^{-1}(\kappa^-)) - \kappa^+\Big)\left[H(\xi^- - \kappa^-) - H(\xi_0^- - \kappa^-)\right], \tag{3.10}$$

where more details on Dirac delta function and step function are given in appendix A. As we see the expressions for derivatives of y_μ are more complex comparing with those in [25], where translational symmetry is present.

Assuming that $C_\mu^{\alpha\beta}$ is an infinitesimal, we can iteratively invert equations of motion (3.7) and (3.8) [20]. Separating variables into two parts, one finite and one infinitesimal

JHEP12(2022)078

proportional to $C_\mu^{\alpha\beta}$, we have

$$
\begin{aligned}
v_-^\nu(\kappa) = & -\frac{1}{2}\bar\Theta_-^{\nu\nu_1}\Big\{\partial_- y_{\nu_1}(\kappa)+\bar\Psi_{\nu_1}^\alpha(F^{-1}(\Delta V))_{\alpha\beta}\partial_-\theta^\beta(\kappa) \\
&+\frac{1}{2}\Psi_{\nu_1}^\alpha(f^{-1})_{\alpha\alpha_1}C_\rho^{\alpha_1\alpha_2}\Delta V^\rho(f^{-1})_{\alpha_2\alpha_3}\Psi_{\nu_2}^{\alpha_3}\bar\Theta_-^{\nu_2\nu_3}\Big(\partial_- y_{\nu_3}(\kappa)+\bar\Psi_{\nu_3}^{\beta_1}(f^{-1})_{\beta_1\beta}\partial_-\theta^\beta(\kappa)\Big) \\
&-\int_\Sigma d^2\xi\Big[\partial_+\bar\theta^\alpha(\xi)+\frac{1}{2}\Big(\partial_+ y_{\mu_1}(\xi)-\partial_+\bar\theta^{\gamma_1}(\xi)(f^{-1})_{\gamma_1\gamma_2}\Psi_{\mu_1}^{\gamma_2}\Big)\bar\Theta_-^{\mu_1\nu_1}\bar\Psi_{\nu_1}^\alpha\Big](f^{-1})_{\alpha\alpha_1}C_{\nu_1}^{\alpha_1\beta_1} \\
&\times(f^{-1})_{\beta_1\beta}N(\kappa^+)\Big[\partial_-\theta^\beta(\xi)-\frac{1}{2}\Psi_{\nu_2}^\beta\bar\Theta_-^{\nu_2\mu_2}\Big(\partial_- y_{\mu_2}(\xi)+\bar\Psi_{\mu_2}^{\gamma_3}(f^{-1})_{\gamma_3\gamma_4}\partial_-\theta^{\gamma_4}(\xi)\Big)\Big]\Big\},
\end{aligned}
\tag{3.11}
$$

$$
\begin{aligned}
v_+^\mu(\kappa) = & \frac{1}{2}\bar\Theta_-^{\mu_1\mu}\Big\{\partial_+ y_{\mu_1}(\kappa)-\partial_+\bar\theta^\alpha(\kappa)(F^{-1}(\Delta V))_{\alpha\beta}\Psi_{\mu_1}^\beta \\
&+\frac{1}{2}\Big(\partial_+ y_{\mu_2}(\kappa)-\partial_+\bar\theta^\alpha(\kappa)(f^{-1})_{\alpha\alpha_1}\Psi_{\mu_2}^{\alpha_1}\Big)\bar\Theta_-^{\mu_2\mu_3}\bar\Psi_{\mu_3}^{\beta_3}(f^{-1})_{\beta_3\beta_2}C_\rho^{\beta_2\beta_1}\Delta V^\rho(f^{-1})_{\beta_1\beta}\Psi_{\mu_1}^\beta \\
&+\int_\Sigma d^2\xi\Big[\partial_+\bar\theta^\alpha(\xi)+\frac{1}{2}\Big(\partial_+ y_{\mu_2}(\xi)-\partial_+\bar\theta^{\gamma_1}(\xi)(f^{-1})_{\gamma_1\gamma_2}\Psi_{\mu_2}^{\gamma_2}\Big)\bar\Theta_-^{\mu_2\nu_1}\bar\Psi_{\nu_1}^\alpha\Big](f^{-1})_{\alpha\alpha_1}C_{\mu_1}^{\alpha_1\beta_1} \\
&\times(f^{-1})_{\beta_1\beta}N(\kappa^-)\Big[\partial_-\theta^\beta(\xi)-\frac{1}{2}\Psi_{\nu_2}^\beta\bar\Theta_-^{\nu_2\mu_3}\Big(\partial_- y_{\mu_3}(\xi)+\bar\Psi_{\mu_3}^{\gamma_3}(f^{-1})_{\gamma_3\gamma_4}\partial_-\theta^{\gamma_4}(\xi)\Big)\Big]\Big\}.
\end{aligned}
\tag{3.12}
$$

Tensor $\bar\Theta_-^{\mu\nu}$ is inverse tensor to $\bar\Pi_{+\mu\nu} = \Pi_{+\mu\nu} + \frac{1}{2}\bar\Psi_\mu^\alpha(f^{-1})_{\alpha\beta}\Psi_\nu^\beta$

$$\bar\Theta_-^{\mu\nu}\bar\Pi_{+\nu\rho} = \delta_\rho^\mu, \tag{3.13}$$

where

$$\bar\Theta_-^{\mu\nu} = \Theta_-^{\mu\nu} - \frac{1}{2}\Theta_-^{\mu\mu_1}\bar\Psi_{\mu_1}^\alpha(\bar f^{-1})_{\alpha\beta}\Psi_{\nu_1}^\beta\Theta_-^{\nu_1\nu}, \tag{3.14}$$

$$\bar f^{\alpha\beta} = f^{\alpha\beta} + \frac{1}{2}\Psi_\mu^\alpha\Theta_-^{\mu\nu}\bar\Psi_\nu^\beta, \tag{3.15}$$

$$\Theta_-^{\mu\nu}\Pi_{+\mu\rho} = \delta_\rho^\mu, \qquad \Theta_- = -4(G_E^{-1}\Pi_- G^{-1})^{\mu\nu}. \tag{3.16}$$

Effective metric tensor is defined as $G_{E\mu\nu} \equiv G_{\mu\nu} - 4(BG^{-1}B)_{\mu\nu}$.

In above expressions ΔV is a quantity in the zeroth order in $C_\mu^{\alpha\beta}$

$$
\begin{aligned}
\Delta V^\rho = & \int d\xi^+ v_+^\rho + \int d\xi^- v_-^\rho \\
= & \frac{1}{2}\int_P d\xi^+\bar\Theta_-^{\rho_1\rho}\Big[\partial_+ y_{\rho_1} - \partial_+\bar\theta^\alpha(f^{-1})_{\alpha\beta}\Psi_{\rho_1}^\beta\Big] - \frac{1}{2}\int_P d\xi^-\bar\Theta_-^{\rho\rho_1}\Big[\partial_- y_{\rho_1} + \bar\Psi_{\rho_1}^\alpha(f^{-1})_{\alpha\beta}\partial_-\theta^\beta\Big].
\end{aligned}
\tag{3.17}
$$

JHEP12(2022)078

Using (3.7) and (3.8) and inserting them into (3.4), we get T-dual action

$$S_{T-dual} = k\int_P d^2\xi\Big[\frac{1}{4}\bar\Theta_-^{\mu\nu}\partial_+ y_\mu\partial_- y_\nu \tag{3.18}$$
$$+\frac{1}{8}\bar\Theta_-^{\mu\mu_1}\bar\Psi^\alpha_{\mu_1}(f^{-1})_{\alpha\alpha_1}C_\rho^{\alpha_1\beta_1}\Delta V^\rho(f^{-1})_{\beta_1\beta}\Psi^\beta_{\nu_1}\bar\Theta_-^{\nu_1\nu}\partial_+ y_\mu\partial_- y_\nu$$
$$+\frac{1}{2}\partial_+\bar\theta^\alpha\Big((F^{-1}(\Delta V))_{\alpha\beta}+\frac{1}{2}(f^{-1})_{\alpha\alpha_1}C_\rho^{\alpha_1\alpha_2}\Delta V^\rho(f^{-1})_{\alpha_2\alpha_3}\Psi^{\alpha_3}_\mu\bar\Theta_-^{\mu\nu}\bar\Psi^{\beta_1}_\nu(f^{-1})_{\beta_1\beta}$$
$$-\frac{1}{2}(f^{-1})_{\alpha\alpha_1}\Psi^{\alpha_1}_\mu\bar\Theta_-^{\mu\nu}\bar\Psi^{\beta_1}_\nu(f^{-1})_{\beta_1\beta}+\frac{1}{2}(f^{-1})_{\alpha\alpha_1}\Psi^{\alpha_1}_\mu\bar\Theta_-^{\mu\nu}\bar\Psi^{\beta_3}_\nu(f^{-1})_{\beta_3\beta_2}C_\rho^{\beta_2\beta_1}\Delta V^\rho(f^{-1})_{\beta_1\beta}$$
$$-\frac{1}{4}(f^{-1})_{\alpha\alpha_1}\Psi^{\alpha_1}_\mu\bar\Theta_-^{\mu\mu_1}\bar\Psi^{\alpha_2}_{\mu_1}(f^{-1})_{\alpha_2\alpha_3}C_\rho^{\alpha_3\beta_3}\Delta V^\rho(f^{-1})_{\beta_3\beta_2}\Psi^{\beta_2}_{\nu_1}\bar\Theta_-^{\nu_1\nu}\bar\Psi^{\beta_1}_\nu(f^{-1})_{\beta_1\beta}\Big)\partial_-\theta^\beta$$
$$+\frac{1}{4}\partial_+ y_\mu\bar\Theta_-^{\mu\mu_1}\bar\Psi^\alpha_{\mu_1}\Big((F^{-1}(\Delta V))_{\alpha\beta}+\frac{1}{2}(f^{-1})_{\alpha\alpha_1}C_\rho^{\alpha_1\beta_3}\Delta V^\rho(f^{-1})_{\beta_3\beta_2}\Psi^{\beta_2}_{\nu_1}\bar\Theta_-^{\nu_1\nu}\bar\Psi^{\beta_1}_\nu(f^{-1})_{\beta_1\beta}\Big)$$
$$\times\partial_-\theta^\beta$$
$$-\frac{1}{4}\partial_+\bar\theta^\alpha\Big((F^{-1}(\Delta V))_{\alpha\beta}+\frac{1}{2}(f^{-1})_{\alpha\alpha_1}\Psi^{\alpha_1}_\mu\bar\Theta_-^{\mu\mu_1}\bar\Psi^{\alpha_2}_{\mu_1}(f^{-1})_{\alpha_2\alpha_3}C_\rho^{\alpha_3\beta_1}\Delta V^\rho(f^{-1})_{\beta_1\beta}\Big)\Psi^\beta_{\nu_1}\bar\Theta_-^{\nu_1\nu}$$
$$\times\partial_- y_\nu\Big].$$

Let us note that above we kept terms up to the first order in $C_\mu^{\alpha\beta}$.

T-dual action contains all terms as initial action (2.15) up to the change $x^\mu\to y_\mu$. Consequently, T-dual background fields are of the form

$$^\star\Pi_+^{\mu\nu}=\frac{1}{4}\bar\Theta_-^{\mu\nu}+\frac{1}{8}\bar\Theta_-^{\mu\mu_1}\bar\Psi^\alpha_{\mu_1}\Big[(F^{-1}(\Delta V))_{\alpha\beta}+(f^{-1})_{\alpha\alpha_1}C_\rho^{\alpha_1\beta_1}\Delta V^\rho(f^{-1})_{\beta_1\beta} \tag{3.19}$$
$$-\frac{1}{2}(f^{-1})_{\alpha\alpha_1}\Psi^{\alpha_1}_{\mu_2}\bar\Theta_-^{\mu_2\nu_2}\bar\Psi^{\beta_1}_{\nu_2}(f^{-1})_{\beta_1\beta}+\frac{1}{2}(f^{-1})_{\alpha\alpha_1}C_\rho^{\alpha_1\alpha_2}\Delta V^\rho(f^{-1})_{\alpha_2\alpha_3}\Psi^{\alpha_3}_{\mu_2}\bar\Theta_-^{\mu_2\nu_2}\bar\Psi^{\beta_1}_{\nu_2}(f^{-1})_{\beta_1\beta}$$
$$+\frac{1}{2}(f^{-1})_{\alpha\alpha_1}\Psi^{\alpha_1}_{\mu_2}\bar\Theta_-^{\mu_2\nu_2}\bar\Psi^{\beta_3}_{\nu_2}(f^{-1})_{\beta_3\beta_2}C_\rho^{\beta_2\beta_1}\Delta V^\rho(f^{-1})_{\beta_1\beta}$$
$$-\frac{1}{4}(f^{-1})_{\alpha\alpha_1}\Psi^{\alpha_1}_{\mu_2}\bar\Theta_-^{\mu_2\mu_3}\bar\Psi^{\alpha_2}_{\mu_3}(f^{-1})_{\alpha_2\alpha_3}C_\rho^{\alpha_3\beta_3}\Delta V^\rho(f^{-1})_{\beta_3\beta_2}\Psi^{\beta_2}_{\nu_3}\bar\Theta_-^{\nu_3\nu_2}\bar\Psi^{\beta_1}_{\nu_2}(f^{-1})_{\beta_1\beta}\Big]\Psi^\beta_{\nu_1}\bar\Theta_-^{\nu_1\nu},$$

$$^\star(F^{-1}(x))_{\alpha\beta}=(F^{-1}(\Delta\bar y))_{\alpha\beta}+\frac{1}{2}(f^{-1})_{\alpha\alpha_1}C_\rho^{\alpha_1\alpha_2}\Delta V^\rho(f^{-1})_{\alpha_2\alpha_3}\Psi^{\alpha_3}_\mu\bar\Theta_-^{\mu\nu}\bar\Psi^{\beta_1}_\nu(f^{-1})_{\beta_1\beta} \tag{3.20}$$
$$-\frac{1}{2}(f^{-1})_{\alpha\alpha_1}\Psi^{\alpha_1}_\mu\bar\Theta_-^{\mu\nu}\bar\Psi^{\beta_1}_\nu(f^{-1})_{\beta_1\beta}+\frac{1}{2}(f^{-1})_{\alpha\alpha_1}\Psi^{\alpha_1}_\mu\bar\Theta_-^{\mu\nu}\bar\Psi^{\beta_3}_\nu(f^{-1})_{\beta_3\beta_2}C_\rho^{\beta_2\beta_1}\Delta V^\rho(f^{-1})_{\beta_1\beta}$$
$$-\frac{1}{4}(f^{-1})_{\alpha\alpha_1}\Psi^{\alpha_1}_\mu\bar\Theta_-^{\mu\mu_1}\bar\Psi^{\alpha_2}_{\mu_1}(f^{-1})_{\alpha_2\alpha_3}C_\rho^{\alpha_3\beta_3}\Delta V^\rho(f^{-1})_{\beta_3\beta_2}\Psi^{\beta_2}_{\nu_1}\bar\Theta_-^{\nu_1\nu}\bar\Psi^{\beta_1}_\nu(f^{-1})_{\beta_1\beta},$$

$$^\star\bar\Psi^{\mu\alpha}=\frac{1}{2}\Theta_-^{\mu\nu}\bar\Psi^\alpha_\nu,\qquad ^\star\Psi_{\nu\beta}=-\frac{1}{2}\Psi^\beta_\mu\Theta_-^{\mu\nu}. \tag{3.21}$$

Comparing background field of T-dual theory with background fields from [25] we immediately notice that background fields have become more complex. However, this is just an illusion. In both cases background field are exactly the same only difference is that here we did not introduce tensor $\breve\Pi_{+\mu\nu}=\Pi_{+\mu\nu}+\frac{1}{2}\bar\Psi^\alpha_\mu F^{-1}(\Delta V)_{\alpha\beta}\Psi^\beta_\nu$ and its inverse, therefore we are missing ingredients to express our fields in more compactified format.

3.2 T-dualization of T-dual theory

Since the initial theory is not symmetric under translations, T-dual action that is obtained from auxiliary action (3.4) and it is now invariant to translations of T-dual coordinates. Consequently, we can dualize T-dual theory by generalized Buscher procedure. We start with the introduction of following substitutions

$$\partial_\pm y_\mu \to D_\pm y_\mu = \partial_\pm y_\mu + u_{\pm\mu} \to D_\pm y_\mu = u_{\pm\mu}, \tag{3.22}$$

$$\Delta\bar{y}^\rho \to \Delta\bar{u}^\rho, \tag{3.23}$$

$$\begin{aligned}\Delta\bar{u}^\rho = &\frac{1}{2}\int_P d\xi^+ \bar{\Theta}_-^{\rho_1\rho}\left[u_{+\rho_1} - \partial_+\bar{\theta}^\alpha (f^{-1})_{\alpha\beta}\Psi^\beta_{\rho_1}\right] \\ &- \frac{1}{2}\int_P d\xi^- \bar{\Theta}_-^{\rho\rho_1}\left[u_{-\rho_1} + \bar{\Psi}^\alpha_{\rho_1}(f^{-1})_{\alpha\beta}\partial_-\theta^\beta\right],\end{aligned} \tag{3.24}$$

$$S \to S + \frac{1}{2}(u_{+\mu}\partial_- x^\mu - u_{-\mu}\partial_+ x^\mu)\,. \tag{3.25}$$

From the first line we see that gauge is fixed choosing $y(\xi) = const$. Inserting these substitutions into (3.18) we obtain

$$\begin{aligned}S_{\text{gauge fix}} = \kappa\int_P d^2\xi\Bigg[&\frac{1}{4}\bar{\Theta}_-^{\mu\nu}u_{+\mu}u_{-\nu} \\
&+ \frac{1}{8}\bar{\Theta}_-^{\mu\mu_1}\bar{\Psi}^\alpha_{\mu_1}(f^{-1})_{\alpha\alpha_1}C_\rho^{\alpha_1\beta_1}\Delta\bar{u}^\rho(f^{-1})_{\beta_1\beta}\Psi^\beta_{\nu_1}\bar{\Theta}_-^{\nu_1\nu}u_{+\mu}u_{-\nu} \\
&+ \frac{1}{2}\partial_+\bar{\theta}^\alpha\Big((F^{-1}(\Delta\bar{u}))_{\alpha\beta} + \frac{1}{2}(f^{-1})_{\alpha\alpha_1}C_\rho^{\alpha_1\alpha_2}\Delta\bar{u}^\rho(f^{-1})_{\alpha_2\alpha_3}\Psi^{\alpha_3}_\mu\bar{\Theta}_-^{\mu\nu}\bar{\Psi}^{\beta_1}_\nu(f^{-1})_{\beta_1\beta} \\
&\quad - \frac{1}{2}(f^{-1})_{\alpha\alpha_1}\Psi^{\alpha_1}_\mu\bar{\Theta}_-^{\mu\nu}\bar{\Psi}^{\beta_1}_\nu(f^{-1})_{\beta_1\beta} + \frac{1}{2}(f^{-1})_{\alpha\alpha_1}\Psi^{\alpha_1}_\mu\bar{\Theta}_-^{\mu\nu}\bar{\Psi}^{\beta_3}_\nu(f^{-1})_{\beta_3\beta_2}C_\rho^{\beta_2\beta_1}\Delta\bar{u}^\rho(f^{-1})_{\beta_1\beta} \\
&\quad - \frac{1}{4}(f^{-1})_{\alpha\alpha_1}\Psi^{\alpha_1}_\mu\bar{\Theta}_-^{\mu\mu_1}\bar{\Psi}^{\alpha_2}_{\mu_1}(f^{-1})_{\alpha_2\alpha_3}C_\rho^{\alpha_3\beta_3}\Delta\bar{u}^\rho(f^{-1})_{\beta_3\beta_2}\Psi^{\beta_2}_{\nu_1}\bar{\Theta}_-^{\nu_1\nu}\bar{\Psi}^{\beta_1}_\nu(f^{-1})_{\beta_1\beta}\Big)\partial_-\theta^\beta \\
&+ \frac{1}{4}u_{+\mu}\bar{\Theta}_-^{\mu\mu_1}\bar{\Psi}^\alpha_{\mu_1}\Big((F^{-1}(\Delta\bar{u}))_{\alpha\beta} + \frac{1}{2}(f^{-1})_{\alpha\alpha_1}C_\rho^{\alpha_1\beta_3}\Delta\bar{u}^\rho(f^{-1})_{\beta_3\beta_2}\Psi^{\beta_2}_{\nu_1}\bar{\Theta}_-^{\nu_1\nu}\bar{\Psi}^{\beta_1}_\nu(f^{-1})_{\beta_1\beta}\Big) \\
&\quad\times\partial_-\theta^\beta \\
&- \frac{1}{4}\partial_+\bar{\theta}^\alpha\Big((F^{-1}(\Delta\bar{u}))_{\alpha\beta} + \frac{1}{2}(f^{-1})_{\alpha\alpha_1}\Psi^{\alpha_1}_\mu\bar{\Theta}_-^{\mu\mu_1}\bar{\Psi}^{\alpha_2}_{\mu_1}(f^{-1})_{\alpha_2\alpha_3}C_\rho^{\alpha_3\beta_1}\Delta\bar{u}^\rho(f^{-1})_{\beta_1\beta}\Big)\Psi^\beta_{\nu_1}\bar{\Theta}_-^{\nu_1\nu} \\
&\quad\times u_{-\nu} \\
&+ \frac{1}{2}(u_{+\mu}\partial_- x^\mu - u_{-\mu}\partial_+ x^\mu)\Bigg].\end{aligned} \tag{3.26}$$

Using equations of motion for Lagrange multipliers, we return to the T-dual action. Finding equations of motion for gauge fields, we have

$$\begin{aligned}u_{+\mu}(\kappa) = \;& 2\bar{\Pi}_{+\nu\mu}\partial_+ x^\nu(\kappa) - \partial_+ x^\nu(\kappa)\bar{\Psi}^\alpha_\nu(f^{-1})_{\alpha\alpha_1}C_\rho^{\alpha_1\beta_1}\Delta x^\rho(f^{-1})_{\beta_1\beta}\Psi^\beta_\mu \\
&+ \partial_+\bar{\theta}^\alpha(\kappa)(F^{-1}(\Delta\bar{x}))_{\alpha\beta}\Psi^\beta_\mu \\
&- \int_\Sigma d^2\xi\Big(\partial_+\bar{\theta}^\alpha(\xi) + \partial_+ x^{\mu_1}(\xi)\bar{\Psi}^\alpha_{\mu_1}\Big)(f^{-1})_{\alpha\alpha_1}C_\mu^{\alpha_1\beta_1}(f^{-1})_{\beta_1\beta}N(\kappa^-) \\
&\times\Big(\partial_-\theta^\beta(\xi) + \Psi^\beta_\nu\partial_- x^\nu(\xi)\Big),\end{aligned} \tag{3.27}$$

JHEP12(2022)078

$$\begin{aligned} u_{-\nu}(\kappa) = & -2\bar{\Pi}_{+\nu\mu}\partial_- x^\mu(\kappa) + \bar{\Psi}^\alpha_\nu (f^{-1})_{\alpha\alpha_1} C^{\alpha_1\beta_1}_\rho \Delta x^\rho (f^{-1})_{\beta_1\beta}\Psi^\beta_\mu \partial_- x^\mu(\kappa) \\ & - \bar{\Psi}^\alpha_\nu (F^{-1}(\Delta\bar{x}))_{\alpha\beta}\partial_-\theta^\beta(\kappa) \\ & + \int_\Sigma d^2\xi \Big(\partial_+\bar{\theta}^\alpha(\xi) + \partial_+ x^\mu(\xi)\bar{\Psi}^\alpha_\mu\Big)(f^{-1})_{\alpha\alpha_1} C^{\alpha_1\beta_1}_\nu (f^{-1})_{\beta_1\beta} N(\kappa^-) \\ & \times \Big(\partial_-\theta^\beta(\xi) + \Psi^\beta_{\nu_1}\partial_- x^{\nu_1}(\xi)\Big)\,. \end{aligned} \tag{3.28}$$

Here we have that $\Delta x^\mu = x(\xi) - x(\xi_0)$, and inserting these equations into the gauge fixed action, keeping all terms linear with respect to $C^{\mu\nu}_\rho$ and selecting ξ_0 such that $x(\xi_0) = 0$, we obtain our original action (2.15).

4 Non-commutative relations

In this section we will establish a relationship between Poisson brackets of original and T-dual theory using results from the previous one. Original theory is a geometric one, which means that canonical variables $x^\mu(\xi)$ and $\pi_\mu(\xi)$ satisfy standard Poisson algebra

$$\{x^\mu(\sigma), \pi_\nu(\bar{\sigma})\} = \delta^\mu_\nu \delta(\sigma - \bar{\sigma}), \quad \{x^\mu(\sigma), x^\nu(\bar{\sigma})\} = 0, \quad \{\pi_\mu(\sigma), \pi_\nu(\bar{\sigma})\} = 0. \tag{4.1}$$

We will find Poisson structure of T-dual theory using relations (3.7) and (3.8) and expressing them in terms of the coordinates and momenta of the initial theory. Replacing gauge fields with solutions of equations of motion for Lagrange multipliers, we get T-dual transformation laws in Lagrangian form. Because we implement here canonical approach, the next step is removing of all terms that are proportional to $\partial_\tau x^\mu(\xi)$. The most of the terms of this type will get incorporated into expression for canonical momenta $\pi_\mu(\xi)$, but term that is non-local and which is dependent on function $N(\xi^\pm)$ remains. One way of removing this term is to first use equations of motion for coordinate $x^\mu(\xi)$, and then replace remaining $\partial_\tau x^\mu$ term with canonical momentum. By doing all the steps that were outlined, we have following relationship between T-dual coordinate and variables of starting theory

$$\begin{aligned} \partial_\sigma y_\nu(\sigma) = & \, 2B_{\nu\mu}\partial_\sigma x^\mu - G_{\nu\mu}(\bar{\Pi}_+ + \bar{\Pi}^T_+)^{-1\nu_1\mu}\Bigg[\frac{\pi_{\nu_1}}{k} - \frac{1}{2}\bar{\Psi}^\alpha_{\nu_1}\Big(F^{-1}(x)\Big)_{\alpha\beta}\partial_-\theta^\beta \\ & - \frac{1}{2}\partial_+\bar{\theta}^\alpha\Big(F^{-1}(x)\Big)_{\alpha\beta}\Psi^\beta_{\nu_1} - \Big[\Pi_{+\mu_1\mu_2} + \frac{1}{2}\bar{\Psi}^\alpha_{\mu_1}\Big(F^{-1}(x)\Big)_{\alpha\beta}\Psi^\beta_{\mu_2}\Big](\delta^{\mu_1}_{\nu_2}\delta^{\mu_2}_{\nu_1} - \delta^{\mu_1}_{\nu_1}\delta^{\mu_2}_{\nu_2})\partial_\sigma x^{\nu_2} \\ & + \frac{1}{2}\bar{\Psi}^\alpha_{\mu_1}(f^{-1})_{\alpha\alpha_1}C^{\alpha_1\beta_1}_\rho x^\rho(\sigma)(f^{-1})_{\beta_1\beta}\Psi^\beta_{\mu_2}(\delta^{\mu_1}_{\nu_2}\delta^{\mu_2}_{\nu_1} - \delta^{\mu_1}_{\nu_1}\delta^{\mu_2}_{\nu_2})(\bar{\Pi}_+ + \bar{\Pi}^T_+)^{-1\rho\nu_2} \\ & \times\Bigg[\frac{\pi_\rho}{k} - \frac{1}{2}\bar{\Psi}^\gamma_\rho(f^{-1})_{\gamma\gamma_1}\partial_-\theta^{\gamma_1} - \frac{1}{2}\partial_+\bar{\theta}^\gamma(f^{-1})_{\gamma\gamma_1}\Psi^{\gamma_1}_\rho + \bar{\Pi}_{+\rho\rho_1}\partial_\sigma x^{\rho_1} - \bar{\Pi}_{+\rho_1\rho}\partial_\sigma x^{\rho_1}\Bigg]\Bigg]. \end{aligned} \tag{4.2}$$

To find Poisson bracket between T-dual coordinates, we can start by finding Poisson bracket of sigma derivatives of T-dual coordinates and then integrating twice (see [25]).

Implementing this procedure we have that Poisson bracket for sigma derivatives is given as

$$\begin{aligned}
&\{\partial_{\sigma_1} y_{\nu_1}(\sigma_1), \partial_{\sigma_2} y_{\nu_2}(\sigma_2)\} = \qquad (4.3)\\
&= \frac{2}{k}(\bar{\Pi}_+ + \bar{\Pi}_+^T)^{-1\mu_1\mu_2}\Big[G_{\nu_1\mu_1}B_{\nu_2\mu_2}\partial_{\sigma_2}\delta(\sigma_1-\sigma_2) - B_{\nu_1\mu_1}G_{\nu_2\mu_2}\partial_{\sigma_1}\delta(\sigma_1-\sigma_2)\Big]\\
&\quad + \frac{1}{k}\bar{\Psi}^\alpha_{\nu_3}(f^{-1})_{\alpha\alpha_1}C^{\alpha_1\beta_1}_\rho(f^{-1})_{\beta_1\beta}\Psi^\beta_{\nu_4}(\delta^{\nu_3}_{\mu_3}\delta^{\nu_4}_{\mu_4} + \delta^{\nu_4}_{\mu_3}\delta^{\nu_3}_{\mu_4})(\bar{\Pi}_+ + \bar{\Pi}_+^T)^{-1\mu_3\mu_1}(\bar{\Pi}_+ + \bar{\Pi}_+^T)^{-1\mu_4\mu_2}\\
&\quad \times\Big[G_{\nu_1\mu_1}B_{\nu_2\mu_2}x^\rho(\sigma_1)\partial_{\sigma_2}\delta(\sigma_1-\sigma_2) - B_{\nu_1\mu_1}G_{\nu_2\mu_2}x^\rho(\sigma_2)\partial_{\sigma_1}\delta(\sigma_1-\sigma_2)\Big].
\end{aligned}$$

Then we integrate with respect to σ_1 (σ_2), where we set boundaries as σ_0 ($\bar{\sigma}_0$) and σ ($\bar{\sigma}$). Extracting only Poisson bracket terms that contain σ and $\bar{\sigma}$, we have

$$\begin{aligned}
&\{y_{\nu_1}(\sigma), y_{\nu_2}(\bar{\sigma})\} = \frac{2}{k}(\bar{\Pi}_+ + \bar{\Pi}_+^T)^{-1\mu_1\mu_2}\Big[G_{\nu_1\mu_1}B_{\nu_2\mu_2} + B_{\nu_1\mu_1}G_{\nu_2\mu_2}\Big]H(\sigma-\bar{\sigma}) \qquad (4.4)\\
&+ \frac{1}{k}\bar{\Psi}^\alpha_{\nu_3}(f^{-1})_{\alpha\alpha_1}G^{\alpha_1\beta_1}_\rho(f^{-1})_{\beta_1\beta}\Psi^\beta_{\nu_4}(\delta^{\nu_3}_{\mu_3}\delta^{\nu_4}_{\mu_4} + \delta^{\nu_4}_{\mu_3}\delta^{\nu_3}_{\mu_4})(\bar{\Pi}_+ + \bar{\Pi}_+^T)^{-1\mu_3\mu_1}(\bar{\Pi}_+ + \bar{\Pi}_+^T)^{-1\mu_4\mu_2}\\
&\times\Big[G_{\nu_1\mu_1}B_{\nu_2\mu_2}x^\rho(\bar{\sigma}) + B_{\nu_1\mu_1}G_{\nu_2\mu_2}x^\rho(\sigma)\Big]H(\sigma-\bar{\sigma}).
\end{aligned}$$

Here, $H(\sigma-\bar{\sigma})$ is same step function defined in appendix A. It should be noted that these Poisson brackets are zero when $\sigma = \bar{\sigma}$. However, in cases where string in curled around compactified dimension, that is cases where $\sigma - \bar{\sigma} = 2\pi$, we have following situation

$$\begin{aligned}
&\{y_{\nu_1}(\sigma+2\pi), y_{\nu_2}(\sigma)\} = \frac{2}{k}(\bar{\Pi}_+ + \bar{\Pi}_+^T)^{-1\mu_1\mu_2}\Big[G_{\nu_1\mu_1}B_{\nu_2\mu_2} + B_{\nu_1\mu_1}G_{\nu_2\mu_2}\Big] \qquad (4.5)\\
&+ \frac{1}{k}\bar{\Psi}^\alpha_{\nu_3}(f^{-1})_{\alpha\alpha_1}C^{\alpha_1\beta_1}_\rho(f^{-1})_{\beta_1\beta}\Psi^\beta_{\nu_4}(\delta^{\nu_3}_{\mu_3}\delta^{\nu_4}_{\mu_4} + \delta^{\nu_4}_{\mu_3}\delta^{\nu_3}_{\mu_4})(\bar{\Pi}_+ + \bar{\Pi}_+^T)^{-1\mu_3\mu_1}(\bar{\Pi}_+ + \bar{\Pi}_+^T)^{-1\mu_4\mu_2}\\
&\times\Big[4\pi G_{\nu_1\mu_1}B_{\nu_2\mu_2}N^\rho + (G_{\nu_1\mu_1}B_{\nu_2\mu_2} + B_{\nu_1\mu_1}G_{\nu_2\mu_2})x^\rho(\sigma)\Big].
\end{aligned}$$

We used fact that $H(2\pi) = 1$. The symbol N^μ denotes winding number around compactified coordinate, if is defined as

$$x^\mu(\sigma+2\pi) - x^\mu(\sigma) = 2\pi N^\mu. \qquad (4.6)$$

Let us note that if we choose $x^\mu(\sigma) = 0$ than Poisson bracket has linear dependence on winding number. In cases where we don't have any winding number, we still have non-commutativity that is proportional to background fields.

Using the expression for sigma derivative of y_ν (4.2) and expression for Poisson bracket of sigma derivatives (4.3), we can find non-associative relations. Procedure is the same as for finding Poisson brackets of T-dual theory, we find Poisson bracket of sigma derivatives and integrate with respect to sigma coordinate, this time integral is done trice. Going along with this procedure we have following final result

$$\begin{aligned}
&\{y_\nu(\sigma), \{y_{\nu_1}(\sigma_1), y_{\nu_2}(\sigma_2)\}\} = \frac{G_{\nu\mu}}{k^2}(\bar{\Pi}_+ + \bar{\Pi}_+^T)^{-1\rho\mu} \qquad (4.7)\\
&\times\bar{\Psi}^\alpha_{\nu_3}(f^{-1})_{\alpha\alpha_1}C^{\alpha_1\beta_1}_\rho(f^{-1})_{\beta_1\beta}\Psi^\beta_{\nu_4}(\delta^{\nu_3}_{\mu_3}\delta^{\nu_4}_{\mu_4} + \delta^{\nu_4}_{\mu_3}\delta^{\nu_3}_{\mu_4})(\bar{\Pi}_+ + \bar{\Pi}_+^T)^{-1\mu_3\mu_1}(\bar{\Pi}_+ + \bar{\Pi}_+^T)^{-1\mu_4\mu_2}\\
&\times\Big[G_{\nu_1\mu_1}B_{\nu_2\mu_2}H(\sigma-\sigma_2) + B_{\nu_1\mu_1}G_{\nu_2\mu_2}H(\sigma-\sigma_1)\Big]H(\sigma_1-\sigma_2).
\end{aligned}$$

Since Jacobi identity is non-zero for T-dual theory we have that coordinate dependent RR field produces non-associative theory. However putting $\sigma_1 = \sigma_2 = \bar{\sigma}$ and $\sigma = \bar{\sigma} + 2\pi$ we have that Jacobi identity disappears

$$\{y_\nu(\bar{\sigma} + 2\pi), \{y_{\nu_1}(\bar{\sigma}), y_{\nu_2}(\bar{\sigma})\}\} = 0. \tag{4.8}$$

5 Conclusion

In this article we examined type II superstring propagating in presence of coordinate dependent RR field. This choice of background was in accordance with consistency conditions for background field and all calculations were made in approximation that are linear with respect to coordinate dependent part of RR field. We have also excluded parts that were non-linear in fermionic coordinates and neglected pure spinor actions. Using equations of motion for fermionic momenta we obtained action that was expressed in terms of bosonic coordinates, their derivatives and derivatives of fermionic coordinates. Unlike [25] we do not impose any conditions on the constant and coordinate dependent part of RR field strength, so, it is not possible to deduce this case to the form of the weakly curved background one.

Action with our choice of background fields did not possess translation symmetry, therefore we needed to use Buscher procedure that was extended to such cases. By substituting starting action with auxiliary action we gave up on locality in order to be able to find T-dual theory. Finding equations of motion of newly introduced Lagrange multipliers we were able to salvage starting action giving us assurance that auxiliary action we selected would produce correct T-dual theory. After this we found equations of motion for gauge fields and by inserting them into action, we found T-dual theory.

Having found T-dual theory, we applied T-dual procedure once again as a more thorough way of checking if action we obtained was in fact correct T-dual of starting action. Unlike starting action, T-dual action possessed translation symmetry and was non-local from the start by virtue of having dual coordinate ΔV^μ. Applying steps of generalized Buscher procedure [16, 19] we obtained starting action, again confirming that our choice of auxiliary action was correct.

We obtained non-commutativity relations in context of T-dual theory, where we used T-dual transformation laws as a bridge between Poisson brackets of starting theory and T-dual theory. T-dual transformation laws were expressed as functions of coordinates and momenta of original theory and using their standard Poisson algebra, we got non-commutativity in T-dual theory. From expression for Poisson brackets (4.4) we can see that non-commutativity is proportional to infinitesimal part of RR field as well as to symmetrised inverse of field $\bar{\Pi}$. Non-commutativity relations are zero in case when $\sigma = \bar{\sigma}$, while in case where $\sigma = \bar{\sigma} + 2\pi$ we see the emergence of winding numbers.

Taking into account Poisson brackets of sigma derivatives and expression for sigma derivative of T-dual coordinate we were able to find non-associative relation for T-dual theory. In general case this relation was non-zero and it was proportional to infinitesimal constant $C_\rho^{\mu\nu}$. In special case when we put $\sigma_1 = \sigma_2 = \bar{\sigma}$ and $\sigma = \bar{\sigma} + 2\pi$ we noticed that non-associativity relation disappears. During the implementation of the T-dualization procedure and calculations, we obtained generalization of β_μ functions in the form of the N-functions.

It should be noted that since we did not preform T-dualization along fermionic coordinates their Poisson structure would remain the same as in original theory. Furthermore, since background fields do not depend on fermionic coordinates it should be expected, as in the case of bosonic coordinates [18], that T-duality would leave Poisson brackets between fermionic fields the same. We expect that, if proposed non-commutative relations from [30, 31] are even possible, we would need at least RR field that depends both on fermionic and bosonic coordinates.

Acknowledgments

Work supported in part by the Serbian Ministry of Education, Science and Technological Development.

A Obtaining $N(\kappa^\pm)$ terms

In this paper function $N(\kappa^\pm)$ emerged in T-dual transformation laws as a consequence of variation of term that was proportional to ΔV. Here we will present derivation of this function.

$$\begin{aligned}
\frac{\delta\left(F^{-1}(\Delta V)\right)_{\alpha\beta}}{\delta v^\mu_+(\kappa)} &= -(f^{-1})_{\alpha\alpha_1} C_l^{\alpha_1\beta_1}(f^{-1})_{\beta_1\beta}\int_P d\xi'^m \frac{\delta v^\rho_m(\xi')}{\delta v^\mu_+(\kappa)} = \qquad (A.1)\\
&= -(f^{-1})_{\alpha\alpha_1} C_\mu^{\alpha_1\beta_1}(f^{-1})_{\beta_1\beta}\int_P d\xi'^+\delta(\xi'^+-\kappa^+)\delta(\xi'^- -\kappa^-)\\
&= -(f^{-1})_{\alpha\alpha_1} C_\mu^{\alpha_1\beta_1}(f^{-1})_{\beta_1\beta}\int_{t_i}^{t_f} dt\frac{d\xi'^+}{dt}\delta(\xi'(t)^+-\kappa^+)\delta(\xi'^-(t)-\kappa^-)\\
&= -(f^{-1})_{\alpha\alpha_1} C_\mu^{\alpha_1\beta_1}(f^{-1})_{\beta_1\beta}\int_{\xi_0{}^+}^{\xi^+} du\delta(u-\kappa^+)\delta\Big(\xi'^-\left((\xi'^+)^{-1}(u)\right)-\kappa^-\Big)\\
&= -(f^{-1})_{\alpha\alpha_1} C_\mu^{\alpha_1\beta_1}(f^{-1})_{\beta_1\beta}\delta\Big(\xi'^-\left((\xi'^+)^{-1}(\kappa^+)\right)-\kappa^-\Big)\Big[H(\xi^+-\kappa^+)-H(\xi_0^+-\kappa^+)\Big]\\
&= -(f^{-1})_{\alpha\alpha_1} G_\mu^{\alpha_1\beta_1}(f^{-1})_{\beta_1\beta}N(\kappa^+).
\end{aligned}$$

In third line we have parametrized the path with parameter t where $\xi'^+(t_i)=\xi_0{}^+$ and $\xi'^+(t_f)=\xi^+$. In fourth line we introduced substitution $u=\xi'^+(t)$, in delta function this substitute is inverted. Fifth line is obtained by using following integration rule for Dirac delta function

$$\int_{\sigma_0}^{\sigma} d\eta f(\eta)\delta(\eta-\bar\eta) = f(\bar\eta)\left[H(\sigma-\bar\eta)-H(\sigma_0-\bar\eta)\right]. \qquad (A.2)$$

Here, $H(x)$ is a step function defined as

$$H(x) = \int_0^x d\eta\delta(\eta) = \frac{1}{2\pi}\Big[x + \sum_{n\geq 2}\frac{1}{n}\sin(nx)\Big]$$
$$= \begin{cases} 0 & \text{if} \quad x = 0 \\ 1/2 & \text{if} \quad 0 < x < 2\pi \\ 1 & \text{if} \quad x = 2\pi \end{cases} \quad . \tag{A.3}$$

Procedure for obtaining $N(\kappa^-)$ is similar.

B Properties of $N(\kappa^\pm)$ terms

Here ve will list some properties of $N(\kappa^\pm)$ function.

$$N(\kappa^+) + N(\kappa^-) = N(\kappa^0), \tag{B.1}$$
$$N(\kappa^+) - N(\kappa^-) = N(\kappa^1), \tag{B.2}$$

Where κ^0 and κ^1 represent τ and σ coordinates respectevly

$$\int_\Sigma d^2\xi\partial_+ N(\kappa^+) = 1, \qquad \int_\Sigma d^2\xi\partial_- N(\kappa^+) = 0, \tag{B.3}$$
$$\int_\Sigma d^2\xi\partial_- N(\kappa^-) = 1, \qquad \int_\Sigma d^2\xi\partial_+ N(\kappa^-) = 0. \tag{B.4}$$

These relationships can be checked directly by applying partial derivatives to expressions from A.

$$\int_\Sigma d^2\xi\partial_+ N(\kappa^+) = \int_\Sigma d^2\xi\delta\Big(\xi'^-((\xi'^+)^{-1}(\kappa^+)) - \kappa^-\Big)\partial_+\Big[H(\xi^+ - \kappa^+) - H(\xi_0^+ - \kappa^+)\Big]$$
$$= \int_\Sigma d^2\xi\delta\Big(\xi'^-((\xi'^+)^{-1}(\kappa^+)) - \kappa^-\Big)\delta(\xi^+ - \kappa^+) = \int d\xi^-\delta\Big(\xi'^-((\xi'^+)^{-1}(\xi^+)) - \kappa^-\Big). \tag{B.5}$$

In appendix A we had following parametrisation of path P: $\xi'^+(t_i) = \xi_0{}^+$ and $\xi'^+(t_f) = \xi^+$. Applying inverse parametrisation we have $(\xi'^+)^{-1}(\xi_0^+) = t_i$ and $(\xi'^+)^{-1}(\xi^+) = t_f$. With these we have

$$\int d\xi^-\delta\Big(\xi'^-((\xi'^+)^{-1}(\xi^+)) - \kappa^-\Big) = \int_\Sigma d\xi^-\delta\Big(\xi'^-(t_f) - \kappa^-\Big)$$
$$= \int_\Sigma d\xi^-\delta\Big(\xi^- - \kappa^-\Big) = 1. \tag{B.6}$$

Same rules apply for $N(\kappa^-)$, $N(\kappa^0)$ and $N(\kappa^1)$. In cases where $F^{-1}(x)_{\alpha\beta}$ is antisymmetric we can transfer partial derivatives from $\partial_\pm V^m u$ to $N(\kappa^\pm)$ and obtain standard $\beta^\pm$ functions.

JHEP12(2022)078

References

[1] M. Evans and B.A. Ovrut, *Symmetry in string theory*, *Phys. Rev. D* **39** (1989) 3016 [INSPIRE].

[2] M. Evans and B.A. Ovrut, *Deformations of conformal field theories and symmetries of the string*, *Phys. Rev. D* **41** (1990) 3149 [INSPIRE].

[3] L. Davidović and B. Sazdović, *The T-dual symmetries of a bosonic string*, *Eur. Phys. J. C* **78** (2018) 600 [arXiv:1806.03138] [INSPIRE].

[4] N. Seiberg and E. Witten, *String theory and noncommutative geometry*, *JHEP* **09** (1999) 032 [hep-th/9908142] [INSPIRE].

[5] B. Sazdovic, *Dilaton field induces commutative Dp-brane coordinate*, *Eur. Phys. J. C* **44** (2005) 599 [hep-th/0408131] [INSPIRE].

[6] B. Nikolic and B. Sazdovic, *Gauge symmetries change the number of Dp-brane dimensions*, *Phys. Rev. D* **74** (2006) 045024 [hep-th/0604129] [INSPIRE].

[7] B. Nikolic and B. Sazdovic, *Noncommutativity in space-time extended by Liouville field*, *Adv. Theor. Math. Phys.* **14** (2010) 1 [arXiv:0711.4463] [INSPIRE].

[8] M.R. Douglas and N.A. Nekrasov, *Noncommutative field theory*, *Rev. Mod. Phys.* **73** (2001) 977 [hep-th/0106048] [INSPIRE].

[9] R.J. Szabo, *Quantum field theory on noncommutative spaces*, *Phys. Rept.* **378** (2003) 207 [hep-th/0109162] [INSPIRE].

[10] D. Latas, V. Radovanović and J. Trampetic, *Non-commutative* SU(N) *gauge theories and asymptotic freedom*, *Phys. Rev. D* **76** (2007) 085006 [hep-th/0703018] [INSPIRE].

[11] E. Alvarez, L. Álvarez-Gaumé and Y. Lozano, *An introduction to T duality in string theory*, *Nucl. Phys. B Proc. Suppl.* **41** (1995) 1 [hep-th/9410237] [INSPIRE].

[12] A. Giveon, M. Porrati and E. Rabinovici, *Target space duality in string theory*, *Phys. Rept.* **244** (1994) 77 [hep-th/9401139] [INSPIRE].

[13] D. Lüst, *T-duality and closed string non-commutative (doubled) geometry*, *JHEP* **12** (2010) 084 [arXiv:1010.1361] [INSPIRE].

[14] T.H. Buscher, *A symmetry of the string background field equations*, *Phys. Lett. B* **194** (1987) 59 [INSPIRE].

[15] T.H. Buscher, *Path integral derivation of quantum duality in nonlinear sigma models*, *Phys. Lett. B* **201** (1988) 466 [INSPIRE].

[16] L. Davidovic and B. Sazdovic, *T-duality in a weakly curved background*, *Eur. Phys. J. C* **74** (2014) 2683 [arXiv:1205.1991] [INSPIRE].

[17] L. Davidović, B. Nikolić and B. Sazdović, *T-duality diagram for a weakly curved background*, *Eur. Phys. J. C* **75** (2015) 576 [arXiv:1406.5364] [INSPIRE].

[18] L. Davidović, B. Nikolić and B. Sazdović, *Canonical approach to the closed string non-commutativity*, *Eur. Phys. J. C* **74** (2014) 2734 [arXiv:1307.6158] [INSPIRE].

[19] L. Davidović and B. Sazdović, *T-dualization in a curved background in absence of a global symmetry*, *JHEP* **11** (2015) 119 [arXiv:1505.07301] [INSPIRE].

[20] D. Andriot, M. Larfors, D. Lüst and P. Patalong, *(Non-)commutative closed string on T-dual toroidal backgrounds*, *JHEP* **06** (2013) 021 [arXiv:1211.6437] [INSPIRE].

JHEP12(2022)078

[21] B. Nikolić and D. Obrić, *Noncommutativity and nonassociativity of closed bosonic string on T-dual toroidal backgrounds*, *Fortsch. Phys.* **66** (2018) 1800009 [arXiv:1801.08772] [INSPIRE].

[22] B. Nikolić and D. Obrić, *Directly from H-flux to the family of three nonlocal R-flux theories*, *JHEP* **03** (2019) 136 [arXiv:1901.01040] [INSPIRE].

[23] K. Becker, M. Becker and J. Schwarz, *String theory and M-theory: a modern introduction*, Cambridge University Press (2015).

[24] M. Green, J. Schwarz and E. Witten, *Superstring theory*, Cambridge University Press (2012).

[25] B. Nikolic, D. Obric and B. Sazdovic, *Noncommutativity and nonassociativity of type II superstring with coordinate dependent RR field*, *Fortsch. Phys.* **70** (2022) 2200048 [arXiv:2203.11651] [INSPIRE].

[26] N. Berkovits and P.S. Howe, *Ten-dimensional supergravity constraints from the pure spinor formalism for the superstring*, *Nucl. Phys. B* **635** (2002) 75 [hep-th/0112160] [INSPIRE].

[27] P.A. Grassi, G. Policastro and P. van Nieuwenhuizen, *The massless spectrum of covariant superstrings*, *JHEP* **11** (2002) 004 [hep-th/0202123] [INSPIRE].

[28] P.A. Grassi, G. Policastro and P. van Nieuwenhuizen, *On the BRST cohomology of superstrings with/without pure spinors*, *Adv. Theor. Math. Phys.* **7** (2003) 499 [hep-th/0206216] [INSPIRE].

[29] P.A. Grassi, G. Policastro and P. van Nieuwenhuizen, *The covariant quantum superstring and superparticle from their classical actions*, *Phys. Lett. B* **553** (2003) 96 [hep-th/0209026] [INSPIRE].

[30] P.A. Grassi and L. Tamassia, *Vertex operators for closed superstrings*, *JHEP* **07** (2004) 071 [hep-th/0405072] [INSPIRE].

[31] J. de Boer, P.A. Grassi and P. van Nieuwenhuizen, *Noncommutative superspace from string theory*, *Phys. Lett. B* **574** (2003) 98 [hep-th/0302078] [INSPIRE].

JHEP12(2022)078

Published for SISSA by Springer

Received: *January 9, 2019*
Revised: *March 1, 2019*
Accepted: *March 11, 2019*
Published: *March 22, 2019*

Directly from H-flux to the family of three nonlocal R-flux theories

B. Nikolić and D. Obrić

Institute of Physics Belgrade, University of Belgrade,
Pregrevica 118, Serbia

E-mail: bnikolic@ipb.ac.rs, dobric@ipb.ac.rs

Abstract: In this article we consider T-dualization of the 3D closed bosonic string in the weakly curved background — constant metric and Kalb-Ramond field with one non-zero component, $B_{xy} = Hz$, where field strength H is infinitesimal. We use standard and generalized Buscher T-dualization procedure and make T-dualization starting from coordinate z, via y and finally along x coordinate. All three theories are *nonlocal*, because variable ΔV, defined as line integral, appears as an argument of background fields. After the first T-dualization we obtain commutative and associative theory, while after we T-dualize along y, we get, κ-Minkowski-like, noncommutative and associative theory. At the end of this T-dualization chain we come to the theory which is both noncommutative and nonassociative. The form of the final T-dual action does not depend on the order of T-dualization while noncommutativity and nonassociativity relations could be obtained from those in the $x \to y \to z$ case by replacing $H \to -H$.

Keywords: Bosonic Strings, String Duality

ArXiv ePrint: 1901.01040

https://doi.org/10.1007/JHEP03(2019)136

JHEP03(2019)136

Contents

JHEP03(2019)136

1 Introduction

Noncommutativity of coordinates has come into focus of physics about hundred years ago when the problem with infinite value of physical quantities occurred. The solution was proposed by Heisenberg in the form of noncommutative coordinates. But after developing of renormalization procedure coordinate noncommutativity was forgotten as a tool for cancelling of infinities.

Commuting of coordinates means that there is no minimal possible length in Nature i.e. that we can measure the position of particle with infinite precision. The return of noncommutativity into physics starts with the article of Hartland Snyder [1]. Usually we treat space-time as continuum but Snyder showed that there is Lorentz invariant discrete space-time. Consequently, this means that commutator of coordinates is nonzero, and noncommutativity parameter dictates the scale at which noncommutativity exists.

In the paper [2] existence of noncommutative manifold was shown using propagators in open bosonic string theory with constant metric and constant Kalb-Ramond field. This result is proven in many articles [3–12] after that but using different mathematical methods. Obtained noncommutativity with constant noncommutativity parameter is known in literature as canonical noncommutativity. Consequently, canonical noncommutativity implies that theory is still associative one.

One of the first application of canonical noncommutativity was in Yang-Mills (YM) theories [13–16]. Noncommutative YM theories are constructed and their renormalisability properties are analyzed. It turned out that some processes forbidden in commutative YM are allowed in noncommutative YM theories. Consequently, cross sections for those decays and processes are calculated [17, 18]. Such predictions offer the possibility of indirect check of idea of noncommutativity.

The next type of noncommuatativity which is considered in literature is Lie-algebraic one, which means that commutator of two coordinates is proportional to the coordinate. The κ-Minkowski space-time is an example of this kind of noncommutativity and it is considered in various contexts [19–24]. The κ-Minkowski space is noncommutative but it is easy to check that is associative one. But, in general, if the commutator of the coordinates is proportional to the some linear combination of coordinates, then the space is nonassociative because jacobiator and associator are nonzero. For example, such spaces are closely related to the L_∞ algebra [25].

The mathematical framework for T-dualization is standard Buscher procedure [26, 27]. It consists of the localization of the shift symmetry and adding a term with Lagrange multiplier in order to make gauge fields unphysical degrees of freedom. Also there is an improvement of standard Buscher procedure developed and applied in refs. [28–31], generalized Buscher procedure. In the application of the generalized procedure of T-dualization there is one additional step with respect to the standard one. We introduce invariant coordinate in order to localize shift symmetry in the coordinate dependent backgrounds.

The first articles addressing the subject of coordinate dependent backgrounds appear in the last ten years [32–43]. A 3-torus with constant metric and Kalb-Ramond field with just one nonzero component, $B_{xy} = Hz$, was considered within standard Buscher procedure [33]. Authors made two successive T-dualzation along isometry directions x and y, and, using nontrivial winding conditions, obtained noncommutativity with parameter proportional to field strength H and winding number N_3.

Using generalized T-duality procedure [30, 44] we obtained coordinate dependent noncommutativity and, consequently, nonassociativity. Also it is shown that final theory is nonlocal. In ref. [30] the bosonic string is considered in the weakly curved background — constant metric and linearly coordinate dependent Kalb-Ramond field with infinitesimal field strength, while in [44] we consider the same model as in [33], but T-dualizing along all three directions and imposing trivial winding conditions. Obtained nonlocality comes from the coordinate dependent background, or more precisely, from invariant coordinates. At the end of T-dualization procedure background fields depend on ΔV, defined as line integral. Nonlocality has been become very important issue in the quantum mechanical considerations [45].

In this article we will deal with closed bosonic string propagating in the constant metric and linear dependent Kalb-Ramond field with $B_{xy} = Hz$, the same background as in [33, 44]. But our goal here is to examine the influence of order of T-dualizations. In ref. [44] we T-dualize first along isometry directions, first along x and then along y, and at

JHEP03(2019)136

the end, along direction z. The first T-dualization produces configuration known as twisted torus which is commutative, and it is globally and locally well defined. After second T-dualization we obtained nongeometric theory with Q flux which is still locally well defined and it is commutative. The final T-dualization along z direction produces nonlocal theory which is noncommutative and nonassociative one. This line of T-dualizations we will call xyz one.

But what it will happen, if we change the order of T-dualizations, regrading (non)locality issue as well as (non)commutativity and (non)associativity? It is quite obvious that nothing will be changed if we T-dualize along line yxz, because the first two directions, which are T-dualized, are isometry ones. Some nontrivial issues could be expected if we T-dualize first along z direction. In this article we will present T-dualization of the model from [33, 44] along the T-dualization line zyx. After every step of T-dualization we will rewrite the T-dual transformation law in canonical form using the expressions for canonical momenta of the initial theory. Also we will check whether the obtained theory is commutative or not and, consequently, we will see whether it is associative or not.

The fact which is quite sure is that all three theories which we will obtain from the T-dualization line zyx are *nonlocal*. The explanation comes from the fact that background field $B_{\mu\nu}$ is z dependent and according to the generalized T-dualization procedure, after T-dualization along z, we obtain quantity ΔV which is defined as line integral. Consequently, the theory is nonlocal. But because y and x T-dualizations do not affect ΔV, all three theories obtained in zyx T-dualization line are nonlocal. That is a difference with respect to the xyz T-dualization line considered in [44].

The interesting thing is that transformation laws can be obtained from the corresponding ones in [44] by replacing $H \to -H$, but because in this article we T-dualize in the opposite direction, that produces theories of the different commutative and associative features with respect to [44]. After first T-dualization we get commutative and associative theory which is the same as in xyz case from [44]. But the second T-dualization here produces *noncommutative* and associative theory of κ-Minkowski type. That is different with respect to the xyz case, where second theory in the line is both commutative and associative. At the end we obtain the same theory as in [44] which is nonassociative and noncommutative. The noncommutativity and nonassociativity parameters have one additional "$-$" sign comparing with the corresponding ones in [44]. In this article as well as in [44], we impose trivial winding conditions which means $x^\mu(\sigma + 2\pi) = x^\mu(\sigma) + 2\pi N^\mu$, where N^μ is a winding number.

At the end we comment some quantum aspects of the problem and add two appendices. The first one contains conventions regarding light-cone coordinates, while the second one is related to the mathematical details concerning derivation of two kinds of Poisson brackets appearing in the article.

2 Bosonic string action and choice of background fields

In this section we will introduce the action for bosonic string propagating in 3D space with constant metric and Kalb-Ramond field which single component is different from zero,

$B_{xy} = Hz$. This model is well known in literature as torus with H-flux. Since we are working with the same model as in [33, 44], for completeness we will repeat most of the steps from introductory part in the [44].

The closed bosonic string which propagates in the presence of the space-time metric $G_{\mu\nu}(x)$, Kalb-Ramond field $B_{\mu\nu}(x)$, and dilaton field $\Phi(x)$ is described by action [46–48]

$$S = \kappa \int_\Sigma d^2\xi \sqrt{-g} \left\{ \left[\frac{1}{2} g^{\alpha\beta} G_{\mu\nu}(x) + \frac{\varepsilon^{\alpha\beta}}{\sqrt{-g}} B_{\mu\nu}(x) \right] \partial_\alpha x^\mu \partial_\beta x^\nu + \Phi(x) R^{(2)} \right\} , \tag{2.1}$$

where world-sheet surface Σ is parameterized by $\xi^\alpha = (\tau, \sigma)$ $[(\alpha = 0, 1),\ \sigma \in (0, \pi)]$, while x^μ $(\mu = 0, 1, 2, \ldots, D-1)$ are space-time coordinates. Intrinsic world sheet metric is denoted by $g_{\alpha\beta}$, and the corresponding scalar curvature with $R^{(2)}$.

Conformal symmetry on the quantum level is not preserved for any choice of background fields. If we want to keep conformal symmetry on the quantum level, background fields must obey the space-time field equations [49]

$$\beta^G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{4} B_{\mu\rho\sigma} B_\nu{}^{\rho\sigma} + 2D_\mu a_\nu = 0 \,, \tag{2.2}$$

$$\beta^B_{\mu\nu} \equiv D_\rho B^\rho{}_{\mu\nu} - 2a_\rho B^\rho{}_{\mu\nu} = 0 \,, \tag{2.3}$$

$$\beta^\Phi \equiv 2\pi\kappa \frac{D-26}{6} - R - \frac{1}{24} B_{\mu\rho\sigma} B^{\mu\rho\sigma} - D_\mu a^\mu + 4a^2 = c \,, \tag{2.4}$$

where c is an arbitrary constant. From

$$D^\nu \beta^G_{\nu\mu} + \partial_\mu \beta^\Phi = 0 \,, \tag{2.5}$$

it follows that third beta function, β^Φ, is equal to an arbitrary constant. Here $R_{\mu\nu}$ and D_μ are Ricci tensor and covariant derivative with respect to the space-time metric $G_{\mu\nu}$. Field strength for Kalb-Ramond field $B_{\mu\nu}$ and dilaton gradient are defined as

$$B_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} + \partial_\nu B_{\rho\mu} + \partial_\rho B_{\mu\nu} \,, \quad a_\mu = \partial_\mu \Phi \,. \tag{2.6}$$

One of the solutions of these equations which is important for us here is the solution where some background fields are coordinate dependent. Let us choose Kalb-Ramond field to be linearly coordinate dependent and dilaton field to be constant. The equation (2.2) turns into

$$R_{\mu\nu} - \frac{1}{4} B_{\mu\rho\sigma} B_\nu{}^{\rho\sigma} = 0 \,. \tag{2.7}$$

If we assume that field strength is infinitesimal, then we take $G_{\mu\nu}$ to be constant in approximation linear in $B_{\mu\nu\rho}$. Consequently, the third equation (2.4) is of the form

$$2\pi\kappa \frac{D-26}{6} = c \,. \tag{2.8}$$

The constant c is arbitrary, and fixing its value at $c = -\frac{23\pi\kappa}{3}$, we obtain $D = 3$, dimension of the space in which we will work further.

JHEP03(2019)136

The choice of background fields in the case we will consider is

$$G_{\mu\nu} = \begin{pmatrix} R_1^2 & 0 & 0 \\ 0 & R_2^2 & 0 \\ 0 & 0 & R_3^2 \end{pmatrix}, \quad B_{\mu\nu} = \begin{pmatrix} 0 & Hz & 0 \\ -Hz & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{2.9}$$

where $R_\mu(\mu = 1,2,3)$ are radii of the compact dimensions. In terms of radii, the imposed condition that H is infinitesimal, can be rewritten as

$$\left(\frac{H}{R_1R_2R_3}\right)^2 = 0\,. \tag{2.10}$$

Physically, infinitesimality of H means that we work with sufficiently large torus (diluted flux approximation). If we rescale the coordinates

$$x^\mu \longmapsto x'^\mu = R_\mu x^\mu\,, \tag{2.11}$$

where indices on the right hand-side of equation are not summed, the form of the metric simplifies

$$G_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{2.12}$$

Taking all assumption into consideration, the action is of the form

$$\begin{aligned} S &= \kappa \int_\Sigma d^2\xi \partial_+ x^\mu \Pi_{+\mu\nu} \partial_- x^\nu \\ &= \kappa \int_\Sigma d^2\xi \left[\frac{1}{2}\left(\partial_+ x\partial_- x + \partial_+ y\partial_- y + \partial_+ z\partial_- z\right) + \partial_+ xHz\partial_- y - \partial_+ yHz\partial_- x\right], \end{aligned} \tag{2.13}$$

where $\partial_\pm = \partial_\tau \pm \partial_\sigma$ is world-sheet derivative with respect to the light-cone coordinates $\xi^\pm = \frac{1}{2}(\tau \pm \sigma)$, $\Pi_{\pm\mu\nu} = B_{\mu\nu} \pm \frac{1}{2}G_{\mu\nu}$ and

$$x^\mu = \begin{pmatrix} x \\ y \\ z \end{pmatrix}. \tag{2.14}$$

T-dualization of dilaton is done within quantum formalism and here it will not be presented.

3 Family of three R flux non-local theories

In this section we will perform T-dualization of closed bosonic string equipped by H-flux torus background fields, one direction at time. T-dualization procedure will go along zyx line. We will show that all three theories are nonlocal with R-flux. Also we will find expressions connecting initial and T-dual variables, so called T-dual transformation laws. Using transformation laws in canonical form, we will check after every step whether obtained theory is (non)commutative and/or (non)associative.

JHEP03(2019)136

3.1 T-dualization along z direction — shortcut to R-flux

Unlike the cases considered in [33, 44], where T-dualization drives along xyz line, let us do that in opposite direction and perform generalized T-dualization [28] of action (2.13) along z direction.

3.1.1 T-dualization procedure

It looks like that this direction is not isometry one. But we can show that it can be treated like isometry direction. Let us consider the global transformation

$$\delta x^\mu = \lambda^\mu \,, \tag{3.1}$$

and vary the action with respect to this transformation

$$\delta S = \frac{\kappa}{3} B_{\mu\nu\rho}\lambda^\rho \int_\Sigma d^2\xi \partial_+ x^\mu \partial_- x^\nu = \frac{2k}{3} B_{\mu\nu\rho}\lambda^\rho \epsilon^{\alpha\beta} \int_\Sigma d^2\xi [\partial_\alpha(x^\mu \partial_\beta x^\nu) - x^\mu(\partial_\alpha\partial_\beta x^\nu)] \,. \tag{3.2}$$

The second term vanishes as a consequence of contraction of antisymmetric ($\epsilon^{\alpha\beta}$) and symmetric ($\partial_\alpha\partial_\beta$) tensors, while the first one, surface term, survives, and it is, in general, different from zero. But, the expression δS is an topological invariant, so it vanishes if the map from the world-sheet to D-dimensional space-time is topologically trivial. Essentially, infinitesimal field strength H does not affect the vanishing of the surface term.

There is one more explanation of vanishing of this surface term. It is more technical and adjusted to the approximation we used in this article which essence is the explanation in paragraph above. Because we work in the approximation up to the linear terms in H, x^μ satisfies equation of motion for constant $G_{\mu\nu}$ and $B_{\mu\nu}$, $\partial_+\partial_- x^\mu = 0$, which solution is well known in literature. If the winding number is equal to zero, it holds $x^\mu(2\pi + \sigma) = x^\mu(\sigma)$, and since the configuration in the initial τ_i and final moment τ_f is fixed, the surface term vanishes.

So, in the weakly curved background case (H-flux torus background is such like that), z direction is an isometry one. Localization of the shift symmetry of the action (2.13) along z starts with introducing the covariant derivative

$$\partial_\pm z \longrightarrow D_\pm z = \partial_\pm z + v_\pm \,, \tag{3.3}$$

where $v_\pm$ is a gauge field. In order to make gauge fields unphysical ones, we introduce term with Lagrange multiplier

$$S_{add} = \frac{\kappa}{2} \int_\Sigma d^2\xi y_3(\partial_+ v_- - \partial_- v_+) \,. \tag{3.4}$$

These two steps are the part of the standard Buscher procedure. Because of coordinate dependent background field $B_{\mu\nu}$, generalized T-dualization procedure has an additional step, introducing of an invariant coordinate

$$z^{inv} = \int_P d\xi^\alpha D_\alpha z = \int_P d\xi^+ D_+ z + \int_P d\xi^- D_- z = z(\xi) - z(\xi_0) + \Delta V \,, \tag{3.5}$$

where

$$\Delta V = \int_P d\xi^\alpha v_\alpha = \int_P (d\xi^+ v_+ + d\xi^- v_-)\,. \tag{3.6}$$

The form of the action is now

$$\bar{S} = \kappa \int_\Sigma d^2\xi \left[H z^{inv}(\partial_+ x \partial_- y - \partial_+ y \partial_- x) + \frac{1}{2}(\partial_+ x \partial_- x + \partial_+ y \partial_- y + D_+ z D_- z) \right.$$
$$\left. + \frac{1}{2}\, y_3(\partial_+ v_- - \partial_- v_+) \right]\,. \tag{3.7}$$

Fixing the gauge, $z(\xi) = z(\xi_0)$, we get gauged fixed action in the form

$$S_{fix} = \kappa \int_\Sigma d^2\xi \left[H\Delta V(\partial_+ x \partial_- y - \partial_+ y \partial_- x) + \frac{1}{2}(\partial_+ x \partial_- x + \partial_+ y \partial_- y + v_+ v_-) \right.$$
$$\left. + \frac{1}{2} y_3(\partial_+ v_- - \partial_- v_+) \right]\,. \tag{3.8}$$

The equation of motion for Lagrange multiplier y_3 obtained from above action (3.8) produces

$$\partial_+ v_- - \partial_- v_+ = 0 \Longrightarrow v_\pm = \partial_\pm z\,, \tag{3.9}$$

which drives us back to the initial action (2.13). On the other side, if we found equations of motion for gauge fields $v_\pm$, we get

$$v_\pm = \pm \partial_\pm y_3 - 2\beta^\mp\,, \tag{3.10}$$

where $\beta^\pm$ functions are defined as

$$\beta^\pm = \mp \frac{1}{2} H(x\partial_\mp y - y \partial_\mp x)\,. \tag{3.11}$$

The $\beta^\pm$ functions stem from the variation of the term containing ΔV. The derivation of beta functions $\beta^\pm$ is based on the relation $\partial_\pm \Delta V = v_\pm$. In the derivation of the beta functions there is one nontrivial technical point and that is vanishing of the surface term after one partial integration. That surface term is of the same form as in eq. (3.2), so the same reasons for surface term vanishing hold here. Mathematical details regarding derivation of $\beta^\pm$ functions can be found in refs. [28–31, 44].

Inserting the relations (3.10) into the gauge fixed action, keeping linear terms in H, we obtain the T-dual action

$$ {}_zS = \kappa \int_\Sigma d^2\xi \partial_+ {}_zX^\mu {}_z\Pi_{+\mu\nu} \partial_- {}_zX^\nu\,, \tag{3.12}$$

where

$$ {}_zX^\mu = \begin{pmatrix} x \\ y \\ y_3 \end{pmatrix}, \qquad {}_z\Pi_{+\mu\nu} = {}_zB_{\mu\nu} + \frac{1}{2} {}_zG_{\mu\nu}\,, \tag{3.13}$$

$$ {}_zB_{\mu\nu} = \begin{pmatrix} 0 & H\Delta V & 0 \\ -H\Delta V & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad {}_zG_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{3.14}$$

Let us note that presence of ΔV, defined as line integral, represents the source of nonlocality of the T-dual theory.

3.1.2 T-dual transformation law

Combining the equations of motion for Lagrange multiplier (3.9) and for gauge fields (3.10), we obtain T-dual transformation laws

$$\partial_\pm z \cong \pm\partial_\pm y_3 \mp H(x\partial_\pm y - y\partial_\pm x)\,, \tag{3.15}$$

where $\cong$ is used here to mark T-dual relation. Momentum of the initial theory (2.13) canonically conjugated to the coordinate z is of the form

$$\pi_z = \frac{\partial\mathcal{L}}{\partial\dot{z}} = \kappa\dot{z}\,, \tag{3.16}$$

where $\mathcal{L}$ is a Lagrangian density defined as $S = \int_\Sigma d^2\xi\mathcal{L}$. Calculating $\dot{z}$ using T-dual transformation law (3.15), we get the T-dual transformation law in canonical form

$$y_3' \cong \frac{1}{\kappa}\pi_z + H(xy' - yx')\,, \tag{3.17}$$

which is of the same form as in the xyz case.

In all further expressions we will keep the symbol ΔV, but we must have in mind that we used equations of motion for Lagrange multipliers (3.9) at the end of T-dulization procedure along z coordinate, so, having in mind (3.6) and (3.15), we get

$$\Delta V = \Delta z \cong \int d\xi^+\partial_+ y_3 - \int d\xi^-\partial_- y_3 \equiv \tilde{y}_3\,. \tag{3.18}$$

The variable ΔV is multiplied by infinitesimal field strength H, so, in the above expression we used $\partial_\pm z \cong \pm\partial_\pm y_3$, as a consequence of diluted flux approximation.

3.1.3 (Non)commutativity and (non)associativity

The initial theory is geometric one and its variables satisfy the standard Poisson algebra

$$\{x^\mu(\sigma), x^\nu(\bar{\sigma})\} = \{\pi_\mu(\sigma), \pi_\nu(\bar{\sigma})\} = 0\,, \quad \{x^\mu, \pi_\nu(\bar{\sigma})\} = \delta^\mu{}_\nu\delta(\sigma - \bar{\sigma})\,, \tag{3.19}$$

where x^μ are the coordinates of the initial theory, while π_μ are their canonically conjugated momenta. Using expression (3.17) and standard Poisson algebra (3.19), we obtain that coordinates of the theory obtained after one T-dualization, ${}_zX^\mu$, are commutative. Consequently, Jacobiator is equal to zero, which means that theory is associative.

Summarizing this first step of T-dualization, obtained theory is *commutative and associative nonlocal R-flux theory*. Comparing with the results of the ref. [44] after first T-dualization, qualitatively we obtain the same result, but with the essential difference that here obtained theory is nonlocal R-flux theory unlike that in [44] which is geometrical one, locally and globally well defined.

3.2 Step 2 — T-dualization along *y* direction

Our starting point is the action given in eq. (3.12). The background fields are independent of y, so, we apply standard Buscher procedure. This means that, unlike the previous case, we perform just first two steps in T-dualization procedure and skip the third one — introducing of invariant coordinate. The T-dualization procedure is already presented, so, we will skip explaining procedure steps further.

JHEP03(2019)136

3.2.1 T-dualization procedure

The gauge fixed action is of the form

$$S_{fix} = \kappa \int_\Sigma d^2\xi \left[\frac{1}{2}\left(\partial_+ x \partial_- x + v_+ v_- + \partial_+ y_3 \partial_- y_3\right) + H\Delta V (v_- \partial_+ x - v_+ \partial_- x)\right]$$
$$+ \frac{\kappa}{2}\int_\Sigma d^2\xi y_2 (\partial_+ v_- - \partial_- v_+) \,. \tag{3.20}$$

Varying with respect to the Lagrange multiplier y_2 we get

$$v_\pm = \partial_\pm y \,, \tag{3.21}$$

while the equations of motion for gauge fields are

$$v_\pm = \pm\partial_\pm y_2 \mp 2H\Delta V \partial_\pm x \,. \tag{3.22}$$

Inserting the expression for gauge fields (3.22) into gauge fixed action (3.20), we obtain the T-dual action

$$_{zy}S = \kappa \int_\Sigma d^2\xi \; \partial_+ \; _{zy}X^\mu \; _{zy}\Pi_{+\mu\nu} \; \partial_- \; _{zy}X^\nu \,, \tag{3.23}$$

where

$$_{zy}X^\mu = \begin{pmatrix} x \\ y_2 \\ y_3 \end{pmatrix} , \qquad _{zy}\Pi_{+\mu\nu} = {}_{zy}B_{\mu\nu} + \frac{1}{2}\, {}_{zy}G_{\mu\nu} \,, \tag{3.24}$$

$$_{zy}B_{\mu\nu} = 0 \,, \qquad _{zy}G_{\mu\nu} = \begin{pmatrix} 1 & -2H\Delta V & 0 \\ -2H\Delta V & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} . \tag{3.25}$$

Let us note that after two T-dualizations in the xyz case in [44] we also obtained that T-dual Kalb-Ramond field is zero.

3.2.2 T-dual transformation law

Combining equations of motion (3.21) and (3.22) we get the corresponding transformation law

$$\partial_\pm y \cong \pm\partial_\pm y_2 \mp 2H\Delta V \partial_\pm x \,. \tag{3.26}$$

Let us now prescribe the transformation law in canonical form. The momentum canonically conjugated to the initial coordinate y is obtained by variation of the initial action (2.13) with respect to the $\dot{y}$ and it is of the form

$$\pi_y = \kappa(\dot{y} + 2Hzx') \,, \tag{3.27}$$

while from transformation law (3.26) we have

$$\dot{y} \cong y_2' - 2H\Delta V x' \,. \tag{3.28}$$

Combining last two equations and using the fact that, in the approximation linear in H, ΔV and z are T-dual to each other, we get

$$y_2' \cong \frac{1}{\kappa}\pi_y \,. \tag{3.29}$$

As we see the transformation law is the same as in the xyz case.

JHEP03(2019)136

3.2.3 (Non)commutativity and (non)associativity

In this paragraph we will calculate Poisson brackets of the coordinates ${}_{zy}X^\mu$ using transformation laws in canonical form given by eqs. (3.17) and (3.29).

With the help of the standard Poisson algebra (3.19) and instructions from appendix B, it is easy to see that

$$\{x(\sigma), x(\bar\sigma)\} = \{y_2(\sigma), y_2(\bar\sigma)\} = \{y_3(\sigma), y_3(\bar\sigma)\} = \{x(\sigma), y_2(\bar\sigma)\} = \{x(\sigma), y_3(\bar\sigma)\} = 0\,. \tag{3.30}$$

The only non-zero Poisson bracket is

$$\{y_2'(\sigma), y_3'(\bar\sigma)\} \cong \frac{H}{\kappa}\left[2x'(\sigma)\delta(\sigma-\bar\sigma) + x(\sigma)\delta'(\sigma-\bar\sigma)\right]\,, \tag{3.31}$$

where $\delta' \equiv \partial_\sigma\delta(\sigma-\bar\sigma)$. This result is obtained by straightforward calculation using T-dual transformation laws, (3.17) and (3.29), and standard Poisson algebra (3.19). The relation (3.31) is of the form (B.1), where $A'(\sigma) = y_2'(\sigma)$, $B'(\bar\sigma) = y_3'(\bar\sigma)$, $U'(\sigma) = \frac{H}{\kappa}2x'(\sigma)$ and $V(\sigma) = \frac{H}{\kappa}x(\sigma)$. With these substitutions in mind, we have that final expression is of the form (B.8)

$$\{y_2(\sigma), y_3(\bar\sigma)\} \cong -\frac{H}{\kappa}\left[2x(\sigma) - x(\bar\sigma)\right]\theta(\sigma-\bar\sigma)\,. \tag{3.32}$$

For $\sigma \to \sigma + 2\pi$ and $\bar\sigma \to \sigma$ we have

$$\{y_2(\sigma+2\pi), y_3(\sigma)\} \cong -\frac{H}{\kappa}\left[x(\sigma) + 4\pi N_x\right]\,, \tag{3.33}$$

because $\theta(2\pi) = 1$ (B.6), while N_x is winding number for x coordinate

$$x(\sigma+2\pi) - x(\sigma) = 2\pi N_x\,. \tag{3.34}$$

As we can see the noncommutativity relation (3.32) is of κ-Minkowski type. It is straightforward to see that

$$\{x(\sigma_1), \{y_2(\sigma_2), y_3(\sigma_3)\}\} + \{y_2(\sigma_2), \{y_3(\sigma_3), x(\sigma_1)\}\} + \{y_3(\sigma_3), \{x(\sigma_1), y_2(\sigma_2)\}\} \cong 0\,. \tag{3.35}$$

Because the Jacobiator is zero, we conclude that this R-flux theory is **noncommutative** and **associative** one.

3.3 Step 3 — T-dualization along x direction

In this subsection we will finish T-dualization procedure not repeating the mathematical details, but giving just the important equations and results.

The gauge fixed action is given by the following equation

$$\begin{aligned} S_{fix} = \kappa \int_\Sigma d^2\xi \left[\frac{1}{2}\left(v_+v_- + \partial_+y_2\partial_-y_2 + \partial_+y_3\partial_-y_3\right) - H\Delta V(v_+\partial_-y_2 + \partial_+y_2\, v_-)\right] \\ + \frac{\kappa}{2}\int_\Sigma d^2\xi y_1(\partial_+v_- - \partial_-v_+)\,. \end{aligned} \tag{3.36}$$

The equations of motion for Lagrange multiplier produces

$$v_\pm = \partial_\pm x \,, \tag{3.37}$$

while the equations of motion for gauge fields $v_\pm$ give

$$v_\pm = \pm\partial_\pm y_1 + 2H\Delta V\, \partial_\pm y_2 \,. \tag{3.38}$$

Inserting expressions for $v_\pm$ into gauge fixed action we get the T-dual action

$${}_{zyx}S = \kappa \int_\Sigma d^2\xi \partial_+\, {}_{zyx}X^\mu\, {}_{zyx}\Pi_{+\mu\nu}\, {}_{zyx}\, X^\nu \,, \tag{3.39}$$

where

$${}_{zyx}X^\mu = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} , \qquad {}_{zyx}\Pi_{+\mu\nu} = {}_{zyx}B_{\mu\nu} + \frac{1}{2}\, {}_{zyx}G_{\mu\nu} \tag{3.40}$$

$${}_{zyx}B_{\mu\nu} = \begin{pmatrix} 0 & -H\Delta V & 0 \\ H\Delta V & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} , \qquad {}_{zyx}G_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} . \tag{3.41}$$

Combining the equations of motion (3.37) and (3.38) we obtain the T-dual transformation law

$$\partial_\pm x \cong \pm\partial_\pm y_1 + 2H\Delta V \partial_\pm y_2 \,. \tag{3.42}$$

It directly follows that

$$\dot{x} \cong y_1' + 2H\Delta V \dot{y}_2 \,. \tag{3.43}$$

From the initial action (2.13) it is obvious that momentum canonically conjugated to x is of the form

$$\pi_x = \kappa\dot{x} - 2\kappa H z y' \,. \tag{3.44}$$

The T-dual transformation law for y (3.26), in the approximation linear in H, produces that $y' \cong \dot{y}_2$. Taking into account the relation (3.43), we get the canonical form of the T-dual transformation law

$$y_1' \cong \frac{1}{\kappa}\pi_x \,. \tag{3.45}$$

As we see the full set of T-dual transformation laws, (3.17), (3.29) and (3.45), are the same as in the case where T-dualization was along xyz line [44] up to $H \to -H$. The full T-dualized theory is of the same form as in [44] with the expressions for **noncommutativity**

$$\{y_1(\sigma), y_3(\bar\sigma)\} \cong \frac{H}{\kappa}\left[2y(\sigma) - y(\bar\sigma)\right]\theta(\sigma - \bar\sigma) \,, \tag{3.46}$$

$$\{y_2(\sigma), y_3(\bar\sigma)\} \cong -\frac{H}{\kappa}\left[2x(\sigma) - x(\bar\sigma)\right]\theta(\sigma - \bar\sigma) \,, \tag{3.47}$$

and **nonassociativity**

$$\begin{aligned} &\{y_1(\sigma_1), y_2(\sigma_2), y_3(\sigma_3)\} \equiv \\ &\{y_1(\sigma_1), \{y_2(\sigma_2), y_3(\sigma_3)\}\} + \{y_2(\sigma_2), \{y_3(\sigma_3), y_1(\sigma_1)\}\} + \{y_3(\sigma_3), \{y_1(\sigma_1), y_2(\sigma_2)\}\} \cong \\ &\frac{2H}{\kappa^2}\left[\theta(\sigma_1 - \sigma_2)\theta(\sigma_2 - \sigma_3) + \theta(\sigma_2 - \sigma_1)\theta(\sigma_1 - \sigma_3) + \theta(\sigma_1 - \sigma_3)\theta(\sigma_3 - \sigma_2)\right] , \end{aligned} \tag{3.48}$$

which can be obtained from the corresponding ones in xyz case [44] by replacing $H \to -H$.

JHEP03(2019)136

4 Quantum aspects of T-dualization in the weakly curved background

In proving isometry and computing the $\beta^{\pm}$ functions we assumed the trivial topology and the surface term occurring there vanishes. Now we want to discuss some quantum aspects of the considered problems in nontrivial topologies. We will consider the action for bosonic string in the weakly curved background — constant metric and Kalb-Ramond field depending on all coordinates and with infinitesimal field strength. Torus with infinitesimal H-flux is special case of this model.

On th classical level there are a few problems in the theory. In order to perform the generalized T-dualization procedure the invariant coordinate x^{μ}_{inv} is introduced. But it is multivalued and the proof of equivalence of gauged and initial theories needs the part considering global characteristics. Moreover, in the quantum theory at higher genus, the holonomies of the world-sheet gauge fields complicate the situation a little bit. Fortunately, these problems can be resolved in Abelian case in the quantum theory [50–52].

First, we make Wick rotation $\tau \to -i\tau$, which makes the term which contains metric tensor $G_{\mu\nu}$ gets multiplier i, while the terms which contain Kalb-Ramond field $B_{\mu\nu}$ and Lagrange multiplier y_μ stay unchanged. Then the partition function is of the form

$$Z = \sum_{g=0}^{\infty} \int \mathcal{D}y\mathcal{D}v\, e^{-\frac{\kappa}{2}\int_\Sigma v\, G\, {}^\star v + i\kappa \int_\Sigma vB[V]v + \frac{i\kappa}{2}\int_\Sigma v dy}\,. \tag{4.1}$$

We use differential forms and omit the space-time indices to simplify writing of equations. The Hodge duality operator is denoted by star. The index g denotes the genus of manifold.

The first step in the calculation process is separation the one form dy into the exact part dy_e (y_e is single valued) and the harmonic part y_h ($dy_h = 0 = d^\dagger y_h$)

$$dy = dy_e + y_h. \tag{4.2}$$

For the closed forms the co-exact term $d^\dagger y_{co}$ in the Hodge decomposition is missing.

The path integral (4.1) goes over all degrees of freedom including local degrees of freedom as well as the sum over different topologies. Consequently, according to the (4.2), we substitute $\mathcal{D}y$ with the path integral over y_e and the sum over all possible topologically nontrivial states contained in y_h (marked by H_y)

$$\mathcal{D}y \to \mathcal{D}y_e \sum_{H_y}. \tag{4.3}$$

The integration over y_e induces vanishing of the field strength

$$Z = \int \mathcal{D}v\, \delta(dv)\, e^{-\frac{\kappa}{2}\int_\Sigma v\, G\, {}^\star v + i\kappa\int_\Sigma vB[V]v} \sum_{H_y} e^{\frac{i\kappa}{2}\int_\Sigma v y_h}. \tag{4.4}$$

The 1-form v can be expressed as sum of exact, co-exact and the harmonic parts

$$v = dx + d^\dagger v_{ce} + v_h, \tag{4.5}$$

which means that

$$\mathcal{D}v \to \mathcal{D}x\mathcal{D}d^\dagger v_{ce}\, dH_v. \tag{4.6}$$

The functional integration over harmonic part v_h drives to the ordinary integration over topologically nontrivial periods (marked by symbol H_v). After integration over $d^\dagger v_{ce}$ we get

$$Z = \int \mathcal{D}x dH_v\, e^{-\frac{\kappa}{2}\int_\Sigma v\, G\, {}^\star v + i\kappa \int_\Sigma vB[V]v} \sum_{H_y} e^{\frac{i\kappa}{2}\int_\Sigma v y_h}. \tag{4.7}$$

The last term in the exponent is responsible for nontrivial holonomies. Eliminating v_{ce} part, the 1-form v becomes closed and the Riemann bilinear relation becomes usable

$$\int_\Sigma v y_h = \sum_{i=1}^{g}\Big[\oint_{a_i} v \oint_{b_i} y_h - \oint_{a_i} y_h \oint_{b_i} v\Big]. \tag{4.8}$$

The symbols $a_i, b_i\, (i = 1, 2, \ldots, g)$ represent the canonical homology basis for the world-sheet. Because of the periodicity of the Lagrange multiplier y, its periods are just the winding numbers around cycles a_i and b_i

$$N_{a_i} = \oint_{a_i} y_h, \quad N_{b_i} = \oint_{b_i} y_h\,. \tag{4.9}$$

Denoting the periods with

$$A_i = \oint_{a_i} v, \quad B_i = \oint_{b_i} v\,, \tag{4.10}$$

we get

$$\int_\Sigma v y_h = \sum_{i=1}^{g}(N_{b_i}A_i - N_{a_i}B_i). \tag{4.11}$$

Now the partition function (4.7) gets the form

$$Z = \int \mathcal{D}x\, dA_i dB_i e^{-\frac{\kappa}{2}\int_\Sigma v\, G\, {}^\star v + i\kappa\int_\Sigma vB[V]v} \sum_{N_{a_i}, N_{b_i}\in Z} e^{\frac{i\kappa}{2}\sum_{i=1}^{g}(N_{b_i}A_i - N_{a_i}B_i)}. \tag{4.12}$$

The periodic delta function is defined as $\delta(x) = \frac{1}{2\pi}\sum_{n\in Z} e^{inx}$, which produces

$$Z = \int \mathcal{D}x\, dA_i dB_i \delta\left(\frac{\kappa}{2}A_i\right)\delta\left(\frac{\kappa}{2}B_i\right)\, e^{-\frac{\kappa}{2}\int_\Sigma v\, G\, {}^\star v + i\kappa\int_\Sigma vB[V]v}\,. \tag{4.13}$$

It is useful to examine the path dependence of the variable V^μ, which form is now

$$V^\mu(\xi) = x^\mu(\xi) - x^\mu(\xi_0) + \int_P v_h^\mu. \tag{4.14}$$

Let us consider two paths, P_1 and P_2, with the same initial ξ_0^α and the final points ξ^α. Now we will subtract from the value of V^μ along P_1 the value along path P_2 and obtain the integral over closed curve $P_1 P_2^{-1}$ of the harmonic form

$$V^\mu[P_1] - V^\mu[P_2] = \oint_{P_1 P_2^{-1}} v_h^\mu. \tag{4.15}$$

Establishing the homology between the closed curve $P_1 P_2^{-1}$ and curve $\sum_i [n_i a_i + m_i b_i]$, $(n_i, m_i \in Z)$ we get finally

$$V^\mu[P_1] = V^\mu[P_2] + \sum_i (n_i A_i^\mu + m_i B_i^\mu). \tag{4.16}$$

The variable $V^\mu(\xi)$ in classical theory is path dependent if holonomies are nontrivial.

Integrating eq. (4.13) over A_i and B_i implies that periods A_i and B_i are zero. Consequently

$$v = dx. \tag{4.17}$$

The variable V^μ becomes single valued, and the initial theory is restored

$$Z = \int \mathcal{D}x e^{-\frac{\kappa}{2}\int_\Sigma dx\, G\, {}^\star dx + i\kappa \int_\Sigma dx B[x] dx} = \int \mathcal{D}x e^{-\kappa \int_\Sigma d^2\xi \partial x \Pi_+[x] \bar\partial x}. \tag{4.18}$$

Consequently, starting with partition function of the gauged fixed action of bosonic string in the weakly curved background, within path integral formalism and in the presence of nontrivial topologies, we came to the partition function of the initial theory. That means that introducing coordinate dependent Kalb-Ramond field is consistent with path integral quantization process.

5 Conclusion

In this article we studied the 3D closed bosonic string propagating in the geometry known as torus with H-flux — constant metric and Kalb-Ramond field with just one nonzero component, $B_{xy} = -B_{yx} = Hz$. The choice of background fields is consistent with the consistency conditions if we work in the diluted flux approximation which assumes that in all calculations we keep just the constant terms and those linear in the infinitesimal field strength H. Our goal was to study the T-dualization line which goes in the opposite direction from the standard one. First, we T-dualize z direction, then y and at the end along x direction — so-called zyx T-dualization line. We analyzed in every step the (non)commutativity and (non)associativity of the obtained theory and made comparisons with the case of xyz T-dualization line considered in [33, 44].

The common fact for all three theories obtained in the process of T-dualization step by step is that all three ones are nonlocal R-flux theories. The nonlocality comes as a result of the first step in T-dualization procedure, T-dualization along z direction. Generalized T-dualization procedure has one additional step with respect to the standard Buscher procedure and that is introduction of invariant coordinate. In the process of T-dualization invariant coordinate turns into variable ΔV which is defined as line integral. Consequently, this means that obtained theory is nonlocal. Further T-dualizations does not affect ΔV and, all three theories are nonlocal ones. As we know, in the case of xyz T-dualization line [44], we obtained three different theories in geometrical sense — twisted torus, Q-flux theory (which is local) and nonlocal R-flux theory.

The dualization along z direction produces nonlocal R-flux theory unlike the xyz case [33, 44] where the theory obtained after first T-dualization is locally and globally well defined. Because initial theory is geometrical one, its variables satisfy standard Poisson algebra (3.19). Using (3.19) and T-dual transformation law written in the canonical form (3.17), we showed that theory obtained after T-dualization along z coordinate (using generalized T-dualization procedure) is *commutative* and, consequently, *associative* one as in [44].

The second step in T-dualization is T-dualization along y direction. Using standard Buscher procedure, we obtained the form of the T-dual theory and the corresponding T-dual transformation law, which is rewritten in the canonical form (3.29) in terms of the coordinates and momenta of the initial theory. Using standard Poisson algebra (3.19) and T-dual transformation laws in canonical form, (3.17) and (3.29), we easily proved that theory after two T-dualizations is *noncommutative*, but it is still *associative* one. In this article we used trivial winding condition (3.34) and showed that T-dual coordinates $y_2(\sigma)$ and $y_3(\bar{\sigma})$ are commutative for equal arguments, $\sigma = \bar{\sigma}$, but they are noncommutative if $\sigma - \bar{\sigma} = 2\pi$. The result is qualitatively similar to the result of [33], where after two T-dualizations the obtained theory is noncommutative one. But, the difference is in the winding condition which is nontrivial in [33], mixing different coordinates. The different winding condition induces the noncommutativity for $\sigma = \bar{\sigma}$ (for more details see [33]). On the other hand in the analysis presented in [44] (xyz T-dualization line) the theory obtained after two T-dualizations is commutative under trivial winding condition.

The final step in T-dualization procedure is T-dualization along x direction. The theory after full T-dualization is the same as in xyz case [44] with the noncommutativity and nonassociativity parameters which can be obtained from those in xyz case [44] adding "$-$" sign. This is a consequence of the fact that the full set of T-dual transformation laws is the same as in [44] up to the replacing $H \to -H$. This difference up to the "$-$" sign stems from the initial actions. In this article we start from (2.13), while in [44] the starting action for z T-dualization is Q-flux action, formally the same as (2.13) up to the replacing $H \to -H$.

Finishing the discussion of the results obtained in this paper it is interesting to make comparison with some similar efforts. We studied the abelian isometries using both standard and generalized T-duality procedure, while in the paper [53] nonabelian isometries using standard Buscher procedure are considered. The authors of [53] showed that spaces with isometry maps to the nonisometry spaces, while in this paper there is isometry in every T- dualization step. One of their conclusions that T-dual transformations are more than continuous isometry can be added to the concluding remarks of this paper. In the ref. [43] generalized T-duality and nongeometric background are considered, but using low energy effective action, unlike here, where we used sigma model action. The paper [54] deals with T-dualizations along nonisometry directions like in [31], using extension of gauge symmetry, while the authors of [31] use the generalized T-dualization procedure introducing invariant coordinates (in [54] they call them "covariant" coordinates). In this paper we use this generalized T-dualization procedure but all directions considered here are isometry ones. It is useful to mention that in the paper [55] bosonic string in the presence of the weakly curved backgrounds is considered using double space formalism as well as the influence of the order of T-dualizations. The double space formalism gives the result which is in accordance with the result of the current paper.

JHEP03(2019)136

Consequently, we conclude that in the case of the full T-dualization the form of the T-dual theory do not depend on the order of T-dualization, while parameters of noncommutativity and nonassociativity change sign.

A Light-cone coordinates

In the paper we often use light-cone coordinates defined as

$$\xi^{\pm} = \frac{1}{2}(\tau \pm \sigma)\,. \tag{A.1}$$

The corresponding partial derivatives are

$$\partial_{\pm} \equiv \frac{\partial}{\partial \xi^{\pm}} = \partial_{\tau} \pm \partial_{\sigma}\,. \tag{A.2}$$

Two dimensional Levi-Civita $\varepsilon^{\alpha\beta}$ is chosen in (τ, σ) basis as $\varepsilon^{\tau\sigma} = -1$. Consequently, in the light-cone basis the form of tensor is

$$\varepsilon^{\alpha\beta}_{lc} = \begin{pmatrix} 0 & \frac{1}{2} \\ -\frac{1}{2} & 0 \end{pmatrix}. \tag{A.3}$$

The flat world-sheet metric is of the form in (τ, σ) and light-cone basis, respectively

$$\eta_{\alpha\beta} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \eta^{lc}_{\alpha\beta} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}. \tag{A.4}$$

Let us stress that in whole article we use standard notation for τ and σ derivatives — $\dot{A} \equiv \partial_{\tau} A$ and $A' \equiv \partial_{\sigma} A$, where A is an arbitrary variable.

B Two types of Poisson brackets used in the paper

In this paper, we have seen that T-dual transformation laws connect derivatives of T-dual coordinates with coordinates and momenta of initial theory. While initial theory satisfies standard Poisson brackets, in order to find Poisson brackets for T-dual theory, we first need to find Poisson brackets between σ derivatives of T-dual coordinates. This type of Poisson bracket will, in general case, be some function of initial coordinates, Dirac delta functions and their derivatives with respect to σ. Having this in mind, general case for our Poisson brackets will have following form

$$\{A'(\sigma), B'(\bar{\sigma})\} = U'(\sigma)\delta(\sigma - \bar{\sigma}) + V(\sigma)\delta'(\sigma - \bar{\sigma})\,, \tag{B.1}$$

where $\delta'(\sigma - \bar{\sigma}) \equiv \partial_{\sigma}\delta(\sigma - \bar{\sigma})$. For terms $A'(\sigma)$, $U'(\sigma)$ and $B'(\bar{\sigma})$, symbol $'$ stands for partial derivative with respect to σ and $\bar{\sigma}$, respectively. If we want to calculate the Poisson bracket

$$\{A(\sigma), B(\bar{\sigma})\}\,,$$

first we have to calculate the following one

$$\{\Delta A(\sigma, \sigma_0), \Delta B(\bar{\sigma}, \bar{\sigma}_0)\}\,, \tag{B.2}$$

JHEP03(2019)136

where

$$\Delta A(\sigma,\sigma_0)=\int_{\sigma_0}^{\sigma}dxA'(x)=A(\sigma)-A(\sigma_0)\,,\quad \Delta B(\bar\sigma,\bar\sigma_0)=\int_{\bar\sigma_0}^{\bar\sigma}dxB'(x)=B(\bar\sigma)-B(\bar\sigma_0)\,. \tag{B.3}$$

Substituting the expressions (B.3) into (B.2), we have

$$\{\Delta A(\sigma,\sigma_0),\Delta B(\bar\sigma,\bar\sigma_0)\}=\int_{\sigma_0}^{\sigma}dx\int_{\bar\sigma_0}^{\bar\sigma}dy\,\left[U'(x)\delta(x-y)+V(x)\delta'(x-y)\right]\,. \tag{B.4}$$

After integration over y we get

$$\begin{aligned}&\{\Delta A(\sigma,\sigma_0),\Delta B(\bar\sigma,\bar\sigma_0)\}=\\ &=\int_{\sigma_0}^{\sigma}dx\{U'(x)\left[\theta(x-\bar\sigma_0)-\theta(x-\bar\sigma)\right]+V(x)\left[\delta(x-\bar\sigma_0)-\delta(x-\bar\sigma)\right]\},\end{aligned} \tag{B.5}$$

where $\theta(x)$ is defined as

$$\theta(x)=\int_0^x d\eta\delta(\eta)=\frac{1}{2\pi}\left[x+2\sum_{n\geq1}\frac{1}{n}\sin(nx)\right]=\begin{cases}0 & \text{if } x=0\\ 1/2 & \text{if } 0<x<2\pi\,,\\ 1 & \text{if } x=2\pi\end{cases} \tag{B.6}$$

where $\delta(x)=\frac{1}{2\pi}\sum_{n\in Z}e^{inx}$. Finally, integrating over x, we obtain

$$\begin{aligned}\{\Delta A(\sigma,\sigma_0),\Delta B(\bar\sigma,\bar\sigma_0)\}=&\,U(\sigma)[\theta(\sigma-\bar\sigma_0)-\theta(\sigma-\bar\sigma)]-U(\sigma_0)[\theta(\sigma_0-\bar\sigma_0)-\theta(\sigma_0-\bar\sigma)]\\ &-U(\bar\sigma_0)[\theta(\sigma-\bar\sigma_0)-\theta(\sigma_0-\bar\sigma_0)]+U(\bar\sigma)[\theta(\sigma-\bar\sigma)-\theta(\sigma_0-\bar\sigma)]\\ &+V(\bar\sigma_0)[\theta(\sigma-\bar\sigma_0)-\theta(\sigma_0-\bar\sigma_0]-V(\bar\sigma)[\theta(\sigma-\bar\sigma)-\theta(\sigma_0-\bar\sigma)].\end{aligned} \tag{B.7}$$

From the last expression, using (B.3), we extract the searched Poisson bracket

$$\{A(\sigma),B(\bar\sigma)\}=-[U(\sigma)-U(\bar\sigma)+V(\bar\sigma)]\theta(\sigma-\bar\sigma)\,. \tag{B.8}$$

In order to calculate Jacobiator we have to find Poisson brackets of type $\{y(\sigma),x(\bar\sigma)\}$, where $y(\sigma)$ is coordinate T-dual to initial one $x(\sigma)$. Having this in mind, we start with the following Poisson bracket

$$\{\Delta y(\sigma,\sigma_0),x(\bar\sigma)\}=\left\{\int_{\sigma_0}^{\sigma}d\eta y'(\eta),x(\bar\sigma)\right\}\,, \tag{B.9}$$

and using T-dual transformation law in canonical form

$$\pi\cong\kappa y'\,, \tag{B.10}$$

we get

$$\{\Delta y(\sigma,\sigma_0),x(\bar\sigma)\}\cong\frac{1}{\kappa}\left\{\int_{\sigma_0}^{\sigma}d\eta\pi(\eta),x(\bar\sigma)\right\}\,, \tag{B.11}$$

where $\pi(\sigma)$ is momentum canonically conjugated to the coordinate $x(\sigma)$. Initial theory is geometric one which variables satisfy standard Poisson algebra, so, the final result is of the form

$$\{\Delta y(\sigma,\sigma_0),x(\bar\sigma)\}\cong-\frac{1}{\kappa}\left[\theta(\sigma-\bar\sigma)-\theta(\sigma_0-\bar\sigma)\right]\quad\Longrightarrow\{y(\sigma),x(\bar\sigma)\}\cong-\frac{1}{\kappa}\theta(\sigma-\bar\sigma)\,. \tag{B.12}$$

Acknowledgments

Work supported in part by the Serbian Ministry of Education, Science and Technological Development, under contract No. 171031. I also want to thank to Prof. Dr. Branislav Sazdović and Dr. Ljubica Davidović from Institute of Physics Belgrade for useful discussions.

References

[1] H.S. Snyder, *Quantized space-time*, Phys. Rev. **71** (1947) 38 [INSPIRE].

[2] N. Seiberg and E. Witten, *String theory and noncommutative geometry*, JHEP **09** (1999) 032 [hep-th/9908142] [INSPIRE].

[3] A. Connes, M.R. Douglas and A.S. Schwarz, *Noncommutative geometry and matrix theory: Compactification on tori*, JHEP **02** (1998) 003 [hep-th/9711162] [INSPIRE].

[4] M.R. Douglas and C.M. Hull, *D-branes and the noncommutative torus*, JHEP **02** (1998) 008 [hep-th/9711165] [INSPIRE].

[5] V. Schomerus, *D-branes and deformation quantization*, JHEP **06** (1999) 030 [hep-th/9903205] [INSPIRE].

[6] F. Ardalan, H. Arfaei and M.M. Sheikh-Jabbari, *Noncommutative geometry from strings and branes*, JHEP **02** (1999) 016 [hep-th/9810072] [INSPIRE].

[7] C.-S. Chu and P.-M. Ho, *Noncommutative open string and D-brane*, Nucl. Phys. **B 550** (1999) 151 [hep-th/9812219] [INSPIRE].

[8] F. Ardalan, H. Arfaei and M.M. Sheikh-Jabbari, *Dirac quantization of open strings and noncommutativity in branes*, Nucl. Phys. **B 576** (2000) 578 [hep-th/9906161] [INSPIRE].

[9] C.-S. Chu and P.-M. Ho, *Constrained quantization of open string in background B field and noncommutative D-brane*, Nucl. Phys. **B 568** (2000) 447 [hep-th/9906192] [INSPIRE].

[10] T. Lee, *Canonical quantization of open string and noncommutative geometry*, Phys. Rev. **D 62** (2000) 024022 [hep-th/9911140] [INSPIRE].

[11] B. Nikolić and B. Sazdović, *Gauge symmetries change the number of Dp-brane dimensions*, Phys. Rev. **D 74** (2006) 045024 [hep-th/0604129] [INSPIRE].

[12] B. Nikolić and B. Sazdović, *Gauge symmetries decrease the number of Dp-brane dimensions, II (Inclusion of the Liouville term)*, Phys. Rev. **D 75** (2007) 085011 [hep-th/0611191] [INSPIRE].

[13] D. Latas, V. Radovanović and J. Trampetić, *Non-commutative* SU(N) *gauge theories and asymptotic freedom*, Phys. Rev. **D 76** (2007) 085006 [hep-th/0703018] [INSPIRE].

[14] M. Burić, V. Radovanović and J. Trampetić, *The One-loop renormalization of the gauge sector in the noncommutative standard model*, JHEP **03** (2007) 030 [hep-th/0609073] [INSPIRE].

[15] D.N. Blaschke, H. Grosse and J.-C. Wallet, *Slavnov-Taylor identities, non-commutative gauge theories and infrared divergences*, JHEP **06** (2013) 038 [arXiv:1302.2903] [INSPIRE].

[16] C.P. Martin and C. Tamarit, *Noncommutative $N = 1$ super Yang-Mills, the Seiberg-Witten map and UV divergences*, *JHEP* **11** (2009) 092 [arXiv:0907.2437] [INSPIRE].

[17] B. Melic, K. Passek-Kumericki and J. Trampetić, *$K \to \pi\gamma$ decay and space-time noncommutativity*, *Phys. Rev.* **D 72** (2005) 057502 [hep-ph/0507231] [INSPIRE].

[18] M. Burić, D. Latas, V. Radovanović and J. Trampetić, *Nonzero $Z \to \gamma\gamma$ decays in the renormalizable gauge sector of the noncommutative standard model*, *Phys. Rev.* **D 75** (2007) 097701 [hep-ph/0611299] [INSPIRE].

[19] S. Meljanac, A. Samsarov, J. Trampetić and M. Wohlgenannt, *Scalar field propagation in the ϕ^4 κ-Minkowski model*, *JHEP* **12** (2011) 010 [arXiv:1111.5553] [INSPIRE].

[20] S. Meljanac, A. Samsarov, M. Stojić and K.S. Gupta, *κ-Minkowski space-time and the star product realizations*, *Eur. Phys. J.* **C 53** (2008) 295 [arXiv:0705.2471] [INSPIRE].

[21] D. Kovačević and S. Meljanac, *Kappa-Minkowski spacetime, Kappa-Poincaré Hopf algebra and realizations*, *J. Phys.* **A 45** (2012) 135208 [arXiv:1110.0944] [INSPIRE].

[22] E. Harikumar, T. Jurić and S. Meljanac, *Electrodynamics on κ-Minkowski space-time*, *Phys. Rev.* **D 84** (2011) 085020 [arXiv:1107.3936] [INSPIRE].

[23] M. Dimitrijević and L. Jonke, *A twisted look on kappa-Minkowski:* U(1) *gauge theory*, *JHEP* **12** (2011) 080 [arXiv:1107.3475] [INSPIRE].

[24] A. Pachol, *κ-Minkowski spacetime: Mathematical formalism and applications in Planck scale physics*, Ph.D. thesis, Wroclaw U., 2011. arXiv:1112.5366 [INSPIRE].

[25] R. Blumenhagen, I. Brunner, V. Kupriyanov and D. Lüst, *Bootstrapping non-commutative gauge theories from L_∞ algebras*, *JHEP* **05** (2018) 097 [arXiv:1803.00732] [INSPIRE].

[26] T.H. Buscher, *A Symmetry of the String Background Field Equations*, *Phys. Lett.* **B 194** (1987) 59 [INSPIRE].

[27] T.H. Buscher, *Path Integral Derivation of Quantum Duality in Nonlinear σ-models*, *Phys. Lett.* **B 201** (1988) 466 [INSPIRE].

[28] L. Davidović and B. Sazdović, *T-duality in a weakly curved background*, *Eur. Phys. J.* **C 74** (2014) 2683 [arXiv:1205.1991] [INSPIRE].

[29] L. Davidović, B. Nikolić and B. Sazdović, *T-duality diagram for a weakly curved background*, *Eur. Phys. J.* **C 75** (2015) 576 [arXiv:1406.5364] [INSPIRE].

[30] L. Davidović, B. Nikolić and B. Sazdović, *Canonical approach to the closed string non-commutativity*, *Eur. Phys. J.* **C 74** (2014) 2734 [arXiv:1307.6158] [INSPIRE].

[31] L. Davidović and B. Sazdović, *T-dualization in a curved background in absence of a global symmetry*, *JHEP* **11** (2015) 119 [arXiv:1505.07301] [INSPIRE].

[32] D. Lüst, *T-duality and closed string non-commutative (doubled) geometry*, *JHEP* **12** (2010) 084 [arXiv:1010.1361] [INSPIRE].

[33] D. Andriot, M. Larfors, D. Lüst and P. Patalong, *(Non-)commutative closed string on T-dual toroidal backgrounds*, *JHEP* **06** (2013) 021 [arXiv:1211.6437] [INSPIRE].

[34] D. Andriot, O. Hohm, M. Larfors, D. Lüst and P. Patalong, *A geometric action for non-geometric fluxes*, *Phys. Rev. Lett.* **108** (2012) 261602 [arXiv:1202.3060] [INSPIRE].

[35] D. Andriot, O. Hohm, M. Larfors, D. Lüst and P. Patalong, *Non-Geometric Fluxes in Supergravity and Double Field Theory*, *Fortsch. Phys.* **60** (2012) 1150 [arXiv:1204.1979] [INSPIRE].

[36] R. Blumenhagen, A. Deser, E. Plauschinn, F. Rennecke and C. Schmid, *The intriguing structure of non-geometric frames in string theory*, Fortsch. Phys. **61** (2013) 893.

[37] R. Blumenhagen, A. Deser, E. Plauschinn and F. Rennecke, *Non-geometric strings, symplectic gravity and differential geometry of Lie algebroids*, JHEP **02** (2013) 122 [arXiv:1211.0030] [INSPIRE].

[38] R. Blumenhagen, A. Deser, E. Plauschinn and F. Rennecke, *A bi-invariant Einstein-Hilbert action for the non-geometric string*, Phys. Lett. **B 720** (2013) 215 [arXiv:1210.1591] [INSPIRE].

[39] D. Lüst, *Twisted Poisson Structures and Non-commutative/non-associative Closed String Geometry*, PoS(CORFU2011)086 (2011) [arXiv:1205.0100] [INSPIRE].

[40] R. Blumenhagen, A. Deser, D. Lüst, E. Plauschinn and F. Rennecke, *Non-geometric Fluxes, Asymmetric Strings and Nonassociative Geometry*, J. Phys. **A 44** (2011) 385401 [arXiv:1106.0316] [INSPIRE].

[41] C. Condeescu, I. Florakis and D. Lüst, *Asymmetric Orbifolds, Non-Geometric Fluxes and Non-Commutativity in Closed String Theory*, JHEP **04** (2012) 121 [arXiv:1202.6366] [INSPIRE].

[42] J. Shelton, W. Taylor and B. Wecht, *Nongeometric flux compactifications*, JHEP **10** (2005) 085 [hep-th/0508133] [INSPIRE].

[43] A. Dabholkar and C. Hull, *Generalised T-duality and non-geometric backgrounds*, JHEP **05** (2006) 009 [hep-th/0512005] [INSPIRE].

[44] B. Nikolić and D. Obrić, *Noncommutativity and nonassociativity of closed bosonic string on T-dual toroidal backgrounds*, Fortsch. Phys. **66** (2018) 1800009.

[45] L.N. Chang, D. Minic, A. Roman, C. Sun and T. Takeuchi, *On the Physics of the Minimal Length: The Question of Gauge Invariance*, Int. J. Mod. Phys. **A 31** (2016) 1630012 [arXiv:1602.07752] [INSPIRE].

[46] K. Becker, M. Becker and J. Schwarz *String Theory and M-Theory: A Modern Introduction*, Cambridge University Press, (2007).

[47] B. Zwiebach, *A First Course in String Theory*, Cambridge University Press, (2004).

[48] J. Polchinski, *String theory*, Volume II, Cambridge University Press, (1998).

[49] C.G. Callan Jr., E.J. Martinec, M.J. Perry and D. Friedan, *Strings in Background Fields*, Nucl. Phys. **B 262** (1985) 593 [INSPIRE].

[50] M. Roček and E.P. Verlinde, *Duality, quotients and currents*, Nucl. Phys. **B 373** (1992) 630 [hep-th/9110053] [INSPIRE].

[51] E. Alvarez, L. Álvarez-Gaumé, J.L.F. Barbon and Y. Lozano, *Some global aspects of duality in string theory*, Nucl. Phys. **B 415** (1994) 71 [hep-th/9309039] [INSPIRE].

[52] C.M. Hull, *Global aspects of T-duality, gauged σ-models and T-folds*, JHEP **10** (2007) 057 [hep-th/0604178] [INSPIRE].

[53] X.C. de la Ossa and F. Quevedo, *Duality symmetries from nonAbelian isometries in string theory*, Nucl. Phys. **B 403** (1993) 377 [hep-th/9210021] [INSPIRE].

[54] A. Chatzistavrakidis, A. Deser and L. Jonke, *T-duality without isometry via extended gauge symmetries of 2D σ-models*, JHEP **01** (2016) 154 [arXiv:1509.01829] [INSPIRE].

[55] B. Sazdović, *T-duality as coordinates permutation in double space for weakly curved background*, JHEP **08** (2015) 055 [arXiv:1503.05580] [INSPIRE].

JHEP03(2019)136

Fortschritte der Physik
Progress of Physics
www.fp-journal.org

Combined Fermionic and Bosonic T-duality of Type II Superstring Theory with Coordinate Dependent RR Field

*B. Nikolić and D. Obrić**

We investigate effects of fermionic T-duality on type II superstring in presence of Ramond-Ramond (RR) field that has infinitesimal linear dependence on bosonic coordinate x^μ. Other fields are assumed to be constant. Procedure that we employ for obtaining fermionic T-dual theory is Buscher procedure, where we will consider two distinct cases. One, where action has not been T-dualized along bosonic coordinates and other where it has. By analyzing these two cases, their actions and T-dual transformation laws, we obtain some insight into how background fields transform and what are necessary ingredients for emergence of fermionic non-commutativity.

1. Introduction

T-duality represents a map that connects different superstring theories, mapping geometry and topology from one theory to another.[1] This symmetry was originally developed with bosonic coordinates in mind, where two theories are connected by transformation laws that establish a link between coordinates.[2] It was not until 2008 that it has been noticed that the same duality can emerge in case of fermionic coordinates. In their paper[3] Berkovits and Maldacena showed that tree level superstring theories in presence of supersymmetric background fields posses new kind of symmetry. Symmetry that maps supersymmetric background fields of one theory to supersymmetric backgrounds of other theory, where dilaton and RR fields are now different. Just like in case of bosonic T-duality, mathematical machinery for obtaining T-dual theories is Buscher procedure[4,5] applied to fermionic coordinates θ^α and $\bar{\theta}^\alpha$.

Busher T-dualization procedure including its extension to fermionic coordinates[6–9] and its generalizations[10–12] mainly follow the same steps. We notice some global symmetry in the theory, usually shift symmetry, which is then localized by replacing partial derivatives with covariant ones. Covariant derivatives come with new gauge fields that insert new degrees of freedom into the theory, these degrees of freedom are eliminated with help of Lagrange multipliers. Next step is utilizing gauge freedom to fix starting coordinates. After that, finding equations of motion for gauge fields and inserting their solutions into the action we obtain T-dual theory. Extension of procedure for fermionic coordinates does not introduce any new steps into the play. However, extending the procedure to include coordinate dependent background fields does introduce one additional step. Namely, we need to replace all coordinates with invariant ones constructed as integrals of covariant derivatives. This step is necessary in order to preserve local shift symmetry.

In our previous paper[13] we demonstrated that T-dual of type II superstring which is moving in coordinate dependent RR field possesses non-commutative Poisson brackets. Since T-duality was performed only along bosonic coordinates it produced non-commutativity only between bosonic T-dual coordinates. In addition to this, we had that background fields that were constants in original theory became functions of both bosonic and fermionic coordinates in dual theory. This has left us with one open question: Would fermionic T-duality of starting theory or even theory that has been dualized along bosonic coordinates produce non-commutative relations between fermionic coordinates? While it has been shown that, in case of closed bosonic string, non-commutativity arises only in coordinates that had appeared in background fields of starting theory[14], it is not clear if that is the case for fermionic coordinates, especially when we have emergence of new coordinate dependence in background fields after bosonic T-duality.

In this article, our goal is to find what effects fermionic T-duality has on action where RR field has dependence on bosonic coordinates and do these effects change for fully dualized action. By obtaining background fields in different stages of T-duality we can determine how geometry of theory changes and when the theory makes the switch from being local to non-local one. At the end we provide few notes on how fermionic T-duality interacts with bosonic T-duality in providing new non-commutative relations.

2. Type II Superstring, Choice of Fields and Bosonic T-Duality

In this section we will present action for type II superstring in pure spinor formulation. We will also define background fields in which string propagates. Finally, we present action that has been T-dualized along bosonic coordinates.

2.1. Type II Superstring in Pure Spinor Formulation

The most general form of type II superstring action in pure spinor formalism[15–19] is given as

$$S = S_0 + V_{SG}. \tag{2.1}$$

B. Nikolić, D. Obrić
Institute of Physics Belgrade
University of Belgrade
Pregrevica 118, Serbia
E-mail: dobric@ipb.ac.rs

DOI: 10.1002/prop.202200160

First term is action for string that propagates in flat background fields.

$$S_0 = \int_\Sigma d^2\xi \left(\frac{k}{2}\eta_{\mu\nu}\partial_m x^\mu \partial_n x^\nu \eta^{mn} - \pi_\alpha \partial_- \theta^\alpha + \partial_+ \bar{\theta}^\alpha \bar{\pi}_\alpha \right) + S_\lambda + S_{\bar{\lambda}}, \tag{2.2}$$

where terms S_λ and $S_{\bar{\lambda}}$represent actions that are composed of pure spinors and their canonical momenta. The pure spinors satisfy pure spinor constraints

$$\lambda^\alpha (\Gamma^\mu)_{\alpha\beta} \lambda^\beta = \bar{\lambda} (\Gamma^\mu)_{\alpha\beta} \bar{\lambda}^\beta = 0. \tag{2.3}$$

All modifications to flat background fields are accomplished by introducing second term in equation (2.1). This term is an integrated vertex operator for massless type II supergravity

$$V_{SG} = \int_\Sigma d^2\xi (X^T)^M A_{MN} \bar{X}^N. \tag{2.4}$$

In general case matrix A_{MN} is composed of physical fields, their curvatures (field strengths) and auxiliary fields that can be expressed with physical ones. These fields are some functions of both bosonic coordinates x^μ and fermionic coordinates θ^α and $\bar{\theta}^\alpha$. Dependence of fields on fermionic coordinates is given as expansion in powers of θ^α and $\bar{\theta}^\alpha$. In our particular case, we will set all background fields except RR field to be constant. Further more, in order to simplify calculations, all terms that are nonlinear in fermionic coordinates θ^α and $\bar{\theta}^\alpha$ will be neglected. With these assumptions in mind we have that vectors X^M and $\bar{X}^M$ and matrix A_{MN} have following form

$$X^M = \begin{pmatrix} \partial_+\theta^\alpha \\ \partial_+ x^\mu \\ \pi_\alpha \\ \frac{1}{2}N_+^{\mu\nu} \end{pmatrix}, \quad \bar{X}^M = \begin{pmatrix} \partial_-\bar{\theta}^\lambda \\ \partial_- x^\mu \\ \bar{\pi}_\lambda \\ \frac{1}{2}\bar{N}_-^{\mu\nu} \end{pmatrix},$$

$$A_{MN} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & k(\frac{1}{2}g_{\mu\nu} + B_{\mu\nu}) & \bar{\Psi}^\beta_\mu & 0 \\ 0 & -\Psi^\alpha_\nu & \frac{2}{k}(f^{\alpha\beta} + C^{\alpha\beta}_\rho x^\rho) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \tag{2.5}$$

Pure spinor contribution to vectors X^M and $\bar{X}^M$ are encoded in

$$N_+^{\mu\nu} = \frac{1}{2}\omega_\alpha (\Gamma^{[\mu\nu]})^\alpha{}_\beta \lambda^\beta, \quad \bar{N}_-^{\mu\nu} = \frac{1}{2}\bar{\omega}_\alpha (\Gamma^{[\mu\nu]})^\alpha{}_\beta \bar{\lambda}^\beta. \tag{2.6}$$

Since this term does not contribute to the vertex operator, we have that pure spinor actions are decoupled from the rest. This allows us to neglect pure spinor parts from now on.

Our choice of matrix A_{MN} is composed of following fields: symmetric tensor $g_{\mu\nu}$, Kalb-Ramon antisymmetric tensor $B_{\mu\nu}$, Mayorana-Weyl gravitino fields Ψ^α_μ and $\bar{\Psi}^\alpha_\mu$, Ramond-Ramond field $\frac{2}{k}(f^{\alpha\beta} + C^{\alpha\beta}_\rho x^\rho)$ where $f^{\alpha\beta}$ and $C^{\alpha\beta}_\rho$ are constant tensors. We have also assumed that dilaton field Φ is constant. This means that factor e^Φ is included in constants $f^{\alpha\beta}$ and $C^{\alpha\beta}_\rho$. This choice of background fields is accompanied with following condition

$$\gamma^\mu_{\alpha\beta} C^{\beta\gamma}_\mu = 0, \quad \gamma^\mu_{\alpha\beta} C^{\gamma\beta}_\mu = 0. \tag{2.7}$$

String propagates in superspace spanned by bosonic coordinates x^μ ($\mu = 0, 1, ..., 9$) and fermionic ones θ^α, $\bar{\theta}^\alpha$ with 16 independent real components each. Fermionic coordinates are accompanied by their canonically conjugated momenta π_α and $\bar{\pi}_\alpha$. Both fermionic coordiantes and their momenta are given as Majorana-Wejl spinors. World sheet Σ that string sweeps in this superspace is parameterized by ξ^m ($\xi^0 = \tau$, $\xi^1 = \sigma$). By combining these parameters we can define light-cone parametrization $\xi^\pm = \frac{1}{2}(\tau \pm \sigma)$ and light-cone partial derivatives $\partial_\pm = \partial_\tau \pm \partial_\sigma$.

Inserting all these assumptions into action (2.1) and integrating out fermionic momenta, we are left with following expression

$$S = k\int_\Sigma d^2\xi \Big[\Pi_{+\mu\nu}\partial_+ x^\mu \partial_- x^\nu + \frac{1}{2}(\partial_+\bar{\theta}^\alpha + \partial_+ x^\mu \bar{\Psi}^\alpha_\mu)\big(F^{-1}(x)\big)_{\alpha\beta}(\partial_-\theta^\beta + \Psi^\beta_\nu \partial_- x^\nu)\Big], \tag{2.8}$$

where we have introduced following tensors

$$\Pi_{\pm\mu\nu} = B_{\mu\nu} \pm \frac{1}{2}G_{\mu\nu}, \tag{2.9}$$

$$F^{\alpha\beta}(x) = f^{\alpha\beta} + C^{\alpha\beta}_\mu x^\mu,$$

$$(F^{-1}(x))_{\alpha\beta} = (f^{-1})_{\alpha\beta} - (f^{-1})_{\alpha\alpha_1} C^{\alpha_1\beta_1}_\rho x^\rho (f^{-1})_{\beta_1\beta}. \tag{2.10}$$

To obtain meaningful T-dual transformation laws we need to assume that x^μ dependent part of tensor $(F^{-1}(x))_{\alpha\beta}$ is antisymmetric and infinitesimal. This additional assumption does not infringe on constraint (2.7).,[15]

Having obtained one of relevant actions , we will now focus on bosonic T-dualization of (2.8) to obtain our second action of interest.

2.2. Bosonic T-dualization

Bosonic T-dualization of action (2.8) is given in detail in [13]. Here we will only summarize the most important results.

One way to obtain T-duality is by Buscher procedure. This procedure is based on localization of translation symmetry. When we localize symmetry we replace all partial derivatives with covariant ones, while in cases where background fields depend on coordinates we also need to introduce invariant coordinate. Invariant coordinate is non-local addition to action and it is the sole reason for emergence of non-commutative behavior in closed strings. Introduction of covariant derivatives and invariant coordinates produces additional gauge fields in action, which in turn add new degrees of freedom to the theory. T-dual and original theory represent same physical system and we expect that those two theories carry exact same degrees of freedom. Because of this, we remove all newly introduced degrees of freedom with Lagrange multipliers. By utilizing gauge freedom of action we can fix bosonic coordinates to be some constant, in essence removing them from action. This gauge fixed action is only a function of gauge fields and Lagrange multipliers. Finding equation of motion for Lagrange multipliers and inserting them into action we can restore original action. On the other hand, finding equations of motion for gauge fields and inserting them into action we obtain T-dual action.

Action (2.8), due to antisymmetric part of tensor $F^{-1}_{\alpha\beta}(x)$ is invariant under global translations of bosonic coordinates. Following steps of Busher procedure, described in preceding paragraph, we obtain following T-dual action

$$ {}^bS = \frac{k}{2}\int_\Sigma d^2\xi\Big[\frac{1}{2}\bar{\Theta}^{\mu\nu}_-\partial_+y_\mu\partial_-y_\nu + \partial_+\bar{\theta}^\alpha\left({}^bF^{-1}(V^{(0)})\right)_{\alpha\beta}\partial_-\theta^\beta + \partial_+y_\mu\,{}^b\bar{\Psi}^{\mu\alpha}\left({}^bF^{-1}(V^{(0)})\right)_{\alpha\beta}\partial_-\theta^\beta + \partial_+\bar{\theta}^\alpha\left({}^bF^{-1}(V^{(0)})\right)_{\alpha\beta}{}^b\Psi^{\nu\beta}\partial_-y_\nu\Big]. \tag{2.11}$$

Here, y_μ is a dual coordinate, left superscript ^{b} denotes bosonic T-duality and V^0 represents following integral

$$\Delta V^{(0)\rho} = \frac{1}{2}\int_P d\xi^+\breve{\Theta}^{\rho_1\rho}_-\left[\partial_+y_{\rho_1} - \partial_+\bar{\theta}^\alpha(f^{-1})_{\alpha\beta}\Psi^\beta_{\rho_1}\right] - \frac{1}{2}\int_P d\xi^-\breve{\Theta}^{\rho\rho_1}_-\left[\partial_-y_{\rho_1} + \bar{\Psi}^\alpha_{\rho_1}(f^{-1})_{\alpha\beta}\partial_-\theta^\beta\right]. \tag{2.12}$$

T-dual tensors that appear in action have following interpretation: $\bar{\Theta}^{\mu\nu}_-$ is inverse tensor of $\bar{\Pi}_{+\mu\nu} = \Pi_{+\mu\nu} + \frac{1}{2}\bar{\Psi}^\alpha_\mu(F^{-1}(x))_{\alpha\beta}\Psi^\beta_\nu = \breve{\Pi}_{+\mu\nu} - \frac{1}{2}\bar{\Psi}^\alpha_\mu(f^{-1})_{\alpha\alpha_1}C^{\alpha_1\beta_1}_\rho x^\rho(f^{-1})_{\beta_1\beta}\Psi^\beta_\nu$, defined as

$$\bar{\Theta}^{\mu\nu}_-\bar{\Pi}_{+\nu\rho} = \delta^\mu_\rho, \tag{2.13}$$

where

$$\bar{\Theta}^{\mu\nu}_- = \breve{\Theta}^{\mu\nu}_- + \frac{1}{2}\breve{\Theta}^{\mu\mu_1}_-\bar{\Psi}^\alpha_{\mu_1}(f^{-1})_{\alpha\alpha_1}C^{\alpha_1\beta_1}_\rho V^{(0)\rho}(f^{-1})_{\beta_1\beta}\Psi^{\beta_1}_{\nu_1}\breve{\Theta}^{\nu_1\nu}_-, \tag{2.14}$$

$$\breve{\Theta}^{\mu\nu}_-\breve{\Pi}_{+\nu\rho} = \delta^\mu_\rho, \qquad \breve{\Theta}^{\mu\nu}_- = \Theta^{\mu\nu}_- - \frac{1}{2}\Theta^{\mu\mu_1}_-\bar{\Psi}^\alpha_{\mu_1}(\bar{f}^{-1})_{\alpha\beta}\Psi^\beta_{\nu_1}\Theta^{\nu_1\nu}_- \tag{2.15}$$

$$\bar{f}^{\alpha\beta} = f^{\alpha\beta} + \frac{1}{2}\Psi^\alpha_\mu\Theta^{\mu\nu}_-\bar{\Psi}^\beta_\nu, \tag{2.16}$$

$$\Theta^{\mu\nu}_-\Pi_{+\mu\rho} = \delta^\mu_\rho, \qquad \Theta_- = -4(G_E^{-1}\Pi_-G^{-1})^{\mu\nu}. \tag{2.17}$$

Tensor $({}^bF^{-1}\$(V^{(0)})\$)_{\alpha\beta}$ is T-dual to $(F^{-1}(x))_{\alpha\beta}$,

$$\left({}^bF^{-1}(V^{(0)})\right)_{\alpha\beta} = \left(F^{-1}(V^{(0)})\right)_{\alpha\beta} - \frac{1}{2}\left(F^{-1}(V^{(0)})\right)_{\alpha\alpha_1}\Psi^{\alpha_1}_\mu\bar{\Theta}^{\mu\nu}_-\bar{\Psi}^{\beta_1}_\nu \times\left(F^{-1}(V^{(0)})\right)_{\beta_1\beta}. \tag{2.18}$$

Finally, ${}^b\bar{\Psi}^{\mu\alpha}$ and ${}^b\Psi^{\nu\beta}$ are T-dual gravitino fields, given as

$${}^b\bar{\Psi}^{\mu\alpha} = \frac{1}{2}\bar{\Theta}^{\mu\nu}_-\bar{\Psi}^\alpha_\nu + \frac{1}{4}\bar{\Theta}^{\mu\mu_1}_-\bar{\Psi}^\beta_{\mu_1}\left(F^{-1}(V^{(0)})\right)_{\beta\beta_1}\Psi^{\beta_1}_\nu\Theta^{\nu\nu_1}_-\bar{\Psi}^\alpha_{\nu_1} = \frac{1}{2}\Theta^{\mu\nu}_-\bar{\Psi}^\alpha_\mu, \tag{2.19}$$

$${}^b\Psi^{\nu\beta} = -\frac{1}{2}\Psi^\beta_\mu\bar{\Theta}^{\mu\nu}_- - \frac{1}{2}\Psi^\beta_\mu\Theta^{\mu\mu_1}_-\bar{\Psi}^\alpha_{\mu_1}\left(F^{-1}(V^{(0)})\right)_{\alpha\alpha_1}\Psi^{\alpha_1}_{\nu_1}\bar{\Theta}^{\nu_1\nu}_- = -\frac{1}{2}\Psi^\beta_\mu\Theta^{\mu\nu}_-. \tag{2.20}$$

Having obtained actions (2.8) and (2.11), we can now consider dualization along fermionic coordinates.

3. Fermionic T-duality

In this section the objectives are to find fermionic T-dual transformation laws and actions that have been T-dualized along fermionic coordinates for case where we performed bosonic T-duality and case where we have not.

Bosonic T-duality relies on utilization of symmetries of action to produce T-dual action and T-dual transformation laws. This task is usually accomplished by utilizing Busher procedure.[4,5,10–12] The main idea of fermionic T-duality is essentially the same, we utilize isometries of fermionic coordinates to generate T-dual action and T-dual transformation laws.[3,6–8] Just like in bosonic case, we localize translational symmetry by introducing covariant derivatives and, in cases where necessary, invariant coordinates. After this we introduce term that eliminates additional degrees of freedom and gauge fix existing symmetry. From this point on, finding equations of motion for gauge fields and inserting those equations of motion into gauge fixed action we obtain T-dual action.

Before proceeding with fermionic variant of Buscher procedure, we can notice that our actions (2.8) and (2.11) do not posses terms proportional to $\partial_+\theta^\alpha$ and $\partial_-\bar{\theta}^\alpha$. This means that our fermionic coordinates have following local symmetry

$$\delta\theta^\alpha = \epsilon^\alpha(\sigma^+), \quad \delta\bar{\theta}^\alpha = \bar{\epsilon}^\alpha(\sigma^-), \quad (\sigma^\pm = \tau \pm \sigma). \tag{3.1}$$

We need to fix this symmetry before obtaining T-dual theory, one way to do this is through BRST formalism. This symmetry has following corresponding BRST transformations for fermionic fields

$$s\theta^\alpha = c^\alpha(\sigma^+), \qquad s\bar{\theta}^\alpha = \bar{c}^\alpha(\sigma^-). \tag{3.2}$$

Here s is BRST nilpotent operator, c^α and $\bar{c}^\alpha$ represent ghost fields that correspond to gauge parameters ϵ^α and $\bar{\epsilon}^\alpha$ respectively. In addition to ghost fields we also have following BRST transformations

$$sC_\alpha = b_{+\alpha}, \quad s\bar{C}_\alpha = \bar{b}_{-\alpha}, \quad sb_{+\alpha} = 0, \quad s\bar{b}_{-\alpha} = 0. \tag{3.3}$$

where $\bar{C}_\alpha$ and C_α are anti-ghosts, $b_{+\alpha}$ and $\bar{b}_{-\alpha}$ are Nakanishi-Lautrup auxiliary fields.

Fixing of gauge symmetry is accomplished by introduction of gauge fermion, where we have decided to follow in the same choice as[9]

$$\Psi = \frac{k}{2}\int_\Sigma d^2\xi\left[\bar{C}_\alpha\left(\partial_+\theta^\alpha + \frac{1}{2}\alpha^{\alpha\beta}b_{+\beta}\right) + \left(\partial_-\bar{\theta}^\alpha + \frac{1}{2}\bar{b}_{-\beta}\alpha^{\beta\alpha}\right)C_\alpha\right], \tag{3.4}$$

here $\alpha^{\alpha\beta}$ is arbitrary invertible matrix.

Applying BRST transformation to gauge fermion we obtain gauge fixed action and Fadeev-Popov action

$$S_{gf} = \frac{k}{2}\int_\Sigma d^2\xi\left[\bar{b}_{-\alpha}\partial_+\theta^\alpha + \partial_-\bar{\theta}^\alpha b_{+\alpha} + \bar{b}_\alpha\alpha^{\alpha\beta}b_{+\beta}\right], \tag{3.5}$$

$$S_{F-P} = \frac{k}{2}\int_\Sigma d^2\xi\left[\bar{C}_\alpha\partial_+c^\alpha + (\partial_-\bar{c}^\alpha)C_\alpha\right]. \tag{3.6}$$

Fadeev-Popov term contains only ghosts and anti-ghosts and it is decoupled from the actions (2.8) and (2.11). From this point on, this term will be ignored. Gauge fixing term contains auxiliary fields $\bar{b}_{-\alpha}$ and $b_{+\alpha}$ that can be removed with equations of motion

$$\bar{b}_{-\alpha} = -\partial_- \bar{\theta}^\beta (\alpha^{-1})_{\beta\alpha}, \qquad b_{+\alpha} = -(\alpha^{-1})_{\alpha\beta}\partial_+\theta^\beta, \tag{3.7}$$

giving us

$$S_{gf} = -\frac{k}{2}\int_\Sigma d^2\xi \partial_- \bar{\theta}^\alpha (\alpha^{-1})_{\alpha\beta}\partial_+\theta^\beta. \tag{3.8}$$

Inserting guage fixing term into (2.8) and (2.11) gives us actions that can be dualized with Buscher procedure.

3.1. Type II superstring - Fermionic T-Duality

Since both action (2.8) and gauge fixing term (3.8) are trivially invariant to global translations of fermionic coordinates, we localize this translational symmetry by replacing partial derivatives with covariant ones

$$\partial_\pm \theta^\alpha \to D_\pm \theta^\alpha = \partial_\pm\theta^\alpha + u^\alpha_\pm, \tag{3.9}$$
$$\partial_\pm \bar{\theta}^\alpha \to D_\pm \bar{\theta}^\alpha = \partial_\pm\bar{\theta}^\alpha + \bar{u}^\alpha_\pm. \tag{3.10}$$

New gauge fields $u^\alpha_\pm$ and $\bar{u}^\alpha_\pm$ introduce new degrees of freedom that are removed by addition of term

$$S_{add} = \frac{k}{2}\int_\Sigma d^2\xi\left[\bar{z}_\alpha(\partial_+ u^\alpha_- - \partial_- u^\alpha_+) + (\partial_+\bar{u}^\alpha_- - \partial_-\bar{u}^\alpha_+)z_\alpha\right]. \tag{3.11}$$

Gauge freedom can be utilized to fix fermionic coordinates such that $\theta^\alpha = \theta^\alpha_0 = const$ and $\bar{\theta}^\alpha = \bar{\theta}^\alpha_0 = const$. This in turn reduces our covariant derivatives to

$$D_\pm\theta^\alpha \to u^\alpha_\pm, \qquad D_\pm\bar{\theta}^\alpha \to \bar{u}^\alpha_\pm. \tag{3.12}$$

With all this in mind, we have following action

$$S_{gf} = k\int_\Sigma d^2\xi\Big[\Pi_{+\mu\nu}\partial_+ x^\mu\partial_- x^\nu + \frac{1}{2}(\bar{u}^\alpha_+ + \partial_+ x^\mu \bar{\Psi}^\alpha_\mu)\left(F^{-1}(x)\right)_{\alpha\beta}$$
$$\times(u^\beta_- + \Psi^\beta_\nu\partial_- x^\nu) - \frac{1}{2}\bar{u}^\alpha_-(\alpha^{-1})_{\alpha\beta}u^\beta_+ + \frac{1}{2}\bar{z}_\alpha(\partial_+ u^\alpha_- - \partial_- u^\alpha_+)$$
$$+\frac{1}{2}(\partial_+\bar{u}^\alpha_- - \partial_-\bar{u}^\alpha_+)z_\alpha\Big]. \tag{3.13}$$

On one side we have equations of motion for Lagrange multipliers $\bar{\chi}_\alpha$ and χ_α

$$\partial_+ u^\alpha_- - \partial_- u^\alpha_+ = 0 \quad\to\quad u^\alpha_\pm = \partial_\pm\theta^\alpha, \tag{3.14}$$
$$\partial_+ \bar{u}^\alpha_- - \partial_- \bar{u}^\alpha_+ = 0 \quad\to\quad \bar{u}^\alpha_\pm = \partial_\pm\bar{\theta}^\alpha. \tag{3.15}$$

Inserting solutions for these equations into action (3.13) we obtain starting action plus gauge fixing term. Variation of action with respect to gauge fields produces following set of equations of motion

$$u^\alpha_- = -\left(F^{\alpha\beta}(x)\partial_- z_\beta + \Psi^\alpha_\mu\partial_- x^\mu\right), \tag{3.16}$$

$$u^\alpha_+ = -\alpha^{\alpha\beta}\partial_+ z_\beta, \tag{3.17}$$

$$\bar{u}^\alpha_+ = \partial_+\bar{z}_\beta F^{\beta\alpha}(x) - \partial_+ x^\mu\bar{\Psi}^\alpha_\mu, \tag{3.18}$$

$$\bar{u}^\alpha_- = \partial_-\bar{z}_\beta\alpha^{\beta\alpha}. \tag{3.19}$$

Utilizing these equations we can remove gauge fields from action, resulting in action that depends only on Lagrange multipliers and bosonic coordinates

$${}^f S = k\int_\Sigma d^2\xi\Big[\Pi_{+\mu\nu}\partial_+ x^\mu\partial_- x^\nu + \frac{1}{2}\partial_+\bar{z}_\alpha\Psi^\alpha_\mu\partial_- x^\mu$$
$$+\frac{1}{2}\partial_+\bar{z}_\alpha F^{\alpha\beta}(x)\partial_- z_\beta - \frac{1}{2}\partial_+ x^\mu\bar{\Psi}^\alpha_\mu\partial_- z_\alpha - \frac{1}{2}\partial_-\bar{z}_\alpha\alpha^{\alpha\beta}\partial_+ z_\beta\Big]. \tag{3.20}$$

Just like in the bosonic case, we have that left superscript ^{f} denotes fermionic T-duality. From here we can deduce background fields of feriomionic T-dual theory

$${}^f\bar{\Pi}_{+\mu\nu} = \Pi_{+\mu\nu}, \tag{3.21}$$

$${}^f\left(F^{-1}(x)\right)^{\alpha\beta} = F^{\alpha\beta}(x), \tag{3.22}$$

$${}^f\bar{\Psi}_{\mu\beta}\,{}^f\left(F^{-1}(x)\right)^{\beta\alpha} = -\bar{\Psi}^\alpha_\mu \quad\to\quad {}^f\bar{\Psi}_{\mu\beta} = -\bar{\Psi}^\alpha_\mu\left(F^{-1}(x)\right)_{\alpha\beta}, \tag{3.23}$$

$${}^f\left(F^{-1}(x)\right)^{\alpha\beta}{}^f\Psi_{\mu\beta} = \Psi^\alpha_\mu \quad\to\quad {}^f\Psi_{\mu\beta} = \left(F^{-1}(x)\right)_{\beta\alpha}\Psi^\alpha_\mu. \tag{3.24}$$

Unlike bosonic case, fermionic T-dual theory is local. This can be attributed to the fact that background fields do not depend on fermionic coordinates. This in turn means that theory is geometric and we should not expect emergence of noncommutative phenomena.

3.2. Type II Superstring - Full T-Duality

To obtain fully dualized theory we start with action that is already T-dualized along bosonic coordinates (2.11). Procedure for fermionic T-duality is mostly the same as described before. The only difference comes from the fact that bosonic T-duality introduced non-local term V^0 which depends on θ^α and $\bar{\theta}^\alpha$ and now we need to introduce invariant fermionic coordinates in order for action to exhibit to local shift symmetry

$$D_\pm\theta^\alpha = \partial_\pm\theta^\alpha + u^\alpha_\pm, \tag{3.25}$$
$$D_\pm\bar{\theta}^\alpha = \partial_\pm\bar{\theta}^\alpha + \bar{u}^\alpha_\pm, \tag{3.26}$$
$$\theta^\alpha_{inv} = \int_P d\xi^m D_m\theta^\alpha = \int_P d\xi^m(\partial_m\theta^\alpha + u^\alpha_m) = \Delta\theta^\alpha + \Delta U^\alpha, \tag{3.27}$$
$$\bar{\theta}^\alpha_{inv} = \int_P d\xi^m D_m\bar{\theta}^\alpha = \int_P d\xi^m(\partial_m\bar{\theta}^\alpha + \bar{u}^\alpha_m) = \Delta\bar{\theta}^\alpha + \Delta\bar{U}^\alpha. \tag{3.28}$$

Fixing gauge symmetry as before, setting fermionic coordinates to constants, we deduce following relations

$$D_\pm\theta^\alpha \to u^\alpha_\pm, \qquad D_\pm\bar\theta^\alpha \to \bar u^\alpha_\pm, \qquad \theta^\alpha_{inv} \to \Delta U^\alpha, \qquad \bar\theta^\alpha_{inv} \to \Delta \bar U^\alpha. \tag{3.29}$$

With these relations we obtain action that is only a function of gauge fields, lagrange multipliers and dual coordinates

$$\begin{aligned}{}^bS_{gf} = \frac{k}{2}\int_\Sigma d^2\xi\Big[&\frac{1}{2}\bar\Theta^{\mu\nu}\partial_+\gamma_\mu\partial_-\gamma_\nu + \bar u^\alpha_+\left({}^bF^{-1}(V^{(0)})\right)_{\alpha\beta}u^\beta_- \\ &+\partial_+\gamma_\mu{}^b\bar\Psi^{\mu\alpha}\left({}^bF^{-1}(V^{(0)})\right)_{\alpha\beta}u^\beta_- + \bar u^\alpha_+\left({}^bF^{-1}(V^{(0)})\right)_{\alpha\beta}{}^b\Psi^{\nu\beta}\partial_-\gamma_\nu \\ &-\bar u^\alpha_-(\alpha^{-1})_{\alpha\beta}u^\beta_+ + \bar z_\alpha(\partial_+u^\alpha_- - \partial_-u^\alpha_+) + (\partial_+\bar u^\alpha_- - \partial_-\bar u^\alpha_+)z_\alpha\Big].\end{aligned} \tag{3.30}$$

In order to simplify calculations we introduce the following two substitutions

$$\begin{aligned}&\left({}^bF^{-1}(V^{(0)})\right)_{\alpha\beta}{}^b\Psi^{\nu\beta}\partial_-\gamma_\nu + \partial_-z_\alpha = Z_{-\alpha}, \\ &\partial_+\gamma_\mu{}^b\bar\Psi^{\mu\alpha}\left({}^bF^{-1}(V^{(0)})\right)_{\alpha\beta} - \partial_+\bar z_\beta = \bar Z_{+\beta}.\end{aligned} \tag{3.31}$$

Now, our action can be expressed as

$$\begin{aligned}{}^bS_{gf} = \frac{k}{2}\int_\Sigma d^2\xi\Big[&\frac{1}{2}\overline\Theta^{\mu\nu}\partial_+\gamma_\mu\partial_-\gamma_\nu + \bar u^\alpha_+\left({}^bF^{-1}(V^{(0)})\right)_{\alpha\beta}u^\beta_- + \bar Z_{+\beta}u^\beta_- \\ &+\bar u^+_\alpha Z_{-\alpha} - \bar u^-_\alpha(\alpha^{-1})_{\alpha\beta}u^+_\beta + \partial_-\bar z_\alpha u^+_\alpha - \bar u^-_\alpha\partial_+z_\alpha\Big].\end{aligned} \tag{3.32}$$

Similar to the first case, we can always revert to starting action by finding equations of motion for Lagrange multipliers and inserting their solutions into the action. In both cases equations of motion are the same so we take the freedom to omit them here.

Equations of motion for gauge fields differ in this case. Since we have that $V^{(0)}$ depends on fermionic coordinates, equations of motion have additional term that depends on invariant coordinate.

$$u^\alpha_+ = -(\alpha)^{\alpha\beta}\partial_+z_\beta, \qquad \bar u^\beta_- = \partial_-\bar z_\alpha(\alpha)^{\alpha\beta}, \tag{3.33}$$

$$\bar u^\alpha_+ = -\bar Z_{+\beta}\,{}^bF^{\beta\alpha}(V^{(0)}) - \beta^-_\nu(V^{(0)},U^{(0)})\,{}^b\bar\Psi^{\nu\alpha}, \tag{3.34}$$

$$u^\beta_- = -{}^bF^{\beta\alpha}(V^{(0)})Z_{-\alpha} - \beta^+_\mu(V^{(0)},U^{(0)})\,{}^b\Psi^{\mu\beta}. \tag{3.35}$$

The beta functions, $\beta^\pm_\mu(V^{(0)},U^{(0)})$, are obtained by varying $V^{(0)}$ (see[13] for more details). They are given as

$$\begin{aligned}\beta^\pm_\mu&(V^{(0)},U^{(0)}) \\ &= \mp\frac{1}{8}\partial_\mp\left[\bar U^\alpha + V^{\nu_1}\bar\Psi^\alpha_{\nu_1}\right](f^{-1})_{\alpha\alpha_1}C^{\alpha_1\beta_1}_\mu(f^{-1})_{\beta_1\beta}\left[U^\beta + \Psi^\beta_{\nu_2}V^{\nu_2}\right] \\ &\quad\pm\frac{1}{8}\left[\bar U^\alpha + V^{\nu_1}\bar\Psi^\alpha_{\nu_1}\right](f^{-1})_{\alpha\alpha_1}C^{\alpha_1\beta_1}_\mu(f^{-1})_{\beta_1\beta}\partial_\mp\left[U^\beta + \Psi^\beta_{\nu_2}V^{\nu_2}\right].\end{aligned} \tag{3.36}$$

Inserting equations of motion for gauge fields into action (3.32) and keeping only terms linear with respect to $C^{\alpha\beta}_\mu$, we obtain fully dualized action

$$\begin{aligned}{}^{bf}S = \frac{k}{2}&\int_\Sigma d^2\xi \\ &\times\left[\frac{1}{2}\bar\Theta^{\mu\nu}_-\partial_+\gamma_\mu\partial_-\gamma_\nu - \bar Z_{+\alpha}{}^bF^{\alpha\beta}(V^{(0)})Z_{-\beta} - \partial_-\bar z_\alpha(\alpha)^{\alpha\beta}\partial_+z_\beta\right].\end{aligned} \tag{3.37}$$

Expanded, we have

$$\begin{aligned}{}^{bf}S = k\int_\Sigma d^2\xi\Big[&\frac{1}{4}\Theta^{\mu\nu}_-\partial_+\gamma_\mu\partial_-\gamma_\nu - \frac{1}{4}\partial_+\gamma_\mu\Theta^{\mu\nu}_-\bar\Psi^\alpha_\nu\partial_-z_\alpha \\ &-\frac{1}{4}\partial_+\bar z_\alpha\Psi^\alpha_\mu\Theta^{\mu\nu}_-\partial_-\gamma_\nu + \frac{1}{2}\partial_+\bar z_\alpha{}^bF^{\alpha\beta}(V^{(0)})\partial_-z_\beta \\ &-\frac{1}{2}\partial_-\bar z_\alpha(\alpha)^{\alpha\beta}\partial_+z_\beta\Big].\end{aligned} \tag{3.38}$$

From here, we can read background fields of T-dual theory

$$\begin{aligned}{}^{bf}\bar\Pi^{\mu\nu}_+ &= \frac{1}{4}\bar\Theta^{\mu\nu}_- - \frac{1}{2}{}^b\bar\Psi^{\mu\alpha}\left({}^bF^{-1}(V^{(0)})\right)_{\alpha\beta}{}^b\Psi^{\nu\beta} = \Theta^{\mu\nu}_-, \\ {}^{bf}\left(F^{-1}(x)\right)^{\alpha\beta} &= {}^bF^{\alpha\beta}(x) = F^{\alpha\beta}(x) + \frac{1}{2}\Psi^\alpha_\mu\Theta^{\mu\nu}_-\bar\Psi^\beta_\nu, \\ {}^{bf}\bar\Psi^\mu_\alpha\,{}^{bf}\left(F^{-1}(x)\right)^{\alpha\beta} &= {}^b\bar\Psi^{\mu\beta} = \frac{1}{2}\Theta^{\mu\nu}_-\bar\Psi^\beta_\nu \\ &\to \quad {}^{bf}\bar\Psi^\mu_\alpha = \frac{1}{2}\bar\Theta^{\mu\nu}_-\bar\Psi^\beta_\nu\left(F^{-1}(x)\right)_{\beta\alpha}, \\ {}^{bf}\left(F^{-1}(x)\right)^{\alpha\beta}\,{}^{bf}\Psi^\nu_\beta &= {}^b\Psi^{\nu\alpha} = -\frac{1}{2}\Psi^\alpha_\mu\Theta^{\mu\nu}_- \\ &\to \quad {}^{bf}\Psi^\nu_\beta = -\frac{1}{2}\left(F^{-1}(x)\right)_{\beta\alpha}\Psi^\alpha_\mu\bar\Theta^{\mu\nu}_-.\end{aligned} \tag{3.39}$$

Comparing background fields in different stages of T-dualization we notice that both fermionic T-duality and bosonic T-duality affect all field, where all T-dual theories now have coordinate dependent fields. It should also be noted that non-commutative relations in theory emerge only after performing bosonic T-duality. Fermionic T-dual coordinates are always only proportional to fermionic momenta therefore Poisson brackets between fermionic coordiantes always remain zero.

3.3. Bosonic T-Duality of Fermionic T-Dual Theory

For completion sake, we will also T-dualize fermionic T-dual action (3.20) along x^μ coordinates. In this specific case, where only RR field depends on bosonic coordinate, we expect that bosonic and ferionic T-dualities commute. Therefore, this section can be thought of as a check for calculations from previous section.

Bosonic T-duality is mostly the same as fermionic one,[10–13] where only difference is the lack of introduction of Fadeev-Popov and gauge fixing actions. We again start by localizing translational symmetry, inserting Lagrange multipliers and fixing gauge fields. This produces following auxiliary action

$$^{f}S_{aux} = \kappa \int d^2\xi \Big[v_+^\mu \Pi_{+\mu\nu} v_-^\nu + \frac{1}{2}\partial_+ \bar{z}_\alpha f^{\alpha\beta}\partial_- z_\beta + \frac{1}{2}\partial_+\bar{z}_\alpha C_\mu^{\alpha\beta}\partial_- z_\beta \Delta V^\mu + \frac{1}{2}\partial_+\bar{z}_\alpha \Psi_\mu^\alpha v_-^\mu - \frac{1}{2} v_+^\mu \bar{\Psi}_\mu^\alpha \partial_- z_\alpha - \frac{1}{2}\partial_-\bar{z}_\alpha \alpha^{\alpha\beta}\partial_+ z_\beta + \frac{1}{2}y_\mu(\partial_+ v_-^\mu - \partial_- v_+^\mu)\Big]. \tag{3.40}$$

Introducing the variables

$$Y_{+\mu} = \partial_+ y_\mu - \partial_+\bar{z}_\alpha \Psi_\mu^\alpha, \quad Y_{-\mu} = \partial_- y_\mu - \bar{\Psi}_\mu^\alpha \partial_- z_\alpha, \tag{3.41}$$

the action (3.40) gets much simpler form

$$^{f}S_{aux} = \kappa \int d^2\xi \Big[v_+^\mu \Pi_{+\mu\nu} v_-^\nu + \frac{1}{2}\partial_+ \bar{z}_\alpha f^{\alpha\beta}\partial_- z_\beta + \frac{1}{2}\partial_+\bar{z}_\alpha C_\mu^{\alpha\beta}\partial_- z_\beta \Delta V^\mu - \frac{1}{2}Y_{+\mu}v_-^\mu + \frac{1}{2}v_+^\mu Y_{-\mu}\Big]. \tag{3.42}$$

Varying the above action with respect to gauge fields v_+^μ and v_-^μ, we get, respectively,

$$\Pi_{+\mu\nu} v_-^\nu = -\Big(\frac{1}{2}Y_{-\mu} + \beta_{+\mu}(V)\Big), \tag{3.43}$$

$$v_+^\nu \Pi_{+\nu\mu} = \frac{1}{2}Y_{+\mu} - \beta_{-\mu}(V), \tag{3.44}$$

where $\beta_{\pm\mu}$ are the beta functions obtained from coordinate dependent term in the action

$$\beta_{\pm\mu} = \mp\frac{1}{8}\Big(\bar{z}_\alpha C_\mu^{\alpha\beta}\partial_\mp z_\beta - \partial_\mp \bar{z}_\alpha C_\mu^{\alpha\beta} z_\beta\Big). \tag{3.45}$$

Inserting (3.43) and (3.44) into the auxiliary action (3.42), keeping the terms linear in $C_\mu^{\alpha\beta}$, we obtain fully T-dualized action (first fermionic, then bosonic T-dualization)

$$^{fb}S = \kappa \int d^2\xi \Big[\frac{1}{2}\partial_+\bar{z}_\alpha F^{\alpha\beta}(\Delta V)\partial_- z_\beta + \frac{1}{4}Y_{+\mu}(\Pi_+^{-1})^{\mu\nu}Y_{-\nu}\Big]. \tag{3.46}$$

Expanding above action we prove that it is identical to one given in (3.38).

4. Few Notes on Non-commutativity

In paper[13] it has been shown that bosonic T-duality produces non-commutative relations between bosonic T-dual coordinates. With this in mind, following question naturally arises: can we expect emergence of same behavior for fermionic coordinates after fermionic T-dualization? To get the answer for this question we have to express fermionic T-dual coordinates as some combination of starting coordinates and their momenta and connect T-dual Poisson brackets with Poisson brackets of original theory. Original theory is geometric theory with regular Poisson structure

$$\{x^\mu(\sigma), \pi_\nu(\bar{\sigma})\} = \delta_\nu^\mu \delta(\sigma - \bar{\sigma}),$$
$$\{\theta^\alpha(\sigma), \pi_\beta(\bar{\sigma})\} = \{\bar{\theta}^\alpha(\sigma), \bar{\pi}_\beta(\bar{\sigma})\} = \delta_\beta^\alpha \delta(\sigma - \bar{\sigma}), \tag{4.1}$$

where all other Poisson brackets vanish.

We start with case that has only been T-dualized along fermionic coordinates. To find how T-dual coordinates depend on starting ones and their momenta we can begin by finding fermionic momenta of starting theory. It is useful to remember that starting theory did not posses terms that are proportional to $\partial_+\theta^\alpha$ and $\partial_-\bar{\theta}^\alpha$ and that this symmetry was fixed with BRST formalism. Addition of gauge fixing term introduced modification to momenta of starting theory and to obtain correct non-commutative relations we should be working with theories that have gauge fixing term in them. With this in mind, it is easy to find fermionic momentum of original theory (3.13)

$$\pi_\beta = -\frac{k}{2}\Big[(\partial_+\bar{\theta}^\alpha + \partial_+ x^\mu \bar{\Psi}_\mu^\alpha)\big(F^{-1}(x)\big)_{\alpha\beta} - \partial_-\bar{\theta}^\alpha(\alpha^{-1})_{\alpha\beta}\Big], \tag{4.2}$$

$$\bar{\pi}_\alpha = \frac{k}{2}\Big[\big(F^{-1}(x)\big)_{\alpha\beta}(\partial_-\theta^\beta + \Psi_\nu^\beta \partial_- x^\nu) - (\alpha^{-1})_{\alpha\beta}\partial_+\theta^\beta\Big]. \tag{4.3}$$

Since we want to obtain Poisson brackets for equal τ we want to find σ partial derivatives of dual coordinate

$$\partial_\sigma z_\alpha = \partial_+ z_\alpha - \partial_- z_\alpha = \frac{2}{k}\bar{\pi}_\alpha, \tag{4.4}$$

$$\partial_\sigma \bar{z}_\alpha = \partial_+ \bar{z}_\alpha - \partial_- \bar{z}_\alpha = -\frac{2}{k}\pi_\alpha. \tag{4.5}$$

Momenta of original theory commute with each other and with x^μ coordinates, therefore we deduce that there has been no change to geometric structure of this theory.

For fully dualized theory, transformation laws (3.33) (3.34) (3.35) all depend on dual bosonic coordinate however, when we insert transformation laws that connect original bosonic coordinates with T-dual ones (more details in[13])

$$\partial_+ y_\mu = 2\Big[\partial_+ x^\nu \bar{\Pi}_{+\nu\mu} + \beta_\mu^-(x)\Big] + \partial_+\bar{\theta}^\alpha\big(F^{-1}(x)\big)_{\alpha\beta}\Psi_\mu^\beta, \tag{4.6}$$

$$\partial_- y_\nu = -2\Big[\bar{\Pi}_{+\mu\nu}\partial_- x^\nu + \beta_\mu^+(x)\Big] - \bar{\Psi}_\nu^\alpha\big(F^{-1}(x)\big)_{\alpha\beta}\partial_-\theta^\beta, \tag{4.7}$$

into transformation laws for fermionic coordinates (3.33), (3.34) and (3.35) we again obtain relations (4.4) and (4.5).

On a first glance it would seem that fermionic T-duality has not produced any new Poisson brackets, however this is not the case. While it is true that there are no modifications to Poisson brackets between fermions, we have new Poisson bracket structure between fermions and bosons. This can be seen from σ derivative of bosonic T-dual coordinate

$$y'_\mu \cong \frac{\pi_\mu}{\kappa} + \beta_\mu^0(x), \tag{4.8}$$

where $\beta_\mu^0(x)$ is combination $\beta_\mu^+(x) + \beta_\mu^-(x)$ given as

$$\beta_\mu^0(x) = \frac{1}{2}\partial_\sigma\Big[\bar{\theta}^\alpha + x^{\nu_1}\bar{\Psi}_{\nu_1}^\alpha\Big](f^{-1})_{\alpha\alpha_1}C_\mu^{\alpha_1\beta_1}(f^{-1})_{\beta_1\beta}\Big[\theta^\beta + \Psi_{\nu_2}^\beta x^{\nu_2}\Big] - \frac{1}{2}\Big[\bar{\theta}^\alpha + x^{\nu_1}\bar{\Psi}_{\nu_1}^\alpha\Big](f^{-1})_{\alpha\alpha_1}C_\mu^{\alpha_1\beta_1}(f^{-1})_{\beta_1\beta}\partial_\sigma\Big[\theta^\beta + \Psi_{\nu_2}^\beta x^{\nu_2}\Big]. \tag{4.9}$$

Finding Poisson brackets between σ derivatives of coordinates and integrating twice we obtain following relations

$$\{y_\mu(\sigma), \bar{z}_\beta(\bar{\sigma})\} = \frac{1}{k}\left[\bar{\theta}^\alpha(\sigma) + x^{\nu_1}(\sigma)\bar{\Psi}^\alpha_{\nu_1} - 2\left(\bar{\theta}^\alpha(\bar{\sigma}) + x^{\nu_1}(\bar{\sigma})\bar{\Psi}^\alpha_{\nu_1}\right)\right] \times (f^{-1})_{\alpha\alpha_1} C_\mu^{\alpha_1\beta_1}(f^{-1})_{\beta_1\beta} H(\sigma - \bar{\sigma}), \tag{4.10}$$

$$\{y_\mu(\sigma), z_\alpha(\bar{\sigma})\} = \frac{1}{k}(f^{-1})_{\alpha\alpha_1} C_\mu^{\alpha_1\beta_1}(f^{-1})_{\beta_1\beta} \times \left[\theta^\beta(\sigma) + \Psi^\beta_{\nu_2} x^{\nu_2}(\sigma) - 2\left(\theta^\beta(\bar{\sigma}) + \Psi^\beta_{\nu_2} x^{\nu_2}(\bar{\sigma})\right)\right] \times H(\sigma - \bar{\sigma}). \tag{4.11}$$

5. Conclusions

In this article we examined effects of fermionic T-duality performed on action of type II superstring in pure spinor formalism. We carried out our investigation in two cases, one where we performed fermionic T-duality on previously non dualized action and in second case where we had action that was already dualized along bosonic coordinates. Starting (non dualized) action that we worked with described closed string that propagates in presence of Ramond-Ramond field with linear coordinate dependence. We made a decision to only consider dependence on bosonic coordinates, furthermore this dependence was tied to infinitesimal antisymmetric term $C_\mu^{\alpha\beta}$. Rest of the background fields were held constant. Terms in action that were non-linearly dependent on fermionic coordinates were neglected. These choices were in accordance with consistency conditions for background fields and were made in order to keep calculations manageable.

On the other hand, bosonic T-duality of starting action provided us with theory that was non-local. Unlike starting theory that only dependent on coordinates through RR field, this theory manifested coordinate dependence on all background fields. Furthermore, bosonic T-dual coordinates now exhibit noncommutative properties.

Before we could start with T-dualization we noticed that both cases posses additional local symmetry which removed terms proportional to $\partial_+\theta^\alpha$ and $\partial_-\bar{\theta}^\alpha$. In order to obtain correct T-dual theory this symmetry was fixed through BRST formalism. In both cases procedure for obtaining fermionic T-duality was the same, we employed Buscher T-dualizing procedure. Procedure is based on localization of translational symmetry where we replace partial derivatives with covariant ones. Introduction of covariant derivatives carries with itself new degrees of freedom in shape of gauge fields. By demanding that starting and T-dual theory give description of same physical system we inevitably demand for both theories to posses same degrees of freedom. Thus, all additional degrees of freedom must be removed with Lagrange multipliers. By utilizing gauge freedom we can also remove all instances of fermionic coordinates in action obtaining action that is only a function of gaguge fields and Lagrange multipliers. Finding equations of motion for gauge fields of this gauge fixed action and inserting their solutions into the action we obtain T-dual theory.

Carrying Buscher procedure for fermionic coordinates of non dualized action we obtain local theory where all fields depend on bosonic coordinates. This theory is commutative, its Poisson brackets are identical to Poisson brackets of starting theory.

Buscher procedure in case of theory that has been dualizied along bosonic coordinates does not change coordinate dependence of the background fields. All fields are still dependent on both bosonic and fermionic coordinates and theory is still non-local. However, this theory posseses two additional non-trivial Poisson brackets. We have emergence of non-commutativity between bosonic and fermionic coordinates, where non-commutativity is proportional to infinitesimal constant $C_\mu^{\alpha\beta}$.

Same result is obtained even in case where we first perform fermionic and then bosonic T-duality. Commutativity between different dualities was expected since fully T-dual theory must be unique. Only distinction between different paths of T-dualization procedures can be noticed in intermediate theories, where most important change is transition of theory from being local to nonlocal.

We suspect that it is possible to obtain T-dual theory that is fully non-commutative, theory that has non-commutativity even between fermionic coordinates, but we would need starting theory that has background fields that depend on both bosonic and fermionic coordinates.

Acknowledgements

Work supported in part by the Serbian Ministry of Education,Science and Technological Development.

Conflict of Interest

The authors have declared no conflict of interest.

Keywords

T-duality, Buscher procedure, non-commutativity, fermionic T-duality, string theory

Received: September 30, 2022

[1] K. Becker, M. Becker, J. Schwarz, *String Theory and M-Theory: A Modern Introduction*, Cambridge University Press, **2015**.
[2] E. Alvarez, L. Alvarez-Gaume, Y. Lozano, *Nucl. Phys. Proc. Suppl.* **1995**, *41*, 1.
[3] N. Berkovits, J. Maldacena, *JHEP* **2008**, *09*, 062.
[4] T. H. Buscher, *Phys. Lett. B* **1987**, *194*, 59.
[5] T. H. Buscher, *Phys. Lett. B* **1988**, *201*, 466.
[6] I. Bakhmatov, D. S. Berman, *Nucl. Phys. B* **2010**, *832*, 89.
[7] K. Sfetsos, K. Siampos, D. C. Thompson, *Class. Quantum Grav.* **2011**, *28*, 05.
[8] N. Beisert, R. Ricci, A. Tseytlin, M. Wolf, *Phys. Rev. D* **2008**, *78*, 126004.
[9] B. Nikolić, B. Sazdović, *Nucl. Phys. B* **2017**, *917*, 105.
[10] Lj. Davidovivć, B. Sazdović, *Eur. Phys. J. C* **2014**, *74*, 2683.
[11] Lj. Davidovivć, B. Nikolić, B. Sazdović, *Eur. Phys. J. C* **2015**, *75*, 576.
[12] Lj. Davidovivć, B. Nikolić, B. Sazdović, *Eur. Phys. J. C* **2014**, *74*, 2734.
[13] B. Nikolić, D. Obrić, B. Sazdović, *Fortschritte der Physik* **2022**, *70*, 2200048.

[14] B. Nikolić, D. Obrić, *JHEP* **2019**, *03*, 136.
[15] P. A. Grassi, L. Tamassia, *JHEP* **2004**, *07*, 071.
[16] N. Berkovits, P. Howe, *Nucl. Phys. B* **2002**, *635*, 75.
[17] P. A. Grassi, G. Policastro, P. van Nieuwenhuizen, *JHEP* **2002**, *11*, 004.
[18] P. A. Grassi, G. Policastro, P. van Nieuwenhuizen, *Adv. Theor. Math. Phys.* **2003**, *7*, 499.
[19] P. A. Grassi, G. Policastro, P. van Nieuwenhuizen, *Phys. Lett. B* **2003**, *553*, 96.

Noncommutativity and Nonassociativity of Type II Superstring with Coordinate Dependent RR Field

B. Nikolić, D. Obrić, and B. Sazdović*

In this paper we will consider noncommutativity that arises from bosonic T-dualization of type II superstring in presence of Ramond-Ramond (RR) field, which linearly depends on the bosonic coordinates x^μ. The derivative of the RR field $C^{\alpha\beta}_\mu$ is infinitesimal. We will employ generalized Buscher procedure that can be applied to cases that have coordinate dependent background fields. Bosonic part of newly obtained T-dual theory is non-local. It is defined in non-geometric space spanned by Lagrange multipliers y_μ. We will apply generalized Buscher procedure once more on T-dual theory and prove that original theory can be salvaged. Finally, we will use T-dual transformation laws along with Poisson brackets of original theory to derive Poisson bracket structure of T-dual theory and nonassociativity relation. Noncommutativity parameter depends on the supercoordinates x^μ, θ^α and $\bar\theta^\alpha$, while nonassociativity parameter is a constant tensor containing infinitesimal $C^{\alpha\beta}_\mu$.

1. Introduction

In 1982 emerged a model[1] that would offer the possibility of obtaining bosonic coordinates of space-time as emergent properties of more fundamental fermionic coordinates. While this model worked with supersymmetric particle, this approach suggested that maybe we can express bosonic coordinates as a Poisson bracket of fermionic coordinates. In addition to these Poisson brackets, Poisson brackets between bosonic coordinates as well as between bosonic and fermionic coordinates would remain zero. Later, in paper,[2] it has been suggested that the same result can be obtained in context of string theory in case of coordinate dependent RR field. It is also suggested that unlike supersymmetric particle model, coordinate dependent RR field would produce full spectrum of non-commutative relations, bosonic coordinates themselves would become non-commutative. In this paper our goal is to determine if coordinate dependent RR field, while remaining background fields are as simple as consistency relations allow, can produce suggested non-commutative relations.

Superstring theory, as a theory of extended objects propagating in space-time, is defined in 10 dimensions.[3,4] In order to establish link between this mathematical model and real world observations, surplus space-like dimensions are compactified on circles of radius R. From this kind of geometry arises new kind of symmetry, T-duality, that links theories that have radii of compactification R with ones that have radii of compactification α'/R.[5,6] Existence of T-duality between different theories implies that those theories are physically equivalent and it gives us a way to explore how geometry and topology of one theory is connected to other. This connection between different geometries makes T-duality a useful tool in examining emergence of non-commutativity in context of closed strings.[7]

While in string theory both open and closed strings, under certain conditions, exhibit emergence of non-commutativity, mechanisms that enable this emergence are different. In case of open string, we have that endpoints of string that propagates in presence of constant metric and Kalb-Ramond field become non-commutative.[8] Basic idea of open string non-commutativity is that initial coordinates can be expressed as linear combination of effective coordinates and momenta by employing boundary conditions. In case of closed string, we do not have string endpoints therefore we don't have emergence of non-commutativity when string propagates in presence of constant background fields. In order to achieve same effect as in case of open string, we have to use coordinate dependent background fields. By finding T-dual of theories with this kind of geometry we obtain T-dual theory in non-geometric background, where T-dual coordinates are expressed as linear combinations of original coordinates and their conjugated momenta.

Mathematical framework for obtaining T-dual theories is standard Buscher procedure.[9,10] Procedure is based on existence of shift symmetry in relevant action and its implementation can be summarized in few steps. First step is localization of translational symmetry by introduction of covariant derivatives and introduction of Lagrange multipliers that make newly introduced gauge fields nonphysical. By gauge fixing and finding equations of motion of both gauge fields and Lagrange multipliers we obtain T-dual transformation laws. These transformation laws inserted into gauge fixed action produce T-dual action. For cases where we have coordinate dependent background fields there exist generalized Buscher procedure,[11–15] this extension has one additional step, replacement of all initial coordinates with invariant coordinates. Further extension of generalized Buscher procedure is possible[16] and it is applicable to theories that do not posses shift symmetry.

B. Nikolić, D. Obrić, B. Sazdović
Institute of Physics Belgrade
University of Belgrade
Pregrevica 118, Serbia
E-mail: dobric@ipb.ac.rs

DOI: 10.1002/prop.202200048

In this article we will deal with closed superstring propagating in presence of linearly coordinate dependent Ramond-Ramond (RR) field using type II superstring model in pure spinor formulation. All calculations we will do in approximation of diluted flux, which means that in all calculations we keep constant and linear terms in infinitesimal derivative of the RR field strength. Rest of the fields, metric, Kalb-Ramond and gravitino fields are constant. Furthermore all dependence of background field on fermionic terms will be neglected for mathematical simplicity. This choice of field configuration is in full accordance with consistency equations for background fields.[17]

Because we are currently only interested in non-commutative relations between bosonic coordinates, T-dualization procedure will be applied only on bosonic part of action. To find T-dual action and T-dual transformation laws we will employ extension of generalized Buscher procedure that works with coordinate dependent background fields.[11] After finding T-dual theory, we will apply Buscher procedure once more to see if we can obtain original theory.

Transformation laws that connect variables from initial with variables from T-dual theory will be written in canonical form, where initial momenta are expressed in terms of the T-dual coordinates. By inverting these transformation laws we obtain how sigma derivatives of T-dual theory depend on linear combinations of coordinates and momenta of original theory. Taking into account that original theory is geometrical, both locally and globally, we have that its coordinate and conjugated momenta satisfy standard Poisson brackets. By using this fact we are able to find Poisson structure of sigma derivatives of T-dual coordinates and by doing integration, Poisson structure of T-dual coordinates is obtained.

The form of obtained non-commutativity is such that non-commutativity exists when arguments are different, $\sigma \neq \bar{\sigma}$. Imposing trivial winding conditions, we obtain string winding numbers from Poisson brackets.

In the end, we give conclusions and in appendix we present some technical details regarding derivation of $\beta^{\pm}_{\mu}$ functions.

2. General Type II Superstring Action and Choice of Background Fields

Starting point of this investigation will be action of type II superstring theory in pure spinor formulation.[18–21] We will present and explain assumed approximations in order to obtain type II pure spinor action with non-constant RR field-strength. It turns out that ghost fields are neglected and only quadratic terms are considered. Final form of this kind of action will be used in subsequent sections.

2.1. General Form of the Pure Spinor Type II Superstring Action

Sigma model of type IIB superstring has the following form[17]

$$S = S_0 + V_{SG}. \tag{2.1}$$

This general form of action is expressed as a sum of the part that describes the motion of string in flat background

$$S_0 = \int_\Sigma d^2\xi \left(\frac{\kappa}{2}\eta_{\mu\nu}\partial_m x^\mu \partial_n x^\nu \eta^{mn} - \pi_\alpha \partial_- \theta^\alpha + \partial_+ \bar{\theta}^\alpha \bar{\pi}_\alpha \right) + S_\lambda + S_{\bar{\lambda}}\,, \tag{2.2}$$

and part that governs the modifications to the background fields

$$V_{SG} = \int_\Sigma d^2\xi (X^T)^M A_{MN} \bar{X}^N. \tag{2.3}$$

Modifications to the flat background are introduced by integrated form of massless type II supergravity vertex operator V_{SG}. The terms S_λ and $S_{\bar{\lambda}}$ in (2.2) are free-field actions for pure spinors

$$S_\lambda = \int_\Sigma d^2\xi \omega_\alpha \partial_- \lambda^\alpha, \quad S_{\bar{\lambda}} = \int_\Sigma d^2\xi \bar{\omega}_\alpha \partial_+ \bar{\lambda}^\alpha\,. \tag{2.4}$$

Here, λ^α and $\bar{\lambda}^\alpha$ are pure spinors whose canonically conjugated momenta are ω_α and $\bar{\omega}_\alpha$, respectively. Pure spinors satisfy pure spinor constraints

$$\lambda^\alpha (\Gamma^\mu)_{\alpha\beta} \lambda^\beta = \bar{\lambda} (\Gamma^\mu)_{\alpha\beta} \bar{\lambda}^\beta = 0\,. \tag{2.5}$$

In general case, vectors X^M and X^N as well as a supermatrix A_{MN} are given by

$$X^M = \begin{pmatrix} \partial_+\theta^\alpha \\ \Pi^\mu_+ \\ d_\alpha \\ \frac{1}{2}N^{\mu\nu}_+ \end{pmatrix}, \quad \bar{X}^M = \begin{pmatrix} \partial_-\bar{\theta}^\lambda \\ \Pi^\mu_- \\ \bar{d}_\lambda \\ \frac{1}{2}\bar{N}^{\mu\nu}_- \end{pmatrix},$$

$$A_{MN} = \begin{bmatrix} A_{\alpha\beta} & A_{\alpha\nu} & E_\alpha{}^\beta & \Omega_{\alpha,\mu\nu} \\ A_{\mu\beta} & A_{\mu\nu} & \bar{E}^\beta_\mu & \Omega_{\mu,\nu\rho} \\ E^\alpha{}_\beta & E^\alpha_\nu & P^{\alpha\beta} & C^\alpha{}_{\mu\nu} \\ \Omega_{\mu\nu,\beta} & \Omega_{\mu\nu,\rho} & \bar{C}^\beta{}_{\mu\nu} & S_{\mu\nu,\rho\sigma} \end{bmatrix}, \tag{2.6}$$

where notation is in accordance with Ref [17]. The components of matrix A_{MN} are generally functions of x^μ, θ^α and $\bar{\theta}^\alpha$. Components themselves are derived as expansions in powers of θ^α and $\bar{\theta}^\alpha$ (for details consult[17]). The superfields $A_{\mu\nu}$, $\bar{E}^\alpha_\mu$, E^α_μ and $P^{\alpha\beta}$ are known as physical superfields, while superfields that are in the first row and the first column are known as auxiliary because they can be expressed in terms of physical ones.[17] Remaining superfields $\Omega_{\mu,\nu\rho}$ ($\Omega_{\mu\nu,\rho}$), $C^\alpha{}_{\mu\nu}$ ($\bar{C}^\beta{}_{\mu\nu}$) and $S_{\mu\nu,\rho\sigma}$, are curvatures (field strengths) for physical fields. Components of vectors X^M and $\bar{X}^N$ are defined as

$$\Pi^\mu_+ = \partial_+ x^\mu + \frac{1}{2}\theta^\alpha (\Gamma^\mu)_{\alpha\beta}\partial_+\theta^\beta, \quad \Pi^\mu_- = \partial_- x^\mu + \frac{1}{2}\bar{\theta}^\alpha (\Gamma^\mu)_{\alpha\beta}\partial_-\bar{\theta}^\beta, \tag{2.7}$$

$$d_\alpha = \pi_\alpha - \frac{1}{2}(\Gamma_\mu \theta)_\alpha \left[\partial_+ x^\mu + \frac{1}{4}(\theta\Gamma^\mu \partial_+\theta)\right],$$

$$\bar{d}_\alpha = \bar{\pi}_\alpha - \frac{1}{2}(\Gamma_\mu \bar{\theta})_\alpha \left[\partial_- x^\mu + \frac{1}{4}(\bar{\theta}\Gamma^\mu \partial_-\bar{\theta})\right], \tag{2.8}$$

$$N_+^{\mu\nu} = \frac{1}{2}\omega_\alpha (\Gamma^{[\mu\nu]})^\alpha{}_\beta \lambda^\beta, \quad \bar{N}_-^{\mu\nu} = \frac{1}{2}\bar{\omega}_\alpha (\Gamma^{[\mu\nu]})^\alpha{}_\beta \bar{\lambda}^\beta. \tag{2.9}$$

The world sheet Σ is parameterized by $\xi^m = (\xi^0 = \tau, \xi^1 = \sigma)$ and world sheet light-cone partial derivatives are defines as $\partial_\pm = \partial_\tau \pm \partial_\sigma$. Superspace in which string propagates is spanned both by bosonic x^μ $(\mu = 0, 1, \dots, 9)$ and fermionic θ^α, $\bar{\theta}^\alpha$ $(\alpha = 1, 2, \dots, 16)$ coordinates. Variables π_α and $\bar{\pi}_\alpha$ represent canonically conjugated momenta of fermionic coordinates θ^α and $\bar{\theta}^\alpha$, respectively. Fermionic coordinates and their canonically conjugated momenta are Majorana-Weyl spinors. It means that each of these spinors has 16 independent real valued components.

2.2. Choice of the Background Fields

In this particular case we will work with the supermatrix A_{MN} where all background fields, except RR field strength $P^{\alpha\beta}$, are constants. RR field strength will have linear coordinate dependence on bosonic coordinate x^μ. With these restrictions in mind, supermatrix A_{MN} has the following form

$$A_{MN} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \kappa\left(\frac{1}{2}g_{\mu\nu} + B_{\mu\nu}\right) & \bar{\Psi}_\mu^\beta & 0 \\ 0 & -\Psi_\nu^\alpha & \frac{2}{\kappa}(f^{\alpha\beta} + C_\rho^{\alpha\beta}x^\rho) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \tag{2.10}$$

Here $g_{\mu\nu}$ is symmetric tensor, $B_{\mu\nu}$ is Kalb-Ramond antisymmetric field, Ψ_μ^α and $\bar{\Psi}_\mu^\alpha$ are Mayorana-Weyl gravitino fields, and finally, $f^{\alpha\beta}$ and $C_\rho^{\alpha\beta}$ are constants. Let us stress that dilaton field Φ is assumed to be constant, so, the factor e^Φ is included in $f^{\alpha\beta}$ and $C_\rho^{\alpha\beta}$. This will be a classical analysis and we will not calculate the dilaton shift under T-duality transformation. Based on the chirality of spinors, there are type IIA superstring theory for opposite chirality and type IIB superstring theory for same chirality.

This particular choice of supermatrix imposes following restriction on background fields

$$\gamma_{\alpha\beta}^\mu C_\mu^{\beta\gamma} = 0, \quad \gamma_{\alpha\beta}^\mu C_\mu^{\gamma\beta} = 0. \tag{2.11}$$

Remaining constraints[17] are trivial and applied only to non-physical fields.

In addition to choice of supermatrix, in order to simplify calculation of bosonic T-duality, because all background fields are expanded in powers of θ^α and $\bar{\theta}^\alpha$, all θ^α and $\bar{\theta}^\alpha$ non-linear terms in X^M and $\bar{X}^N$ will be neglected. With this in mind, components of these two vectors reduce into the following form

$$\Pi_\pm^\mu \to \partial_\pm x^\mu, \quad d_\alpha \to \pi_\alpha, \quad \bar{d}_\alpha \to \bar{\pi}_\alpha. \tag{2.12}$$

Taking into account all these assumptions, the action (2.1) takes the form

$$S = \int_\Sigma d^2\xi \left[\frac{\kappa}{2}\Pi_{+\mu\nu}\partial_+ x^\mu \partial_- x^\nu - \pi_\alpha(\partial_-\theta^\alpha + \Psi_\nu^\alpha \partial_- x^\nu) \right.$$
$$\left. + (\partial_+\bar{\theta}^\alpha + \partial_+ x^\mu \bar{\Psi}_\mu^\alpha)\bar{\pi}_\alpha + \frac{2}{\kappa}\pi_\alpha(f^{\alpha\beta} + C_\rho^{\alpha\beta}x^\rho)\bar{\pi}_\beta\right]. \tag{2.13}$$

Here, new tensor $\Pi_{\pm\mu\nu} = B_{\mu\nu} \pm \frac{1}{2}G_{\mu\nu}$ is introduced, where $G_{\mu\nu} = \eta_{\mu\nu} + g_{\mu\nu}$ is metric tensor. Terms for S_λ and $S_{\bar{\lambda}}$ are fully decoupled from action and they will not be considered from now on.

Before considering T-duality, we can notice that fermionic momenta act as auxiliary fields in full actions. These fields can be integrated out and final action will be function of only coordinates and their derivatives. Finding equations for motion for both π_α and $\bar{\pi}_\alpha$ we get following two equations

$$\bar{\pi}_\beta = \frac{\kappa}{2}\left(F^{-1}(x)\right)_{\beta\alpha}\left(\partial_-\theta^\alpha + \Psi_\nu^\alpha \partial_- x^\nu\right), \tag{2.14}$$

$$\pi_\alpha = -\frac{\kappa}{2}\left(\partial_+\bar{\theta}^\beta + \partial_+ x^\mu \bar{\Psi}_\mu^\beta\right)\left(F^{-1}(x)\right)_{\beta\alpha}, \tag{2.15}$$

where $F^{\alpha\beta}(x)$ and $(F^{-1}(x))_{\alpha\beta}$ are of the form

$$F^{\alpha\beta}(x) = f^{\alpha\beta} + C_\mu^{\alpha\beta}x^\mu,$$
$$(F^{-1}(x))_{\alpha\beta} = (f^{-1})_{\alpha\beta} - (f^{-1})_{\alpha\alpha_1}C_\mu^{\alpha_1\beta_1}(f^{-1})_{\beta_1\beta}\, x^\mu. \tag{2.16}$$

In order to invert previous equations and T-dual transformation laws, as well as to simplify calculations, we take two additional assumptions. First assumption is that $C_\mu^{\alpha\beta}$ is infinitesimal. Second assumption is that $(f^{-1})_{\alpha\alpha_1}C_\mu^{\alpha_1\beta_1}(f^{-1})_{\beta_1\beta}$ is antisymmetric under exchange of first and last index. In other words, tensor $(F^{-1}(x))_{\alpha\beta}$ has only antysimetric part that depends on x^μ and it is infinitesimal. These assumptions are in full accordance with constraints.[17]

Substituting equations (2.14) and (2.15) into (2.13) the final form of action is

$$S = \kappa \int_\Sigma d^2\xi \left[\Pi_{+\mu\nu}\partial_+ x^\mu \partial_- x^\nu \right.$$
$$\left. + \frac{1}{2}(\partial_+\bar{\theta}^\alpha + \partial_+ x^\mu \bar{\Psi}_\mu^\alpha)\left(F^{-1}(x)\right)_{\alpha\beta}(\partial_-\theta^\beta + \Psi_\nu^\beta \partial_- x^\nu)\right]. \tag{2.17}$$

In the following sections, this form of action will be used for investigation of bosonic T-duality and for obtaining transformation laws between starting and T-dual coordinates.

3. T-dualization

In this section T-duality will be performed along all bosonic coordinates in order to find relations that connect T-dual coordinates with coordinates and momenta of original theory. These transformation laws will then be used in subsequent chapters to find non-commutativity relations between coordinates of T-dual theory.

Starting point for considering T-duality will be generalized Buscher T-dualization procedure.[11] Standard Buscher procedure[9,10] is designed to be applied along isometry directions on which background fields do not depend. Generalized Buscher procedure can be applied to theories with coordinate dependent background fields. The shift symmetry in the generalized procedure is localized by introduction of covariant derivatives, invariant coordinates and additional gauge fields. These newly introduced gauge fields produce additional degrees of freedom. Since we expect that starting and T-dual theory have exactly the same number of degrees of freedom we need to eliminate all excessive degrees of freedom. This is accomplished by demanding that field strength of gauge fields ($F_{+-} = \partial_+ v_- - \partial_- v_+$) vanishes by addition of Lagrange multipliers. Next step in procedure is fixing the gauge symmetry such that starting coordinates are constant and action is only left with gauge fields and its derivatives. From this gauge fixed action, finding equations of motion for gauge fields, expressing gauge fields as function of Lagrange multipliers and inserting those equations into action we can obtain T-dual action, were Lagrange multipliers of original theory now play the role of T-dual coordinates.

In cases where shift symmetry is absent, T-duality can still be performed by extending generalized Buscher procedure.[16] This extension is based on replacing original action with translation invariant auxiliary action. Form of this auxiliary action is exactly the same as the form of action where translation symmetry was localized and gauged fixed, that is, derivatives have been replaced with gauge fields and coordinates with integrals of gauge fields. Auxiliary action gives correct T-dual theory only if original action can be salvaged from it. In cases where this is possible, original theory is obtained by finding equations of motion with respect to Lagrange multipliers and inserting their solutions into auxiliary action.

Action (2.17) is invariant to translation symmetry, by the virtue of antisymmetric part of $F^{-1}_{\alpha\beta}$, tensor $(f^{-1}C_\mu f^{-1})_{\alpha\beta}$. Following antisymmetricity of this tensor, we can rewrite the action (2.17) in the following way

$$S = \kappa \int_\Sigma d^2\xi \Big[\Pi_{+\mu\nu}\partial_+ x^\mu \partial_- x^\nu + \frac{1}{2}\epsilon^{mn}\partial_m(\bar\theta^\alpha + x^\mu \bar\Psi^\alpha_\mu)\big(F^{-1}(x)\big)_{\alpha\beta}\partial_n(\theta^\beta + \Psi^\beta_\nu x^\nu)\Big]. \tag{3.1}$$

Let us now consider the global shift symmetry $\delta x^\mu = \lambda^\mu$ and vary the action (3.1)

$$\delta S = -\frac{\kappa}{2}(f^{-1}C_\mu f^{-1})_{\alpha\beta}\lambda^\mu \int_\Sigma d^2\xi \epsilon^{mn}\partial_m(\bar\theta^\alpha + \bar\Psi^\alpha_\nu x^\nu)\partial_n(\theta^\beta + \Psi^\beta_\rho x^\rho), \tag{3.2}$$

where m, n are indices of the twodimensional worldsheet. After one partial integration, we obtain one surface term and one term which is identically zero because it is summation of symmetric, $\partial_m\partial_n$, and antisymmetric, ϵ^{mn}, tensor. The surface term is zero for trivial topology. So, the shift isometry exists.

In order to find T-dual action we have to implement following substitutions

$$\partial_\pm x^\mu \to D_\pm x^\mu = \partial_\pm x^\mu + v^\mu_\pm, \tag{3.3}$$

$$x^\rho \to x^\mu_{inv} = \int_P d\xi^m D_m x^\mu = x^\mu(\xi) - x^\mu(\xi_0) + \Delta V^\mu, \quad \Delta V^\mu = \int_P d\xi^m v^\mu_m(\xi), \tag{3.4}$$

$$S \to S + \frac{\kappa}{2}\int_\Sigma d^2\xi\big[v^\mu_+\partial_- y_\mu - v^\mu_-\partial_+ y_\mu\big]. \tag{3.5}$$

Because of the shift symmetry we fix the gauge, $x^\mu(\xi) = x^\mu(\xi_0)$ and, inserting these substitutions into action (2.17), we obtain auxiliary action suitable for T-dualization

$$S_{aux} = \kappa \int_\Sigma d^2\xi \Big[\Pi_{+\mu\nu}v^\mu_+ v^\nu_- + \frac{1}{2}(\partial_+\bar\theta^\alpha + v^\mu_+\bar\Psi^\alpha_\mu)\big(F^{-1}(\Delta V)\big)_{\alpha\beta} \times (\partial_-\theta^\beta + \Psi^\beta_\nu v^\nu_-) + \frac{1}{2}(v^\mu_+\partial_- y_\mu - v^\mu_-\partial_+ y_\mu)\Big]. \tag{3.6}$$

It should be noted that, path P that is taken in expression for ΔV^ρ goes from some starting point ξ_0 to end point ξ. Introduction of this element makes this action non-local, however, this is a necessary step in order to find T-dual theory of coordinate dependent background fields.[11]

In order to check if substitutions we had introduced are valid and that they will lead to correct T-dual theory of starting action, we need to be able to obtain original action by finding solutions to equations of motion for Lagrange multipliers. Equations of motion for Lagrange multipliers give us

$$\partial_- v^\mu_+ - \partial_+ v^\mu_- = 0 \quad \Rightarrow \quad v^\mu_\pm = \partial_\pm x^\mu. \tag{3.7}$$

Inserting this result into (3.4) we get the following

$$\Delta V^\rho = \int_P d\xi'^m \partial_m x^\rho(\xi') = x^\rho(\xi) - x^\rho(\xi_0) = \Delta x^\rho. \tag{3.8}$$

Since, we had shift symmetry in original action, we can let $x^\rho(\xi_0)$ be any arbitrary constant. Taking all this into account and inserting (3.7), (3.8) into (3.6) we obtain our starting action (2.17).

Before we obtain equations for motion for gauge fields, we would like to make following substitution in action

$$Y_{+\mu} = \partial_+ y_\mu - \partial_+\bar\theta^\alpha\big(F^{-1}(\Delta V)\big)_{\alpha\beta}\Psi^\beta_\mu, \quad Y_{-\mu} = \partial_- y_\mu + \bar\Psi^\alpha_\mu\big(F^{-1}(\Delta V)\big)_{\alpha\beta}\partial_-\theta^\beta, \tag{3.9}$$

$$\bar\Pi_{+\mu\nu} = \Pi_{+\mu\nu} + \frac{1}{2}\bar\Psi^\alpha_\mu\big(F^{-1}(\Delta V)\big)_{\alpha\beta}\Psi^\beta_\nu = \breve\Pi_{+\mu\nu} - \frac{1}{2}\bar\Psi^\alpha_\mu(f^{-1})_{\alpha\alpha_1}C^{\alpha_1\beta_1}_\rho(f^{-1})_{\beta_1\beta}\Psi^\beta_\nu\Delta V^\rho, \tag{3.10}$$

$$\breve\Pi_{+\mu\nu} \equiv \Pi_{+\mu\nu} + \frac{1}{2}\bar\Psi^\alpha_\mu(f^{-1})_{\alpha\beta}\Psi^\beta_\nu. \tag{3.11}$$

With these substitutions in mind we have that auxiliary action takes the following form

$$S_{aux} = \kappa \int_\Sigma d^2\xi \Big[\bar\Pi_{+\mu\nu}v^\mu_+ v^\nu_- + \frac{1}{2}v^\mu_+ Y_{-\mu} - \frac{1}{2}v^\mu_- Y_{+\mu} + \frac{1}{2}\partial_+\bar\theta^\alpha\big(F^{-1}(\Delta V)\big)_{\alpha\beta}\partial_-\theta^\beta\Big]. \tag{3.12}$$

This action produces following equations of motion for gauge fields

$$\bar\Pi_{+\mu\nu}v^\nu_- = -(\frac{1}{2}Y_{-\mu} + \beta^+_\mu(V)), \qquad \bar\Pi_{+\mu\nu}v^\mu_+ = \frac{1}{2}Y_{+\nu} - \beta^-_\nu(V). \tag{3.13}$$

Here, function $\beta^\pm(V)$ is obtained from variation of term containing ΔV^ρ in expression for $F^{-1}(\Delta V)$ (details are presented in

Appendix A)

$$\beta^-_\mu(V) = \frac{1}{4}\partial_+\left[\bar\theta^\alpha + V^{\nu_1}\bar\Psi^\alpha_{\nu_1}\right](f^{-1})_{\alpha\alpha_1}C^{\alpha_1\beta_1}_\mu(f^{-1})_{\beta_1\beta}\left[\theta^\beta + \Psi^\beta_{\nu_2}V^{\nu_2}\right] - \frac{1}{4}\left[\bar\theta^\alpha + V^{\nu_1}\bar\Psi^\alpha_{\nu_1}\right](f^{-1})_{\alpha\alpha_1}C^{\alpha_1\beta_1}_\mu(f^{-1})_{\beta_1\beta}\partial_+\left[\theta^\beta + \Psi^\beta_{\nu_2}V^{\nu_2}\right], \tag{3.14}$$

$$\beta^+_\mu(V) = \frac{1}{4}\left[\bar\theta^\alpha + V^{\nu_1}\bar\Psi^\alpha_{\nu_1}\right](f^{-1})_{\alpha\alpha_1}C^{\alpha_1\beta_1}_\mu(f^{-1})_{\beta_1\beta}\partial_-\left[\theta^\beta + \Psi^\beta_{\nu_2}V^{\nu_2}\right] - \frac{1}{4}\partial_-\left[\bar\theta^\alpha + V^{\nu_1}\bar\Psi^\alpha_{\nu_1}\right](f^{-1})_{\alpha\alpha_1}C^{\alpha_1\beta_1}_\mu(f^{-1})_{\beta_1\beta}\left[\theta^\beta + \Psi^\beta_{\nu_2}V^{\nu_2}\right]. \tag{3.15}$$

Here we have took advantage of the fact that $\partial_\pm V^\mu = v^\mu_\pm$ (more details in Appendix A). Let us note that V^μ in the expressions for beta functions is actually $V^{(0)\mu}$ because it stands besides $C^{\alpha\beta}_\mu$. We omit index (0) just in order to simplify the form of the expressions.

In order to find how gauge fields depend on Lagrange multipliers, we need to invert equations of motion (3.13). Since $C^{\alpha\beta}_\mu$ is an infinitesimal constant, these equations can be inverted iteratively.[22] We separate variables into two parts, one finite and one proportional to $C^{\alpha\beta}_\mu$. After doing this we have

$$v^\nu_- = -\bar\Theta^{\nu\mu}_-\left[\frac{1}{2}Y_{-\mu} + \beta^+_\mu(V^{(0)})\right], \qquad v^\mu_+ = \left[\frac{1}{2}Y_{+\nu} - \beta^-_\nu(V^{(0)})\right]\bar\Theta^{\nu\mu}_-. \tag{3.16}$$

Functions $\beta_{\pm\mu}(V^{(0)})$ are obtained by substituting first order of expression for $v_\pm$ into $\beta_{\pm\mu}(V)$, where $V^{(0)}$ is given by

$$\Delta V^{(0)\rho} = \int_P d\xi^m v^{(0)\rho}_m = \frac{1}{2}\int_P d\xi^+\check\Theta^{\rho_1\rho}_-\left[\partial_+ y_{\rho_1} - \partial_+\bar\theta^\alpha(f^{-1})_{\alpha\beta}\Psi^\beta_{\rho_1}\right] - \frac{1}{2}\int_P d\xi^-\check\Theta^{\rho\rho_1}_-\left[\partial_- y_{\rho_1} + \bar\Psi^\alpha_{\rho_1}(f^{-1})_{\alpha\beta}\partial_-\theta^\beta\right]. \tag{3.17}$$

Where $\bar\Theta^{\mu\nu}_-$ is inverse tensor of $\bar\Pi_{+\mu\nu} = \Pi_{+\mu\nu} + \frac{1}{2}\bar\Psi^\alpha_\mu(F^{-1}(\Delta V))_{\alpha\beta}\Psi^\beta_\nu$, defined as

$$\bar\Theta^{\mu\nu}_-\bar\Pi_{+\nu\rho} = \delta^\mu_\rho, \tag{3.18}$$

where

$$\bar\Theta^{\mu\nu}_- = \check\Theta^{\mu\nu}_- + \frac{1}{2}\check\Theta^{\mu\mu_1}_-\bar\Psi^\alpha_{\mu_1}(f^{-1})_{\alpha\alpha_1}C^{\alpha_1\beta_1}_\rho V^{(0)\rho}(f^{-1})_{\beta_1\beta}\Psi^{\beta_1}_{\nu_1}\check\Theta^{\nu_1\nu}_-, \tag{3.19}$$

$$\check\Theta^{\mu\nu}_-\check\Pi_{\nu\rho} = \delta^\mu_\rho, \qquad \check\Theta^{\mu\nu}_- = \Theta^{\mu\nu}_- - \frac{1}{2}\Theta^{\mu\mu_1}_-\bar\Psi^\alpha_{\mu_1}(\bar f^{-1})_{\alpha\beta}\Psi^\beta_{\nu_1}\Theta^{\nu_1\nu}_- \tag{3.20}$$

$$\bar f^{\alpha\beta} = f^{\alpha\beta} + \frac{1}{2}\Psi^\alpha_\mu\Theta^{\mu\nu}_-\bar\Psi^\beta_\nu, \tag{3.21}$$

$$\Theta^{\mu\nu}_-\Pi_{+\mu\rho} = \delta^\mu_\rho, \qquad \Theta_- = -4(G_E^{-1}\Pi_-G^{-1})^{\mu\nu}. \tag{3.22}$$

Tensor $G_{E\mu\nu} \equiv G_{\mu\nu} - 4(BG^{-1}B)_{\mu\nu}$ is known in the literature as the effective metric.

Inserting equations (3.16) into (3.6), keeping only terms that are linear in $C^{\alpha\beta}_\mu$ we obtain T-dual action

$$S^\star = \frac{\kappa}{2}\int_\Sigma\left[\frac{1}{2}\bar\Theta^{\mu\nu}_-Y_{+\mu}Y_{-\nu} + \partial_+\bar\theta^\alpha\left(F^{-1}(\Delta V)\right)_{\alpha\beta}\partial_-\theta^\beta\right]. \tag{3.23}$$

Comparing starting action (2.17) with T-dual action, were we note that $\partial_\pm x^\mu$ transforms into $\partial_\pm y_\mu$ and x^μ transforms into $V^{(0)}$, we can deduce that T-dual action has following arguments.

$${}^\star\bar\Pi^{\mu\nu}_+ = \frac{1}{4}\bar\Theta^{\mu\nu}_-, \tag{3.24}$$

$$\left({}^\star F^{-1}(V^{(0)})\right)_{\alpha\beta} = \left(F^{-1}(V^{(0)})\right)_{\alpha\beta} - \frac{1}{2}\left(F^{-1}(V^{(0)})\right)_{\alpha\alpha_1}\Psi^{\alpha_1}_\mu\bar\Theta^{\mu\nu}_-\bar\Psi^{\beta_1}_\nu\left(F^{-1}(V^{(0)})\right)_{\beta_1\beta}, \tag{3.25}$$

$${}^\star\bar\Psi^{\mu\alpha}\left({}^\star F^{-1}(V^{(0)})\right)_{\alpha\beta} = \frac{1}{2}\bar\Theta^{\mu\nu}_-\bar\Psi^\alpha_\nu\left(F^{-1}(V^{(0)})\right)_{\alpha\beta}, \tag{3.26}$$

$$\left({}^\star F^{-1}(V^{(0)})\right)_{\alpha\beta}{}^\star\Psi^{\nu\beta} = -\frac{1}{2}\left(F^{-1}(V^{(0)})\right)_{\alpha\beta}\Psi^\beta_\mu\bar\Theta^{\mu\nu}_-. \tag{3.27}$$

In order to express T-dual gravitino background fields in terms of its components, it is useful to calculate inverse of field ${}^\star F^{-1}_{\alpha\beta}$

$${}^\star F^{\alpha\beta}(V^{(0)}) = F^{\alpha\beta}(V^{(0)}) + \frac{1}{2}\Psi^\alpha_\mu\Theta^{\mu\nu}_-\bar\Psi^\beta_\nu. \tag{3.28}$$

With this equation at hand it is straightforward to obtain T-dual gravitino fields. Here we present T-dual gravitino fields expanded in terms of their components

$${}^\star\bar\Psi^{\mu\alpha} = \frac{1}{2}\bar\Theta^{\mu\nu}\bar\Psi^\alpha_\nu + \frac{1}{4}\bar\Theta^{\mu\mu_1}_-\bar\Psi^\beta_{\mu_1}\left(F^{-1}(V^{(0)})\right)_{\beta\beta_1}\Psi^{\beta_1}_\nu\Theta^{\nu\nu_1}_-\bar\Psi^\alpha_{\nu_1}, \tag{3.29}$$

$${}^\star\Psi^{\nu\beta} = -\frac{1}{2}\Psi^\beta_\mu\bar\Theta^{\mu\nu}_- - \frac{1}{2}\Psi^\beta_\mu\Theta^{\mu\mu_1}_-\bar\Psi^\alpha_{\mu_1}\left(F^{-1}(V^{(0)})\right)_{\alpha\alpha_1}\Psi^{\alpha_1}_{\nu_1}\bar\Theta^{\nu_1\nu}_-. \tag{3.30}$$

The general conclusion is that all background fields get the linear corrections in $C^{\alpha\beta}_\mu$ comparing with the results of the case with constant background fields.[23] Also the coordinate dependence is present in all T-dual background fields.

From the above equations we see how background fields of original theory transform under T-duality. It should be noted that these actions are of the same form taking into account that initial coordinates x^μ are replaced by y_μ after T-dualization.

4. T-dualization of T-dual Theory

From requirement that original theory and T-dual theory be physically equivalent, it should be possible to obtain original theory from T-dual one by applying T-duality procedure a second time. Since original action possessed translation symmetry, we have that this symmetry is inherited by T-dual action. T-dual theory is invariant to translations of T-dual coordinate. However, even in cases where starting action is not invariant to translation symmetry we can expect emergence of this symmetry in T-dual theory. This is a natural consequence of introducing $\Delta V^{(0)}$ and of the fact that T-dual theory is intrinsically a non-local one. T-dualization of T-dual theory is obtained with generalized Buscher procedure and steps are identical as before.

$$\partial_\pm y_\mu \to D_\pm y_\mu = \partial_\pm y_\mu + u_{\pm\mu} \to D_\pm y_\mu = u_{\pm\mu}, \tag{4.1}$$

$$\Delta V^{(0)\rho} \to \Delta U^{(0)\rho}, \tag{4.2}$$

$$\Delta U^{(0)\rho} = \frac{1}{2}\int_P d\xi^+ \check{\Theta}_-^{\rho_1\rho}\Big[u_{+\rho_1} - \partial_+\bar\theta^\alpha (f^{-1})_{\alpha\beta}\Psi^\beta_{\rho_1}\Big] - \frac{1}{2}\int_P d\xi^- \check{\Theta}_-^{\rho\rho_1}\Big[u_{-\rho_1} + \bar\Psi^\alpha_{\rho_1}(f^{-1})_{\alpha\beta}\partial_-\theta^\beta\Big], \tag{4.3}$$

$$Y_{+\mu} \to U_{+\mu} = u_{+\mu} - \partial_+\bar\theta^\alpha\big(F^{-1}(\Delta U^{(0)})\big)_{\alpha\beta}\Psi^\beta_\mu \tag{4.4}$$

$$Y_{-\mu} \to U_{-\mu} = u_{-\mu} + \bar\Psi^\alpha_\mu\big(F^{-1}(\Delta U^{(0)})\big)_{\alpha\beta}\partial_-\theta^\beta \tag{4.5}$$

$$^\star S \to {}^\star S + \frac{\kappa}{2}\int_\Sigma d^2\xi (u_{+\mu}\partial_- x^\mu - u_{-\mu}\partial_+ x^\mu). \tag{4.6}$$

In first line we immediately fixed gauge by choosing $y(\xi) = const$. Inserting these substitutions into (3.23) we get

$$^\star S_{gfix} = \frac{k}{2}\int_\Sigma d^2\xi \Big[\frac{1}{2}\bar\Theta_-^{\mu\nu} U_{+\mu}U_{-\nu} + \partial_+\bar\theta^\alpha\big(F^{-1}(\Delta U^{(0)})\big)_{\alpha\beta}\partial_-\theta^\beta + (u_{+\mu}\partial_- x^\mu - u_{-\mu}\partial_+ x^\mu)\Big]. \tag{4.7}$$

Finding equations of motion for Lagrange multipliers and inserting solution to those equations into gauge fixed action we return to the starting point of this chapter, T-dual action. On the other hand, finding equations of motion for gauge fields

$$u_{+\mu} = 2\Big[\partial_+ x^\nu \bar\Pi_{+\nu\mu} + \beta^-_\mu(x)\Big] + \partial_+\bar\theta^\alpha\big(F^{-1}(x)\big)_{\alpha\beta}\Psi^\beta_\mu, \tag{4.8}$$

$$u_{-\nu} = -2\Big[\bar\Pi_{+\mu\nu}\partial_- x^\nu + \beta^+_\mu(x)\Big] - \bar\Psi^\alpha_\nu\big(F^{-1}(x)\big)_{\alpha\beta}\partial_-\theta^\beta, \tag{4.9}$$

and inserting these equations into the gauge fixed action, keeping all terms linear with respect to $C^{\mu\nu}_\rho$, we obtain our original action (2.17). Here we use the freedom to choose $\Delta x^\mu = x(\xi) - x(\xi_0)$, with $x(\xi_0) = 0$.

5. Non-commutative Relations

Having found T-dual action and equations that link T-dual coordinate with original coordinates in previous chapters, in this chapter we will focus on establishing a relationship between Poisson brackets of original and T-dual theory. Furthermore, we will mainly focus on Poisson brackets between bosonic variables and their momenta. Original theory is a geometric one with variables $x^\mu(\xi)$ and $\pi_\mu(\xi)$. Therefore, it is natural to impose standard Poisson structure on original theory

$$\{x^\mu(\sigma), \pi_\nu(\bar\sigma)\} = \delta^\mu_\nu \delta(\sigma - \bar\sigma), \quad \{x^\mu(\sigma), x^\nu(\bar\sigma)\} = 0, \quad \{\pi_\mu(\sigma), \pi_\nu(\bar\sigma)\} = 0. \tag{5.1}$$

In order to find Poisson brackets of T-dual theory, we need to find T-dual transformation laws which connect the initial and T-dual coordinates. Starting with relations (4.8) and (4.9) and using equations of motion for Lagrange multipliers x^μ, $u_{\pm\mu} = \partial_\pm y_\mu$, we obtain T-dual transformation laws

$$\partial_+ y_\mu \cong 2\Big[\partial_+ x^\nu \bar\Pi_{+\nu\mu} + \beta^-_\mu(x)\Big] + \partial_+\bar\theta^\alpha\big(F^{-1}(x)\big)_{\alpha\beta}\Psi^\beta_\mu, \tag{5.2}$$

$$\partial_- y_\mu \cong -2\Big[\bar\Pi_{+\mu\nu}\partial_- x^\nu + \beta^+_\mu(x)\Big] - \bar\Psi^\alpha_\nu\big(F^{-1}(x)\big)_{\alpha\beta}\partial_-\theta^\beta, \tag{5.3}$$

where symbol $\cong$ denotes T-dual transformation. Subtracting these two equations, we get

$$y'_\mu \cong \bar\Pi_{+\mu\nu}\partial_- x^\nu + \partial_+ x^\nu \bar\Pi_{+\nu\mu} + \beta^+_\mu + \beta^-_\mu + \frac{1}{2}\partial_+\bar\theta^\alpha\big(F^{-1}(x)\big)_{\alpha\beta}\Psi^\beta_\mu + \frac{1}{2}\bar\Psi^\alpha_\nu\big(F^{-1}(x)\big)_{\alpha\beta}\partial_-\theta^\beta. \tag{5.4}$$

Taking into account that bosonic momenta, π_μ of original theory are of the form

$$\pi_\mu = \kappa\Big[\bar\Pi_{+\mu\nu}\partial_- x^\nu + \partial_+ x^\nu \bar\Pi_{+\nu\mu} + \frac{1}{2}\bar\Psi^\alpha_\mu\big(F^{-1}(x)\big)_{\alpha\beta}\partial_-\theta^\beta + \frac{1}{2}\partial_+\bar\theta^\alpha\big(F^{-1}(x)\big)_{\alpha\beta}\Psi^\beta_\nu\Big], \tag{5.5}$$

and $\beta^0_\mu = \beta^+_\mu + \beta^-_\mu$, we obtain

$$y'_\mu \cong \frac{\pi_\mu}{\kappa} + \beta^0_\mu(x). \tag{5.6}$$

Here $\beta^0_\mu(x)$ is given by

$$\beta^0_\mu(x) = \frac{1}{2}\partial_\sigma\Big[\bar\theta^\alpha + x^{\nu_1}\bar\Psi^\alpha_{\nu_1}\Big](f^{-1})_{\alpha\alpha_1}C^{\alpha_1\beta_1}_\mu(f^{-1})_{\beta_1\beta}\Big[\theta^\beta + \Psi^\beta_{\nu_2}x^{\nu_2}\Big] - \frac{1}{2}\Big[\bar\theta^\alpha + x^{\nu_1}\bar\Psi^\alpha_{\nu_1}\Big](f^{-1})_{\alpha\alpha_1}C^{\alpha_1\beta_1}_\mu(f^{-1})_{\beta_1\beta}\partial_\sigma\Big[\theta^\beta + \Psi^\beta_{\nu_2}x^{\nu_2}\Big]. \tag{5.7}$$

To find Poisson bracket between T-dual coordinates, we can start by finding Poisson bracket of sigma derivatives of T-dual coordinates and then integrating twice (see [13, 24, 25], B). Implementing this procedure we have that Poisson bracket is given as

$$\{y_{\nu_1}(\sigma), y_{\nu_2}(\bar\sigma)\} \cong \frac{1}{2k}[2\delta^{\mu_1}_{\nu_1}\delta^{\mu_2}_{\nu_2} - \delta^{\mu_2}_{\nu_1}\delta^{\mu_1}_{\nu_2}]\big[K_{\mu_1\mu_2}(\bar\sigma) + K_{\mu_2\mu_1}(\sigma)\big]H(\sigma - \bar\sigma), \tag{5.8}$$

where, for the sake of simplicity, we introduced

$$K_{\mu\nu}(\sigma) = \Big(\bar\theta^\alpha(\sigma) + x^{\mu_1}(\sigma)\bar\Psi^\alpha_{\mu_1}\Big)(f^{-1})_{\alpha\alpha_1}C^{\alpha_1\beta_1}_\mu(f^{-1})_{\beta_1\beta}\Psi^\beta_\nu - \bar\Psi^\alpha_\nu(f^{-1})_{\alpha\alpha_1}C^{\alpha_1\beta_1}_\mu(f^{-1})_{\beta_1\beta}\Big(\theta^\beta(\sigma) + \Psi^\beta_{\nu_1}x^{\nu_1}(\sigma)\Big). \tag{5.9}$$

Here, $H(\sigma - \bar\sigma)$ is same step function defined in Appendix B. It should be noted that these Poisson brackets are zero when $\sigma = \bar\sigma$. However, in cases where string in curled around compactified dimension, that is cases where $\sigma - \bar{\sigma} = 2\pi$, we have following situation

$$\{y_{\nu_1}(\sigma + 2\pi), y_{\nu_2}(\sigma)\}$$
$$\cong \frac{1}{2k}[2\delta^{\mu_1}_{\nu_1}\delta^{\mu_2}_{\nu_2} - \delta^{\mu_2}_{\nu_1}\delta^{\mu_1}_{\nu_2}]\left[K_{\mu_1\mu_2}(\sigma) + K_{\mu_2\mu_1}(\sigma)\right]$$
$$+ \frac{\pi}{k}N^{\mu}\bar{\Psi}^{\alpha}_{\mu_1}(f^{-1})_{\alpha\alpha_1}C^{\alpha_1\beta_1}_{\mu_2}(f^{-1})_{\beta_1\beta}\Psi^{\beta}_{\mu_3}[\delta^{\mu_1}_{\mu}\delta^{\mu_2}_{\nu_1}\delta^{\mu_3}_{\nu_2} - \delta^{\mu_1}_{\nu_2}\delta^{\mu_2}_{\nu_1}\delta^{\mu_3}_{\mu}$$
$$+ \delta^{\mu_1}_{\mu}\delta^{\mu_2}_{\nu_2}\delta^{\mu_3}_{\nu_1} - \delta^{\mu_1}_{\nu_1}\delta^{\mu_2}_{\nu_2}\delta^{\mu_3}_{\mu}]. \tag{5.10}$$

Here we used fact that $H(2\pi) = 1$, while N^{ρ} is winding number around compactified coordinate defined as

$$x^{\mu}(\sigma + 2\pi) - x^{\mu}(\sigma) = 2\pi N^{\mu}. \tag{5.11}$$

From this relation we can see that if we choose $x^{\mu}(\sigma) = 0$ than Poisson bracket has linear dependence on winding number. In cases where we do not have any winding number, we still have non-commutativity that is proportional to background fields.

Using the expression for sigma derivative of y_{ν} (5.6) and expression for Poisson bracket of T-dual coordinates (5.8), we can find non-associative relations. Procedure is the same as for finding Poisson brackets of T-dual theory, we find Poisson bracket of sigma derivative and integrate with respect to sigma coordinate, this time integration is done once. Going along with this procedure we have the final result

$$\{y_{\nu}(\sigma), \{y_{\nu_1}(\sigma_1), y_{\nu_2}(\sigma_2)\}\}$$
$$\cong \frac{1}{2k}H(\sigma_1 - \sigma_2)\bar{\Psi}^{\alpha}_{\mu_1}(f^{-1})_{\alpha\alpha_1}C^{\alpha_1\beta_1}_{\mu_2}(f^{-1})_{\beta_1\beta}\Psi^{\beta}_{\mu_3}$$
$$\times\Bigg[H(\sigma_1 - \sigma)[2\delta^{\mu_1}_{\nu}\delta^{\mu_2}_{\nu_2}\delta^{\mu_3}_{\nu_1} - 2\delta^{\mu_1}_{\nu_1}\delta^{\mu_2}_{\nu_2}\delta^{\mu_3}_{\nu} - \delta^{\mu_1}_{\nu}\delta^{\mu_2}_{\nu_1}\delta^{\mu_3}_{\nu_2} + \delta^{\mu_1}_{\nu_2}\delta^{\mu_2}_{\nu_1}\delta^{\mu_3}_{\nu}]$$
$$+H(\sigma_2 - \sigma)[2\delta^{\mu_1}_{\nu}\delta^{\mu_2}_{\nu_1}\delta^{\mu_3}_{\nu_2} - 2\delta^{\mu_1}_{\nu_2}\delta^{\mu_2}_{\nu_1}\delta^{\mu_3}_{\nu} - \delta^{\mu_1}_{\nu}\delta^{\mu_2}_{\nu_2}\delta^{\mu_3}_{\nu_1} + \delta^{\mu_1}_{\nu_1}\delta^{\mu_2}_{\nu_2}\delta^{\mu_3}_{\nu}]\Bigg]. \tag{5.12}$$

Since Jacobi identity is non-zero for T-dual theory we have that coordinate dependent RR field produces non-associative theory. However putting $\sigma = \sigma_2 = \bar{\sigma}$ and $\sigma_1 = \bar{\sigma} + 2\pi$ we have following Jacobi identity

$$\{y_{\nu}(\bar{\sigma}), \{y_{\nu_1}(\bar{\sigma} + 2\pi), y_{\nu_2}(\bar{\sigma})\}\}$$
$$\cong \bar{\Psi}^{\alpha}_{\mu_1}(f^{-1})_{\alpha\alpha_1}C^{\alpha_1\beta_1}_{\mu_2}(f^{-1})_{\beta_1\beta}\Psi^{\beta}_{\mu_3}[2\delta^{\mu_1}_{\nu}\delta^{\mu_2}_{\nu_2}\delta^{\mu_3}_{\nu_1} - 2\delta^{\mu_1}_{\nu_1}\delta^{\mu_2}_{\nu_2}\delta^{\mu_3}_{\nu}$$
$$- \delta^{\mu_1}_{\nu}\delta^{\mu_2}_{\nu_1}\delta^{\mu_3}_{\nu_2} + \delta^{\mu_1}_{\nu_2}\delta^{\mu_2}_{\nu_1}\delta^{\mu_3}_{\nu}]. \tag{5.13}$$

Examining equation (5.6), we notice that $\partial_{\sigma}y_{\mu}$ is not only a linear combination of initial coordinate and its momenta but also has terms that are proportional to fermionic coordinates. This might lead us to believe that T-dual theory would have nontrivial Poisson bracket between T-dual coordinate and fermionic coordinates. However, this is not the case, and it can be directly calculated by finding Poisson bracket between sigma derivative of T-dual coordinate and fermion coordinates (more details in B).

$$\{\theta^{\alpha}(\sigma), y_{\mu}(\bar{\sigma})\} \cong 0, \qquad \{\bar{\theta}^{\alpha}(\sigma), y_{\mu}(\bar{\sigma})\} \cong 0. \tag{5.14}$$

6. Conclusion

In this article we examined type II superstring propagating in presence of coordinate dependent RR field. This choice of background was in accordance with consistency conditions for background field and all calculations were made in approximation that are linear with respect to the space-time derivative of the RR field, $C^{\alpha\beta}_{\mu}$, which is infinitesimal one. We have also excluded parts that were non-linear in fermionic coordinates and neglected pure spinor actions. Using equations of motion for fermionic momenta we obtained action that was expressed in terms of bosonic coordinates, their derivatives and derivatives of fermionic coordinates.

Action with our choice of background fields possessed translation symmetry, therefore we use generalized Buscher procedure that was developed for such cases. By substituting starting action with auxiliary action we gave up on locality in order to be able to find T-dual theory. Finding equations of motion of newly introduced Lagrange multipliers we were able to salvage starting action giving us assurance that auxiliary action we selected would produce correct T-dual theory. After this we found equations of motion for gauge fields and by inserting them into action, we found T-dual theory.

Having found T-dual theory, we applied T-dual procedure once again as a more thorough way of checking if action we obtained was in fact correct T-dual of starting action. Unlike starting action, T-dual action was non-local from the start by virtue of containing $V^{(0)}$ term. Applying steps of generalized Buscher procedure we obtained starting action, again confirming that our choice of auxiliary action was correct.

We obtained non-commutativity relations in context of T-dual theory, where we used T-dual transformation laws as a bridge between Poisson brackets of starting theory and T-dual theory. T-dual transformation laws were expressed in terms of coordinates and momenta of original theory, which produced non-commutativity in T-dual theory. From expression for Poisson brackets (5.8) we can see that non-commutativity is proportional to infinitesimal part of RR field. Non-commutativity relations are zero in case when $\sigma = \bar{\sigma}$, while in case where $\sigma = \bar{\sigma} + 2\pi$ we see the emergence of winding numbers. Noncommutativity parameters are linearly dependent on bosonic coordinates x^{μ} as well as on fermionic ones, θ^{α} and $\bar{\theta}^{\alpha}$.

Taking into account Poisson brackets of T-dual coordinates and expression for sigma derivative of T-dual coordinate we were able to find non-associative relation for T-dual theory. In general case this relation was non-zero and it was proportional to infinitesimal constant, which is proportional to $C^{\alpha\beta}_{\mu}$. In special case when we put $\sigma_1 = \sigma_2 = \bar{\sigma}$ and $\sigma_3 = \bar{\sigma} + 2\pi$ we noticed that non-associativity relation remains constant.

It should be noted that since we did not preform T-dualization along fermionic coordinates their Poisson structure would remain the same as in original theory. However, unlike original theory, T-dual coordinates depend on sigma derivatives of fermionic coordinates. This dependence does not affect the Poisson brackets of the T-dual coordinates and fermionic coordinates (5.14). So, T-dual SUSY algebra has non zero Poisson bracket of the bosonic coordinates, while the rest ones are zero. In further investigation we will study fermionic T-dualization and we expect the effect on the algebra of the fermionic coordinates.

Appendix A: Obtaining $\beta_\mu^\pm$ Terms

In this paper function $\beta_\mu^\pm(V)$ emerged in T-dual transformation laws as a consequence of variation of term that was proportional to ΔV. Here we will present derivation of this function.

Here we will use substitutions $\partial_+\bar{\Theta}^\alpha = \partial_+\bar{\theta}^\alpha + v_+^{\nu_1}\bar{\Psi}_{\nu_1}^\alpha$, $\partial_-\Theta^\beta = \partial_-\theta^\beta + \Psi_{\nu_2}^\beta v_-^{\nu_2}$, also we will use $F_{\alpha\beta\rho}$ to represent term containing infinitesimal constant

$$\int_\Sigma d^2\xi \partial_+\bar{\Theta}^\alpha F_{\alpha\beta\rho}\Delta V^{(0)\rho}\partial_-\Theta^\beta = \int_\Sigma d^2\xi \epsilon^{mn}\partial_m\bar{\Theta}^\alpha F_{\alpha\beta\rho}\Delta V^{(0)\rho}\partial_n\Theta^\beta$$
$$= \int_\Sigma d^2\xi \Big[\frac{1}{2}\epsilon^{mn}\partial_m\bar{\Theta}^\alpha F_{\alpha\beta\rho}\Delta V^{(0)\rho}\partial_n\Theta^\beta - \frac{1}{2}\epsilon^{mn}\partial_n\bar{\Theta}^\alpha F_{\alpha\beta\rho}\Delta V^{(0)\rho}\partial_m\Theta^\beta\Big]$$
$$= -\frac{1}{2}\int_\Sigma d^2\xi \Big[\epsilon^{mn}\bar{\Theta}^\alpha F_{\alpha\beta\rho}\partial_m\Delta V^{(0)\rho}\partial_n\Theta^\beta + \frac{1}{2}\epsilon^{mn}\partial_n\bar{\Theta}^\alpha F_{\alpha\beta\rho}\partial_m\Delta V^{(0)\rho}\Theta^\beta\Big]$$
$$= -\frac{1}{2}\int_\Sigma d^2\xi \epsilon^{mn}\partial_m\Delta V^{(0)\rho}\left[\bar{\Theta}^\alpha F_{\alpha\beta\rho}\partial_n\Theta^\beta - \partial_n\bar{\Theta}^\alpha F_{\alpha\beta\rho}\Theta^\beta\right]$$
$$= -\frac{1}{2}\int_\Sigma d^2\xi \epsilon^{mn} v_m^\rho\left[\bar{\Theta}^\alpha F_{\alpha\beta\rho}\partial_n\Theta^\beta - \partial_n\bar{\Theta}^\alpha F_{\alpha\beta\rho}\Theta^\beta\right]$$
$$= \int_\Sigma d^2\xi v_m^\rho \beta_\rho^m. \tag{A.1}$$

Variation with respect to gauge field $v_\pm^\rho$, and setting $F_{\alpha\beta\rho} = -(f^{-1})_{\alpha\alpha_1} C_\mu^{\alpha_1\beta_1}(f^{-1})_{\beta_1\beta}$ produces desired $\beta_\rho^\pm$ functions (3.14), (3.15) in equations of motion (3.13). Here we have used the property that $(f^{-1})_{\alpha\alpha_1} C_\mu^{\alpha_1\beta_1}(f^{-1})_{\beta_1\beta}$, ie. $F_{\alpha\beta\rho}$, is antisymmetric under exchange of α and β, this, in combination with the fact that we can express $\partial_+\bar{\Theta}\partial_-\Theta$ as $\epsilon^{nm}\partial_n\bar{\Theta}\partial_m\Theta$, removes all terms proportional to $\partial_+\partial_-$, using identity $\epsilon^{mn}\bar{\Theta}\partial_m\partial_n\Theta = 0$.

It should be noted that $\beta_{\pm\mu}(V)$ functions are not unique, we could have obtained different function simply by not using using symmetrization in (A.1). In case of non-symmetric $\beta_{\pm\mu}(V)$, all results that have been obtained would take a simpler form. We have chosen to work with symmetric function because results that are deduced from this case can be easily reduced, by neglecting terms, to simpler case.

Appendix B: Poisson Bracket Between Sigma Derivatives of T-dual Coordinate

In this article, in order to find Poisson brackets of the T-dual coordinates we had to find first Poisson brackets of the sigma derivative of T-dual coordinates, $y'_\mu \equiv \partial_\sigma y_\mu(\sigma)$. In this section we will demonstrate how to obtain Poisson brackets from Poisson brackets that contain sigma derivatives. We will use canonical form of the T-dual transformation law (5.6) and standard Poisson algebra, because the initial theory is geometric one. First, we have to calculate the following Poisson bracket

$$\{\partial_{\sigma_1} y_{\nu_1}(\sigma_1), \partial_{\sigma_2} y_{\nu_2}(\sigma_2)\}$$
$$= \frac{1}{2k}\Big[K_{\nu_2\nu_1}(\sigma_2)\partial_{\sigma_2}\delta(\sigma_2-\sigma_1) - K_{\nu_1\nu_2}(\sigma_1)\partial_{\sigma_1}\delta(\sigma_1-\sigma_2) + \partial_{\sigma_1}K_{\nu_1\nu_2}(\sigma_1)\delta(\sigma_1-\sigma_2) - \partial_{\sigma_2}K_{\nu_2\nu_1}(\sigma_2)\delta(\sigma_2-\sigma_1)\Big], \tag{B.1}$$

where $K_{\mu\nu}(\sigma)$ is given by (5.9).

On the other side we have

$$\{\Delta y_{\nu_1}(\sigma_0,\sigma), \Delta y_{\nu_2}(\bar{\sigma}_0,\bar{\sigma})\}$$
$$= \int_{\sigma_0}^{\sigma} d\sigma_1 \int_{\bar{\sigma}_0}^{\bar{\sigma}} d\sigma_2 \{\partial_{\sigma_1} y_{\nu_1}(\sigma_1), \partial_{\sigma_2} y_{\nu_2}(\sigma_2)\}$$
$$= \{y_{\nu_1}(\sigma), y_{\nu_2}(\bar{\sigma})\} - \{y_{\nu_1}(\sigma), y_{\nu_2}(\bar{\sigma}_0)\} - \{y_{\nu_1}(\sigma_0), y_{\nu_2}(\bar{\sigma})\} + \{y_{\nu_1}(\sigma_0), y_{\nu_2}(\bar{\sigma}_0)\}, \tag{B.2}$$

where

$$\Delta y_\mu(\sigma_0,\sigma) \equiv \int_{\sigma_0}^{\sigma} d\sigma_1 \partial_{\sigma_1} y_\mu(\sigma_1) = y_\mu(\sigma) - y_\mu(\sigma_0). \tag{B.3}$$

Combining the equations (B.1) and (B.2) we have

$$\{\Delta y_{\nu_1}(\sigma_0,\sigma), \Delta y_{\nu_2}(\bar{\sigma}_0,\bar{\sigma})\} = \frac{1}{2k}\int_{\sigma_0}^{\sigma} d\sigma_1 \int_{\bar{\sigma}_0}^{\bar{\sigma}} d\sigma_2 \times \Big[K_{\nu_2\nu_1}(\sigma_2)\partial_{\sigma_2}\delta(\sigma_2-\sigma_1) - K_{\nu_1\nu_2}(\sigma_1)\partial_{\sigma_1}\delta(\sigma_1-\sigma_2) + \partial_{\sigma_1}K_{\nu_1\nu_2}(\sigma_1)\delta(\sigma_1-\sigma_2) - \partial_{\sigma_2}K_{\nu_2\nu_1}(\sigma_2)\delta(\sigma_2-\sigma_1)\Big]. \tag{B.4}$$

By applying partial integration, it is straightforward to extract the Poisson bracket of T-dual coordinates given by (5.8).

In paper[13] it has been shown that Poisson brackets between σ derivatives of coordinates have following form

$$\{\partial_{\sigma_1} X_\mu(\sigma_1), \partial_{\sigma_2} Y_\nu(\sigma_2)\} \cong \partial_{\sigma_1} K_{\mu\nu}(\sigma_1)\delta(\sigma_1-\sigma_2) + L_{\mu\nu}(\sigma_1)\partial_{\sigma_1}\delta(\sigma_1-\sigma_2). \tag{B.5}$$

Applying integrating twice and using partial integration this equation reduces to

$$\{X_\mu(\sigma_1), Y_\nu(\sigma_2)\} \cong -\left[K_{\mu\nu}(\sigma_1) - K_{\mu\nu}(\sigma_2) + L_{\mu\nu}(\sigma_2)\right]\delta(\sigma_1-\sigma_2). \tag{B.6}$$

In our case, we can bring equation (B.1) to the form of equation (B.5) by making following substitutions

$$\partial_{\sigma_1} K_{\nu_1\nu_2}(\sigma_1) = \partial_{\sigma_1} K_{\nu_1\nu_2}(\sigma_1), \qquad \partial_{\sigma_2} K_{\nu_2\nu_1}(\sigma_2) = \partial_{\sigma_2} K_{\nu_2\nu_1}(\sigma_2),$$
$$K_{\nu_1\nu_2}(\sigma_1) = -L_{\nu_1\nu_2}(\sigma_1), \qquad K_{\nu_2\nu_1}(\sigma_2) = -L_{\nu_2\nu_1}(\sigma_2). \tag{B.7}$$

Because we chose to work with symmetric $\beta^{\pm}_{\mu}$ function we obtain duplicated terms in (B.5).

Same procedure can be applied to find Poisson bracket between T-dual coordinate and fermionic momenta. That is, we start from Poisson bracket for sigma derivative of T-dual coordinate and fermionic momenta, then integrate once and compare left and right hand sides.

The step function $H(x)$ is defined as

$$H(x) = \int_0^x ds\delta(s) = \frac{1}{2\pi}\Sigma_{n\in Z}\int_0^x e^{ins} = \begin{cases} 0 & \text{if} \quad x = 0 \\ 1/2 & \text{if} \quad 0 < x < 2\pi \\ 1 & \text{if} \quad x = 2\pi \end{cases}. \tag{B.8}$$

Acknowledgements

Work supported in part by the Serbian Ministry of Education, Science and Technological Development.

Keywords

string theory, noncommutativity, nonassociativity, T-duality

Received: March 27, 2022
Revised: March 27, 2022
Published online: April 12, 2022

[1] J. H. Schwarz, P. van Nieuwenhuizen, *Lett. Nuovo Cimento* **1982**, *34*, 21.
[2] J. de Boer, P. A. Grassi, P. van Nieuwenhuizen, *Phys. Lett. B* **2003**, *574*, 98.
[3] K. Becker, M. Becker, J. Schwarz, *String Theory and M-Theory: A Modern Introduction*, Cambridge University Press, **2015**.
[4] M. Green, J. Schwarz, E. Witten, *Superstring Theory*, Cambridge University Press, **2012**.
[5] E. Alvarez, L. Alvarez-Gaume, Y. Lozano, *Nucl. Phys. Proc. Suppl.* **1995**, *41*, 1.
[6] A. Giveon, M. Porrati, E. Rabinovici, *Phys. Rep.* **1994**, *244*, 77.
[7] D. Lüst, *JHEP* **2010**, *12*, 084.
[8] N. Seiberg, E. Witten, *JHEP* **1999**, *09*, 032.
[9] T. H. Buscher, *Phys. Lett. B* **1987**, *194*, 59.
[10] T. H. Buscher, *Phys. Lett. B* **1988**, *201*, 466.
[11] Lj. Davidovivć, B. Sazdović, *Eur. Phys. J. C* **2014**, *74*, 2683.
[12] Lj. Davidovivć, B. Nikolić, B. Sazdović, *Eur. Phys. J. C* **2015**, *75*, 576.
[13] Lj. Davidovivć, B. Nikolić, B. Sazdović, *Eur. Phys. J. C* **2014**, *74*, 2734.
[14] L. Cornalba, R. Schiappa, *Commun. Math. Phys.* **2002**, *225*, 33.
[15] L. Cornalba, M. S. Costa, R. Schiappa, *Adv. Theor. Math. Phys.* **2005**, *9*, 355.
[16] Lj. Davidović, B. Sazdović, *JHEP* **2015**, *11*, 119.
[17] P. A. Grassi, L. Tamassia, *JHEP* **2004**, *07*, 071.
[18] N. Berkovits, P. Howe, *Nucl. Phys.* **2002**, *B635*, 75.
[19] P. A. Grassi, G. Policastro, P. van Nieuwenhuizen, *JHEP* **2002**, *11*, 004.
[20] P. A. Grassi, G. Policastro, P. van Nieuwenhuizen, *Adv. Theor. Math. Phys.* **2003**, *7*, 499.
[21] P. A. Grassi, G. Policastro, P. van Nieuwenhuizen, *Phys. Lett.* **2003**, *B553*, 96.
[22] D. Andriot, M. Larfors, D. Lust, P. Patalong, *JHEP* **2013**, *06*, 021.
[23] B. Nikolic, B. Sazdovic, *Nucl. Phys. B* **2010**, *836*, 100.
[24] B. Nikolić, D. Obrić, *Fortschritte der Physik* **2018**, *66*, 1800009.
[25] B. Nikolić, D. Obrić, *JHEP* **2019**, *03*, 136.

Eur. Phys. J. C (2015) 75:576
DOI 10.1140/epjc/s10052-015-3808-8

THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

T-duality diagram for a weakly curved background

Ljubica Davidović[a], **Bojan Nikolić**[b], **Branislav Sazdović**[c]

Institute of Physics, University of Belgrade, Belgrade, Serbia

Received: 18 June 2014 / Accepted: 23 November 2015 / Published online: 7 December 2015

Abstract In one of our previous papers we generalized the Buscher T-dualization procedure. Here we will investigate the application of this procedure to the theory of a bosonic string moving in the weakly curved background. We obtain the complete T-dualization diagram, connecting the theories which are the result of the T-dualizations over all possible choices of the coordinates. We distinguish three forms of the T-dual theories: the initial theory, the theory obtained T-dualizing some of the coordinates of the initial theory and the theory obtained T-dualizing all of the initial coordinates. While the initial theory is geometric, all the other theories are non-geometric and additionally non-local. We find the T-dual coordinate transformation laws connecting these theories and show that the set of all T-dualizations forms an Abelian group.

1 Introduction

T-duality is a property of string theory that was not encountered in any point particle theory [1–4]. Its discovery was surprising, because it implies that there exist theories, defined for essentially different geometries of the compactified dimensions, which are physically equivalent. The origin of T-duality is seen in the possibility that, unlike a point particle, the string can wrap around compactified dimensions. But, no matter if one dimension is compactified on a circle of radius R or rather on a circle of radius l_s^2/R, where l_s is the fundamental string length scale, the theory will describe the string with the same physical properties. The investigation of T-duality does not cease to provide interesting new physical implications.

The prescription for obtaining the equivalent T-dual theories is given by the Buscher T-dualization procedure [5,6]. The procedure is applicable along the isometry directions, which allows the investigation of the backgrounds which do not depend on some coordinates. It is found that T-duality transforms geometric backgrounds to the non-geometric backgrounds with Q flux which are locally well defined, and these to different types of non-geometric backgrounds, backgrounds with R flux which are not well defined even locally [7,8]. A similar prescription can be used to obtain fermionic T-duality [9,10]. It is argued that the better understanding of T-duality should be sought for by doubling the coordinates, investigating the theories in which the background fields depend on both the usual space-time coordinates and their doubles [11–14], which would make the T-duality a manifest symmetry.

T-duality enables the investigation of the closed string non-commutativity. The coordinates of the closed string are commutative when the string moves in a constant background. In a 3-dimensional space with the Kalb–Ramond field depending on one of the coordinates, successive T-dualizations along isometry directions lead to a theory with Q flux and the non-commutative coordinates [15–17]. The novelty in the research is the generalized T-dualization procedure, realized in [18], addressing the bosonic string moving in the weakly curved background–constant gravitational field and coordinate dependent Kalb–Ramond field with an infinitesimal field strength. The non-commutativity characteristics of a closed string moving in the weakly curved background was considered in [19].

The generalized procedure is applicable to all the space-time coordinates on which the string backgrounds depend. In Ref. [18], it was first applied to all initial coordinates, which produces a T-dual theory; it was then applied to all the T-dual coordinates and the initial theory was obtained. In this paper, we will investigate the application of the generalized T-dualization procedure to an arbitrary set of coordinates. Let us denote the T-dualization along the direction x^μ by T^μ and

Work supported in part by the Serbian Ministry of Education, Science and Technological Development, under contract No. 171031.

[a] e-mail: ljubica@ipb.ac.rs
[b] e-mail: bnikolic@ipb.ac.rs
[c] e-mail: sazdovic@ipb.ac.rs

the T-dualization along dual direction y_μ by T_μ. Choosing d arbitrary directions, we denote

$$\mathcal{T}^a = \circ_{n=1}^{d} T^{\mu_n}, \quad \mathcal{T}^i = \circ_{n=d+1}^{D} T^{\mu_n}, \quad \mathcal{T} = \circ_{n=1}^{D} T^{\mu_n}, \quad (1)$$

$$\mathcal{T}_a = \circ_{n=1}^{d} T_{\mu_n}, \quad \mathcal{T}_i = \circ_{n=d+1}^{D} T_{\mu_n}, \quad \widetilde{\mathcal{T}} = \circ_{n=1}^{D} T_{\mu_n}, \qquad (2)$$

where $\mu_n \in (0, 1, \ldots, D-1)$, and $\circ$ denotes the composition of T-dualizations. We will apply T-dualizations (1) to the initial theory, and T-dualizations (2) to its completely T-dual theory (obtained in [18]). We will prove the following composition laws:

$$\mathcal{T}^i \circ \mathcal{T}^a = \mathcal{T}, \quad \mathcal{T}_i \circ \mathcal{T}_a = \widetilde{\mathcal{T}}, \quad \mathcal{T}_a \circ \mathcal{T}^a = 1, \qquad (3)$$

where 1 denotes the identical transformation (T-dualization not performed). Therefore, the elements 1, $\mathcal{T}^a$ and $\mathcal{T}_a$, with $d = 1, \ldots, D$, form an Abelian group. We will find the explicit form of the resulting theories and the corresponding T-dual coordinate transformation laws. These results complete the T-dualization diagram connecting all the theories T-dual to the initial theory.

Throughout the whole article (except for Sect. 9) we assume that the Kalb–Ramond field depends on all coordinates. In that case all T-dual theories, except the initial theory, are non-geometric and non-local because they depend on the variable V^μ, which is a line integral of the derivatives of the dual coordinates. To all of these theories there corresponds a flux which is of the same type as the R flux unlike the non-geometric theories with Q flux, which have a local geometric description.

In Sects. 9.1 and 9.2, we present an example of the 3-dimensional torus, T^3 with H-flux, where Kalb–Ramond field depends only on coordinate x^3. Then T-dualizations along the isometry directions x^1 and x^2 lead to geometric background and the T-dualization along x^3 leads to non-geometric background. In Sect. 9.1 putting $D = 3$, $d = 1, 2$ with $B_{\mu\nu}$ depending on x^3 we reproduce the T-duality chain of Refs. [15–17].

In Sect. 9.2 we will compare the results of our paper with those of Ref. [8]. In our manuscript, the background fields' argument, the variable V^μ, incorporates all features of the non-geometric spaces. First, as pointed out in Ref. [8] it "eludes a geometric description even locally" because it is a line integral of the derivative. Second, we obtain non-associativity and breaking of Jacobi identity typical for the so called R-flux backgrounds. In Sect. 9.3 we present example of the 4-dimensional torus T^4 to generalize the case of Ref. [20] to critical surface.

The generalized T-dualization procedure originates from the Buscher T-dualization procedure. The first rule in the prescription is to replace the derivatives with the covariant derivatives. The new point in the prescription is the replacement of the coordinates in the background fields' argument with the invariant coordinates. The invariant coordinates are defined as the line integrals of the covariant derivatives of the original coordinates. Both covariant derivatives and invariant coordinates are defined using the gauge fields. These fields should be nonphysical, so one requires that their field strength should be zero. This is realized by adding the corresponding Lagrange multipliers' terms. As a consequence of the translational symmetry one can fix the coordinates along which the T-dualization is performed and obtain a gauge fixed action. An important cross-way in the T-dualization procedure is determined by the equations of motion of the gauge fixed action. Two equations of motion obtained varying this action are used to direct the procedure either back to the initial action or forward to the T-dual action. For the equation of motion obtained varying the action over the Lagrange multipliers, the gauge fixed action reduces to the initial action. For the equation of motion obtained varying the action over the gauge fields one obtains the T-dual theory. Comparing the solutions for the gauge fields in these two directions, one obtains the T-dual coordinate transformation laws.

2 T-duality in the weakly curved background

Let us consider the closed bosonic string propagating in the background with metric field $G_{\mu\nu}$, Kalb–Ramond field $B_{\mu\nu}$ and a dilaton field Φ, described by the action [3,4]

$$S[x] = \kappa \int_\Sigma d^2\xi \sqrt{-g}\Big[\Big(\frac{1}{2} g^{\alpha\beta} G_{\mu\nu}(x) + \frac{\varepsilon^{\alpha\beta}}{\sqrt{-g}} B_{\mu\nu}(x)\Big) \times \partial_\alpha x^\mu \partial_\beta x^\nu + \frac{1}{4\pi\kappa}\Phi(x) R^{(2)}\Big]. \qquad (4)$$

The integration goes over a 2-dimensional world-sheet Σ parametrized by ξ^α ($\xi^0 = \tau$, $\xi^1 = \sigma$), $g_{\alpha\beta}$ is the intrinsic world-sheet metric, $R^{(2)}$ corresponding 2-dimensional scalar curvature, $x^\mu(\xi)$, $\mu = 0, 1, \ldots, D-1$ are the coordinates of the D-dimensional space-time, $\kappa = \frac{1}{2\pi\alpha'}$ with α' being the Regge slope parameter and $\varepsilon^{01} = -1$.

2.1 Weakly curved background

The requirement of the quantum conformal invariance of the world-sheet results in the space-time equations of motion for the background fields. In the lowest order in the slope parameter α' these equations are

$$\begin{aligned} &R_{\mu\nu} - \frac{1}{4} B_{\mu\rho\sigma} B_\nu^{\ \rho\sigma} + 2D_\mu \partial_\nu \Phi = 0, \\ &D_\rho B^\rho_{\ \mu\nu} - 2\partial_\rho \Phi B^\rho_{\ \mu\nu} = 0, \\ &4(\partial\Phi)^2 - 4D_\mu \partial^\mu \Phi + \frac{1}{12} B_{\mu\nu\rho} B^{\mu\nu\rho} \\ &\quad + 4\pi\kappa(D-26)/3 - R = 0. \end{aligned} \qquad (5)$$

Here $B_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} + \partial_\nu B_{\rho\mu} + \partial_\rho B_{\mu\nu}$ is the field strength of the field $B_{\mu\nu}$, and $R_{\mu\nu}$ and D_μ are the Ricci tensor and the covariant derivative with respect to the space-time metric. We will consider one of the simplest coordinate dependent solutions of (5), the weakly curved background. This background was considered in Refs. [21–23], where the influence of the boundary conditions on the non-commutativity of the open bosonic string has been investigated. The same approximation was considered in [16,19] in context of the closed string non-commutativity.

The weakly curved background is defined by

$$G_{\mu\nu}(x) = \text{const},$$
$$B_{\mu\nu}(x) = b_{\mu\nu} + \frac{1}{3}B_{\mu\nu\rho}x^\rho \equiv b_{\mu\nu} + h_{\mu\nu}(x),$$
$$\Phi(x) = \text{const}, \tag{6}$$

with $b_{\mu\nu}, B_{\mu\nu\rho} = \text{const}$. This background is the solution of the space-time equations of motion if the constant $B_{\mu\nu\rho}$ is taken to be infinitesimal and all the calculations are done in the first order in $B_{\mu\nu\rho}$, so that the curvature $R_{\mu\nu}$ can be neglected as the infinitesimal of the second order. Through the whole manuscript (with the exception of Sect. 9) we assume that the background has the topology of D-dimensional torus T^D, where the Kalb–Ramond field depends on all coordinates. In Sects. 9.1 and 9.2 we give an example of the 3-dimensional torus, T^3, with H-flux, where the Kalb–Ramond field depends only on the coordinate x^3, while in Sect. 9.3 we give an example of the 4-dimensional torus T^4 with constant background fields.

The assumption that $B_{\mu\nu\rho}$ is infinitesimal means that we consider the D-dimensional torus so large that for any choice of indices

$$\frac{B_{\mu\nu\rho}}{R_\mu R_\nu R_\rho} \ll 1 \tag{7}$$

holds [16], where R_μ are the radii of the torus. The H-flux background, considered in Refs. [8,16], is of the same type as the weakly curved background. However, this background depends just on x^3 and corresponds to the examples addressed in Sect. 9 of our paper. The background considered in the rest of the article depends on all coordinates.

In this paper we will investigate the T-dualization properties of the action (4) describing the closed string moving in the weakly curved background. Taking the conformal gauge $g_{\alpha\beta} = e^{2F}\eta_{\alpha\beta}$, the action (4) becomes

$$S[x] = \kappa \int_\Sigma d^2\xi \; \partial_+ x^\mu \Pi_{+\mu\nu}(x)\partial_- x^\nu, \tag{8}$$

with the background field composition equal to

$$\Pi_{\pm\mu\nu}(x) = B_{\mu\nu}(x) \pm \frac{1}{2}G_{\mu\nu}(x), \tag{9}$$

and the light-cone coordinates given by

$$\xi^\pm = \frac{1}{2}(\tau \pm \sigma), \qquad \partial_\pm = \partial_\tau \pm \partial_\sigma. \tag{10}$$

2.2 Complete T-dualization

The T-dualization of the closed string theory in the weakly curved background was presented in [18]. The procedure is related to a global symmetry of the theory

$$\delta x^\mu = \lambda^\mu. \tag{11}$$

The symmetry still exists in the presence of the nontrivial Kalb–Ramond field (6), but only in the case of the trivial mapping of the world-sheet into the space-time, because in that case the variation of the action (8)

$$\delta S = \frac{\kappa}{3}\varepsilon^{\alpha\beta}B_{\mu\nu\rho}\lambda^\rho \int d^2\xi\, \partial_\alpha x^\mu \partial_\beta x^\nu \tag{12}$$

after partial integration, using the identity $\varepsilon^{\alpha\beta}\partial_\alpha\partial_\beta = 0$, becomes

$$\delta S = \frac{\kappa}{3}B_{\mu\nu\rho}\lambda^\rho\epsilon^{\alpha\beta} \int d^2\xi\, \partial_\alpha(x^\mu\partial_\beta x^\nu), \tag{13}$$

which is equal to zero. This means that classically, directions which appear in the argument of Kalb–Ramond field are also Killing directions. However, the standard Buscher procedure cannot be applied to them, because background fields depend on the coordinates but not on their derivatives.

The T-dual picture of the theory, obtained on applying the T-dualization procedure to all the coordinates, is given by

$$S[y] = \kappa \int d^2\xi \; \partial_+ y_\mu \, {}^\star\Pi_+^{\mu\nu}\big(\Delta V(y)\big)\, \partial_- y_\nu$$
$$= \frac{\kappa^2}{2}\int d^2\xi \; \partial_+ y_\mu \Theta_-^{\mu\nu}\big(\Delta V(y)\big)\partial_- y_\nu, \tag{14}$$

where

$$\Theta_\pm^{\mu\nu} \equiv -\frac{2}{\kappa}(G_E^{-1}\Pi_\pm G^{-1})^{\mu\nu} = \theta^{\mu\nu} \mp \frac{1}{\kappa}(G_E^{-1})^{\mu\nu}, \tag{15}$$

with

$$G_{E\mu\nu} \equiv G_{\mu\nu} - 4(BG^{-1}B)_{\mu\nu},$$
$$\theta^{\mu\nu} \equiv -\frac{2}{\kappa}(G_E^{-1}BG^{-1})^{\mu\nu}, \tag{16}$$

being the effective metric and the non-commutativity parameter in Seiberg–Witten terminology of the open bosonic string theory [24]. The T-dual background fields are equal to

$${}^\star G^{\mu\nu}\big(\Delta V(y)\big) = (G_E^{-1})^{\mu\nu}\big(\Delta V(y)\big),$$
$${}^\star B^{\mu\nu}\big(\Delta V(y)\big) = \frac{\kappa}{2}\theta^{\mu\nu}\big(\Delta V(y)\big), \tag{17}$$

and their argument is given by

$$\Delta V^\mu(y) = -\frac{\kappa}{2}\left(\Theta_{0-}^{\mu\nu} + \Theta_{0+}^{\mu\nu}\right)\Delta y_\nu + \frac{\kappa}{2}\left(\Theta_{0-}^{\mu\nu} - \Theta_{0+}^{\mu\nu}\right)\Delta \tilde{y}_\nu = -\kappa\theta_0^{\mu\nu}\Delta y_\nu + (g^{-1})^{\mu\nu}\Delta\tilde{y}_\nu. \tag{18}$$

Here $\Theta_{0\pm}^{\mu\nu}$ is the zeroth order value of the field composition $\Theta_\pm^{\mu\nu}$ defined in (15) and $g_{\mu\nu} = G_{\mu\nu} - 4b^2_{\mu\nu}$ and $\theta_0^{\mu\nu} = -\frac{2}{\kappa}(g^{-1}bG^{-1})^{\mu\nu}$ are the zeroth order values of the effective fields (16). The variable $\Delta\tilde{y}_\mu$ is the double of the dual variable $\Delta y_\mu = y_\mu(\xi) - y_\mu(\xi_0)$, defined as the following line integral:

$$\Delta\tilde{y}_\mu = \int_P (d\tau y'_\mu + d\sigma \dot{y}_\mu) = \int_P d\xi^\alpha \varepsilon^\beta{}_\alpha \partial_\beta y_\mu, \tag{19}$$

taken along the path P, from the point $\xi_0^\alpha(\tau_0,\sigma_0)$ to the point $\xi^\alpha(\tau,\sigma)$.

The fact that we are working with the weakly curved background ensures that the T-dual background fields are the solution of the space-time equations (5). Because both dual metric ${}^\star G^{\mu\nu}$ and dual Kalb–Ramond field ${}^\star B^{\mu\nu}$ are linear in coordinates with infinitesimal coefficients, the dual Christoffel symbol ${}^\star\Gamma^{\nu\rho}_\mu$ and dual field strength ${}^\star B^{\mu\nu\rho}$ are constant and infinitesimal. In Eq. (114) of Sect. 8 we will show that T-dual dilaton field is ${}^\bullet\Phi = \Phi - \ln\det\sqrt{2\Pi_+}$, where Φ is constant and Π_+ is linear in coordinates with infinitesimal coefficients. So, ${}^\bullet\Phi$ is also linear in coordinates with infinitesimal coefficients, and $\partial_\mu{}^\bullet\Phi$ is constant and infinitesimal. Consequently, $D_\mu\partial_\nu{}^\bullet\Phi$, $\partial_\rho{}^\bullet\Phi B^\rho{}_{\mu\nu}$ and $(\partial_\mu{}^\bullet\Phi)^2$ are infinitesimals of the second order. So, all T-dual space-time equations, for the metric, for the Kalb–Ramond field and for dilaton field, are infinitesimals of the second order and as such are neglected.

The initial theory (8) and its completely T-dual theory (14) are connected by the T-dual coordinate transformation laws (eq. (42) of Ref. [18])

$$\partial_\pm x^\mu = -\kappa\Theta^{\mu\nu}(\Delta V)\partial_\pm y_\nu \mp 2\kappa\Theta_{0\pm}^{\mu\nu}\beta_\nu^\mp(V), \tag{20}$$

and its inverse (eq. (66) of Ref. [18])

$$\partial_\pm y_\mu \cong -2\Pi_{\mp\mu\nu}(\Delta x)\partial_\pm x^\nu \mp 2\beta_\mu^\mp(x), \tag{21}$$

where $\beta_\mu^\pm(x) = \mp\frac{1}{2}h_{\mu\nu}(x)\partial_\mp x^\nu$. It is shown that

$$\mathcal{T} : S[x^\mu] \to S[y_\mu], \quad \widetilde{\mathcal{T}} : S[y_\mu] \to S[x^\mu], \tag{22}$$

and therefore

$$\mathcal{T}\circ\widetilde{\mathcal{T}} = 1. \tag{23}$$

3 T-dualization along arbitrary subset of coordinates $\mathcal{T}^a : S[x^\mu] \to S[x^i, y_a]$

In this section, we will learn what theory is obtained if one chooses to apply the T-dualization procedure to the action (8), along arbitrary d coordinates x^a, $\mathcal{T}^a : S[x^\mu] \to S[x^i, y_a]$, with $\mathcal{T}^a = \circ_{n=1}^d T^{\mu_n}$, $\mu_n \in (0,1,\dots,D-1)$.

The closed string action in the weakly curved background (6) has a global symmetry (11). One localizes the symmetry for the coordinates x^a, by introducing the gauge fields v^a_α and substituting the ordinary derivatives with the covariant derivatives

$$\partial_\alpha x^a \to D_\alpha x^a = \partial_\alpha x^a + v^a_\alpha. \tag{24}$$

The covariant derivatives are invariant under standard gauge transformations

$$\delta v^a_\alpha = -\partial_\alpha\lambda^a. \tag{25}$$

In the case of the weakly curved background, in order to obtain the gauge invariant action one should additionally substitute the coordinates x^a in the argument of the background fields with their invariant extension, defined by

$$\Delta x^a_{inv} \equiv \int_P d\xi^\alpha D_\alpha x^a = \int_P (d\xi^+ D_+x^a + d\xi^- D_-x^a) = x^a - x^a(\xi_0) + \Delta V^a, \tag{26}$$

where

$$\Delta V^a \equiv \int_P d\xi^\alpha v^a_\alpha = \int_P (d\xi^+ v^a_+ + d\xi^- v^a_-). \tag{27}$$

To preserve the physical equivalence between the gauged and the original theory, one introduces the Lagrange multipliers y_a and adds term $\frac{1}{2}y_aF^a_{+-}$ to the Lagrangian, which will force the field strength $F^a_{+-} \equiv \partial_+v^a_- - \partial_-v^a_+ = -2F^a_{01}$ to vanish. In this way, the gauge invariant action

$$\begin{aligned} S_{inv}[x^\mu, x^a_{inv}, y_a] &= \kappa\int d^2\xi\Big[\partial_+x^i\Pi_{+ij}\left(x^i, \Delta x^a_{inv}\right)\partial_-x^j \\ &+ \partial_+x^i\Pi_{+ia}\left(x^i,\Delta x^a_{inv}\right)D_-x^a \\ &+ D_+x^a\Pi_{+ai}\left(x^i,\Delta x^a_{inv}\right)\partial_-x^i \\ &+ D_+x^a\Pi_{+ab}\left(x^i,\Delta x^a_{inv}\right)D_-x^b \\ &+ \frac{1}{2}(v^a_+\partial_-y_a - v^a_-\partial_+y_a)\Big] \end{aligned} \tag{28}$$

is obtained, where the last term is equal to $\frac{1}{2}y_aF^a_{+-}$ up to the total divergence. Now, we can fix the gauge taking $x^a(\xi) = x^a(\xi_0)$ and obtain the gauge fixed action

$S_{fix}[x^i, v^a_\pm, y_a]$

$$= \kappa \int d^2\xi \Big[\partial_+ x^i \Pi_{+ij}\big(x^i, \Delta V^a\big)\partial_- x^j + \partial_+ x^i \Pi_{+ia}\big(x^i, \Delta V^a\big)v_-^a + v_+^a \Pi_{+ai}\big(x^i, \Delta V^a\big)\partial_- x^i + v_+^a \Pi_{+ab}\big(x^i, \Delta V^a\big)v_-^b + \frac{1}{2}(v_+^a \partial_- y_a - v_-^a \partial_+ y_a)\Big]. \tag{29}$$

This action reduces to the initial one for the equations of motion obtained varying over the Lagrange multipliers. The T-dual action is obtained for the equations of motion for the gauge fields.

3.1 Regaining the initial action

Varying the gauge fixed action (29) over the Lagrange multipliers y_a one obtains the equations of motion

$$\partial_+ v_-^a - \partial_- v_+^a = 0, \tag{30}$$

which have the solution

$$v_\pm^a = \partial_\pm x^a. \tag{31}$$

On this solution the background fields' argument ΔV^a defined in (27) is path independent and reduces to

$$\Delta V^a(\xi) = x^a(\xi) - x^a(\xi_0). \tag{32}$$

The gauge fixed action (29) reduces to the initial action (8), but the background fields' argument is ΔV^a instead of x^i. However, the action (8) is invariant under the constant shift of coordinates, so shifting coordinates by $x^a(\xi_0)$ one obtains the exact form of the initial action.

3.2 The T-dual action

Using the equations of motion for the gauge fields, we eliminate them and obtain the T-dual action.

The equations of motion obtained varying the gauge fixed action (29) over the gauge fields $v_\pm^a$ are

$$\Pi_{\pm ai}\big(x^i, \Delta V^a\big)\partial_\mp x^i + \Pi_{\pm ab}\big(x^i, \Delta V^a\big)v_\mp^b + \frac{1}{2}\partial_\mp y_a = \pm\beta_a^\pm\big(x^i, V^a\big), \tag{33}$$

where

$$\beta_a^\pm\big(x^i, V^a\big) = \mp\frac{1}{2}\Big[h_{ai}(x^i)\partial_\mp x^i + h_{ab}(x^i)\partial_\mp V^b + h_{ai}\big(V^a\big)\partial_\mp x^i + h_{ab}\big(V^a\big)\partial_\mp V^b\Big] \tag{34}$$

is the contribution from the background fields' argument ΔV^a, defined in a same way as in Ref. [18], by $\delta_V S_{\text{fix}} = -\kappa \int d^2\xi(\beta_a^+ \delta v_+^a + \beta_a^- \delta v_-^a)$. If the initial background $\Pi_{+\mu\nu}$ does not depend on the coordinates x^a, the corresponding beta functions are zero $\beta_a^\pm = 0$.

Multiplying Eq. (33) by $2\kappa\tilde{\Theta}_\mp^{ab}$, defined in (A.7), the inverse of the background fields composition $\Pi_{\pm ab}$, one obtains

$$v_\mp^a = -2\kappa\tilde{\Theta}_\mp^{ab}\big(x^i, \Delta V^a\big)\Big[\Pi_{\pm bi}\big(x^i, \Delta V^a\big)\partial_\mp x^i + \frac{1}{2}\partial_\mp y_b \mp\beta_b^\pm\big(x^i, V^a\big)\Big]. \tag{35}$$

Substituting (35) into the action (29), we obtain the T-dual action

$$S[x^i, y_a] = \kappa \int d^2\xi \Big[\partial_+ x^i \overline{\Pi}_{+ij}\big(x^i, \Delta V^a(x^i, y_a)\big)\partial_- x^j - \kappa\, \partial_+ x^i \Pi_{+ia}\big(x^i, \Delta V^a(x^i, y_a)\big) \times \tilde{\Theta}_-^{ab}\big(x^i, \Delta V^a(x^i, y_a)\big)\partial_- y_b + \kappa\, \partial_+ y_a \tilde{\Theta}_-^{ab}\big(x^i, \Delta V^a(x^i, y_a)\big) \times \Pi_{+bi}\big(x^i, \Delta V^a(x^i, y_a)\big)\partial_- x^i + \frac{\kappa}{2}\partial_+ y_a \tilde{\Theta}_-^{ab}\big(x^i, \Delta V^a(x^i, y_a)\big)\partial_- y_b\Big], \tag{36}$$

where

$$\overline{\Pi}_{+ij} \equiv \Pi_{+ij} - 2\kappa\Pi_{+ia}\tilde{\Theta}_-^{ab}\Pi_{+bj}. \tag{37}$$

In order to find the explicit value of the background fields argument $\Delta V^a(x^i, y_a)$, it is enough to consider the zeroth order of the equations of motion for the gauge fields $v_\pm^a$ (35)

$$v_\pm^{(0)a} = -2\kappa\tilde{\Theta}_{0\pm}^{ab}\Big[\Pi_{0\mp bi}\partial_\pm x^{(0)i} + \frac{1}{2}\partial_\pm y_b^{(0)}\Big]. \tag{38}$$

Here $\tilde{\Theta}_{0\pm}^{ab}$ and $\Pi_{0\mp bi}$ stand for the zeroth order values of $\tilde{\Theta}_\pm^{ab}$ and $\Pi_{\mp bi}$, and they are defined in (A.11).

Substituting (38) into (27) we obtain

$$\Delta V^{(0)a}(x^i, y_a) = -\kappa\Big[\tilde{\Theta}_{0+}^{ab}\Pi_{0-bi} + \tilde{\Theta}_{0-}^{ab}\Pi_{0+bi}\Big]\Delta x^{(0)i} - \kappa\Big[\tilde{\Theta}_{0+}^{ab}\Pi_{0-bi} - \tilde{\Theta}_{0-}^{ab}\Pi_{0+bi}\Big]\Delta\tilde{x}^{(0)i} - \frac{\kappa}{2}\Big[\tilde{\Theta}_{0+}^{ab} + \tilde{\Theta}_{0-}^{ab}\Big]\Delta y_b^{(0)} - \frac{\kappa}{2}\Big[\tilde{\Theta}_{0+}^{ab} - \tilde{\Theta}_{0-}^{ab}\Big]\Delta\tilde{y}_b^{(0)}. \tag{39}$$

Here

$$\Delta\tilde{y}_a^{(0)} = \int_P (d\tau\, y_a^{(0)\prime} + d\sigma\, \dot{y}_a^{(0)}),$$
$$\Delta\tilde{x}^{(0)i} = \int_P (d\tau\, x^{(0)\prime i} + d\sigma\, \dot{x}^{(0)i}), \tag{40}$$

are the variables T-dual to the coordinates y_a and x^i in the zeroth order in $B_{\mu\nu\rho}$, for $b_{\mu\nu} = 0$, which we call the double variables.

Thus, we obtain the explicit form of the T-dual action and conclude that it is given in terms of the original coordinates x^i and the dual coordinates y_a originating from the Lagrange multipliers. However, the background fields' argument depends not only on these variables but on their doubles as well. Because of this the theory is non-local as the double variables $\tilde{x}^i$ and $\tilde{y}_a$ are defined as line integrals.

The action (36) can be obtained from the initial action (8) under the following substitutions of the coordinate derivatives and the background fields:

$$\partial_\pm x^i \to \partial_\pm x^i, \quad \partial_\pm x^a \to \partial_\pm y_a, \tag{41}$$

$$\Pi_{+ij} \to {}^\bullet\Pi_{+ij}, \quad \Pi_{+ia} \to {}^\bullet\Pi_{+i}{}^a, \\ \Pi_{+ai} \to {}^\bullet\Pi^a_{+i}, \quad \Pi_{+ab} \to {}^\bullet\Pi^{ab}_+, \tag{42}$$

where the dual background fields are

$$ {}^\bullet\Pi_{+ij} = \overline{\Pi}_{+ij}, \quad {}^\bullet\Pi_{+i}{}^a = -\kappa\Pi_{+ib}\tilde{\Theta}^{ba}_-, \\ {}^\bullet\Pi^a_{+i} = \kappa\tilde{\Theta}^{ab}_-\Pi_{+bi}, \quad {}^\bullet\Pi^{ab}_+ = \frac{\kappa}{2}\tilde{\Theta}^{ab}_-, \tag{43}$$

with $\overline{\Pi}_{+ij}$, $\Pi_{+\mu\nu}$, and $\tilde{\Theta}^{ab}_-$ defined in (37), (9), and (A.7). The argument of all T-dual background fields is $[x^i, V^a(x^i, y_a)]$. According to (27) and (39), it is non-local and consequently non-geometric. Calculating the symmetric and antisymmetric part of the T-dual field compositions (43), we find that the T-dual metric and Kalb–Ramond field are equal to

$$\begin{aligned}
{}^\bullet G_{ij} &= \overline{G}_{ij} = G_{ij} - G_{ia}(\tilde{G}_E^{-1})^{ab}G_{bj} \\
&\quad -2\kappa\Big(B_{ia}\tilde{\theta}^{ab}G_{bj} + G_{ia}\tilde{\theta}^{ab}B_{bj}\Big) - 4B_{ia}(\tilde{G}_E^{-1})^{ab}B_{bj}, \\
{}^\bullet B_{ij} &= \overline{B}_{ij} = B_{ij} - \frac{\kappa}{2}G_{ia}\tilde{\theta}^{ab}G_{bj} - B_{ia}(\tilde{G}_E^{-1})^{ab}G_{bj} \\
&\quad -G_{ia}(\tilde{G}_E^{-1})^{ab}B_{bj} - 2\kappa B_{ia}\tilde{\theta}^{ab}B_{bj}, \\
{}^\bullet G^{ab} &= (\tilde{G}_E^{-1})^{ab}, \\
{}^\bullet B^{ab} &= \frac{\kappa}{2}\tilde{\theta}^{ab}, \\
{}^\bullet G^a{}_i &= \kappa\tilde{\theta}^{ab}G_{bi} + 2(\tilde{G}_E^{-1})^{ab}B_{bi}, \\
{}^\bullet B^a{}_i &= \kappa\tilde{\theta}^{ab}B_{bi} + \frac{1}{2}(\tilde{G}_E^{-1})^{ab}G_{bi},
\end{aligned} \tag{44}$$

where $\tilde{G}_{Eab}$ and $\tilde{\theta}^{ab}$ are defined in (A.6) and (A.10). The T-dual background fields have the same form as in the flat background [1,5,25] but in the present case fields $B_{\mu\nu}$, $\tilde{G}_E^{-1ab}$ and $\tilde{\theta}^{ab}$ are coordinate dependent.

Comparing the solutions for the gauge fields (31) and (35), we obtain the T-dual coordinate transformation law

$$\begin{aligned}
\partial_\mp x^a \cong & -2\kappa\tilde{\Theta}^{ab}_\mp\big(x^i, \Delta V^a(x^i, y_a)\big) \\
&\times\Big[\Pi_{\pm bi}\big(x^i, \Delta V^a(x^i, y_a)\big)\partial_\mp x^i + \frac{1}{2}\partial_\mp y_b \\
&\mp\beta^\pm_b\big(x^i, V^a(x^i, y_a)\big)\Big].
\end{aligned} \tag{45}$$

4 Inverse T-dualization $\mathcal{T}_a : S[x^i, y_a] \to S[x^\mu]$

In this section we will show that T-dualization of the action $S[x^i, y_a]$, given by (36), along already treated directions y_a leads to the original action.

So, let us localize the global symmetry of the coordinates y_a

$$\delta y_a = \lambda_a, \tag{46}$$

of the action (36). Note that this is the symmetry, despite the coordinate dependence of the metric (44), due to the invariance of the background fields' argument [18]. Following the T-dualization procedure, we substitute the ordinary derivatives with the covariant ones

$$D_\pm y_a = \partial_\pm y_a + u_{\pm a}, \tag{47}$$

where $u_{\pm a}$ are gauge fields which transform as $\delta u_{\pm a} = -\partial_\pm\lambda_a$. We also substitute coordinates y_a in the background fields' argument with the invariant coordinates

$$\begin{aligned}
y_a^{\text{inv}} &= \int_P (d\xi^+ D_+ y_a + d\xi^- D_- y_a) \\
&= y_a(\xi) - y_a(\xi_0) + \Delta U_a,
\end{aligned} \tag{48}$$

where

$$\Delta U_a = \int_P (d\xi^+ u_{+a} + d\xi^- u_{-a}). \tag{49}$$

In this way, adding the Lagrange multiplier term which makes the introduced gauge fields nonphysical, we obtain the gauge invariant action

$$\begin{aligned}
&S_{\text{inv}}[x^i, y_a, y_a^{\text{inv}}, z^a] \\
&= \kappa\int d^2\xi\Big[\partial_+ x^i\overline{\Pi}_{+ij}\big(x^i, \Delta V^a(x^i, y_a^{\text{inv}})\big)\partial_- x^j \\
&\quad -\kappa\,\partial_+ x^i\Pi_{+ia}\big(x^i, \Delta V^a(x^i, y_a^{\text{inv}})\big) \\
&\quad \times\tilde{\Theta}^{ab}_-\big(x^i, \Delta V^a(x^i, y_a^{\text{inv}})\big)D_- y_b \\
&\quad +\kappa\, D_+ y_a\tilde{\Theta}^{ab}_-\big(x^i, \Delta V^a(x^i, y_a^{\text{inv}})\big) \\
&\quad \times\Pi_{+bi}\big(x^i, \Delta V^a(x^i, y_a^{\text{inv}})\big)\partial_- x^i \\
&\quad +\frac{\kappa}{2}D_+ y_a\tilde{\Theta}^{ab}_-\big(x^i, \Delta V^a(x^i, y_a^{\text{inv}})\big)D_- y_b \\
&\quad +\frac{1}{2}(u_{+a}\partial_- z^a - u_{-a}\partial_+ z^a)\Big],
\end{aligned} \tag{50}$$

which after fixing the gauge by $y_a(\xi) = y_a(\xi_0)$ becomes

$$\begin{aligned}
&S_{\text{fix}}[x^i, u_{\pm a}, z^a] \\
&= \kappa\int d^2\xi\Big[\partial_+ x^i\overline{\Pi}_{+ij}\big(x^i, \Delta V^a(x^i, \Delta U_a)\big)\partial_- x^j \\
&\quad -\kappa\,\partial_+ x^i\Pi_{+ia}\big(x^i, \Delta V^a(x^i, \Delta U_a)\big) \\
&\quad \times\tilde{\Theta}^{ab}_-\big(x^i, \Delta V^a(x^i, \Delta U_a)\big)u_{-b}
\end{aligned}$$

$$+\kappa\, u_{+a}\tilde{\Theta}_-^{ab}\big(x^i, \Delta V^a(x^i, \Delta U_a)\big) \times \Pi_{+bi}\big(x^i, \Delta V^a(x^i, \Delta U_a)\big)\partial_- x^i + \frac{\kappa}{2}u_{+a}\tilde{\Theta}_-^{ab}\big(x^i, \Delta V^a(x^i, \Delta U_a)\big)u_{-b} + \frac{1}{2}(u_{+a}\partial_- z^a - u_{-a}\partial_+ z^a)\Big], \quad (51)$$

where ΔV^a is defined in (39) and ΔU_a in (49).

4.1 Regaining the T-dual action

The equations of motion obtained varying the gauge fixed action (51) over the Lagrange multipliers z^a

$$\partial_+ u_{-a} - \partial_- u_{+a} = 0, \quad (52)$$

have the solution

$$u_{\pm a} = \partial_\pm y_a. \quad (53)$$

On this solution the variable ΔU_a defined by (49) is path independent and reduces to

$$\Delta U_a(\xi) = y_a(\xi) - y_a(\xi_0), \quad (54)$$

and the gauge fixed action (51) reduces to the action (36).

4.2 Regaining the initial action

The equations of motion obtained varying the gauge fixed action (51) over the gauge fields $u_{\pm a}$ are

$$\kappa\tilde{\Theta}_\mp^{ab}\big(x^i, \Delta V^a(x^i, \Delta U_a)\big) \times\Big[\frac{1}{2}u_{\mp b} + \Pi_{\pm bi}\big(x^i, \Delta V^a(x^i, \Delta U_a)\big)\partial_\mp x^i\Big] + \frac{1}{2}\partial_\mp z^a = \pm\kappa\,\tilde{\Theta}_{0\mp}^{ab}\beta_b^\pm\big(x^i, V^a(x^i, U_a)\big), \quad (55)$$

where terms $\tilde{\Theta}_{0\mp}^{ab}\beta_b^\pm$ are the contribution from the variation over the background field argument

$$\delta_U S_{\rm fix} = -\kappa^2\int d^2\xi\,\big(\delta u_{+a}\tilde{\Theta}_{0-}^{ab}\beta_b^+ + \delta u_{-a}\tilde{\Theta}_{0+}^{ab}\beta_b^-\big). \quad (56)$$

Here $\beta_a^\pm$ is of the same form as (34) and $\tilde{\Theta}_{0\mp}^{ab}$ is defined in (A.11).

Let us show that for the equations of motion (55), the gauge fixed action (51) will reduce to the initial action (8). Using the fact that $\tilde{\Theta}_\mp^{ab}$ is inverse to $2\kappa\Pi_{\pm ab}$, these equations of motion can be rewritten as

$$u_{\mp a} = -2\Pi_{\pm ai}\big(x^i, \Delta V^a(x^i, \Delta U_a)\big)\partial_\mp x^i -2\Pi_{\pm ab}\big(x^i, \Delta V^a(x^i, \Delta U_a)\big)\partial_\mp z^b \pm 2\beta_a^\pm\big(x^i, V^a(x^i, U_a)\big). \quad (57)$$

Substituting (57) into (51), using the definition (37) and the first relation in (A.22) one obtains

$$S[x^i, z^a] = \kappa\int_\Sigma d^2\xi\Big[\partial_+ x^i\Pi_{+ij}\partial_- x^j + \partial_+ x^i\Pi_{+ia}\partial_- z^a + \partial_+ z^a\Pi_{+ai}\partial_- x^i + \partial_+ z^a\Pi_{+ab}\partial_- z^b\Big]. \quad (58)$$

The explicit form of the argument of the background fields is obtained substituting the zeroth order of Eq. (57) into (49)

$$U_a^{(0)} = -2b_{ai}x^{(0)i} + G_{ai}\tilde{x}^{(0)i} - 2b_{ab}z^{(0)b} + G_{ab}\tilde{z}^{(0)b}. \quad (59)$$

Consequently, the argument of the background fields ΔV^a, defined in (39), is just

$$V^{(0)a}(x^i, U_a) = z^a. \quad (60)$$

So, the action (58) is equal to the initial action (8) with $x^\mu = (x^i, z^a)$.

Comparing the solutions for the gauge fields (53) and (57), we obtain the T-dual transformation law

$$\partial_\mp y_a \cong -2\Pi_{\pm ai}(x^i, z^a)\partial_\mp x^i - 2\Pi_{\pm ab}(x^i, z^a)\partial_\mp z^b \pm 2\beta_a^\pm\big(x^i, z^a\big). \quad (61)$$

Substituting $\partial_\mp y_a$ to (45) with the help of (60) one finds $\partial_\pm x^a = \partial_\pm z^a$. Therefore, (61) is the transformation inverse to (45), which confirms the relation $\mathcal{T}^a\circ\mathcal{T}_a = 1$.

5 T-dualization along all undualized coordinates $\mathcal{T}^i : S[x^i, y_a] \to S[y_\mu]$

In this section we will T-dualize the action (36), applying the T-dualization procedure to the undualized coordinates x^i. Substituting the ordinary derivatives $\partial_\pm x^i$ with the covariant derivatives

$$D_\pm x^i = \partial_\pm x^i + w_\pm^i, \quad (62)$$

where the gauge fields $w_\pm^i$ transform as $\delta w_\pm^i = -\partial_\pm\lambda^i$, substituting the coordinates x^i in the background field arguments with

$$\Delta x_{\rm inv}^i = \int_P (d\xi^+ D_+ x^i + d\xi^- D_- x^i), \quad (63)$$

and adding the Lagrange multiplier term, we obtain the gauge invariant action

$$S_{\rm inv}[x^i, x_{\rm inv}^i, y] = \kappa\int d^2\xi\Big[D_+ x^i\overline{\Pi}_{+ij}\big(\Delta x_{\rm inv}^i, \Delta V^a(\Delta x_{\rm inv}^i, y_a)\big)D_- x^j - \kappa\, D_+ x^i\,\Pi_{+ia}\big(\Delta x_{\rm inv}^i, \Delta V^a(\Delta x_{\rm inv}^i, y_a)\big)$$

$$
\begin{aligned}
&\times \tilde{\Theta}_-^{ab}\big(\Delta x^i_{\rm inv}, \Delta V^a(\Delta x^i_{\rm inv}, y_a)\big)\partial_- y_b \\
&+ \kappa\, \partial_+ y_a \tilde{\Theta}_-^{ab}\big(\Delta x^i_{\rm inv}, \Delta V^a(\Delta x^i_{\rm inv}, y_a)\big) \\
&\times \Pi_{+bi}\big(\Delta x^i_{\rm inv}, \Delta V^a(\Delta x^i_{\rm inv}, y_a)\big) D_- x^i \\
&+ \frac{\kappa}{2}\, \partial_+ y_a \tilde{\Theta}_-^{ab}\big(\Delta x^i_{\rm inv}, \Delta V^a(\Delta x^i_{\rm inv}, y_a)\big)\partial_- y_b \\
&+ \frac{1}{2}(w^i_+ \partial_- y_i - w^i_- \partial_+ y_i)\Big]. \qquad (64)
\end{aligned}
$$

Substituting the gauge fixing condition $x^i(\xi) = x^i(\xi_0)$ one obtains

$$
\begin{aligned}
&S_{\rm fix}[x^i, w^i_\pm, y] \\
&= \kappa \int d^2\xi \Big[w^i_+ \overline{\Pi}_{+ij}(\Delta W) w^j_- \\
&- \kappa\, w^i_+ \Pi_{+ia}(\Delta W)\tilde{\Theta}_-^{ab}(\Delta W)\partial_- y_b \\
&+ \kappa\, \partial_+ y_a \tilde{\Theta}_-^{ab}(\Delta W)\Pi_{+bi}(\Delta W) w^i_- \\
&+ \frac{\kappa}{2}\, \partial_+ y_a \tilde{\Theta}_-^{ab}(\Delta W)\partial_- y_b \\
&+ \frac{1}{2}(w^i_+ \partial_- y_i - w^i_- \partial_+ y_i)\Big], \qquad (65)
\end{aligned}
$$

where $\Delta W^\mu = \big[\Delta W^i, \Delta V^a(\Delta W^i, y_a)\big]$ with ΔW^i defined by

$$\Delta W^i \equiv \int_P (d\xi^+ w^i_+ + d\xi^- w^i_-), \qquad (66)$$

and $\Delta V^a = \Delta V^a(\Delta W^i, y_a)$ is defined in (39), where argument x^i is replaced by ΔW^i.

5.1 Regaining the T-dual action

The equations of motion for the Lagrange multipliers y_i are

$$\partial_+ w^i_- - \partial_- w^i_+ = 0, \qquad (67)$$

and they have the solution

$$w^i_\pm = \partial_\pm x^i. \qquad (68)$$

For this solution the background field argument ΔW^i defined in (66) reduces to

$$\Delta W^i(\xi) = x^i(\xi) - x^i(\xi_0), \qquad (69)$$

so that the argument ΔV^a becomes

$$\Delta V^a(\Delta W^i, y^a) = \Delta V^a(x^i, y^a), \qquad (70)$$

and therefore the gauge fixed action (65) reduces to the action (36).

5.2 From the gauge fixed action to the completely T-dual action

The equations of motion obtained varying the gauge fixed action (65) over $w^i_\pm$ are

$$
\begin{aligned}
&\overline{\Pi}_{\pm ij}(\Delta W) w^j_\mp - \kappa\, \Pi_{\pm ia}(\Delta W)\tilde{\Theta}_\mp^{ab}(\Delta W)\partial_\mp y_b + \frac{1}{2}\partial_\mp y_i \\
&= \pm 2\kappa\, \overline{\Pi}_{\pm ij}\Theta_\mp^{j\mu}\beta^\pm_\mu(W), \qquad (71)
\end{aligned}
$$

where

$$\beta^\pm_\mu(V) = \mp\frac{1}{2} h_{\mu\nu}(V)\partial_\mp V^\nu. \qquad (72)$$

Terms $\overline{\Pi}_{\pm ij}\Theta_\mp^{j\mu}\beta^\pm_\mu(W)$ are the contribution from the background fields' argument, defined by

$$
\begin{aligned}
\delta_U S_{\rm fix} = -2\kappa^2 \int d^2\xi \Big(&\delta w^i_+ \overline{\Pi}_{+ij}\Theta_-^{j\mu}\beta^+_\mu \\
&+ \delta w^i_- \overline{\Pi}_{-ij}\Theta_+^{j\mu}\beta^-_\mu\Big), \qquad (73)
\end{aligned}
$$

calculated using (A.15), (A.16), and (39).

Using the fact that the background field composition $\overline{\Pi}_{\pm ij}$ is invese to $2\kappa\Theta_\mp^{ij}$ defined by (A.22), we can rewrite the equation of motion (71) expressing the gauge fields as

$$
\begin{aligned}
w^i_\mp = 2\kappa\Theta_\mp^{ij}(\Delta W)\Big[&\kappa\, \Pi_{\pm ja}(\Delta W)\tilde{\Theta}_\mp^{ab}(\Delta W)\partial_\mp y_b \\
&- \frac{1}{2}\partial_\mp y_j\Big] \pm 2\kappa\Theta_{0\pm}^{i\mu}\beta^\pm_\mu(W). \qquad (74)
\end{aligned}
$$

Using the second relation in (A.23), we obtain

$$w^i_\mp = -\kappa\Theta_\mp^{i\mu}(\Delta W)\Big[\partial_\mp y_\mu \mp 2\beta^\pm_\mu(W)\Big]. \qquad (75)$$

Substituting (75) into the gauge fixed action (65), we obtain

$$
\begin{aligned}
&S[y] \\
&= \kappa \int d^2\xi \Big[\partial_+ y_i\Big(\kappa\Theta_-^{ij} - \kappa^2\Theta_-^{ik}\overline{\Pi}_{+kl}\Theta_-^{lj}\Big)\partial_- y_j \\
&+ \Big(-\kappa^2\Theta_-^{ij}\overline{\Pi}_{+jk}\Theta_-^{ka} + \frac{\kappa}{2}\Theta_-^{ia} - \kappa^2\Theta_-^{ij}\Pi_{+jb}\tilde{\Theta}_-^{ba}\Big) \\
&\times \partial_+ y_i\, \partial_- y_a \\
&+ \Big(-\kappa^2\Theta_-^{aj}\overline{\Pi}_{+jk}\Theta_-^{ki} + \frac{\kappa}{2}\Theta_-^{ai} - \kappa^2\tilde{\Theta}_-^{ab}\Pi_{+bj}\Theta_-^{ji}\Big) \\
&\times \partial_+ y_a\, \partial_- y_i + \partial_+ y_a\Big(\frac{\kappa}{2}\tilde{\Theta}_-^{ab} - \kappa^2\Theta_-^{ai}\overline{\Pi}_{+ij}\Theta_-^{jb} \\
&- \kappa^2\Theta_-^{ai}\Pi_{+ic}\tilde{\Theta}_-^{cb} - \kappa^2\tilde{\Theta}_-^{ac}\Pi_{+ci}\Theta_-^{ib}\Big)\partial_- y_b\Big]. \qquad (76)
\end{aligned}
$$

Using (A.22), (A.27), and (A.29) one can rewrite this action as

$$S[y] = \frac{\kappa^2}{2}\int d^2\xi\, \partial_+ y_\mu \Theta_-^{\mu\nu}(\Delta W)\partial_- y_\nu. \qquad (77)$$

In order to find the background fields' argument ΔW^i, we consider the zeroth order of Eqs. (75), and we conclude that

$$\Delta W^i = -\kappa\theta_0^{i\mu}\Delta y_\mu + (g^{-1})^{i\mu}\Delta\tilde{y}_\mu. \tag{78}$$

Using (A.28) and (A.23), we find that $\Delta V^a(\Delta W^i, y^a)$ defined in (39) equals

$$\Delta V^a(\Delta W^i, y_a) = -\kappa\theta_0^{a\mu}\Delta y_\mu + (g^{-1})^{a\mu}\Delta\tilde{y}_\mu. \tag{79}$$

Therefore, we conclude that the background fields' argument is equal to (18), so that the action (77) is the completely T-dual action (14), which is in agreement with Ref. [18]. Comparing the solutions for the gauge fields (68) and (75), we obtain the T-dual transformation law

$$\partial_\mp x^i \cong -\kappa\Theta_\mp^{i\mu}(\Delta V(y))\Big[\partial_\mp y_\mu \mp 2\beta_\mu^\pm(V(y))\Big]. \tag{80}$$

One can verify that two successive T-duality transformations (45) and (80) correspond to the total T-duality transformation (20). Indeed, the relation (80) is just the ith component of this transformation. Substituting $\partial_\pm x^i$ from (80) into (45), using (A.25) and (A.29), we obtain

$$\partial_\pm x^a = -\kappa\Theta_\pm^{a\mu}(\Delta V)\Big[\partial_\pm y_\mu \pm 2\beta_\mu^\mp(V)\Big],$$

which is just the ath component of the complete T-duality transformation. So, we confirm that $\mathcal{T}^a \circ \mathcal{T}^i = \mathcal{T}$.

6 Inverse T-dualization along arbitrary subset of the dual coordinates $\mathcal{T}_i : S[y_\mu] \to S[x^i, y_a]$

Finally, in this section we will show that the T-dualization of the completely T-dual action (14), along arbitrary subset of the dual coordinates y_i leads to T-dual action (36). So, let us start with the T-dual action

$$S[y] = \frac{\kappa^2}{2}\int d^2\xi\ \partial_+ y_\mu\Theta_-^{\mu\nu}(\Delta V(y))\partial_- y_\nu, \tag{81}$$

which is globally invariant to the constant shift of coordinates y_μ

$$\delta y_\mu = \lambda_\mu. \tag{82}$$

We localize this symmetry for the coordinates y_i and obtain the locally invariant action

$$\begin{aligned}
&S_{\text{inv}}[y, y_i^{\text{inv}}, z^i]\\
&= \frac{\kappa^2}{2}\int d^2\xi\Big[D_+y_i\Theta_-^{ij}(\Delta V(y_i^{\text{inv}}, y_a))D_-y_j\\
&\quad + D_+y_i\Theta_-^{ia}(\Delta V(y_i^{\text{inv}}, y_a))\partial_-y_a\\
&\quad +\partial_+y_a\Theta_-^{ai}(\Delta V(y_i^{\text{inv}}, y_a))D_-y_i\\
&\quad +\partial_+y_a\Theta_-^{ab}(\Delta V(y_i^{\text{inv}}, y_a))\partial_-y_b\\
&\quad +\frac{1}{\kappa}(u_{+i}\partial_-z^i - u_{-i}\partial_+z^i)\Big],
\end{aligned} \tag{83}$$

where $D_\pm y_i = \partial_\pm y_i + u_{\pm i}$ are the covariant derivatives. The gauge fields $u_{\pm i}$ transform as $\delta u_{\pm i} = -\partial_\pm\lambda_i$ and the invariant coordinates are defined by $y_i^{\text{inv}} = \int_P(d\xi^+ D_+y_i + d\xi^- D_-y_i)$. After fixing the gauge by $y_i(\xi) = y_i(\xi_0)$, the action becomes

$$\begin{aligned}
&S_{\text{fix}}[y_a, u_{\pm i}, z^i]\\
&= \frac{\kappa^2}{2}\int d^2\xi\Big[u_{+i}\Theta_-^{ij}(\Delta V(\Delta U_i, y_a))u_{-j}\\
&\quad + u_{+i}\Theta_-^{ia}(\Delta V(\Delta U_i, y_a))\partial_-y_a\\
&\quad +\partial_+y_a\Theta_-^{ai}(\Delta V(\Delta U_i, y_a))u_{-i}\\
&\quad +\partial_+y_a\Theta_-^{ab}(\Delta V(\Delta U_i, y_a))\partial_-y_b\\
&\quad +\frac{1}{\kappa}(u_{+i}\partial_-z^i - u_{-i}\partial_+z^i)\Big],
\end{aligned} \tag{84}$$

where $\Delta U_i = \int_P(d\xi^+ u_{+i} + d\xi^- u_{-i})$.

6.1 Regaining the T-dual action

The equations of motion obtained varying the gauge fixed action (84) over the Lagrange multipliers

$$\partial_+u_{-i} - \partial_-u_{+i} = 0, \tag{85}$$

have the solution

$$u_{\pm i} = \partial_\pm y_i. \tag{86}$$

On this solution the variable ΔU_i reduces to

$$\Delta U_i(\xi) = y_i(\xi) - y_i(\xi_0), \tag{87}$$

and therefore

$$\Delta V^\mu(\Delta U_i, y_a) = \Delta V^\mu(y). \tag{88}$$

So, the action (84) becomes the action (81).

6.2 Obtaining the T-dual action

The equations of motion obtained varying the action (84) over $u_{\pm i}$ are

$$\begin{aligned}
&\kappa\Theta_\mp^{ij}(\Delta V(\Delta U_i, y_a))u_{\mp j} + \kappa\Theta_\mp^{ia}(\Delta V(\Delta U_i, y_a))\partial_\mp y_a\\
&\quad +\partial_\mp z^i = \pm 2\kappa\Theta_{0\mp}^{i\mu}\beta_\mu^\pm(V(U_i, y_a)),
\end{aligned} \tag{89}$$

where $\beta_\mu^\pm$ are given by (72). The terms with beta function come from the variation over the argument U_i

$$\delta_U S_{\text{fix}} = -\kappa^2\int d^2\xi(\delta u_{+i}\Theta_{0-}^{i\mu}\beta_\mu^+ + \delta u_{-i}\Theta_{0+}^{i\mu}\beta_\mu^-), \tag{90}$$

and are calculated using (A.15) and (18). Using the fact that $2\kappa\overline{\Pi}_{\mp ij}$ is the inverse of $\Theta^{ij}_{\pm}$, the equation (89) can be rewritten as

$$u_{\mp i} = -2\overline{\Pi}_{\pm ij}\big(\Delta V(\Delta U_i, y_a)\big)\Big[\kappa\Theta^{ja}_{\mp}\big(\Delta V(\Delta U_i, y_a)\big) \times\partial_{\mp}y_a + \partial_{\mp}z^j \mp 2\kappa\Theta^{j\mu}_{0\mp}\beta^{\pm}_{\mu}\big(V(U_i, y_a)\big)\Big]. \tag{91}$$

Substituting (91) into the gauge fixed action (84), using (A.25) we obtain

$$S[z^i, y_a] = \frac{\kappa^2}{2}\int d^2\xi\Big[\frac{2}{\kappa}\partial_+ z^i\overline{\Pi}_{+ij}\partial_- z^j + 2\partial_+ z^i\overline{\Pi}_{+ij}\Theta^{jb}_{-}\partial_- y_b - 2\partial_+ y_a\Theta^{ai}_{-}\overline{\Pi}_{+ij}\partial_- z^j + \partial_+ y_a\tilde{\Theta}^{ab}_{-}\partial_- y_b\Big], \tag{92}$$

which with the help of (A.29) becomes

$$S[z^i, y_a] = \frac{\kappa^2}{2}\int d^2\xi\Big[\frac{2}{\kappa}\partial_+ z^i\overline{\Pi}_{+ij}\partial_- z^j -2\partial_+ z^i\Pi_{+ia}\tilde{\Theta}^{ab}_{-}\partial_- y_b + 2\partial_+ y_a\tilde{\Theta}^{ab}_{-}\Pi_{+bj}\partial_- z^j +\partial_+ y_a\tilde{\Theta}^{ab}_{-}\partial_- y_b\Big]. \tag{93}$$

In order to find the argument of the background fields $\Delta V(\Delta U_i, y_a)$, one considers the zeroth order of Eqs. (91) and obtains

$$\Delta U^{(0)}_i = -\Big[\overline{\Pi}_{0+ij} + \overline{\Pi}_{0-ij}\Big]\Delta z^{(0)j} +\Big[\overline{\Pi}_{0+ij} - \overline{\Pi}_{0-ij}\Big]\Delta\tilde{z}^{(0)j} -\kappa\Big[\overline{\Pi}_{0+ij}\Theta^{ja}_{0-} + \overline{\Pi}_{0-ij}\Theta^{ja}_{0+}\Big]\Delta y^{(0)}_a +\kappa\Big[\overline{\Pi}_{0+ij}\Theta^{ja}_{0-} - \overline{\Pi}_{0-ij}\Theta^{ja}_{0+}\Big]\Delta\tilde{y}^{(0)}_a, \tag{94}$$

where the double variables are defined in analogy with (40). Substituting (94) into (18), we obtain

$$\Delta V^i(\Delta U_i, y_a) = \Delta z^i, \tag{95}$$

and

$$\Delta V^a(\Delta U_i, y_a) = -\kappa\Big[\tilde{\Theta}^{ab}_{0+}\Pi_{0-bi} + \tilde{\Theta}^{ab}_{0-}\Pi_{0+bi}\Big]\Delta z^{(0)i} -\kappa\Big[\tilde{\Theta}^{ab}_{0+}\Pi_{0-bi} - \tilde{\Theta}^{ab}_{0-}\Pi_{0+bi}\Big]\Delta\tilde{z}^{(0)i} -\frac{\kappa}{2}\Big[\tilde{\Theta}^{ab}_{0+} + \tilde{\Theta}^{ab}_{0-}\Big]\Delta y^{(0)}_b -\frac{\kappa}{2}\Big[\tilde{\Theta}^{ab}_{0+} - \tilde{\Theta}^{ab}_{0-}\Big]\Delta\tilde{y}^{(0)}_b, \tag{96}$$

which is exactly (39) with $z^i = x^i$. So, we can conclude that the action (93) is equal to the T-dual action (36).

Comparing the solutions for the gauge fields (86) and (91), we obtain the T-dual transformation law

$$\partial_{\mp}y_i \cong -2\overline{\Pi}_{\pm ij}\big(\Delta z^i, \Delta V^a(\Delta U_i(z^i, y_a), y_a)\big) \times\Big[\kappa\Theta^{ja}_{\mp}\big(\Delta z^i, \Delta V^a(\Delta U_i(z^i, y_a), y_a)\big)\partial_{\mp}y_a + \partial_{\mp}z^j \mp 2\kappa\Theta^{j\mu}_{0\mp}\beta^{\pm}_{\mu}\big(z^i, V^a(U_i(z^i, y_a), y_a)\big)\Big]. \tag{97}$$

These transformations are inverse to (80), so that $\mathcal{T}^i \circ \mathcal{T}_i = 1$. Successively applying (97) and (61), using (A.29) and (A.25), we obtain the ith component of the inverse law of the total T-dualization (21). Its ath component is (61), so we confirm that $\mathcal{T}_a \circ \mathcal{T}_i = \tilde{\mathcal{T}}$.

7 Group of the T-dual transformation laws

In this section we will recapitulate the coordinate transformation laws between the theories considered. In Sect. 3, we performed the T-dualization procedure along the coordinates x^a

$$\mathcal{T}^a : S[x^\mu] \to S[x^i, y_a], \tag{98}$$

and obtained the following coordinate transformation law: (45)

$$\partial_{\mp}x^a \cong -2\kappa\tilde{\Theta}^{ab}_{\mp}\big(x^i, \Delta V^a(x^i, y_a)\big) \times\Big[\Pi_{\pm bi}\big(x^i, \Delta V^a(x^i, y_a)\big)\partial_{\mp}x^i + \frac{1}{2}\partial_{\mp}y_b \mp\beta^{\pm}_b\big(x^i, V^a(x^i, y_a)\big)\Big] \tag{99}$$

where V^a and $\beta^{\pm}_a$ are given by (39) and (34). In the zeroth oder this law implies

$$x^{(0)a} \cong V^{(0)a}(x^i, y_a). \tag{100}$$

In Sect. 4, starting from the action $S[x^i, y_a]$ we performed the T-dualization procedure along the coordinates y_a

$$\mathcal{T}_a : S[x^i, y_a] \to S[x^\mu], \tag{101}$$

and obtained the transformation law (61)

$$\partial_{\mp}y_a \cong -2\Pi_{\pm a\mu}(x)\partial_{\mp}x^\mu \pm 2\beta^{\pm}_a(x), \tag{102}$$

which is the law inverse to (99) and in the zeroth order it implies

$$y^{(0)}_a \cong U^{(0)}_a(x). \tag{103}$$

Multiplying the transformation law (99) from the left side by $\Pi_{\pm ca}(x) \cong \Pi_{\pm ca}\big(x^i, \Delta V^a(x^i, y_a)\big)$, using (100), we obtain the transformation law (102). So, we confirm that $\mathcal{T}^a \circ \mathcal{T}_a = 1$.

In Sect. 5, starting once again from the action $S[x^i, y_a]$, we performed the T-dualization procedure along the undualized coordinates x^i

$$\mathcal{T}^i : S[x^i, y_a] \to S[y_\mu], \tag{104}$$

and obtained the coordinate transformation law (80)

$$\partial_\mp x^i \cong -\kappa \Theta^{i\mu}_{\mp}\big(\Delta V(y)\big)\Big[\partial_\mp y_\mu \mp 2\beta^\pm_\mu\big(V(y)\big)\Big], \tag{105}$$

where V^μ and $\beta^\pm_\mu$ are given by (18) and (72). In the zeroth order it gives

$$x^{(0)i} \cong V^{(0)i}(y). \tag{106}$$

The two successive T-duality transformations (99) and (105) give the complete transformation (20), so that $\mathcal{T}^a \circ \mathcal{T}^i = \mathcal{T}$.

In Sect. 6, starting from the completely T-dual action $S[y]$, we performed the T-dualization procedure along the coordinates y_i

$$\mathcal{T}_i : S[y_\mu] \to S[x^i, y_a], \tag{107}$$

and obtained (97)

$$\begin{aligned}\partial_\mp y_i \cong& -2\overline{\Pi}_{\pm ij}\big(\Delta x^i, \Delta V^a(\Delta U_i(x^i, y_a), y_a)\big)\\ &\times\Big[\kappa\Theta^{ja}_{\mp}\big(\Delta x^i, \Delta V^a(\Delta U_i(x^i, y_a), y_a)\big)\partial_\mp y_a + \partial_\mp x^j\\ &\mp 2\kappa\Theta^{j\mu}_{0\mp}\beta^\pm_\mu\big(x^i, V^a(U_i(x^i, y_a), y_a)\big)\Big],\end{aligned} \tag{108}$$

with V^a, U_i, and $\beta^\pm_\mu$ given by (79), (94), and (72). In the zeroth order this law implies

$$y_i^{(0)} \cong U_i^{(0)}(x^i, y_a). \tag{109}$$

Multiplying (108) from the left by

$$\Theta^{ki}_{\mp}\big(\Delta x^i, \Delta V^a(y)\big) \cong \Theta^{ki}_{\mp}\big(\Delta x^i, \Delta V^a(\Delta U_i(x^i, y_a), y_a)\big),$$

using (106), we obtain the transformation law (105), so that $\mathcal{T}^i \circ \mathcal{T}_i = 1$. Successively applying (108) and (102), using (A.29) and (A.25), we obtain the ith component of the inverse law of the complete T-dualization (21). Its ath component is (102), so we confirm that $\mathcal{T}_a \circ \mathcal{T}_i = \tilde{\mathcal{T}}$.

We can conclude that the elements $1, \mathcal{T}^a$ and $\mathcal{T}_a$, with $d = 1, \ldots, D$, form an Abelian group. The element $\mathcal{T}^a$ is the inverse of the element $\mathcal{T}_a$.

8 Dilaton field in the weakly curved background

The T-duality transformation of the dilaton field in the weakly curved background was considered in Ref. [26]. For completeness and further use, we give here a brief recapitulation of some basic steps of the treatment.

It is well known that a dilaton transformation has a quantum origin. So, let us start with the path integral for the gauge fixed action

$$\mathcal{Z} = \int \mathrm{d}v^\mu_+ \mathrm{d}v^\mu_- \mathrm{d}y_\mu e^{iS_{\text{fix}}(v_\pm, \partial_\pm y)}, \tag{110}$$

where

$$S_{\text{fix}}(v_\pm, \partial_\pm y) = S_0 + S_1\,, \tag{111}$$

with S_1 being the infinitesimal part of the action

$$\begin{aligned}S_0 &= \kappa\int \mathrm{d}^2\xi\,[v^\mu_+\Pi_{0+\mu\nu}v^\nu_- + \frac{1}{2}(v^\mu_+\partial_- y_\mu - v^\mu_-\partial_+ y_\mu)]\,,\\ S_1 &= \kappa\int \mathrm{d}^2\xi\; v^\mu_+ h_{\mu\nu}(V)v^\nu_-\,.\end{aligned} \tag{112}$$

For a constant background ($S_1 = 0$) the path integral is Gaussian and it equals $(\det \Pi_{0+\mu\nu})^{-1}$. In our case the background is coordinate dependent and thus the integral is not Gaussian. The fact that we work with an infinitesimal parameter enables us to show that the final result is formally the same as in the flat case [26],

$$\mathcal{Z} = \int \mathrm{d}y_\mu \,\frac{1}{\det(\Pi_{+\mu\nu}(V))}\, e^{i\,{}^\star S(y)}, \tag{113}$$

where ${}^\star S(y) = \frac{\kappa^2}{2}\int \mathrm{d}^2\xi\; \partial_+ y_\mu\, \Theta^{\mu\nu}_-(V)\partial_- y_\nu$ is the complete T-dual action and $\Pi_{+\mu\nu}(V) = B_{\mu\nu}(V) + \frac{1}{2}G_{\mu\nu}$. Consequently, although for the weakly curved background the functional integration over $v_\pm$ is of the third degree, it produces formally the same result as in the flat space (where the action is Gaussian),

$${}^\bullet\Phi = \Phi - \ln\det\sqrt{2\Pi_{+ab}}. \tag{114}$$

Using the expressions for T-dual fields (43) we can find the relations between the determinants

$$\begin{aligned}\det(2\Pi_{\pm ab}) &= \frac{1}{\det(2\,{}^\bullet\Pi^{ab}_\pm)} = \sqrt{\frac{\det G_{ab}}{\det {}^\bullet G^{ab}}}\\ &= \sqrt{\frac{\det G_{\mu\nu}}{\det {}^\bullet G_{\mu\nu}}},\end{aligned} \tag{115}$$

where because of the relation $\Pi_{\pm ab} = B_{ab} \pm \frac{1}{2}G_{ab}$ we put in the factor 2 for convenience. The symbol ${}^\bullet G_{\mu\nu}$ denotes metric in the whole space-time after partial T-dualization along x^a directions. With the help of last relation we can show that the change of space-time measure in the path integral is correct

$$\begin{aligned}\sqrt{\det G_{\mu\nu}}\,\mathrm{d}x^i\mathrm{d}x^a &\to \sqrt{\det G_{\mu\nu}}\,\mathrm{d}x^i\,\frac{1}{\det(2\,\Pi_{+ab})}\,\mathrm{d}y_a\\ &= \sqrt{\det {}^\bullet G_{\mu\nu}}\,\mathrm{d}x^i\,\mathrm{d}y_a,\end{aligned} \tag{116}$$

when we performed T-dualization T^a along x^a directions.

9 Comparison with the existing facts

9.1 T-dualization chain for the background with H flux

In this section we will compare our results with the T-dualization chain of Ref. [16]. The coordinates of the $D = 3$-dimensional torus will be denoted by x^1, x^2, x^3. Because of the different notation, the background fields considered in this paper and those considered in [16], which will be denoted $\mathcal{G}$ and $\mathcal{B}$, are related by

$$\mathcal{B}_{\mu\nu} = -2B_{\mu\nu}, \quad \mathcal{G}_{\mu\nu} = G_{\mu\nu}, \quad \mu,\nu = 1,2,3. \tag{117}$$

Nontrivial components of the background considered in Ref. [16] are

$$\mathcal{G}_{\mu\nu} = \delta_{\mu\nu}, \quad \mathcal{B}_{12} = Hx^3, \tag{118}$$

which in our notation corresponds to the background fields

$$G_{\mu\nu} = \delta_{\mu\nu}, \quad B_{12} = -\frac{1}{2}Hx^3. \tag{119}$$

Let us first compare the results in the case $d = 1$, corresponding to the transition

T^1 : torus with H-flux → twisted torus.

To do so, let us perform T-dualization along the direction x^1, $T^1 : S[x] \to S[y_1, x^2, x^3]$, for the string moving in the background (119). The indices take the values $a, b \in \{1\}$ and $i, j \in \{2, 3\}$. Because the only nontrivial component of the Kalb–Ramond field is $B_{ai} = -\frac{1}{2}Hx^3\delta_{i2}$, the effective fields are just $\tilde{G}^E_{\mu\nu} = \delta_{\mu\nu}$ and $\tilde{\theta}^{ab} = 0$. So, the T-dual background fields (44), in the linear order in H, are

$$\begin{aligned} &{}^\bullet G_{ij} = \delta_{ij}, \quad {}^\bullet B_{ij} = 0,\\ &{}^\bullet G^{ab} = \delta^{ab}, \quad {}^\bullet B^{ab} = 0,\\ &{}^\bullet G^a{}_i = -Hx^3\delta_{i2}, \quad {}^\bullet B^a{}_i = 0. \end{aligned} \tag{120}$$

Therefore

$${}^\bullet G_{\mu\nu} = \begin{pmatrix} 1 & -Hx^3 & 0 \\ -Hx^3 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = {}^\bullet\mathcal{G}_{\mu\nu}, \tag{121}$$

and

$${}^\bullet B_{\mu\nu} = 0 = {}^\bullet\mathcal{B}_{\mu\nu}, \tag{122}$$

so our result is in agreement with that of Ref. [16].

Now, let us make the comparison in the case $d = 2$, which corresponds to the transition

$T^1 \circ T^2$: torus with H-flux → Q-flux non-geometry.

Instead to perform T^2 dualization, from twisted torus to Q-flux non-geometry as in [16], we will start from the initial background with H-flux and perform T-dualizations along x^1 and x^2, $T^1 \circ T^2 : S[x] \to S[y_1, y_2, x^3]$. The indices take the values $a, b \in \{1, 2\}$ and $i, j \in \{3\}$. Because the only nontrivial contribution to the Kalb–Ramond field B_{ab} is $B_{12} = -\frac{1}{2}Hx^3$, the effective background fields are $\tilde{G}^E_{ab} = \delta_{ab}$, $\bar{G}^E_{ij} = \delta_{ij}$, and the only nonzero component of $\tilde{\theta}^{ab}$ is $\tilde{\theta}^{12} = \frac{1}{\kappa}Hx^3$. The T-dual background fields linear in H are therefore

$${}^\bullet G_{ij} = \delta_{ij}, \quad {}^\bullet G^{ab} = \delta^{ab}, \quad {}^\bullet G^a i = 0, \tag{123}$$

and

$${}^\bullet B_{ij} = 0, \quad {}^\bullet B^{12} = \frac{1}{2}Hx^3, \quad {}^\bullet B^a{}_i = 0. \tag{124}$$

Consequently

$${}^\bullet G_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = {}^\bullet\mathcal{G}_{\mu\nu}, \tag{125}$$

$${}^\bullet\mathcal{B}_{\mu\nu} = -2\,{}^\bullet B_{\mu\nu} = \begin{pmatrix} 0 & -Hx^3 & 0 \\ Hx^3 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{126}$$

so the results of this paper and [16] in this case coincide.

9.2 Non-associativity of R-flux background and breaking of Jacobi identity

In Refs. [18,19] we obtained T-dual transformation laws connecting T-dual coordinates y_μ with the initial coordinates x^μ. Here we will reduce our case to the 3-dimensional torus with H-flux considered in [8]. Then, the full T-dualization along all coordinates corresponds to the so-called R-flux. So, we are going to calculate its characteristic features: non-associativity relation and breaking of Jacobi identity.

We will work in the background of Sect. 9.1 consisting of euclidean flat metric $G_{\mu\nu}$ and Kalb–Ramond field with one nontrivial component $B_{12} = -\frac{1}{2}Hx^3$. T-dual transformation laws for coordinates y_μ $(\mu = 1, 2, 3)$ are of the form

$$y_1' \cong \frac{1}{\kappa}\pi_1 + \frac{1}{2}Hx^3x'^2, \tag{127}$$

$$y_2' \cong \frac{1}{\kappa}\pi_2 - \frac{1}{2}Hx^3x'^1, \tag{128}$$

$$y_3' \cong \frac{1}{\kappa}\pi_3, \tag{129}$$

where π_1, π_2, π_3 are canonically conjugated momenta for coordinates x^1, x^2, x^3, respectively. The initial space is a geometric one, so, the standard Poisson algebra is satisfied,

$$\{x^\mu(\sigma), \pi_\nu(\bar\sigma)\} = \delta^\mu{}_\nu \delta(\sigma - \bar\sigma),$$
$$\{x^\mu, x^\nu\} = \{\pi_\mu, \pi_\nu\} = 0. \quad (130)$$

From (127)–(129) we obtain

$$\{y'_\mu(\sigma), y'_\nu(\bar\sigma)\} = -\frac{1}{2\kappa} H \varepsilon_{\mu\nu\rho} x'^\rho \delta(\sigma - \bar\sigma), \quad (131)$$

which, after two partial integrations, produces

$$\{y_\mu(\sigma), y_\nu(\bar\sigma)\} = \frac{1}{2\kappa} H \varepsilon_{\mu\nu\rho} \left[x^\rho(\sigma) - x^\rho(\bar\sigma)\right] \theta(\sigma - \bar\sigma), \quad (132)$$

where $\varepsilon_{\mu\nu\rho}$ is the 3-dimensional Levi-Civita tensor ($\varepsilon_{123} = 1$) and the function $\theta(\sigma)$ is defined as

$$\theta(\sigma) \equiv \begin{cases} 0 & \text{if} \quad \sigma = 0 \\ 1/2 & \text{if} \quad 0 < \sigma < 2\pi, \quad \sigma \in [0, 2\pi]. \\ 1 & \text{if} \quad \sigma = 2\pi \end{cases} \quad (133)$$

Using the standard Poisson algebra (130) and transformation laws (127)–(129), after one partial integration, we get

$$\{\{y_\mu(\sigma_1), y_\nu(\sigma_2)\}, y_\rho(\sigma_3)\}$$
$$= \frac{1}{2\kappa^2} H \varepsilon_{\mu\nu\rho} [\theta(\sigma_2 - \sigma_1)\theta(\sigma_1 - \sigma_3)$$
$$+ \theta(\sigma_1 - \sigma_2)\theta(\sigma_2 - \sigma_3)], \quad (134)$$

Now we have all ingredients to calculate the non-associativity relation

$$\{\{y_\mu(\sigma_1), y_\nu(\sigma_2)\}, y_\rho(\sigma_3)\} - \{y_\mu(\sigma_1), \{y_\nu(\sigma_2), y_\rho(\sigma_3)\}\}$$
$$= \frac{1}{2\kappa^2} H \varepsilon_{\mu\nu\rho} [2\theta(\sigma_3 - \sigma_2)\theta(\sigma_2 - \sigma_1)$$
$$+ \theta(\sigma_1 - \sigma_3)\theta(\sigma_3 - \sigma_2) + \theta(\sigma_3 - \sigma_1)\theta(\sigma_1 - \sigma_2)] \quad (135)$$

and breaking of Jacobi identity

$$\{y_\mu(\sigma_1), y_\nu(\sigma_2), y_\rho(\sigma_3)\}$$
$$\equiv \{\{y_\mu(\sigma_1), y_\nu(\sigma_2)\}, y_\rho(\sigma_3)\} + \{\{y_\nu(\sigma_2), y_\rho(\sigma_3)\}, y_\mu(\sigma_1)\}$$
$$+ \{\{y_\rho(\sigma_3), y_\mu(\sigma_1)\}, y_\nu(\sigma_2)\}$$
$$= \frac{1}{\kappa^2} H \varepsilon_{\mu\nu\rho} [\theta(\sigma_1 - \sigma_2)\theta(\sigma_2 - \sigma_3)$$
$$+ \theta(\sigma_3 - \sigma_1)\theta(\sigma_1 - \sigma_2) + \theta(\sigma_2 - \sigma_3)\theta(\sigma_3 - \sigma_1)]. \quad (136)$$

For example, for $\sigma_1 = 2\pi + \sigma$ and $\sigma_2 = \sigma_3 = \sigma$ one has

$$\{y_\mu(2\pi + \sigma), y_\nu(\sigma), y_\rho(\sigma)\} = -\frac{1}{\kappa^2} H \varepsilon_{\mu\nu\rho}. \quad (137)$$

In the approach of this article, the background of the T-dual theory depends on the non-local variable V^μ, which incorporates the main features of the non-geometric spaces. Reducing our procedure to three dimensions and using the backgrounds of Refs. [8,16,27], we showed that our structure of arguments of background fields proves the proposal of Refs. [8,27] that non-associativity and breaking of Jacobi identity are features of R-flux background.

9.3 Critical surface

Let us generalize the discussion of Ref. [20] where the critical surface, which separates equivalent sections of background fields, generalizes the critical radius. Using the dilaton field analysis, namely the relation (115), we can conclude that T-duality maps the theories with a given

$$\det(2\Pi_{\pm ab})$$

into the theories with

$$1/\det(2\Pi_{\pm ab}),$$

so that all different theories are in the region

$$\det(2\Pi_{\pm ab}) \leq 0.$$

The theories which background fields satisfy the condition $\det(2\Pi_{\pm ab}) = 1$, are mapped into each other under T-duality. This is a generalization of the critical radius and can be consider as a critical surface. So, relation (115) implies $\sqrt{\det G_{ab}} = \sqrt{\det {}^\bullet G^{ab}}$, which means that a dual volume is equal to the initial one. At the critical surface the extended symmetry should be expected.

Let us, following [20], give an example of the relation between the original and T-dual background fields. We will consider the initial background in the 4-dimensional torus T^4 given by

$$G_{\mu\nu} = g\delta_{\mu\nu}, \quad B_{\mu\nu} = b^i E^i_{\mu\nu}, \quad (138)$$

where

$$E^1 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}, \quad E^2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix},$$
$$E^3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}, \quad (139)$$

satisfies

$$E^i E^j = -\delta^{ij} I + \varepsilon^{ijk} E^k, \qquad \varepsilon^{123} = 1. \quad (140)$$

The zero modes of the T-dual metric and T-dual Kalb–Ramond field (17) for the initial fields (138) are

$$^\star G^{\mu\nu} = (G_E^{-1})^{\mu\nu} = \frac{g}{g^2+b^2} I \tag{141}$$

and

$$^\star B^{\mu\nu} = \frac{\kappa}{2}\theta^{\mu\nu} = -\frac{1}{2}\frac{b^i}{g^2+b^2} E^i , \tag{142}$$

with $b^2 = b^i b^i$. They have the same form as the initial fields (138)

$$^\star G_{\mu\nu} = {}^\star g \delta_{\mu\nu}, \quad {}^\star B_{\mu\nu} = {}^\star b^i E^i_{\mu\nu}, \tag{143}$$

with

$$^\star g = \frac{g}{g^2+b^2}, \quad {}^\star b = -\frac{b^i}{g^2+b^2}. \tag{144}$$

One easily shows

$$^\star g^2 + {}^\star b^2 = \frac{1}{g^2+b^2}. \tag{145}$$

In spheric coordinates one has

$$(g, b^1, b^2, b^3) = (r\cos\theta, r\sin\theta\cos\varphi, \\ r\sin\theta\sin\varphi\cos\varphi_1, r\sin\theta\sin\varphi\sin\varphi_1), \tag{146}$$

so $g^2 + b^2 = r^2$, and using (144) one obtains

$$({}^\star g, {}^\star b^1, {}^\star b^2, {}^\star b^3) \\ = \left(\frac{1}{r}\cos\theta, -\frac{1}{r}\sin\theta\cos\varphi, -\frac{1}{r}\sin\theta\sin\varphi\cos\varphi_1, \\ -\frac{1}{r}\sin\theta\sin\varphi\sin\varphi_1\right). \tag{147}$$

Therefore, T-duality transforms $(r, \theta, \varphi, \varphi_1)$ to

$$({}^\star r, {}^\star\theta, {}^\star\varphi, {}^\star\varphi_1) = \left(\frac{1}{r}, -\theta, \varphi, \varphi_1\right). \tag{148}$$

From the relation $\Pi_\pm G^{-1}\Pi_\mp = -\frac{1}{4}G_E$ we find

$$\det(2\Pi_{\pm\mu\nu}) = \frac{g^2}{{}^\star g^2} = (g^2+b^2)^2 = r^4 . \tag{149}$$

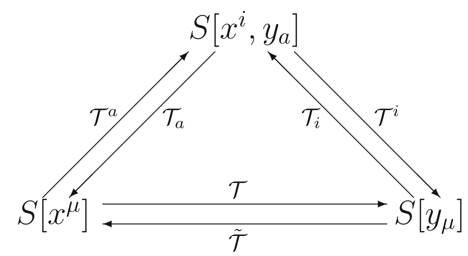
Backgrounds corresponding to $r = 1$ are mapped into themselves. The subset of this is the fixed surface with the condition

$$\det(2\Pi_{\pm\mu\nu}) = r^4 = 1 , \theta = 0$$

or $g = 1,\ b^i = 0$.

10 Conclusion

In this paper, we considered the closed string propagating in the weakly curved background (6), composed of a constant metric $G_{\mu\nu}$ and a linearly coordinate dependent Kalb–Ramond field $B_{\mu\nu}$, with infinitesimal field strength. We investigated the application of the generalized T-dualization procedure on the arbitrary set of coordinates and obtained the following T-duality diagram:



Let us stress that generalized T-dualization procedure enables the T-dualization along arbitrary direction, even if the background fields depend on these directions. The consequence of this procedure is that the arguments of the background fields, such as ΔV^a, are non-local. They are non-local by definition, as they are the line integrals of the gauge fields. Once the explicit form is obtained the non-locality is seen in a fact that they depend on double coordinates $\tilde{x}$ and $\tilde{y}$, which are the line integrals of the τ and σ derivatives of the original coordinates. To all the theories considered, except the initial theory, there corresponds the non-geometric, non-local flux.

The generalized T-dualization procedure was first applied along arbitrary d ($d = 1, \ldots, D-1$) coordinates $x^a = \{x^{\mu_1}, \ldots, x^{\mu_d}\}$. We obtained the T-dual action $S[x^i, y_a]$, given by Eq. (36) with the dual background fields equal to

$$^\bullet\Pi_{+ij} = \overline{\Pi}_{+ij}, \quad {}^\bullet\Pi_{+i}{}^a = -\kappa\Pi_{+ib}\tilde{\Theta}_-^{ba}, \\ {}^\bullet\Pi^a_{+i} = \kappa\tilde{\Theta}_-^{ab}\Pi_{+bi}, \quad {}^\bullet\Pi_+^{ab} = \frac{\kappa}{2}\tilde{\Theta}_-^{ab}. \tag{150}$$

The argument of all background fields, $[x^i, V^a(x^i, y_a)]$, depends nonlinearly on coordinates x^i, y_a through their doubles $\tilde{x}^i$, $\tilde{y}_a$ [see (39) and (40)]. All actions $S[x^i, y_a]$ are physically equivalent, but they are described with coordinates $x^i = \{x^{\mu_{d+1}}, \ldots, x^{\mu_D}\}$, for the untreated directions and dual coordinates $y_a = \{y_{\mu_1}, \ldots, y_{\mu_d}\}$, for the dualized directions. The case $d = D$ corresponds to the completely T-dual action with the T-dual fields $\frac{\kappa}{2}\Theta_-^{\mu\nu}\big(V(y)\big)$ and the case $d = 0$ to the initial action with the background $\Pi_{+\mu\nu}(x)$.

Applying the procedure to the T-dual action along dual directions $y_a = \{y_{\mu_1}, \ldots, y_{\mu_d}\}$ we obtained the initial theory, and applying it to the untreated directions $x^i = \{x^{\mu_{d+1}}, \ldots, x^{\mu_D}\}$ we obtained the completely T-dual theory. All these derivations confirmed that the set of all T-dualizations forms an Abelian group. The neutral element of the group is the unexecuted T-dualization, while the T-dualizations along some subset of original directions $\mathcal{T}^a$ is inverse to the T-dualizations along the set of the corresponding dual directions $\mathcal{T}_a$.

Funded by SCOAP[3].

Appendix A: The background field compositions

The background field compositions $\Pi_{\pm\mu\nu}$ of the initial theory are

$$\Pi_{\pm\mu\nu} = B_{\mu\nu} \pm \frac{1}{2}G_{\mu\nu}, \tag{A.1}$$

where $G_{\mu\nu}$ and $B_{\mu\nu}$ are the initial metric and the initial Kalb–Ramond field. The background field compositions $\Theta_\pm^{\mu\nu}$ of the T-dual theory are

$$\Theta_\pm^{\mu\nu} \equiv -\frac{2}{\kappa}(G_E^{-1}\Pi_\pm G^{-1})^{\mu\nu} = \theta^{\mu\nu} \mp \frac{1}{\kappa}(G_E^{-1})^{\mu\nu}, \tag{A.2}$$

with $G_{E\mu\nu}$ being the effective metric

$$G_{E\mu\nu} \equiv G_{\mu\nu} - 4(BG^{-1}B)_{\mu\nu}, \tag{A.3}$$

and $\theta^{\mu\nu}$ being the parameter of non-commutativity

$$\theta^{\mu\nu} \equiv -\frac{2}{\kappa}(G_E^{-1}BG^{-1})^{\mu\nu}. \tag{A.4}$$

These background field compositions satisfy

$$\Pi_{\pm\mu\nu}\Theta_\mp^{\nu\rho} = \Theta_\pm^{\rho\nu}\Pi_{\mp\nu\mu} = \frac{1}{2\kappa}\delta_\mu^\rho. \tag{A.5}$$

Let us define the analogs of $\Theta_\pm^{\mu\nu}$ in the d- and $D-d$-dimensional subspaces determined by coordinates $x^a = \{x^{\mu_1}, \dots, x^{\mu_d}\}$ and $x^i = \{x^{\mu_{d+1}}, \dots, x^{\mu_D}\}$, where $d = 1, 2, \dots, D-1$. The effective metrics in these subspaces are defined by

$$\tilde{G}_{Eab} \equiv G_{ab} - 4B_{ac}(\tilde{G}^{-1})^{cd}B_{db},$$
$$\bar{G}_{Eij} \equiv G_{ij} - 4B_{ik}(\bar{G}^{-1})^{kl}B_{lj}, \tag{A.6}$$

where $\tilde{G}_{ab} \equiv G_{ab}$ and $\bar{G}_{ij} \equiv G_{ij}$. Using these we define the following field compositions:

$$\tilde{\Theta}_\pm^{ab} \equiv -\frac{2}{\kappa}(\tilde{G}_E^{-1})^{ac}\Pi_{\pm cd}(\tilde{G}^{-1})^{db},$$
$$\bar{\Theta}_\pm^{ij} \equiv -\frac{2}{\kappa}(\bar{G}_E^{-1})^{ik}\Pi_{\pm kl}(\bar{G}^{-1})^{lj}, \tag{A.7}$$

which are in fact the inverses of $2\kappa\Pi_{\mp ab}$ and $2\kappa\Pi_{\mp ij}$

$$\tilde{\Theta}_\pm^{ab}\Pi_{\mp bc} = \Pi_{\mp cb}\tilde{\Theta}_\pm^{ba} = \frac{1}{2\kappa}\delta_c^a,$$
$$\bar{\Theta}_\pm^{ij}\Pi_{\mp jk} = \Pi_{\mp kj}\bar{\Theta}_\pm^{ji} = \frac{1}{2\kappa}\delta_k^i. \tag{A.8}$$

Analogously as the fields theta $\Theta_\pm^{\mu\nu}$ defined in the whole space by (A.2), the theta fields defined in the subspaces can be separated into antisymmetric and symmetric parts as

$$\tilde{\Theta}_\pm^{ab} = \tilde{\theta}^{ab} \mp \frac{1}{\kappa}(\tilde{G}_E^{-1})^{ab},$$
$$\bar{\Theta}_\pm^{ij} = \bar{\theta}^{ij} \mp \frac{1}{\kappa}(\bar{G}_E^{-1})^{ij}, \tag{A.9}$$

where

$$\tilde{\theta}^{ab} \equiv -\frac{2}{\kappa}(\tilde{G}_E^{-1})^{ac}B_{cd}(\tilde{G}^{-1})^{db},$$
$$\bar{\theta}^{ij} \equiv -\frac{2}{\kappa}(\bar{G}_E^{-1})^{ik}B_{kl}(\bar{G}^{-1})^{lj}. \tag{A.10}$$

In the zeroth order the quantities $\Pi_{\pm\mu\nu}$, $\Theta_\pm^{\mu\nu}$, $\tilde{\Theta}_\pm^{ab}$, and $\bar{\Theta}_\pm^{ij}$ reduce to

$$\Pi_{0\pm\mu\nu} = b_{\mu\nu} \pm \frac{1}{2}G_{\mu\nu},$$
$$\Theta_{0\pm}^{\mu\nu} = -\frac{2}{\kappa}(g^{-1})^{\mu\rho}\Pi_{0\pm\rho\sigma}(G^{-1})^{\sigma\nu} = \theta_0^{\mu\nu} \mp \frac{1}{\kappa}(g^{-1})^{\mu\nu},$$
$$\tilde{\Theta}_{0\pm}^{ab} = -\frac{2}{\kappa}(\tilde{g}^{-1})^{ac}\,\Pi_{0\pm cd}(\tilde{G}^{-1})^{db} = \tilde{\theta}_0^{ab} \mp \frac{1}{\kappa}(\tilde{g}^{-1})^{ab},$$
$$\bar{\Theta}_{0\pm}^{ij} = -\frac{2}{\kappa}(\bar{g}^{-1})^{ik}\,\Pi_{0\pm kl}(\bar{G}^{-1})^{lj} = \bar{\theta}_0^{ij} \mp \frac{1}{\kappa}(\bar{g}^{-1})^{ij}, \tag{A.11}$$

where the zeroth order effective metrics are

$$g_{\mu\nu} = G_{\mu\nu} - 4b_{\mu\rho}(G^{-1})^{\rho\sigma}b_{\sigma\nu},$$
$$\tilde{g}_{ab} = G_{ab} - 4b_{ac}(\tilde{G}^{-1})^{cd}b_{db},$$
$$\bar{g}_{ij} = G_{ij} - 4b_{ik}(\bar{G}^{-1})^{kl}b_{lj}, \tag{A.12}$$

and the zeroth order non-commutativity parameters are

$$\theta_0^{\mu\nu} = -\frac{2}{\kappa}(g^{-1})^{\mu\rho}b_{\rho\sigma}(G^{-1})^{\sigma\nu},$$
$$\tilde{\theta}_0^{ab} = -\frac{2}{\kappa}(\tilde{g}^{-1})^{ac}\,b_{cd}(\tilde{G}^{-1})^{db}$$
$$\bar{\theta}_0^{ij} = -\frac{2}{\kappa}(\bar{g}^{-1})^{ik}\,b_{kl}(\bar{G}^{-1})^{lj}. \tag{A.13}$$

Quantities $\Pi_{0\pm\mu\nu}$, $\Theta^{\mu\nu}_{0\pm}$, $\tilde{\Theta}^{ab}_{0\pm}$, and $\bar{\Theta}^{ij}_{0\pm}$ satisfy

$$\Pi_{0\pm\mu\nu}\Theta^{\nu\rho}_{0\mp} = \Theta^{\rho\nu}_{0\pm}\Pi_{0\mp\nu\mu} = \frac{1}{2\kappa}\delta^{\rho}_{\mu},$$
$$\Pi_{0\pm ab}\tilde{\Theta}^{bc}_{0\mp} = \tilde{\Theta}^{cb}_{0\pm}\Pi_{0\mp ba} = \frac{1}{2\kappa}\delta^{c}_{a},$$
$$\Pi_{0\pm ij}\bar{\Theta}^{jk}_{0\mp} = \bar{\Theta}^{kj}_{0\pm}\Pi_{0\mp ji} = \frac{1}{2\kappa}\delta^{k}_{i}. \quad (A.14)$$

The non-commutativity parameters theta $\Theta^{\mu\nu}_{\pm}$, $\tilde{\Theta}^{ab}_{\pm}$, and $\bar{\Theta}^{ij}_{\pm}$ can be expressed as

$$\Theta^{\mu\nu}_{\pm} = \Theta^{\mu\nu}_{0\pm} - 2\kappa\Theta^{\mu\rho}_{0\pm}h_{\rho\sigma}\Theta^{\sigma\nu}_{0\pm},$$
$$\tilde{\Theta}^{ab}_{\pm} = \tilde{\Theta}^{ab}_{0\pm} - 2\kappa\tilde{\Theta}^{ac}_{0\pm}h_{cd}\tilde{\Theta}^{db}_{0\pm},$$
$$\bar{\Theta}^{ij}_{\pm} = \bar{\Theta}^{ij}_{0\pm} - 2\kappa\bar{\Theta}^{ik}_{0\pm}h_{kl}\bar{\Theta}^{lj}_{0\pm}. \quad (A.15)$$

Appendix A.1: Relations between field compositions

In Sect. 3.2 we introduced the background field composition

$$\overline{\Pi}_{\pm ij} \equiv \Pi_{\pm ij} - 2\kappa\Pi_{\pm ia}\tilde{\Theta}^{ab}_{\mp}\Pi_{\pm bj}, \quad (A.16)$$

and analogously we define

$$\widetilde{\Pi}_{\pm ab} \equiv \Pi_{\pm ab} - 2\kappa\Pi_{\pm ai}\bar{\Theta}^{ij}_{\mp}\Pi_{\pm jb}. \quad (A.17)$$

Here we will show that these quantities are the inverses of the ordinary non-commutativity parameters theta, projected to the i- and a-subspaces [see (A.22)].

Let us express the tensors $\Pi_{\pm\mu\nu}$ and $\Theta^{\mu\nu}_{\pm}$, which satisfy (A.5), in a block-wise form as

$$\Pi_{\pm\mu\nu} = \begin{pmatrix} \Pi_{\pm ij} & \Pi_{\pm ib} \\ \Pi_{\pm aj} & \Pi_{\pm ab} \end{pmatrix}, \quad \Theta^{\mu\nu}_{\pm} = \begin{pmatrix} \Theta^{ij}_{\pm} & \Theta^{ib}_{\pm} \\ \Theta^{aj}_{\pm} & \Theta^{ab}_{\pm} \end{pmatrix}. \quad (A.18)$$

We will use the definition of block-wise inversion, which states that the inverse of the matrix of the form

$$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \quad (A.19)$$

equals

$$M^{-1} = \begin{pmatrix} (A - BD^{-1}C)^{-1} & -A^{-1}B(D - CA^{-1}B)^{-1} \\ -D^{-1}C(A - BD^{-1}C)^{-1} & (D - CA^{-1}B)^{-1} \end{pmatrix}. \quad (A.20)$$

Applying (A.20) to the first matrix in (A.18), Eq. (A.5) implies

$$2\kappa\Theta^{ij}_{\mp} = \left(\Pi_{\pm ij} - 2\kappa\Pi_{\pm ia}\tilde{\Theta}^{ab}_{\mp}\Pi_{\pm bj}\right)^{-1},$$
$$2\kappa\Theta^{ib}_{\mp} = -2\kappa\bar{\Theta}^{ij}_{\mp}\Pi_{\pm ja}(\Pi_{\pm ab} - 2\kappa\Pi_{\pm ak}\bar{\Theta}^{kl}_{\mp}\Pi_{\pm lb})^{-1},$$
$$2\kappa\Theta^{aj}_{\mp} = -2\kappa\tilde{\Theta}^{ab}_{\mp}\Pi_{\pm bi}(\Pi_{\pm ij} - 2\kappa\Pi_{\pm ic}\tilde{\Theta}^{cd}_{\mp}\Pi_{\pm dj})^{-1},$$
$$2\kappa\Theta^{ab}_{\mp} = (\Pi_{\pm ab} - 2\kappa\Pi_{\pm ai}\bar{\Theta}^{ij}_{\mp}\Pi_{\pm jb})^{-1}, \quad (A.21)$$

and we can conclude that (A.16) and (A.17) are the inverses of $2\kappa\Theta^{ij}_{\mp}$ and $2\kappa\Theta^{ab}_{\mp}$, respectively. So, we can write

$$\overline{\Pi}_{\pm ij}\Theta^{jk}_{\mp} = \Theta^{kj}_{\mp}\overline{\Pi}_{\pm ji} = \frac{1}{2\kappa}\delta^{k}_{i},$$
$$\widetilde{\Pi}_{\pm ab}\Theta^{bc}_{\mp} = \Theta^{cb}_{\mp}\widetilde{\Pi}_{\pm ba} = \frac{1}{2\kappa}\delta^{c}_{a}, \quad (A.22)$$

and

$$\Theta^{ib}_{\mp} = -2\kappa\bar{\Theta}^{ij}_{\mp}\Pi_{\pm ja}\Theta^{ab}_{\mp},$$
$$\Theta^{aj}_{\mp} = -2\kappa\tilde{\Theta}^{ab}_{\mp}\Pi_{\pm bi}\Theta^{ij}_{\mp}. \quad (A.23)$$

Applying (A.20) to the second matrix in (A.18), Eq. (A.5) implies

$$2\kappa\Pi_{\mp ij} = (\Theta^{ij}_{\pm} - 2\kappa\Theta^{ia}_{\pm}\widetilde{\Pi}_{ab\mp}\Theta^{bj}_{\pm})^{-1},$$
$$2\kappa\Pi_{\mp ib} = -2\kappa\overline{\Pi}_{\mp ij}\Theta^{ja}_{\pm}(\Theta^{ab}_{\pm} - 2\kappa\Theta^{ak}_{\pm}\overline{\Pi}_{\mp kl}\Theta^{lb}_{\pm})^{-1},$$
$$2\kappa\Pi_{\mp aj} = -2\kappa\widetilde{\Pi}_{ab\mp}\Theta^{bi}_{\pm}(\Theta^{ij}_{\pm} - 2\kappa\Theta^{ic}_{\pm}\widetilde{\Pi}_{\mp cd}\Theta^{dj}_{\pm})^{-1},$$
$$2\kappa\Pi_{\mp ab} = (\Theta^{ab}_{\pm} - 2\kappa\Theta^{ai}_{\pm}\overline{\Pi}_{\mp ij}\Theta^{jb}_{\pm})^{-1}, \quad (A.24)$$

so using (A.8) we conclude that

$$\bar{\Theta}^{ij}_{\pm} = \Theta^{ij}_{\pm} - 2\kappa\Theta^{ia}_{\pm}\widetilde{\Pi}_{ab\mp}\Theta^{bj}_{\pm},$$
$$\tilde{\Theta}^{ab}_{\pm} = \Theta^{ab}_{\pm} - 2\kappa\Theta^{ai}_{\pm}\overline{\Pi}_{\mp ij}\Theta^{jb}_{\pm}, \quad (A.25)$$

and

$$\Pi_{\mp ib} = -2\kappa\overline{\Pi}_{\mp ij}\Theta^{ja}_{\pm}\Pi_{\mp ab},$$
$$\Pi_{\mp aj} = -2\kappa\widetilde{\Pi}_{\mp ab}\Theta^{bi}_{\pm}\Pi_{\mp ij}. \quad (A.26)$$

Let us derive some useful relations between these quantities. Equation (A.5), for $\mu = a$, $\nu = i$ and $\mu = i$, $\nu = a$, becomes

$$\Pi_{\pm ab}\Theta^{bi}_{\mp} = -\Pi_{\pm aj}\Theta^{ji}_{\mp},$$
$$\Pi_{\pm ij}\Theta^{ja}_{\mp} = -\Pi_{\pm ib}\Theta^{ba}_{\mp}, \quad (A.27)$$

while taking $\mu = a$, $\nu = b$ and $\mu = i$, $\nu = j$ we obtain

$$\Pi_{\pm ac}\Theta^{cb}_{\mp} + \Pi_{\pm ai}\Theta^{ib}_{\mp} = \frac{1}{2\kappa}\delta^{b}_{a},$$
$$\Pi_{\pm ia}\Theta^{aj}_{\mp} + \Pi_{\pm ik}\Theta^{kj}_{\mp} = \frac{1}{2\kappa}\delta^{j}_{i}. \quad (A.28)$$

Multiplying Eq. (A.27) from the left with $\tilde{\Theta}^{ca}_{\mp}$ and from the right with $\overline{\Pi}_{\mp ik}$ we get the relation

$$\Theta^{ci}_{\mp}\overline{\Pi}_{\pm ik} = -\tilde{\Theta}^{ca}_{\mp}\Pi_{\pm ak}, \quad (A.29)$$

while multiplying Eq. (A.28) from the right with $\bar{\Theta}^{ki}_{\mp}$ and from the left with $\tilde{\Pi}_{\pm ac}$, we obtain

$$\Theta^{ka}_{\mp}\widetilde{\Pi}_{\pm ac} = -\bar{\Theta}^{ki}_{\mp}\Pi_{\pm ic}. \quad (A.30)$$

References

1. A. Giveon, M. Porrati, E. Rabinovici, Phys. Rep. **244**, 77–202 (1994)
2. E. Alvarez, L. Alvarez-Gaume, Y. Lozano, Nucl. Phys. Proc. Suppl. **41**, 1 (1995)
3. B. Zwiebach, *A First Course in String Theory*, 2nd edn. (Cambridge University Press, 2009)
4. K. Becker, M. Becker, J.H. Schwarz, *String Theory and M-Theory—A Modern Introdution* (Cambridge University Press, 2007)
5. T.H. Buscher, Phys. Lett. B **201**(4), 466 (1988)
6. M. Roček, E. Verlinde, Nucl. Phys. B **373**, 630 (1992)
7. J. Shelton, W. Taylor, B. Wecht, JHEP **10**, 085 (2005)
8. R. Blumenhagen, A. Deser, D. Lüst, E. Plauschinn, F. Rennecke, J. Phys. A **44**, 385401 (2011)
9. B. Nikolić, B. Sazdović, Phys. Rev. D **84**, 065012 (2011)
10. B. Nikolić, B. Sazdović, JHEP **06**, 101 (2012)
11. O. Hohm, Prog. Theor. Phys. Suppl. **188**, 116–125 (2011)
12. C. Hull, B. Zwiebach, JHEP **09**, 099 (2009)
13. O. Hohm, C. Hull, B. Zwiebach, JHEP **08**, 008 (2010)
14. O. Hohm, C. Hull, B. Zwiebach, JHEP **07**, 016 (2010)
15. D. Lüst, JHEP **12**, 084 (2010)
16. D. Andriot, M. Larfors, D. Lüst, P. Patalong, JHEP **06**, 021 (2013)
17. D. Andriot, O. Hohm, M. Larfors, D. Lüst, P. Patalong, Phys. Rev. Lett. **108**, 261602 (2012)
18. Lj. Davidović, B. Sazdović, EPJC **74**(1), 2683 (2014)
19. Lj. Davidović, B. Nikolić, B. Sazdović, EPJC **74**(1), 2734 (2014)
20. A. Giveon, E. Rabinovici, G. Veneciano, Nucl. Phys. B **323**, 167 (1989)
21. Lj. Davidović, B. Sazdović, Phys. Rev. D **83**, 066014 (2011)
22. Lj. Davidović, B. Sazdović, JHEP **08**, 112 (2011)
23. Lj. Davidović, B. Sazdović, EPJC **72**(11), 2199 (2012)
24. N. Seiberg, E. Witten, JHEP **09**, 032 (1999)
25. B. Nikolić, B. Sazdović, Nucl. Phys. B **836**, 100 (2010)
26. B. Sazdović, JHEP **08**, 055 (2015)
27. R. Blumenhagen, E. Plauschinn, J. Phys. A Math. Theor. **44**, 015401 (2011)

Eur. Phys. J. C (2014) 74:2734
DOI 10.1140/epjc/s10052-014-2734-5

THE EUROPEAN
PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

Canonical approach to the closed string non-commutativity

Lj. Davidović[a], **B. Nikolić**[b], **B. Sazdović**[c]

Institute of Physics, University of Belgrade, P.O.Box 57, 11001 Belgrade, Serbia

Received: 27 November 2013 / Accepted: 20 December 2013 / Published online: 28 January 2014

Abstract We consider the closed string moving in a weakly curved background and its totally T-dualized background. Using T-duality transformation laws, we find the structure of the Poisson brackets in the T-dual space corresponding to the fundamental Poisson brackets in the original theory. From this structure we see that the commutative original theory is equivalent to the non-commutative T-dual theory, whose Poisson brackets are proportional to the background fluxes times winding and momentum numbers. The non-commutative theory of the present article is more nongeometrical than T-folds and in the case of three space-time dimensions corresponds to the nongeometric space-time with R-flux.

1 Introduction

It is well known that the open string endpoints, attached to a Dp-brane, are non-commutative [1–12]. The non-commutativity is implied by the fact that for the solution of the boundary conditions the initial coordinate is given as a linear combination of the effective coordinate and the effective momentum, which have a nonzero Poisson bracket (PB). In the constant background case, the coefficient in front of the momenta is proportional to the Kalb–Ramond field $B_{\mu\nu}$, whose presence is crucial in gaining the non-commutativity.

The closed string does not have endpoints and in the flat space the boundary conditions are satisfied automatically. But, to understand the closed string non-commutativity, we are going to use a explanation similar to the open string case. We will express the closed string coordinates in terms of the coordinates and momenta of some other space. The relation between different spaces will be established using the T-duality transformations.

The T-dualization along isometry directions, and the construction of T-dual theory was first realized through a Buscher procedure [13,14]. The procedure is in fact a localization of the translation invariance symmetry, in which beside the covariantization of derivatives one adds the Lagrangian multiplier term to the action, which ensures the physical equivalence of the initial and the T-dual theory.

In flat space, T-duality relates σ-derivatives of the coordinates of the original theory with the momenta of its T-dual theory, and vice versa. As the momenta of the original theory are taken to be commutative, it follows that the coordinates commute as well. So, in flat space there is no non-commutativity of the closed string T-dual coordinates. This is in agreement with the fact that T-duality is a canonical transformation in the flat space, and with the fact that PB's are invariant under such transformations.

The closed string non-commutativity was first observed in the papers [15], and investigated further in [16–20], where it was found that the commutators of the coordinates are proportional to the flux and the winding number.

Let us briefly describe the result of Ref. [16], following its notation. After T_1-dualization along the X^1 coordinate, one obtains the twisted torus with coordinates $Y^a (a = 1, 2, 3)$ and f-flux. After additional T_2-dualization along $X^2 = Y^2$ one obtains the nongeometric background with coordinates Z^a and Q-flux. Using the standard Buscher prescription one cannot perform T_3-dualization along the coordinate $X^3 = Y^3 = Z^3$ because the Kalb–Ramond field B_{ab} depends on Z^3. But it is argued in Refs. [16,21,22] that T_3-dualization leads to a nongeometric background with R-flux configuration and W^a coordinates presented in the T-duality chain,

$$H_{abc},\ X^a \xrightarrow{T_1} f^a_{bc},\ Y^a \xrightarrow{T_2} Q^{ab}_{c},\ Z^a \xrightarrow{T_3} R^{abc},\ W^a. \quad (1.1)$$

In the paper [16], the non-commutativity of the nongeometric background (Z^a with Q-flux) has been obtained using its T_2-duality connection $Z^a = Z^a(Y^a)$ with the geometric background (twisted torus with Y^a and f-flux).

[a] e-mail: ljubica@ipb.ac.rs
[b] e-mail: bnikolic@ipb.ac.rs
[c] e-mail: sazdovic@ipb.ac.rs

In our paper [23], we performed a generalized Buscher T-dualization procedure along all the coordinate directions. It corresponds to the $T = T_1 \circ T_2 \circ \cdots \circ T_D$-duality relation $y_\mu = y_\mu(x^\mu)$, connecting the beginning and the end of the T-duality chain:

$$H_{\mu\nu\rho},\ x^\mu \xrightarrow{T_1} (f_1)_{\mu\nu\rho},\ x_1^\mu \xrightarrow{T_2} (f_2)_{\mu\nu\rho},\ x_2^\mu \xrightarrow{T_3} \cdots \xrightarrow{T_D} (f_D)_{\mu\nu\rho},\ x_D^\mu = y_\mu, \tag{1.2}$$

where $(f_i)_{\mu\nu\rho}$ and x_i^μ, $(i = 1, 2, \ldots, D)$ are fluxes and the coordinates of the corresponding configuration. In D-dimensional space-time it is possible to perform T-duality along any subset of coordinates. For simplicity, in the present article we will T-dualize all the directions. The general case will be published separately.

We considered the bosonic string moving in a background with constant metric $G_{\mu\nu}$ = const and the linear Kalb–Ramond field $B_{\mu\nu} = b_{\mu\nu} + \frac{1}{3}B_{\mu\nu\rho}x^\rho$, where the field strength of the Kalb–Ramond field $B_{\mu\nu\rho}$ is infinitesimally small (for more details see the introductory part of Sect. 2). The T-dual theory obtained is of the same form as the initial theory, so that the T-dual string moves in the T-dual background, but in the doubled space given by the coordinates y_μ, $\tilde{y}_\mu$. The dual coordinates satisfy the following conditions: $\dot{y}_\mu = \tilde{y}'_\mu$, $y'_\mu = \dot{\tilde{y}}_\mu$. The improvement, in comparison to the standard Buscher procedure, is the covariantization of the coordinates x^μ. In fact, because x^μ is gauge dependent, it is replaced by the gauge invariant expression $\Delta x^\mu_{inv} = \int d\xi^\alpha D_\alpha x^\mu$. As pointed out in [21,22], the T-dual background of the present paper is of the 'new class that is even more nongeometrical than T-folds'. Unlike the T-folds, this background is not a standard manifold even locally. In our formulation, this stems from the fact that the argument of the background fields Δx^μ_{inv} is the line integral. Some authors argued that such a spaces (for $D = 3$ known as R-flux background) involve nonassociative geometries [24].

In the canonical formalism, the T-dual variables can be expressed in terms of the original ones in the simple form $y'_\mu \cong \frac{1}{\kappa}\pi_\mu - \beta^0_\mu[x]$ and ${}^\star\pi^\mu \cong \kappa x'^\mu + \kappa^2\theta_0^{\mu\nu}\beta^0_\nu[x]$. The infinitesimal expression β^0_μ is an improvement in comparison to the flat background case. Because the coordinates and momenta of the original theory do not commute, β^0_μ is the source of the closed string non-commutativity.

We will follow the main idea of Ref. [16], using the T-duality transformation laws between the T-dual backgrounds in order to study the non-commutativity of the coordinates. In the paper [16], the T_2-duality connects coordinates $Z^a = Z^a(Y^a)$ of the nongeometric background (Z^a with Q-flux) and the geometric background (twisted torus with Y^a and f-flux). We performed the T-dualization procedure along all the coordinates, and we obtained the T-duality transformation $y_\mu = y_\mu(x^\mu)$ of the locally nongeometric background (the end of the chain (1.2) with y_μ and f_D-flux) and the geometric background (torus with H-flux in the beginning of the chain (1.2)). In both approaches it was assumed that the geometric backgrounds (described by Y^a in [16] and by X^a in our paper) have the standard commutation relations. The PB between the y_μ is proportional to the flux $B_{\mu\nu\rho}$ and the winding number N^μ of the initial theory. In addition, we obtain the complete algebra of the T-dual coordinates and momenta in terms of the fluxes.

For $D = 3$, the case of the present article corresponds to T-duality, $T = T_1 \circ T_2 \circ T_3$, which connects the coordinates $W^a = W^a(X^a)$ of the nongeometric background (W^a with R-flux) and the geometric background (torus with X^a and H-flux). In comparison to Ref. [16], this procedure contains one T-dualization more, T_3-dualization along the coordinate $X^3 = Y^3 = Z^3$, which cannot be done using the standard Buscher prescription because the Kalb–Ramond field B_{ab} depends on Z^3. Thus, in terms of Ref. [16], we obtained the non-commutativity of the nongeometric background, with R-flux configuration. This background does not look like the conventional space even locally.

At the end we give three appendices. In the first one we derive in detail the expression for the dual momentum ${}^\star\pi^\mu$, while in the second one we present a list of the fluxes used in the paper. The third appendix contains the mathematical details regarding the transition from PB $\{\Delta X, \Delta Y\}$ to PB $\{X, Y\}$.

2 Bosonic string in the weakly curved background and its T-dual picture

Let us consider the closed string moving in the D-dimensional space-time, in the coordinate $x^\mu(\tau, \sigma)$, $\mu = 0, \ldots, D-1$ dependent background, described by the action

$$S[x] = \kappa \int_\Sigma d^2\xi\ \partial_+ x^\mu \Pi_{+\mu\nu}[x]\,\partial_- x^\nu. \tag{2.1}$$

We suppose that all the coordinates are compact, with radii R_μ. The background is defined by the space-time metric $G_{\mu\nu}$ and the antisymmetric Kalb–Ramond field $B_{\mu\nu}$,

$$\Pi_{\pm\mu\nu}[x] = B_{\mu\nu}[x] \pm \frac{1}{2}G_{\mu\nu}[x]. \tag{2.2}$$

The light-cone coordinates are

$$\xi^\pm = \frac{1}{2}(\tau \pm \sigma), \qquad \partial_\pm = \partial_\tau \pm \partial_\sigma, \tag{2.3}$$

and the action is given in the conformal gauge (the world-sheet metric is taken to be $g_{\alpha\beta} = e^{2F}\eta_{\alpha\beta}$).

World-sheet conformal invariance is required as a condition of having a consistent theory on the quantum level [25–28]. This results in the following space-time equations for the background fields:

$$R_{\mu\nu} - \frac{1}{4} B_{\mu\rho\sigma} B_{\nu}^{\ \rho\sigma} = 0, \quad D_\rho B^\rho_{\ \mu\nu} = 0, \tag{2.4}$$

in the lowest order in the slope parameter α' and for the constant dilaton field $\Phi = \text{const}$. Here

$$B_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} + \partial_\nu B_{\rho\mu} + \partial_\rho B_{\mu\nu} \tag{2.5}$$

is the field strength of the field $B_{\mu\nu}$, and $R_{\mu\nu}$ and D_μ are Ricci tensor and the covariant derivative with respect to the space-time metric.

We will consider a weakly curved background [11,12,16, 23,29–31] defined by

$$G_{\mu\nu}[x] = \text{const},$$
$$B_{\mu\nu}[x] = b_{\mu\nu} + h_{\mu\nu}[x] = b_{\mu\nu} + \frac{1}{3} B_{\mu\nu\rho} x^\rho,$$
$$b_{\mu\nu}, B_{\mu\nu\rho} = \text{const}. \tag{2.6}$$

Here, the constant $B_{\mu\nu\rho}$ is infinitesimally small, which, according to [15,16,18–20], means that we will assume that the D-dimensional torus is so large that for any μ, ν, ρ

$$\frac{B_{\mu\nu\rho}}{R_\mu R_\nu R_\rho} \ll 1, \tag{2.7}$$

where $R_\mu (\mu = 0, 1, \dots D-1)$ are the radii of the torus. For simplicity we will take $R_0 = R_1 = \cdots = R_{D-1}$ and rescale the background fields according to Appendix A of Ref. [16]. The background (2.6) is the solution of Eq. (2.4) in the first order in the $B_{\mu\nu\rho}$ approximation of closed string theory of Eq. (2.1).

2.1 T-dual bosonic string

The T-dualization of closed string theory in a weakly curved background was the subject of investigation in [23]. There we presented the T-dualization procedure performed along all the coordinates, in a background which depends on these coordinates. Here we will give a short overview of the most important results.

The T-dual picture of the theory is given by

$$^\star S[y] = \kappa \int d^2\xi \ \partial_+ y_\mu \, ^\star\Pi_+^{\mu\nu} [\Delta V[y]] \, \partial_- y_\nu$$
$$= \frac{\kappa^2}{2} \int d^2\xi \ \partial_+ y_\mu \Theta_-^{\mu\nu} [\Delta V[y]] \, \partial_- y_\nu, \tag{2.8}$$

with

$$\Theta_\pm^{\mu\nu} \equiv -\frac{2}{\kappa} (G_E^{-1} \Pi_\pm G^{-1})^{\mu\nu} = \theta^{\mu\nu} \mp \frac{1}{\kappa} (G_E^{-1})^{\mu\nu},$$
$$G_{E\mu\nu} \equiv G_{\mu\nu} - 4(BG^{-1}B)_{\mu\nu}. \tag{2.9}$$

The dual background fields, defined in analogy with Eq. (2.2) as $^\star\Pi_\pm^{\mu\nu} = {}^\star B^{\mu\nu} \pm \frac{1}{2} {}^\star G^{\mu\nu}$, have the form

$$^\star G^{\mu\nu}[\Delta V[y]] = (G_E^{-1})^{\mu\nu}[\Delta V[y]],$$
$$^\star B^{\mu\nu}[\Delta V[y]] = \frac{\kappa}{2} \theta^{\mu\nu}[\Delta V[y]]. \tag{2.10}$$

Using the terminology introduced in the open string case, they are equal to the inverse of the effective metric $G^E_{\mu\nu}$ and proportional to the non-commutativity parameter $\theta^{\mu\nu}$. Their argument is given by

$$\Delta V^\mu[y] = -\kappa \theta_0^{\mu\nu} \Delta y_\nu + (g^{-1})^{\mu\nu} \Delta \tilde{y}_\nu, \tag{2.11}$$

where

$$\Delta y_\mu = \int_P (d\tau \, \dot{y}_\mu + d\sigma \, y'_\mu) = y_\mu(\xi) - y_\mu(\xi_0),$$
$$\Delta \tilde{y}_\mu = \int_P (d\tau \, y'_\mu + d\sigma \, \dot{y}_\mu), \tag{2.12}$$

and

$$g_{\mu\nu} = G_{\mu\nu} - 4(bG^{-1}b)_{\mu\nu}, \quad \theta_0^{\mu\nu} = -\frac{2}{\kappa}(g^{-1}bG^{-1})^{\mu\nu} \tag{2.13}$$

are constant finite parts of the effective metric and the non-commutativity parameter. The variable $\Delta\tilde{y}_\mu$ is path independent on the zeroth order equation of motion. T-dual theory is defined in the doubled space, defined by the two coordinates y_μ and $\tilde{y}_\mu$, related by the expressions $\dot{y}_\mu = \tilde{y}'_\mu$, $y'_\mu = \dot{\tilde{y}}_\mu$.

2.2 Transformation laws

The T-duality transformation connecting the variables of the closed string theory in the weakly curved background and its T-dualized string theory is [23]

$$\partial_\pm x^\mu \cong -\kappa \Theta_\pm^{\mu\nu}[\Delta V] \left[\partial_\pm y_\nu \pm 2\beta_\nu^\mp[V]\right], \tag{2.14}$$

with

$$\beta_\mu^\pm[x] = \frac{1}{2}(\beta_\mu^0 \pm \beta_\mu^1) = \mp \frac{1}{2} h_{\mu\nu}[x] \partial_\mp x^\nu,$$
$$\beta_\mu^0[x] = h_{\mu\nu}[x] x'^\nu, \ \beta_\mu^1[x] = -h_{\mu\nu}[x] \dot{x}^\nu. \tag{2.15}$$

From Eq. (2.14) we can find the transformation law for $\dot{x}^\mu$ and x'^μ:

$$\dot{x}^\mu \cong -\kappa \theta^{\mu\nu}[\Delta V] \dot{y}_\nu + (G_E^{-1})^{\mu\nu}[\Delta V] y'_\nu + (g^{-1})^{\mu\nu} \beta_\nu^0[V]$$
$$+ \kappa \theta_0^{\mu\nu} \beta_\nu^1[V] \tag{2.16a}$$

$$x'^\mu \cong (G_E^{-1})^{\mu\nu}[\Delta V] \dot{y}_\nu - \kappa \theta^{\mu\nu}[\Delta V] y'_\nu - \kappa \theta_0^{\mu\nu} \beta_\nu^0[V]$$
$$-(g^{-1})^{\mu\nu} \beta_\nu^1[V]. \tag{2.16b}$$

Using the expression for the canonical momentum of the original theory,

$$\pi_\mu = \frac{\delta S}{\delta \dot{x}^\mu} = \kappa \left[G_{\mu\nu} \dot{x}^\nu - 2B_{\mu\nu}[x] x'^\nu\right], \tag{2.17}$$

and the T-dual canonical momentum,

$$\star\pi^\mu = \frac{\delta \star S}{\delta \dot{y}_\mu} = \kappa (G_E^{-1})^{\mu\nu} [\Delta V[y]]\, \dot{y}_\nu - \kappa^2 \theta^{\mu\nu} [\Delta V[y]]\, y'_\nu - \kappa (g^{-1})^{\mu\nu} \beta^1_\nu [V[y]], \tag{2.18}$$

derived in Appendix A, we rewrite the above transformations in canonical form:

$$x'^\mu \cong \frac{1}{\kappa} \star\pi^\mu - \kappa \theta_0^{\mu\nu} \beta^0_\nu [V], \tag{2.19a}$$

$$\pi_\mu \cong \kappa y'_\mu + \kappa \beta^0_\mu [V], \tag{2.19b}$$

with $\beta^0_\mu[V]$ defined in Eq. (2.15). It is shown in Ref. [23] that the T-dual of the T-dual action is the original one. The corresponding T-dual transformation of the variables law is the inverse of Eq. (2.14),

$$\partial_\pm y_\mu \cong -2\Pi_{\mp\mu\nu} [\Delta x]\, \partial_\pm x^\nu \mp 2\beta^\mp_\mu [x], \tag{2.20}$$

and so the transformation laws for $\dot{y}_\mu$ and y'_μ are equal to

$$\dot{y}_\mu \cong -2B_{\mu\nu}[x]\, \dot{x}^\nu + G_{\mu\nu} x'^\nu + \beta^1_\mu [x], \tag{2.21a}$$

$$y'_\mu \cong G_{\mu\nu} \dot{x}^\nu - 2B_{\mu\nu}[x]\, x'^\nu - \beta^0_\mu [x]. \tag{2.21b}$$

Using Eqs. (2.17) and (2.18) we obtain the canonical form of the T-dual transformations,

$$y'_\mu \cong \frac{1}{\kappa} \pi_\mu - \beta^0_\mu [x], \tag{2.22a}$$

$$\star\pi^\mu \cong \kappa x'^\mu + \kappa^2 \theta_0^{\mu\nu} \beta^0_\nu [x]. \tag{2.22b}$$

In the zeroth order one has $x^{(0)\mu} \cong V^\mu$, and it is easy to see that Eq. (2.22) is the inverse of Eq. (2.17).

Because the T-dual theory is defined in the doubled space, we will need the canonical expression for $\tilde{y}'_\mu = \dot{y}_\mu$. Using Eqs. (2.21a) and (2.17), we obtain

$$\tilde{y}'_\mu \cong -\frac{2}{\kappa} \left(B[\Delta x] + \frac{1}{2} h[x] \right)_{\mu\nu} (G^{-1})^{\nu\rho} \pi_\rho + \left(G^E[\Delta x] - 2h[x]\, G^{-1} b \right)_{\mu\nu} x'^\nu. \tag{2.23}$$

3 Non-commutativity relations between canonical variables

We want to establish the relation between the Poisson structures of the original and T-dual theory. The initial theory is the geometric one, described by the canonical variables x^μ and π_μ. Thus, we choose the standard form of the PB's in the original space, which are

$$\{x^\mu(\sigma), \pi_\nu(\bar\sigma)\} = \delta^\mu_\nu \delta(\sigma - \bar\sigma), \quad \{x^\mu(\sigma), x^\nu(\bar\sigma)\} = 0, \quad \{\pi_\mu(\sigma), \pi_\nu(\bar\sigma)\} = 0. \tag{3.1}$$

The T-dual theory is the nongeometric one, defined in the doubled space, with two coordinates y_μ and $\tilde{y}_\mu$, connected by relations $\dot{y}_\mu = \tilde{y}'_\mu$, $y'_\mu = \dot{\tilde{y}}_\mu$. Using the T-duality transformation laws, we search for the corresponding Poisson structure in T-dual theory i.e. the expressions for the PB's between the T-dual string coordinates $y_\mu(\sigma)$, $\tilde{y}_\mu(\sigma)$ and momenta $\star\pi^\mu(\sigma)$. This is done considering the brackets between

$$\Delta Y_\mu(\sigma, \sigma_0) = \int_{\sigma_0}^{\sigma} d\eta\, Y'_\mu(\eta) = Y_\mu(\sigma) - Y_\mu(\sigma_0) \tag{3.2}$$

$Y_\mu = y_\mu, \tilde{y}_\mu$ and calculating the equal time commutators. The fact that T-dual coordinates under T-duality transform to both coordinate and momentum dependent expressions enables non-commutativity. The relation of the form

$$\{X'_\mu(\sigma), Y'_\nu(\bar\sigma)\} \cong K'_{\mu\nu}(\sigma)\delta(\sigma - \bar\sigma) + L_{\mu\nu}(\sigma)\delta'(\sigma - \bar\sigma) \tag{3.3}$$

implies the following relation (derived in Appendix C) between the coordinates

$$\{X_\mu(\tau, \sigma), Y_\nu(\tau, \bar\sigma)\} \cong -\left[K_{\mu\nu}(\sigma) - K_{\mu\nu}(\bar\sigma) + L_{\mu\nu}(\bar\sigma)\right]\theta(\sigma - \bar\sigma), \tag{3.4}$$

where $\theta(\sigma)$ is the step function defined in Eq. (8.6).

In flat space the coordinate dependent part of the Kalb–Ramond field is absent, $h_{\mu\nu} = 0$, and consequently $\beta^0_\mu = 0$. Thus, from Eqs. (2.22a) and (2.22b) follows $y'_\mu \cong \frac{1}{\kappa}\pi_\mu$ and $\star\pi^\mu \cong \kappa x'^\mu$. Therefore, the PB of the canonical variables of the T-dual theory remain the standard ones, the same as in the original theory. So, the nontrivial infinitesimal expression β^0_μ, which exists only in the coordinate dependent backgrounds, is the source of the closed string non-commutativity.

Using the transformation laws (2.22a) and (2.23), we calculate the PB's $\{y'_\mu, y'_\nu\}$, $\{y'_\mu(\sigma), \tilde{y}'_\nu(\bar\sigma)\}$ and $\{\tilde{y}'_\mu(\sigma), \tilde{y}'_\nu(\bar\sigma)\}$ and express them in the form of Eq. (3.3) with K and L equal:

1. $\{y'_\mu, y'_\nu\}$

$$K_{\mu\nu}[x] = \frac{3}{\kappa} h_{\mu\nu}[x] = \frac{1}{\kappa} B_{\mu\nu\rho} x^\rho, \quad L_{\mu\nu} = 0, \tag{3.5}$$

2. $\{y'_\mu, \tilde{y}'_\nu\}$

$$K_{\mu\nu}[x, \tilde{x}] = \frac{3}{\kappa} h_{\mu\nu}[\tilde{x}] - \frac{6}{\kappa} \left[h[x]\, G^{-1} b + b G^{-1} h[x] \right]_{\mu\nu},$$
$$L_{\mu\nu}[x] = \frac{1}{\kappa} g_{\mu\nu} - \frac{6}{\kappa} \left[h[x]\, G^{-1} b + b G^{-1} h[x] \right]_{\mu\nu}, \tag{3.6}$$

with

$$\tilde{x}'^\mu = \frac{1}{\kappa} (G^{-1})^{\mu\nu} \pi_\nu + 2(G^{-1}B)^\mu{}_\nu x'^\nu. \tag{3.7}$$

Using Eqs. (2.6) and (7.2), expressions (3.6) can be rewritten in terms of the fluxes:

$$K_{\mu\nu}\left[x,\tilde{x}\right]=\frac{1}{\kappa}B_{\mu\nu\rho}\tilde{x}^{\rho}-\frac{3}{2\kappa}\Gamma^{E}_{\rho,\mu\nu}x^{\rho},$$
$$L_{\mu\nu}\left[x\right]=\frac{1}{\kappa}g_{\mu\nu}-\frac{3}{2\kappa}\Gamma^{E}_{\rho,\mu\nu}x^{\rho}, \tag{3.8}$$

3. $\{\tilde{y}'_{\mu},\tilde{y}'_{\nu}\}$

$$K_{\mu\nu}\left[x\right]=\frac{3}{\kappa}h_{\mu\nu}\left[x\right]+\frac{24}{\kappa}\left[bh\left[x\right]b\right]_{\mu\nu}$$
$$+\frac{6}{\kappa}\left[h\left[\tilde{x}\right]b-bh\left[\tilde{x}\right]\right]_{\mu\nu},\quad L_{\mu\nu}=0. \tag{3.9}$$

In terms of fluxes it becomes

$$K_{\mu\nu}=-\frac{1}{\kappa}\left[B_{\mu\nu\rho}-6g_{\mu\alpha}Q^{\alpha\beta}{}_{\rho}g_{\beta\nu}\right]x^{\rho}$$
$$+\left[-\frac{3}{2\kappa}\left(\Gamma^{E}_{\mu,\nu\rho}-\Gamma^{E}_{\nu,\mu\rho}\right)+\frac{4}{\kappa}B_{\mu\nu\sigma}(G^{-1}b)^{\sigma}{}_{\rho}\right]\tilde{x}^{\rho}, \tag{3.10}$$

where $\Gamma^{E}_{\nu,\mu\rho}$ and $Q_{\mu\nu\rho}$ are defined in Eqs. (7.1) and (7.5).

For the above values of K and L, the relation (3.4) gives

$$\{y_{\mu}(\sigma),y_{\nu}(\bar{\sigma})\}\cong-\frac{1}{\kappa}B_{\mu\nu\rho}\left[x^{\rho}(\sigma)-x^{\rho}(\bar{\sigma})\right]\theta(\sigma-\bar{\sigma}), \tag{3.11}$$

$$\{y_{\mu}(\sigma),\tilde{y}_{\nu}(\bar{\sigma})\}\cong-\Big\{\frac{1}{\kappa}B_{\mu\nu\rho}\left[\tilde{x}^{\rho}(\sigma)-\tilde{x}^{\rho}(\bar{\sigma})\right]$$
$$-\frac{3}{2\kappa}\Gamma^{E}_{\rho,\mu\nu}\left[x^{\rho}(\sigma)-x^{\rho}(\bar{\sigma})\right]$$
$$+\frac{1}{\kappa}g_{\mu\nu}-\frac{3}{2\kappa}\Gamma^{E}_{\rho,\mu\nu}x^{\rho}(\bar{\sigma})\Big\}\theta(\sigma-\bar{\sigma}), \tag{3.12}$$

$$\{\tilde{y}_{\mu}(\sigma),\tilde{y}_{\nu}(\bar{\sigma})\}\cong-\Big\{-\frac{1}{\kappa}\left[B_{\mu\nu\rho}-6g_{\mu\alpha}Q^{\alpha\beta}{}_{\rho}g_{\beta\nu}\right]\left[x^{\rho}(\sigma)-x^{\rho}(\bar{\sigma})\right]$$
$$+\left[-\frac{3}{2\kappa}\left(\Gamma^{E}_{\mu,\nu\rho}-\Gamma^{E}_{\nu,\mu\rho}\right)+\frac{4}{\kappa}B_{\mu\nu\sigma}(G^{-1}b)^{\sigma}{}_{\rho}\right]$$
$$\times\left[\tilde{x}^{\rho}(\sigma)-\tilde{x}^{\rho}(\bar{\sigma})\right]\Big\}\theta(\sigma-\bar{\sigma}). \tag{3.13}$$

After two-dimensional reparametrization, the σ dependent part takes the form

$$\left[X^{\mu}(f(\sigma))-X^{\mu}(f(\bar{\sigma}))\right]\theta\left[f(\sigma)-f(\bar{\sigma})\right],$$

where $f(\sigma)$ is a monotonically increasing function with properties $f(0)=0$ and $f(2\pi)=2\pi$. Therefore, the PB between different points is not reparametrization invariant. For fixed points, it can be fit to be arbitrary small, by the appropriate choice of the function $f(\sigma)$. So, only PB's at the same point are physically significant.

Taking $\sigma=\bar{\sigma}$ we find that all PB's vanish, and consequently the coordinates commute. But, taking $\sigma=\bar{\sigma}+2\pi$ in the non-commutativity relation between the dual coordinates y's (3.11), we obtain the *closed string non-commutativity relation*

$$\{y_{\mu}(\sigma+2\pi),y_{\nu}(\sigma)\}\cong-\frac{2\pi}{\kappa}B_{\mu\nu\rho}N^{\rho}. \tag{3.14}$$

Here, $N^{\mu}=\frac{1}{2\pi}\left[x^{\mu}(\sigma+2\pi)-x^{\mu}(\sigma)\right]$ is the winding number of the original coordinates. In Sec. 4, we will compare this relation with the result of Refs. [16,18–20].

Similarly, from Eqs. (3.12) and (3.13), we obtain

$$\{y_{\mu}(\sigma+2\pi),\tilde{y}_{\nu}(\sigma)\}+\{y_{\mu}(\sigma),\tilde{y}_{\nu}(\sigma+2\pi)\}\cong$$
$$-\frac{4\pi}{\kappa^{2}}B_{\mu\nu\rho}p^{\rho}+\frac{\pi}{\kappa}\left(3\Gamma^{E}_{\rho,\mu\nu}-8B_{\mu\nu\lambda}b^{\lambda}{}_{\rho}\right)N^{\rho}, \tag{3.15}$$

and

$$\{\tilde{y}_{\mu}(\sigma+2\pi),\tilde{y}_{\nu}(\sigma)\}$$
$$\cong\frac{2\pi}{\kappa}\Big[-B_{\mu\nu\rho}-6g_{\mu\alpha}Q^{\alpha\beta}{}_{\rho}g_{\beta\nu}+2B_{\mu\nu}{}^{\lambda}g_{\lambda\rho}$$
$$+3\left(\Gamma^{E}_{\mu,\nu\lambda}-\Gamma^{E}_{\nu,\mu\lambda}\right)b^{\lambda}{}_{\rho}\Big]N^{\rho}$$
$$+\frac{\pi}{\kappa^{2}}\left[3\left(\Gamma^{E}_{\mu,\nu\rho}-\Gamma^{E}_{\nu,\mu\rho}\right)p^{\rho}-8B_{\mu\nu\lambda}b^{\lambda}{}_{\rho}\right]p^{\rho}. \tag{3.16}$$

Using Eq. (3.7) and integrating from σ to $\sigma+2\pi$ we have

$$\frac{1}{2\pi}\left[\tilde{x}^{\mu}(\sigma+2\pi)-\tilde{x}^{\mu}(\sigma)\right]$$
$$=\frac{1}{\kappa}(G^{-1})^{\mu\nu}p_{\nu}+2(G^{-1})^{\mu\rho}b_{\rho\lambda}N^{\lambda}, \tag{3.17}$$

where

$$p_{\mu}=\frac{1}{2\pi}\int_{\sigma}^{\sigma+2\pi}d\eta\pi_{\mu}(\eta). \tag{3.18}$$

To complete the algebra, using the expressions (2.22) and (2.23) and after one σ integration, we find that the algebra of y_{μ}, $\tilde{y}_{\mu}$ and ${}^{\star}\pi^{\mu}$ is of the following form:

$$\left\{y_{\mu}(\sigma),{}^{\star}\pi^{\nu}(\bar{\sigma})\right\}\cong\delta_{\mu}{}^{\nu}\delta(\sigma-\bar{\sigma})+\kappa h_{\mu\rho}\left[x(\sigma)\right]\theta_{0}^{\rho\nu}\times\delta$$
$$(\sigma-\bar{\sigma})+\kappa h_{\mu\rho}\left[x'(\bar{\sigma})\right]\theta_{0}^{\rho\nu}\theta(\sigma-\bar{\sigma}), \tag{3.19}$$

$$\left\{\tilde{y}_{\mu}(\sigma),{}^{\star}\pi^{\nu}(\bar{\sigma})\right\}$$
$$\cong\left[-2bG^{-1}-3h\left[x(\sigma)\right]G^{-1}-2\kappa bh\left[x(\sigma)\right]\theta_{0}\right]_{\mu}^{\ \nu}\delta(\sigma-\bar{\sigma})$$
$$-\left[3h\left[x'(\bar{\sigma})\right]G^{-1}+2\kappa bh\left[x'(\bar{\sigma})\right]\theta_{0}\right]_{\mu}^{\ \nu}\theta(\sigma-\bar{\sigma}), \tag{3.20}$$

$$\{{}^{\star}\pi^{\mu}(\sigma),{}^{\star}\pi^{\nu}(\bar{\sigma})\}\cong0. \tag{3.21}$$

Note that at the zeroth order one has $\{y_{\mu}(\sigma),{}^{\star}\pi^{\nu}(\bar{\sigma})\}=\delta_{\mu}^{\nu}\delta(\sigma-\bar{\sigma})$ and $\{\tilde{y}_{\mu}(\sigma),{}^{\star}\pi^{\nu}(\bar{\sigma})\}=-2b_{\mu}{}^{\nu}\delta(\sigma-\bar{\sigma})$, so both doubled space variables y_{μ} and $\tilde{y}_{\mu}$ have a nontrivial PB with ${}^{\star}\pi^{\mu}$.

4 Comparison with the previous results

Let us mention that the case considered in the present paper is different from that of Ref. [16]. In Ref. [16], the non-commutativity relations in the nongeometric background with Q-flux where established, which are given in terms of winding numbers on the twisted torus $N^3 = \frac{1}{2\pi}\left(Y^3(\sigma+2\pi)-Y^3(\sigma)\right)$. In the present article, the non-commutativity of the nongeometric background, which is not standard even locally and for $D=3$ turns to R-flux background, was obtained in terms of the winding numbers on the torus with H-flux $N^\mu = \frac{1}{2\pi}\left(X^\mu(\sigma+2\pi)-X^\mu(\sigma)\right)$.

4.1 The brief overview of the results of Ref. [16]

Before comparing the results of our paper with those of Ref. [16] let us shortly reexpress the result of Ref. [16] using its notation. From the last identification in Eq. (2.17) and the first relation in (2.25) of Ref. [16] it follows that

$$Y_H^1 = Y_0^2 Y_0^3 + \cdots . \tag{4.1}$$

Using the expression for $G_{ab}(Y_3)$ for the twisted torus (Table 1) of Ref. [16] we find

$$\pi_1 = \dot{Y}^1 - HY_0^3\dot{Y}_0^2, \quad \pi_2 = \dot{Y}^2 - HY_0^3\dot{Y}_0^1, \tag{4.2}$$

and consequently

$$\pi_{01} = \dot{Y}_0^1, \quad \pi_{H2} = \dot{Y}_H^2 - Y_0^3\dot{Y}_0^1 = \dot{Y}_H^2 - Y_0^3\pi_{01}. \tag{4.3}$$

The T_2-duality along Y^2, from the twisted torus to the nongeometric background produces

$$Z^1 \cong Y^1 = Y_0^1 + HY_0^2Y_0^3, \quad Z^{2'} \cong \dot{Y}^2 - HY_0^3\dot{Y}_0^1 \\ = \pi_2 = \pi_{02} + H\left(\dot{Y}_H^2 - Y_0^3\pi_{01}\right). \tag{4.4}$$

Thus, we find the PB

$$\{Z^1(\sigma), Z^{2'}(\bar\sigma)\} \cong \{Y^1(\sigma), \pi_2(\bar\sigma)\} \\ = H\left[Y_0^3(\sigma) - Y_0^3(\bar\sigma)\right]\delta_{2\pi}(\sigma-\bar\sigma). \tag{4.5}$$

Note that $\delta_{2\pi}(\sigma-\bar\sigma)$ is a 2π periodic δ-function, $\delta_{2\pi}(\alpha) = \sum_{n\in Z}\delta(\alpha - 2\pi n)$, so the periodic parts in the bracket in front of the δ-function disappear and we obtain

$$\{Z^1(\sigma), Z'^2(\bar\sigma)\} = HN^3(\sigma-\bar\sigma)\delta_{2\pi}(\sigma-\bar\sigma). \tag{4.6}$$

Here N^3 is the winding number of Y_0^3, which has the general form

$$Y_0^3(\sigma) = N^3\sigma + Y_{\text{periodic}}^3(\sigma). \tag{4.7}$$

The expression $\alpha\delta_{2\pi}(\alpha)$ is zero for $\alpha = 0$, but it is different from zero for $\alpha = 2n\pi$ ($n \in Z, n \neq 0$).

The integration over $\bar\sigma$, from $\bar\sigma_0$ to $\bar\sigma$, produces

$$\{Z^1(\sigma), Z^2(\bar\sigma)\} - \{Z^1(\sigma), Z^2(\bar\sigma_0)\} \\ = -\frac{1}{2\pi}HN^3\left[F(\sigma-\bar\sigma) - F(\sigma-\bar\sigma_0)\right], \tag{4.8}$$

where

$$2\pi\int_{\alpha_0}^{\alpha} d\eta\,\eta\,\delta_{2\pi}(\eta) = F(\alpha) - F(\alpha_0), \tag{4.9}$$

and

$$F(\alpha) = \sum_{n\neq 0}\frac{1}{n^2}e^{-in\alpha} + i\alpha\sum_{n\neq 0}\frac{1}{n}e^{-in\alpha} + \frac{\alpha^2}{2}. \tag{4.10}$$

The function $F(\alpha)$ is even, $F(-\alpha) = F(\alpha)$, and $F(0) = \frac{\pi^2}{3}$.

So, the result for the PB itself,

$$\{Z^1(\sigma), Z^2(\bar\sigma)\} = -\frac{1}{2\pi}HN^3\left[F(\sigma-\bar\sigma) + C\right], \tag{4.11}$$

is in fact equation (4.41) of Ref. [16] up to some integration constant C. The undetermined constant C corresponds to the contribution of the zero modes of the undetermined commutators, because one started with the σ-derivative of the coordinate Z^2. The choice of Ref. [16] in subsection 4.4.2 is $C = 0$, which produces the expression (4.41) of Ref. [16] and the non-commutativity at the same point, $\sigma = \bar\sigma$,

$$\{Z^1(\sigma), Z^2(\sigma)\} = -\frac{1}{2\pi}HN^3F(0) = -\frac{\pi}{6}HN^3. \tag{4.12}$$

As was pointed out in Ref. [16], 'other reasonings could as well be pursued'. Following the line of our paper one may require that coordinates are commutative at the same point ($\sigma = \bar\sigma$), which produces

$$C = -F(0) = -\frac{\pi^2}{3}. \tag{4.13}$$

Thus, with this choice one has

$$\{Z^1(\sigma), Z^2(\bar\sigma)\} = HN^3\left[F(\sigma-\bar\sigma) - \frac{\pi^2}{3}\right], \tag{4.14}$$

and one obtains the non-commutativity for $\sigma = 2\pi + \bar\sigma$,

$$\{Z^1(\sigma+2\pi), Z^2(\sigma)\} = \pi HN^3. \tag{4.15}$$

4.2 Similarities and differences

Although we analyzed the different cases, let us compare some general features of the results considered. In both approaches the commutators are infinitesimally small and they close on some winding numbers. Note that, in general, we can connect any geometric background with every nongeometric background from the chain of T-duality (1.2). Using the T-duality transformations we can calculate the non-commutativity of the coordinates of the nongeometric background in terms of the winding numbers of the geometrical background.

For arbitrary σ and $\bar\sigma$, the σ-dependence is different. In Ref. [16], up to the integration constant C, it is equal to

$$F(\sigma-\bar\sigma) + C,$$

and in the present article, up to the integration constant C_1, it is

$$\left[x^\mu(\sigma) - x^\mu(\bar{\sigma})\right]\theta(\sigma - \bar{\sigma}) + C_1.$$

The constants appear because in both approaches we started with the sigma derivatives of the coordinates. In the papers considered, the values of the constants are taken to be $C = 0$ and $C_1 = 0$. For these choices, the non-commutativity appears for $\sigma = \bar{\sigma}$ in Ref. [16] and for $\sigma = \bar{\sigma} + 2\pi$ in the present article. For the other choice, $C = -F(0) = -\frac{\pi^2}{3}$ and $C_1 = 0$, in both cases the coordinates commute at the same point $\sigma = \bar{\sigma}$ and have nontrivial PB for $\sigma = \bar{\sigma} + 2\pi$.

The main difference between the two approaches is the origin of non-commutativity. The nontrivial boundary conditions given in Eq. (2.25) of Ref. [16] are the source of the non-commutativity in that article. Because Ref. [16] does not consider T_3-dualization, the β^0_μ-functions (introduced in Eq. (2.15)) are zero and there is no non-commutativity of this kind. On the other hand, in the case considered in this paper, just these β^0_μ functions are the sources of the non-commutativity, even in the absence of the nontrivial boundary conditions of Ref. [16]. For complete non-commutativity relations one should take into account both kinds of non-commutativity.

5 Concluding remarks

In the present article we derived the closed string non-commutativity relations. We considered the theory describing a string moving in a weakly curved background. Its T-dual theory is obtained performing the T-dualization procedure along all the coordinates [23]. The T-dual transformation laws play a central role in our approach. These laws connect the world-sheet derivatives of the coordinates and momenta in the original and the T-dual theory. The zero orders are transformation laws of the constant background and they do not lead to the non-commutativity. The term β^0_μ, which is infinitesimally small and bilinear in the x^μ coordinates, plays a key role in obtaining the non-commutativity relations.

In the original space we choose the standard Poisoon brackets. The T-dual coordinates y_μ have two terms: one linear in the original momenta and the other bilinear in the original coordinates. This explains the nontrivial PB $\{y_\mu, y_\nu\}$ of Eq. (3.11), which is linear in the coordinates. Note that in the case of an open string moving in the flat background coordinate is linear function in both effective momenta and coordinates. Therefore, the corresponding PB is constant.

The T-dual momenta ${}^\star\pi^\mu$ are bilinear expressions in the original coordinates. Thus, the PB of the T-dual momenta vanishes, see Eq. (3.21), but the PB between the T-dual coordinates and the momenta (3.19) obtained an additional term linear in the coordinates.

In the doubled space there exists the additional coordinate $\tilde{y}_\mu$. It consists of a term linear in the original momenta, but with the coefficient linear in the original coordinate and the other terms bilinear in the original coordinates. Thus, it produces a nontrivial PB with all variables $(y_\mu, \tilde{y}_\mu, {}^\star\pi^\mu)$, see Eqs. (3.12), (3.13), and (3.20).

The general structure of the non-commutativity relations is

$$\{Y_\mu(\sigma), Y_\nu(\bar{\sigma})\} = \{F_{\mu\nu\rho}\left[x^\rho(\sigma) - x^\rho(\bar{\sigma})\right] + \tilde{F}_{\mu\nu\rho}\left[\tilde{x}^\rho(\sigma) - \tilde{x}^\rho(\bar{\sigma})\right]\}\theta(\sigma - \bar{\sigma}), \tag{5.1}$$

where $Y_\mu = (y_\mu, \tilde{y}_\nu)$ and $F_{\mu\nu\rho}$ and $\tilde{F}_{\mu\nu\rho}$ are the constant and infinitesimally small fluxes. At the same points, for $\sigma = \bar{\sigma}$ all PB's are zero. In the important particular case for $\sigma = \bar{\sigma} + 2\pi$ we get

$$\{Y_\mu(\sigma + 2\pi), Y_\nu(\sigma)\} = 2\pi\left[(F_{\mu\nu\rho} + 2\tilde{F}_{\mu\nu\alpha}b^\alpha_\rho)N^\rho + \frac{1}{\kappa}\tilde{F}_{\mu\nu}{}^\rho p_\rho\right], \tag{5.2}$$

where N^μ and p_μ are the winding numbers and momenta of the original theory. We rewrite it in the form

$$\{Y_\mu(\sigma + 2\pi), Y_\nu(\sigma)\} = \oint_{C_\rho} F_{\mu\nu\rho}\mathrm{d}x^\rho + \oint_{\tilde{C}_\rho} \tilde{F}_{\mu\nu\rho}\mathrm{d}\tilde{x}^\rho, \tag{5.3}$$

where C_ρ and $\tilde{C}_\rho$ are cycles around which the closed string is wrapped. Note that the 'wrapping' of the auxiliary coordinate $\tilde{x}^\mu$ is in accordance with Eq. (3.17) and represents a linear combination of momenta p_μ and winding numbers N^μ. This generalizes the conjecture of Ref. [32] on the relation between the closed string non-commutativity and fluxes.

In terms of Ref. [16] for the three-dimensional torus $x^\mu \to X^a$, $(a = 1, 2, 3)$ our case corresponds to the non-commutativity of the nongeometric background with W^a coordinates and R-fluxes obtained after the successive performation of all three T-dualizations along all three coordinates. It relates the W^a with the X^a coordinates of the torus with H-flux, and so the PB closes on the winding number of the X^a-coordinates. We hope that these results will contribute to a better understanding of the strangest, uncommon R-flux configurations where the non-commutativity appears as a consequence of the nontrivial β^0_μ-functions. Note that Ref. [16] uses T_2-duality (performed along Y^2) and the relation $Z^a = Z^a(Y^a)$ to obtain the non-commutativity of the nongeometric background with Q-flux in terms of the winding of the Y^a-coordinates. There the non-commutativity originates from the nontrivial boundary conditions. To obtain the general structure of the closed string non-commutativity for arbitrary background of the chain (1.2) one should find its

T-duality transformations with all other backgrounds of the chain and calculate both kinds of non-commutativity originating from nontrivial boundary conditions as well as from nontrivial β^0_μ functions.

The term of the action with the constant part of the Kalb–Ramond field $b_{\mu\nu}$ is topological. Thus, it does not contribute to the equations of motion. In the open string case it contributes to the boundary conditions and it is a source of the open string non-commutativity. In the closed string case it is absent from boundary conditions as well. Classically, we can gauge it away and the Kalb–Ramond field becomes infinitesimally small. But if $b_{\mu\nu} = 0$ one loses topological contributions. In order to investigate the global structure of the theory with holonomies of the world-sheet gauge fields in quantum theory we should preserve such a term.

Putting $b_{\mu\nu} = 0$ the non-commutativity relations (3.14), (3.15), and (3.16) get a simpler form,

$$\begin{aligned}
\{y_\mu(\sigma+2\pi), y_\nu(\sigma)\} &= -\frac{2\pi}{\kappa} B_{\mu\nu\rho} N^\rho, \\
\{y_\mu(\sigma+2\pi), \tilde{y}_\nu(\sigma)\} &= -\frac{1}{\kappa} G_{\mu\nu} - \frac{2\pi}{\kappa^2} B_{\mu\nu}{}^\rho p_\rho, \\
\{\tilde{y}_\mu(\sigma+2\pi), \tilde{y}_\nu(\sigma)\} &= -\frac{6\pi}{\kappa} B_{\mu\nu\rho} N^\rho. \qquad (5.4)
\end{aligned}$$

Acknowledgments Work supported in part by the Serbian Ministry of Education, Science and Technological Development, under contract No. 171031.

Appendix A: The momentum in T-dual theory

Let us here calculate the T-dual momentum given in Eq. (2.18). The T-dual theory depends on the two variables y_μ, $\tilde{y}_\mu$, which are connected by the relations $\dot{y}_\mu = \tilde{y}'_\mu$, $y'_\mu = \dot{\tilde{y}}_\mu$. Therefore, to obtain the momentum canonically conjugated to y_μ, we should vary the action with respect to both $\dot{y}_\mu$ and $\tilde{y}'_\mu$.

First, let us calculate the contribution from the background fields argument. With the help of the relation

$$\Theta^{\mu\nu}_-[x] = \Theta^{\mu\nu}_{0-} - 2\kappa\Theta^{\mu\rho}_{0-} h_{\rho\sigma}[x]\Theta^{\sigma\nu}_{0-}, \qquad (6.1)$$

we rewrite the T-dual action (2.8) as

$$\begin{aligned}
{}^\star S[y] &= {}^\star S_0 - \kappa^3 \int d^2\xi\, \partial_+ y_\mu \Theta^{\mu\rho}_{0-} h_{\rho\sigma}[\Delta V[y]]\Theta^{\sigma\nu}_{0-}\partial_- y_\nu, \\
{}^\star S_0 &= \frac{\kappa^2}{2}\int d^2\xi\, \partial_+ y_\mu \Theta^{\mu\nu}_{0-}\partial_- y_\nu. \qquad (6.2)
\end{aligned}$$

Using the expression

$$\partial_\pm V^\mu = -\kappa\Theta^{\mu\nu}_{0\pm}\partial_\pm y^{(0)}_\nu, \qquad (6.3)$$

we obtain

$$\begin{aligned}
{}^\star S[y] &= {}^\star S_0 + \kappa\int d^2\xi\, \partial_+ V^\mu h_{\mu\nu}[\Delta V]\partial_- V^\nu \\
&= {}^\star S_0 + \kappa\int d^2\xi\, \Delta V^\mu h_{\mu\nu}\left[\partial_- V\right]\partial_+ V^\nu. \qquad (6.4)
\end{aligned}$$

Because of the relation

$$h_{\mu\nu}\left[\partial_- V\right]\partial_+ V^\nu = \partial_0\beta^0_\mu[V] + \partial_1\beta^1_\mu[V], \qquad (6.5)$$

the action becomes

$$\begin{aligned}
{}^\star S[y] = {}^\star S_0 + \kappa\int d^2\xi &\left[\kappa\Delta y_\mu\theta^{\mu\nu}_0 + \Delta\tilde{y}_\mu(g^{-1})^{\mu\nu}\right] \\
&\times\left(\partial_0\beta^0_\nu[V] + \partial_1\beta^1_\nu[V]\right). \qquad (6.6)
\end{aligned}$$

So, the contribution to the T-dual momentum coming from the T-dual background fields argument is obtained from Eq. (6.6), integrating over σ by parts in $\Delta\tilde{y}_\mu(g^{-1})^{\mu\nu}\partial_1\beta^1_\nu$. Using $\tilde{y}'_\mu = \dot{y}_\mu$ we obtain

$$\Delta{}^\star\pi^\mu = -\kappa(g^{-1})^{\mu\nu}\beta^1_\nu[V]. \qquad (6.7)$$

Therefore, the total T-dual momentum is

$$\begin{aligned}
{}^\star\pi^\mu &= \kappa(G^{-1}_E)^{\mu\nu}[\Delta V[y]]\,\dot{y}_\nu \\
&\quad -\kappa^2\theta^{\mu\nu}[\Delta V[y]]\,y'_\nu - \kappa(g^{-1})^{\mu\nu}\beta^1_\nu[V[y]]. \qquad (6.8)
\end{aligned}$$

Appendix B: Fluxes

The field strength of the original Kalb–Ramond field is given by Eq. (2.5). The original metric $G_{\mu\nu}$ is constant, and therefore the corresponding Christoffel connection is zero. The effective metric $G^E_{\mu\nu}$ is linear in the coordinates and the corresponding Christoffel connection,

$$\begin{aligned}
\Gamma^E_{\mu,\nu\rho} &= \frac{1}{2}\left(\partial_\nu G^E_{\mu\rho} + \partial_\rho G^E_{\mu\nu} - \partial_\mu G^E_{\nu\rho}\right) \\
&= -\frac{4}{3}\left(B_{\mu\sigma\nu}(G^{-1}b)^\sigma{}_\rho + B_{\mu\sigma\rho}(G^{-1}b)^\sigma{}_\nu\right), \qquad (7.1)
\end{aligned}$$

is an infinitesimally small constant. It will be used in the following form:

$$\Gamma^E_{\mu,\nu\rho}x^\mu = 4\left(h[x]G^{-1}b + bG^{-1}h[x]\right)_{\nu\rho} \qquad (7.2)$$

and

$$\begin{aligned}
(\Gamma^E_{\mu,\nu\rho} - \Gamma^E_{\nu,\mu\rho})x^\rho &= 8h_{\mu\nu}[bx] \\
&\quad -4\left(h[x]G^{-1}b - bG^{-1}h[x]\right)_{\mu\nu}. \qquad (7.3)
\end{aligned}$$

We express the dual Kalb–Ramond field [23] as

$${}^\star B^{\mu\nu}[\Delta V] = {}^\star b^{\mu\nu} + Q^{\mu\nu}{}_\rho\Delta V^\rho, \qquad (7.4)$$

where ${}^\star b^{\mu\nu} = \frac{\kappa}{2}\theta_0^{\mu\nu}$ and

$$Q^{\mu\nu}{}_\rho = -\frac{1}{3}\left[(g^{-1})^{\mu\sigma}(g^{-1})^{\nu\tau} - \kappa^2\theta_0^{\mu\sigma}\theta_0^{\nu\tau}\right]B_{\sigma\tau\rho}. \quad (7.5)$$

This will be used as

$$Q^{\mu\nu}{}_\rho x^\rho = -(g^{-1})^{\mu\rho}\left[h[x] + 4bG^{-1}h[x]G^{-1}b\right]_{\rho\sigma}(g^{-1})^{\sigma\nu}$$
$$= -\left[g^{-1}h[x]g^{-1} + \kappa^2\theta_0 h[x]\theta_0\right]^{\mu\nu}. \quad (7.6)$$

Appendix C: PB's between pure coordinates

Starting with the PB of the σ derivatives of the coordinates

$$\{X'_\mu(\sigma), Y'_\nu(\bar\sigma)\} \cong K'_{\mu\nu}(\sigma)\delta(\sigma-\bar\sigma) + L_{\mu\nu}(\sigma)\delta'(\sigma-\bar\sigma), \quad (8.1)$$

let us find the expression for the PB between the coordinates, $\{X_\mu(\sigma), Y_\nu(\bar\sigma)\}$. From Eq. (8.1) it follows that $\Delta X_\mu(\sigma,\sigma_0)$ and $\Delta Y_\mu(\sigma,\sigma_0)$ defined by

$$\Delta X_\mu(\sigma,\sigma_0) = \int_{\sigma_0}^{\sigma} d\eta\, X'_\mu(\eta) = X_\mu(\sigma) - X_\mu(\sigma_0),$$
$$\Delta Y_\mu(\sigma,\sigma_0) = \int_{\sigma_0}^{\sigma} d\eta\, Y'_\mu(\eta) = Y_\mu(\sigma) - Y_\mu(\sigma_0) \quad (8.2)$$

satisfy

$$\{\Delta X_\mu(\sigma,\sigma_0), \Delta Y_\nu(\bar\sigma,\bar\sigma_0)\} \cong \int_{\sigma_0}^{\sigma} d\eta \int_{\bar\sigma_0}^{\bar\sigma} d\bar\eta\left[K'_{\mu\nu}(\eta)\delta(\eta-\bar\eta) + L_{\mu\nu}(\eta)\delta'(\eta-\bar\eta)\right]. \quad (8.3)$$

Integrating over $\bar\eta$ and using

$$\int_{\sigma_0}^{\sigma} d\eta f(\eta)\delta(\eta-\bar\sigma) = f(\bar\sigma)\left[\theta(\sigma-\bar\sigma) - \theta(\sigma_0-\bar\sigma)\right], \quad (8.4)$$

we obtain

$$\{\Delta X_\mu(\sigma,\sigma_0), \Delta Y_\nu(\bar\sigma,\bar\sigma_0)\} \cong \int_{\sigma_0}^{\sigma} d\eta\left[K'_{\mu\nu}(\eta)\left[\theta(\eta-\bar\sigma_0) - \theta(\eta-\bar\sigma)\right] + L_{\mu\nu}(\eta)\left[\delta(\eta-\bar\sigma_0) - \delta(\eta-\bar\sigma)\right]\right], \quad (8.5)$$

where the function $\theta(\sigma)$ is defined as

$$\theta(\sigma) \equiv \int_0^{\sigma} d\eta\delta(\eta) = \frac{1}{2\pi}\left(\sigma + 2\sum_{n\geq 1}\frac{1}{n}\sin n\sigma\right) = \begin{cases} 0 & \text{if } \sigma = 0 \\ 1/2 & \text{if } 0<\sigma<2\pi, \quad \sigma\in[0.2\pi]. \\ 1 & \text{if } \sigma = 2\pi \end{cases} \quad (8.6)$$

Integrating by parts over η and using Eq. (8.4) we get

$$\{\Delta X_\mu(\sigma,\sigma_0), \Delta Y_\nu(\bar\sigma,\bar\sigma_0)\}$$
$$\cong K_{\mu\nu}(\sigma)\left[\theta(\sigma-\bar\sigma_0) - \theta(\sigma-\bar\sigma)\right]$$
$$-K_{\mu\nu}(\sigma_0)\left[\theta(\sigma_0-\bar\sigma_0) - \theta(\sigma_0-\bar\sigma)\right]$$
$$-K_{\mu\nu}(\bar\sigma_0)\left[\theta(\sigma-\bar\sigma_0) - \theta(\sigma_0-\bar\sigma_0)\right]$$
$$+K_{\mu\nu}(\bar\sigma)\left[\theta(\sigma-\bar\sigma) - \theta(\sigma_0-\bar\sigma)\right]$$
$$+L_{\mu\nu}(\bar\sigma_0)\left[\theta(\sigma-\bar\sigma_0) - \theta(\sigma_0-\bar\sigma_0)\right]$$
$$-L_{\mu\nu}(\bar\sigma)\left[\theta(\sigma-\bar\sigma) - \theta(\sigma_0-\bar\sigma)\right]. \quad (8.7)$$

The relation

$$\{X_\mu(\tau,\sigma), Y_\nu(\tau,\bar\sigma)\} \cong -\left[K_{\mu\nu}(\sigma) - K_{\mu\nu}(\bar\sigma) + L_{\mu\nu}(\bar\sigma)\right]\times\theta(\sigma-\bar\sigma) \quad (8.8)$$

solves Eq. (8.7), up to additive constant.

For $X_\mu = Y_\mu$, the antisymmetry of the left hand side under the replacement $\mu\leftrightarrow\nu$ and $\sigma\leftrightarrow\bar\sigma$ produces the conditions $L_{\mu\nu} = L_{\nu\mu}$ and $K_{\mu\nu} + K_{\nu\mu} = L_{\mu\nu}$.

References

1. F. Ardalan, H. Arfaei, M.M. Sheikh-Jabbari, JHEP **02**, 016 (1999)
2. C.S. Chu, P.M. Ho, Nucl. Phys. B **550**, 151 (1999)
3. N. Seiberg, E. Witten, JHEP **09**, 032 (1999)
4. F. Ardalan, H. Arfaei, M.M. Sheikh-Jabbari, Nucl. Phys. B **576**, 578 (2000)
5. C.S. Chu, P.M. Ho, Nucl. Phys. B **568**, 447 (2000)
6. T. Lee, Phys. Rev. D **62**, 024022 (2000)
7. B. Sazdović, Eur. Phys. J. C **44**, 599 (2005)
8. B. Nikolić, B. Sazdović, Phys. Rev. D **74**, 045024 (2006)
9. B. Nikolić, B. Sazdović, Phys. Rev. D **75**, 085011 (2007)
10. B. Nikolić, B. Sazdović, Adv. Theor. Math. Phys. **14**, 1 (2010)
11. Lj. Davidović, B. Sazdović, Phys. Rev. D **83**, 066014 (2011)
12. Lj. Davidović, B. Sazdović, JHEP **08**, 112 (2011)
13. T.H. Buscher, Phys. Lett. B **194**, 59 (1987)
14. T.H. Buscher, Phys. Lett. B **201**, 466 (1988)
15. D. Luest, JHEP **12**, 084 (2010)
16. D. Andriot, M. Larfors, D. Luest, P. Patalong, JHEP **06**, 021 (2013)
17. D. Andriot, O. Hohm, M. Larfors, D. Luest, P. Patalong, Phys. Rev. Lett. **108**, 261602 (2012)
18. D. Luest, arxiv:1205.0100 [hep-th]
19. R. Blumenhagen, A. Deser, D. Luest, E. Plauschinn, F. Rennecke, J. Phys. A **44**, 385401 (2011)
20. C. Condeescu, I. Florakis, D. Luest, JHEP **04**, 121 (2012)
21. J. Shelton, W. Taylor, B. Wecht, JHEP **10**, 085 (2005)
22. A. Dabholkar, C. Hull, JHEP **05**, 009 (2006)
23. Lj. Davidović, B. Sazdović, Eur. Phys. J. C **74** (1), (2014)

24. R. Blumenhagen, E. Plauschinn, J. Phys. A Math. Theor. **44**, 015401 (2011)
25. E.S. Fradkin, A.A. Tseytlin, Phys. Lett. B **158**, 316 (1985)
26. E.S. Fradkin, A.A. Tseytlin, Nucl. Phys. B **261**, 1 (1985)
27. K. Becker, M. Becker, J. Schwarz, *String Theory and M-Theory: A Modern Introduction*. (Cambridge University Press, Cambridge, 2006)
28. B. Zwiebach, *A First Course in String Theory*. (Cambridge University Press, Cambridge, 2004)
29. V. Schomerus, Class. Quant. Grav. **19**, 5781 (2002)
30. L. Cornalba, R. Schiappa, Commun. Math. Phys. **225**, 33 (2002)
31. Lj. Davidović, B. Sazdović, EPJ C **72** (11), 2199 (2012)
32. D. Andriot, O. Hohm, M. Larfors, D. Luest,, P. Patalong, Fort. Phys. **60** (2012)

Eur. Phys. J. C (2019) 79:819
https://doi.org/10.1140/epjc/s10052-019-7338-7

THE EUROPEAN PHYSICAL JOURNAL C

Letter

Advantage of the second-order formalism in double space T-dualization of type II superstring

B. Nikolić[a], **B. Sazdović**[b]

Institute of Physics, University of Belgrade, P.O. Box 57, 11001 Belgrade, Serbia

Received: 11 July 2019 / Accepted: 21 September 2019 / Published online: 4 October 2019

Abstract In this article we present bosonic T-dualization in double space of the type II superstring theory in the pure spinor formulation. We use the action with constant background fields obtained from the general case under some physically and mathematically justified assumptions. Unlike Nikolić and Sazdović (EPJ C 77:197, 2017), where we used the first-order theory, in this article fermionic momenta are integrated out. Full T-dualization in double space is represented as a permutation of the initial x^μ and T-dual coordinates y_μ. Requiring that a T-dual transformation law of the T-dual double coordinate ${}^\star Z^M = (y_\mu, x^\mu)$ to be of the same form as for initial one $Z^M = (x^\mu, y_\mu)$, we obtain the form of the T-dual background fields in terms of the initial ones. The advantage of using the action with integrated fermionic momenta is that it gives all T-dual background fields in terms of the initial ones. In the case of the first-order theory Nikolić and Sazdović (2017) a T-dual R-R field strength was obtained out of the double space formalism under additional assumptions.

1 Introduction

T-duality is a feature which cannot be met in the point-particle theory and represents a novelty brought about by string theory [2–9]. The basic mathematical framework, in which T-dualization is performed, is the Buscher procedure [5,6]. The starting point of the procedure is the existence of global isometries along some directions. In the next step we localize that symmetry introducing world-sheet covariant derivatives (instead of ordinary ones) and gauge fields. In order to make the gauge field an unphysical degree of freedom, a term with Lagrange multipliers is added to the action. The final phase of the procedure is using gauge freedom to fix the initial coordinates. Variation of the gauge fixed action with respect to the Lagrange multipliers produces the initial action, while variation with respect to the gauge fields gives a T-dual action. Combining these equations of motion the relations connecting initial and T-dual coordinates are obtained. These relations are known in the literature as T-dual transformation laws.

Why is T-duality so important? The answer is concerned with M-theory. Five consistent superstring theories are connected by a web of T- and S-dualities. It is a well-known fact in the case of type II superstring theories that T-dualization along one spatial dimension transforms type IIA(B) to type IIB(A) theory, while T-dualization along the time-like direction produces type II* theory, of which the R-R field strength is the initial one multiplied by the imaginary unit [1,10]. Using double space enables one to unify all three theories. This could be a way toward better understanding M-theory.

The basic presumption for implementing the Buscher T-dualization procedure is the existence of global isometry along some directions. Effectively, it means that we can find the coordinate basis in which background fields do not depend on those directions [5–9,11,12].

Except the standard Buscher procedure, there is a generalized Buscher procedure dealing with T-dualization along directions on which the background fields depend. In the generalized Buscher procedure, to be compared with the standard one, an additional ingredient is present and that is an invariant coordinate, $x^\mu_{inv} = \int d\xi^\alpha D_\alpha x^\mu$, where D_α is a world-sheet covariant derivative. So far the generalized procedure was applied in two cases: the bosonic string moving in the weakly curved background [13–15] and the case where the metric is quadratic in the coordinates and the Kalb–Ramond field is a linear function of the coordinates [16]. In the first case isometry is not obvious but actually exists, while in the second case isometry is absent.

This work was supported in part by the Serbian Ministry of Education, Science and Technological Development, under contract no. 171031.

[a] e-mail: bnikolic@ipb.ac.rs
[b] e-mail: sazdovic@ipb.ac.rs

The Buscher T-dualization procedure was used in Refs. [17–24] in the context of closed string noncommutativity. In these articles there was considered the coordinate dependent background—a constant metric and a Kalb–Ramond field with only one nonzero component, $B_{xy} = Hz$, where the field strength H is infinitesimal.

The Buscher T-dualization procedure can be considered as a definition of T-dualization. But there is a illuminating way of representing T-duality representation using double space and a permutation group. The name "double" comes from the way it is constructed. The double space coordinate Z^M consists of initial coordinates x^μ and their T-dual ones, y_μ, $Z^M = (x^\mu, y_\mu)$ ($\mu = 0, 1, 2, \dots, D-1$). The formalism emerged about 20 years ago and it was addressed in Refs. [25–29]. In recent years the interest for this formalism was revived [30–37]. In these recent articles T-duality is related with $O(d, d)$ transformations. On the other hand, in Refs. [1,25,38–40], T-dualization along some subset of directions is represented as a permutation of that subset of initial coordinates and the corresponding T-dual ones. T-duality becomes a symmetry transformation in double space.

In Ref. [1] we demonstrated the equivalence of the Buscher approach and the double space one for type II superstring theory. But there is one detail which has to be emphasized. In the mentioned article we used the pure spinor type II superstring action with constant background fields in the form of the first-order theory, i.e., the fermionic momenta, π_α and $\bar\pi_\alpha$, are not integrated out. In that case the process of T-dualization is mathematically simple, but the price to pay is that the T-dual R-R field strength $P^{\alpha\beta}$ could not be obtained within the double space formalism. The reason is that the R-R field strength is coupled only with the fermionic degrees of freedom which are not dualized. To reproduce the Buscher form of the T-dual R-R field strength we made some additional assumptions.

In this article we will integrate out the momenta and obtain the theory in terms of the derivatives of the bosonic coordinates, x^μ, and the fermionic coordinates, θ^α and $\bar\theta^\alpha$. After the fermionic momenta are integrated out, the R-R field strength is coupled with $\partial_\pm x^\mu$. It turns out that Buscher T-dualization with such an action is slightly more complicated, but, as expected, gives the same result as in the case of the first-order theory. The mathematical framework for double space T-dualization is the same as in [1]. We rewrite the T-dual transformation laws in terms of the double space coordinates Z^M, introducing the generalized metric $\check{\mathcal{H}}_{MN}$, the generalized current $\check{J}_{\pm M}$ and the permutation matrix $\mathcal{T}^M{}_N$, which swaps the initial coordinates x^μ and T-dual ones y_μ. Requiring that the T-dual double space coordinates, ${}^\star Z^M = \mathcal{T}^M{}_N Z^N$, satisfy the transformation law of the same form as the initial coordinates, Z^M, we obtain the expressions for the T-dual generalized metric, ${}^\star\check{\mathcal{H}}_{MN} = (\mathcal{T}\check{\mathcal{H}}\mathcal{T})_{MN}$, and T-dual current, ${}^\star\check{J}_{\pm M} = (\mathcal{T}\check{J}_\pm)_M$.

There is an advantage when we perform T-duality within the double space formalism. The main benefit of the using the action with integrated fermionic momenta is that we get all T-dual background fields.

2 Buscher T-dualization of type II superstring theory with integrated fermionic momenta

In this section we will introduce the type II superstring action in a pure spinor formulation [41–48] in the approximation of constant background fields and up to the quadratic terms. Then we will integrate out the fermionic momenta and apply the standard Buscher procedure. This leads to more complicated calculations, but in double space an advantage occurs.

2.1 Type II superstring in the pure spinor formulation

The general form of the action is borrowed from [49] and it is of the form

$$S = \int_\Sigma d^2\xi (X^T)^M A_{MN} \bar X^N + S_\lambda + S_{\bar\lambda}, \tag{2.1}$$

where the vectors X^M and $\bar X^N$ are the left and right chiral supersymmetric variables

$$X^M = \begin{pmatrix} \partial_+\theta^\alpha \\ \Pi_+^\mu \\ d_\alpha \\ \frac{1}{2}N_+^{\mu\nu} \end{pmatrix}, \quad \bar X^M = \begin{pmatrix} \partial_-\bar\theta^\alpha \\ \Pi_-^\mu \\ \bar d_\alpha \\ \frac{1}{2}\bar N_-^{\mu\nu} \end{pmatrix}, \tag{2.2}$$

of which the components are defined as

$$\Pi_+^\mu = \partial_+ x^\mu + \frac{1}{2}\theta^\alpha(\Gamma^\mu)_{\alpha\beta}\partial_+\theta^\beta,$$
$$\Pi_-^\mu = \partial_- x^\mu + \frac{1}{2}\bar\theta^\alpha(\Gamma^\mu)_{\alpha\beta}\partial_-\bar\theta^\beta, \tag{2.3}$$

$$d_\alpha = \pi_\alpha - \frac{1}{2}(\Gamma_\mu\theta)_\alpha\left[\partial_+ x^\mu + \frac{1}{4}(\theta\Gamma_\mu\partial_+\theta)\right],$$
$$\bar d_\alpha = \bar\pi_\alpha - \frac{1}{2}(\Gamma_\mu\bar\theta)_\alpha\left[\partial_- x^\mu + \frac{1}{4}(\bar\theta\Gamma_\mu\partial_-\bar\theta)\right], \tag{2.4}$$

$$N_+^{\mu\nu} = \frac{1}{2}w_\alpha(\Gamma^{[\mu\nu]})^\alpha{}_\beta\lambda^\beta, \quad \bar N_-^{\mu\nu} = \frac{1}{2}\bar w_\alpha(\Gamma^{[\mu\nu]})^\alpha{}_\beta\bar\lambda^\beta. \tag{2.5}$$

The supermatrix A_{MN} is of the form

$$A_{MN} = \begin{pmatrix} A_{\alpha\beta} & A_{\alpha\nu} & E_\alpha{}^\beta & \Omega_{\alpha,\mu\nu} \\ A_{\mu\beta} & A_{\mu\nu} & \bar E_\mu^\beta & \Omega_{\mu,\nu\rho} \\ E^\alpha{}_\beta & E^\alpha_\nu & P^{\alpha\beta} & C^\alpha{}_{\mu\nu} \\ \Omega_{\mu\nu,\beta} & \Omega_{\mu\nu,\rho} & \bar C_{\mu\nu}{}^\beta & S_{\mu\nu,\rho\sigma} \end{pmatrix}. \tag{2.6}$$

The world sheet Σ is parameterized by $\xi^m = (\xi^0 = \tau, \xi^1 = \sigma)$ and $\partial_\pm = \partial_\tau \pm \partial_\sigma$. Superspace is spanned by the bosonic coordinates x^μ ($\mu = 0, 1, 2, \dots, 9$) and the fermionic ones θ^α and $\bar\theta^\alpha$ ($\alpha = 1, 2, \dots, 16$). The variables π_α and $\bar\pi_\alpha$ are canonically conjugate momenta to θ^α and $\bar\theta^\alpha$, respectively. The actions for the pure spinors, S_λ and $S_{\bar\lambda}$, are the free field actions

$$S_\lambda = \int d^2\xi\, w_\alpha \partial_- \lambda^\alpha, \quad S_{\bar\lambda} = \int d^2\xi\, \bar w_\alpha \partial_+ \bar\lambda^\alpha, \tag{2.7}$$

where λ^α and $\bar\lambda^\alpha$ are pure spinors and w_α and $\bar w_\alpha$ are their canonically conjugate momenta, respectively. The pure spinors satisfy the so-called pure spinor constraints

$$\lambda^\alpha (\Gamma^\mu)_{\alpha\beta} \lambda^\beta = \bar\lambda^\alpha (\Gamma^\mu)_{\alpha\beta} \bar\lambda^\beta = 0. \tag{2.8}$$

This action (2.1) for type II superstring in the pure spinor formulation is general and it is constructed as an expansion in powers of θ^α and $\bar\theta^\alpha$ (for details see [49]).

Our plan is to implement full T-dualization, which means that we T-dualize along all bosonic directions x^μ. Consequently, we will assume that the background fields do not depend on them. On the other hand, because of the way how the action is constructed, for practical reasons (mathematical simplification), we will consider just the first components in the expansion in powers of θ^α and $\bar\theta^\alpha$. Effectively, this means that the nonzero background fields are constant. The background fields from the first and last columns and rows in the matrix A_{MN} are zero (a detailed explanation could be found in [1,49]). The fields surviving these approximations are known in the literature as physical superfields because their first components are supergravity fields.

Finally, all our assumptions produce

$$\Pi^\mu_\pm \to \partial_\pm x^\mu, \quad d_\alpha \to \pi_\alpha, \quad \bar d_\alpha \to \bar\pi_\alpha, \tag{2.9}$$

where the physical superfields take the form

$$A_{\mu\nu} = \kappa\left(\frac{1}{2} g_{\mu\nu} + B_{\mu\nu}\right) + \frac{1}{4\pi}\eta_{\mu\nu}\Phi, \quad E^\alpha_\nu = -\Psi^\alpha_\nu,$$
$$\bar E^\alpha_\mu = \bar\Psi^\alpha_\mu, \quad \mathrm{P}^{\alpha\beta} = \frac{1}{2\kappa} P^{\alpha\beta}. \tag{2.10}$$

Here $g_{\mu\nu}$ is a symmetric and $B_{\mu\nu}$ is an antisymmetric tensor. Consequently, the full action S is

$$S = \kappa \int_\Sigma d^2\xi \left[\partial_+ x^\mu \Pi_{+\mu\nu} \partial_- x^\nu + \frac{1}{4\pi\kappa}\Phi R^{(2)}\right]$$
$$+ \int_\Sigma d^2\xi \Big[-\pi_\alpha \partial_- \left(\theta^\alpha + \Psi^\alpha_\mu x^\mu\right)$$
$$+ \partial_+ \left(\bar\theta^\alpha + \bar\Psi^\alpha_\mu x^\mu\right)\bar\pi_\alpha + \frac{1}{2\kappa}\pi_\alpha P^{\alpha\beta}\bar\pi_\beta\Big], \tag{2.11}$$

where $G_{\mu\nu} = \eta_{\mu\nu} + g_{\mu\nu}$ is the metric tensor and

$$\Pi_{\pm\mu\nu} = B_{\mu\nu} \pm \frac{1}{2} G_{\mu\nu}. \tag{2.12}$$

We will neglect the Tseytlin term in the further analysis because, for a constant dilaton field Φ, it is proportional to the Euler characteristic. Consequently, on some given manifold that term is constant. The actions S_λ and $S_{\bar\lambda}$ are decoupled from the rest and the action, in its final form, is ghost independent.

2.2 Full bosonic T-dualization using Buscher rules

Let us now integrate out the fermionic momenta from the action (2.11) and obtain the theory expressed in terms of the supercoordinates $(x^\mu, \theta^\alpha, \bar\theta^\alpha)$ and their world-sheet derivatives. For the equations of motion for fermionic momenta π_α and $\bar\pi_\alpha$,

$$\pi_\alpha = -2\kappa\, \partial_+ \left(\bar\theta^\beta + \bar\Psi^\beta_\mu x^\mu\right)(P^{-1})_{\beta\alpha},$$
$$\bar\pi_\alpha = 2\kappa (P^{-1})_{\alpha\beta}\partial_- \left(\theta^\beta + \Psi^\beta_\mu x^\mu\right), \tag{2.13}$$

the action gets the form

$$S = \kappa \int_\Sigma d^2\xi\, \partial_+ x^\mu \left[\Pi_{+\mu\nu} + 2\bar\Psi^\alpha_\mu (P^{-1})_{\alpha\beta}\Psi^\beta_\nu\right]\partial_- x^\nu$$
$$+ 2\kappa \int_\Sigma d^2\xi \Big[\partial_+\bar\theta^\alpha (P^{-1})_{\alpha\beta}\partial_-\theta^\beta$$
$$+ \partial_+\bar\theta^\alpha (P^{-1}\Psi)_{\alpha\mu}\partial_- x^\nu + \partial_+ x^\mu (\bar\Psi P^{-1})_{\mu\alpha}\partial_-\theta^\alpha\Big]. \tag{2.14}$$

We will perform bosonic T-dualization of the action (2.14) along all directions x^μ using the Buscher T-dualization rules. In order to gauge global symmetry $\delta x^\mu = \lambda^\mu$, we introduce covariant derivatives, $D_\pm x^\mu = \partial_\pm x^\mu + v^\mu_\pm$, instead of the ordinary ones, $\partial_\pm x^\mu$, where $v^\mu_\pm$ are gauge fields. Fixing the gauge ($x^\mu = const.$) means effectively that ordinary derivatives $\partial_\pm x^\mu$ are replaced with gauge fields $v^\mu_\pm$ in the initial action (2.14), while, in order to make $v^\mu_\pm$ unphysical degrees of freedom, we add to the action

$$S_{add} = \frac{\kappa}{2}\int d^2\xi \left(v^\mu_+ \partial_- y_\mu - v^\mu_- \partial_+ y_\mu\right). \tag{2.15}$$

The gauged fixed action is of the form

$$S_{fix} = S + S_{add}$$
$$= \kappa \int d^2\xi \Big[v^\mu_+ \Pi_{+\mu\nu} v^\nu_- + 2\left(\partial_+\bar\theta^\alpha + \bar\Psi^\alpha_\mu v^\mu_+\right)(P^{-1})_{\alpha\beta}$$
$$\left(\partial_-\theta^\beta + \Psi^\beta_\nu v^\nu_-\right) + \frac{1}{2}(v^\mu_+\partial_- y_\mu - v^\mu_-\partial_+ y_\mu)\Big]. \tag{2.16}$$

Varying the gauge fixed action (2.16) with respect to the Lagrange multipliers y_μ, we find that the field strength for gauge fields $v^\mu_\pm$ is equal to zero,

$$\partial_+ v_-^\mu - \partial_- v_+^\mu = 0 \Rightarrow v_\pm^\mu = \partial_\pm x^\mu. \tag{2.17}$$

In this way we restore the initial theory from the gauge fixed action. Let us note that we omitted the dilaton term because this is a classical analysis while the dilaton is treated within the quantum formalism.

Varying the gauge fixed action with respect to the gauge fields v_+^μ and v_-^μ, we get the equations, respectively,

$$\Pi_{+\mu\nu} v_-^\nu + 2\bar{\Psi}^\alpha{}_\mu (P^{-1})_{\alpha\beta} \left(\partial_- \theta^\beta + \Psi^\beta{}_\nu v_-^\nu\right) + \frac{1}{2}\partial_- y_\mu = 0, \tag{2.18}$$

$$v_+^\nu \Pi_{+\nu\mu} + 2\left(\partial_+ \bar{\theta}^\alpha + \bar{\Psi}^\alpha{}_\nu v_+^\nu\right)(P^{-1})_{\alpha\beta} \Psi^\beta{}_\mu - \frac{1}{2}\partial_+ y_\mu = 0. \tag{2.19}$$

Here we introduce the notation

$$\check{\Pi}_{+\mu\nu} \equiv \Pi_{+\mu\nu} + 2\bar{\Psi}^\alpha{}_\mu (P^{-1})_{\alpha\beta} \Psi^\beta{}_\nu = \check{B}_{\mu\nu} + \frac{1}{2}\check{G}_{\mu\nu}, \tag{2.20}$$

where $\check{B}_{\mu\nu}$ and $\check{G}_{\mu\nu}$ are the antisymmetric and symmetric parts of $\check{\Pi}_{+\mu\nu}$, respectively. These expressions are in fact the Kalb–Ramond field $B_{\mu\nu}$ and metric $G_{\mu\nu}$ improved by some expressions consisting of the NS-R and R-R background fields. The above expressions for the gauge fields can be rewritten in the form

$$\check{\Pi}_{+\mu\nu} v_-^\nu + 2\bar{\Psi}^\alpha{}_\mu (P^{-1})_{\alpha\beta} \partial_- \theta^\beta + \frac{1}{2}\partial_- y_\mu = 0, \tag{2.21}$$

$$v_+^\nu \check{\Pi}_{+\nu\mu} + 2\partial_+ \bar{\theta}^\alpha (P^{-1})_{\alpha\beta} \Psi^\beta{}_\mu - \frac{1}{2}\partial_+ y_\mu = 0. \tag{2.22}$$

Using the relations

$$\check{\Theta}_-^{\mu\nu} \check{\Pi}_{+\nu\rho} = \frac{1}{2\kappa}\delta^\mu{}_\rho, \quad \check{\Theta}_-^{\mu\nu} = -\frac{2}{\kappa}\left(\check{G}_E^{-1} \check{\Pi}_- \check{G}^{-1}\right)^{\mu\nu}, \tag{2.23}$$

where

$$\check{\Theta}_-^{\mu\nu} = \Theta_-^{\mu\nu} - 4\kappa \Theta_-^{\mu\rho} \bar{\Psi}^\alpha{}_\rho (\tilde{P}^{-1})_{\alpha\beta} \Psi^\beta{}_\lambda \Theta_-^{\lambda\nu} = \check{\Theta}^{\mu\nu} + \frac{1}{\kappa}\left(\check{G}_E^{-1}\right)^{\mu\nu}, \tag{2.24}$$

$$\tilde{P}^{\alpha\beta} \equiv P^{\alpha\beta} + 4\kappa \Psi^\alpha{}_\mu \Theta_-^{\mu\nu} \bar{\Psi}^\beta{}_\nu, \tag{2.25}$$

$$\Theta_-^{\mu\nu} = -\frac{2}{\kappa}\left(G_E^{-1} \Pi_- G^{-1}\right)^{\mu\nu}, \quad \Theta_-^{\mu\rho} \Pi_{+\rho\nu} = \frac{1}{2\kappa}\delta^\mu{}_\nu, \tag{2.26}$$

we get

$$v_-^\mu = -\kappa \check{\Theta}_-^{\mu\nu} \partial_- \left[y_\nu + 4\bar{\Psi}^\alpha{}_\mu (P^{-1})_{\alpha\beta} \theta^\beta\right], \tag{2.27}$$

$$v_+^\mu = \kappa \partial_+ \left[y_\nu - 4\bar{\theta}^\alpha (P^{-1})_{\alpha\beta} \Psi^\beta{}_\nu\right] \check{\Theta}_-^{\nu\mu}. \tag{2.28}$$

Equation (2.23) is proved by direct calculation and using the definition of $\tilde{P}^{\alpha\beta}$.

Inserting the expressions (2.27) and (2.28) into the expression for the gauge fixed action (2.16) we obtain the T-dual action

$$\begin{aligned} {}^\star S = \kappa \int d^2\xi \Big[&\frac{\kappa}{2}\partial_+ y_\mu \check{\Theta}_-^{\mu\nu} \partial_- y_\nu \\ &+ 2\kappa \partial_+ y_\mu \check{\Theta}_-^{\mu\nu} \bar{\Psi}^\alpha{}_\nu (P^{-1})_{\alpha\beta} \partial_- \theta^\beta \\ &- 2\kappa \partial_+ \bar{\theta}^\alpha (P^{-1})_{\alpha\beta} \Psi^\beta{}_\mu \check{\Theta}_-^{\mu\nu} \partial_- y_\nu \\ &+ 2\partial_+ \bar{\theta}^\alpha (P^{-1} - 4\kappa P^{-1} \Psi \check{\Theta}_- \bar{\Psi} P^{-1})_{\alpha\beta} \partial_- \theta^\beta \Big]. \end{aligned} \tag{2.29}$$

Introducing T-dual background fields marked by $\star$, we write the T-dual action in the form of the initial action (2.14)

$$\begin{aligned} {}^\star S = \kappa \int d^2\xi \Big[&\partial_+ y_\mu \left({}^\star\Pi_+ + 2{}^\star\bar{\Psi}{}^\star P^{-1}{}^\star\Psi\right)^{\mu\nu} \partial_- y_\nu \\ &+ 2\partial_+ y_\mu ({}^\star\bar{\Psi}{}^\star P^{-1})^\mu{}_\alpha \partial_- \theta^\alpha \\ &+ 2\partial_+ \bar{\theta}^\alpha ({}^\star P^{-1}{}^\star\Psi)_\alpha{}^\mu \partial_- y_\mu + 2\partial_+ \bar{\theta}^\alpha ({}^\star P^{-1})_{\alpha\beta} \partial_- \theta \Big]. \end{aligned} \tag{2.30}$$

Comparing the last two equations, we get the T-dual background fields in terms of the initial ones,

$${}^\star\Pi_+^{\mu\nu} + 2{}^\star\bar{\Psi}^{\mu\alpha} ({}^\star P^{-1})_{\alpha\beta} {}^\star\Psi^{\beta\nu} = \frac{\kappa}{2}\check{\Theta}_-^{\mu\nu}, \tag{2.31}$$

$${}^\star\bar{\Psi}^{\mu\beta} ({}^\star P^{-1})_{\beta\alpha} = \kappa \check{\Theta}_-^{\mu\nu} \bar{\Psi}^\beta{}_\nu (P^{-1})_{\beta\alpha}, \quad ({}^\star P^{-1})_{\alpha\beta} {}^\star\Psi^{\beta\mu} = -\kappa (P^{-1})_{\alpha\beta} \Psi^\beta{}_\nu \check{\Theta}_-^{\nu\mu}, \tag{2.32}$$

$$({}^\star P^{-1})_{\alpha\beta} = (P^{-1})_{\alpha\beta} - 4\kappa (P^{-1})_{\alpha\gamma} \Psi^\gamma{}_\mu \check{\Theta}_-^{\mu\nu} \bar{\Psi}^\delta{}_\nu (P^{-1})_{\delta\beta}. \tag{2.33}$$

By direct calculation, solving the above four equations, we finally get

$${}^\star\Pi_+^{\mu\nu} = \frac{\kappa}{2}\Theta_-^{\mu\nu}, \tag{2.34}$$

$${}^\star\Psi^{\alpha\mu} = -\kappa \Psi^\alpha{}_\nu \Theta_-^{\nu\mu}, \quad {}^\star\bar{\Psi}^{\mu\alpha} = \kappa \Theta_-^{\mu\nu} \bar{\Psi}^\alpha{}_\nu, \tag{2.35}$$

$${}^\star P^{\alpha\beta} = \tilde{P}^{\alpha\beta}, \tag{2.36}$$

which is in full agreement with the case where we T-dualize the same model in the form of the first-order theory.

Combining the equations of motion for the Lagrange multiplier (2.17) with the equations of motion for the gauge fields, (2.28) and (2.28), we obtain the relation between the initial x^μ and T-dual coordinates y_μ

$$\partial_\pm x^\mu \cong -\kappa \check{\Theta}_\pm^{\mu\nu} \left[\partial_\pm y_\nu + 4\Psi^\alpha_{\pm\nu} (P_\mp^{-1})_{\alpha\beta} \partial_\pm \theta_\mp^\beta\right]. \tag{2.37}$$

The inverse of this relation is also useful and it is of the form

$$\partial_\pm y_\mu \cong -2\check{\Pi}_{\mp\mu\nu} \partial_\pm x^\nu - 4\Psi^\alpha_{\pm\mu} (P_\mp^{-1})_{\alpha\beta} \partial_\pm \theta_\mp^\beta. \tag{2.38}$$

Here we use the notation

$$\theta_+^\alpha \equiv \theta^\alpha, \quad \theta_-^\alpha \equiv \bar\theta^\alpha, \tag{2.39}$$
$$P_+^{\alpha\beta} \equiv P^{\alpha\beta}, \quad P_-^{\alpha\beta} \equiv P^{\beta\alpha}, \tag{2.40}$$
$$\Psi_{+\mu}^\alpha \equiv \Psi^\alpha{}_\mu, \quad \Psi_{-\mu}^\alpha \equiv \bar\Psi^\alpha{}_\mu, \tag{2.41}$$
$$\check\Theta_+^{\mu\nu} \equiv -\check\Theta_-^{\nu\mu}. \tag{2.42}$$

As we see, the two chirality sectors transform differently under T-dualization. The form of the T-dualization transformation laws is of the same form as in [1]. Consequently, in accordance with the results of Refs. [1,50,51], we introduce the proper fermionic coordinates

$$^\bullet\theta_+^\alpha = \theta_+^\alpha, \quad ^\bullet\bar\theta_-^\alpha = -(\Gamma_{11}\bar\theta_-)^\alpha, \tag{2.43}$$

and the correct form of the T-dual fields is

$$^\star\Pi_+^{\mu\nu} = \frac{\kappa}{2}\Theta_-^{\mu\nu}, \tag{2.44}$$
$$^\star\Psi^{\alpha\mu} = -\kappa\Psi^\alpha{}_\nu\Theta_-^{\nu\mu}, \quad ^\star\bar\Psi^{\mu\alpha} = -\kappa\Theta_-^{\mu\nu}(\Gamma_{11}\bar\Psi)^\alpha{}_\nu, \tag{2.45}$$
$$^\star P^{\alpha\beta} = -(\tilde P\Gamma_{11})^{\alpha\beta}. \tag{2.46}$$

3 T-dualization of type II superstring in double space

In this section we will demonstrate another framework in which we can perform the T-dualization procedure. Unlike the case of the T-dualization of type II superstring theory in the form of the first-order theory [1] where the T-dual R-R field strength is not obtained within double space framework, here we will see that, when fermionic momenta are integrated out, the double space formalism gives all T-dual background fields. Before the T-dualization procedure we will introduce double space and the corresponding quantities.

3.1 T-dual transformation law in double space

Let us introduce the double space coordinate

$$Z^M = \begin{pmatrix} x^\mu \\ y_\mu \end{pmatrix}, \tag{3.1}$$

and rewrite the T-dual transformation laws (2.37)–(2.38) in the form

$$\pm\partial_\pm y_\mu \cong \check G^E_{\mu\nu}\partial_\pm x^\nu + \kappa\check G^E_{\mu\rho}\check\Theta^{\rho\nu}\partial_\pm y_\nu + 4\kappa G^E_{\mu\rho}\Theta_\pm^{\rho\lambda}\Psi^\alpha_{\pm\lambda}(P_\mp^{-1})_{\alpha\beta}\partial_\pm\theta_\mp^\beta, \tag{3.2}$$
$$\pm\partial_\pm x^\mu \cong (\check G^{-1})^{\mu\nu}\partial_\pm y_\nu + 2(\check G^{-1}\check B)^\mu{}_\nu\partial_\pm x^\nu + 4(\check G^{-1})^{\mu\rho}\Psi^\alpha_{\pm\rho}(P_\mp^{-1})_{\alpha\beta}\partial_\pm\theta_\mp^\beta. \tag{3.3}$$

These two equations can be rewritten in double space as

$$\pm\Omega_{MN}\partial_\pm Z^N \cong \check{\mathcal H}_{MN}\partial_\pm Z^N + \check J_{\pm M}, \tag{3.4}$$

where the generalized metric is of the form

$$\check{\mathcal H}_{MN} = \begin{pmatrix} \check G^E_{\mu\nu} & \kappa\check G^E_{\mu\rho}\check\Theta^{\rho\nu} \\ 2(\check G^{-1})^{\mu\rho}\check B_{\rho\nu} & (\check G^{-1})^{\mu\nu} \end{pmatrix}, \tag{3.5}$$

and the double current is

$$\check J_{\pm M} = 4\begin{pmatrix} \kappa\check G^E_{\mu\rho}\check\Theta_\pm^{\rho\nu} \\ (\check G^{-1})^{\mu\nu} \end{pmatrix} J_{\pm\mu}, \quad J_{\pm\mu} = \Psi^\alpha_{\pm\mu}(P_\mp^{-1})_{\alpha\beta}\partial_\pm\theta_\mp^\beta. \tag{3.6}$$

The matrix

$$\Omega = \begin{pmatrix} 0 & 1_D \\ 1_D & 0 \end{pmatrix}, \tag{3.7}$$

where 1_D denotes the unity matrix in D dimensions, known in double field theory (DFT) as the invariant $SO(D,D)$ metric.

Note that generalized metric is not of the standard form because its components contain the improved Kalb–Ramond field $\check B_{\mu\nu}$ and the improved metric $\check G_{\mu\nu}$. Those additional factors in $\check B_{\mu\nu}$ and $\check G_{\mu\nu}$ have a bilinear form in the NS-R fields $\Psi^\alpha{}_\mu$ and $\bar\Psi^\alpha{}_\mu$. Still, we have

$$\check{\mathcal H}^T\Omega\check{\mathcal H} = \Omega, \quad \Omega^2 = 1, \quad \det\check{\mathcal H} = 1, \tag{3.8}$$

which means that $\check{\mathcal H} \in SO(D,D)$.

3.2 Full T-dualization in double space

Let us introduce the permutation matrix

$$\mathcal T^M{}_N = \begin{pmatrix} 0 & 1_D \\ 1_D & 0 \end{pmatrix}, \tag{3.9}$$

and define the T-dual double coordinate $^\star Z^M$ as

$$^\star Z^M = \mathcal T^M{}_N Z^N. \tag{3.10}$$

We require that the T-dual transformation law for the T-dual double coordinate $^\star Z^M$ has the same form as for initial double coordinate Z^M (3.4)

$$\pm\Omega_{MN}\partial_\pm{}^\star Z^N \cong {}^\star\check{\mathcal H}_{MQ}\partial_\pm{}^\star Z^Q + {}^\star\check J_{\pm M}, \tag{3.11}$$

which implies that T-dual generalized metric and double current are of the form, respectively,

$$^\star\check{\mathcal H}_{MN} = \mathcal T_M{}^P\check{\mathcal H}_{PQ}\mathcal T^Q{}_N, \quad ^\star\check J_{\pm M} = \mathcal T_M{}^N\check J_{\pm N}. \tag{3.12}$$

Let us make explicit the first equation in (3.12):

$$\begin{pmatrix} ^\star\check G_E^{\mu\nu} & \kappa{}^\star\check G_E^{\mu\rho}{}^\star\check\Theta_{\rho\nu} \\ 2({}^\star\check G^{-1})_{\mu\rho}{}^\star\check B^{\rho\nu} & ({}^\star\check G^{-1})_{\mu\nu} \end{pmatrix}$$

$$= \begin{pmatrix} (\check{G}^{-1})^{\mu\nu} & 2(\check{G}^{-1})^{\mu\rho}\check{B}_{\rho\nu} \\ \kappa \check{G}^E_{\mu\rho}\check{\Theta}^{\rho\nu} & \check{G}^E_{\mu\nu} \end{pmatrix}. \tag{3.13}$$

Equating the (2, 2) block components we get

$$({}^\star\check{G}^{-1})_{\mu\nu} = \check{G}^E_{\mu\nu}, \tag{3.14}$$

which produces

$${}^\star\check{G}^{\mu\nu} = \left(\check{G}_E^{-1}\right)^{\mu\nu}. \tag{3.15}$$

Using (2, 1) the block components equation

$$2({}^\star\check{G}^{-1})_{\mu\rho}{}^\star\check{B}^{\rho\nu} = \kappa \check{G}^E_{\mu\rho}\check{\Theta}^{\rho\nu}, \tag{3.16}$$

combining with (3.14), we obtain

$${}^\star\check{B}^{\mu\nu} = \frac{\kappa}{2}\check{\Theta}^{\mu\nu}. \tag{3.17}$$

Using these two results we have

$${}^\star\check{\Pi}^{\mu\nu}_\pm = {}^\star B^{\mu\nu} \pm \frac{1}{2}{}^\star\check{G}^{\mu\nu} = \frac{\kappa}{2}\left[\check{\Theta}^{\mu\nu} \pm \frac{1}{\kappa}\left(\check{G}_E^{-1}\right)^{\mu\nu}\right] = \frac{\kappa}{2}\check{\Theta}^{\mu\nu}_\mp. \tag{3.18}$$

The obtained equation coincides with the equation obtained by the standard Buscher procedure (2.31). The block components (1, 1) and (1, 2) give, respectively,

$${}^\star\check{G}^{\mu\nu}_E = (\check{G}^{-1})^{\mu\nu}, \quad {}^\star\check{G}^{\mu\rho}_E{}^\star\check{\Theta}_{\rho\nu} = \frac{2}{\kappa}(\check{G}^{-1})^{\mu\rho}\check{B}_{\rho\nu}. \tag{3.19}$$

Combining the last two equations produces

$${}^\star\check{\Theta}_{\mu\nu} = \frac{2}{\kappa}\check{B}_{\mu\nu}, \tag{3.20}$$

and, furthermore, we have

$${}^\star\check{\Theta}_{-\mu\nu} = \frac{2}{\kappa}\check{\Pi}_{+\mu\nu}. \tag{3.21}$$

Using the relations between the initial and T-dual NS-NS background fields obtained above and the second equation in (3.12), we get

$${}^\star J^\mu_\pm = \kappa\check{\Theta}^{\mu\nu}_\pm J_{\pm\nu}, \tag{3.22}$$

where the T-dual current ${}^\star J^\mu_\pm$ has the same form as the initial one but in terms of the T-dual background fields and proper fermionic coordinates (for details see [1])

$${}^\star J^\mu_\pm \equiv {}^\star\Psi^{\alpha\mu}_\pm({}^\star P^{-1}_\mp)_{\alpha\beta}\partial_\pm{}^\bullet\theta^\beta_\mp. \tag{3.23}$$

The proper fermionic coordinates are defined as

$${}^\bullet\theta^\alpha_+ = \theta^\alpha_+, \quad {}^\bullet\theta^\alpha_- = -(\Gamma_{11}\theta_-)^\alpha. \tag{3.24}$$

From Eq. (3.23) we have

$${}^\star\check{\Pi}^{\mu\nu}_+ = {}^\star\Pi^{\mu\nu}_+ + 2{}^\star\bar{\Psi}^{\alpha\mu}({}^\star P^{-1})_{\alpha\beta}{}^\star\Psi^{\beta\nu}, \tag{3.25}$$

$${}^\star\check{\Theta}_{-\mu\nu} = {}^\star\Theta_{-\mu\nu} - 4\kappa{}^\star\Theta_{-\mu\rho}{}^\star\bar{\Psi}^{\rho\alpha}({}^\star\tilde{P}^{-1})_{\alpha\beta}{}^\star\Psi^{\beta\lambda}{}^\star\Theta_{-\lambda\nu}, \tag{3.26}$$

and, solving these equations, we get

$${}^\star\Psi^{\alpha\mu} = \pm\kappa\Psi^\alpha{}_\nu\Theta^{\nu\mu}_-, \quad {}^\star\bar{\Psi}^{\alpha\mu} = \pm\kappa\Theta^{\mu\nu}_-(\Gamma_{11}\bar{\Psi})^\alpha_\nu,$$
$${}^\star P^{\alpha\beta} = -\left(P^{\alpha\gamma} + 4\kappa\Psi^\alpha{}_\mu\Theta^{\mu\nu}\bar{\Psi}^\gamma{}_\nu\right)(\Gamma_{11})_\gamma{}^\beta. \tag{3.27}$$

Here the double space formalism produces (3.18) and (3.27) i.e. all relations (2.44)–(2.46), up to the sign of the T-dual NS-R background fields. This uncertainty in sign is a consequence of the fact that in both equations, (3.25) and (3.26), the NS-R fields, ${}^\star\Psi^{\alpha\mu}$ and ${}^\star\bar{\Psi}^{\alpha\mu}$, appear in a bilinear combination. In comparison with the case where the fermionic momenta are not integrated out [1], this is an improvement because the double formalism gives all T-dual background fields. In Ref. [1] we did not obtain the relation for the T-dual R-R background field and we had to impose additional conditions. Here the calculation is slightly more complicated, but we obtain all fields within one formalism. The reason is that in [1] the R-R field is coupled by fermionic momenta which are not T-dualized. Consequently, after integration of the fermionic momenta, there appears a coupling between the R-R field strength with bosonic coordinates x^μ which results in Eq. (3.27).

4 Conclusion

In this article we considered the type II superstring theory in a pure spinor formulation with constant background fields. We integrated out the fermionic momenta and obtained the theory quadratic in world-sheet derivatives of bosonic and fermionic coordinates. Our goal was to show the advantage of the T-dualization within the double space formalism comparing to the first-order theory [1].

At the beginning we explained how we obtained the action with constant background field from the general one derived in [49]. The assumed shift symmetry along the bosonic directions x^μ means that the background fields do not depend on x^μ. On the other hand, for technical simplicity of the calculations, we take just the first terms in the expansions of background fields in powers of θ^α and $\bar{\theta}^\alpha$. All these assumptions result in constant background fields. In the final form of the action, just physical superfields are present, while the auxiliary fields and field strengths are zero.

The main mathematical difference from Ref. [1] is that the fermionic momenta are integrated out. In this way we obtained a theory which is quadratic in the world-sheet derivatives of the coordinates, $\partial_\pm x^\mu$, $\partial_\pm \theta^\alpha$ and $\partial_\pm \bar\theta^\alpha$. It is important to emphasize that in such a formulation R-R field strength $P^{\alpha\beta}$ is coupled with the derivatives of the bosonic coordinates $\partial_\pm x^\mu$. Then we applied the Buscher procedure and, beside some slightly more complicated mathematical calculations, we obtained the same result as in the case for the first-order theory [1].

Our contribution was to show the benefit in performing a T-dualization procedure in double space using the action (2.14), to be compared with the results obtained for the action (2.11) in Ref. [1].

The double space is spanned by the coordinates $Z^M = (x^\mu, y_\mu)$, where x^μ are initial bosonic coordinates and y_μ are corresponding the T-dual ones. The T-dual transformation laws are rewritten in terms of the double space coordinates introducing the generalized metric $\check{\mathcal{H}}_{MN}$ and the current $\check{J}_{\pm M}$. Note that their components are expressed in terms of the improved Kalb–Ramond field and the metric containing additional terms bilinear in the NS-R background fields Ψ^α_μ and $\bar\Psi^\alpha_\mu$. Requiring that T-dual double space coordinates ${}^\star Z^M = \mathcal{T}^M{}_N Z^N$ satisfy a transformation law of the same form as the initial coordinates Z^M we found the T-dual generalized metric ${}^\star\check{\mathcal{H}}_{MN}$ and the T-dual current ${}^\star J_{\pm M}$. The T-dual generalized metric should have the same form as the initial ones, so, in this way we obtain relations which produce the expressions for T-dual background fields in terms of the initial ones, which agrees with that obtained applying the Buscher procedure.

In Ref. [1] we obtained the expressions for the T-dual NS-NS background fields as well as for the NS-R fields. But because we T-dualized along the bosonic directions which are not coupled with R-R field strength, the double space formalism did not give us the expression for the T-dual R-R field strength. These expressions were obtained under some additional assumptions out of the double space formalism.

Here we succeeded in obtaining the expressions for all T-dual background fields, which showed that there is an advantage in performing double space T-dualization in the second-order theory where the fermionic momenta are integrated out.

After having summarized the results of this article it is interesting to discuss their significance and relation to the results of other articles addressing the same or similar subjects. For example, in Ref. [52] the authors construct the type II model with T-duality as a manifest symmetry. The way of construction is partially similar to the one from [49] used in this paper. In [49] they used (anti)holomorphicity and nilpotency conditions, while in [52] instead of nilpotency conditions they used conditions originating from κ symmetry. But in [52] they do not study the problem of the RR field strength as well as an interchange between types IIA/B in the T-dualization process, which are the subjects addressed in this article. Furthermore, in [53] one version of the doubled superspace is discussed. It is pretty similar to the space we used in this article, but it is obtained by multiplication of the left and right chiral sectors with $\mathcal{N} = 1$ supersymmetry in $D = 10$. The space which is obtained is spanned by the initial bosonic coordinates, their T-dual ones and two fermionic coordinates. Our doubled coordinate contains just the bosonic part of this doubled supercoordinate because we consider here just bosonic T-dualization and, consequently, we do not consider a fermionic sector. The obvious difference is that in [53] they obtained a type II action in the Green–Schwarz formalism, while we study here a pure spinor action. In [54] the geometry of superspace is developed with a type II model as the main example. One of the things discussed is the relation of some sectors with pure spinor fields. Finnaly, it is useful to mention also Ref. [55]. In this article a reduction of type IIA/B superstring theory from 10d to 9d is done, which effectively could be T-dualization. The L_∞ isomorphisms relate two coefficient L_∞ algebras. Also one derived the Buscher rules for the RR field strength, which is done in this article using simpler mathematical methods.

Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors' comment: This paper does not include any additional information.]

Funded by SCOAP^3.

References

1. B. Nikolić, B. Sazdović, EPJ C **77**, 197 (2017)
2. K. Becker, M. Becker, J. Schwarz, String Theory and M-Theory: A Modern Introduction
3. B. Zwiebach, *A First Course in String Theory* (Cambridge University Press, Cambridge, 2004)
4. J. Polchinski, *String theory*, vol. II (Cambridge University Press, Cambridge, 1998)
5. T. Buscher, Phys. Lett. B **194**, 59 (1987)
6. T. Buscher, Phys. Lett. B **201**, 466 (1988)
7. M. Roček, E. Verlinde, Nucl. Phys. B **373**, 630 (1992)
8. A. Giveon, M. Porrati, E. Rabinovici, Phys. Rep. **244**, 77 (1994)
9. E. Alvarez, L. Alvarez-Gaume, J. Barbon, Y. Lozano, Nucl. Phys. B **415**, 71 (1994)
10. C.M. Hull, JHEP **07**, 021 (1998)
11. B. Nikolić, B. Sazdović, Nucl. Phys. B **836**, 100 (2010)
12. D.S. Berman, D.C. Thompson, Phys. Rept. **566**, 1–60 (2014)
13. Lj Davidović, B. Sazdović, EPJ C **74**, 2683 (2014)
14. Lj Davidović, B. Nikolić, B. Sazdović, EPJ C **75**, 576 (2015)
15. Lj Davidović, B. Nikolić, B. Sazdović, EPJ C **74**, 2734 (2014)

16. Lj Davidović, B. Sazdović, JHEP **11**, 119 (2015)
17. D. Lust, JHEP **12**, 084 (2010)
18. D. Andriot, M. Larfors, D. Luest, P. Patalong, JHEP **06**, 021 (2013)
19. D. Andriot, O. Hohm, M. Larfors, D. Lust, P. Patalong, Phys. Rev. Lett. **108**, 261602 (2012)
20. D. Luest, arXiv:1205.0100 [hep-th]
21. R. Blumenhagen, A. Deser, D. Luest, E. Plauschinn, F. Rennecke, J. Phys. A **44**, 385401 (2011)
22. C. Condeescu, I. Florakis, D. Luest, JHEP **04**, 121 (2012)
23. J. Shelton, W. Taylor, B. Wecht, JHEP **10**, 085 (2005)
24. A. Dabholkar, C. Hull, JHEP **05**, 009 (2006)
25. M. Duff, Nucl. Phys. B **335**, 610 (1990)
26. A.A. Tseytlin, Phys. Lett. B **242**, 163 (1990)
27. A.A. Tseytlin, Nucl. Phys. B **350**, 395 (1991)
28. W. Siegel, Phys. Rev. D **48**, 2826 (1993)
29. W. Siegel, Phys. Rev. D **47**, 5453 (1993)
30. C.M. Hull, JHEP **10**, 065 (2005)
31. C.M. Hull, JHEP **10**, 057 (2007)
32. C.M. Hull, JHEP **07**, 080 (2007)
33. D.S. Berman, M. Cederwall, M.J. Perry, JHEP **09**, 066 (2014)
34. D.S. Berman, C.D.A. Blair, E. Malek, M.J. Perry, Int. J. Mod. Phys. A **29**(15), 1450080 (2014)
35. C.D.A. Blair, E. Malek, A.J. Routh, Class. Quant. Grav. **31**(20), 205011 (2014)
36. C.M. Hull, R.A. Reid-Edwards, JHEP **09**, 014 (2009)
37. O. Hohm, B. Zwiebach, JHEP **11**, 075 (2014)
38. B. Sazdović, Chin. Phys. C **41**, 053101 (2017)
39. B. Sazdović, JHEP **08**, 055 (2015)
40. B. Sazdović, EPJ C **77**, 634 (2017)
41. N. Berkovits, arXiv:hep-th/0209059
42. N. Berkovits, arXiv:0902.4032
43. P.A. Grassi, G. Policastro, P. van Nieuwenhuizen, JHEP **10**, 054 (2002)
44. P.A. Grassi, G. Policastro, P. van Nieuwenhuizen, JHEP **11**, 004 (2002)
45. P.A. Grassi, G. Policastro, P. van Nieuwenhuizen, Adv. Theor. Math. Phys. **7**, 499 (2003)
46. P.A. Grassi, G. Policastro, P. van Nieuwenhuizen, Phys. Lett. B **553**, 96 (2003)
47. J. de Boer, P.A. Grassi, P. van Nieuwenhuizen, Phys. Lett. B **574**, 98 (2003)
48. N. Berkovits, P. Howe, Nucl. Phys. B **635**, 75 (2002)
49. P.A. Grassi, L. Tamassia, JHEP **07**, 071 (2004)
50. S.F. Hassan, Nucl. Phys. B **568**, 145 (2000)
51. R. Benichou, G. Policastro, J. Troost, Phys. Lett. B **661**, 129 (2008)
52. M. Hatsuda, K. Kamimura, W. Siegel, JHEP **06**, 039 (2014)
53. I. Bandos, Phys. Lett. B **751**, 408–412 (2015)
54. M. Cederwall, JHEP **06**, 155 (2016)
55. D. Fiorenza, H. Sati, U. Schreiber, ATMP **22** No.5 (2018)

P r o c e e d i n g s

of the

10th MATHEMATICAL PHYSICS MEETING: School and Conference on Modern Mathematical Physics

September 9 – 14, 2019, Belgrade, Serbia

Editors

B. Dragovich, I. Salom and M. Vojinović

Institute of Physics
Belgrade, 2020
SERBIA

Autor: Grupa autora

Naslov: 10th MATHEMATICAL PHYSICS MEETING: SCHOOL AND CONFERENCE ON MODERN MATHEMATICAL PHYSICS (Deseti naučni skup iz matematičke fizike: škola i konferencija iz savremene matematičke fizike)

Izdavač: Institut za fiziku, Beograd, Srbija

Izdanje: Prvo izdanje
(SFIN year XXXIII Series A: Conferences, No. A1 (2020))

Štampa: Zemunplast, Beograd

Tiraž: 150
ISBN: 978-86-82441-51-9

1. Dragović Branko
Matematička fizika-Zbornici

CIP – Каталогизација у публикацији
Народна библиотека Србије, Београд

51-7:53(082)

MATHEMATICAL Physics Meeting: School and Conference on Modern Mathematical Physics (10 ; 2019 ; Beograd)
Proceedings of the 10th Mathematical Physics Meeting: School and Conference on Modern Mathematical Physics, September 9-14, 2019, Belgrade, Serbia / [organizers Institute of Physics, Belgrade ... [et al.]] ; editors B. [Branko] Dragovich, I. [Igor] Salom and M. [Marko] Vojinović. - 1. izd. - Belgrade : Institute of Physics, 2020 (Beograd : Zemunplast). - XII, 383 str. : ilustr. ; 28 cm. - (SFIN ; year 33. Ser. A, Conferences, ISSN 0354-9291 ; n° A1, (2020))

Nasl. u kolofonu: Deseti naučni skup iz matematičke fizike: škola i konferencija iz savremene matematičke fizike. - Tiraž 150. - Str. VII: Preface / editors. - Napomene i bibliografske reference uz radove. - Bibliografija uz svaki rad.

ISBN 978-86-82441-51-9

1. Dragović, Branko, 1945- [уредник] [аутор додатног текста]
a) Математичка физика -- Зборници

COBISS.SR-ID 13561865

P R E F A C E

This volume contains some reviews and original research contributions, which are related to the **10th Mathematical Physics Meeting: School and Conference on Modern Mathematical Physics**, organized by the Institute of Physics, Belgrade (Serbia), September 9–14, 2019. The programme of this meeting was mainly oriented towards some recent developments in gravity and cosmology, string and quantum field theory, and some relevant mathematical methods. We hope that articles presented here will be valuable literature not only for the participants of this meeting but also for many other PhD students and researchers in modern mathematical and theoretical physics. We are grateful to all authors for writing their contributions for these proceedings.

The previous nine meetings in this series of schools and conferences on modern mathematical physics were also held in Serbia: Sokobanja 2001, Kopaonik 2002, Zlatibor 2004, Belgrade 2006, 2008, 2010, 2012, 2014, and 2017. The corresponding proceedings of all these meetings were published by the Institute of Physics Belgrade, and are available in the printed form as well as online at the websites. According to an agreement with the journal Symmetry, several papers are published in the special issue "Selected Papers: 10th Mathematical Physics Meeting".

This jubilary tenth meeting took place at two different venues — the opening and the first day of lectures was held in the grand lecture hall of the Serbian Academy of Sciences and Arts, while the lectures for the remaining five days were held at the Mathematical Institute. Both venues are located in Belgrade downtown, across the road of each other. We hope that all attendees of this meeting will recall it as a useful and pleasant event, and will wish to participate again in the future.

We wish to thank all lecturers and other speakers for their interesting and valuable talks. We also thank all participants for their active participation. Financial support of our sponsors, *Ministry of Education, Science and Technological Development of the Republic of Serbia, Belgrade; Telekom Srbija; Open access journal "Symmetry"*, and the support of our media partner, *Open access journal "Entropy"*, were very significant for realization of this activity.

April 2020

E d i t o r s

B. Dragovich
I. Salom
M. Vojinović

10th MATHEMATICAL PHYSICS MEETING:
School and Conference on
Modern Mathematical Physics

Belgrade, September 9 - 14, 2019

Organizers

- **Institute of Physics, Belgrade, Serbia**
- **Faculty of Mathematics, Belgrade, Serbia**
- **Mathematical Institute, Belgrade, Serbia**
- **Faculty of Science, Kragujevac, Serbia**
- **Serbian Academy of Sciences and Arts, Serbia**

Co-organizers

- **Institute of Nuclear Sciences "Vinča", Belgrade, Serbia**
- **Institute of Physics, Kragujevac, Serbia**
- **Faculty of Physics, Belgrade, Serbia**
- **Faculty of Sciences, Novi Sad, Serbia**
- **School of Electrical Engineering, Belgrade, Serbia**
- **SEENET-MTP, Niš, Serbia**

International Advisory Committee

International Organizing Committee

Vladimir Dobrev (Sofia, Bulgaria)
Radu Constantinescu (Craiova, Romania)
Branko Dragovich (Belgrade, Serbia)
Goran Đorđević (Niš, Serbia)
Zoran Rakić (Belgrade, Serbia)
Đorđe Šijački (Belgrade, Serbia)
Marko Vojinović (Belgrade, Serbia)
Igor Volovich (Moscow, Russia)

Local Organizing Committee

Branko Dragovich (Co-Chairman, Institute of Physics, Belgrade)
Marko Vojinović (Co-Chairman, Institute of Physics, Belgrade)
Sanja Ćirković (Institute of Physics, Belgrade)
Ljubica Davidović (Institute of Physics, Belgrade)
Ivan Dimitrijević (Faculty of Mathematics, Belgrade, Serbia)
Ilija Ivanišević (Institute of Physics, Belgrade)
Danijel Obrić (Institute of Physics, Belgrade)
Tijana Radenković (Institute of Physics, Belgrade)
Igor Salom (Institute of Physics, Belgrade)
Dejan Simić (Institute of Physics, Belgrade)

Sponsors

- **Ministry of Education, Science and Technological Development, Republic of Serbia**

МИНИСТАРСТВО ПРОСВЕТЕ,
НАУКЕ И ТЕХНОЛОШКОГ РАЗВОЈА

- **Telekom Srbija**

Telekom Srbija

- **Open Access Journal "Symmetry" (MDPI, Switzerland)**

symmetry

Media Partner

- **Open Access Journal "Entropy" (MDPI, Switzerland)**

entropy
an Open Access Journal by MDPI

CONTENTS

Review and Research Works

From 3D torus with H-flux to torus with R-flux and back*

Bojan Nikolić†
Institute of Physics Belgrade, University of Belgrade
Pregrevica 118, Belgrade, Serbia

Danijel Obrić‡
Institute of Physics Belgrade, University of Belgrade
Pregrevica 118, Belgrade, Serbia

Abstract

In this article we study 3D closed bosonic string propagating in the constant metric and Kalb-Ramond field with one non-zero component, $B_{xy} = Hz$, where field strength H is infinitesimal. We will T-dualize along line $x \to y \to z$, which means that we T-dualize first along x coordinate, then along y and, finally, along z coordinate. After first two T-dualizations we obtain Q flux theory which is just locally well defined, while after all three T-dualizations we obtain nonlocal R flux theory. The Q flux theory is commutative one and the R flux theory is noncommutative and nonassociative one. After that we reverse the T-dualization line and T-dualize along $z \to y \to x$. All three theories are nonlocal, but after the first T-dualization we obtain commutative and associative theory, while after we T-dualize along y, we get noncommutative and associative theory. T-dualizing along x, we come to the theory which is both noncommutative and nonassociative. The form of the final T-dual action does not depend on the order of T-dualization while noncommutativity and nonassociativity relations could be obtained from those in the $x \to y \to z$ case by replacing $H \to -H$.

1. Introduction

Heisenberg suggested coordinate noncommutativity in order to solve the problem of infinities before developing of renormalization procedure. In the paper [1] discrete Lorentz invariant space-time is constructed, which means that coordinates are noncommutative.

Noncommutativity came into the focus of interest with the appearance of the paper [2], where it is shown that open string endpoints in the presence

* This work has been supported by the Serbian Ministry of Science and Technological Development, under contract No. 171031.

† e-mail address: bnikolic@ipb.ac.rs

‡ e-mail address: dobric@ipb.ac.rs

of the constant metric and Kalb-Ramond field became noncommutative. After this article many others [3] appeared addressing the same subject but using different approaches.

The closed bosonic string in the presence of constant metric and Kalb-Ramond filed remains commutative, because there are no boundary conditions constraining string dynamics. Noncommutativity could be achieved [4] but using T-duality procedure and coordinate dependent Kalb-Ramond field.

T-duality as a fundamental feature of string theory [5, 6, 7, 8, 9, 10, 11] is realized within Buscher T-dualization procedure [6] which can considered as definition of T-dualization. It is applicable along directions on which background fields do not depend. In order to work with coordinate dependent backgrounds generalized T-dualization procedure is developed [12, 13, 14].

Here we will study closed bosonic string in the presence of the constant metric and linear dependent Kalb-Ramond field with just one nonzero component, $B_{xy} = Hz$, the background already analyzed in [15]. In all calculations we keep constant and linear terms in infinitesimal field strength H. We will use transformation laws, relations which connect initial and T-dual variables, in canonical form, expressed in terms of the coordinates and momenta. Our task is to T-dualize along T-dualization chain $x \to y \to z$ and in opposite direction and examine the influence of the T-dualization sequence on the form of the final theory (theory obtained after three T-dualizations) as well as on the noncommutativity and nonassociativity parameters.

T-dualizations along x and y produces the Q-flux background [15], which is still locally well defined, but the theory is commutative. Applying the generalized T-duality procedure [12, 13, 14], z T-dualization gives R flux theory which is nonlocal one because it depends on the non-locally defined variable ΔV. Nonzero Poisson brackets of the T-dual coordinates shows that there is a connection of non-locality and closed string noncommutativity.

The form of noncommutativity is proportional to the infinitesimal field strength H and difference of the initial coordinates. When arguments of the coordinates are different, $\sigma \neq \bar{\sigma}$, there exists noncommutativity. The consequence of the coordinate dependent noncommutativity relations is broken Jacobi identity - nonassociativity occurs. Nonassociativity parameter is proportional to the field strength H.

In the second part of the article we will T-dualize first along z and then along isometry directions y and finally along x. After first T-dualization we get commutative and associative theory as in xyz case. The second T-dualization produces noncommutative and associative theory. In the xyz case, theory second in the T-dualization chain is both commutative and associative. The action of the final theory is the same as in xyz case which is nonassociative and noncommutative. The noncommutativity and nonassociativity parameters have one additional "$-$" sign comparing with the corresponding ones in [16].

2. Action and T-dualization procedure

In this section we will present the construction of the model and give some important details of the T-dualization procedure.

2.1. Model

The closed bosonic string action is of the form [5]

$$S = \kappa \int_\Sigma d^2\xi \sqrt{-g} \left\{ \left[\frac{1}{2} g^{\alpha\beta} G_{\mu\nu}(x) + \frac{\varepsilon^{\alpha\beta}}{\sqrt{-g}} B_{\mu\nu}(x) \right] \partial_\alpha x^\mu \partial_\beta x^\nu + \Phi(x) R^{(2)} \right\} , \tag{1}$$

where world-sheet surface Σ is parameterized by $\xi^\alpha = (\tau , \sigma)$ $[(\alpha = 0 , 1)$, $\sigma \in (0 , \pi)]$, while x^μ $(\mu = 0, 1, 2, \ldots , D - 1)$ are space-time coordinates. Intrinsic world sheet metric is denoted by $g_{\alpha\beta}$, and the corresponding scalar curvature with $R^{(2)}$. Here $G_{\mu\nu}$ is, in the general case, coordinate dependent metric, $B_{\mu\nu}$ is coordinate dependent Kalb-Ramond field, while Φ is dilaton field.

If we intend to have conformal symmetry on the quantum level, background fields are not arbitrarily chosen i.e. they must obey the space-time field equations [17]

$$\beta^G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{4} B_{\mu\rho\sigma} B_\nu{}^{\rho\sigma} + 2 D_\mu a_\nu = 0 , \tag{2}$$

$$\beta^B_{\mu\nu} \equiv D_\rho B^\rho{}_{\mu\nu} - 2 a_\rho B^\rho{}_{\mu\nu} = 0 , \tag{3}$$

$$\beta^\Phi \equiv 2\pi\kappa \frac{D - 26}{6} - R - \frac{1}{24} B_{\mu\rho\sigma} B^{\mu\rho\sigma} - D_\mu a^\mu + 4a^2 = c , \tag{4}$$

where c is an arbitrary constant. It holds

$$D^\nu \beta^G_{\nu\mu} + \partial_\mu \beta^\Phi = 0 , \tag{5}$$

which means that third beta function, β^Φ, can be zero or nonzero constant. The notation is in the standard form: $R_{\mu\nu}$ and D_μ are Ricci tensor and covariant derivative with respect to the space-time metric $G_{\mu\nu}$, while field strength for Kalb-Ramond field $B_{\mu\nu}$ and dilaton gradient are defined as

$$B_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} + \partial_\nu B_{\rho\mu} + \partial_\rho B_{\mu\nu} , \quad a_\mu = \partial_\mu \Phi . \tag{6}$$

Choosing Kalb-Ramond field to be linearly coordinate dependent and dilaton field to be constant, we obtain (2)

$$R_{\mu\nu} - \frac{1}{4} B_{\mu\rho\sigma} B_\nu{}^{\rho\sigma} = 0 . \tag{7}$$

Assuming that Kalb-Ramond field strength is infinitesimal we can take $G_{\mu\nu}$ to be constant but in approximation linear in $B_{\mu\nu\rho}$. As a consequence the third equation (4) becomes

$$2\pi\kappa\frac{D-26}{6}=c\,. \tag{8}$$

The arbitrary constant c can be fixed, $c=-\frac{23\pi}{3}$, which gives $D=3$.

The background fields are of the form

$$G_{\mu\nu}=\begin{pmatrix} R_1^2 & 0 & 0 \\ 0 & R_2^2 & 0 \\ 0 & 0 & R_3^2 \end{pmatrix}\,,\quad B_{\mu\nu}=\begin{pmatrix} 0 & Hz & 0 \\ -Hz & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\,, \tag{9}$$

where $R_\mu(\mu=1,2,3)$ are radii of the compact dimensions. If we rescale the coordinates

$$x^\mu \longmapsto x'^\mu = R_\mu x^\mu\,, \tag{10}$$

the form of the metric simplifies

$$G_{\mu\nu}=\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\,. \tag{11}$$

After all the action gets the form

$$\begin{aligned} S &= \kappa\int_\Sigma d^2\xi\partial_+x^\mu\Pi_{+\mu\nu}\partial_-x^\nu \\ &= \kappa\int_\Sigma d^2\xi\left[\frac{1}{2}\left(\partial_+x\partial_-x+\partial_+y\partial_-y+\partial_+z\partial_-z\right)\right. \\ &+ \left.\partial_+xHz\partial_-y-\partial_+yHz\partial_-x\right]\,, \end{aligned} \tag{12}$$

where $\partial_\pm=\partial_\tau\pm\partial_\sigma$, and

$$\Pi_{\pm\mu\nu}=B_{\mu\nu}\pm\frac{1}{2}G_{\mu\nu}\,,\quad x^\mu=\begin{pmatrix} x \\ y \\ z \end{pmatrix}\,. \tag{13}$$

2.2. T-dualization procedure

In the standard Buscher procedure, the starting point is assumption that the target space has isometries. Our background is coordinate dependent so it is useful to examine if the isometry exists. Let us start with the coordinate shift

$$\delta x^\mu=\lambda^\mu=const, \tag{14}$$

and assume that all the coordinates are compact. As $B_{\mu\nu}$ is linear in coordinate, we have

$$\begin{aligned} \delta S &= \frac{\kappa}{3} B_{\mu\nu\rho} \lambda^\rho \int d^2\xi \partial_+ x^\mu \partial_- x^\nu \\ &= \frac{\kappa}{3} B_{\mu\nu\rho} \lambda^\rho \epsilon^{\alpha\beta} \int d^2\xi \partial_\alpha x^\mu \partial_\beta x^\nu. \end{aligned} \tag{15}$$

This is proportional to the total divergence

$$\delta S = \frac{\kappa}{3} B_{\mu\nu\rho} \lambda^\rho \epsilon^{\alpha\beta} \int d^2\xi \partial_\alpha (x^\mu \partial_\beta x^\nu) = 0, \tag{16}$$

which vanishes in the case of the closed string and the topologically trivial mapping of the world-sheet into the space-time. So, the isometry exists even in the case we have chosen.

To localize the global symmetry, we introduce the gauge fields v^μ_α and substitute the ordinary derivatives with the covariant ones

$$\partial_\alpha x^\mu \to D_\alpha x^\mu = \partial_\alpha x^\mu + v^\mu_\alpha. \tag{17}$$

The covariant derivatives are gauge invariant under the following transformation law for the gauge fields

$$\delta v^\mu_\alpha = -\partial_\alpha \lambda^\mu, \quad (\lambda^\mu = \lambda^\mu(\tau, \sigma)). \tag{18}$$

This replacement is not sufficient to make the action locally invariant because the background field $B_{\mu\nu}$ is coordinate dependent. The coordinate x^μ, should be replaced with the invariant coordinate

$$\begin{aligned} \Delta x^\mu_{inv} &\equiv \int_P d\xi^\alpha\, D_\alpha x^\mu = \int_P (d\xi^+ D_+ x^\mu + d\xi^- D_- x^\nu) \\ &= x^\mu - x^\mu(\xi_0) + \Delta V^\mu, \end{aligned} \tag{19}$$

where

$$\Delta V^\mu \equiv \int_P d\xi^\alpha v^\mu_\alpha = \int_P (d\xi^+ v^\mu_+ + d\xi^- v^\mu_-). \tag{20}$$

In order to make gauge fields v^μ_α nonphysical degrees of freedom, the corresponding field strength

$$F^\mu_{\alpha\beta} \equiv \partial_\alpha v^\mu_\beta - \partial_\beta v^\mu_\alpha, \tag{21}$$

must vanish. Technically, this can be achieved by introducing the Lagrange multiplier y_μ, and the appropriate additional term in the Lagrangian

$$S_{add} = \frac{\kappa}{2} \int d^2 (v^\mu_+ \partial_- y_\mu - v^\mu_- \partial_+ y_\mu), \tag{22}$$

where the last term is equal $\frac{1}{2} y_\mu F^\mu_{+-}$ up to the total divergence. At the end of procedure we fix gauge freedom in the way that $x^\mu(\xi) = x^\mu(\xi_0)$.

3. T-dualization along chain $x \to y \to z$

In this section we will make T-dualization along chain $x \to y \to z$, step by step. Our goal is to find transformation laws, and using them we will calculate noncommutativity and nonassociativity relations.

3.1. Twisted torus geometry from torus with H-flux

Because background fields do not depend on coordinate x, T-dualization along direction x is performed within standard Buscher procedure (without introduction of invariant coordinate). Here we will repeat the standard and generalized Buscher procedure explained above, while in other cases we will just give the final results.

Since x direction is an isometry one, action has a global shift symmetry, $x \longrightarrow x+a$. Localizing this symmetry we replace ordinary derivatives with the covariant ones

$$\partial_\pm x \longrightarrow D_\pm x = \partial_\pm x + v_\pm \,, \tag{23}$$

where $v_\pm$ are gauge field. In order to have T-dual action with the same number of degrees of freedom as initial one, we have to add following term to the action

$$S_{add} = \frac{\kappa}{2}\int_\Sigma d^2\xi y_1(\partial_+ v_- - \partial_- v_+)\,, \tag{24}$$

where y_1 is a Lagrange multiplier. Symmetry enables us to fix gauge, $x = const.$, which produces

$$\begin{aligned} S_{fix} &= \kappa\int d^2\xi\left[\frac{1}{2}\left(v_+v_- + \partial_+ y\partial_- y + \partial_+ z\partial_- z\right)\right.\\ &+ v_+ Hz\partial_- y - \partial_+ y Hz v_- \\ &+ \left.\frac{1}{2}y_1(\partial_+ v_- - \partial_- v_+)\right]\,. \end{aligned} \tag{25}$$

On the equations of motion for y_1 field strength for the gauge field $v_\pm$ is equal to zero

$$F_{+-} = \partial_+ v_- - \partial_- v_+ = 0\,. \tag{26}$$

Vanishing of the field strength gives us the solution

$$v_\pm = \partial_\pm x\,. \tag{27}$$

Applying this solution from gauge fixed action (25) we restore initial action (12). Varying the gauge fixed action with respect to the gauge fields we get

$$v_- = -\partial_- y_1 - 2Hz\partial_- y\,, \tag{28}$$

$$v_+ = \partial_+ y_1 + 2Hz\partial_+ y\,. \tag{29}$$

Using (28) and (29) from gauge fixed action (25) we get the T-dual action

$$ {}_xS = \kappa \int_\Sigma d^2\xi \partial_+ ({}_xX)^\mu {}_x\Pi_{+\mu\nu} \partial_- ({}_xX)^\nu \,, \tag{30} $$

where

$$ {}_xX^\mu = \begin{pmatrix} y_1 \\ y \\ z \end{pmatrix} , \tag{31} $$

and T-dual background fields

$$ {}_x\Pi_{+\mu\nu} = {}_xB_{\mu\nu} + \frac{1}{2} {}_xG_{\mu\nu} \,, \quad {}_xB_{\mu\nu} = 0 \,, \quad {}_xG_{\mu\nu} = \begin{pmatrix} 1 & 2Hz & 0 \\ 2Hz & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} . \tag{32} $$

Geometry described by background fields (32) defines so called *twisted torus geometry*. String theory after one T-dualization is geometrical (flux H takes the role of connection).

Combining two sets of equations of motion, (27), (28) and (29), we get the transformation laws

$$ \partial_\pm x \cong \pm \partial_\pm y_1 \pm 2Hz\partial_\pm y \,, \tag{33} $$

where $\cong$ denotes T-duality relation. From the initial action (12) we can easily find canonical momentum π_x

$$ \pi_x = \frac{\delta S}{\delta \dot{x}} = \kappa(\dot{x} - 2Hzy') \,, \tag{34} $$

where $\dot{A} \equiv \partial_\tau A$ and $A' \equiv \partial_\sigma A$. Transformation law (33) produces the relation

$$ \dot{x} \cong y_1' + 2Hzy' \,, \tag{35} $$

which, combined with the expression for π_x, enables us to find transformation law in canonical form

$$ \pi_x \cong \kappa y_1' \,. \tag{36} $$

The initial theory described by action (12) is geometrical one and their coordinates and canonical momenta satisfy standard Poisson algebra

$$ \{x^\mu(\sigma), \pi_\nu(\bar{\sigma})\} = \delta^\mu{}_\nu \delta(\sigma - \bar{\sigma}) \,, \quad \{x^\mu, x^\nu\} = \{\pi_\mu, \pi_\nu\} = 0 \,. \tag{37} $$

Using this algebra it is simply to show that obtained theory (30) is commutative one

$$ \{{}_xX^\mu, {}_xX^\nu\} = 0 \,. \tag{38} $$

3.2. The second step - Q-flux theory

In this subsection the starting action is the action obtained after T-dualization along x (30). Because y is an isometry direction T-dualization along y direction will be performed according to standard Buscher procedure.

Let us construct the gauge fixed action starting with (30)

$$\begin{aligned} S_{fix} &= \kappa \int_\Sigma d^2\xi \left[\frac{1}{2}\left(\partial_+ y_1 \partial_- y_1 + v_+ v_- + \partial_+ z \partial_- z\right)\right. \\ &+ \left.\partial_+ y_1 H z v_- + v_+ H z \partial_- y_1 + \frac{1}{2} y_2 (\partial_+ v_- - \partial_- v_+)\right] . \end{aligned} \tag{39}$$

The equation of motion for Lagrange multiplier y_2 produces

$$\partial_+ v_- - \partial_- v_+ = 0 \longrightarrow v_\pm = \partial_\pm y \,. \tag{40}$$

Application of these equations of motions transfers action (39) to (30). Varying the gauge fixed action with respect to the gauge fields are

$$v_\pm = \pm \partial_\pm y_2 - 2Hz\partial_\pm y_1 \,. \tag{41}$$

Putting these expressions into gauge fixed action, we get T-dual action

$${}_{xy}S = \kappa \int d^2\xi \partial_+ ({}_{xy}X)^\mu {}_{xy}\Pi_{+\mu\nu} \partial_- ({}_{xy}X)^\nu \,, \tag{42}$$

where the background fields are

$${}_{xy}B_{\mu\nu} = \begin{pmatrix} 0 & -Hz & 0 \\ Hz & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = -B_{\mu\nu} \,, \quad {}_{xy}G_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \,, \tag{43}$$

and

$$({}_{xy}X)^\mu = \begin{pmatrix} y_1 \\ y_2 \\ z \end{pmatrix} \,, \quad {}_{xy}\Pi_{+\mu\nu} = {}_{xy}B_{\mu\nu} + \frac{1}{2} {}_{xy}G_{\mu\nu} = \begin{pmatrix} \frac{1}{2} & -Hz & 0 \\ Hz & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} \end{pmatrix} . \tag{44}$$

We see that background fields look like those of torus with H flux (12). Their global properties are non-trivial and that is a reason why the term nongeometry is introduced.

Using T-dual transformation laws in Lagrangian form

$$\partial_\pm y \cong \pm \partial_\pm y_2 - 2Hz\partial_\pm y_1 \,, \tag{45}$$

in combination with the expression for canonical momentum of the initial theory

$$\pi_y = \frac{\delta S}{\delta \dot{y}} = \kappa(\dot{y} + 2Hzx') \,. \tag{46}$$

we get T-dual transformation law in canonical form

$$\pi_y \cong \kappa y_2' \,. \tag{47}$$

From standard Poisson algebra (37) it follows that theory obtained by two T-dualizations along isometry directions is still commutative

$$\{{}_{xy}X^\mu, {}_{xy}X^\nu\} = 0 \,. \tag{48}$$

3.3. Full T-dualized theory

Kalb-Ramond field is dependent on z but this direction is still isometry one (see subsection 2.2). For T-dualization along z we use generalized T-dualization procedure [12, 13, 14].

Starting action is the one obtained in the previous step (42). The first step in the T-dualization procedure is localizing shift symmetry of the action (42) along z direction. This means that we have to introduce covariant derivative

$$\partial_\pm z \longrightarrow D_\pm z = \partial_\pm z + v_\pm \,. \tag{49}$$

Then we introduce the invariant coordinate as line integral

$$z^{inv} = \int_P d\xi^\alpha D_\alpha z = \int_P d\xi^+ D_+ z + \int_P d\xi^- D_- z = z(\xi) - z(\xi_0) + \Delta V \,, \tag{50}$$

where

$$\Delta V = \int_P d\xi^\alpha v_\alpha = \int_P (d\xi^+ v_+ + d\xi^- v_-) \,. \tag{51}$$

In order to make $v_\pm$ to be nonphysical degrees of freedom we add to the action term with Lagrange multiplier

$$S_{add} = \frac{\kappa}{2} \int_\Sigma d^2\xi \; y_3(\partial_+ v_- - \partial_+ v_-) \,. \tag{52}$$

The final form of the action is

$$\begin{aligned} \bar{S} &= \kappa \int_\Sigma d^2\xi \Big[-H z^{inv}(\partial_+ y_1 \partial_- y_2 - \partial_+ y_2 \partial_- y_1) \\ &+ \frac{1}{2}(\partial_+ y_1 \partial_- y_1 + \partial_+ y_2 \partial_- y_2 + D_+ z D_- z) + \frac{1}{2}\, y_3(\partial_+ v_- - \partial_- v_+) \Big] \,. \end{aligned} \tag{53}$$

The final step in the procedure is gauge fixing, $z(\xi) = z(\xi_0)$, and then the gauge fixed action is of the form

$$\begin{aligned} S_{fix} &= \kappa \int_\Sigma d^2\xi \, [-H \Delta V (\partial_+ y_1 \partial_- y_2 - \partial_+ y_2 \partial_- y_1) \\ &+ \frac{1}{2}(\partial_+ y_1 \partial_- y_1 + \partial_+ y_2 \partial_- y_2 + v_+ v_-) + \frac{1}{2} y_3(\partial_+ v_- - \partial_- v_+) \Big] \,. \end{aligned} \tag{54}$$

We restore initial theory (42) from the gauge fixed action using equation of motion for Lagrange multiplier y_3

$$\partial_+ v_- - \partial_- v_+ = 0 \Longrightarrow v_\pm = \partial_\pm z\,, \quad \Delta V = \Delta z\,. \tag{55}$$

Varying the gauge fixed action (54) with respect to the gauge fields we get

$$v_\pm = \pm\partial_\pm y_3 - 2\beta^\mp\,, \tag{56}$$

where $\beta^\pm$ are functions defined as

$$\beta^\pm = \pm\frac{1}{2}H(y_1\partial_\mp y_2 - y_2\partial_\mp y_1)\,. \tag{57}$$

They are a result of the variation of the term containing ΔV. Using relations (56) from the gauge fixed action, we obtain the T-dual action

$${}_{xyz}S = \kappa\int_\Sigma d^2\xi\partial_{+\,xyz}X^\mu\,{}_{xyz}\Pi_{+\mu\nu}\partial_{-\,xyz}X^\nu\,, \tag{58}$$

where background fields are

$${}_{xyz}B_{\mu\nu} = \begin{pmatrix} 0 & -H\Delta\tilde{y}_3 & 0 \\ H\Delta\tilde{y}_3 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad {}_{xyz}G_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{59}$$

and

$${}_{xyz}X^\mu = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}. \tag{60}$$

The double coordinate $\tilde{y}_3$ is defined as

$$\partial_\pm y_3 \equiv \pm\partial_\pm\tilde{y}_3\,, \tag{61}$$

while, because it stands together with H, we calculate ΔV in the zeroth order

$$\Delta V = \int d\xi^+\partial_+ y_3 - \int d\xi^-\partial_- y_3\,. \tag{62}$$

Because ΔV is defined as line integral, the nonlocality occures in the T-dual theory. The result of the three T-dualization is a theory with R flux.

Lagrangian form of the T-dual transformation law is

$$\partial_\pm z \cong \pm\partial_\pm y_3 - 2\beta^\mp\,. \tag{63}$$

while its canonical form is

$$y_3' \cong \frac{1}{\kappa}\pi_z - H(xy' - yx')\,. \tag{64}$$

4. Noncommutativity and nonassociativity

In this section we will calculate noncommutativity and nonassociativity relations using canonical forms of the transformation laws.

4.1. Noncommutativity relations

Rewriting the relations (36), (47) and (64) as

$$y_1' \cong \frac{1}{\kappa}\pi_x\,, \quad y_2' \cong \frac{1}{\kappa}\pi_y\,, \quad y_3' \cong \frac{1}{\kappa}\pi_z - H(xy' - yx')\,. \tag{65}$$

we conclude that nontrivial Poisson brackets will be $\{y_1(\sigma), y_3(\bar\sigma)\}$ and $\{y_2(\sigma), y_3(\bar\sigma)\}$. Using the result presented in the Appendix A and the relations

$$\{y_1'(\sigma), y_3'(\bar\sigma)\} \cong \frac{2}{\kappa}Hy'(\sigma)\delta(\sigma-\bar\sigma) + \frac{1}{\kappa}Hy(\sigma)\delta'(\sigma-\bar\sigma)\,, \tag{66}$$

$$\{y_2'(\sigma), y_3'(\bar\sigma)\} \cong -\frac{2}{\kappa}Hx'(\sigma)\delta(\sigma-\bar\sigma) - \frac{1}{\kappa}Hx(\sigma)\delta'(\sigma-\bar\sigma)\,, \tag{67}$$

we get the Poisson brackets of the T-dual coordinates

$$\{y_1(\sigma), y_3(\bar\sigma)\} \cong -\frac{H}{\kappa}\left[2y(\sigma) - y(\bar\sigma)\right]\theta(\sigma-\bar\sigma)\,, \tag{68}$$

$$\{y_2(\sigma), y_3(\bar\sigma)\} \cong \frac{H}{\kappa}\left[2x(\sigma) - x(\bar\sigma)\right]\theta(\sigma-\bar\sigma)\,. \tag{69}$$

If $\sigma = \bar\sigma$ then these two Poisson brackets are zero. But if we choose that $\sigma - \bar\sigma = 2\pi$ then $\theta(2\pi) = 1$ and it follows

$$\{y_1(\sigma+2\pi), y_3(\sigma)\} \cong -\frac{H}{\kappa}\left[4\pi N_y + y(\sigma)\right]\,, \tag{70}$$

$$\{y_2(\sigma+2\pi), y_3(\sigma)\} \cong \frac{H}{\kappa}\left[4\pi N_x + x(\sigma)\right]\,, \tag{71}$$

where N_x and N_y are winding numbers defined as

$$x(\sigma+2\pi) - x(\sigma) = 2\pi N_x\,, \quad y(\sigma+2\pi) - y(\sigma) = 2\pi N_y\,. \tag{72}$$

4.2. Nonassociativity

Let us start calculating Poisson brackets $\{y_1(\sigma), x(\bar\sigma)\}$ and $\{y_2(\sigma), y(\bar\sigma)\}$. Similar to the calculations presented in Appendix A, we start with

$$\{\Delta y_1(\sigma,\sigma_0), x(\bar\sigma)\} = \{\int_{\sigma_0}^{\sigma} d\eta y_1'(\eta), x(\bar\sigma)\}\,. \tag{73}$$

Using the T-dual transformation in canonical form (36), we obtain

$$\{\Delta y_1(\sigma,\sigma_0), x(\bar\sigma)\} \cong \frac{1}{\kappa}\{\int_{\sigma_0}^{\sigma} d\eta \pi_x(\eta), x(\bar\sigma)\}\,, \tag{74}$$

which produces further

$$\begin{aligned} &\{\Delta y_1(\sigma,\sigma_0), x(\bar\sigma)\} \cong -\frac{1}{\kappa}\left[\theta(\sigma-\bar\sigma)-\theta(\sigma_0-\bar\sigma)\right] \\ \Longrightarrow\quad &\{y_1(\sigma), x(\bar\sigma)\} \cong -\frac{1}{\kappa}\theta(\sigma-\bar\sigma)\,. \end{aligned} \tag{75}$$

The relation $\{y_2(\sigma), y(\bar\sigma)\}$ can be obtained in the same way

$$\{y_2(\sigma), y(\bar\sigma)\} \cong -\frac{1}{\kappa}\theta(\sigma-\bar\sigma)\,. \tag{76}$$

Now it is straightforward to calculate Jacobi identity using (68), (69) and above Poisson brackets

$$\begin{aligned} &\{y_1(\sigma_1), y_2(\sigma_2), y_3(\sigma_3)\} \equiv \\ &\{y_1(\sigma_1), \{y_2(\sigma_2), y_3(\sigma_3)\}\} + \{y_2(\sigma_2), \{y_3(\sigma_3), y_1(\sigma_1)\}\} \\ + \quad &\{y_3(\sigma_3), \{y_1(\sigma_1), y_2(\sigma_2)\}\} \cong \\ &-\frac{2H}{\kappa^2}\left[\theta(\sigma_1-\sigma_2)\theta(\sigma_2-\sigma_3)+\theta(\sigma_2-\sigma_1)\theta(\sigma_1-\sigma_3)\right. \\ + \quad &\left.\theta(\sigma_1-\sigma_3)\theta(\sigma_3-\sigma_2)\right]\,. \end{aligned} \tag{77}$$

Jacobi identity is nonzero which means that theory with R-flux is in general nonassociative. For $\sigma_2=\sigma_3=\sigma$ and $\sigma_1=\sigma+2\pi$ we get

$$\{y_1(\sigma+2\pi), y_2(\sigma), y_3(\sigma)\} \cong \frac{2H}{\kappa^2}\,. \tag{78}$$

5. Reversed T-dualization chain $z \to y \to x$

Our intention is to T-dualize (12) in opposite direction to find the form of noncommutativity and nonassociativity relations. Because we T-dualize first along z, all three theories are R-flux ones.

The T-dualization procedure is presented and already applied three times. So, omitting all technical steps, the T-dual action is of the form

$${}_zS = \kappa\int_\Sigma d^2\xi\,\partial_+ {}_zX^\mu\, {}_z\Pi_{+\mu\nu}\partial_- {}_zX^\nu\,, \tag{79}$$

where

$${}_zX^\mu = \begin{pmatrix} x \\ y \\ y_3 \end{pmatrix}\,,\quad {}_z\Pi_{+\mu\nu} = {}_zB_{\mu\nu} + \frac{1}{2}{}_zG_{\mu\nu}\,, \tag{80}$$

$$ {}_zB_{\mu\nu} = \begin{pmatrix} 0 & H\Delta V & 0 \\ -H\Delta V & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad {}_zG_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{81} $$

T-dual transformation laws are

$$ \partial_\pm z \cong \pm\partial_\pm y_3 \mp H(x\partial_\pm y - y\partial_\pm x)\,. \tag{82} $$

Momentum of the initial theory (12) canonically conjugated to the coordinate z is of the form

$$ \pi_z = \kappa\dot{z}\,, \tag{83} $$

so, the T-dual transformation law in canonical form is

$$ y_3' \cong \frac{1}{\kappa}\pi_z + H(xy' - yx')\,. \tag{84} $$

From the expressions (84) and (37), we get that coordinates ${}_zX^\mu$ are commutative. Consequently, Jacobiator is equal to zero, which means that theory is associative.

After T-dualization along y direction the T-dual action is

$$ {}_{zy}S = \kappa\int_\Sigma d^2\xi\; \partial_+\,{}_{zy}X^\mu\;{}_{zy}\Pi_{+\mu\nu}\;\partial_-\,{}_{zy}X^\nu\,, \tag{85} $$

where

$$ {}_{zy}X^\mu = \begin{pmatrix} x \\ y_2 \\ y_3 \end{pmatrix}, \quad {}_{zy}\Pi_{+\mu\nu} = {}_{zy}B_{\mu\nu} + \frac{1}{2}\,{}_{zy}G_{\mu\nu}\,, \tag{86} $$

$$ {}_{zy}B_{\mu\nu} = 0\,, \quad {}_{zy}G_{\mu\nu} = \begin{pmatrix} 1 & -2H\Delta V & 0 \\ -2H\Delta V & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{87} $$

The canonical form of the T-dual transformation law is

$$ y_2' \cong \frac{1}{\kappa}\pi_y\,. \tag{88} $$

The only non-zero Poisson bracket is

$$ \{y_2'(\sigma), y_3'(\bar\sigma)\} \cong \frac{H}{\kappa}\left[2x'(\sigma)\delta(\sigma-\bar\sigma) + x(\sigma)\delta'(\sigma-\bar\sigma)\right]. \tag{89} $$

Using the instructions from Appendix A, the solution is of the form (107)

$$ \{y_2(\sigma), y_3(\bar\sigma)\} \cong -\frac{H}{\kappa}\left[2x(\sigma) - x(\bar\sigma)\right]\theta(\sigma-\bar\sigma)\,. \tag{90} $$

For $\sigma - \bar{\sigma} = 2\pi$, we have

$$\{y_2(\sigma + 2\pi), y_3(\sigma)\} \cong -\frac{H}{\kappa}\left[x(\sigma) + 4\pi N_x\right] , \tag{91}$$

where N_x is winding number for x coordinate defined as

$$x(\sigma + 2\pi) - x(\sigma) = 2\pi N_x . \tag{92}$$

It is straightforward to calculate the Jacobiator

$$\begin{aligned} &\{x(\sigma_1), \{y_2(\sigma_2), y_3(\sigma_3)\}\} + \{y_2(\sigma_2), \{y_3(\sigma_3), x(\sigma_1)\}\} \\ + \;&\{y_3(\sigma_3), \{x(\sigma_1), y_2(\sigma_2)\}\} \cong 0 . \end{aligned} \tag{93}$$

Because it is zero, we conclude that this R-flux theory is noncommutative and associative one.

The full T-dualozed action is

$$_{zyx}S = \kappa \int_\Sigma d^2\xi \partial_+ \;_{zyx}X^\mu \;_{zyx}\Pi_{+\mu\nu} \;_{zyx}\; X^\nu , \tag{94}$$

where

$$_{zyx}X^\mu = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} , \quad _{zyx}\Pi_{+\mu\nu} = \,_{zyx}B_{\mu\nu} + \frac{1}{2}\,_{zyx}G_{\mu\nu} \tag{95}$$

$$_{zyx}B_{\mu\nu} = \begin{pmatrix} 0 & -H\Delta V & 0 \\ H\Delta V & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} , \quad _{zyx}G_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} . \tag{96}$$

The canonical form of the T-dual transformation law is the same as in the case $x \to y \to z$

$$y_1' \cong \frac{1}{\kappa}\pi_x . \tag{97}$$

The set of T-dual transformation laws, (84), (88) and (97), is the same as in the xyz case up to $H \to -H$. Consequently, we have

$$\{y_1(\sigma), y_3(\bar{\sigma})\} \cong \frac{H}{\kappa}\left[2y(\sigma) - y(\bar{\sigma})\right]\theta(\sigma - \bar{\sigma}) , \tag{98}$$

$$\{y_2(\sigma), y_3(\bar{\sigma})\} \cong -\frac{H}{\kappa}\left[2x(\sigma) - x(\bar{\sigma})\right]\theta(\sigma - \bar{\sigma}) , \tag{99}$$

and nonassociativity

$$\begin{aligned} &\{y_1(\sigma_1), y_2(\sigma_2), y_3(\sigma_3)\} \equiv \\ &\{y_1(\sigma_1), \{y_2(\sigma_2), y_3(\sigma_3)\}\} + \{y_2(\sigma_2), \{y_3(\sigma_3), y_1(\sigma_1)\}\} \\ + \;&\{y_3(\sigma_3), \{y_1(\sigma_1), y_2(\sigma_2)\}\} \cong \\ &\frac{2H}{\kappa^2}\left[\theta(\sigma_1 - \sigma_2)\theta(\sigma_2 - \sigma_3) + \theta(\sigma_2 - \sigma_1)\theta(\sigma_1 - \sigma_3)\right. \\ + \;&\left.\theta(\sigma_1 - \sigma_3)\theta(\sigma_3 - \sigma_2)\right] . \end{aligned} \tag{100}$$

6. Conclusion

In this article we considered closed bosonic string propagating in the constant metric and Kalb-Ramond field with just one nonzero component, $B_{xy} = -B_{yx} = Hz$. We worked in the approximation linear in H. We T-dualized along chain xyz and in opposite direction. Our goal was to examine the influence of the sequence of T-dualizations on the full T-dualized theory, geometrical features and noncommutativity and nonassociativity of the coordinates.

T-dualizing along chain xyz we concluded that the first theory is geometrical, commutative and associative, the second one is nongeometrical, commutative and associative, while the full T-dualized theory is nongeometrical R flux theory which is noncommuattive and nonassociative. When we reversed the direction of T-dualizations we obtained that all three theories are non geometrical R flux ones. But the first one is both commutative and associative, the second one is noncommutative and associative, while the third one is both nonassociative and noncommutative. The corresponding parameters can be obtained from those in xyz case by replacing $H \to -H$. The form of the full T-dualized theory does not depend on the sequence of T-dualizations.

A Important Poisson bracket

Let us start with the Poisson bracket of the σ derivatives of two arbitrary functions in the form

$$\{A'(\sigma), B'(\bar{\sigma})\} = U'(\sigma)\delta(\sigma - \bar{\sigma}) + V(\sigma)\delta'(\sigma - \bar{\sigma}), \tag{101}$$

where $\delta'(\sigma - \bar{\sigma}) \equiv \partial_\sigma \delta(\sigma - \bar{\sigma})$. Our task is to find

$$\{A(\sigma), B(\bar{\sigma})\},$$

from

$$\{\Delta A(\sigma, \sigma_0), \Delta B(\bar{\sigma}, \bar{\sigma}_0)\},$$

where

$$\Delta A(\sigma, \sigma_0) = \int_{\sigma_0}^{\sigma} dx A'(x) = A(\sigma) - A(\sigma_0),$$
$$\Delta B(\bar{\sigma}, \bar{\sigma}_0) = \int_{\bar{\sigma}_0}^{\bar{\sigma}} dx B'(x) = B(\bar{\sigma}) - B(\bar{\sigma}_0). \tag{102}$$

It is obvious that

$$\{\Delta A(\sigma, \sigma_0), \Delta B(\bar{\sigma}, \bar{\sigma}_0)\} = \int_{\sigma_0}^{\sigma} dx \int_{\bar{\sigma}_0}^{\bar{\sigma}} dy \ \left[U'(x)\delta(x - y) + V(x)\delta'(x - y)\right], \tag{103}$$

and after integration over y we get

$$\begin{aligned}&\{\Delta A(\sigma,\sigma_0),\Delta B(\bar\sigma,\bar\sigma_0)\} \\ =& \int_{\sigma_0}^{\sigma} dx\{U'(x)\,[\theta(x-\bar\sigma_0)-\theta(x-\bar\sigma)]+V(x)\,[\delta(x-\bar\sigma_0)-\delta(x-\bar\sigma)]\},\end{aligned} \tag{104}$$

where $\theta(x)$ is defined as

$$\theta(x)=\int_0^x d\eta\delta(\eta)=\frac{1}{2\pi}\left[x+2\sum_{n\geq 1}\frac{1}{n}\sin(nx)\right]=\begin{cases}0 & \text{if } x=0\\ 1/2 & \text{if } 0<x<2\pi\\ 1 & \text{if } x=2\pi\end{cases}. \tag{105}$$

Integrating over x, finally we get

$$\begin{aligned}&\{\Delta A(\sigma,\sigma_0),\Delta B(\bar\sigma,\bar\sigma_0)\} = \\ &U(\sigma)[\theta(\sigma-\bar\sigma_0)-\theta(\sigma-\bar\sigma)]-U(\sigma_0)[\theta(\sigma_0-\bar\sigma_0)-\theta(\sigma_0-\bar\sigma)]\\ -\ &U(\bar\sigma_0)[\theta(\sigma-\bar\sigma_0)-\theta(\sigma_0-\bar\sigma_0)]+U(\bar\sigma)[\theta(\sigma-\bar\sigma)-\theta(\sigma_0-\bar\sigma)]\\ +\ &V(\bar\sigma_0)[\theta(\sigma-\bar\sigma_0)-\theta(\sigma_0-\bar\sigma_0]-V(\bar\sigma)[\theta(\sigma-\bar\sigma)-\theta(\sigma_0-\bar\sigma)]\end{aligned} \tag{106}$$

From the last expression, we extract the desired Poisson bracket

$$\{A(\sigma),B(\bar\sigma)\}=-[U(\sigma)-U(\bar\sigma)+V(\bar\sigma)]\theta(\sigma-\bar\sigma)\,. \tag{107}$$

References

[1] H. S. Snyder, *Phys.Rev.* **71** (1947) 38.

[2] N. Seiberg and E. Witten, *JHEP* **09** (1999) 032.

[3] A. Connes, M. R. Douglas and A. Schwarz, *JHEP* **02** (1998) 003; M. R. Douglas and C. Hull, *JHEP* **02** (1998) 008; V. Schomerus, *JHEP* **06** (1999) 030; F. Ardalan, H.Arfaei and M. M. Sheikh-Jabbari, *JHEP* **02** (1999) 016; C. S. Chu and P. M. Ho, *Nucl. Phys.* **B550** (1999) 151; F. Ardalan, H. Arfaei and M. M. Sheikh-Jabbari, *Nucl. Phys.* **B576** (2000) 578; C. S. Chu and P. M. Ho, *Nucl. Phys.* **B568** (2000) 447; T.Lee, *Phys. Rev.* **D62** (2000) 024022; B. Nikolić and B. Sazdović, *Phys.Rev.* **D74** (2006) 045024; *Phys.Rev.* **D75** (2007) 085011.

[4] D. Lust *JHEP* **12** (2010) 084.

[5] K. Becker, M. Becker and J. Schwarz *String Theory and M-Theory: A Modern Introduction*; B. Zwiebach, *A First Course in String Theory*, Cambridge University Press, 2004; J. Polchinski, *String theory - Volume II*, Cambridge University Press, 1998.

[6] T. Buscher, *Phys. Lett.* **B 194** (1987) 59; **201** (1988) 466.

[7] M. Roček and E. Verlinde, *Nucl.Phys.* **B 373** (1992) 630.

[8] A. Giveon, M. Porrati and E. Rabinovici, *Phys. Rep.* **244** (1994) 77.

[9] E. Alvarez, L. Alvarez-Gaume, J. Barbon and Y. Lozano, *Nucl. Phys.* **B 415** (1994) 71.

[10] B. Nikolić and B. Sazdović, *Nucl. Phys.* **B 836** (2010) 100.

[11] D. S. Berman, D. C. Thompson, *Phys.Rept.* **566** (2014) 1-60.

[12] Lj. Davidović and B. Sazdović, *EPJ* **C 74** (2014) 2683.

[13] Lj. Davidović, B. Nikolić and B. Sazdović, *EPJ* **C 75** (2015) 576.

[14] Lj. Davidović, B. Sazdović, *JHEP* **11** (2015) 119.

[15] D. Andriot, M. Larfors, D. Luest, P. Patalong, *JHEP* **06** (2013) 021.

[16] B. Nikolić and D. Obrić, *Fortschr. Phys.* **66** (2018) 040009.

[17] C. G. Callan, D. Friedan, E. J. Martinec and M. J. Perry, *Nucl. Phys.* **B262** (1985) 593-609.

P r o c e e d i n g s

of the

9th MATHEMATICAL PHYSICS MEETING:
School and Conference on
Modern Mathematical Physics

September 18 – 23, 2017, Belgrade, Serbia

Editors

B. Dragovich, I. Salom and M. Vojinović

Institute of Physics
Belgrade, 2018
SERBIA

Autor: Grupa autora

Naslov: 9th MATHEMATICAL PHYSICS MEETING: SCHOOL AND CONFERENCE ON MODERN MATHEMATICAL PHYSICS
(Deveti naučni skup iz matematičke fizike: škola i konferencija iz savremene matematičke fizike)

Izdavač: Institut za fiziku, Beograd, Srbija

Izdanje: Prvo izdanje
(SFIN year XXXI Series A: Conferences No. A1 (2018))

Štampa: Dual Mode, Beograd

Tiraž: 150
ISBN: 978-86-82441-48-9

1. Dragović Branko
Matematička fizika-Zbornici

CIP- Каталогизација у публикацији
Народна библиотека Србије

51-73:53(082)

SCHOOL and Conference on Modern Mathematical Physics (9 ; 2017 ; Beograd)
Proceedings of the 9th Mathematical Physics Meeting: School and Conference on Modern Mathematical Physics, September 18-23, 2017, Belgrade, Serbia / [organizers Institute of Physics, Belgrade ... et al.] ; editors B. [Branko] Dragovich, I. [Igor] Salom and M. [Marko] Vojinović. - 1. izd. - Belgrade : Institute of Physics, 2018 (Beograd : Dual Mode). - VIII, 324 str. : ilustr. ; 25 cm. - (Sveske fizičkih nauka : SFIN ; year 31. Ser. A, Conferences, ISSN 0354-9291 ; n^o A1, (2018))

U kolofonu: Deveti naučni skup iz matematičke fizike: škola i konferencija iz savremene matematičke fizike. - Tiraž 150. - Str. V: Preface / editors. - Napomene i bibliografske reference uz tekst. - Bibliografija uz svaki rad.

ISBN 978-86-82441-48-9

1. Dragović, Branko, 1945- [уредник] [аутор додатног текста] 2. Institut za fiziku (Beograd)

а) Математичка физика - Зборници

COBISS.SR-ID 265404172

P R E F A C E

This volume contains some reviews and original research contributions, which are related to the **9th Mathematical Physics Meeting: School and Conference on Modern Mathematical Physics**, organized by the Institute of Physics, Belgrade (Serbia), September 18–23, 2017. The programme of this meeting was mainly oriented towards some recent developments in gravity and cosmology, string and quantum field theory, and some relevant mathematical methods. We hope that articles presented here will be valuable literature not only for the participants of this meeting but also for many other PhD students and researchers in modern mathematical and theoretical physics. We are grateful to all authors for writing their contributions for these proceedings.

The previous eight meetings in this series of schools and conferences on modern mathematical physics were also held in Serbia: Sokobanja, 13–25 August 2001; Kopaonik, 1–12 September 2002; Zlatibor, 20–31 August 2004; Belgrade, 3–14 September 2006; Belgrade, 6–17 July 2008; Belgrade, 14–23 September 2010; Belgrade, 9–19 September 2012; and Belgrade, 24–31 August 2014. The corresponding proceedings of all these meetings were published by the Institute of Physics, Belgrade, and are available in the printed form as well as online at the websites.

This ninth meeting took place at the Mathematical Institute, Serbian Academy of Sciences and Arts, located in Belgrade downtown. We hope that all attending this meeting will recall it as a useful and pleasant event, and will wish to participate again in the future.

We wish to thank all lecturers and other speakers for their interesting and valuable talks. We also thank all participants for their active participation. Financial support of our sponsors, *Ministry of Education, Science and Technological Development of the Republic of Serbia, Belgrade; and the Abdus Salam International Centre for Theoretical Physics* was very significant for realization of this activity.

April 2018

E d i t o r s

B. Dragovich

I. Salom

M. Vojinović

9th MATHEMATICAL PHYSICS MEETING: School and Conference on Modern Mathematical Physics

Belgrade, September 18 - 23, 2017

Organizers

- **Institute of Physics, Belgrade, Serbia**
- **Mathematical Institute, Belgrade, Serbia**
- **Faculty of Mathematics, Belgrade, Serbia**

International Advisory Committee

Ignatios Antoniadis (Paris, France)
Irina Arefeva (Moscow, Russia)
Alexander Belavin (Moscow, Russia)
Loriano Bonora (Trieste, Italy)
Maja Buric (Belgrade, Serbia)
Nemanja Kaloper (Davis, USA)
Viatcheslav Mukhanov (Munich, Germany)
Emil Nissimov (Sofia, Bulgaria)
Sergei Odintsov (Barcelona, Spain)
Dmitri Polyakov (Chengdu, China; Moscow, Russia)
Branislav Sazdovic (Belgrade, Serbia)
Djordje Sijacki (Belgrade, Serbia)
Alexei Starobinsky (Moscow, Russia)
Dejan Stojkovic (Buffalo, USA)
Mihai Visinescu (Bucharest, Romania)

International Organizing Committee

Vladimir Dobrev (Sofia, Bulgaria)
Branko Dragovich (Belgrade, Serbia)
Goran Djordjevic (Nis, Serbia)
Zoran Rakic (Belgrade, Serbia)
Igor Volovich (Moscow, Russia)

Local Organizing Committee

Branko Dragovich (Co-Chairman, Institute of Physics, Belgrade)
Igor Salom (Co-Chairman, Institute of Physics, Belgrade)
Sanja Ćirković (Institute of Physics, Belgrade)
Ljubica Davidović (Institute of Physics, Belgrade)
Marko Vojinović (Institute of Physics, Belgrade)

Sponsors

- **Ministry of Education, Science and Technological Development, Republic of Serbia**
- **The Abdus Salam International Centre for Theoretical Physics**

CONTENTS

Review and Research Works

Representation of T-duality of type II pure spinor superstring in double space*

Bojan Nikolić†
Branislav Sazdović‡
Institute of Physics Belgrade, University of Belgrade
Pregrevica 118, 11080 Belgrade, Serbia

Abstract

In this article we present a new way of representation of T-duality using double space. Double space is obtained by adding T-dual coordinates to the initial ones. T-dualization in double space is represented by permutation of appropriate directions from initial and T-dual space. We did this both for bosonic and fermionic T-dualization of type II superstring theory propagating in the constant background fields. Obtained results are in full correspondence with the results of standard Buscher procedure.

1. Introduction

T-duality as a fundamental characteristic of string dynamics [1, 2, 3, 4, 5], unexprerienced by point-like particle, makes that there is no difference in physical content between string theories compactified on a circle of radius R and circle of radius $1/R$. It is very important for understanding M-theory, because five consistent superstring theories are connected by web of T and S dualities.

Buscher T-dualization procedure [2] represents a mathematical environment for performing T-duality. In order to make T-dualization along some directions, they should be isometry ones. Effectively, this means that background fields do not depend on those coordinates [2, 3, 4, 5, 6, 7]. Further, we localize noticed symmetry in standard way introducing world-sheet covariant derivatives, $\partial_\pm x^\mu \to D_\pm x^\mu = \partial_\pm x^\mu + v_\pm^\mu$, where $v_\pm^\mu$ are gauge fields. In order to have the same number of degrees of freedom in T-dual theory as in the initial one, the new term with Lagrange multipliers is added to the action. Using gauge freedom we fix initial coordinates and get the gauge

* This work has been supported by the Serbian Ministry of Education, Science and Technological Development, under contract No. 171031.
† e-mail address: bnikolic@ipb.ac.rs
‡ e-mail address: sazadovic@ipb.ac.rs

fixed action. Varying gauge fixed action with respect to the Lagrange multipliers one gets the initial action and varying with respect to the gauge fields one gets T-dual action.

This standard T-dualization procedure was used in the papers [8, 9, 10, 11, 12] in the context of closed string noncommutativity. There is a generalized Buscher procedure which deals with background fields depending on all coordinates. The new step in this procedure is introducing of gauge invariant coordinate for the directions on which background fields depend on. The generalized procedure was applied to the case of bosonic string moving in the weakly curved background [13, 14]. It leads directly to closed string noncommutativity [15].

Double space formalism is a framework in which we can represent T-dualization in a simple and elegant way. It is spanned by double coordinates $Z^M = (x^\mu, y_\mu)$ $(\mu = 0, 1, 2, \dots, D-1)$, where x^μ and y_μ are the coordinates of the D-dimensional initial and T-dual space-time, respectively. It was the subject of the articles about twenty years ago [16, 17, 18, 19, 20], but interest for it occured again [21, 22, 23, 24, 25]. In all these papers T-duality is represented as $O(d, d)$ symmetry transformations.

In Refs.[26, 27] we doubled all bosonic coordinates and obtain the theory which contains the initial and all corresponding T-dual theories. In such theory partial T-dualization (T-dualization along some of the initial directions x^a) is represented as permutation of the corresponding coordinate subsets, x^a and y_a, which is a generalization of ideas given in [16].

When one says T-duality, one means bosonic T-duality. But since recently we can also speak about fermionic T-duality. Analyzing the gluon scattering amplitudes in $N = 4$ super Yang-Mills theory fermionic T-duality was discovered [28, 29]. Mathematically, fermionic T-duality is realized within Buscher procedure, except that dualization is performed along fermionic directions, θ^α and $\bar{\theta}^\alpha$.

In the present paper we are going to extend approach of double space to the type II theories both in the case of bosonic and fermionic T-duality [28, 30, 31]. We will show that double space method gives the same results as in the case of applying of standard Buscher procedure.

Here we start applying the approach of Refs.[26, 27] in the case of partial bosonic T-dualization of the type II superstring theory [1] and then in the case of fermionic T-duality. Generally, we use type II pure spinor action from [32]. This action is given in the form of an expansion in powers of fermionic coordinates θ^α and $\bar{\theta}^\alpha$. In both cases we simplify the action using different assumptions explained in the paper. As a result in both cases we obtain same action, which describes ghost free type II superstring theory in pure spinor formulation [33, 34, 35] in the approximation of constant background fields and up to the quadratic terms.

Bosonic T-dual transformation laws can be rewritten via double space coordinates Z^M. In order to achieve that we introduce the generalized metric $\mathcal{H}_{MN}$ and the generalized current $J_{\pm M}$. The permutation matrix $(\mathcal{T}^a)^M{}_N$ makes permutation of x^a and y_a, where index a marks the directions along which we make partial bosonic T-dualization. The T-dual coordinate is defined as ${}_aZ^M = (\mathcal{T}^a)^M{}_N Z^N$ and it has to obey the T-dual transformation law of the same form as initial coordinates, Z^M. This demand produces the expressions for T-dual generalized metric, ${}_a\mathcal{H}_{MN} = (\mathcal{T}^a \mathcal{H} \mathcal{T}^a)_{MN}$, and T-dual current, ${}_aJ_{\pm M} = (\mathcal{T}^a J_\pm)_M$. From transformation of the generalized metric we obtain T-dual NS-NS background fields and from transformation of the current we obtain T-dual NS-R fields. The transformation law for R-R field strength we get imposing additional assumtions because it is coupled by fermionic degrees of freedom along which we do not dualize.

Further we apply the method in the case of fermionic T-dualization. We are going to double fermionic sector of type II theories adding to the coordinates θ^α and $\bar\theta^\alpha$ their fermionic T-duals, ϑ_α and $\bar\vartheta_\alpha$, where $\alpha = 1, 2, \dots, 16$. Rewriting T-dual transformation laws in terms of the double coordinates, $\Theta^A = (\theta^\alpha, \vartheta_\alpha)$ and $\bar\Theta^A = (\bar\theta^\alpha, \bar\vartheta_\alpha)$, we define the "fermionic generalized metric" $\mathcal{F}_{AB}$ and the generalized currents $\bar{\mathcal{J}}_{+A}$ and $\mathcal{J}_{-A}$. The permutation matrix $\mathcal{T}^A{}_B$ exchanges $\bar\theta^\alpha$ and θ^α with their T-dual partners, $\bar\vartheta_\alpha$ and ϑ_α, respectively. From the requirement that fermionic T-dual coordinates, ${}^\star\Theta^A = \mathcal{T}^A{}_B \Theta^B$ and ${}^\star\bar\Theta^A = \mathcal{T}^A{}_B \bar\Theta^B$, have the same T-dual transformation law as initial ones, Θ^A and $\bar\Theta^A$, we obtain the expressions for fermionic T-dual generalized metric, ${}^\star\mathcal{F}_{AB} = (\mathcal{T}\mathcal{F}\mathcal{T})_{AB}$, and T-dual currents, ${}^\star\bar{\mathcal{J}}_{+A} = \mathcal{T}_A{}^B \bar{\mathcal{J}}_{+B}$ and ${}^\star\mathcal{J}_{-A} = \mathcal{T}_A{}^B \mathcal{J}_{-B}$, in terms of the initial ones. These expressions produce the expressions for fermionic T-dual NS-R fields and R-R field strength. Expressions for fermionic T-dual metric and Kalb-Ramond field are obtained separately under some assumptions.

In this article we will not present explicitely the transformation of dilaton field, which demands quantum treatment.

2. Pure spinor action of type II action

The sigma model of pure spinor type II superstring action for type II superstring [32] is of the form

$$\begin{aligned} S &= \int d^2\xi \,\big[\partial_+\theta^\alpha A_{\alpha\beta}\partial_-\bar\theta^\beta + \partial_+\theta^\alpha A_{\alpha\mu}\bar\Pi^\mu + \Pi^\mu A_{\mu\alpha}\partial_-\bar\theta^\alpha + \Pi^\mu A_{\mu\nu}\bar\Pi^\nu \\ &+ d_\alpha E^\alpha{}_\beta \partial_-\bar\theta^\beta + d_\alpha E^\alpha{}_\mu \bar\Pi^\mu + \partial_+\theta^\alpha E_\alpha{}^\beta \bar d_\beta + \Pi^\mu \bar E_\mu{}^\beta \bar d_\beta + d_\alpha \mathrm{P}^{\alpha\beta}\bar d_\beta \\ &+ \frac{1}{2}N^{\mu\nu}\Omega_{\mu\nu,\beta}\partial_-\bar\theta^\beta + \frac{1}{2}N^{\mu\nu}\Omega_{\mu\nu,\rho}\bar\Pi^\rho + \frac{1}{2}\partial_+\theta^\alpha\Omega_{\alpha,\mu\nu}\bar N^{\mu\nu} + \frac{1}{2}\Pi^\mu\Omega_{\mu,\nu\rho}\bar N^{\nu\rho} \\ &+ \frac{1}{2}N^{\mu\nu}\bar C_{\mu\nu}{}^\beta \bar d_\beta + \frac{1}{2}d_\alpha C^\alpha{}_{\mu\nu}\bar N^{\mu\nu} + \frac{1}{4}N^{\mu\nu}S_{\mu\nu,\rho\sigma}\bar N^{\rho\sigma}\Big] + S_\lambda + S_{\bar\lambda}\,, \end{aligned} \tag{1}$$

where

$$\Pi^\mu = \partial_+ x^\mu + \frac{1}{2}\theta^\alpha(\Gamma^\mu)_{\alpha\beta}\partial_+\theta^\beta\,, \quad \bar\Pi^\mu = \partial_- x^\mu + \frac{1}{2}\bar\theta^\alpha(\Gamma^\mu)_{\alpha\beta}\partial_-\bar\theta^\beta\,, \tag{2}$$

$$d_\alpha = \pi_\alpha - \frac{1}{2}(\Gamma_\mu\theta)_\alpha \left[\partial_+ x^\mu + \frac{1}{4}(\theta\Gamma_\mu\partial_+\theta)\right] ,$$
$$\bar{d}_\alpha = \bar{\pi}_\alpha - \frac{1}{2}(\Gamma_\mu\bar{\theta})_\alpha \left[\partial_- x^\mu + \frac{1}{4}(\bar{\theta}\Gamma_\mu\partial_-\bar{\theta})\right] , \tag{3}$$

$$N^{\mu\nu} = \frac{1}{2} w_\alpha (\Gamma^{[\mu\nu]})^\alpha{}_\beta \lambda^\beta , \quad \bar{N}^{\mu\nu} = \frac{1}{2}\bar{w}_\alpha(\Gamma^{[\mu\nu]})^\alpha{}_\beta \bar{\lambda}^\beta . \tag{4}$$

Type II superfields generally depends on x^μ, θ^α and $\bar{\theta}^\alpha$. The superfields $A_{\mu\nu}$, $\bar{E}_\mu{}^\alpha$, $E^\alpha{}_\mu$ and $\mathrm{P}^{\alpha\beta}$ are physical superfields, because their first components are supergravity fields. The fields $\Omega_{\mu,\nu\rho}(\Omega_{\mu\nu,\rho})$, $C^\alpha{}_{\mu\nu}(\bar{C}_{\mu\nu}{}^\alpha)$ and $S_{\mu\nu,\rho\sigma}$, are curvatures (field strengths) for physical superfields. The rest fields are auxiliary superfirlds because they can be expressed in terms of the physical ones [32].

The world sheet Σ is parameterized by $\xi^m = (\xi^0 = \tau\,, \xi^1 = \sigma)$ and $\partial_\pm = \partial_\tau \pm \partial_\sigma$. Superspace is spanned by bosonic coordinates x^μ ($\mu = 0, 1, 2, \dots, 9$) and fermionic ones θ^α and $\bar{\theta}^\alpha$ ($\alpha = 1, 2, \dots, 16$). The variables π_α and $\bar{\pi}_\alpha$ are canonically conjugated momenta to θ^α and $\bar{\theta}^\alpha$, respectively. The actions for pure spinors, S_λ and $S_{\bar{\lambda}}$, are free field actions

$$S_\lambda = \int d^2\xi w_\alpha \partial_- \lambda^\alpha , \quad S_{\bar{\lambda}} = \int d^2\xi \bar{w}_\alpha \partial_+ \bar{\lambda}^\alpha , \tag{5}$$

where λ^α and $\bar{\lambda}^\alpha$ are pure spinors and w_α and $\bar{w}_\alpha$ are their canonically conjugated momenta, respectively. The pure spinors satisfy so called pure spinor constraints

$$\lambda^\alpha(\Gamma^\mu)_{\alpha\beta}\lambda^\beta = \bar{\lambda}^\alpha(\Gamma^\mu)_{\alpha\beta}\bar{\lambda}^\beta = 0 . \tag{6}$$

3. Bosonic T-dualization

In this section we will introduce approximated action and then apply standard Buscher procedure on some subset of coordinates x^a. Then we will compare obtained result with the result following from double space formalism.

3.1. Simplification of action

The action (1) could be considered as an expansion in powers of θ^α and $\bar{\theta}^\alpha$. For computational simplicity, in the first step we neglect all terms in the action containing θ^α and $\bar{\theta}^\alpha$. As a consequence θ^α and $\bar{\theta}^\alpha$ terms disappear from $\Pi^\mu_\pm$, d_α and $\bar{d}_\alpha$ and in the solutions for physical superfields, just x-dependence of the supergravity fields survives.

Let us make T-dualization along some subset of bosonic coordinates x^a. So, we will assume that these directions are isometry ones. It essentially means that corresponding superfields (A_{ab}, $\bar{E}_a{}^\alpha$, $E^\alpha{}_a$, $\mathrm{P}^{\alpha\beta}$) should not depend on x^a. This assumption could be extended on all space-time directions x^μ which means that physical superfields are constant. According to [32], auxiliary superfields are zero, because all physical superfields are constant. Further, constant physical superfields means that their field strengths, $\Omega_{\mu,\nu\rho}(\Omega_{\mu\nu,\rho})$, $C^\alpha{}_{\mu\nu}(\bar{C}_{\mu\nu}{}^\alpha)$ and $S_{\mu\nu,\rho\sigma}$, are zero.

Background fields obey space-time field equations [36], which are some kind of consistency conditions. The equation (B.7) from this set of equations represents the backreaction of $\mathrm{P}^{\alpha\beta}$ on the metric $G_{\mu\nu}$. If we take constant dilaton Φ and constant antisymmetric NS-NS field $B_{\mu\nu}$ we obtain that

$$R_{\mu\nu} - \frac{1}{2} G_{\mu\nu} R \sim (\mathrm{P}^{\alpha\beta})^2_{\mu\nu} \,. \tag{7}$$

If we choose the background field $\mathrm{P}^{\alpha\beta}$ to be constant, in general, we will have constant Ricci tensor which means that metric tensor is quadratic function of space-time coordinates. If one wants to cancel non-quadratic terms originating from back-reaction, additional conditions must be imposed on R-R field strength (see the first reference in [33]).

Taking into account above analysis and arguments, our approximation is realized in the following way

$$\Pi^\mu_\pm \to \partial_\pm x^\mu \,, \quad d_\alpha \to \pi_\alpha \,, \quad \bar{d}_\alpha \to \bar{\pi}_\alpha \,, \tag{8}$$

and physical superfields take the form

$$\begin{aligned} &A_{\mu\nu} = \kappa(\frac{1}{2} G_{\mu\nu} + B_{\mu\nu}) \,, \quad E^\alpha_\nu = -\Psi^\alpha_\nu \,, \quad \bar{E}^\alpha_\mu = \bar{\Psi}^\alpha_\mu \,, \\ &\mathrm{P}^{\alpha\beta} = \frac{2}{\kappa} P^{\alpha\beta} = \frac{2}{\kappa} e^{\frac{\Phi}{2}} F^{\alpha\beta} \,, \end{aligned} \tag{9}$$

where $G_{\mu\nu}$ is metric tensor and $B_{\mu\nu}$ is antisymmetric NS-NS background field.

Consequently, the full action S is

$$\begin{aligned} S = \kappa \int_\Sigma d^2\xi \left[\partial_+ x^\mu \Pi_{+\mu\nu} \partial_- x^\nu + \frac{1}{4\pi\kappa} \Phi R^{(2)} \right] \qquad (10) \\ + \int_\Sigma d^2\xi \left[-\pi_\alpha \partial_-(\theta^\alpha + \Psi^\alpha_\mu x^\mu) + \partial_+(\bar{\theta}^\alpha + \bar{\Psi}^\alpha_\mu x^\mu)\bar{\pi}_\alpha + \frac{2}{\kappa}\pi_\alpha P^{\alpha\beta} \bar{\pi}_\beta \right] \,, \end{aligned}$$

where

$$\Pi_{\pm\mu\nu} = B_{\mu\nu} \pm \frac{1}{2} G_{\mu\nu} \,. \tag{11}$$

Actions S_λ and $S_{\bar{\lambda}}$ are decoupled from the rest and can be neglected in the further analysis.

3.2. Buscher T-dualization

Let some shift symmetry along x^a directions exists. The first step in Buscher procedure is that we localize the noticed shift symmetry. We substitute the ordinary derivatives with covariant ones, introducing gauge fields v^a_α. Then we add the term $\frac{1}{2}y_a F^a_{+-}$ to the Lagrangian in order to force the field strength F^a_{+-} to vanish and preserve equivalence between original and T-dual theories. Finally, we fix the gauge $x^a = 0$ and obtain gauge fixed action

$$\begin{aligned} & S_{fix}(v^a_\pm, x^i, \theta^\alpha, \bar\theta^\alpha, \pi_\alpha, \bar\pi_\alpha) = \\ & \int_\Sigma d^2\xi \left[\kappa v^a_+ \Pi_{+ab} v^b_- + \kappa v^a_+ \Pi_{+aj}\partial_- x^j + \kappa \partial_+ x^i \Pi_{+ib} v^b_- + \right. \\ + \quad & \kappa \partial_+ x^i \Pi_{+ij}\partial_- x^j + \frac{1}{4\pi}\Phi R^{(2)} - \pi_\alpha \Psi^\alpha_b v^b_- \\ + \quad & v^a_+ \bar\Psi^\alpha_a \bar\pi_\alpha - \pi_\alpha \partial_-(\theta^\alpha + \Psi^\alpha_i x^i) + \partial_+(\bar\theta^\alpha + \bar\Psi^\alpha_i x^i)\bar\pi_\alpha + \frac{1}{2\kappa} e^{\frac{\Phi}{2}} \pi_\alpha F^{\alpha\beta}\bar\pi_\beta \\ + \quad & \left. \frac{\kappa}{2}(v^a_+ \partial_- y_a - v^a_- \partial_+ y_a)\right] . \end{aligned} \tag{12}$$

Varying the gauge fixed action with respect to the Lagrange multipliers y_a we get the solution for gauge fields in the form

$$v^a_\pm = \partial_\pm x^a \,, \tag{13}$$

which produces the initial action, while varying with respect to the gauge fields $v^a_\pm$ we have

$$v^a_\pm = -2\kappa \hat\theta^{ab}_\pm \Pi_{\mp bi}\partial_\pm x^i - \kappa \hat\theta^{ab}_\pm \partial_\pm y_b \pm 2\hat\theta^{ab}_\pm \Psi^\alpha_{\pm b}\pi_{\pm\alpha} \,. \tag{14}$$

Substituting $v^a_\pm$ in (12) we find

$$\begin{aligned} & S_{fix}(y_a, x^i, \theta^\alpha, \bar\theta^\alpha, \pi_\alpha, \bar\pi_\alpha) = \\ & \int_\Sigma d^2\xi \left[\frac{\kappa^2}{2}\partial_+ y_a \hat\theta^{ab}_- \partial_- y_b + \kappa^2 \partial_+ y_a \hat\theta^{ab}_- \Pi_{+bj}\partial_- x^j - \kappa^2 \partial_+ x^i \Pi_{+ia}\hat\theta^{ab}_- \partial_- y_b \right. \\ + \quad & \kappa \partial_+ x^i (\Pi_{+ij} - 2\kappa \Pi_{+ia}\hat\theta^{ab}_- \Pi_{+bj})\partial_- x^j + \frac{1}{4\pi}\Phi R^{(2)} \\ - \quad & \pi_\alpha \partial_-(\theta^\alpha + \Psi^\alpha_i x^i - 2\kappa \Psi^\alpha_a \hat\theta^{ab}_- \Pi_{+bj} x^j - \kappa \Psi^\alpha_a \hat\theta^{ab}_- y_b) \\ + \quad & \partial_+(\bar\theta^\alpha + \bar\Psi^\alpha_i x^i + 2\kappa \bar\Psi^\alpha_a \hat\theta^{ab}_+ \Pi_{-bj} x^j + \kappa \bar\Psi^\alpha_a \hat\theta^{ab}_+ y_b)\bar\pi_\alpha \\ + \quad & \left. 2\pi_\alpha \Psi^\alpha_a \hat\theta^{ab}_- \bar\Psi^\beta_b \bar\pi_\beta + \frac{1}{2\kappa} e^{\frac{\Phi}{2}} \pi_\alpha F^{\alpha\beta}\bar\pi_\beta \right] . \end{aligned} \tag{15}$$

Combining two solutions for gauge fields (13) and (14) we obtain transformation law between initial x^a and T-dual coordinates y_a

$$\partial_+ ({}_aX)_{\hat\mu} = (\bar Q^{-1T})_{\hat\mu\nu}\partial_+ x^\nu + J_{+\hat\mu} \,, \quad \partial_- \, ({}_aX)_{\hat\mu} = (Q^{-1T})_{\hat\mu\nu}\partial_- x^\nu + J_{-\hat\mu} \,, \tag{16}$$

where we introduced the T-dual variables ${}_aX_{\hat\mu} = \{y_a,\, x^i\}$. For coordinates which contain both x^i and y_a we will use "hat" indices $\hat\mu, \hat\nu$. The matrices

$$Q^{\hat\mu\nu} = \begin{pmatrix} \kappa\hat\theta^{ab}_{+} & 0 \\ -2\kappa\Pi_{-ic}\hat\theta^{cb}_{+} & \delta^i_j \end{pmatrix}, \qquad \bar Q^{\hat\mu\nu} = \begin{pmatrix} \kappa\hat\theta^{ab}_{-} & 0 \\ -2\kappa\Pi_{+ic}\hat\theta^{cb}_{-} & \delta^i_j \end{pmatrix}, \tag{17}$$

and theirs inverse

$$Q^{-1}_{\mu\hat\nu} = \begin{pmatrix} 2\Pi_{-ab} & 0 \\ 2\Pi_{-ib} & \delta^j_i \end{pmatrix}, \qquad \bar Q^{-1}_{\mu\hat\nu} = \begin{pmatrix} 2\Pi_{+ab} & 0 \\ 2\Pi_{+ib} & \delta^j_i \end{pmatrix}, \tag{18}$$

perform T-dualization for vector indices. Here we introduced the currents $J_{\pm\hat\mu}$

$$J_{+\hat\mu} = \begin{pmatrix} J_{+a} \\ 0 \end{pmatrix}, \quad J_{-\hat\mu} = \begin{pmatrix} J_{-a} \\ 0 \end{pmatrix}, \tag{19}$$

where

$$J_{\pm\mu} = \pm\frac{2}{\kappa}\Psi^\alpha_{\pm\mu}\pi_{\pm\alpha}\,, \tag{20}$$

$$\Psi^\alpha_{+\mu} \equiv \Psi^\alpha_\mu\,, \quad \Psi^\alpha_{-\mu} \equiv \bar\Psi^\alpha_\mu\,, \quad \pi_{+\alpha} \equiv \pi_\alpha\,, \quad \pi_{-\alpha} \equiv \bar\pi_\alpha\,, \tag{21}$$

and $\hat\theta^{ab}_{\pm}$ is defined by the relation

$$\hat\theta^{ac}_{\pm}\Pi_{\mp cb} = \frac{1}{2\kappa}\delta^a{}_b\,. \tag{22}$$

Note that different chiralities transform with different matrices $Q^{\hat\mu\nu}$ and $\bar Q^{\hat\mu\nu}$. So, there are two types of T-dual vielbeins

$${}_ae^{\underline{a}\hat\mu} = e^{\underline{a}}{}_\nu(Q^T)^{\nu\hat\mu}\,, \qquad {}_a\bar e^{\underline{a}\hat\mu} = e^{\underline{a}}{}_\nu(\bar Q^T)^{\nu\hat\mu}\,, \tag{23}$$

with the same T-dual metric

$${}_aG^{\hat\mu\hat\nu} \equiv ({}_ae^T\eta\,{}_ae)^{\hat\mu\hat\nu} = (QGQ^T)^{\hat\mu\hat\nu} = {}_a\bar G^{\hat\mu\hat\nu} \equiv ({}_a\bar e^T\eta\,{}_a\bar e)^{\hat\mu\hat\nu} = (\bar QG\bar Q^T)^{\bar\mu\hat\nu}\,. \tag{24}$$

The two T-dual vielbeins are related by particular local Lorentz transformation

$${}_a\bar e^{\underline{a}\hat\mu} = \Lambda^{\underline{a}}{}_{\underline{b}}\,{}_ae^{b\hat\mu}\,, \qquad \Lambda^{\underline{a}}{}_{\underline{b}} = e^{\underline{a}}{}_\mu(Q^{-1}\bar Q)^{T\mu}{}_\nu(e^{-1})^\nu{}_{\underline{b}}\,. \tag{25}$$

From (17) and (18) we have

$$(Q^{-1}\bar Q)^{T\mu}{}_\nu = \begin{pmatrix} \delta^a{}_b + 2\kappa\hat\theta^{ac}_{+}G_{cb} & 2\kappa\hat\theta^{ac}_{+}G_{cj} \\ 0 & \delta^i_j \end{pmatrix}, \tag{26}$$

which produces

$$\Lambda^{\underline{a}}{}_{\underline{b}} = \delta^{\underline{a}}{}_{\underline{b}} - 2\omega^{\underline{a}}{}_{\underline{b}}\,, \qquad \omega^{\underline{a}}{}_{\underline{b}} = -\kappa e^{\underline{a}}{}_a\hat\theta^{ab}_{+}(e^T)_b{}^{\underline{c}}\eta_{\underline{cb}}\,. \tag{27}$$

It satisfies definition of Lorentz transformations

$$\Lambda^T \eta \Lambda = \eta \quad \Longrightarrow \quad \det \Lambda^{\underline{a}}{}_{\underline{b}} = \pm 1 \,, \tag{28}$$

and it holds $\det \Lambda^{\underline{a}}{}_{\underline{b}} = (-1)^d$, where d is the number of dimensions along which we perform T-duality.

Existence of the local Lorentz transformation which connects two sets of vielbeins means that in T-dual picture we can take that, for example, nonbar variables remain the same while bar variables must be multiplied by matrix ${}_a\Omega$, which is a spinorial representation of that local Lorentz symmetry. Skipping technical details explained in details in [37], we give here the final form of the matrix ${}_a\Omega$

$${}_a\Omega \;=\sqrt{\prod_{i=1}^{d} G_{a_i a_i}}\; {}_a\Gamma\,(i\,\Gamma^{11})^d \,, \tag{29}$$

where

$${}_a\Gamma \;= (i)^{\frac{d(d-1)}{2}} \prod_{i=1}^{d} \Gamma^{a_i} = (i)^{\frac{d(d-1)}{2}}\, \Gamma^{a_1}\Gamma^{a_2}\cdots\Gamma^{a_d} \,. \tag{30}$$

It is easy to check that ${}_a\Omega^2 = 1$.

Let we introduce proper fermionic variables

$${}_a\theta^\alpha = \theta^\alpha \,, \quad {}_a\pi_\alpha = \pi_\alpha \,, \quad {}^\bullet\bar\theta^\alpha \equiv {}_a\Omega^\alpha{}_\beta\, {}_a\bar\theta^\beta \,, \quad {}^\bullet\bar\pi_\alpha \equiv {}_a\Omega_\alpha{}^\beta\, {}_a\bar\pi_\beta \,. \tag{31}$$

Using the action (15) and proper fermionic variables, we read T-dual background fields

$${}_a\Pi_\pm^{ab} = \frac{\kappa}{2}\hat\theta_\mp^{ab} \,, \tag{32}$$

$${}_a\Pi_{\pm i}{}^a = -\kappa \Pi_{\pm ib}\hat\theta_\mp^{ba} \,, \quad {}_a(\Pi_\pm)^a{}_i = \kappa\hat\theta_\mp^{ab}\Pi_{\pm bi} \,, \tag{33}$$

$${}_a\Pi_{\pm ij} = \Pi_{\pm ij} - 2\kappa\Pi_{\pm ia}\hat\theta_\mp^{ab}\Pi_{\pm bj} \,, \tag{34}$$

$${}_a\Psi^{\alpha a} = \kappa\hat\theta_+^{ab}\Psi_b^\alpha \,, \quad {}_a\bar\Psi^{\alpha a} = \kappa\, {}_a\Omega^\alpha{}_\beta\, \hat\theta_-^{ab}\bar\Psi_b^\beta \,, \tag{35}$$

$${}_a\Psi_i^\alpha = \Psi_i^\alpha - 2\kappa\Pi_{-ib}\hat\theta_+^{ba}\Psi_a^\alpha \,, \quad {}_a\bar\Psi_i^\alpha = {}_a\Omega^\alpha{}_\beta(\bar\Psi_i^\beta - 2\kappa\Pi_{+ib}\hat\theta_-^{ba}\bar\Psi^\beta) \,, \tag{36}$$

$$e^{\frac{{}_a\Phi}{2}}\, {}_aF^{\alpha\beta} = (e^{\frac{\Phi}{2}}\, F^{\alpha\gamma} + 4\kappa\Psi_a^\alpha\hat\theta_-^{ab}\bar\Psi_b^\gamma)\, {}_a\Omega_\gamma{}^\beta \,. \tag{37}$$

3.3. Double space

Above expressions for T-dual background fields in the case of full T-dualization are

$${}^\star\Pi_\pm^{\mu\nu} \equiv {}^\star B^{\mu\nu} \pm \frac{1}{2}{}^\star G^{\mu\nu} = \frac{\kappa}{2}\Theta_\mp^{\mu\nu} \,, \tag{38}$$

$${}^\star\Psi^{\alpha\mu} = \kappa\Theta_+^{\mu\nu}\Psi_\nu^\alpha \,, \quad {}^\star\bar\Psi^{\alpha\mu} = \kappa\, {}^\star\Omega^\alpha{}_\beta\, \Theta_-^{\mu\nu}\bar\Psi_\nu^\beta \,, \tag{39}$$

$$e^{\frac{\star\Phi}{2}}\,{}^\star F^{\alpha\beta} = (e^{\frac{\Phi}{2}} F^{\alpha\gamma} + 4\kappa \Psi^\alpha_\mu \Theta^{\mu\nu}_- \bar\Psi^\gamma_\nu)^\star \Omega_\gamma{}^\beta\,. \tag{40}$$

Here we use the notation

$$G^E_{\mu\nu} = G_{\mu\nu} - 4(BG^{-1}B)_{\mu\nu}\,, \quad \Theta^{\mu\nu} = -\frac{2}{\kappa}(G^{-1}_E BG^{-1})^{\mu\nu}\,, {}^\star\Omega = -\Gamma^{11}, \tag{41}$$

and

$$\Theta^{\mu\nu}_\pm = -\frac{2}{\kappa}(G^{-1}_E \Pi_\pm G^{-1})^{\mu\nu} = \Theta^{\mu\nu} \mp \frac{1}{\kappa}(G^{-1}_E)^{\mu\nu}\,. \tag{42}$$

In this case the T-dual transformation laws (16) obtain the form

$$\partial_\pm x^\mu \cong -\kappa\Theta^{\mu\nu}_\pm \partial_\pm y_\nu + \kappa\Theta^{\mu\nu}_\pm J_{\pm\nu}\,, \qquad \partial_\pm y_\mu \cong -2\Pi_{\mp\mu\nu}\partial_\pm x^\nu + J_{\pm\mu}\,. \tag{43}$$

In terms of double coordinates

$$Z^M = \begin{pmatrix} x^\mu \\ y_\mu \end{pmatrix}, \tag{44}$$

the relations (16) are replaced by one

$$\partial_\pm Z^M \cong \pm\Omega^{MN}\left(\mathcal{H}_{NP}\partial_\pm Z^P + J_{\pm N}\right), \tag{45}$$

where the matrix $\mathcal{H}_{MN}$ is generalized metric and has the form

$$\mathcal{H}_{MN} = \begin{pmatrix} G^E_{\mu\nu} & -2\,B_{\mu\rho}(G^{-1})^{\rho\nu} \\ 2(G^{-1})^{\mu\rho}\,B_{\rho\nu} & (G^{-1})^{\mu\nu} \end{pmatrix}. \tag{46}$$

The double current $J_{\pm M}$ is defined as

$$J_{\pm M} = \begin{pmatrix} 2(\Pi_\pm G^{-1})_\mu{}^\nu J_{\pm\nu} \\ -(G^{-1})^{\mu\nu}J_{\pm\nu} \end{pmatrix}, \tag{47}$$

and

$$\Omega^{MN} = \begin{pmatrix} 0 & 1_D \\ 1_D & 0 \end{pmatrix}, \tag{48}$$

is constant symmetric matrix. Here 1_D denotes the identity operator in D dimensions.

It is known that equations of motion of initial theory are Bianchi identities in T-dual picture and vice versa [9, 13, 16, 38]. From Bianchi identity

$$\partial_+\partial_- Z^M - \partial_-\partial_+ Z^M = 0\,, \tag{49}$$

and relation (45), we obtain the consistency condition

$$\partial_+\left[\mathcal{H}_{MN}\partial_- Z^N + J_{-M}\right] + \partial_-\left[\mathcal{H}_{MN}\partial_+ Z^N + J_{+M}\right] = 0\,. \tag{50}$$

The equation (50) is equation of motion of the following action

$$S = \frac{\kappa}{4}\int d^2\xi \left[\partial_+ Z^M \mathcal{H}_{MN}\partial_- Z^N + \partial_+ Z^M J_{-M} + J_{+M}\partial_- Z^M + L(\pi_\alpha, \bar{\pi}_\alpha)\right] , \tag{51}$$

where $L(\pi_\alpha, \bar{\pi}_\alpha)$ is arbitrary functional of fermionic momenta.

Let us split coordinate index μ into a and i ($a = 0, \cdots, d-1$, $i = d, \cdots, D-1$) and denote T-dualization along direction x^a and y_a as

$$\mathcal{T}^a = T^a \circ T_a\,, \quad T^a \equiv T^0 \circ T^1 \circ \cdots \circ T^{d-1}\,, \quad T_a \equiv T_0 \circ T_1 \circ \cdots \circ T_{d-1}\,, \tag{52}$$

where $\circ$ marks the operation of composition of T-dualizations. Permutation of the initial coordinates x^a with its T-dual y_a we realize by multiplying double space coordinate by the constant symmetric matrix $(\mathcal{T}^a)^M{}_N$

$${}_aZ^M \equiv \begin{pmatrix} y_a \\ x^i \\ x^a \\ y_i \end{pmatrix} = (\mathcal{T}^a)^M{}_N Z^N \equiv \begin{pmatrix} 0 & 0 & 1_a & 0 \\ 0 & 1_i & 0 & 0 \\ 1_a & 0 & 0 & 0 \\ 0 & 0 & 0 & 1_i \end{pmatrix} \begin{pmatrix} x^a \\ x^i \\ y_a \\ y_i \end{pmatrix} , \tag{53}$$

where 1_a and 1_i are identity operators in the subspaces spanned by x^a and x^i, respectively. We demand that double T-dual coordinate ${}_aZ^M$ satisfy the T-duality transformations of the form as initial one Z^M (45)

$$\partial_\pm\, {}_aZ^M \cong \pm\, \Omega^{MN} \left({}_a\mathcal{H}_{NK}\, \partial_\pm\, {}_aZ^K + {}_aJ_{\pm N}\right) . \tag{54}$$

From this relation we find the T-dual generalized metric

$${}_a\mathcal{H}_{MN} = (\mathcal{T}^a)_M{}^K \mathcal{H}_{KL} (\mathcal{T}^a)^L{}_N\,, \tag{55}$$

and T-dual current

$${}_aJ_{\pm M} = (\mathcal{T}^a)_M{}^N J_{\pm N}\,. \tag{56}$$

Demanding that the T-dual generalized metric ${}_a\mathcal{H}_{MN}$ has the same form as the initial one $\mathcal{H}_{MN}$ (46)

$${}_a\mathcal{H}_{MN} = \begin{pmatrix} {}_aG_E^{\mu\nu} & -2({}_aB\, {}_aG^{-1})^\mu{}_\nu \\ 2({}_aG^{-1}\, {}_aB)_\mu{}^\nu & ({}_aG^{-1})_{\mu\nu} \end{pmatrix} , \tag{57}$$

we obtain the T-dual NS-NS background fields

$${}_a\Pi_\pm^{ab} = \tfrac{\kappa}{2}\hat{\theta}_\mp^{ab}\,, \qquad {}_a\Pi^a_{\pm}{}_i = \kappa\hat{\theta}_\mp^{ab}\Pi_{\pm bi}\,, \tag{58}$$

$${}_a\Pi_{\pm i}{}^a = -\kappa\Pi_{\pm ib}\hat{\theta}_\mp^{ba}\,, \quad {}_a\Pi_{\pm ij} = \Pi_{\pm ij} - 2\kappa\Pi_{\pm ia}\hat{\theta}_\mp^{ab}\Pi_{\pm bj}\,, \tag{59}$$

which are in full agreement with those from the Refs.[6, 14].

The T-dual current ${}_aJ_{\pm M}$ (56) should have the same form as initial one (47) but in terms of the T-dual background fields

$$\begin{pmatrix} 2({}_a\Pi_{\pm}\,{}_aG^{-1})^a{}_b\,({}_aJ)^b_{\pm} + 2({}_a\Pi_{\pm}\,{}_aG^{-1})^{ai}\,({}_aJ)_{\pm i} \\ 2({}_a\Pi_{\pm}\,{}_aG^{-1})_{ia}\,({}_aJ)^a_{\pm} + 2({}_a\Pi_{\pm}\,{}_aG^{-1})_i{}^j\,({}_aJ)_{\pm j} \\ -({}_aG^{-1})_{ab}\,({}_aJ)^b_{\pm} - ({}_aG^{-1})_a{}^i\,({}_aJ)_{\pm i} \\ -({}_aG^{-1})^i{}_a\,({}_aJ)^a_{\pm} - ({}_aG^{-1})^{ij}\,({}_aJ)_{\pm j} \end{pmatrix} = \begin{pmatrix} -(G^{-1})^{a\mu}J_{\pm\mu} \\ 2(\Pi_{\pm}G^{-1})_i{}^{\mu}J_{\pm\mu} \\ 2(\Pi_{\pm}G^{-1})_a{}^{\mu}J_{\pm\mu} \\ -(G^{-1})^{i\mu}J_{\pm\mu} \end{pmatrix}. \tag{60}$$

From the lower D components of the above equation, after straightforward calculation we get

$${}_a\Psi^{\alpha a} = \kappa\hat{\theta}^{ab}_{+}\Psi^{\alpha}_b\,, \quad {}_a\bar{\Psi}^{\alpha a} = \kappa\,{}_a\Omega^{\alpha}{}_{\beta}\hat{\theta}^{ab}_{-}\bar{\Psi}^{\beta}_b\,. \tag{61}$$

$${}_a\Psi^{\alpha}_i = \Psi^{\alpha}_i - 2\kappa\Pi_{-ib}\hat{\theta}^{ba}_{+}\Psi^{\alpha}_a\,, \quad {}_a\bar{\Psi}^{\alpha}_i = {}_a\Omega^{\alpha}{}_{\beta}(\bar{\Psi}^{\beta}_i - 2\kappa\Pi_{+ib}\hat{\theta}^{ba}_{-}\bar{\Psi}^{\beta}_a)\,. \tag{62}$$

which is in full agreement with results obtained applying standard Buscher procedure. The upper D components of Eq.(60) produce the same result for T-dual background fields.

The R-R field strength $F^{\alpha\beta}$ appears in the action (10) coupled with fermionic momenta π_α and $\bar{\pi}_\alpha$ along which we do not perform T-dualization. Let us suppose that fermionic term $L(\pi_\alpha, \bar{\pi}_\alpha)$ (51) in the form

$$L = e^{\frac{\Phi}{2}}\,\pi_\alpha\,F^{\alpha\beta}\bar{\pi}_\beta + e^{\frac{{}_a\Phi}{2}}\,{}_a\pi_\alpha\,{}_aF^{\alpha\beta}\,{}_a\bar{\pi}_\beta \equiv \mathcal{L} + {}_a\mathcal{L}\,, \tag{63}$$

for some $F^{\alpha\beta}$ and ${}_aF^{\alpha\beta}$. This term should be invariant under T-dual transformation

$${}_a\mathcal{L} = \mathcal{L} + \Delta\mathcal{L}\,. \tag{64}$$

Taking into account the fact that two successive T-dualization are identity transformation, we obtain that the T-dual R-R field strength has the form

$$e^{\frac{{}_a\Phi}{2}}\,{}_aF^{\alpha\beta} = (e^{\frac{\Phi}{2}}F^{\alpha\gamma} + c\Psi^{\alpha}_a\hat{\theta}^{ab}_{-}\bar{\Psi}^{\gamma}_b)\,{}_a\Omega_{\gamma}{}^{\beta}\,. \tag{65}$$

For $c = 4\kappa$ we obtain the agreement with the expression (37).

4. Fermionic T-dualization

4.1. Action

We start with the action (1). In order to perform fermionic T-duality we must avoid explicit dependence of background fields on the fermionic coordinates θ^α and $\bar{\theta}^\alpha$ and allow only dependence on the σ and τ derivatives of these coordinates. This assumption produces that the auxiliary superfields are zero what can be seen from Eq.(5.5) of Ref.[32].

We choose that $G_{\mu\nu}$, $B_{\mu\nu}$, Φ, Ψ^α_μ and $\bar{\Psi}^\alpha_\mu$ are constant, and corresponding field strengths, $\Omega_{\mu,\nu\rho}(\Omega_{\mu\nu,\rho})$, $C^\alpha{}_{\mu\nu}(\bar{C}_{\mu\nu}{}^\alpha)$ and $S_{\mu\nu,\rho\sigma}$, are zero. The only nontrivial contribution of the quadratic terms in equations of motion (see

[36]) comes from constant field strength $\mathrm{P}^{\alpha\beta}$. In order to analyze this issue we will use relations from Eq.(3.6) of Ref.[32] labeled by $(\frac{1}{2},\frac{3}{2},\frac{3}{2})$

$$D_\alpha \mathrm{P}^{\beta\gamma} - \frac{1}{4}(\Gamma^{\mu\nu})_\alpha{}^\beta \bar{C}_{\mu\nu}{}^\gamma = 0\,, \quad \bar{D}_\alpha \mathrm{P}^{\beta\gamma} - \frac{1}{4}(\Gamma^{\mu\nu})_\alpha{}^\gamma C^\beta{}_{\mu\nu} = 0\,. \tag{66}$$

Here

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + \frac{1}{2}(\Gamma^\mu\theta)_\alpha\frac{\partial}{\partial x^\mu}\,, \quad \bar{D}_\alpha = \frac{\partial}{\partial\bar{\theta}^\alpha} + \frac{1}{2}(\Gamma^\mu\bar{\theta})_\alpha\frac{\partial}{\partial x^\mu}\,, \tag{67}$$

are superspace covariant derivatives and $C^\alpha{}_{\mu\nu}$ and $\bar{C}_{\mu\nu}{}^\alpha$ are field strengths for gravitino fields Ψ^α_μ and $\bar{\Psi}^\alpha_\mu$, respectively. In order to perform fermionic T-dualization along all fermionic directions, θ^α and $\bar{\theta}^\alpha$, we assume that they are Killing spinors which means

$$\frac{\partial \mathrm{P}^{\beta\gamma}}{\partial\theta^\alpha} = \frac{\partial \mathrm{P}^{\beta\gamma}}{\partial\bar{\theta}^\alpha} = 0\,. \tag{68}$$

From the equations (66) it follows

$$(\Gamma^\mu)_{\alpha\delta}\partial_\mu \mathrm{P}^{\beta\gamma} = 0\,. \tag{69}$$

Our choice of constant $\mathrm{P}^{\alpha\beta}$ is consistent with this condition.

Under these assumptions the final form of the action is given by the expression (10). The fermionic part of the action (10) has the form of the first order theory. On the equations of motion for fermionic momenta π_α and $\bar{\pi}_\alpha$,

$$\pi_\alpha = -\frac{\kappa}{2}\partial_+\left(\bar{\theta}^\beta + \bar{\Psi}^\beta_\mu x^\mu\right)(P^{-1})_{\beta\alpha}\,, \quad \bar{\pi}_\alpha = \frac{\kappa}{2}(P^{-1})_{\alpha\beta}\partial_-\left(\theta^\beta + \Psi^\beta_\mu x^\mu\right)\,, \tag{70}$$

the action gets the form

$$\begin{aligned} & S(\partial_\pm x, \partial_-\theta, \partial_+\bar{\theta}) = \kappa\int_\Sigma d^2\xi\partial_+x^\mu\Pi_{+\mu\nu}\partial_-x^\nu + \frac{1}{4\pi}\int_\Sigma d^2\xi\Phi R^{(2)} \\ + \quad & \frac{\kappa}{2}\int_\Sigma d^2\xi\partial_+\left(\bar{\theta}^\alpha + \bar{\Psi}^\alpha_\mu x^\mu\right)(P^{-1})_{\alpha\beta}\partial_-\left(\theta^\beta + \Psi^\beta_\nu x^\nu\right) \\ = \quad & \kappa\int_\Sigma d^2\xi\partial_+x^\mu\left[\Pi_{+\mu\nu} + \frac{1}{2}\bar{\Psi}^\alpha_\mu(P^{-1})_{\alpha\beta}\Psi^\beta_\nu\right]\partial_-x^\nu + \frac{1}{4\pi}\int_\Sigma d^2\xi\Phi R^{(2)} \\ + \quad & \frac{\kappa}{2}\int_\Sigma d^2\xi\left[\partial_+\bar{\theta}^\alpha(P^{-1})_{\alpha\beta}\partial_-\theta^\beta + \partial_+\bar{\theta}^\alpha(P^{-1}\Psi)_{\alpha\mu}\partial_-x^\nu\right. \\ + \quad & \left.\partial_+x^\mu(\bar{\Psi}P^{-1})_{\mu\alpha}\partial_-\theta^\alpha\right]\,. \end{aligned} \tag{71}$$

We notice that theory has a local symmetry

$$\delta\theta^\alpha = \varepsilon^\alpha(\sigma^+)\,, \quad \delta\bar{\theta}^\alpha = \bar{\varepsilon}^\alpha(\sigma^-)\,, \quad (\sigma^\pm = \tau \pm \sigma)\,. \tag{72}$$

The corresponding BRST transformations are

$$s\theta^\alpha = c^\alpha(\sigma^+)\,, \quad s\bar{\theta}^\alpha = \bar{c}^\alpha(\sigma^-)\,, \tag{73}$$

where for each gauge parameter $\varepsilon^\alpha(\sigma^+)$ and $\bar{\varepsilon}^\alpha(\sigma^-)$ we introduced the ghost fields $c^\alpha(\sigma^+)$ and $\bar{c}^\alpha(\sigma^-)$, respectively. Here s denotes BRST nilpotent operator.

To fix gauge freedom we introduce gauge fermion with ghost number -1

$$\Psi = \frac{\kappa}{2}\int d^2\xi \left[\bar{C}_\alpha\left(\partial_+\theta^\alpha + \frac{\alpha^{\alpha\beta}}{2}b_{+\beta}\right) + \left(\partial_-\bar{\theta}^\alpha + \frac{1}{2}\bar{b}_{-\beta}\alpha^{\beta\alpha}\right)C_\alpha\right]\,, \tag{74}$$

where $\alpha^{\alpha\beta}$ is arbitrary non singular matrix, $\bar{C}_\alpha$ and C_α are antighost fields, while $b_{+\alpha}$ and $\bar{b}_{-\alpha}$ are Nakanishi-Lautrup auxillary fields which satisfy

$$sC_\alpha = b_{+\alpha}\,, \quad s\bar{C}_\alpha = \bar{b}_{-\alpha}\,, \quad sb_{+\alpha} = 0 \quad s\bar{b}_{-\alpha} = 0\,. \tag{75}$$

BRST transformation of gauge fermion Ψ produces the gauge fixed and Fadeev-Popov action

$$\begin{aligned} s\Psi &= S_{gf} + S_{FP}\,,\\ S_{gf} &= \frac{\kappa}{2}\int d^2\xi\left[\bar{b}_{-\alpha}\partial_+\theta^\alpha + \partial_-\bar{\theta}^\alpha b_{+\alpha} + \bar{b}_{-\alpha}\alpha^{\alpha\beta}b_{+\beta}\right]\,,\\ S_{FD} &= \frac{\kappa}{2}\int d^2\xi\left[\bar{C}_\alpha\partial_+c^\alpha + (\partial_-\bar{c}^\alpha)C_\alpha\right]\,. \end{aligned} \tag{76}$$

The Fadeev-Popov action is decoupled from the rest and, consequently, it can be omitted in further analysis. On the equations of motion for b-fields

$$b_{+\alpha} = -(\alpha^{-1})_{\alpha\beta}\partial_+\theta^\alpha\,, \quad \bar{b}_{-\alpha} = -\partial_-\bar{\theta}^\beta(\alpha^{-1})_{\beta\alpha}\,, \tag{77}$$

we obtain the final form of the BRST gauge fixed action

$$S_{gf} = -\frac{\kappa}{2}\int d^2\xi\partial_-\bar{\theta}^\alpha(\alpha^{-1})_{\alpha\beta}\partial_+\theta^\beta\,. \tag{78}$$

4.2. Fermionic T-duality using Buscher rules

As in the bosonic case we introduce gauge fields $v_\pm^\alpha$ and $\bar{v}_\pm^\alpha$ and replace ordinary world-sheet derivatives with covariant ones

$$\partial_\pm\theta^\alpha \to D_\pm\theta^\alpha \equiv \partial_\pm\theta^\alpha + v_\pm^\alpha\,, \quad \partial_\pm\bar{\theta}^\alpha \to D_\pm\bar{\theta}^\alpha \equiv \partial_\pm\bar{\theta}^\alpha + \bar{v}_\pm^\alpha\,. \tag{79}$$

In order to make the fields $v_\pm^\alpha$ and $\bar v_\pm^\alpha$ to be unphysical we add the following terms in the action

$$S_{gauge}(\vartheta, v_\pm, \bar\vartheta, \bar v_\pm) = \frac{1}{2}\kappa\int_\Sigma d^2\xi \bar\vartheta_\alpha(\partial_+ v_-^\alpha - \partial_- v_+^\alpha) + \frac{1}{2}\kappa\int_\Sigma d^2\xi(\partial_+\bar v_-^\alpha - \partial_-\bar v_+^\alpha)\vartheta_\alpha\,, \tag{80}$$

where ϑ_α and $\bar\vartheta_\alpha$ are Lagrange multipliers. The full gauge invariant action is of the form

$$\begin{aligned} S_{inv}(x,\theta,\bar\theta,\vartheta,\bar\vartheta,v_\pm,\bar v_\pm) &= S(\partial_\pm x, D_-\theta, D_+\bar\theta) \\ + \quad S_{gf}(D_-\theta, D_+\bar\theta) &+ S_{gauge}(\vartheta,\bar\vartheta,v_\pm,\bar v_\pm)\,. \end{aligned} \tag{81}$$

Fixing θ^α and $\bar\theta^\alpha$ to zero we obtain the gauge fixed action

$$\begin{aligned} S_{fix} &= \kappa\int_\Sigma d^2\xi\partial_+ x^\mu\left[\Pi_{+\mu\nu} + \frac{1}{2}\bar\Psi_\mu^\alpha(P^{-1})_{\alpha\beta}\Psi_\nu^\beta\right]\partial_- x^\nu + \frac{1}{4\pi}\int_\Sigma d^2\xi\Phi R^{(2)} \\ &+\frac{\kappa}{2}\int_\Sigma\Big[\bar v_+^\alpha(P^{-1})_{\alpha\beta}v_-^\beta + \bar v_+^\alpha(P^{-1})_{\alpha\beta}\Psi_\nu^\beta\partial_- x^\nu \\ +&\quad \partial_+ x^\mu\bar\Psi_\mu^\alpha(P^{-1})_{\alpha\beta}v_-^\beta - \bar v_-^\alpha(\alpha^{-1})_{\alpha\beta}v_+^\beta\Big] \\ &+\frac{\kappa}{2}\int_\Sigma d^2\xi\bar\vartheta_\alpha(\partial_+ v_-^\alpha - \partial_- v_+^\alpha) + \frac{\kappa}{2}\int_\Sigma d^2\xi(\partial_+\bar v_-^\alpha - \partial_-\bar v_+^\alpha)\vartheta_\alpha\,. \end{aligned} \tag{82}$$

Varying the above action with respect to the Lagrange multipliers we obtain the initial action (71) because

$$\partial_+ v_-^\alpha - \partial_- v_+^\alpha = 0 \Longrightarrow v_\pm^\alpha = \partial_\pm\theta^\alpha\,, \quad \partial_+\bar v_-^\alpha - \partial_-\bar v_+^\alpha = 0 \Longrightarrow \bar v_\pm^\alpha = \partial_\pm\bar\theta^\alpha\,. \tag{83}$$

On the other side, the equations of motion for $v_\pm^\alpha$ and $\bar v_\pm^\alpha$ give

$$\bar v_-^\alpha = \partial_-\bar\vartheta_\beta\alpha^{\beta\alpha}\,, \quad \bar v_+^\alpha = \partial_+\bar\vartheta_\beta P^{\beta\alpha} - \partial_+ x^\mu\bar\Psi_\mu^\alpha\,, \tag{84}$$

$$v_+^\alpha = -\alpha^{\alpha\beta}\partial_+\vartheta_\beta\,, \quad v_-^\alpha = -P^{\alpha\beta}\partial_-\vartheta_\beta - \Psi_\mu^\alpha\partial_- x^\mu\,. \tag{85}$$

Substituting these expressions in the action S_{fix} we obtain the fermionic T-dual action

$$\begin{aligned} {}^\star S(\partial_\pm x, \partial_-\vartheta, \partial_+\bar\vartheta) &= \kappa\int_\Sigma d^2\xi\partial_+ x^\mu\Pi_{+\mu\nu}\partial_- x^\nu + \frac{1}{4\pi}\int_\Sigma d^2\xi\ {}^\star\Phi R^{(2)}\,, \\ &+\frac{\kappa}{2}\int_\Sigma d^2\xi\left[\partial_+\bar\vartheta_\alpha P^{\alpha\beta}\partial_-\vartheta_\beta\right. \\ -&\quad \left.\partial_+ x^\mu\bar\Psi_\mu^\alpha\partial_-\vartheta_\alpha + \partial_+\bar\vartheta_\alpha\Psi_\mu^\alpha\partial_- x^\mu - \partial_-\bar\vartheta_\alpha\alpha^{\alpha\beta}\partial_+\vartheta_\beta\right]\,. \end{aligned} \tag{86}$$

We read fermionic T-dual background fields

$${}^\star\Psi_{\alpha\mu} = (P^{-1}\Psi)_{\alpha\mu}\,,\ {}^\star\bar\Psi_{\mu\alpha} = -(\bar\Psi P^{-1})_{\mu\alpha}\,, \tag{87}$$

$$^\star P_{\alpha\beta} = (P^{-1})_{\alpha\beta}\,, \quad ^\star\alpha_{\alpha\beta} = (\alpha^{-1})_{\alpha\beta}\,. \tag{88}$$

From the condition

$$^\star\Pi_{+\mu\nu} + \frac{1}{2}{}^\star\bar{\Psi}_{\alpha\mu}\,({}^\star P^{-1})^{\alpha\beta}\,{}^\star\Psi_{\beta\nu} = \Pi_{+\mu\nu}\,, \tag{89}$$

the fermionic T-dual metric and Kalb-Ramond field are

$$\begin{aligned} ^\star G_{\mu\nu} &= G_{\mu\nu} + \frac{1}{2}\left[(\bar{\Psi}P^{-1}\Psi)_{\mu\nu} + (\bar{\Psi}P^{-1}\Psi)_{\nu\mu}\right]\,, \\ ^\star B_{\mu\nu} &= B_{\mu\nu} + \frac{1}{4}\left[(\bar{\Psi}P^{-1}\Psi)_{\mu\nu} - (\bar{\Psi}P^{-1}\Psi)_{\nu\mu}\right]\,. \end{aligned} \tag{90}$$

We obtain T-dual transformation laws combining the different solutions of equations of motion for $v^\alpha_\pm$ and $\bar{v}^\alpha_\pm$ (83) and (84)-(85)

$$\partial_-\theta^\alpha \cong -P^{\alpha\beta}\partial_-\vartheta_\beta - \Psi^\alpha_\mu\partial_- x^\mu\,, \quad \partial_+\bar{\theta}^\alpha \cong \partial_+\bar{\vartheta}_\beta P^{\beta\alpha} - \partial_+ x^\mu\bar{\Psi}^\alpha_\mu\,, \tag{91}$$

$$\partial_+\theta^\alpha \cong -\alpha^{\alpha\beta}\partial_+\vartheta_\beta\,, \quad \partial_-\bar{\theta}^\alpha \cong \partial_-\bar{\vartheta}_\beta\alpha^{\beta\alpha}\,. \tag{92}$$

From these relations we can obtain inverse transformation rules

$$\begin{aligned} \partial_-\vartheta_\alpha &\cong -(P^{-1})_{\alpha\beta}\partial_-\theta^\beta - (P^{-1})_{\alpha\beta}\Psi^\beta_\mu\partial_- x^\mu\,, \\ \partial_+\bar{\vartheta}_\alpha &\cong \partial_+\bar{\theta}^\beta(P^{-1})_{\beta\alpha} + \partial_+ x^\mu\bar{\Psi}^\beta_\mu(P^{-1})_{\beta\alpha}\,, \end{aligned} \tag{93}$$

$$\partial_+\vartheta_\alpha \cong -(\alpha^{-1})_{\alpha\beta}\partial_+\theta^\beta\,, \quad \partial_-\bar{\vartheta}_\alpha \cong \partial_-\bar{\theta}^\beta(\alpha^{-1})_{\beta\alpha}\,. \tag{94}$$

4.3. Fermionic T-dualization using double space

Now we will extend the meaning of the double space and double both fermionic coordinate as

$$\Theta^A = \begin{pmatrix} \theta^\alpha \\ \vartheta_\alpha \end{pmatrix}\,, \quad \bar{\Theta}^A = \begin{pmatrix} \bar{\theta}^\alpha \\ \bar{\vartheta}_\alpha \end{pmatrix}\,. \tag{95}$$

The transformation laws, (91)-(94), can be rewritten in the form

$$\partial_-\Theta^A \cong -\Omega^{AB}\left[\mathcal{F}_{BC}\partial_-\Theta^C + \mathcal{J}_{-B}\right]\,, \quad \partial_+\bar{\Theta}^A \cong \left[\partial_+\bar{\Theta}^C\mathcal{F}_{CB} + \bar{\mathcal{J}}_{+B}\right]\Omega^{BA}\,, \tag{96}$$

$$\partial_+\Theta^A \cong -\Omega^{AB}\mathcal{A}_{BC}\partial_+\Theta^C\,, \quad \partial_-\bar{\Theta}^A \cong \partial_-\bar{\Theta}^C\mathcal{A}_{CB}\Omega^{BA}\,, \tag{97}$$

where "fermionic generalized metric" $\mathcal{F}_{AB}$ has the form

$$\mathcal{F}_{AB} = \begin{pmatrix} (P^{-1})_{\alpha\beta} & 0 \\ 0 & P^{\gamma\delta} \end{pmatrix}\,, \tag{98}$$

and

$$\mathcal{A}_{AB} = \begin{pmatrix} (\alpha^{-1})_{\alpha\beta} & 0 \\ 0 & \alpha^{\gamma\delta} \end{pmatrix} . \tag{99}$$

The double currents, $\bar{\mathcal{J}}_{+A}$ and $\mathcal{J}_{-A}$, are of the form

$$\bar{\mathcal{J}}_{+A} = \begin{pmatrix} (\bar\Psi P^{-1})_{\mu\alpha}\partial_+ x^\mu \\ -\bar\Psi^\alpha_\mu \partial_+ x^\mu \end{pmatrix} , \quad \mathcal{J}_{-A} = \begin{pmatrix} (P^{-1}\Psi)_{\alpha\mu}\partial_- x^\mu \\ \Psi^\alpha_\mu \partial_- x^\mu \end{pmatrix} . \tag{100}$$

Let us introduce the permutation matrix

$$\mathcal{T}^A{}_B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} , \tag{101}$$

so that double T-dual coordinates are

$${}^\star\Theta^A = \mathcal{T}^A{}_B \Theta^B , \quad {}^\star\bar\Theta^A = \mathcal{T}^A{}_B \bar\Theta^B . \tag{102}$$

As in the case of bosonic T-dualization, from demand that T-dual transformation laws for T-dual coordinates ${}^\star\Theta^A$ and ${}^\star\bar\Theta^A$ have the same form as for initial ones Θ^A and $\bar\Theta^A$ we get the fermionic T-dual "generalized metric" ${}^\star\mathcal{F}_{AB}$ and T-dual currents, ${}^\star\bar{\mathcal{J}}_{+A}$ and ${}^\star\mathcal{J}_{-A}$

$${}^\star\mathcal{F}_{AB} = \mathcal{T}_A{}^C \mathcal{F}_{CD} \mathcal{T}^D{}_B , \quad {}^\star\bar{\mathcal{J}}_{+A} = \mathcal{T}_A{}^B \bar{\mathcal{J}}_{+B} , \quad {}^\star\mathcal{J}_{-A} = \mathcal{T}_A{}^B \mathcal{J}_{-B} . \tag{103}$$

The matrix $\mathcal{A}_{AB}$ transforms as

$${}^\star\mathcal{A}_{AB} = \mathcal{T}_A{}^C \mathcal{A}_{CD} \mathcal{T}^D{}_B = (\mathcal{A}^{-1})_{AB} . \tag{104}$$

From the first relation in (103) we obtain the form of the fermionic T-dual R-R background field

$${}^\star P_{\alpha\beta} = (P^{-1})_{\alpha\beta} , \tag{105}$$

while from the second and third equation we obtain the form of the fermionic T-dual NS-R background fields

$${}^\star\Psi_{\alpha\mu} = (P^{-1})_{\alpha\beta}\Psi^\beta_\mu , \quad {}^\star\bar\Psi_{\alpha\mu} = -\bar\Psi^\beta_\mu (P^{-1})_{\beta\alpha} . \tag{106}$$

The non singular matrix $\alpha^{\alpha\beta}$ transforms as

$$({}^\star\alpha)_{\alpha\beta} = (\alpha^{-1})_{\alpha\beta} . \tag{107}$$

As in the case of bosonic T-dualization, in the same way how we obtained the double action (51), we get the double action corresponding to the fermionic T-dual transformation law

$$\begin{aligned} & S_{double}(\Theta, \bar\Theta) = \qquad\qquad (108) \\ = \ & \frac{\kappa}{2} \int d^2\xi \Big[\partial_+ \bar\Theta^A \mathcal{F}_{AB} \partial_- \Theta^B + \bar{\mathcal{J}}_{+A} \partial_- \Theta^A + \partial_+ \bar\Theta^A \mathcal{J}_{-A} \\ - \ & \partial_- \bar\Theta^A \mathcal{A}_{AB} \partial_+ \Theta^B + L(x) \Big] , \end{aligned}$$

where $L(x)$ is arbitrary functional of the bosonic coordinates. In order to find fermionic T-dual metric and Kalb-Ramond field we suppose that term $L(x)$ has the form

$$L(x) = 2\partial_+ x^\mu \left(\Pi_{+\mu\nu} + {}^\star\Pi_{+\mu\nu}\right) \partial_- x^\nu \equiv \mathcal{L} + {}^\star\mathcal{L}\,. \tag{109}$$

This term should be invariant under T-dual transformation

$${}^\star\mathcal{L} = \mathcal{L} + \Delta\mathcal{L}\,. \tag{110}$$

Using the fact that two successive T-dualization are identity transformation, we obtain

$$\mathcal{L} = {}^\star\mathcal{L} + {}^\star\Delta\mathcal{L}\,. \tag{111}$$

Combining last two relations we get

$${}^\star\Delta\mathcal{L} = -\Delta\mathcal{L}\,. \tag{112}$$

If $\Delta\mathcal{L} = 2\partial_+ x^\mu \Delta_{\mu\nu} \partial_- x^\nu$, we obtain the condition for $\Delta_{\mu\nu}$

$${}^\star\Delta_{\mu\nu} = -\,\Delta_{\mu\nu}\,. \tag{113}$$

Using the relations (87) and (88) we realize that, up to multiplication constant, combination

$$\Delta_{\mu\nu} = \bar\Psi_\mu^\alpha (P^{-1})_{\alpha\beta} \Psi_\nu^\beta\,, \tag{114}$$

satisfies the condition (113). So, we conclude that

$${}^\star\Pi_{+\mu\nu} = \Pi_{+\mu\nu} + c\bar\Psi_\mu^\alpha (P^{-1})_{\alpha\beta} \Psi_\nu^\beta\,, \tag{115}$$

where c is an arbitrary constant. For $c = \frac{1}{2}$ we obtain the relations (90).

5. Conclusions

In this article we showed that the new interpretation of T-dualization procedure in double space formalism offered in [26, 27] is also valid in the case of type II superstring theory - both for bosonic and fermionic T-dualization. We used the ghost free action of type II superstring theory in pure spinor formulation in the approximation of quadratic terms and constant background fields. One can obtain this action from action (1) under some set of assumptions.

We introduced the double space coordinate $Z^M = (x^\mu, y_\mu)$ adding to all bosonic initial coordinates, x^μ, the T-dual ones, y_μ. Then we rewrote the T-dual transformation laws (43) in terms of double space variables (45) introducing the generalized metric $\mathcal{H}_{MN}$ and the current $J_{\pm M}$. Further, we split initial coordinates x^μ in two parts: x^a are directions along which we made T-dulization and the rest ones x^i.

T-dualization is realized as permutation of the subsets x^a and y_a in the double space coordinate Z^M. Demanding that T-dual double space coordinates ${}_aZ^M = (\mathcal{T}^a)^M{}_N Z^N$ satisfy the T-dual transformation law of the same form as the initial coordinates Z^M we found the T-dual generalized metric ${}_a\mathcal{H}_{MN}$ and the T-dual current ${}_aJ_{\pm M}$. Consequently, we obtained the form of NS-NS and NS-R T-dual background fields in terms of the initial ones which are in full accordance with the results obtained by Buscher T-dualization procedure [6, 7].

In order to obtain T-dual R-R field strength $F^{\alpha\beta}$ we should make some additional assumptions. Supposing that term $L(\pi_\alpha, \bar{\pi}_\alpha)$ (51) is T-dual invariant and taking into account that two successive T-dualizations act as identity operator, we found the form of T-dual R-R field strength up to one arbitrary constant c. For $c = 4\kappa$ we get the T-dual R-R field strength ${}_aF^{\alpha\beta}$ as in Buscher procedure [6].

In the case of fermionic T-duality, using equations of motion with respect to the fermionic momenta π_α and $\bar{\pi}_\alpha$, we eliminated them from the action. Then we fixed local chiral gauge invariance using BRST approach.

Using the Buscher approach we performed fermionic T-duality procedure and obtained the form of the fermionic T-dual background fields. Analogously with the bosonic case we introduced double fermionic space doubling the initial coordinates θ^α and $\bar{\theta}^\alpha$ with their fermionic T-duals, ϑ_α and $\bar{\vartheta}_\alpha$. Double fermionic space is spanned by the coordinates $\Theta^A = (\theta^\alpha, \vartheta_\alpha)$ and $\bar{\Theta}^A = (\bar{\theta}^\alpha, \bar{\vartheta}_\alpha)$. Demanding that T-dual transformation laws for fermionic T-dual double coordinates, ${}^\star\Theta^A = \mathcal{T}^A{}_B \Theta^B$ and ${}^\star\bar{\Theta}^A = \mathcal{T}^A{}_B \bar{\Theta}^B$, are of the same form as those for Θ^A and $\bar{\Theta}^A$, we obtained the form of the fermionic T-dual NS-R and R-R background fields which are in full accordance with the results obtained by standard Buscher procedure. The expressions for T-dual metric ${}^\star G_{\mu\nu}$ and Kalb-Ramond field ${}^\star B_{\mu\nu}$ cannot be found from double space formalism because they do not appear in the T-dual transformation laws. These expressions, up to arbitrary constant, are obtained assuming that two successive T-dualization act as identity transformation.

References

[1] K. Becker, M. Becker and J. Schwarz *String Theory and M-Theory: A Modern Introduction*; B. Zwiebach, *A First Course in String Theory*, Cambridge University Press, 2004; J. Polchinski, *String theory - Volume II*, Cambridge University Press, 1998.

[2] T. Buscher, *Phys. Lett.* **B 194** (1987) 59; **201** (1988) 466.

[3] M. Roček and E. Verlinde, *Nucl.Phys.* **B 373** (1992) 630.

[4] A. Giveon, M. Porrati and E. Rabinovici, *Phys. Rep.* **244** (1994) 77.

[5] E. Alvarez, L. Alvarez-Gaume, J. Barbon and Y. Lozano, *Nucl. Phys.* **B 415** (1994) 71.

[6] B. Nikolić and B. Sazdović, *Nucl. Phys.* **B 836** (2010) 100.

[7] D. S. Berman, D. C. Thompson, *Phys.Rept.* **566** (2014) 1-60.

[8] D. Lust *JHEP* **12** (2010) 084.

[9] D. Andriot, M. Larfors, D. Luest, P. Patalong, *JHEP* **06** (2013) 021.

[10] D. Andriot, O. Hohm, M. Larfors, D. Lust, P. Patalong, *Phys. Rev. Lett.* 108 (2012) 261602.

[11] D. Luest, arxiv:1205.0100 [hep-th]; R. Blumenhagen, A. Deser, D. Luest, E. Plauschinn, and F. Rennecke, *J.Phys.* **A44** (2011) 385401; C. Condeescu, I. Florakis, and D. Luest, *JHEP* **04** (2012) 121.

[12] J. Shelton, W. Taylor, and B. Wecht, *JHEP* **10** (2005) 085; A. Dabholkar and C. Hull, *JHEP* **05** (2006) 009.

[13] Lj. Davidović and B. Sazdović, *EPJ* **C 74** (2014) 2683.

[14] Lj. Davidović, B. Nikolić and B. Sazdović, *EPJ* **C 75** (2015) 576.

[15] Lj. Davidović, B. Nikolić and B. Sazdović, *EPJ* **C 74** (2014) 2734.

[16] M. Duff, *Nucl. Phys.* **B 335** (1990) 610.

[17] A. A. Tseytlin, *Phys.Lett.* **B 242** (1990) 163.

[18] A. A. Tseytlin, *Nucl. Phys.* **B 350** (1991) 395.

[19] W. Siegel, *Phys.Rev.* **D 48** (1993) 2826.

[20] W. Siegel, *Phys.Rev.* **D 47** (1993) 5453.

[21] C. M. Hull, *JHEP* **10** (2005) 065.

[22] C. M. Hull, *JHEP* **10** (2007) 057; **07** (2007) 080.

[23] D. S. Berman, M. Cederwall and M. J. Perry, *JHEP* **09** (2014) 066; D. S. Berman, C. D. A. Blair, E. Malek and M. J. Perry, *Int.J.Mod.Phys.* **A29** (2014) 15, 1450080; C. D. A. Blair, E. Malek and A. J. Routh, *Class.Quant.Grav.* **31** (2014) 20, 205011.

[24] C.M. Hull and R.A. Reid-Edwards, *JHEP* **09** (2009) 014.

[25] O. Hohm and B. Zwiebach, *JHEP* **11** (2014) 075.

[26] B. Sazdović, *T-duality as coordinates permutation in double space*, arxiv: 1501.01024.

[27] B. Sazdović, *JHEP* **08** (2015) 055.

[28] N. Berkovits and J. Maldacena, *JHEP* **09** (2008) 062; I. Bakhmatov and D. S. Berman, *Nucl. Phys.* B**832** (2010) 89-108; K. Sfetsos, K. Siampos and D. C. Thompson, QMUL-PH-10-08, arXiv:1007.5142; I. Bakhmatov, *Fermionic T-duality and U-duality in type II supergravity*, arXiv:1112.1983.

[29] N. Beisert, R. Ricci, A. A. Tseytlin and M. Wolf, *Phys.Rev.* **D78** (2008) 126004; R. Ricci, A. A. Tseytlin and M. Wolf, *JHEP* **12** (2007) 082.

[30] N. Berkovits and J. Maldacena, *JHEP* **09** (2008) 062; K. Sfetsos, K. Siampos and D. C. Thompson, *Class.Quant.Grav.* **28** 2011, 055010; I. Bakhmatov and D. S. Berman, *Nucl. Phys.* B**832** (2010) 89-108; N. Beisert, R. Ricci, A. A. Tseytlin and M. Wolf, *Phys.Rev.* **D78** (2008) 126004; R. Ricci, A. A. Tseytlin and M. Wolf, *JHEP* **12** (2007) 082; B. Nikolić and B. Sazdović, *Phys. Rev.* **D84** (2011) 065012, *JHEP* **06** (2012) 101.

[31] M. Hatsuda, K. Kamimura, W. Siegel, *JHEP* **06** (2014) 039.

[32] P. A. Grassi, L. Tamassia, *JHEP* **07** (2004) 071.

[33] N. Berkovits, hep-th/0209059; N. Berkovits, arxiv:0902.4032; P. A. Grassi, G. Policastro and P. van Nieuwenhuizen, *JHEP* **10** (2002) 054; P. A. Grassi, G. Policastro and P. van Nieuwenhuizen, *JHEP* **11** (2002) 004; P. A. Grassi, G. Policastro and P. van Nieuwenhuizen, *Adv. Theor. Math. Phys.* **7** (2003) 499; P. A. Grassi, G. Policastro and P. van Nieuwenhuizen, *Phys. Lett.* **B553** (2003) 96.

[34] J. de Boer, P. A. Grassi and P. van Nieuwenhuizen, *Phys. Lett.* **B574** (2003) 98.

[35] N. Berkovits and P. Howe, *Nucl. Phys.* **B635** (2002) 75.

[36] M. J. Duff, arxiv:hep-th/9912164v2.

[37] S. F. Hassan, *Nucl. Phys.* **B 568** (2000) 145; B. Nikolić and B. Sazdović, *EPJ* **C 77** (2017) 197.

[38] A. Giveon and M. Roček, *Nucl. Phys.* **B 421** (1994) 173.

Journal of Physics: Conference Series

PAPER • OPEN ACCESS

Higher category theory and n-groups as gauge symmetries for quantum gravity

To cite this article: Bojan Nikolić *et al* 2023 *J. Phys.: Conf. Ser.* **2667** 012019

View the article online for updates and enhancements.

Higher category theory and n-groups as gauge symmetries for quantum gravity

Bojan Nikolić, Danijel Obrić, Tijana Radenković, Igor Salom and Marko Vojinović

Institute of Physics, University of Belgrade, Pregrevica 118, 11080 Belgrade, Serbia

E-mail: vmarko@ipb.ac.rs

Abstract. Higher category theory can be employed to generalize the notion of a gauge group to the notion of a gauge n-group. This novel algebraic structure is designed to generalize notions of connection, parallel transport and holonomy from curves to manifolds of dimension higher than one. Thus it generalizes the concept of gauge symmetry, giving rise to a topological action called nBF action, living on a corresponding n-principal bundle over a spacetime manifold. Similarly as for the Plebanski action, one can deform the topological nBF action by adding appropriate simplicity constraints, in order to describe the correct dynamics of both gravity and matter fields. Specifically, one can describe the whole Standard Model coupled to gravity as a constrained $3BF$ or $4BF$ action. The split of the full action into a topological sector and simplicity constraints sector is adapted to the spinfoam quantization technique, with the aim to construct a full model of quantum gravity with matter. In addition, the properties of the gauge n-group structure open up a possibility of a nontrivial unification of all fields. An n-group naturally contains additional novel gauge groups which specify the spectrum of matter fields present in the theory, in a similar way to the ordinary gauge group that prescribes the spectrum of gauge vector bosons in the Yang-Mills theory. The presence and the properties of these new gauge groups has the potential to explain fermion families, and other structure in the matter spectrum of the theory.

1. Introduction

The formulation of a quantum theory of gravity represents one of the fundamental open problems in modern theoretical physics. Among the many approaches to this problem, some have developed into vast research frameworks, such as Loop Quantum Gravity, which aims to formulate a model of quantum gravity (QG) in a nonperturbative fashion, both canonically and covariantly [1, 2, 3]. The covariant approach aims to give a tentative rigorous definition of the path integral for the gravitational field,

$$Z = \int \mathcal{D}g \; e^{iS[g]} \,. \tag{1}$$

One of the essential assumptions is a triangulation of a spacetime manifold, and the path integral is introduced as a discrete state sum of the gravitational field configurations, living on the simplicial complex structure. This approach to quantization of gravity is usually called the *spinfoam* quantization method. It is performed via the following three steps:

(1) one reformulates the classical action $S[g]$ as a constrained BF action, separating the topological BF part and the constraint part of the action;
(2) one employs the underlying Lie group structure of the BF sector of the action, in order to define a triangulation-independent state sum Z;
(3) finally, one deforms the topological state sum by applying the simplicity constraints, and therefore redefining it into a triangulation-dependent state sum, which plays the role of a definition for the path integral (1).

This type of quantization prescription has been implemented in a number of cases, for various choices of the gravitational action, of the Lie group, and of the spacetime dimension. Historically the first spinfoam model was the Ponzano-Regge model [4], defined in 3 spacetime dimensions. In 4 dimensions multiple models have been formulated, differing in the choice of the Lie group and the way one imposes the simplicity constraints [5, 6, 7, 8, 9]. While all these models do represent definitions of the gravitational path integral, none of them are able to include matter fields in a seamless way. Introducing the latter into a spinfoam QG model has so far had only limited success [10], predominantly due to the lack of the tetrad fields in the topological part of the model.

Recently, a new approach has been developed to address the issue of matter fields, which employs the framework of *higher gauge theory* (see [11] for a review). Specifically, one uses the notion of a *categorical ladder* to generalize the BF action (based on a Lie group) to a $2BF$ action (based on the so-called 2-group structure), and further to a $3BF$ action (based on a 3-group structure). A convenient choice of the *Poincaré 2-group* gives rise to the needed tetrad fields in the topological sector of the action [12], while an additional extension to the 3-group naturally introduces the matter fields (fermions and scalars) into the model [13]. The steps of the categorical ladder and their corresponding structures are summarized as follows:

categorical structure	algebraic structure	linear structure	topological action	degrees of freedom
Lie group	Lie group	Lie algebra	BF theory	gauge fields
Lie 2-group	Lie crossed module	differential Lie crossed module	$2BF$ theory	tetrad fields
Lie 3-group	Lie 2-crossed module	differential Lie 2-crossed module	$3BF$ theory	scalar and fermion fields

The main aim of this work is to provide a short review of the classical pure BF, $2BF$ and $3BF$ actions, in order to demonstrate the categorical ladder procedure and the construction of higher gauge theories. In other words, we mainly focus on the step 1 of the spinfoam quantization programme, with a very short review of step 2 of the programme.

The layout of the paper is as follows. Section 2 deals with first three examples of nBF theories, namely BF, $2BF$ and $3BF$ actions, and their construction using the categorical ladder. After this, in Section 3 we briefly present an application of a $3BF$ theory to the Standard Model of elementary particles coupled to Einstein-Cartan gravity. As it turns out, the scalar and fermion fields are *naturally associated to a new gauge group*, generalizing the role of an ordinary gauge group in the Yang-Mills theory. This opens up a possibility of an algebraic classification of matter fields, and (more speculatively) a possibility of the explanation of the three fermion families. Finally, Section 4 contains some discussion and our conclusions.

The notation and conventions are as follows. Spacetime indices are denoted by the Greek letters $\mu, \nu, \dots$, and are raised and lowered by the spacetime metric $g_{\mu\nu} = \eta_{ab} e^a{}_\mu e^b{}_\nu$, where $e^a{}_\mu$ are the tetrad fields. The inverse tetrad is denoted as $e^\mu{}_a$. The local Lorentz indices are denoted by the Latin letters $a, b, c, \dots$, take values $0, 1, 2, 3$, and are raised and lowered using the Minkowski metric η_{ab} with signature $(-, +, +, +)$. All other indices that appear in the paper depend on the context, and their use is explicitly defined in the text where they appear. We

Journal of Physics: Conference Series **2667** (2023) 012019 doi:10.1088/1742-6596/2667/1/012019

work in the natural system of units where $c = \hbar = 1$, and $G = l_p^2$, where l_p is the Planck length. The exterior product in the space of differential forms is denoted with the standard "wedge" symbol, $\wedge$.

2. nBF theories

We begin by giving a short review of nBF theories, for $n = 1, 2, 3$, which represent the most interesting cases for physics.

2.1. BF theory

A BF theory and its various applications in physics are already well known in the literature, see for example [14, 15, 16], so here we merely give a brief definition. Given a Lie group G, and its corresponding Lie algebra as $\mathfrak{g}$, one defines the BF action in the form (we discuss only the 4-dimensional spacetime manifolds $\mathcal{M}_4$):

$$S_{BF} = \int_{\mathcal{M}_4} \langle B \wedge \mathcal{F} \rangle_{\mathfrak{g}} \,. \tag{2}$$

Here, $\mathcal{F} \equiv \mathrm{d}\alpha + \alpha \wedge \alpha$ is the curvature 2-form for the $\mathfrak{g}$-valued connection 1-form $\alpha \in \Lambda^1(\mathcal{M}_4) \otimes \mathfrak{g}$, while $B \in \Lambda^2(\mathcal{M}_4) \otimes \mathfrak{g}$ is a $\mathfrak{g}$-valued Lagrange multiplier 2-form. Also, $\langle \_ , \_ \rangle_{\mathfrak{g}}$ denotes a G-invariant nondegenerate symmetric bilinear form over $\mathfrak{g}$.

Varying the action (2) with respect to B and α, one obtains the equations of motion:

$$\mathcal{F} = 0 \,, \qquad \nabla B \equiv \mathrm{d}B + \alpha \wedge B = 0 \,. \tag{3}$$

The first equation implies that α is a flat connection, in the sense that $\alpha = 0$ up to gauge transformations. The second equation then implies that B is covariantly constant. From these one can deduce that there are no local propagating degrees of freedom, and therefore the theory is said to be *topological*.

2.2. 2BF theory

Once we have introduced the BF model, we procceed to first step of the *categorical ladder*, generalizing the algebraic notion of a group to the notion of a 2-group. This leads to the generalization of the BF theory to the $2BF$ theory, also sometimes called $BFCG$ theory [11, 17, 18, 19].

The categorical ladder is a procedure of generalizing various notions in mathematics, using the framework of category theory, and works as follows. One starts from the notion of a group as an algebraic structure, and notes that it can be understood as a category with only one object and invertible morphisms [11]. Then, one employs the fundamental idea that a category can be generalized to the so-called *higher categories*, which have not only objects and morphisms, but also 2-morphisms (maps between morphisms), 3-morphisms (maps between 2-morphisms), and so on. This tower of n-categories is known as the *categorical ladder*. Applying the construction to groups, it is straightforward to introduce the notion of a *2-group* as a 2-category consisting of only one object, where all the morphisms and all 2-morphisms are invertible. It was demonstrated that every strict 2-group is equivalent to a *crossed module* $(H \stackrel{\partial}{\to} G \,, \triangleright)$, see [13] for detailed definitions. Here G and H are groups, ∂ is a homomorphism from H to G, while $\triangleright : G \times H \to H$ is an action of G on H.

Just like an ordinary Lie group G has a naturally associated connection α and gives rise to a BF theory, a Lie 2-group has a naturally associated 2-connection $(\alpha \,, \beta)$, described by the usual $\mathfrak{g}$-valued 1-form $\alpha \in \Lambda^1(\mathcal{M}_4) \otimes \mathfrak{g}$ and an $\mathfrak{h}$-valued 2-form $\beta \in \Lambda^2(\mathcal{M}_4) \otimes \mathfrak{h}$, where $\mathfrak{h}$ is a

Lie algebra of the Lie group H. This 2-connection gives rise to the so-called *fake* 2*-curvature* $(\mathcal{F},\mathcal{G})$, defined as

$$\mathcal{F} = \mathrm{d}\alpha + \alpha\wedge\alpha - \partial\beta\,, \qquad \mathcal{G} = d\beta + \alpha\wedge^{\triangleright}\beta\,. \tag{4}$$

Here $\alpha\wedge^{\triangleright}\beta$ means that α and β are multiplied as forms using $\wedge$, and simultaneously multiplied as algebra elements using $\triangleright$, see [13]. The curvature pair $(\mathcal{F},\mathcal{G})$ is called "fake" due of the presence of the additional term $\partial\beta$ in the definition of $\mathcal{F}$ [11].

Using the structure of a 2-group, or equivalently the crossed module, one can introduce the so-called $2BF$ action, as a generalization of the BF action, as follows [17, 18]:

$$S_{2BF} = \int_{\mathcal{M}_4} \langle B\wedge\mathcal{F}\rangle_{\mathfrak{g}} + \langle C\wedge\mathcal{G}\rangle_{\mathfrak{h}}\,. \tag{5}$$

Here the 2-form $B\in\Lambda^2(\mathcal{M}_4)\otimes\mathfrak{g}$ and the 1-form $C\in\Lambda^1(\mathcal{M}_4)\otimes\mathfrak{h}$ are Lagrange multipliers. Also, $\langle\_\,,\_\rangle_{\mathfrak{g}}$ and $\langle\_\,,\_\rangle_{\mathfrak{h}}$ denote the G-invariant nondegenerate symmetric bilinear forms over the Lie algebras $\mathfrak{g}$ and $\mathfrak{h}$, respectively. As a consequence of the axiomatic structure of a crossed module (see [13]), the bilinear form $\langle\_\,,\_\rangle_{\mathfrak{h}}$ is H-invariant as well. See [17, 18] for review and references.

The equations of motion for a $2BF$ theory are an extension of the equations of motion of a BF theory. Varying with respect to B and C one obtains

$$\mathcal{F}^{\alpha} = 0\,, \qquad \mathcal{G}^{a} = 0\,, \tag{6}$$

while varying with respect to α and β one obtains the equations for the multipliers,

$$\nabla B + C\wedge^{\mathcal{T}}\beta = 0\,, \qquad \nabla C - \partial B = 0\,. \tag{7}$$

Here the map $\mathcal{T}$ is defined in [13]. A rigorous Hamiltonian analysis of the model demonstrates that in this case as well there are no local propagating degrees of freedom [20, 21] (see also [22]). Therefore the $2BF$ theory is also topological.

2.3. $3BF$ *theory*

When constructing more realistic (nontopological) models by adding constraints to BF and $2BF$ models, it becomes apparent that the group G with a constrained BF action can successfully describe ordinary gauge vector bosons, while the so-called Poincaré 2-group with a constrained $2BF$ action can successfully describe general relativity. However, neither of these can suitably accomodate matter fields, such as fermions or scalars. Nevertheless, it turns out that this can be remedied if we make one further step in the categorical ladder, passing from the notion of a 2-group to the notion of a 3-group. As we shall see in the next Section, the notion of a 3-group will prove to be an excellent structure for the description of all fields that are present in the Standard Model, coupled to Einstein-Cartan gravity. Moreover, a 3-group contains one more gauge group, which is novel and specifies the spectrum of scalar and fermion fields present in the theory. This is an unexpected and beautiful result, absent from ordinary gauge theory.

Applying the categorical ladder once more, one can introduce the notion of a 3-group in the framework of higher category theory, as a 3-category with only one object where all the morphisms, 2-morphisms and 3-morphisms are invertible. Also, the equivalence between a 2-group and a crossed module has been generalized to the equivalence between a strict 3-group and a 2-*crossed module* [23]. A Lie 2-crossed module, denoted as $(L\overset{\delta}{\to}H\overset{\partial}{\to}G\,,\triangleright\,,\{\_\,,\_\})$, is an algebraic structure specified by three Lie groups G, H and L, together with the homomorphisms δ and ∂, an action $\triangleright$ of the group G on all three groups, and a G-equivariant map

$$\{\_\,,\_\} : H\times H\to L\,.$$

called the Peiffer lifting. The maps ∂, δ, $\triangleright$ and the Peiffer lifting satisfy certain axioms, so that the resulting structure is equivalent to a 3-group [13].

Based on a given 2-crossed module $(L \stackrel{\delta}{\to} H \stackrel{\partial}{\to} G\,, \triangleright\,, \{\_\,,\_\})$, one can introduce a gauge invariant topological $3BF$ action over the manifold $\mathcal{M}_4$ as follows. Denoting $\mathfrak{g}$, $\mathfrak{h}$ and $\mathfrak{l}$ as Lie algebras corresponding to the groups G, H and L, respectively, the Lie 3-group structure allows one to introduce a 3-connection (α,β,γ) given by the algebra-valued differential forms $\alpha \in \Lambda^1(\mathcal{M}_4)\otimes\mathfrak{g}$, $\beta \in \Lambda^2(\mathcal{M}_4)\otimes\mathfrak{h}$ and $\gamma \in \Lambda^3(\mathcal{M}_4)\otimes\mathfrak{l}$. The corresponding fake 3-curvature $(\mathcal{F}\,,\mathcal{G}\,,\mathcal{H})$ is then defined as

$$\begin{gathered} \mathcal{F} = \mathrm{d}\alpha + \alpha\wedge\alpha - \partial\beta\,, \qquad \mathcal{G} = \mathrm{d}\beta + \alpha\wedge^{\triangleright}\beta - \delta\gamma\,, \\ \mathcal{H} = \mathrm{d}\gamma + \alpha\wedge^{\triangleright}\gamma + \{\beta\wedge\beta\}\,, \end{gathered} \tag{8}$$

see [23, 24] for details. Note that γ is a 3-form, while its corresponding field strength $\mathcal{H}$ is a 4-form, requiring that the spacetime manifold be at least 4-dimensional. Also, for this reason, going beyond 3-groups and 4-groups in the categorical ladder does not have many applications in realistic 4-dimensional physics. A $3BF$ action is defined as

$$S_{3BF} = \int_{\mathcal{M}_4} \langle B\wedge\mathcal{F}\rangle_{\mathfrak{g}} + \langle C\wedge\mathcal{G}\rangle_{\mathfrak{h}} + \langle D\wedge\mathcal{H}\rangle_{\mathfrak{l}}\,, \tag{9}$$

where $B\in\Lambda^2(\mathcal{M}_4)\otimes\mathfrak{g}$, $C\in\Lambda^1(\mathcal{M}_4)\otimes\mathfrak{h}$ and $D\in\Lambda^0(\mathcal{M}_4)\otimes\mathfrak{l}$ are Lagrange multipliers valued in the respective algebras. Note that exclusively in 4 spacetime dimensions the Lagrange multiplier D corresponding to $\mathcal{H}$ is a 0-form, i.e. a scalar function. As before, the bilinear forms $\langle\_\,,\_\rangle_{\mathfrak{g}}$, $\langle\_\,,\_\rangle_{\mathfrak{h}}$ and $\langle\_\,,\_\rangle_{\mathfrak{l}}$ are G-invariant, nondegenerate and symmetric, over the algebras $\mathfrak{g}$, $\mathfrak{h}$ and $\mathfrak{l}$, respectively.

The equations of motion can be obtained by varying the action with respect to the multipliers B, C and D,

$$\mathcal{F} = 0\,, \qquad \mathcal{G} = 0\,, \qquad \mathcal{H} = 0\,, \tag{10}$$

and by varying with respect to the connections α, β and γ,

$$\nabla B + C\wedge^{\mathcal{T}}\beta - D\wedge^{\mathcal{S}}\gamma = 0\,, \qquad \nabla C - \partial B - D\wedge^{(\mathcal{X}_1+\mathcal{X}_2)}\beta = 0\,, \qquad \nabla D + \delta C = 0\,. \tag{11}$$

See [13] for the detailed definitions of the maps $\mathcal{T}$, $\mathcal{S}$, $\mathcal{X}_1$ and $\mathcal{X}_2$.

3. The Standard Model 3-group

At this point we are finally ready to construct a realistic classical action, featuring the full Standard Model of elementary particles coupled to Einstein-Cartan gravity. The action is based on a so-called Standard Model 3-group, which is a 2-crossed module $(\, L \stackrel{\delta}{\to} H \stackrel{\partial}{\to} G\,,\ \triangleright\,,\ \{\_\,,\_\}\,)$ with a following choices for the Lie groups:

$$G = SO(3,1)\times SU(3)\times SU(2)\times U(1)\,, \qquad H = \mathbb{R}^4\,,$$

$$L = \mathbb{C}^4\times\mathbb{G}^{64}\times\mathbb{G}^{64}\times\mathbb{G}^{64}\,.$$

We choose the group G as a product of the Lorentz group and the usual internal gauge symmetry group of the Standard Model. The group H is chosen to be the group of spacetime translations, motivated by the Poincaré 2-group construction [12]. Finally, we choose the group L as a product of $\mathbb{C}^4$ accouning for the doublet of complex scalar fields, and three copies of the 64-dimensional Grassmann algebra $\mathbb{G}^{64}$, representing three families of fermions. The maps δ, ∂ and $\{\_\,,\_\}$ are trivial, while the map $\triangleright$ is chosen in a natural way, in accord with the usual action of the gauge group G onto translations and various components of matter fields. It is defined in detail in [13].

Once the 3-group has been completely specified, the corresponding action can be written as a $3BF$ action with suitable constraint terms, as follows:

$$
\begin{array}{rcll}
S & = & \displaystyle\int \overbrace{B_\alpha \wedge F^\alpha + B^{[ab]} \wedge R_{[ab]}}^{\langle B\wedge\mathcal{F}\rangle} + \overbrace{e_a \wedge \nabla\beta^a}^{\langle C\wedge\mathcal{G}\rangle} + \overbrace{\phi^A (\nabla\gamma)_A + \bar\psi_A (\overrightarrow{\nabla}\gamma)^A - (\bar\gamma \overleftarrow{\nabla})_A \psi^A}^{\langle D\wedge\mathcal{H}\rangle} & 3BF \\
& - & \displaystyle\int \lambda_{[ab]} \wedge \left(B^{[ab]} - \frac{1}{16\pi l_p^2}\, \varepsilon^{[ab]cd}\, e_c \wedge e_d \right) + \frac{1}{96\pi l_p^2}\, \Lambda\, \varepsilon_{abcd}\, e^a \wedge e^b \wedge e^c \wedge e^d & \text{GR and CC} \\
& + & \displaystyle\int \lambda^\alpha \wedge \left(B_\alpha - 12\, C_\alpha{}^\beta M_{\beta ab}\, e^a \wedge e^b \right) + \zeta^{\alpha ab} \left(M_{\alpha ab}\, \varepsilon_{cdef}\, e^c \wedge e^d \wedge e^e \wedge e^f - F_\alpha \wedge e_a \wedge e_b \right) & \text{YM} \\
& + & \displaystyle\int \lambda^A \wedge \left(\gamma_A - H_{abcA}\, e^a \wedge e^b \wedge e^c \right) + \Lambda^{abA} \wedge \left(H_{abcA}\, \varepsilon^{cdef}\, e_d \wedge e_e \wedge e_f - (\nabla\phi)_A \wedge e_a \wedge e_b \right) & \text{Higgs} \\
& - & \displaystyle\int \frac{1}{12}\, \chi \left(\phi^A \phi_A - v^2 \right)^2 \varepsilon_{abcd}\, e^a \wedge e^b \wedge e^c \wedge e^d & \text{Higgs potential} \\
& + & \displaystyle\int \bar\lambda_A \wedge \left(\gamma^A + \frac{i}{6}\, \varepsilon_{abcd}\, e^a \wedge e^b \wedge e^c \left(\gamma^d \psi \right)^A \right) - \lambda^A \wedge \left(\bar\gamma_A - \frac{i}{6}\, \varepsilon_{abcd}\, e^a \wedge e^b \wedge e^c \left(\bar\psi \gamma^d \right)_A \right) & \text{Dirac} \\
& - & \displaystyle\int \frac{1}{12}\, Y_{ABC}\, \bar\psi^A \psi^B \phi^C\, \varepsilon_{abcd}\, e^a \wedge e^b \wedge e^c \wedge e^d & \text{Yukawa} \\
& + & \displaystyle\int 2\pi i\, l_p^2\, \bar\psi_A \gamma_5 \gamma^a \psi^A\, \varepsilon_{abcd}\, e^b \wedge e^c \wedge \beta^d \,. & \text{spin-torsion}
\end{array}
$$

Here the first row represents the topological $3BF$ part, while the remaining rows represent various constraint terms, each corresponding to one sector of the theory. Taking all together, the equations of motion obtained from the action S are equivalent to the full set of equations of motion for all Standard Model fields, coupled to the Einstein-Cartan theory of gravity.

The key novelty of the above structure is the role of the group L, which prescribes the spectrum of scalar and fermion fields present in the theory, via the $\langle D\wedge\mathcal{H}\rangle$ term in the topological sector of the action.

4. Conclusions

Let us summarize the results of the paper. In Section 2 we have introduced the nBF theories for $n = 1, 2, 3$, and explanied in brief terms how the categorical ladder procedure can be applied to generalize the notion of a group to the notions of a 2-group and a 3-group, which represent more powerful ways to describe the gauge symmetry of a physical theory. These structures were employed in Section 3 to construct the constrained $3BF$ action for the Standard Model of elementary particles coupled to the Einstein-Cartan gravity in the usual way. Within that framework, the spectrum of scalar and fermion fields happens to be determined by a *new gauge group*, in a way similar to that of the ordinary gauge group determining the spectrum of gauge vector bosons in Yang-Mills theory. This opens up a very interesting possibility of applying the structure of a 3-group to classify matter fields, and possibly gain some insight into why there are three families of fermions.

These results complete the first step of the spinfoam quantization programme, as outlined in the Introduction. The second step has also been performed in [25], for a general case of a Lie 2-crossed module $(L \xrightarrow{\delta} H \xrightarrow{\partial} G\,, \triangleright\,, \{\_\,,\_\})$. The resulting state sum is a novel topological invariant of a 4-dimensional manifold, and has the following form:

$$
\begin{aligned}
Z = & |G|^{-|\Lambda_0|+|\Lambda_1|-|\Lambda_2|}|H|^{|\Lambda_0|-|\Lambda_1|+|\Lambda_2|-|\Lambda_3|}\,|L|^{-|\Lambda_0|+|\Lambda_1|-|\Lambda_2|+|\Lambda_3|-|\Lambda_4|} \\
& \times \prod_{(jk)\in\Lambda_1} \int_G dg_{jk} \prod_{(jk\ell)\in\Lambda_2} \int_H dh_{jk\ell} \prod_{(jk\ell m)\in\Lambda_3} \int_L dl_{jk\ell m} \\
& \times \prod_{(jk\ell)\in\Lambda_2} \delta_G\Big(\partial(h_{jk\ell})\, g_{k\ell}\, g_{jk}\, g_{j\ell}^{-1}\Big) \prod_{(jk\ell m)\in\Lambda_3} \delta_H\Big(\delta(l_{jk\ell m}) h_{j\ell m}\,(g_{\ell m}\rhd h_{jk\ell})\, h_{k\ell m}^{-1}\, h_{jkm}^{-1}\Big) \\
& \times \prod_{(jk\ell mn)\in\Lambda_4} \delta_L\Big(l_{j\ell mn}^{-1}\, h_{j\ell n} \rhd' \{h_{\ell mn}, (g_{mn}g_{\ell m)} \rhd h_{jk\ell}\}_{\mathrm{p}}\, l_{jk\ell n}^{-1}(h_{jkn} \rhd' l_{k\ell mn}) l_{jkmn} h_{jmn} \rhd' (g_{mn} \rhd l_{jk\ell m})\Big).
\end{aligned}
\tag{12}
$$

Here $g_{ij}, h_{ijk}, l_{ijkl}$ are elements from groups G, H, L, respectively, which are assigned to simplices of the triangulation whose vertices are numerated by indices $i, j, \dots$ In other words, g_{ij} are assigned to edges, h_{ijk} are assigned to triangles, and l_{ijkl} are assigned to tetrahedra of the simplicial complex representing a compact 4-manifold, which has a total number of Λ_0 vertices, Λ_1 edges, Λ_2 triangles, Λ_3 tetrahedra, and Λ_4 4-simplices.

Of course, when building a realistic theory, we are in fact not interested in a topological theory, but instead in a theory which contains local propagating degrees of freedom. Thus the state sum Z should be appropriately deformed. This is the task of step 3 of the spinfoam quantization programme, by imposing the simplicity constraints on Z. The classical action from Section 3 manifestly distinguishes the topological sector from the simplicity constraints. Imposing those constraints should thus complete the spinfoam quantization programme, and would ultimately lead us to a tentative model of quantum gravity with matter, by providing a rigorous definition for the path integral

$$
Z = \int \mathcal{D}g \int \mathcal{D}\phi \; e^{iS[g,\phi]} \,, \tag{13}
$$

which is a generalization of (1) in the sense that it contains matter fields as well as gravity, at the quantum level.

In addition to the construction of a full quantum theory of gravity, there are also many additional possible studies of the classical constrained $3BF$ action. For example, a full Hamiltonian analysis of the $3BF$ action has been done for the example of scalar electrodynamics [26], and then also for a general choice of a Lie 3-group [27], and the complete gauge symmetry group has been discussed in detail [27, 28]. Also, it is worth looking into the idea of imposing the simplicity constraints using a spontaneous symmetry breaking mechanism, and some work has already begun in this area. Finally, one can also study in more depth the mathematical structure and properties of the simplicity constraints. The list is not conclusive, and there may be many other interesting topics to study.

Acknowledgments

This research was supported by the Ministry of Science, Technological Development and Innovations of the Republic of Serbia, and by the Science Fund of the Republic of Serbia, grant 7745968, “Quantum Gravity from Higher Gauge Theory 2021” — QGHG-2021. The contents of this publication are the sole responsibility of the authors and can in no way be taken to reflect the views of the Science Fund of the Republic of Serbia.

References

[1] Rovelli C 2004 *Quantum Gravity* (Cambridge University Press: Cambridge)
[2] Rovelli C and Vidotto F 2014 *Covariant Loop Quantum Gravity* (Cambridge University Press: Cambridge)
[3] Thiemann T 2007 *Modern Canonical Quantum General Relativity* (Cambridge University Press: Cambridge)
[4] Ponzano G and Regge T 1968 *Semiclassical limit of Racah coefficients* (Spectroscopic and Group Theoretical Methods in Physics) ed F Bloch (Amsterdam: North Holland)

[5] Barrett J W and Crane L 1998 *J. Math. Phys.* **39** 3296
[6] Barrett J W and Crane L 2000 *Class. Quant. Grav.* **17** 3101
[7] Ooguri H 1992 *Mod. Phys. Lett. A* **7** 279
[8] Engle J, Livine E R, Pereira R and Rovelli C 2008 *Nucl. Phys.* **B799** 136
[9] Freidel L and Krasnov K 2008 *Class. Quant. Grav.* **25** 125018
[10] Bianchi E, Han M, Magliaro E, Perini C, Rovelli C and Wieland W 2013 *Class. Quantum Grav.* **30** 235023
[11] Baez J C and Huerta J 2011 *Gen. Relativ. Gravit.* **43** 2335
[12] Miković A and Vojinović M 2012 *Class. Quant. Grav.* **29** 165003
[13] Radenković T and Vojinović M 2019 *JHEP* **10** 222
[14] Celada M, González D and Montesinos M 2016 *Class. Quant. Grav.* **33** 213001
[15] Rovelli C 2011 Zakopane lectures on loop gravity *Preprint* arXiv:1102.3660
[16] Plebanski J F 1977 *J. Math. Phys.* **18** 2511
[17] Girelli F, Pfeiffer H and Popescu E M 2008 *Jour. Math. Phys.* **49** 032503
[18] Martins J F and Miković A 2011 *Adv. Theor. Math. Phys.* **15** 1059
[19] Crane L and Sheppeard M D 2003 2-categorical Poincare Representations and State Sum Applications *Preprint* math/0306440
[20] Miković A and Oliveira M A 2015 *Gen. Relativ. Gravit.* **47** 58
[21] Miković A, Oliveira M A and Vojinović M 2016 Hamiltonian analysis of the BFCG theory for a generic Lie 2-group *Preprint* arXiv:1610.09621
[22] Miković A, Oliveira M A and Vojinović M 2019 *Class. Quant. Grav.* **36** 015005
[23] Martins J F and Picken R 2011 *Differ. Geom. Appl. Journal* **29** 179
[24] Wang W 2014 *Jour. Math. Phys.* **55** 043506
[25] Radenković T and Vojinović M 2022 *JHEP* **7** 105
[26] Radenković T and Vojinović M 2020 *Symmetry* **12** 620
[27] Radenković T and Vojinović M 2022 *Class. Quant. Grav.* **39** 135009
[28] Djordjević M, Radenković T, Stipsić P and Vojinović M 2023 *Universe* **9** 281

P r o c e e d i n g s

of the

8th MATHEMATICAL PHYSICS MEETING: Summer School and Conference on Modern Mathematical Physics

August 24 – 31, 2014, Belgrade, Serbia

Editors

B. Dragovich, I. Salom

Institute of Physics
Belgrade, 2015
SERBIA

Autor: Grupa autora

Naslov: 8th MATHEMATICAL PHYSICS MEETING: SUMMER SCHOOL AND CONFERENCE ON MODERN MATHEMATICAL PHYSICS

(Osmi naučni skup iz matematičke fizike: letnja škola i konferencija iz savremene matematičke fizike)

Izdavač: Institut za fiziku, Beograd, Srbija

Izdanje: Prvo izdanje
(SFIN year XXVIII Series A: Conferences No. A1 (2015))

Štampar: Ton Plus, Beograd

Tiraž: 150
ISBN: 978-86-82441-43-4

1. Dragović Branko
Matematička fizika-Zbornici

CONTENTS

Review and Research Works

Weakly curved background T-duals*

Lj. Davidović[†]
Institute of Physics, University of Belgrade, Serbia
B. Nikolić[‡]
Institute of Physics, University of Belgrade, Serbia
B. Sazdović[§]
Institute of Physics, University of Belgrade, Serbia

Abstract

We discuss the generalized T-dualization procedure, its connection to the standard procedure, and the results of its application to the arbitrary set of coordinates of the closed string moving in the weakly curved background. This background consists of a constant metric and linearly coordinate dependent Kalb-Ramond field with infinitesimal strength. We unite all the results into a T-dualization diagram, representing all T-dual theories, the ways to obtain the theories from one another and the T-dual coordinate transformation laws connecting the corresponding coordinates.

* Work supported in part by the Serbian Ministry of Education, Science and Technological Development, under contract No. 171031.

[†] e-mail address: ljubica@ipb.ac.rs
[‡] e-mail address: bnikolic@ipb.ac.rs
[§] e-mail address: sazdovic@ipb.ac.rs

T-duality is a symmetry seen in the string spectrum. It was observed that for toroidal compactifications [1], where one dimension is compactified on a circle of radius R and the corresponding dual dimension is compactified on a circle of radius $1/R$, one obtains the discription of a string with the same physical properties. So, there exist different string theories, describing the string in the geometrically different backgrounds, with the same predictions. Such a symmetry is not present in any point particle theory [2, 3, 4, 5], and because of that the explanation for T-duality was sought for in a fact that strings can wrap around compactified dimensions. Investigation of T-duality lead to a discovery of a Buscher T-dualization procedure [6, 7], which gave a prescription how to find the T-dual theory for same known theory. The procedure is applicable along isometry directions, what allowed the investigation of a backgrounds which do not depend on some coordinates. This procedure enabled the investigation of the properties of the background connected by T-duality. It was discovered that geometric backgrounds transform to the non-geometric backgrounds and these to different non-geometric backgrounds, which differ in a form of the background fluxes, some of which are not locally well defined [8, 9]. T-duality is also investigated for the double string theories, where T-duality is a manifest symmetry [10, 11, 12, 13].

In this talk we will discuss the results of T-dualizations done for the closed string moving in the weakly curved background, using the generalized T-dualization procedure, defined in our paper [14]. This background depends on all the space-time coordinates and as such was not a candidate for T-dualization using the standard T-dualization procedures. In paper [14], we presented generalized T-dualization procedure applicable to all space-time directions regardless of the possible background coordinate dependence. We obtained the T-dual theory which is a result of T-dualizing all the initial coordinates. In paper [15], we broadened the investigation by considering T-dualization of an arbitrary set of coordinates of both initial and its completely T-dual theory. We will recapitulate the results here and discuss further investigations. We obtained the T-dualization diagram describing the relation between all string theories T-dual to the string moving in a weakly curved background, their backgrounds and giving the T-duality laws connecting the corresponding coordinates.

So, let us start by the action describing a closed string moving in a coordinate dependent background

$$S[x] = \kappa \int_\Sigma d^2\xi \; \partial_+ x^\mu \Pi_{+\mu\nu}(x) \partial_- x^\nu, \quad \partial_\pm = \partial_\tau \pm \partial_\sigma \tag{1}$$

given in the conformal gauge $g_{\alpha\beta} = e^{2F}\eta_{\alpha\beta}$. The background field composition is defined by

$$\Pi_{\pm\mu\nu}(x) = B_{\mu\nu}(x) \pm \frac{1}{2} G_{\mu\nu}(x). \tag{2}$$

It consists of a symmetric metric tensor $G_{\mu\nu} = G_{\nu\mu}$ and an antisymmetric Kalb-Ramond field $B_{\mu\nu} = -B_{\nu\mu}$. The background must obey the following space-time equations of motion

$$R_{\mu\nu} - \frac{1}{4}B_{\mu\rho\sigma}B_{\nu}{}^{\rho\sigma} = 0, \quad D_\rho B^\rho{}_{\mu\nu} = 0, \tag{3}$$

in order to have a consistent quantum theory. We will consider one of the simplest coordinate dependent solutions, the weakly curved background, composed of a constant metric and linearly coordinate dependent Kalb-Ramond field which has an infinitesimal field strength

$$G_{\mu\nu}(x) = const,\ B_{\mu\nu}(x) = b_{\mu\nu} + \frac{1}{3}B_{\mu\nu\rho}x^\rho, \quad b_{\mu\nu}, B_{\mu\nu\rho} = const. \tag{4}$$

What are the backgrounds T-dual to this background? As, the standard T-dualization procedure is applicable to coordinate directions which do not appear as background field arguments, and the weakly curved background depends on all space-time coordinates, this procedure could not provide the answer to this question. So, a generalization of a T-dualization procedure which does not have this limitation had to be made. The main difference between the procedures obviously must be connected to background fields argument. We presented the new T-dualization procedure in [14].

Both procedures are built as a localization of a global coordinate shift symmetry $\delta x^\mu = \lambda^\mu = const$. One introduces the gauge fields v^μ_α and substitutes the ordinary derivatives with the covariant ones

$$\partial_\alpha x^\mu \to D_\alpha x^\mu = \partial_\alpha x^\mu + v^\mu_\alpha. \tag{5}$$

Imposing the following transformation law for the gauge fields

$$\delta v^\mu_\alpha = -\partial_\alpha \lambda^\mu, \quad (\lambda^\mu = \lambda^\mu(\tau,\sigma)) \tag{6}$$

one obtains that $\delta D_\alpha x^\mu = 0$. If the background does not depend on the coordinates which are T-dualized the gauge invariant action is already obtained. But, what if the background depends on all the coordinates? The additional step must be introduced. It consists of a substitution of background field argument (the coordinate x^μ), by the invariant argument (invariant coordinate) defined as a line integral of the covariant derivatives of the original coordinate

$$\Delta x^\mu_{inv} \equiv \int_P d\xi^\alpha\, D_\alpha x^\mu = x^\mu - x^\mu(\xi_0) + \Delta V^\mu, \tag{7}$$

where

$$\Delta V^\mu \equiv \int_P d\xi^\alpha v^\mu_\alpha. \tag{8}$$

Consequently the arguments of the background fields will be nonlocal. Here, they are defined as the line integrals of the gauge fields, and as such are nonlocal. Later, once the explicit form of T-dual theories is obtained the non locality will appear as dependence on double coordinates.

In order to obtain the physically equivalent theories, one must make the introduced gauge fields nonphysical which is done by requiring that there field strength

$$F^{\mu}_{\alpha\beta} \equiv \partial_\alpha v^\mu_\beta - \partial_\beta v^\mu_\alpha \tag{9}$$

must be zero. This is achieved by adding the Lagrange multiplier y_μ term to the Lagrangian. Finally, the gauge invariant action, physically equivalent to the initial action is

$$S_{inv} = \kappa \int d^2\xi \Big[D_+ x^\mu \Pi_{+\mu\nu}(\Delta x_{inv}) D_- x^\nu + \frac{1}{2}(v^\mu_+ \partial_- y_\mu - v^\mu_- \partial_+ y_\mu)\Big]. \tag{10}$$

Fixing the gauge $x^\mu(\xi) = x^\mu(\xi_0)$, one obtains

$$S_{fix}[y, v_\pm] = \kappa \int d^2\xi \Big[v^\mu_+ \Pi_{+\mu\nu}(\Delta V) v^\nu_- + \frac{1}{2}(v^\mu_+ \partial_- y_\mu - v^\mu_- \partial_+ y_\mu)\Big]. \tag{11}$$

The gauge fixed action is the main crossway of the procedure, for an appropriate equation of motion it can transform both to initial action and to the T-dual action. For the equation of motion obtained varying the action over the Lagrange multipliers $\partial_+ v^\mu_- - \partial_- v^\mu_+ = 0$, with solution $v^\mu_\pm = \partial_\pm x^\mu$, the gauge fixed action reduces to the initial action. For the equation of motion obtained varying the action over the gauge fields $\Pi_{\mp\mu\nu}[\Delta V] v^\nu_\pm + \frac{1}{2}\partial_\pm y_\mu = \mp \beta^\mp_\mu[V]$, where $\beta^\alpha_\mu[V] \equiv \partial_\mu B_{\nu\rho} \epsilon^{\alpha\beta} V^\nu \partial_\beta V^\rho$, one obtains the T-dual theory. Comparing the solutions for the gauge fields in these two directions, one obtains the T-dual coordinate transformation laws. So, for application along all directions we obtain the following connection

$$S[x] = \kappa \int_\Sigma d^2\xi\, \partial_+ x^\mu \Pi_{+\mu\nu}(x) \partial_- x^\nu \Leftrightarrow {}^\star S[y] = \frac{\kappa^2}{2} \int d^2\xi\, \partial_+ y_\mu \Theta^{\mu\nu}_-(\Delta V) \partial_- y_\nu, \tag{12}$$

with $\Delta V^\mu = V^\mu(\xi) - V^\mu(\xi_0)$, $\quad V^\mu = (g^{-1})^{\mu\nu}[(2bG^{-1})_\nu{}^\rho y_\rho + \tilde{y}_\nu]$. The dual background field composition is defined by

$$\Theta^{\mu\nu}_\pm = -\frac{2}{\kappa}(G^{-1}_E \Pi_\pm G^{-1})^{\mu\nu} = \theta^{\mu\nu} \mp \frac{1}{\kappa}(G^{-1}_E)^{\mu\nu}, \tag{13}$$

and consequently the T-dual background are

$$\begin{aligned} G_{\mu\nu} \quad &\Leftrightarrow \quad {}^\star G^{\mu\nu}(y, \tilde{y}) = (G^{-1}_E)^{\mu\nu}(\Delta V), \\ B_{\mu\nu}(x) \quad &\Leftrightarrow \quad {}^\star B^{\mu\nu}(y, \tilde{y}) = \frac{\kappa}{2}\theta^{\mu\nu}(\Delta V), \end{aligned} \tag{14}$$

where $G_{E\mu\nu}$ and $\theta^{\mu\nu}$ are the effective metric and the noncommutativity parameter for open bosonic string, which are

$$G_{E\mu\nu} = G_{\mu\nu} - 4(BG^{-1}B)_{\mu\nu},\; \theta^{\mu\nu} = -\frac{2}{\kappa}(G^{-1}_E B G^{-1})^{\mu\nu}. \tag{15}$$

But what if one does not consider T-dualization over all coordinates, but only some set of coordinates. To investigate this problem, let us mark a T-dualization along direction x^μ by T^μ and a T-dualization along dual direction y_μ by T_μ. Also mark the T-dualizations along some d initial directions, all other $D-d$ initial directions, and all initial directions by

$$\mathcal{T}^a = \circ_{n=1}^{d} T^{\mu_n}, \qquad \mathcal{T}^i = \circ_{n=d+1}^{D} T^{\mu_n}, \qquad \mathcal{T} = \circ_{n=1}^{D} T^{\mu_n} \tag{16}$$

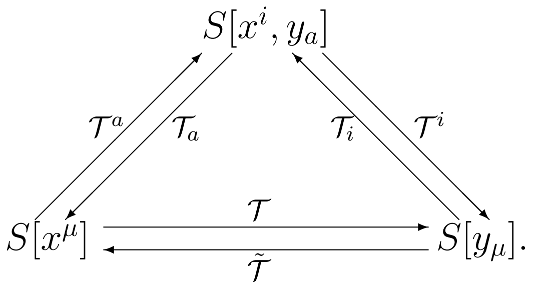
and T-dualizations along corresponding dual directions by

$$\mathcal{T}_a = \circ_{n=1}^{d} T_{\mu_n}, \qquad \mathcal{T}_i = \circ_{n=d+1}^{D} T_{\mu_n}, \qquad \widetilde{\mathcal{T}} = \circ_{n=1}^{D} T_{\mu_n} \tag{17}$$

$\mu_n \in (0, 1, \ldots, D-1)$. We showed in [15] that these T-dualizations form an Abelian group

$$\mathcal{T}^i \circ \mathcal{T}^a = \mathcal{T}, \quad \mathcal{T}_i \circ \mathcal{T}_a = \widetilde{\mathcal{T}}, \quad \mathcal{T}_a \circ \mathcal{T}^a = 1. \tag{18}$$

We showed that all the theories T-dual to the theory of the closed bosonic string are the part of the T-dualization diagram, given by



This diagram clearly describes the connection between arbitrary theory and the initial and completely T-dual theory. The explicit form of the theory obtained T-dualizing some set (marked by a) of the initial coordinates is the following

$$\begin{aligned} S[x^i, y_a] \;=\; \kappa \int d^2\xi \Big[& \partial_+ x^i \overline{\Pi}_{+ij}(x^i, \Delta V^a(x^i, y_a)) \partial_- x^j \\ & -\kappa\, \partial_+ x^i \Pi_{+ia}(x^i, \Delta V^a(x^i, y_a)) \tilde{\Theta}_-^{ab}(x^i, \Delta V^a(x^i, y_a)) \partial_- y_b \\ & +\kappa\, \partial_+ y_a \tilde{\Theta}_-^{ab}(x^i, \Delta V^a(x^i, y_a)) \Pi_{+bi}(x^i, \Delta V^a(x^i, y_a)) \partial_- x^i \\ & +\frac{\kappa}{2}\, \partial_+ y_a \tilde{\Theta}_-^{ab}(x^i, \Delta V^a(x^i, y_a)) \partial_- y_b \Big]. \end{aligned} \tag{19}$$

The new background field compositions $\overline{\Pi}_{\pm ij}$ and $\tilde{\Theta}_{\pm}^{ab}$ are defined as the inveres of the ordinary background field compositions $\Theta_{\mp}^{jk}$ and $\Pi_{\mp bc}$ reduced to the appropriate d and $D-d$ dimensional subspaces

$$\overline{\Pi}_{\pm ij} \Theta_{\mp}^{jk} = \Theta_{\mp}^{kj} \overline{\Pi}_{\pm ji} = \frac{1}{2\kappa} \delta_i^k, \tag{20}$$

$$\tilde{\Theta}^{ab}_{\pm}\Pi_{\mp bc} = \Pi_{\mp cb}\tilde{\Theta}^{ba}_{\pm} = \frac{1}{2\kappa}\delta^a_c. \tag{21}$$

It can be shown that

$$\overline{\Pi}_{+ij} \equiv \Pi_{+ij} - 2\kappa\Pi_{+ia}\tilde{\Theta}^{ab}_{-}\Pi_{+bj}. \tag{22}$$

The argument of the background fields is

$$\begin{aligned}\Delta V^{(0)a}(x^i, y_a) &= -\kappa\Big[\tilde{\Theta}^{ab}_{0+}\Pi_{0-bi} + \tilde{\Theta}^{ab}_{0-}\Pi_{0+bi}\Big]\Delta x^{(0)i} \\ &- \kappa\Big[\tilde{\Theta}^{ab}_{0+}\Pi_{0-bi} - \tilde{\Theta}^{ab}_{0-}\Pi_{0+bi}\Big]\Delta\tilde{x}^{(0)i} \\ &- \frac{\kappa}{2}\Big[\tilde{\Theta}^{ab}_{0+} + \tilde{\Theta}^{ab}_{0-}\Big]\Delta y^{(0)}_b - \frac{\kappa}{2}\Big[\tilde{\Theta}^{ab}_{0+} - \tilde{\Theta}^{ab}_{0-}\Big]\Delta\tilde{y}^{(0)}_b, \end{aligned} \tag{23}$$

where $\Delta x^\mu(\xi) = x^\mu(\xi) - x^\mu(\xi_0)$ and $\Delta y_\mu(\xi) = y_\mu(\xi) - y_\mu(\xi_0)$ while $\Delta\tilde{x}^\mu(\xi)$ and $\Delta\tilde{y}_\mu(\xi)$ are their duals, defined by

$$\Delta\tilde{x}^\mu(\xi) = \int_P d\xi^\alpha\, \varepsilon^\beta{}_\alpha\, \partial_\beta x^\mu, \quad \Delta\tilde{y}(\xi) = \int_P d\xi^\alpha\, \varepsilon^\beta{}_\alpha\, \partial_\beta y_\mu. \tag{24}$$

Calculating the symmetric and antisymmetric part of the background fields we obtain the T-dual metric and Kalb-Ramond field:

$$\begin{aligned}&\bullet G_{ij} = \overline{G}_{ij} = G_{ij} - G_{ia}(\tilde{G}^{-1}_E)^{ab}G_{bj} \\ &\quad -2\kappa\Big(B_{ia}\tilde{\theta}^{ab}G_{bj} + G_{ia}\tilde{\theta}^{ab}B_{bj}\Big) - 4B_{ia}(\tilde{G}^{-1}_E)^{ab}B_{bj} \\ &\bullet B_{ij} = \overline{B}_{ij} = B_{ij} - \frac{\kappa}{2}G_{ia}\tilde{\theta}^{ab}G_{bj} - B_{ia}(\tilde{G}^{-1}_E)^{ab}G_{bj} \\ &\quad -G_{ia}(\tilde{G}^{-1}_E)^{ab}B_{bj} - 2\kappa B_{ia}\tilde{\theta}^{ab}B_{bj} \\ &\bullet G^{ab} = (\tilde{G}^{-1}_E)^{ab} \\ &\bullet B^{ab} = \frac{\kappa}{2}\tilde{\theta}^{ab} \\ &\bullet G^a{}_i = \kappa\tilde{\theta}^{ab}G_{bi} + 2(\tilde{G}^{-1}_E)^{ab}B_{bi} \\ &\bullet B^a{}_i = \kappa\tilde{\theta}^{ab}B_{bi} + \frac{1}{2}(\tilde{G}^{-1}_E)^{ab}G_{bi}. \end{aligned} \tag{25}$$

As the constituents of the dual background field there appear the effective metric in the d-dimensional subspace a, defined by

$$\tilde{G}_{Eab} \equiv G_{ab} - 4B_{ac}(\tilde{G}^{-1})^{cd}B_{db}, \tag{26}$$

the non-commutativity parameter in the same subspace

$$\tilde{\theta}^{ab} \equiv -\frac{2}{\kappa}(\tilde{G}^{-1}_E)^{ac}B_{cd}(\tilde{G}^{-1})^{db}, \tag{27}$$

which combined give the new theta function $\tilde{\Theta}^{ab}_{\pm} = \tilde{\theta}^{ab} \mp \frac{1}{\kappa}(\tilde{G}^{-1}_{E})^{ab}$.

Every arrow in the T-duality diagram is accompanied with the appropriate T-dual coordinate transformation law. These are obtained comparing the solutions for the gauge fields in a T-dualization procedures performed between two actions in both directions. The laws for transitions

$$\mathcal{T}^a : S[x^\mu] \to S[x^i, y_a], \quad \mathcal{T}_a : S[x^i, y_a] \to S[x^\mu],$$

which are inverse to each other, are given by

$$\begin{aligned} \partial_\mp x^a &\cong -2\kappa \tilde{\Theta}^{ab}_{\mp}(x^i, \Delta V^a(x^i, y_a)) \cdot \\ &\cdot \Big[\Pi_{\pm bi}(x^i, \Delta V^a(x^i, y_a))\partial_\mp x^i + \frac{1}{2}\partial_\mp y_b \mp \beta^\pm_b(x^i, V^a(x^i, y_a))\Big] \\ x^{(0)a} &\cong V^{(0)a}(x^i, y_a) \end{aligned} \tag{28}$$

and its inverse

$$\begin{aligned} \partial_\mp y_a &\cong -2\Pi_{\pm a\mu}(x)\partial_\mp x^\mu \pm 2\beta^\pm_a(x), \\ y^{(0)}_a &\cong U^{(0)}_a(x). \end{aligned} \tag{29}$$

These relations enable an investigation of the closed string non-commutativity and other geometric properties of the T-dual backgrounds. One can determine the geometric structure for an arbitrary sigma model in a T-duality diagram, find the connection between the Poisson structures of T-dual theories and the relations between non-commutativity parameters. The coordinates of the closed string are commutative when the string moves in a constant background. In a three dimensional space with the Kalb-Ramond field depending on one of the coordinates, successive T-dualizations along isometry directions lead to a theory with Q flux and the non-commutative coordinates [16, 17, 18]. Using the generalized T-dualization procedure, we found the non-commutativity characteristics of a closed string moving in the weakly curved background [15] comparing the initial and completely T-daul theory. One can expect the further investigations will reveal novelties regarding the form of the fluxes of all T-dual background forming a diagram. For now it is known that all fluxes are of type R.

References

[1] L.Brink, M.B. Green, J.H. Schwarz, *Nucl. Phys.* **B 198** (1982) 474; K. Kikkawa and M.Yamasaki, *Phys. Lett.* **B 149** (1984) 357; N. Sakai and I. Senda, *Prog. Theor. Phys.* **75** (1984) 692.

[2] A. Giveon, M. Porrati, E. Rabinovici, Phys. Rep. **244**, 77-202 (1994).

[3] E. Alvarez, L. Alvarez-Gaume and Y. Lozano, Nucl. Phys. Proc. Suppl. **41**, 1 (1995).

[4] B. Zwiebach, *A First Course in String Theory*, Second Edition, Cambridge University Press (2009).

[5] K. Becker, M. Becker and J. H. Schwarz, *String Theory and M-Theory - A Modern Introdution*, Cambridge University Press (2007).

[6] T. H. Buscher, Phys. Lett. **B 201** No. 4, 466 (1988).

[7] M. Roček and E. Verlinde, Nucl. Phys. **B 373**, 630 (1992).

[8] J. Shelton, W. Taylor and B. Wecht, JHEP **10**, 085 (2005).

[9] R. Blumenhagen, A. Deser, D. Lüst, E. Plauschinn, F. Rennecke, J. Phys. **A 44**, 385401 (2011).

[10] O. Hohm, Prog. Theor. Phys. Suppl. **188**, 116-125 (2011).

[11] C. Hull and B. Zwiebach, JHEP **09**, 099 (2009).

[12] O. Hohm, C. Hull and B. Zwiebach, JHEP **08**, 008 (2010).

[13] O. Hohm, C. Hull and B. Zwiebach, JHEP **07**, 016 (2010).

[14] Lj. Davidović and B. Sazdović, EPJ**C 74**(1), 2683 (2014).

[15] Lj. Davidović, B. Nikolić and B. Sazdović, *T-duality diagram for a weakly curved background*, arXiv:1406.5364 [hep-th].

[16] D. Lüst, JHEP **12**, 084 (2010).

[17] D. Andriot, M. Larfors, D. Lüst, P. Patalong, JHEP **06**, 021 (2013).

[18] D. Andriot, O. Hohm, M. Larfors, D. Lüst, P. Patalong, Phys. Rev. Lett. **108** , 261602 (2012).

Closed string noncommutativity in the weakly curved background*

Lj. Davidović, B. Nikolić and B. Sazdović†
Institute of Physics, University of Belgrade
11001 Belgrade, P.O.Box 57, Serbia

Abstract

We consider the closed bosonic string moving in the weakly curved background. Using T-duality transformation laws we calculate the Poisson brackets of the coordinates in the T-dual space assuming that initial theory is geometric one, which means that standard Poisson algebra is obeyed. The result is that the commutative initial theory is equivalent to the non-commutative T-dual theory. All noncommutativity parameters are infinitesimal and proportional to the $B_{\mu\nu\rho}$, field strength of Kalb-Ramond field $B_{\mu\nu}$. In addition we find the algebra of the T-dual winding numbers and momenta in terms of the winding numbers and momenta of the initial theory.

1. Introduction

In order to obtain noncommutativity in the open string case it is enough to consider the open string in the presence of the *constant* gravitational $G_{\mu\nu}$ and Kalb-Ramond field $B_{\mu\nu}$ and use the boundary conditions [1, 2]. Treating boundary conditions as canonical constraints and solving them, one gets the initial coordinates expressed in terms of the Ω even effective coordinates and momenta, where Ω is world-sheet parity transformation $\Omega : \sigma \to -\sigma$. Because effective variables have nonzero Poisson bracket (PB), the PB between initial coordinates is also nonzero. The noncommutativity parameter is proportional to the Kalb-Ramond field $B_{\mu\nu}$.

There is one interesting thing which we noted in the open string case. The effective metric and the noncommutativity parameter are (up to some constants) the backgroud fields of the T-dual theory. As we know T-dual theory is physically equivalent to the initial one in the sense they have the same degrees of freedom - one at the scale R and the T-dual one at the scale $1/R$. The mathematical realization of the T-duality goes through Buscher procedure [3]. As a result of the procedure we get the relation between initial and T-dual variables which we call *transformation laws.*

* Work supported in part by the Serbian Ministry of Education and Science, under contract number No.171031.

† e-mail address: ljubica, bnikolic, sazdovic@ipb.ac.rs

The closed strings do not have endpoints, so in the constant background there are no boundary conditions. To obtain noncommutativity in the closed string case we have to use T-duality as a helping tool. But, in the constant background case, T-duality relates σ-derivatives of the coordinates of one theory with the momenta of its T-dual one. Assuming that momenta of the initial theory commute (geometric theory) it follows that the T-dual coordinates commute as well. Consequently, in the constant background case there is no closed string non-commutativity.

It is obvious that T-duality is just one part of the solution in order to get the closed string noncommutativity. The second part is coordinate dependent background obeying the space-time field equations [4, 5]. Considering the closed string in the constant gravitational field $G_{\mu\nu}$ and Kalb-Ramond field depending on one coordinate, the closed string non-commutativity was first observed in the paper [6], and investigated further in [7, 8, 9]. In these articles 3-torus is considered, where $B_{\mu\nu}$ depends on one coordinate and T-dualization is performed along two other coordinates (isometry directions) using standard Buscher procedure [3].

One can ask if it is possible to do that in the background where $B_{\mu\nu}$ depends on all space-time coordinates. The answer is affirmative but in order to achieve that we have to use the generalized T-duality procedure presented in details in [10] and to apply it to the weakly curved background. The weakly curved background used in the present article is defined by constant gravitational $G_{\mu\nu} = const$ and the linear Kalb-Ramond field $B_{\mu\nu} = b_{\mu\nu} + \frac{1}{3}B_{\mu\nu\rho}x^\rho$, where the field strength $B_{\mu\nu\rho}$ is supposed to be infinitesimal. Such background obeys space-time field equations [4, 5] in the linear approximation in $B_{\mu\nu\rho}$.

We perform the generalized T-dualization procedure [10] along all the coordinates and obtain the T-duality transformation law, $\partial_\pm y_\mu = \partial_\pm y_\mu(\partial_\pm x)$, where $\partial_\pm$ are world-sheet partial derivatives. Using canonical formalism, the T-dual coordinates are expressed in terms of the original variables, $y'_\mu \cong \frac{1}{\kappa}\pi_\mu - \beta^0_\mu[x]$, where π_μ are canonically conjugated momenta to the coordinates x^μ. The infinitesimal expression β^0_μ is the correction in comparison to the flat background case. Assuming that the coordinates and momenta of the original theory satisfy standard Poisson algebra (initial theory is geometric one), we get the coordinate noncommutativity relations in the T-dual picture. In addition, we obtain the complete algebra of the T-dual winding numbers and momenta.

2. Generalized T-duality and noncommutativity

We consider the closed bosonic string moving in the D-dimensional space-time described by the action

$$S[x] = \kappa \int_\Sigma d^2\xi \; \partial_+ x^\mu \left(B_{\mu\nu}[x] + \frac{1}{2} G_{\mu\nu}[x] \right) \partial_- x^\nu \,, \tag{1}$$

where the light-cone coordinates are defined as $\xi^\pm = \frac{1}{2}(\tau \pm \sigma)$ and the corresponding derivatives $\partial_\pm = \partial_\tau \pm \partial_\sigma$. In order to keep conformal invariance on the quantum level, the background fields have to obey the following one-loop consistency conditions [4, 5]

$$R_{\mu\nu} - \frac{1}{4} B_{\mu\rho\sigma} B_{\nu}{}^{\rho\sigma} = 0, \quad D_{\rho} B^{\rho}{}_{\mu\nu} = 0\,. \tag{2}$$

Here $B_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} + \partial_\nu B_{\rho\mu} + \partial_\rho B_{\mu\nu}$ is the field strength of the field $B_{\mu\nu}$, and $R_{\mu\nu}$ and D_μ are Ricci tensor and the covariant derivative with respect to the space-time metric.

The solution of the equations in the first order in $B_{\mu\nu\rho}$, so called the weakly curved background, [7, 10, 11, 12], is defined by

$$\begin{aligned} G_{\mu\nu}[x] &= const, \\ B_{\mu\nu}[x] &= b_{\mu\nu} + h_{\mu\nu}[x] = b_{\mu\nu} + \frac{1}{3} B_{\mu\nu\rho} x^\rho, \qquad b_{\mu\nu}, B_{\mu\nu\rho} = const. \end{aligned} \tag{3}$$

Here, the field strength $B_{\mu\nu\rho}$ is infinitesimal.

Applying the generalized T-dualization procedure [10] on the closed string propagating in the weakly curved background, we obtain the T-dual action

$$^\star S[y] = \frac{\kappa^2}{2} \int d^2\xi \; \partial_+ y_\mu \Theta_-^{\mu\nu}[\Delta V[y]] \partial_- y_\nu, \tag{4}$$

where

$$\begin{aligned} \Theta_\pm^{\mu\nu} &\equiv -\frac{2}{\kappa} (G_E^{-1} \Pi_\pm G^{-1})^{\mu\nu} = \theta^{\mu\nu} \mp \frac{1}{\kappa} (G_E^{-1})^{\mu\nu}\,, \\ G_{E\mu\nu} &\equiv G_{\mu\nu} - 4(BG^{-1}B)_{\mu\nu}\,, \quad \Pi_{\pm\mu\nu} = B_{\mu\nu} \pm \frac{1}{2} G_{\mu\nu}\,. \end{aligned} \tag{5}$$

The argument ΔV is defined nonlocally as

$$\Delta V^\mu[y] = -\kappa \theta_0^{\mu\nu} \Delta y_\nu + (g^{-1})^{\mu\nu} \Delta \tilde{y}_\nu\,, \tag{6}$$

where

$$\Delta y_\mu = \int_P (d\tau \dot{y}_\mu + d\sigma y'_\mu) = y_\mu(\xi) - y_\mu(\xi_0), \quad \Delta \tilde{y}_\mu = \int_P (d\tau y'_\mu + d\sigma \dot{y}_\mu), \tag{7}$$

and

$$g_{\mu\nu} = G_{\mu\nu} - 4(bG^{-1}b)_{\mu\nu}, \quad \theta_0^{\mu\nu} = -\frac{2}{\kappa} (g^{-1} b G^{-1})^{\mu\nu}\,. \tag{8}$$

It is obvious from the definitions (7) that these two coordinates are related by the following expressions, $\dot{y}_\mu = \tilde{y}'_\mu$, $y'_\mu = \dot{\tilde{y}}_\mu$.

The transformation laws connecting initial and T-dual coordinates play the key role in our considerations. To be more precise, we obtain from T-dualization procedure the relations between world-sheet derivatives of the initial and T-dual coordinates

$$\partial_\pm x^\mu \cong -\kappa \Theta_\pm^{\mu\nu}[\Delta V] \Big[\partial_\pm y_\nu \pm 2\beta_\nu^\mp[V] \Big], \tag{9}$$

where

$$\beta^{\pm}_{\mu}[x] = \frac{1}{2}(\beta^0_\mu \pm \beta^1_\mu) = \mp\frac{1}{2}h_{\mu\nu}[x]\partial_{\mp}x^\nu\,,$$
$$\beta^0_\mu[x] = h_{\mu\nu}[x]x'^\nu,\ \beta^1_\mu[x] = -h_{\mu\nu}[x]\dot{x}^\nu\,. \tag{10}$$

Because we use the canonical formalism, we must have these transformation laws in the canonical form

$$x'^\mu \cong \frac{1}{\kappa}{}^\star\pi^\mu - \kappa\theta_0^{\mu\nu}\beta^0_\nu[V], \tag{11}$$
$$\pi_\mu \cong \kappa y'_\mu + \kappa\beta^0_\mu[V]\,, \tag{12}$$

where π_μ and ${}^\star\pi^\mu$ are canonically conjugated momenta to the coordinates x^μ and y_μ, respectively. It is shown in Ref. [10] that the T-dual of the T-dual action is the initial one. If we want to have T-dual coordinates in terms of the initial ones, we just have to invert the relation (9)

$$\partial_\pm y_\mu \cong -2\Pi_{\mp\mu\nu}[\Delta x]\partial_\pm x^\nu \mp 2\beta^\mp_\mu[x]. \tag{13}$$

The canonical form of the T-dual transformations is

$$y'_\mu \cong \frac{1}{\kappa}\pi_\mu - \beta^0_\mu[x], \tag{14}$$
$${}^\star\pi^\mu \cong \kappa x'^\mu + \kappa^2\theta_0^{\mu\nu}\beta^0_\nu[x]. \tag{15}$$

Our intention is to calculate the PB's of the T-dual variables y_μ and $\tilde{y}_\mu$ using PB algebra of the initial variables. Consequently, we assume that initial theory is geometric which means that coordinates x^μ and momenta π_ν satisfy standard PB algebra

$$\{x^\mu(\sigma),\pi_\nu(\bar\sigma)\} = \delta^\mu_\nu\delta(\sigma-\bar\sigma),\quad \{x^\mu(\sigma),x^\nu(\bar\sigma)\}=0,\quad \{\pi_\mu(\sigma),\pi_\nu(\bar\sigma)\}=0. \tag{16}$$

In this article we will calculate, besides already mentioned PB algebra of the T-dual coordinates, also the algebra of the T-dual winding numbers and momenta. For both purposes, the first step is introducing the quantity

$$\Delta Y_\mu(\sigma,\sigma_0) = \int_{\sigma_0}^{\sigma} d\eta\, Y'_\mu(\eta) = Y_\mu(\sigma) - Y_\mu(\sigma_0), \tag{17}$$

where $Y_\mu = (y_\mu,\tilde{y}_\mu)$. The second step is to calculate their PB's. It is obvious that key relation which we have to calculate is PB between σ derivatives of Y's. When we calculate it in three possible cases it turns out that it can be written in the form

$$\{X'_\mu(\sigma),Y'_\nu(\bar\sigma)\} \cong K'_{\mu\nu}(\sigma)\delta(\sigma-\bar\sigma) + L_{\mu\nu}(\sigma)\delta'(\sigma-\bar\sigma)\,. \tag{18}$$

Integrating this relation by parts over σ and $\bar\sigma$, after straightforward calculation, we extract PB we are searching for

$$\{X_\mu(\tau,\sigma), Y_\nu(\tau,\bar\sigma)\} \cong -\left[K_{\mu\nu}(\sigma) - K_{\mu\nu}(\bar\sigma) + L_{\mu\nu}(\bar\sigma)\right]\theta(\sigma-\bar\sigma)\,, \tag{19}$$

where $\theta(\sigma)$ is the step function defined as

$$\theta(\sigma) = \begin{cases} 0 & \text{if } \sigma = 0 \\ 1/2 & \text{if } 0<\sigma<2\pi, \\ 1 & \text{if } \sigma = 2\pi \end{cases} \quad \sigma\in[0,2\pi]. \tag{20}$$

This is a general form of the relation. Using transforation laws we calculate PB's in three cases: $\{y'_\mu(\sigma), y'_\nu(\bar\sigma)\}$, $\{y'_\mu(\sigma), \tilde y'_\nu(\bar\sigma)\}$ and $\{\tilde y'_\mu(\sigma), \tilde y'_\nu(\bar\sigma)\}$, and express them in the form of (18). Reading the corresponding values of K and L and using (19), we get the noncommutativity relations for T-dual closed string coordinates

$$\{y_\mu(\sigma), y_\nu(\bar\sigma)\} \cong -\frac{1}{\kappa}B_{\mu\nu\rho}[x^\rho(\sigma)-x^\rho(\bar\sigma)]\theta(\sigma-\bar\sigma), \tag{21}$$

$$\begin{aligned}\{y_\mu(\sigma), \tilde y_\nu(\bar\sigma)\} \cong &-\Big\{\frac{1}{\kappa}B_{\mu\nu\rho}[\tilde x^\rho(\sigma)-\tilde x^\rho(\bar\sigma)] - \frac{3}{2\kappa}\Gamma^E_{\rho,\mu\nu}[x^\rho(\sigma)-x^\rho(\bar\sigma)]\\ &+\frac{1}{\kappa}g_{\mu\nu} - \frac{3}{2\kappa}\Gamma^E_{\rho,\mu\nu}\,x^\rho(\bar\sigma)\Big\}\theta(\sigma-\bar\sigma),\end{aligned} \tag{22}$$

$$\begin{aligned}\{\tilde y_\mu(\sigma), \tilde y_\nu(\bar\sigma)\} \cong &-\Big\{-\frac{1}{\kappa}\Big[B_{\mu\nu\rho} - 6g_{\mu\alpha}Q^{\alpha\beta}{}_\rho g_{\beta\nu}\Big][x^\rho(\sigma)-x^\rho(\bar\sigma)]\\ &+\Big[-\frac{3}{2\kappa}\Big(\Gamma^E_{\mu,\nu\rho}-\Gamma^E_{\nu,\mu\rho}\Big)+\frac{4}{\kappa}B_{\mu\nu\sigma}(G^{-1}b)^\sigma{}_\rho\Big][\tilde x^\rho(\sigma)-\tilde x^\rho(\bar\sigma)]\Big\}\theta(\sigma-\bar\sigma)\,,\end{aligned} \tag{23}$$

where

$$\tilde x'^\mu = \frac{1}{\kappa}(G^{-1})^{\mu\nu}\pi_\nu + 2(G^{-1}B)^\mu{}_\nu x'^\nu\,. \tag{24}$$

Here the infinitesimal fluxes are defined as

$$\Gamma^E_{\mu,\nu\rho} = \frac{1}{2}\Big(\partial_\nu G^E_{\mu\rho}+\partial_\rho G^E_{\mu\nu}-\partial_\mu G^E_{\nu\rho}\Big) = -\frac{4}{3}\Big(B_{\mu\sigma\nu}(G^{-1}b)^\sigma{}_\rho + B_{\mu\sigma\rho}(G^{-1}b)^\sigma{}_\nu\Big), \tag{25}$$

$$Q^{\mu\nu}{}_\rho = -\frac{1}{3}\Big[(g^{-1})^{\mu\sigma}(g^{-1})^{\nu\tau} - \kappa^2\theta_0^{\mu\sigma}\theta_0^{\nu\tau}\Big]B_{\sigma\tau\rho}. \tag{26}$$

For $\sigma=\bar\sigma$ we obtain that all PB's vanish, and consequently, coordinates commute. Also we can consider $\sigma=\bar\sigma+2\pi$, which is the same point on the world-sheet as our first choice $\sigma=\bar\sigma$. Taking $\sigma=\bar\sigma+2\pi$, three noncommutativity relations take the form

$$\{y_\mu(\sigma+2\pi), y_\nu(\sigma)\} \cong -\frac{2\pi}{\kappa}B_{\mu\nu\rho}N^\rho\,, \tag{27}$$

$$
\begin{aligned}
&\{y_\mu(\sigma+2\pi), \tilde{y}_\nu(\sigma)\} + \{y_\mu(\sigma), \tilde{y}_\nu(\sigma+2\pi)\} \cong -\frac{4\pi}{\kappa^2} B_{\mu\nu\rho} p^\rho \\
+ \quad & \frac{\pi}{\kappa}\left(3\Gamma^E_{\rho,\mu\nu} - 8B_{\mu\nu\lambda} b^\lambda{}_\rho\right) N^\rho,
\end{aligned}
\tag{28}
$$

and

$$
\begin{aligned}
&\{\tilde{y}_\mu(\sigma+2\pi), \tilde{y}_\nu(\sigma)\} \cong \\
\cong \quad & \frac{2\pi}{\kappa}\left[-B_{\mu\nu\rho} - 6g_{\mu\alpha} Q^{\alpha\beta}{}_\rho g_{\beta\nu} + 2B_{\mu\nu}{}^\lambda g_{\lambda\rho} + 3\left(\Gamma^E_{\mu,\nu\lambda} - \Gamma^E_{\nu,\mu\lambda}\right) b^\lambda{}_\rho\right] N^\rho \\
+ \quad & \frac{\pi}{\kappa^2}\left[3\left(\Gamma^E_{\mu,\nu\rho} - \Gamma^E_{\nu,\mu\rho}\right) p^\rho - 8B_{\mu\nu\lambda} b^\lambda{}_\rho\right] p^\rho \,,
\end{aligned}
\tag{29}
$$

where $N^\mu = \frac{1}{2\pi}\left[x^\mu(\sigma+2\pi) - x^\mu(\sigma)\right]$ is winding number of the initial coordinates and

$$
p_\mu = \frac{1}{2\pi}\int_\sigma^{\sigma+2\pi} d\eta \pi_\mu(\eta) \,, \tag{30}
$$

is mean value of the momentum π_μ. Note that all three PB's are proportional to the Kalb-Ramond field strength which means they are infinitesimal.

In addition we can obtain the algebra of the T-dual winding number and momenta defined as

$$
\Delta y_\mu(2\pi, 0) = 2\pi {}^\star N_\mu \,, \quad \Delta\tilde{y}_\mu(2\pi, 0) = 2\pi {}^\star P_\mu \,, \tag{31}
$$

while we introduced earlier

$$
\Delta x^\mu(2\pi, 0) = 2\pi N^\mu \,, \quad \Delta\tilde{x}^\mu(2\pi, 0) = 2\pi P^\mu \,. \tag{32}
$$

Using (17), (18), transformation laws and above definitions we have

$$
\{{}^\star N_\mu, {}^\star N_\nu\} = \frac{1}{\pi\kappa} B_{\mu\nu\rho} N^\rho \,, \tag{33}
$$

$$
\{{}^\star N_\mu, {}^\star P_\nu\} = \frac{1}{\pi\kappa} B_{\mu\nu\rho} P^\rho - \frac{3}{4\pi\kappa}\Gamma^E_{\rho,\mu\nu} N^\rho \,, \tag{34}
$$

$$
\begin{aligned}
\{{}^\star P_\mu, {}^\star P_\nu\} \quad = \quad & -\frac{1}{\pi\kappa}\left(B_{\mu\nu\rho} - 6g_{\mu\alpha} Q^{\alpha\beta}{}_\rho g_{\beta\nu}\right) N^\rho \\
+ \quad & \frac{1}{\pi}\left[-\frac{3}{2\kappa}(\Gamma^E_{\mu,\nu\rho} - \Gamma^E_{\nu,\mu\rho}) + \frac{4}{\kappa} B_{\mu\nu\sigma} (G^{-1}b)^\sigma{}_\rho\right] P^\rho \,.
\end{aligned}
$$

3. Concluding remarks

In the present article we considered the theory describing the closed bosonic string moving in the weakly curved background and derived the non-commutativity relations using canonical approach.

We applied generalized T-duality procedure and obtained the transformation laws connecting the initial and T-dual variables. They, expressed in the canonical form, have the central role in calculation of the PB's of the T-dual coordinates y_μ and $\tilde{y}_\mu$. Infinitesimal Kalb-Ramond field strength, as a part of the function β_μ, gives the main contribution to the noncommutativity parameters. The result is that we showed the physical equivalence of the commutative initial theory and noncommutative T-dual one in linear approximation in the field strength $B_{\mu\nu\rho}$.

The general structure of the non-commutativity relations is

$$\{Y_\mu(\sigma), Y_\nu(\bar{\sigma})\} = \{F_{\mu\nu\rho}\,[x^\rho(\sigma) - x^\rho(\bar{\sigma})] + \tilde{F}_{\mu\nu\rho}\,[\tilde{x}^\rho(\sigma) - \tilde{x}^\rho(\bar{\sigma})]\}\theta(\sigma - \bar{\sigma})\,, \tag{35}$$

where $Y_\mu = (y_\mu, \tilde{y}_\nu)$ and $F_{\mu\nu\rho}$ and $\tilde{F}_{\mu\nu\rho}$ are the constant and infinitesimally small fluxes. At the same points, for $\sigma = \bar{\sigma}$ all PB's are zero. In the important particular case for $\sigma = \bar{\sigma} + 2\pi$ we get

$$\{Y_\mu(\sigma + 2\pi), Y_\nu(\sigma)\} = 2\pi\left[(F_{\mu\nu\rho} + 2\tilde{F}_{\mu\nu\alpha}b^\alpha_\rho)N^\rho + \frac{1}{\kappa}\tilde{F}_{\mu\nu}{}^\rho p_\rho\right]\,, \tag{36}$$

where N^μ and p_μ are winding numbers and momenta of the original theory. In addition we calculated the PB algebra of the T-dual winding numbers and momenta in terms of the initial ones.

References

[1] F. Ardalan, H. Arfaei and M. M. Sheikh-Jabbari, *JHEP* **02** (1999) 016; C. S. Chu and P. M. Ho, *Nucl. Phys.* **B550** (1999) 151; N. Seiberg and E. Witten, *JHEP* **09** (1999) 032; F. Ardalan, H. Arfaei and M. M. Sheikh-Jabbari *Nucl. Phys.* **B576** (2000) 578; C. S. Chu and P. M. Ho, *Nucl. Phys.* **B568** (2000) 447; T.Lee, *Phys. Rev.* **D62** (2000) 024022.

[2] B. Sazdović, Eur. Phys. J. **C44** (2005) 599; B. Nikolić and B. Sazdović, *Phys. Rev.* **D 74** (2006) 045024; B. Nikolić and B. Sazdović, *Phys. Rev.* **D 75** (2007) 085011; B. Nikolić and B. Sazdović, *Adv. Theor. Math. Phys.* **14** (2010) 1, Lj. Davidovic and B. Sazdovic, *Phys. Rev.* **D 83** (2011) 066014; *JHEP* **08** (2011)112.

[3] T. H. Buscher, *Phys. Lett.* **B194** (1987) 59; T. H. Buscher, *Phys. Lett.* **B201** (1988) 466.

[4] E. S. Fradkin and A. A. Tseytlin, *Phys. Lett.* **B 158** (1985) 316; *Nucl. Phys.* **B 261** (1985) 1.

[5] K. Becker, M. Becker and J. Schwarz *String Theory and M-Theory: A Modern Introduction*, Cambridge University Press 2006; B. Zwiebach, *A First Course in String Theory*, Cambridge University Press, 2004.

[6] D. Luest, *JHEP* **12** (2010) 084.

[7] D. Andriot, M. Larfors, D. Luest, P. Patalong, *JHEP* **06** (2013) 021.

[8] D. Andriot, O. Hohm, M. Larfors, D. Luest, P. Patalong, *Phys. Rev. Lett.* 108 (2012) 261602.

[9] D. Luest, arxiv:1205.0100 [hep-th]; R. Blumenhagen, A. Deser, D. Luest, E. Plauschinn, and F. Rennecke, *J.Phys.* **A44** (2011) 385401; C. Condeescu, I. Florakis, and D. Luest, *JHEP* **04** (2012) 121.

[10] Lj. Davidović and B. Sazdović, arXiv:1205.1991 [hep-th].

[11] V. Schomerus, *Class. Quant. Grav.* **19** (2002) 5781; L.Cornalba and R.Schiappa, *Commun. Math. Phys.* **225** (2002) 33.

[12] Lj. Davidović and B. Sazdović, *Phys. Rev.* **D 83** (2011) 066014; *JHEP* **08** (2011) 112; *EPJ* **C 72** No. 11 (2012) 2199.

Springer Proceedings in Mathematics & Statistics

Vladimir Dobrev *Editor*

Lie Theory and Its Applications in Physics

Varna, Bulgaria, June 2013

Springer

Springer Proceedings in Mathematics & Statistics

Volume 111

More information about this series at http://www.springer.com/series/10533

Springer Proceedings in Mathematics & Statistics

This book series features volumes composed of select contributions from workshops and conferences in all areas of current research in mathematics and statistics, including operation research and optimization. In addition to an overall evaluation of the interest, scientific quality, and timeliness of each proposal at the hands of the publisher, individual contributions are all refereed to the high quality standards of leading journals in the field. Thus, this series provides the research community with well-edited, authoritative reports on developments in the most exciting areas of mathematical and statistical research today.

Vladimir Dobrev
Editor

Lie Theory and Its Applications in Physics

Varna, Bulgaria, June 2013

Springer

Editor
Vladimir Dobrev
Institute for Nuclear Research
and Nuclear Energy
Bulgarian Academy of Sciences
72 Tsarigradsko Chaussee
Sofia, Bulgaria

ISSN 2194-1009 ISSN 2194-1017 (electronic)
ISBN 978-4-431-55284-0 ISBN 978-4-431-55285-7 (eBook)
DOI 10.1007/978-4-431-55285-7
Springer Tokyo Heidelberg New York Dordrecht London

Library of Congress Control Number: 2014958024

Printed on acid-free paper

Springer is part of Springer Science+Business Media (www.springer.com)

Preface

The Workshop series 'Lie Theory and Its Applications in Physics' is designed to serve the community of theoretical physicists, mathematical physicists and mathematicians working on mathematical models for physical systems based on geometrical methods and in the field of Lie theory.

The series reflects the trend towards a geometrisation of the mathematical description of physical systems and objects. A geometric approach to a system yields in general some notion of symmetry which is very helpful in understanding its structure. Geometrisation and symmetries are meant in their widest sense, i.e., representation theory, algebraic geometry, infinite-dimensional Lie algebras and groups, superalgebras and supergroups, groups and quantum groups, noncommutative geometry, symmetries of linear and nonlinear PDE, special functions. Furthermore we include the necessary tools from functional analysis and number theory. This is a big interdisciplinary and interrelated field.

The first three workshops were organised in Clausthal (1995, 1997, 1999), the 4th was part of the 2nd Symposium 'Quantum Theory and Symmetries' in Cracow (2001), the 5th, 7th, 8th and 9th were organised in Varna (2003, 2007, 2009, 2011), the 6th was part of the 4th Symposium 'Quantum Theory and Symmetries' in Varna (2005), but has its own volume of Proceedings.

The 10th Workshop of the series (LT-10) was organized by the Institute of Nuclear Research and Nuclear Energy of the Bulgarian Academy of Sciences (BAS) in June 2013 (17–23), at the Guest House of BAS near Varna on the Bulgarian Black Sea Coast.

The overall number of participants was 71 and they came from 21 countries.

The scientific level was very high as can be judged by the speakers. The *plenary speakers* were: Loriano Bonora (Trieste), Branko Dragovich (Belgrade), Ludvig Faddeev (St. Petersburg), Malte Henkel (Nancy), Evgeny Ivanov (Dubna), Toshiyuki Kobayashi (Tokyo), Ivan Kostov (Saclay), Karl-Hermann Neeb (Erlangen), Eric Ragoucy (Annecy), Ivan Todorov (Sofia), Joris Van Der Jeugt (Ghent), George Zoupanos (Athens).

The topics covered the most modern trends in the field of the Workshop: Symmetries in String Theories and Gravity Theories, Conformal Field Theory, Integrable Systems, Representation Theory, Supersymmetry, Quantum Groups, Vertex Algebras and Superalgebras, Quantum Computing.

There is some similarity with the topics of preceding workshops, however, the comparison shows how certain topics evolve and that new structures were found and used. For the present workshop we mention more emphasis on: representation theory, quantum groups, integrable systems, vertex algebras and superalgebras, on conformal field theories, applications to the minimal supersymmetric standard model.

The International Organizing Committee was: V.K. Dobrev (Sofia) and H.-D. Doebner (Clausthal) in collaboration with G. Rudolph (Leipzig).

The Local Organizing Committee was: V.K. Dobrev (Chairman), V.I. Doseva, A.Ch. Ganchev, S.G. Mihov, D.T. Nedanovski, T.V. Popov, T.P. Stefanova, M.N. Stoilov, N.I. Stoilova, S.T. Stoimenov.

Acknowledgments

We express our gratitude to the

– Institute of Nuclear Research and Nuclear Energy
– Abdus Salam International Centre for Theoretical Physics

for financial help. We thank the Bulgarian Academy of Sciences for providing its Guest House which contributed very much to the stimulating and pleasant atmosphere during the Workshop. We thank the Publisher, Springer Japan, represented by Ms. Chino Hasebe (Executive Editor in Mathematics, Statistics, Business, Economics, Computer Science) and Mr. Masayuki Nakamura (Editorial Department), for assistance in the publication. Last, but not least, I thank the members of the Local Organizing Committee who, through their efforts, made the workshop run smoothly and efficiently.

Sofia, Bulgaria Vladimir Dobrev
May 2014

Contents

Part I Plenary Talks

List of Participants

Ana Agore Vrije Universiteit Brussel, Belgium

Iana Anguelova College of Charleston, USA

Simona Babeti Politehnica University Timisoara, Romania

Katrina Barron University of Notre Dame, USA

Elaine Beltaos Grant MacEwan University, Edmonton, Canada

Agata Bezubik University of Bialystok, Poland

Loriano Bonora SISSA, Trieste, Italy

Cestmir Burdik Czech Technical University in Prague, Czech Republic

Jamil Daboul Ben-Gurion University of the Negev, Beer Sheva, Israel

Ljubica Davidovic Institute of Physics, Belgrade, Serbia

Ivan Dimitrijevic University of Belgrade, Serbia

Vladimir Dobrev INRNE, BAS, Sofia, Bulgaria

Branko Dragovich Institute of Physics, Belgrade, Serbia

Charalambos Evripidou University of Cyprus, Nicosia, Cyprus

Ludvig Faddeev Steklov Mathematical Institute of Russian Academy of Sciences, St. Petersburg, Russia

Alon Faraggi University of Liverpool, UK

Laszlo Feher Hungarian Academy of Sciences, Budapest, Hungary

Uwe Franz Université de Franche-Comté, Besançon, France

Paolo Furlan Trieste University, Italy

Akito Futaki University of Tokyo, Japan

Alexander Ganchev INRNE, BAS, Sofia, Bulgaria

Lachezar Georgiev INRNE, BAS, Sofia, Bulgaria

Tetiana Gerasymova Taras Shevchenko National University, Kiev, Ukraine

Joanna Gonera University of Lodz, Poland

Jelena Grujic University of Belgrade, Serbia

Leszek Hadasz Jagiellonian University, Krakow, Poland

Ludmil Hadjiivanov INRNE, BAS, Sofia, Bulgaria

Malte Henkel Universite de Lorraine, Nancy, France

Jiri Hrivnak Czech Technical University in Prague, Czech Republic

Evgeny Ivanov Joint Institute for Nuclear Research, Dubna, Russian Federation

Elchin Jafarov Azerbaijan National Academy of Sciences, Baku, Azerbaijan

Masao Jinzenji Hokkaido University, Japan

Dalibor Karasek Czech Technical University in Prague, Czech Republic

Mariana Kirchbach Autonomous University at San Luis Potosi, Mexico

Toshiyuki Kobayashi University of Tokyo, Japan

Takeo Kojima Yamagata University, Yonezawa, Japan

Alexey Koshelev Vrije Universiteit Brussel, Belgium

Ivan Kostov CEA Saclay, Gif sur Yvette, France

Oksana Kuriksha Petro Mohyla Black Sea State University, Ukraine

Gigel Militaru University of Bucharest, Romania

Lenka Motlochova Universite de Montreal, Canada

Stefan Mihov INRNE, BAS, Sofia, Bulgaria

Patrick Moylan Pennsylvania State University, Abington, USA

Dimitar Nedanovski INRNE, BAS, Sofia, Bulgaria

Karl-Hermann Neeb Friedrich-Alexander University Erlangen-Nuremberg, Germany

Nikolay Nikolov INRNE, BAS, Sofia, Bulgaria

Teodor Nikolov University of Edinburgh, UK

Emil Nissimov INRNE, BAS, Sofia, Bulgaria

Petr Novotny Czech Technical University in Prague, Czech Republic

Svetlana Pacheva INRNE, BAS, Sofia, Bulgaria

Tchavdar Palev INRNE, BAS, Sofia, Bulgaria

Valentina Petkova INRNE, BAS, Sofia, Bulgaria

Elena Poletaeva University of Texas-Pan American, Edinburg, USA

Todor Popov INRNE, BAS, Sofia, Bulgaria

Severin Posta Czech Technical University in Prague, Czech Republic

Helene Quintard Université de Rouen, Saint-Étienne-du-Rouvray, France

Eric Ragoucy LAPTh-CNRS, Annecy-le-Vieux, France

Zoran Rakic University of Belgrade, Serbia

Igor Salom Institute of Physics, Belgrade, Serbia

Michail Stoilov INRNE, BAS, Sofia, Bulgaria

Neli Stoilova INRNE, BAS, Sofia, Bulgaria

Stoimen Stoimenov INRNE, BAS, Sofia, Bulgaria

Fumihiko Sugino Okayama Institute for Quantum Physics, Japan

Tetsuya Taniguchi Kitasato University, Kanagawa, Japan

Ivan Todorov INRNE, BAS, Sofia, Bulgaria

Valeriy Tolstoy Lomonosov Moscow State University, Russian Federation

Joris Van Der Jeugt Ghent University, Belgium

Olena Vaneeva National Academy of Sciences of Ukraine, Kiev, Ukraine

Milen Yakimov Louisiana State University, Baton Rouge, USA

Iryna Yehorchenko National Academy of Sciences of Ukraine, Kiev, Ukraine

George Zoupanos National Technical University in Athens, Greece

Complete T-Dualization of a String in a Weakly Curved Background

Lj. Davidović, B. Nikolić, and B. Sazdović

Abstract We apply the generalized Buscher procedure, to a subset of the initial coordinates of the bosonic string moving in the weakly curved background, composed of a constant metric and a linearly coordinate dependent Kalb-Ramond field with the infinitesimal strength. In this way we obtain the partially T-dualized action. Applying the procedure to the rest of the original coordinates we obtain the totally T-dualized action. This derivation allows the investigation of the relations between the Poisson structures of the original, the partially T-dualized and the totally T-dualized theory.

1 Bosonic String in the Weakly Curved Background

Let us consider the closed string moving in the coordinate dependent background, described by the action [1]

$$S[x] = \kappa \int_{\Sigma} d^2\xi \ \partial_+ x^\mu \Pi_{+\mu\nu}[x] \partial_- x^\nu. \tag{1}$$

The background is defined by the space-time metric $G_{\mu\nu}$ and the antisymmetric Kalb-Ramond field $B_{\mu\nu}$

$$\Pi_{\pm\mu\nu}[x] = B_{\mu\nu}[x] \pm \frac{1}{2} G_{\mu\nu}[x]. \tag{2}$$

The light-cone coordinates are

$$\xi^{\pm} = \frac{1}{2}(\tau \pm \sigma), \qquad \partial_{\pm} = \partial_\tau \pm \partial_\sigma, \tag{3}$$

and the action is given in the conformal gauge (the world-sheet metric is taken to be $g_{\alpha\beta} = e^{2F}\eta_{\alpha\beta}$).

Lj. Davidović (✉) • B. Nikolić • B. Sazdović
Institute of Physics, Pregrevica 118, 11080 Belgrade, Serbia
e-mail: ljubica@ipb.ac.rs; bnikolic@ipb.ac.rs; sazdovic@ipb.ac.rs

V. Dobrev (ed.), *Lie Theory and Its Applications in Physics*, Springer Proceedings in Mathematics & Statistics 111, DOI 10.1007/978-4-431-55285-7_2

The world-sheet conformal invariance is required, as a condition of having a consistent theory on a quantum level. This leads to the space-time equations for the background fields, which equal

$$R_{\mu\nu} - \frac{1}{4} B_{\mu\rho\sigma} B_{\nu}^{\ \rho\sigma} = 0, \quad D_\rho B^\rho_{\ \mu\nu} = 0, \tag{4}$$

in the lowest order in slope parameter α' and for the constant dilaton field $\Phi = const$. Here $B_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} + \partial_\nu B_{\rho\mu} + \partial_\rho B_{\mu\nu}$ is the field strength of the field $B_{\mu\nu}$, and $R_{\mu\nu}$ and D_μ are Ricci tensor and covariant derivative with respect to the space-time metric.

We will consider a weakly curved background [2, 3], defined by

$$\begin{aligned} &G_{\mu\nu}[x] = const, \\ &B_{\mu\nu}[x] = b_{\mu\nu} + h_{\mu\nu}[x] = b_{\mu\nu} + \frac{1}{3} B_{\mu\nu\rho} x^\rho, \qquad b_{\mu\nu}, B_{\mu\nu\rho} = const. \end{aligned} \tag{5}$$

Here, the constant $B_{\mu\nu\rho}$ is infinitesimal. The background (5) is the solution of the field equations (4) in the first order in $B_{\mu\nu\rho}$.

2 Partial T-Dualization

In the paper [3], we generalized the Buscher prescription for a construction of a T-dual theory. This prescription, unlike the standard one [4], is applicable to the string backgrounds depending on all the space-time coordinates, such as the weakly curved background. We performed the procedure along all the coordinates and obtained T-dual theory. The noncommutativity of the T-dual coordinates we investigated in [5]. In the present paper we consider the partial T-dualization, i.e. the application of the procedure to some without subset of the coordinates. We construct the partially T-dualized theory. The noncommutativity of the coordinates in similar theories was considered in [6].

Let us mark the T-dualization along the coordinate x^μ by T_μ, and separate the coordinates into two subsets (x^i, x^a) with $i = 0, \dots, d-1$ and $a = d, \dots, D-1$ and mark the T-dualizations along these subsets of coordinates by

$$T^i \equiv T_0 \circ \cdots \circ T_{d-1}, \quad T^a \equiv T_d \circ \cdots \circ T_{D-1}. \tag{6}$$

In this section we will find the partially T-dualized action performing T-dualization along coordinates x^a, $\mathcal{T}^a : S$.

The closed string action in the weakly curved background has a global symmetry

$$\delta x^\mu = \lambda^\mu. \tag{7}$$

Let us localize this symmetry for the coordinates x^a

$$\delta x^a = \lambda^a(\tau,\sigma), \quad a = d,\dots,D-1, \tag{8}$$

by introducing the gauge fields v^a_α and substituting the ordinary derivatives with the covariant ones

$$\partial_\alpha x^a \to D_\alpha x^a = \partial_\alpha x^a + v^a_\alpha. \tag{9}$$

The gauge invariance of the covariant derivatives is obtained by imposing the following transformation law for the gauge fields

$$\delta v^a_\alpha = -\partial_\alpha \lambda^a. \tag{10}$$

Also, substitute x^a in the argument of the background fields with its invariant extension, defined by

$$\begin{aligned}\Delta x^a_{inv} &\equiv \int_P d\xi^\alpha\, D_\alpha x^a = \int_P (d\xi^+ D_+ x^a + d\xi^- D_- x^a)\\ &= x^a - x^a(\xi_0) + \Delta V^a,\end{aligned} \tag{11}$$

where

$$\Delta V^a \equiv \int_P d\xi^\alpha v^a_\alpha = \int_P (d\xi^+ v^a_+ + d\xi^- v^a_-). \tag{12}$$

The line integral is taken along the path P, from the initial point $\xi^\alpha_0(\tau_0,\sigma_0)$ to the final one $\xi^\alpha(\tau,\sigma)$. To preserve the physical equivalence between the gauged and the original theory, one introduces the Lagrange multiplier y_a and adds the term $\frac{1}{2} y_a F^a_{+-}$ to the Lagrangian, which will force the field strength $F^a_{+-} \equiv \partial_+ v^a_- - \partial_- v^a_+ = -2F^a_{01}$ to vanish. In this way, we obtain the gauge invariant action

$$\begin{aligned}S_{inv} = \kappa \int d^2\xi \Big[&\partial_+ x^i \Pi_{+ij}[x^i, \Delta x^a_{inv}]\partial_- x^j + \partial_+ x^i \Pi_{+ia}[x^i, \Delta x^a_{inv}] D_- x^a\\ &+ D_+ x^a \Pi_{+ai}[x^i, \Delta x^a_{inv}]\partial_- x^i + D_+ x^a \Pi_{+ab}[x^i, \Delta x^a_{inv}] D_- x^b\\ &+ \frac{1}{2}(v^a_+ \partial_- y_a - v^a_- \partial_+ y_a)\Big],\end{aligned} \tag{13}$$

where the last term is equal to $\frac{1}{2} y_a F^a_{+-}$ up to the total divergence. Now, we can use the gauge freedom to fix the gauge $x^a(\xi) = x^a(\xi_0)$. The gauge fixed action equals

$$S_{fix} = \kappa \int d^2\xi \Big[\partial_+ x^i \Pi_{+ij}[x^i, \Delta V^a] \partial_- x^j + \partial_+ x^i \Pi_{+ia}[x^i, \Delta V^a] v_-^a + v_+^a \Pi_{+ai}[x^i, \Delta V^a] \partial_- x^i + v_+^a \Pi_{+ab}[x^i, \Delta V^a] v_-^b + \frac{1}{2}(v_+^a \partial_- y_a - v_-^a \partial_+ y_a) \Big]. \tag{14}$$

The equations of motion for the Lagrange multiplier y_a, $\partial_+ v_-^a - \partial_- v_+^a = 0$, have a solution $v_\pm^a = \partial_\pm x^a$, which turns the gauge fixed action to the initial one.

2.1 The Partially T-Dualized Action

The partially T-dualized action will be obtained after elimination of the gauge fields from the gauge fixed action (14), using their equations of motion. Varying over the gauge fields $v_\pm^a$ one obtains

$$\Pi_{\pm ai}[x^i, \Delta V^a] \partial_\mp x^i + \Pi_{\pm ab}[x^i, \Delta V^a] v_\mp^b + \frac{1}{2} \partial_\mp y_a = \pm \beta_a^\pm [x^i, V^a], \tag{15}$$

where $\beta_a^\pm[x^i, V^a]$ is the infinitesimal contribution from the background fields argument. Using the inverse of the background fields composition $2\kappa \Pi_{\pm ab}$, defined by $\tilde{\Theta}_\pm^{ab} \equiv -\frac{2}{\kappa}(\tilde{G}_E^{-1})^{ac} \Pi_{\pm cd} (\tilde{G}^{-1})^{db}$, where $\tilde{G}_{ab} \equiv G_{ab}$ and $\tilde{G}_{Eab} \equiv G_{ab} - 4B_{ac}(\tilde{G}^{-1})^{cd} B_{db}$, we can extract the gauge fields $v_\pm^a$ from Eq. (15)

$$v_\mp^a = -2\kappa \tilde{\Theta}_\mp^{ab}[x^i, \Delta V^a] \Big[\Pi_{\pm bi}[x^i, \Delta V^a] \partial_\mp x^i + \frac{1}{2} \partial_\mp y_b \mp \beta_b^\pm [x^i, V^a] \Big]. \tag{16}$$

Substituting (16) into the action (14), we obtain the partially T-dualized action

$$S_\pi[x^i, y_a] = \kappa \int d^2\xi \Big[\partial_+ x^i \bar{\Pi}_{+ij}[x^i, \Delta V^a(x^i, y^a)] \partial_- x^j + \frac{\kappa}{2} \partial_+ y_a \tilde{\Theta}_-^{ab}[x^i, \Delta V^a(x^i, y^a)] \partial_- y_b - \kappa \, \partial_+ x^i \Pi_{+ia}[x^i, \Delta V^a(x^i, y^a)] \tilde{\Theta}_-^{ab}[x^i, \Delta V^a(x^i, y^a)] \partial_- y_b + \kappa \, \partial_+ y_a \tilde{\Theta}_-^{ab}[x^i, \Delta V^a(x^i, y^a)] \Pi_{+bi}[x^i, \Delta V^a(x^i, y^a)] \partial_- x^i \Big], \tag{17}$$

where

$$\bar{\Pi}_{+ij} \equiv \Pi_{+ij} - 2\kappa \Pi_{+ia} \tilde{\Theta}_-^{ab} \Pi_{+bj}. \tag{18}$$

In order to find the explicit value of the background fields argument $\Delta V^a(x^i, y^a)$, one substitutes the zeroth order of the equations of motion (16) into (12) and obtains

$$\begin{aligned}\Delta V^{(0)a} &= -\kappa\Big[\tilde{\Theta}^{ab}_{0+}\Pi_{0-bi} + \tilde{\Theta}^{ab}_{0-}\Pi_{0+bi}\Big]\Delta x^{(0)i} \\ &\quad - \kappa\Big[\tilde{\Theta}^{ab}_{0+}\Pi_{0-bi} - \tilde{\Theta}^{ab}_{0-}\Pi_{0+bi}\Big]\Delta \tilde{x}^{(0)i} \\ &\quad - \frac{\kappa}{2}\Big[\tilde{\Theta}^{ab}_{0+} + \tilde{\Theta}^{ab}_{0-}\Big]\Delta y^{(0)}_b - \frac{\kappa}{2}\Big[\tilde{\Theta}^{ab}_{0+} - \tilde{\Theta}^{ab}_{0-}\Big]\Delta \tilde{y}^{(0)}_b, \end{aligned} \tag{19}$$

where $\tilde{\Theta}^{ab}_{0\pm}$ stands for the zeroth order value of $\tilde{\Theta}^{ab}_{\pm}$, which can be written as

$$\tilde{\Theta}^{ab}_{0\pm} \equiv -\frac{2}{\kappa}(\tilde{g}^{-1})^{ac}\,\Pi_{0\pm cd}(\tilde{G}^{-1})^{db} = \tilde{\theta}^{ab}_0 \mp \frac{1}{\kappa}(\tilde{g}^{-1})^{ab}, \tag{20}$$

where $\tilde{g}_{ab} = G_{ab} - 4b_{ac}(\tilde{G}^{-1})^{cd}b_{db}$; $\tilde{\theta}^{ab}_0 \equiv -\frac{2}{\kappa}(\tilde{g}^{-1})^{ac}\,b_{cd}(\tilde{G}^{-1})^{db}$ and

$$\Delta \tilde{y}^{(0)}_a = \int (d\tau y'^{(0)}_a + d\sigma \dot{y}^{(0)}_a), \quad \Delta \tilde{x}^{(0)i} = \int (d\tau x'^{(0)i} + d\sigma \dot{x}^{(0)i}). \tag{21}$$

Initial theory, the partially T-dualized theory and the totally T-dualized theory obtained in [3] are physically equivalent theories. In the next section we will partially T-dualize the partially T-dualized theory.

3 The Total T-Dualization of the Initial Action

The T-dual theory, derived in [3], a result of T-dualization of the initial action along all the coordinates, is given by

$${}^\star S[y] = \kappa\int d^2\xi\,\partial_+ y_\mu\,{}^\star\Pi^{\mu\nu}_+[\Delta V(y)]\,\partial_- y_\nu = \frac{\kappa^2}{2}\int d^2\xi\,\partial_+ y_\mu \Theta^{\mu\nu}_-[\Delta V(y)]\partial_- y_\nu, \tag{22}$$

with

$$\Theta^{\mu\nu}_\pm \equiv -\frac{2}{\kappa}(G_E^{-1}\Pi_\pm G^{-1})^{\mu\nu} = \theta^{\mu\nu} \mp \frac{1}{\kappa}(G_E^{-1})^{\mu\nu}, \tag{23}$$

where

$$G_{E\mu\nu} \equiv G_{\mu\nu} - 4(BG^{-1}B)_{\mu\nu}, \quad \theta^{\mu\nu} \equiv -\frac{2}{\kappa}(G_E^{-1}BG^{-1})^{\mu\nu}. \tag{24}$$

The T-dual background fields are equal to

$$^\star G^{\mu\nu}[\Delta V(y)] = (G_E^{-1})^{\mu\nu}[\Delta V(y)], \quad ^\star B^{\mu\nu}[\Delta V(y)] = \frac{\kappa}{2}\theta^{\mu\nu}[\Delta V(y)]. \tag{25}$$

The argument of the background fields is given by

$$\Delta V^\mu(y) = -\kappa\theta_0^{\mu\nu}\Delta y_\nu + (g^{-1})^{\mu\nu}\Delta\tilde{y}_\nu, \tag{26}$$

where $\Delta y_\mu = y_\mu(\xi) - y_\mu(\xi_0)$ and $\tilde{y}_\mu = \int(d\tau y'_\mu + d\sigma\dot{y}_\mu)$, while $g_{\mu\nu} = G_{\mu\nu} - 4b^2_{\mu\nu}$ and $\theta_0^{\mu\nu} = -\frac{2}{\kappa}(g^{-1}bG^{-1})^{\mu\nu}$.

Let us now show that the same result will be obtained applying the T-dualization procedure to the coordinates x^i of the partially T-dualized theory (17), $\mathcal{T}^i : S_\pi[x^i, y_a]$. Substituting the ordinary derivatives $\partial_\pm x^i$ with the covariant derivatives

$$D_\pm x^i = \partial_\pm x^i + v^i_\pm, \tag{27}$$

where the gauge fields $v^i_\pm$ transform as $\delta v^i_\pm = -\partial_\pm\lambda^i$, and substituting the coordinates x^i in the background field arguments by

$$\Delta x^i_{inv} = \int_P (d\xi^+ D_+ x^i + d\xi^- D_- x^i), \tag{28}$$

we obtain the gauge invariant action, which after fixing the gauge by $x^i(\xi) = x^i(\xi_0)$ becomes

$$\begin{aligned} S_\pi^{fix} = \kappa\int d^2\xi\Big[& v^i_+\bar{\Pi}_{+ij}[\Delta V^\mu]v^j_- + \frac{\kappa}{2}\partial_+ y_a\tilde{\Theta}_-^{ab}[\Delta V^\mu]\partial_- y_b \\ & -\kappa\, v^i_+\Pi_{+ia}[\Delta V^\mu]\tilde{\Theta}_-^{ab}[\Delta V^\mu]\partial_- y_b + \kappa\,\partial_+ y_a\tilde{\Theta}_-^{ab}[\Delta V^\mu]\Pi_{+bi}[\Delta V^\mu]v^i_- \\ & +\frac{1}{2}(v^i_+\partial_- y_i - v^i_-\partial_+ y_i)\Big]. \end{aligned} \tag{29}$$

Here ΔV^i is defined by

$$\Delta V^i \equiv \int_P (d\xi^+ v^i_+ + d\xi^- v^i_-), \tag{30}$$

and ΔV^a is defined in (19), whose arguments are in this case ΔV^i and y^a.

The totally T-dualized action will be obtained by eliminating the gauge fields from the gauge fixed action, using their equations of motion. Varying the action (29) over the gauge fields $v^i_\pm$ one obtains

$$\bar{\Pi}_{\pm ij}v^j_\mp - \kappa\Pi_{\pm ia}\tilde{\Theta}_\mp^{ab}\partial_\mp y_b + \frac{1}{2}\partial_\mp y_i = \pm\beta_i^\pm. \tag{31}$$

Using the fact that the background field composition $\bar{\Pi}_{\pm ij}$ is inverse to $2\kappa\Theta^{ij}_{\mp}$, we can rewrite the equation of motion (31) expressing the gauge fields as

$$v^i_{\mp} = 2\kappa\Theta^{ij}_{\mp}\Big[\kappa\Pi_{\pm ja}\tilde{\Theta}^{ab}_{\mp}\partial_{\mp}y_b - \frac{1}{2}\partial_{\mp}y_j \pm \beta^{\pm}_j\Big]. \tag{32}$$

Using $\Pi_{\pm ab}\Theta^{bi}_{\mp} = -\Pi_{\pm aj}\Theta^{ji}_{\mp}$, we note that

$$\Theta^{ij}_{\mp}\Pi_{\pm ja}\tilde{\Theta}^{ab}_{\mp} = -\Theta^{ic}_{\mp}\Pi_{\pm ca}\tilde{\Theta}^{ab}_{\mp} = -\frac{1}{2\kappa}\Theta^{ib}_{\mp}, \tag{33}$$

and obtain

$$v^i_{\mp} = -\kappa\Theta^{i\mu}_{\mp}\partial_{\mp}y_\mu \pm 2\kappa\Theta^{ij}_{\mp}\beta^{\pm}_j. \tag{34}$$

Substituting (34) into (29), the action becomes

$$\begin{aligned} S = \kappa\int d^2\xi\Big[&\partial_+y_i\Big(\kappa\Theta^{ij}_- - \kappa^2\Theta^{ik}_-\bar{\Pi}_{+kl}\Theta^{lj}_-\Big)\partial_-y_j \\ &+ \partial_+y_a\Big(-\kappa^2\Theta^{aj}_-\bar{\Pi}_{+jk}\Theta^{ki}_- + \frac{\kappa}{2}\Theta^{ai}_- - \kappa^2\tilde{\Theta}^{ab}_-\Pi_{+bj}\Theta^{ji}_-\Big)\partial_-y_i \\ &+ \partial_+y_i\Big(-\kappa^2\Theta^{ij}_-\bar{\Pi}_{+jk}\Theta^{ka}_- + \frac{\kappa}{2}\Theta^{ia}_- - \kappa^2\Theta^{ij}_-\Pi_{+jb}\tilde{\Theta}^{ba}_-\Big)\partial_-y_a \\ &+ \partial_+y_a\Big(\frac{\kappa}{2}\tilde{\Theta}^{ab}_- - \kappa^2\Theta^{ai}_-\bar{\Pi}_{+ij}\Theta^{jb}_- - \kappa^2\Theta^{ai}_-\Pi_{+ic}\tilde{\Theta}^{cb}_- - \kappa^2\tilde{\Theta}^{ac}_-\Pi_{+ci}\Theta^{ib}_-\Big)\partial_-y_b\Big]. \end{aligned} \tag{35}$$

Using $\bar{\Pi}_{\pm ij}\Theta^{jk}_{\mp} = \Theta^{kj}_{\mp}\bar{\Pi}_{\pm ji} = \frac{1}{2\kappa}\delta^k_i$; $\tilde{\Pi}_{\pm ab}\Theta^{bc}_{\mp} = \Theta^{cb}_{\mp}\tilde{\Pi}_{\pm ba} = \frac{1}{2\kappa}\delta^c_a$; $\Pi_{\pm ab}\Theta^{bi}_{\mp} = -\Pi_{\pm aj}\Theta^{ji}_{\mp}$; $\Pi_{\pm ij}\Theta^{ja}_{\mp} = -\Pi_{\pm ib}\Theta^{ba}_{\mp}$ and $\Theta^{ci}_{\mp}\bar{\Pi}_{\pm ik} = -\tilde{\Theta}^{ca}_{\mp}\Pi_{\pm ak}$, one can rewrite this action as

$$S = \frac{\kappa^2}{2}\int d^2\xi\,\partial_+y_\mu\Theta^{\mu\nu}_-\partial_-y_\nu. \tag{36}$$

In order to find the background fields argument ΔV^i, we consider the zeroth order of Eq. (34)

$$v^i_{0\mp} = -\kappa\Theta^{i\mu}_{0\mp}\partial_{\mp}y_\mu, \tag{37}$$

and conclude that

$$\Delta V^i = -\kappa\theta^{i\mu}_0\Delta y_\mu + (g^{-1})^{i\mu}\Delta\tilde{y}_\mu. \tag{38}$$

Using the integral form of the variables and the relations $\Pi_{\pm ac}\Theta^{cb}_{\mp} + \Pi_{\pm ai}\Theta^{ib}_{\mp} = \frac{1}{2\kappa}\delta^b_a$; $\Theta^{ib}_{\mp} = -2\kappa\bar{\Theta}^{ij}_{\mp}\Pi_{\pm ja}\Theta^{ab}_{\mp}$; $\Theta^{aj}_{\mp} = -2\kappa\tilde{\Theta}^{ab}_{\mp}\Pi_{\pm bi}\Theta^{ij}_{\mp}$, we obtain that $\Delta V^a(\Delta V^i, y^a)$ defined in (19) equals

$$\Delta V^a(\Delta V^i, y_a) = -\kappa\theta_0^{a\mu}\Delta y_\mu + (g^{-1})^{a\mu}\Delta\tilde{y}_\mu. \tag{39}$$

Therefore, we conclude that action (36) is the totally T-dualized action (22).

In this paper we performed the partial T-dualizations and obtained the T-duality chain

$$S[x^\mu] \xrightarrow{\mathcal{T}^a} S_\pi[x^i, y_a] \xrightarrow{\mathcal{T}^i} {}^\star S[y_\mu]. \tag{40}$$

The first action describes the geometrical background, while the second and the third describe the non-geometrical backgrounds with nontrivial fluxes. From this chain one can find the relations between the arbitrary two coordinates in the chain. These general T-duality coordinate transformation laws are used in the investigation of the relations between the Poisson structures of the original, the partially T-dualized and the totally T-dualized theory [5]. Their canonical form will be used in deriving the complete closed string non-commutativity relations, which are the important features of the non-geometrical backgrounds.

Acknowledgements Work supported in part by the Serbian Ministry of Education, Science and Technological Development, under contract No. 171031.

References

1. Becker, K., Becker, M., Schwarz, J.: String Theory and M-Theory: A Modern Introduction. Cambridge University Press, Cambridge (2007); Zwiebach, B.: A First Course in String Theory. Cambridge University Press, Cambridge (2004)
2. Davidović, Lj., Sazdović, B.: Phys. Rev. **D83**, 066014 (2011); J. High Energy Phys. **08**, 112 (2011); Eur. Phys. J. **C72**(11), 2199 (2012)
3. Davidović, Lj., Sazdović, B.: Eur. Phys. J. **C74**(1), 2683 (2014)
4. Buscher, T.: Phys. Lett. **B194**, 51 (1987); **201**, 466 (1988); Ročer, M., Verlinde, E.: Nucl. Phys. **B373**, 630 (1992)
5. Davidović, Lj., Nikolić, B., Sazdović, B.: Eur. Phys. J. **C74**(1), 2734 (2014)
6. Lust, D.: J. High Energy Phys. **12**, 084 (2010); Andriot, D., Larfors, M., Lust, D., Patalong, P.: J. High Energy Phys. **06**, 021 (2013)

T-dualization of a weakly curved background

2017 J. Phys.: Conf. Ser. 804 012014

(http://iopscience.iop.org/1742-6596/804/1/012014)

T-dualization of a weakly curved background

Lj. Davidović, B. Nikolić and B. Sazdović

Institute of Physics, University of Belgrade, Belgrade, Serbia

Abstract. We consider a string moving in a weakly curved background, composed of a constant metric and a linearly coordinate dependent Kalb-Ramond field with an infinitesimal strength. We discuss the T-dualization procedure which we developed for a closed bosonic string moving in a weakly curved background. The procedure is a generalization of a Buscher T-dualization procedure and enables the T-dualization of the nonisometry directions. The same procedure is used to investigate the T-duals of an open bosonic string as well. The generalized T-dualizations give insight to the connection between the geometrical properties of the T-dual spaces.

1. Introduction

In string theory there exists a symmetry, T-duality, which allows the physical equivalence of the string living on the different geometrical structures of the compactified dimensions. The string living in a space with one dimension compactified on a radius R, has the same physical features as a string leaving in a space with one dimension compactified on a radius $\frac{\alpha'}{R}$, where α' is a Regge slope parameter. T-duality was first described in the context of toroidal compactification in [1] (thoroughly explained in [2]), it can be generalized to the arbitrary toroidal compactification [3], and extended to the non-flat conformal backgrounds [4]. The origin of T-duality is seen in a possibility that, unlike a point particle, the string can wrap around compactified dimensions.

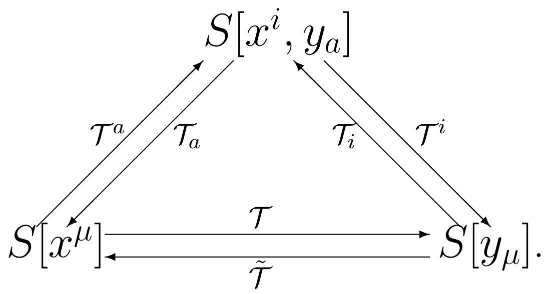
The first T-dualization procedure, the prescription for obtaining a theory which is T-dual of a given theory, was defined by Buscher [5]. The procedure was done for a string sigma model, describing a string moving in a background composed of a metric $G_{\mu\nu}$, an antisymmetric field $B_{\mu\nu}$ and a dilaton field Φ. It is required that the metric admits at least one continuous abelian isometry which leaves the action invariant. The procedure is founded in gauging the isometry by introducing the gauge fields v^μ_α. In order to preserve the physical content of the original theory, one requires that the new fields v^μ_α are nonphysical, which is achieved by the requirement that the gauge fields have a vanishing field strength $F^\mu_{\alpha\beta} = \partial_\alpha v^\mu_\beta - \partial_\beta v^\mu_\alpha$. This requirement is included in the theory by adding the Lagrange multiplier term $y_\mu F^\mu_{01}$ into the Lagrangian. Fixing the gauge one obtains the gauge fixed Lagrangian which carries the information on both initial and a T-dual theory. The integration over the Lagrange multipliers y_μ, simply recovers the original theory. The integration over the gauge fields v^μ_α, produces the T-dual theory.

The standard T-dualization procedure is applicable along directions which do not appear as the background field arguments. The generalized T-dualization procedure which is applicable along an arbitrary coordinate was done in Refs. [6, 7, 8]. The procedure is founded in the standard procedure and keeps the main rules of the standard procedure. In order to gauge the global isometry, one introduces the gauge fields v^μ_α, as usual. The replacement of the derivatives $\partial_\alpha x^\mu$ with the covariant ones $D_\alpha x^\mu$, does not as before make the whole action invariant. The obstacle is the background field $B_{\mu\nu}$ depending on x^μ, which is not locally gauge invariant. So, as a new rule we substitute the argument of the background fields by an invariant argument Δx^μ_{inv}, defined as the line integral of the covariant derivatives of the original argument. As before, in order to obtain the theory physically equivalent to the original one, we add the Lagrange multiplier term. Using the local gauge freedom we fix the gauge taking $x^\mu(\xi) = x^\mu(\xi_0)$. The obtained gauge fixed action reduces to to the original action for the equations of motion for the Lagrange multiplier. The T-dual theory is obtained for the equations of motion for the gauge fields v^μ_α.

The generalized T-dualization procedure was investigated for a string moving in a weakly curved background composed of a constant metric, a linearly coordinate dependent Kalb-Ramond field with an infinitesimal filed strength and a constant dilaton field. It was first applied to all space-time coordinates in Ref. [6], and a T-dual was obtained. In Ref. [8], the procedure was applied to an arbitrary set of the initial coordinates. Choosing d arbitrary directions, we denote $\mathcal{T}^a = \circ_{n=1}^d T^{\mu_n}$, $\mathcal{T}^i = \circ_{n=d+1}^D T^{\mu_n}$, and $\mathcal{T} = \circ_{n=1}^D T^{\mu_n}$, where T^μ stands for a T-dualization along direction x^μ and $\mathcal{T}_a = \circ_{n=1}^d T_{\mu_n}$, $\mathcal{T}_i = \circ_{n=d+1}^D T_{\mu_n}$, $\tilde{\mathcal{T}} = \circ_{n=1}^D T_{\mu_n}$, where T_μ stands for the T-dualization along a dual direction y_μ. Performing the generalized procedure we proved the following composition laws:

$$\mathcal{T}^i \circ \mathcal{T}^a = \mathcal{T}, \quad \mathcal{T}_i \circ \mathcal{T}_a = \tilde{\mathcal{T}}, \quad \mathcal{T}_a \circ \mathcal{T}^a = 1, \tag{1}$$

where 1 denotes the identical transformation (T-dualization not performed). We found the explicit forms of the resulting theories and the corresponding T-dual coordinate transformation laws. These results complete the T-dualization diagram connecting all the theories T-dual to the initial theory.



The initial theory, describing the bosonic string moving in the weakly curved background is defined on the geometrical space. All its T-dual theories are non-geometric and non-local because they depend on variable V^μ, which is a line integral of the derivatives of the dual coordinates. To all of these theories there corresponds a flux which is of the same type as the R flux unlike the non-geometric theories with Q flux, which have a local geometric description.

2. Bosonic string action

Let us consider the action [9, 10] describing the propagation of the bosonic string in a background composed of a space-time metric $G_{\mu\nu}$, a Kalb-Ramond field $B_{\mu\nu}$ and a dilaton field Φ

$$S[x] = \kappa \int_\Sigma d^2\xi \sqrt{-g} \Big[\Big(\frac{1}{2} g^{\alpha\beta} G_{\mu\nu}(x) + \frac{\varepsilon^{\alpha\beta}}{\sqrt{-g}} B_{\mu\nu}(x) \Big) \partial_\alpha x^\mu \partial_\beta x^\nu + \frac{1}{4\pi\kappa} \Phi(x) R^{(2)} \Big]. \tag{2}$$

The integration goes over two-dimensional world-sheet Σ parametrized by ξ^α ($\xi^0 = \tau,\ \xi^1 = \sigma$), $g_{\alpha\beta}$ is intrinsic world-sheet metric, $R^{(2)}$ corresponding 2-dimensional scalar curvature, $x^\mu(\xi)$, $\mu = 0, 1, ..., D-1$ are the coordinates of the D-dimensional space-time, $\kappa = \frac{1}{2\pi\alpha'}$ and $\varepsilon^{01} = -1$.

In order to have a world-sheet conformal invariance on the quantum level, the background fields have to obey the space-time equations of motion which in the lowest order in slope parameter α', have the following form

$$\begin{aligned} &R_{\mu\nu} - \frac{1}{4} B_{\mu\rho\sigma} B_{\nu}{}^{\rho\sigma} + 2 D_\mu \partial_\nu \Phi = 0, \\ &D_\rho B^\rho{}_{\mu\nu} - 2\partial_\rho \Phi B^\rho{}_{\mu\nu} = 0, \\ &4(\partial\Phi)^2 - 4D_\mu \partial^\mu \Phi + \frac{1}{12} B_{\mu\nu\rho} B^{\mu\nu\rho} - R + 4\pi\kappa \frac{D-26}{3} = 0\,, \end{aligned} \tag{3}$$

where $B_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} + \partial_\nu B_{\rho\mu} + \partial_\rho B_{\mu\nu}$ is the field strength of the field $B_{\mu\nu}$, and $R_{\mu\nu}$ and D_μ are Ricci tensor and covariant derivative with respect to space-time metric. We consider the weakly curved background, defined by

$$G_{\mu\nu} = const, \quad B_{\mu\nu}(x) = b_{\mu\nu} + \frac{1}{3} B_{\mu\nu\rho} x^\rho \equiv b_{\mu\nu} + h_{\mu\nu}(x), \quad \Phi = const. \tag{4}$$

The Kalb-Ramond field strength $B_{\mu\nu\rho}$ is taken to be infinitesimal. All the calculations are done in the first order in $B_{\mu\nu\rho}$. In this approximation the weakly curved background is the solution of the space-time equations of motion (3).

Introducing the light-cone coordinates

$$\xi^\pm = \frac{1}{2}(\tau \pm \sigma)$$

and their derivatives $\partial_\pm = \partial_\tau \pm \partial_\sigma$, taking a conformal gauge $g_{\alpha\beta} = e^{2F}\eta_{\alpha\beta}$, the action (2) can be written as

$$S[x] = \kappa \int_\Sigma d^2\xi \; \partial_+ x^\mu \Pi_{+\mu\nu}(x) \partial_- x^\nu, \tag{5}$$

where

$$\Pi_{\pm\mu\nu}(x) = B_{\mu\nu}(x) \pm \frac{1}{2} G_{\mu\nu}(x). \tag{6}$$

3. The Generalized Buscher T-dualization procedure

The standard T-dualization procedure, enables one to find a T-dual of a given theory, applying the procedure to the coordinate directions which do not appear as the background field arguments. The generalized T-dualization procedure does not have this limitation. Both procedures are grounded in a localization of a global coordinate shift symmetry $\delta x^\mu = \lambda^\mu = const$. The first rule of the procedures is the introduction of the gauge fields v^μ_α and the substitution of the ordinary derivatives with the covariant derivatives, defined by

$$\partial_\alpha x^\mu \to D_\alpha x^\mu = \partial_\alpha x^\mu + v^\mu_\alpha. \tag{7}$$

If one imposes the following transformation law for the gauge fields

$$\delta v^\mu_\alpha = -\partial_\alpha \lambda^\mu, \quad (\lambda^\mu = \lambda^\mu(\tau,\sigma)) \tag{8}$$

one obtains $\delta D_\alpha x^\mu = 0$. In the case when the background does not depend on the coordinates, along which the T-dualization is performed, the first step is sufficient to obtain the gauge invariant action. However if the background depends on all the coordinates, an additional rule must be introduced. The new rule reads: *Substitute the background field argument (the* *coordinate x^μ), by the invariant argument (invariant coordinate)*, defined as a line integral of the covariant derivatives of the original coordinate

$$\Delta x^\mu_{inv} \equiv \int_P d\xi^\alpha\, D_\alpha x^\mu = x^\mu - x^\mu(\xi_0) + \Delta V^\mu, \quad \Delta V^\mu \equiv \int_P d\xi^\alpha v^\mu_\alpha. \tag{9}$$

The invariant coordinate is by definition nonlocal. The consequence of this will be a nonlocal T-dual theory, defined on the doubled geometrical space composed of the dual coordinate y_μ and its double $\tilde{y}_\mu$.

The common rule of the procedures is the addition of the Lagrange multiplier term which makes the introduced gauge fields nonphysical, by requiring that there field strength

$$F^\mu_{\alpha\beta} \equiv \partial_\alpha v^\mu_\beta - \partial_\beta v^\mu_\alpha \tag{10}$$

must be zero. This enables the physical equivalence of the theories. Following these rules we built the gauge invariant action.

The main object and the main crossway of the procedure are the gauge fixed action and their equations of motion, because for the equation of motion obtained varying the action over the Lagrange multipliers, one returns to the initial action. On the other hand for the equation of motion obtained varying the gauge fixed action over the gauge fields one obtains the T-dual theory. Comparing the solutions for the gauge fields in these two directions, one obtains the T-dual coordinate transformation laws. These laws are used in investigation of the relations between the non-commutativity characteristics of the spaces connected by T-duality.

The generalized procedure, can be generalized once more in order to allow the T-dualization of the backgrounds which do not have a global symmetry. The generalization was made in Ref. [7] for a bosonic string moving in a weakly curved background of the second order, which consists of the coordinate dependent metric and Kalb-Ramond field. One postulates the auxiliary action which inherits two important features of the gauge fixed action. It reduces to the initial theory for the equations of motion for the Lagrange multipliers and to the T-dual action for the equations of motion for the auxiliary fields.

3.1. Complete T-dualization

If one applies the T-dualization procedure to all coordinates, one obtains a following gauge invariant action

$$S_{inv} = \kappa \int d^2\xi \Big[D_+ x^\mu \Pi_{+\mu\nu}(\Delta x_{inv}) D_- x^\nu + \frac{1}{2}(v^\mu_+ \partial_- y_\mu - v^\mu_- \partial_+ y_\mu)\Big], \tag{11}$$

which is physically equivalent to the initial action. Fixing the gauge by $x^\mu(\xi) = x^\mu(\xi_0)$, one obtains the gauge fixed action

$$S_{fix}[y, v_\pm] = \kappa \int d^2\xi \Big[v^\mu_+ \Pi_{+\mu\nu}(\Delta V) v^\nu_- + \frac{1}{2}(v^\mu_+ \partial_- y_\mu - v^\mu_- \partial_+ y_\mu)\Big]. \tag{12}$$

In order to find a T-dual action one has to integrate out the gauge fields from (12).

The equations of motion with respect to the gauge fields $v^\mu_\pm$ are

$$\Pi_{\mp\mu\nu}(\Delta V) v^\nu_\pm + \frac{1}{2}\partial_\pm y_\mu = \mp \beta^\mp_\mu(V), \tag{13}$$

with the right hand side coming from the variation of the background fields argument, with $\beta^\pm_\mu(x) = \mp\frac{1}{2} h_{\mu\nu}[x]\partial_\mp x^\nu$. The equation of motion can be rewritten as

$$v^\mu_\pm(y) \;=\; -\kappa\, \Theta^{\mu\nu}_\pm[\Delta V(y)]\Big[\partial_\pm y_\nu \pm 2\beta^\mp_\nu[V(y)]\Big], \tag{14}$$

where

$$\Theta_\pm^{\mu\nu}[\Delta V] = -\frac{2}{\kappa}(G_E^{-1}\Pi_\pm G^{-1})^{\mu\nu} = \theta^{\mu\nu}[\Delta V] \mp \frac{1}{\kappa}(G_E^{-1})^{\mu\nu}[\Delta V], \tag{15}$$

and $G^E_{\mu\nu} \equiv [G - 4BG^{-1}B]_{\mu\nu}$, $\theta^{\mu\nu} \equiv -\frac{2}{\kappa}(G_E^{-1}BG^{-1})^{\mu\nu}$ are the open string background fields: the effective metric and the non-commutativity parameter respectively. They are defined in analogy with the flat space-time open string background fields introduced in [11]. Tensors $\Pi_{\mp\mu\nu}$ and $\Theta_\pm^{\mu\nu}$ are connected by $\Theta_\pm^{\mu\nu}\Pi_{\mp\nu\rho} = \frac{1}{2\kappa}\delta^\mu_\rho$. Substituting (14) into the action (12), we obtain T-dual action

$${}^\star S[y] \equiv S_{fix}[y] = \frac{\kappa^2}{2}\int d^2\xi\ \partial_+ y_\mu \Theta_-^{\mu\nu}[\Delta V^{(0)}(y)]\partial_- y_\nu, \tag{16}$$

where we neglected the term $\beta^-_\mu\beta^+_\nu$ as the infinitesimal of the second order, and the argument is given by

$$\Delta V^{(0)\mu}(y) = -\kappa\theta_0^{\mu\nu}\Delta y_\nu^{(0)} + (g^{-1})^{\mu\nu}\Delta\tilde{y}_\nu^{(0)}. \tag{17}$$

Comparing the initial action (5) with the T-dual action (16), we see that they are equal under following transformations $\partial_\pm x^\mu \to \partial_\pm y_\mu$ and $\Pi_{+\mu\nu}[x] \to \frac{\kappa}{2}\Theta_-^{\mu\nu}[\Delta V^{(0)}]$, which implies

$$\begin{aligned} G_{\mu\nu} &\to {}^\star G^{\mu\nu} = (G_E^{-1})^{\mu\nu}[\Delta V^{(0)}], \\ B_{\mu\nu}[x] &\to {}^\star B^{\mu\nu} = \frac{\kappa}{2}\theta^{\mu\nu}[\Delta V^{(0)}], \end{aligned} \tag{18}$$

where $(G_E^{-1})^{\mu\nu}$ and $\theta^{\mu\nu}$ are introduced in (15).

The initial background consisted of a constant metric and a linearly coordinate dependent Kalb-Ramond field with an infinitesimal field strength. The T-dual background consists of coordinate dependent metric and Kalb-Ramond field, with the argument ΔV^μ, which is the linear combination of y_μ and its double $\tilde{y}_\mu$. Note that the variable V^μ and consequently T-dual action is not defined on the geometrical space (defined by the coordinate y_μ) but on the so called doubled target space [12] composed of both y_μ and $\tilde{y}_\mu$.

3.2. Partial T-dualization

If one choses only a subset of the initial coordinates, say d coordinates x^a, and performs T-dualization procedure along these coordinates, one obtains the following gauge invariant action

$$\begin{aligned} S_{inv}[x^\mu, x^a_{inv}, y_a] &= \kappa\int d^2\xi\Big[\partial_+ x^i\Pi_{+ij}(x^i,\Delta x^a_{inv})\partial_- x^j \\ &+ \partial_+ x^i \Pi_{+ia}(x^i,\Delta x^a_{inv})D_- x^a + D_+ x^a\Pi_{+ai}(x^i,\Delta x^a_{inv})\partial_- x^i \\ &+ D_+ x^a \Pi_{+ab}(x^i,\Delta x^a_{inv})D_- x^b + \frac{1}{2}(v^a_+\partial_- y_a - v^a_-\partial_+ y_a)\Big]. \end{aligned} \tag{19}$$

This action is obtained localizing the global shift symmetry only for the coordinates x^a, by introducing the gauge fields v^a_α. The ordinary derivatives $\partial_\alpha x^a$ were substituted by the covariant derivatives $D_\alpha x^a = \partial_\alpha x^a + v^a_\alpha$. The covariant derivatives are invariant under the standard gauge transformations $\delta v^a_\alpha = -\partial_\alpha\lambda^a$. The coordinates x^a in the argument of the background fields were substituted by their invariant extension, defined by $\Delta x^a_{inv} \equiv \int_P d\xi^\alpha D_\alpha x^a = x^a - x^a(\xi_0) + \Delta V^a$, where $\Delta V^a \equiv \int_P d\xi^\alpha v^a_\alpha$. The physical equivalence is preserved by adding the Lagrange multiplier term (the last term in the action). Fixing the gauge by $x^a(\xi) = x^a(\xi_0)$ one obtains the gauge fixed action

$$\begin{aligned} S_{fix}[x^i, v_\pm^a, y_a] &= \kappa \int d^2\xi \Big[\partial_+ x^i \Pi_{+ij}(x^i, \Delta V^a)\partial_- x^j \\ &+ \partial_+ x^i \Pi_{+ia}(x^i, \Delta V^a) v_-^a + v_+^a \Pi_{+ai}(x^i, \Delta V^a)\partial_- x^i \\ &+ v_+^a \Pi_{+ab}(x^i, \Delta V^a) v_-^b + \frac{1}{2}(v_+^a \partial_- y_a - v_-^a \partial_+ y_a)\Big]. \end{aligned} \tag{20}$$

This action reduces to the initial one for the equations of motion obtained varying over the Lagrange multipliers. The T-dual action is obtained for the equations of motion for the gauge fields. It reads

$$\begin{aligned} S[x^i, y_a] &= \kappa \int d^2\xi \Big[\partial_+ x^i \overline{\Pi}_{+ij}(x^i, \Delta V^a(x^i, y_a))\partial_- x^j \\ &-\kappa\, \partial_+ x^i \Pi_{+ia}(x^i, \Delta V^a(x^i, y_a))\tilde{\Theta}_-^{ab}(x^i, \Delta V^a(x^i, y_a))\partial_- y_b \\ &+\kappa\, \partial_+ y_a \tilde{\Theta}_-^{ab}(x^i, \Delta V^a(x^i, y_a))\Pi_{+bi}(x^i, \Delta V^a(x^i, y_a))\partial_- x^i \\ &+\frac{\kappa}{2}\, \partial_+ y_a \tilde{\Theta}_-^{ab}(x^i, \Delta V^a(x^i, y_a))\partial_- y_b\Big]. \end{aligned} \tag{21}$$

The T-dual background fields compositions are the inverses of the already known background compositions, divided into two coordinate subspaces, the subspace formed by the coordinates we T-dualize and the subspace formed by the rest of the coordinates. The background field compositions $\overline{\Pi}_{\pm ij}$ and $\tilde{\Theta}_\pm^{ab}$ are defined as the inverses of the background field compositions $\Theta_\mp^{jk}$ and $\Pi_{\mp bc}$, which are the parts of $\Theta_\mp^{\mu\nu}$ and $\Pi_{\mp\mu\nu}$ in an appropriate subspace

$$\begin{aligned} \overline{\Pi}_{\pm ij}\Theta_\mp^{jk} &= \Theta_\mp^{kj}\overline{\Pi}_{\pm ji} = \frac{1}{2\kappa}\delta_i^k, \\ \tilde{\Theta}_\pm^{ab}\Pi_{\mp bc} &= \Pi_{\mp cb}\tilde{\Theta}_\pm^{ba} = \frac{1}{2\kappa}\delta_c^a. \end{aligned} \tag{22}$$

It can be shown that

$$\overline{\Pi}_{+ij} \equiv \Pi_{+ij} - 2\kappa \Pi_{+ia}\tilde{\Theta}_-^{ab}\Pi_{+bj}. \tag{23}$$

The argument of the background fields is

$$\begin{aligned} \Delta V^{(0)a}(x^i, y_a) &= -\kappa\Big[\tilde{\Theta}_{0+}^{ab}\Pi_{0-bi} + \tilde{\Theta}_{0-}^{ab}\Pi_{0+bi}\Big]\Delta x^{(0)i} \\ &- \kappa\Big[\tilde{\Theta}_{0+}^{ab}\Pi_{0-bi} - \tilde{\Theta}_{0-}^{ab}\Pi_{0+bi}\Big]\Delta\tilde{x}^{(0)i} \\ &- \frac{\kappa}{2}\Big[\tilde{\Theta}_{0+}^{ab} + \tilde{\Theta}_{0-}^{ab}\Big]\Delta y_b^{(0)} - \frac{\kappa}{2}\Big[\tilde{\Theta}_{0+}^{ab} - \tilde{\Theta}_{0-}^{ab}\Big]\Delta\tilde{y}_b^{(0)}. \end{aligned} \tag{24}$$

Calculating the symmetric and antisymmetric part of the background fields we obtain a T-dual metric and a T-dual Kalb-Ramond field

$$\begin{aligned} &\bullet G_{ij} = \overline{G}_{ij} = G_{ij} - G_{ia}(\tilde{G}_E^{-1})^{ab}G_{bj} \\ &\quad -2\kappa\Big(B_{ia}\tilde{\theta}^{ab}G_{bj} + G_{ia}\tilde{\theta}^{ab}B_{bj}\Big) - 4B_{ia}(\tilde{G}_E^{-1})^{ab}B_{bj}, \\ &\bullet B_{ij} = \overline{B}_{ij} = B_{ij} - \frac{\kappa}{2}G_{ia}\tilde{\theta}^{ab}G_{bj} - B_{ia}(\tilde{G}_E^{-1})^{ab}G_{bj} \\ &\quad -G_{ia}(\tilde{G}_E^{-1})^{ab}B_{bj} - 2\kappa B_{ia}\tilde{\theta}^{ab}B_{bj}, \\ &\bullet G^{ab} = (\tilde{G}_E^{-1})^{ab}, \\ &\bullet B^{ab} = \frac{\kappa}{2}\tilde{\theta}^{ab}, \\ &\bullet G^a{}_i = \kappa\tilde{\theta}^{ab}G_{bi} + 2(\tilde{G}_E^{-1})^{ab}B_{bi}, \\ &\bullet B^a{}_i = \kappa\tilde{\theta}^{ab}B_{bi} + \frac{1}{2}(\tilde{G}_E^{-1})^{ab}G_{bi}. \end{aligned} \tag{25}$$

As the constituents of the T-dual background field there appear the effective metric in the subspace a, defined by $\tilde{G}_{Eab} \equiv G_{ab} - 4B_{ac}(\tilde{G}^{-1})^{cd}B_{db}$, the non-commutativity parameter in the same subspace $\tilde{\theta}^{ab} \equiv -\frac{2}{\kappa}(\tilde{G}_E^{-1})^{ac}B_{cd}(\tilde{G}^{-1})^{db}$, which combined give the new theta function $\tilde{\Theta}_{\pm}^{ab} = \tilde{\theta}^{ab} \mp \frac{1}{\kappa}(\tilde{G}_E^{-1})^{ab}$.

4. Open string T-dualization

In paper [13] we investigated a T-duality of an open string moving in a weakly curved background. The open string moving in a weakly curved background was a subject of investigation in our papers [14, 15, 16]. Solving the boundary conditions at the open string end-points, one obtains the effective closed string described by the effective closed string theory S^{eff}, defined on the doubled space $(q^\mu, \tilde{q}^\mu)$. As the effective theory is closed string theory, one can try to apply the generalized T-dualization procedure to this theory. The effective theory is defined on the doubled theory, just as the T-duals of the closed string theory moving in the weakly curved background. So, the application in this case resembles the application of the T-dualization procedure to the T-dual theories.

The effective theory of the open string moving in the weakly curved background, obtained for the solution of the boundary conditions equals

$$S^{eff} = \kappa \int d\tau \int_{-\pi}^{\pi} d\sigma\, \partial_+ q^\mu\, \Pi^{eff}_{+\mu\nu}(q, 2b\tilde{q})\, \partial_- q^\nu, \tag{26}$$

where

$$\Pi^{eff}_{\pm\mu\nu}(q, 2b\tilde{q}) \equiv B^{eff}_{\mu\nu}(2b\tilde{q}) \pm \frac{1}{2} G^{eff}_{\mu\nu}(q). \tag{27}$$

The effective variable is $q^\mu(\sigma)$, an even part of the initial coordinate. The effective metric and the Kalb-Ramond field are explicitly given by

$$\begin{aligned} G^{eff}_{\mu\nu}(q) &= G^E_{\mu\nu}(q) := (G - 4B^2(q))_{\mu\nu}, \\ B^{eff}_{\mu\nu}(2b\tilde{q}) &= -\frac{\kappa}{2}\Big(g_E \Delta\theta(2b\tilde{q}) g_E\Big)_{\mu\nu}, \end{aligned} \tag{28}$$

where $\Delta\theta^{\mu\nu}$ is the infinitesimal part of the non-commutativity parameter $\theta^{\mu\nu} = -\frac{2}{\kappa}\Big[G_E^{-1} B G^{-1}\Big]^{\mu\nu} = \theta_0^{\mu\nu} - \frac{2}{\kappa}\Big[g_E^{-1}(h + 4bhb)g_E^{-1}\Big]^{\mu\nu}$. In paper [13] we applied the generalized Buscher T-dualization procedure, to the effective theory along all effective directions q^μ. Following the procedure we find the gauge fixed action

$$\mathcal{S}_{fix} = \kappa \int d^2\xi \Big[v_+^\mu \Pi^{eff}_{+\mu\nu}(\Delta V, 2b\Delta\tilde{V}) v_-^\nu + \frac{1}{2}(v_+^\mu \partial_- \varrho_\mu - v_-^\mu \partial_+ \varrho_\mu)\Big], \tag{29}$$

obtained from the effective action (26), by substituting the light-cone derivatives $\partial_\pm q^\mu$ with the covariant derivatives $D_\pm q^\mu = \partial_\pm q^\mu + v_\pm^\mu$, where $v_\pm^\mu$ are the gauge fields, which transform as $\delta v_\pm^\mu = -\partial_\pm \lambda^\mu$. The argument of the background fields is substituted with an invariant argument, which is obtained substituting the effective coordinate q^μ and its double $\tilde{q}^\mu$ with an invariant effective coordinate and its double, defined by the following line integrals of the gauge fields $\Delta V^\mu = \int_P (d\xi^+ v_+^\mu + d\xi^- v_-^\mu)$, and $\Delta\tilde{V}^\mu = \int_P (d\xi^+ v_+^\mu - d\xi^- v_-^\mu)$. The physical equivalence was achieved by adding the Lagrange multiplier term $\frac{1}{2}(v_+^\mu \partial_- \varrho_\mu - v_-^\mu \partial_+ \varrho_\mu)$ and the gauge is fixed with $q^\mu(\xi) = q^\mu(\xi_0)$.

The T-dual theory was obtained for the equation of motion for the gauge fields. The T-dual action reads

$$^\star\mathcal{S} = \kappa \int d^2\xi \partial_+ \varrho_\mu \frac{\kappa}{2} (\Theta_-^{eff})^{\mu\nu}(\Delta V(\varrho), 2b\Delta\tilde{V}(\varrho)) \partial_- \varrho_\nu, \tag{30}$$

where

$$(\Theta^{eff}_{\pm})^{\mu\nu}(x,y) \equiv \Theta^{\mu\nu}_{\pm}(G_{eff}(x), B_{eff}(y)) = \theta^{\mu\nu}_{eff}(y) \mp \frac{1}{\kappa}(G_E^{-1})^{\mu\nu}(x), \tag{31}$$

$\theta^{\mu\nu}_{eff} := \theta^{\mu\nu}(G_{eff}(x), B_{eff}(y)) = -\frac{2}{\kappa}\Big(G_E^{-1}(G_{eff}(x), B_{eff}(y))B_{eff}(y)G^{-1}_{eff}(x)\Big)^{\mu\nu}$ and the argument is

$$\begin{aligned} V_0^{\mu}(\varrho) &= (g_E^{-1})^{\mu\nu}(G^{eff}, B^{eff})\tilde{\varrho}_\nu = (g_E^{-1})^{\mu\nu}\tilde{\varrho}_\nu, \\ \tilde{V}_0^{\mu}(\varrho) &= (g_E^{-1})^{\mu\nu}(G^{eff}, B^{eff})\varrho_\nu = (g_E^{-1})^{\mu\nu}\varrho_\nu. \end{aligned} \tag{32}$$

The T-dual metric ${}^\star G^{\mu\nu}$ which depends on the first variable ΔV^μ and the T-dual Kalb-Ramond field ${}^\star B^{\mu\nu}$, which depends on the second variable $2b^\mu{}_\nu \Delta\tilde{V}^\nu$ are

$$\begin{aligned} {}^\star G^{\mu\nu} &= (G_E^{-1})^{\mu\nu}(\Delta V), \\ {}^\star B^{\mu\nu} &= \frac{\kappa}{2}(\theta^{eff})^{\mu\nu}(2b\Delta\tilde{V}) = \frac{\kappa}{2}\Delta\theta^{\mu\nu}(2b\Delta\tilde{V}). \end{aligned} \tag{33}$$

We see, that the effective metric has transformed to its inverse and that the Kalb-Ramond field has transformed to the infinitesimal part of the non-commutativity parameter.

Finally, we searched for the open string theory $\tilde{S}$ such that its effective theory is ${}^\star S^{eff}$ exactly. We found

$$\widetilde{S}[y] = \kappa \int_\Sigma d^2\xi\ \partial_+ y_\mu \widetilde{\Pi}^{\mu\nu}_+(y)\partial_- y_\nu, \tag{34}$$

with

$$\begin{aligned} \widetilde{G} &= -(C^T)^{-1}GC^{-1}, \\ \widetilde{B}(y) &= \pm(C^T)^{-1}(b - h(C^{-1}y))C^{-1}, \end{aligned} \tag{35}$$

where C makes a connection between the variables of the effective theory of $\tilde{S}$ and the T-dual theory (30)

$$\begin{aligned} q_\mu(y) &= C_{\mu\nu}(g_E^{-1})^{\nu\rho}\tilde{\varrho}_\rho\,, \\ \bar{q}_\mu(y) &= \mp C_{\mu\nu}2(G^{-1}bg_E^{-1})^{\nu\rho}\varrho_\rho. \end{aligned} \tag{36}$$

In the closed string moving in the weakly curved background case, the T-duality transforms the geometrical background into a doubled non-geometrical background. It transforms a constant metric to a coordinate dependent effective metric inverse, while the linearly coordinate dependent Kalb-Ramond field is transformed into a coordinate dependent non-commutativity parameter. In the open string case, the T-dual theory remains geometric. T-duality transforms the constant metric of the weakly curved background to a constant T-dual metric, while the coordinate dependent Kalb-Ramond field transforms again to the coordinate dependent field.

In paper [17] a generalization of the standard analysis of the open bosonic string moving in a flat background is addressed. The T-dualization was performed in two ways, first in terms of non-constant vector fields in which case the Buscher T-dualization procedure can not be applied and second in terms of the field strengths of the gauge fields. The role of the gauge fields, which live on the string boundary, is to restore the symmetries of the closed string: the local gauge symmetry of the Kalb-Ramond field and the general coordinate transformations, at the string end-points. The investigation lead to a discovery of the geometrical features of the non-geometry.

Conclusion

The generalized T-dualization procedure, enabled T-dualization over the non isometry directions. It gives the new insights into a connection between the spaces connected by T-duality. It enabled further investigations of the closed string non-commutativity [18]. Comparing the solutions for the gauge fields which transform the gauge fixed actions into the initial or the T-dual actions, one obtains the T-dual coordinate transformation laws. Using these laws one can find how does for example a standard Poisson bracket transform. It is obtained that the original theory which is commutative is equivalent to the non-commutative T-dual theory, whose Poisson brackets are proportional to the background fluxes times winding and momentum numbers. The obtained results add novelty to the form and the origin of different non-commutative structures.

Acknowledgments

Research is supported in part by the Serbian Ministry of Education, Science and Technological Development (project No. 171031) and by The National Scholarship L'Oreal-UNESCO "For Women in Science" which Lj.D. obtained in 2015.

References

[1] L.Brink, M.B. Green, J.H. Schwarz, *Nucl. Phys.* **B 198** (1982) 474; K. Kikkawa and M.Yamasaki, *Phys. Lett.* **B 149** (1984) 357; N. Sakai and I. Senda, *Prog. Theor. Phys.* **75** (1984) 692.
[2] E. Alvarez, L. Alvarez-Gaume and Y. Lozano, *Nucl. Phys. Proc. Suppl.* **41** (1995) 1; A. Giveon, M.Parrati and E. Rabinovici, *Phys. Rep.* **244** (1994) 77.
[3] K.Narain, H. Sarmadi and E. Witten, *Nucl. Phys.* **B 279** (1987) 369.
[4] T. Buscher, *Phys. Lett.* **B 194** (1987) 51; **201** (1988) 466.
[5] T. Buscher, Phys. Lett. **B 194** (1987) 59; M. Roček and E.P. Verlinde, Nucl. Phys. **B 373** (1992) 630.
[6] Lj. Davidović and B. Sazdović, EPJ**C 74** No. 1 (2014) 2683.
[7] Lj. Davidović and B. Sazdović, EPJ**C 75** No. 12 (2015) 576.
[8] Lj. Davidović, B. Nikolić and B. Sazdović, JHEP **11** (2015) 119.
[9] B. Zwiebach, *A First Course in String Theory*, (Cambridge University Press, 2004).
[10] K. Backer, M. Backer and J. Schwarz, *String Theory and M-theory*, (Cambridge University Press, 2007).
[11] N. Seiberg, E. Witten, J. High Energy Phys. **09** (1999) 032.
[12] C. Hull, *JHEP* **10** (2005) 065; A. Dabholkar and C. Hull, *JHEP* **05** (2006) 009; C. Hull, *JHEP* **07** (2007) 080; J. Shelton, W. Taylor and B. Wecht, *JHEP* **10** (2005) 085; A. Dabholkar and C. Hull, *JHEP* **09** (2003) 054; A. Flournoy and B. Williams, *JHEP* **01** (2006) 166; T. Kugo, B. Zwiebach, *Prog. Theor. Phys.* **87** (1992) 801; O. Hohm, C. Hull and B. Zwiebach, *JHEP* **08** (2010) 008; O. Hohm, C. Hull and B. Zwiebach, *JHEP* **07** (2010) 016; B. Wecht, *Class.Quant.Grav.* **24 21** (2007) S773; D. Lust, *JHEP* **12** (2010) 084; D.Mylonas, P. Schupp and R. Szabo, arXiv:1207.0926 [hep-th].
[13] Lj. Davidović, EPJ**C 76** (2016) 660.
[14] Lj. Davidović and B. Sazdović, Phys. Rev. **D 83** (2011) 066014.
[15] Lj. Davidović and B. Sazdović, JHEP **08** (2011) 112.
[16] Lj. Davidović and B. Sazdović, EPJ**C 72** No. 11 (2012) 2199.
[17] B. Sazdović, arXiv:1606.01938 [hep-th].
[18] Lj. Davidović, B. Nikolić and B. Sazdović, EPJ**C 74** (2014) 2734.

BOOK OF ABSTRACTS

10th MATHEMATICAL PHYSICS MEETING:
School and Conference on Modern
Mathematical Physics

9 – 14 September 2019, Belgrade, Serbia

MΩΦ

www.mphys10.ipb.ac.rs

Organizers

Institute of Physics (University of Belgrade)
Belgrade, Serbia

Faculty of Mathematics (University of Belgrade)
Belgrade, Serbia

Mathematical Institute (Serbian Academy of Sciences and Arts)
Belgrade, Serbia

Faculty of Science (University of Kragujevac)
Kragujevac, Serbia

Co-organizers

Institute of Nuclear Sciences "Vinča" (University of Belgrade)
Belgrade, Serbia

Institute of Physics (University of Kragujevac)
Kragujevac, Serbia

Faculty of Physics (University of Belgrade)
Belgrade, Serbia

Faculty of Sciences (University of Novi Sad)
Novi Sad, Serbia

School of Electrical Engineering (University of Belgrade)
Belgrade, Serbia

Southeastern European Network in Mathematical and Theoretical Physics (SEENET-MTP)
Nis, Serbia

Sponsors

Ministry of Education, Science and Technological Development, Serbia

МИНИСТАРСТВО ПРОСВЕТЕ,
НАУКЕ И ТЕХНОЛОШКОГ РАЗВОЈА

Telekom Srbija

Telekom Srbija

Open Access Journal "Symmetry" (MDPI, Switzerland)

symmetry

Media partner

Open Access Journal "Entropy" (MDPI, Switzerland)

entropy
an Open Access Journal by MDPI

International Advisory Committee

Ignatios Antoniadis (Paris, France)
Irina Arefeva (Moscow, Russia)
Milutin Blagojević (Belgrade, Serbia)
Loriano Bonora (Trieste, Italy)
Martin Cederwall (Goteborg, Sweden)
Branislav Cvetković (Belgrade, Serbia)
Marija Dimitrijević Ćirić (Belgrade, Serbia)
Nemanja Kaloper (Davis, USA)
Đorđe Minić (Blacksburg, USA)
Viatcheslav Mukhanov (Munich, Germany)
Emil Nissimov (Sofia, Bulgaria)
Sergei Odintsov (Barcelona, Spain)
Dmitri Polyakov (Chengdu, China; Moscow, Russia)
Branislav Sazdović (Belgrade, Serbia)
Paul Sorba (Annecy, France)
Alexei Starobinsky (Moscow, Russia)
Dejan Stojković (Buffalo, USA)
Mihai Visinescu (Bucharest, Romania)
Rade Živaljević (Belgrade, Serbia)
George Zoupanos (Athens, Greece)

International Organizing Committee

Vladimir Dobrev (Sofia, Bulgaria)
Radu Constantinescu (Craiova, Romania)
Branko Dragovich (Belgrade, Serbia)
Goran Đorđević (Nis, Serbia)
Zoran Rakić (Belgrade, Serbia)
Marko Vojinović (Belgrade, Serbia)
Igor Volovich (Moscow, Russia)

Local Organizing Committee

Branko Dragovich (Co-Chairman, Institute of Physics, Belgrade)
Marko Vojinović (Co-Chairman, Institute of Physics, Belgrade)
Sanja Ćirković (Institute of Physics, Belgrade)
Ljubica Davidović (Institute of Physics, Belgrade)
Ivan Dimitrijević (Faculty of Mathematics, Belgrade, Serbia)

Bojan Nikolić

From $3D$ torus with H-flux to torus with R-flux and back

We consider 3D closed bosonic string propagating in the constant metric and Kalb-Ramond field with one non-zero component, $B_{xy} = Hz$, where field strength H is infinitesimal. In the first part of the article, applying Buscher T-dualization procedure and generalized one, we T-dualize along line $x \to y \to z$, which means that we T-dualize first along x coordinate, then along y and, finally, along z coordinate. After first two T-dualizations we obtain Q flux theory which is just locally well defined, while after all three T-dualizations we obtain non-local R flux theory. Origin of non-locality is variable ΔV defined as line integral, which appears as an argument of the background fields. Rewriting T-dual transformation laws in the canonical form and using standard Poisson algebra, we obtained that Q flux theory is commutative one and the R flux theory is noncommutative and nonassociative one.

In the second part of the article, we reverse the T-dualization line and T-dualize along $z \to y \to x$. All three theories are nonlocal, because variable ΔV appears as an argument of background fields. After the first T-dualization we obtain commutative and associative theory, while after we T-dualize once more, along y, we get noncommutative and associative theory. At the end, dualizing along x, we come to the theory which is both noncommutative and nonassociative. The form of the final T-dual action does not depend on the order of T-dualization while noncommutativity and nonassociativity relations could be obtained from those in the $x \to y \to z$ case by replacing $H \to -H$.

100 ГОДИНА ОПШТЕ ТЕОРИЈЕ РЕЛАТИВНОСТИ

САНУ, БЕОГРАД, 23. ЈУН 2015.

www.gravity.ipb.ac.rs/gr100


ЖИВОТ
ЈЕ КАО ВОЖЊА
БИЦИКЛА
ДА БИ ОСТАО
У РАВНОТЕЖИ
МОРАШ СЕ
КРЕТАТИ

100 ГОДИНА ОПШТЕ ТЕОРИЈЕ РЕЛАТИВНОСТИ

Скуп 100 ГОДИНА ОПШТЕ ТЕОРИЈЕ РЕЛАТИВНОСТИ организовали су чланови пројекта основних истраживања ОН171031 “Физичке импликације модификованог простор-времена” Министарства просвете, науке и технолошког развоја Републике Србије. Група која се бави проблемима гравитације у Београду постоји отприлике од 1979. године и за тих 36 година кроз њу је прошло више од 36 студената и истраживача: данас је број сарадника на пројекту 18 а њихове главне теме истраживања су градијентне теорије гравитације, динамика струна и брана, некомутативна геометрија и некомутативна теорија поља и “loop”-квантна гравитација.

Захваљујемо се МПНТР Републике Србије и Српској академији наука и уметности на финансијској и организационој помоћи.

Научни одбор:
Милутин Благојевић
Маја Бурић
Воја Радовановић
Бранислав Саздовић
Ђорђе Шијачки

Организациони одбор:
Маја Бурић
Марко Војиновић
Љубица Давидовић
Никола Коњик
Душко Латас
Драган Прекрат

Садржај

Гравитациони таласи - шта се то таласа?

Бојан Николић

Институт за физику
Универзитет у Београду
мејл: bnikolic@ipb.ac.rs

100 година опште теорије релативности 2015.
4:43–50

Кључни појмови

гравитациони таласи, ОТР, детекција гравитационих таласа

Резиме

У тренутку настанка опште теорије релативности (ОТР) у физици су били познати механички и електромагнетни таласи. Једна од последица ОТР је постојање гравитационих таласа. Свака маса која врши неко периодично кретање је извор таласа, али су они недетектабилни због високог нивоа шума на фреквенцијама на којима се очекује њихова детекција. Извори гравитационих таласа би могли бити бинарни системи неутронских звезда (ротирају релативно великом фреквенцијом) као и догађаји у којима долази до значајне прерасподеле маса (судари галаксија и сл.). Чињеница је да гравитациони таласи до данас нису директно детектовани али постоје велики експерименти који покушавају то да остваре (LIGO и његови "наследници").

1. Увод

Камен бачен у воду изазива појаву трансверзалних таласа на њеној површини, треперење гласних жица омогућава да чујемо саговорника, земљотреси изазивају појаву цунами таласа итд. Све наведено су примери *механичких* таласа. За простирање механичких таласа је потребна материјална средина. Поремећај настао на једном месту преноси се таласом кроз материјалну средину. Механички таласи се не простиру кроз вакуум. Положај честице средине у датом тренутку t у тачки $\vec{r}$ (колоквијално "поремећај") $u(\vec{r}, t)$ задовољава хомогену *таласну једначину*

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{1}{v^2}\frac{\partial^2}{\partial t^2} \right) u(\vec{r}, t) = 0 \,. \tag{1}$$

Величина v представља брзину таласа у датој материјалној средини (није једнака брзини честице материјалне средине). Поремећај $u(\vec{r}, t)$ може бити ортогоналан на правац простирања таласа (трансверзални талас) или колинеаран са правцем простирања таласа (лонгитудиналан талас).

У другој половини 19. века енглески физичар Џејмс Кларк Мексвел је, објединиавајући дотадашња експериментална сазнања, написао једначине електромагнетног поља познате у литератури као Мексвелове једначине. Једноставна анализа тих једначина показује да у простору где нема наелектрисања и струја електрично $\vec{E}$ и магнетно поље $\vec{B}$ задовољавају хомогене таласне једначине. Простије речено, око простора у коме су задате расподеле наелектрисања и струја постоји електромагнетно (ЕМ) поље. Енергија ЕМ поља се преноси дуж правца који је ортогоналан на векторе јачине електричног и магнетног поља - ЕМ талас је трансверзалан. За разлику од механичких таласа, ЕМ таласи се простиру и кроз вакуум и то највећом брзином у природи $c = 3 \times 10^8 m/s$. Херцовим експериментом (1888) потврђено је постојање ЕМ таласа. Питање које су поставили научници тога времена тицало се средине кроз коју се ЕМ талас простире. По аналогији са механичким таласима морала је постојати нека средина која преноси таласе. Тада је уведен појам етера. Међутим, Мајкелсон-Морлијев експеримент као и многа унапређења овог експеримента потврдили су да је брзина светлости иста у свим правцима и да не зависи од избора референтног система - једноставније речено, етера, у облику како су га научници замишљали, нема. И тада (1905) се родила специјална теорија релативности (СТР).

Почетак 20. века физика је "сачекала" са познавањем две врсте таласа. Али човек који је формулисао СТР, Алберт Ајнштајн, вредно је радио на новој теорији гравитације полазећи од једног основног захтева - закони физике морају бити инваријантни на избор референтног система (инерцијалног или неинерцијалног). И дошао је до теорије која је на фундаментално нов начин интерпретирала гравитацију - формулисана је Општа теорија релативности (ОТР) 1915. године.

2. Основе ОТР

У Њутновој теорији гравитације маса је извор гравитационог поља. Свако друго тело одређене масе које се нађе у датом гравитационом пољу је изложено деловању привлачне силе. Њутн је дао аналитички облик за гравитациону интеракцију две тачкасте масе (сила којом тело 1 делује на тело 2)

$$\vec{F} = -G\frac{m_1 m_2}{|\vec{r}_2 - \vec{r}_1|^3}(\vec{r}_2 - \vec{r}_1)\,, \tag{2}$$

где је $G = 6.67 \times 10^{-11} \mathrm{Nm^2/kg^2}$ Њутнова гравитациона константа. Из Њутновог закона гравитације се види да је сила пропорционална масама тела, обрнуто пропорционална међусобном растојању, привлачна и колинеарна са вектором релативног положаја.

По Њутновој теорији гравитација је СИЛА. И по том питању није било никаквих квалитативних помака до почетка 20. века. А онда се појавио Алберт Ајнштајн, прво са својом специјалном теоријом релативности. Једна од револуционарних ствари коју је Ајнштајн тада увео у физику је обједињеност простора и времена у један просторно-временски континуум тј. време више није параметар већ координата. Један од два постулата СТР захтева инваријантност физичких закона у односу на избор инерцијалног референтног система (други се тиче брзине светлости).

Сам Ајнштајн није био задовољан. Сматрао је да физички закони морају бити инваријантни и односу на избор инерцијалног и неинерцијалног система тј. увидео је да у "причу" мора да укључи и гравитацију. Математичким језиком речено, какву год трансформацију координата да направимо закони физике морају очувати свој облик (строго речено инваријантност на дифеоморфизме).

Овде нећемо улазити у суптилне детаље извођења Ајнштајнових једначина за гравитационо поље. Ајнштајн је једначине извео користећи се законом одржања тензора енргије-импулса као и особинама неких геометријских величина. У савременој литератури извођење иде из одговарајућег дејства применом методе минимума дејства. Било како било, једначине за гравитационо поље су облика

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}\,, \qquad [\mu = 0, 1, 2, 3] \tag{3}$$

где је, најгрубље речено, на левој страни ГЕОМЕТРИЈА, а на десној страни МАТЕРИЈА. Прецизније речено, лева страна једначине је комбинација Ричијевог тензора кривине и скаларне кривине, док је на десној страни тензор енергије-импулса материје/енергије (може се односити и на електромагнетно поље).

Ова једначина успоставља везу између геометрије простор-времена и материје која својим присуством "закривљује" тај простор-време. У Ајнштајновој слици гравитација није сила већ ГЕОМЕТРИЈА простор-времена. Наравно, добра физичка теорија има особину да објашњава познате феномене и предвиђа неке нове. У оквиру Ајнштајнове теорије успешно је објашњена појава скретања светлосних зрака који пролазе близу Сунца, затим прецесија Меркуровог перихела. Теорија предвиђа постојање сингуларитета (основ за теорију Великог праска) као и црних рупа, за чије постојање постоје индиректни докази. Такође једна од последица ОТР је и постојање *гравитационих таласа*.

3. Гравитациони таласи у ОТР

Метод који користимо при математичком опису таласа је метод позадинског поља. Талас (мала пертурбација) се простире кроз простор описан одређеном метриком која задовољава Ајнштајнове једначине.

Математички доказ постојања гравитационих таласа у ОТР је врло једноставан. Уколико посматрамо Ајнштајнове једначине (3) далеко од извора поља (тј. маса) и метрику узмемо у облику

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + h_{\mu\nu}\,, \tag{4}$$

где је $h_{\mu\nu}$ мала пертурбација (талас) и $g^{(0)}_{\mu\nu}$ метрика простора у коме посматрамо талас, тада Ајнштајнове једначине (уз још неке додатне претпоставке) имају облик таласне једначине

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} \right) h_{\mu\nu} = 0\,, \tag{5}$$

где је c брзина светлости у вакууму. Позадинска метрика $g^{(0)}_{\mu\nu}$ може бити и метрика равног простора као и метрика закривљеног простора[1]. У оба случаја исход је је исти, таласна једначина за малу пертурбацију метрике $h_{\mu\nu}$. Може се даље показати да је талас трансверзалан. Математички делује све поприлично једноставно. Гравитациони таласи постоје уколико је ОТР "добра" теорија. Потврде ОТР у случајевима скретања светлосног зрака, прецесије Меркуровог перихела и индиректан доказ постојања црних рупа нас охрабрују да верујемо у ОТР.

Наравно, логично питање је шта се то таласа. Механичи талас је пренос осцилација кроз околну материјалну средину. Електромагнетни талас је пренос електричног и магнетног поља наелектрисане честице која се креће убрзано (нпр. осцилује). У случају гравитационих таласа, математички речено, таласа се метрика. Растојање између две инфинитезимално блиске тачке у простору са метриком $g_{\mu\nu}(x)$ је

$$ds^2 = \Sigma^3_{\mu=0}\Sigma^3_{\nu=0} g_{\mu\nu}(x) dx^\mu dx^\nu \, . \tag{6}$$

Уколико десну страну ставимо под квадратни корен и интегралимо од тачке A до тачке B, добићемо растојање између тачака у закривљеном простору. "Таласање" метрике слободније речено значи и таласање растојање између две тачке. Уколико бисмо детектовали те осцилације растојања између тела то би значило и да смо успели да детектујемо гравитационе таласе.

4. Детекција гравитационих таласа

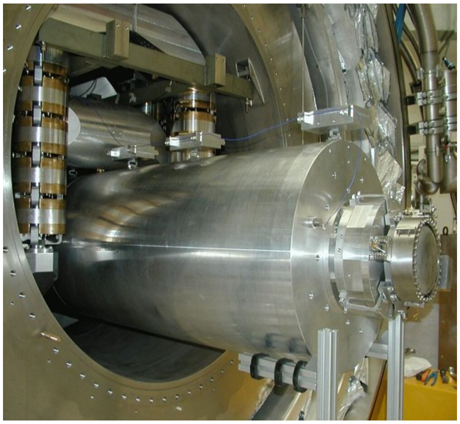
Директна детекција гравитационих таласа је веома компликована због изузетно слабог ефекта који таласи изазивају у детекторима. Амплитуда гравитационог таласа опада са растојањем као $\sim 1/r$. Тако да је веома тешко детектовати и таласе који настају услед спајања црних рупа јер им је амплитуда занемарљиво мала када дођу до Земље. До 2014. године није остварена директна детекција гравитационих таласа. Међутим, постоји низ експеримената који указују на то да гравитациони таласи заиста постоје. На пример, еволуција орбитирања бинарних пулсара потпуно је је у складу са губицима енергије кроз гравитационо зрачење које предвиђа ОТР. За откриће те посебне врсте пулсара додељена је и Нобелова награда 1993. године (Расел Халс и Џозеф Тејлор Јуниор).

Људи непрекидно раде на разним видовима детектора којима би регистровали гравитационе таласе. Први и најједноставнији детектори су тзв. Веберове шипке. Друга група детектора су интерферометарски детектори, а у новије време се развијају и граде високофреквентни детектори гравитационих таласа.

4.1. Веберове шипке

Једноставан уређај за детекцију очекиваног таласног кретања је тзв. Веберова шипка - велика, чврста метална шипка изолована од спољашњих вибрација. Овај тип детектора је био први који је коришћен. Принцип рада овог детектора је једноставан. Упадни гравитациони талас побуђује резонантно осциловање шипке, а шипка онда својим осциловањем појачава тај ефекат на детектабилни ниво. Савремене варијанте оваквих детектора су охлађене до екстремно ниских температура и опремљене квантним интерференционим уређајима за детекцију вибрација (на

[1] Математички, закривљеност простора подразумева да је тензор кривине различит од нуле. Самим тим су и компоненте метрике функције координата простор-времена. Постоји могућност да је тензор кривине нула а да је метрика зависна од координате. У том случају простор је раван и погодним избором координата можемо прећи на метрику равног простора.



Слика 1

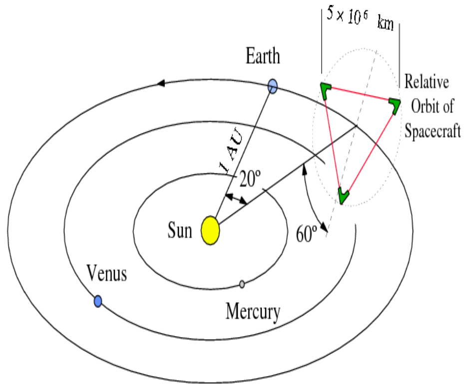
AURIGA детектор

Слика 2

MiniGRAIL детектор

пример, ALLEGRO). Проблем са овим детекторима што се они могу користити само за врло јаке гравитационе таласе.

MiniGRAIL је антена за детекцију гравитационих таласа сферног облика. Ова антена се налази на Универзитету у Лајдену (Холандија), а састоји се од сфере масе 1150 килограма охлађене на температуру 20 mK. Облик сфере омогућава детекцију из свих праваца. Фреквенције које овај детектор најбоље ”хвата” су интервалу 2-4 kHz, па је погодан за детекцију гравитационих таласа који настају у бинарним пулсарима и спајањем мањих црних рупа. Сличног типа је ултрахладни детектор AURIGA који се налази на INFN-у у Италији. Он се састоји од алуминијумског цилиндра дужине 3 метра који је охлађен на температуру реда величине $\sim$ mK.



Слика 3

LISA

4.2. Интерферометри

Ова група детектора користи ласерску интерферометрију за детекцију гравитационих таласа. Светлост крећући се кроз простор прати закривљење просторно-временског континума. Принцип рада ових детектора је да се измери ефекат интерференције ласерских зрака при чему је путна разлика настала ”скраћивањем” или ”издуживањем” простора.

Данас постоје само интерферометри на Земљи. Тренутно најосетљивији интерферометарски детектор је LIGO (Laser Interferometer Gravitational Wave Observatory). LIGO има три детектора: један је у Ливингстону (држава Луизијана) а друга два су у Хенфорду (држава Вашингтон). Сви они се састоје од по два велика крака дужине 2-4 километра који су под правим углом. Ласерски зраци путују унутар кракова у цевима дијаметра 1 метар. Промене у дужини које ласерски зрак прелази услед проласка гравитационог таласа би у принципу требало да региструје детектор у виду неке (ласерске) интерференционе слике. Нажалост, LIGO није успео да детектује гравитационе таласе. Очекује се да унапређене верзије овог детектора веће осетљивости (VIRGO, GEO 600, TAMA 300) доведу до детекције гравитационих таласа.

Интерферометарски детектори имају и своја ограничења. Прва од њих је шум који настаје као последица тога што ласерски извор производи фотоне у произвољним тренуцима. Ако уз то користимо и мало јачи ласер онда сами фотони својим импулсом могу да уздрмају детекторска огледала. Други проблем је проблем Брауновог кретања, а ни сеизмички шум се не може занемарити.

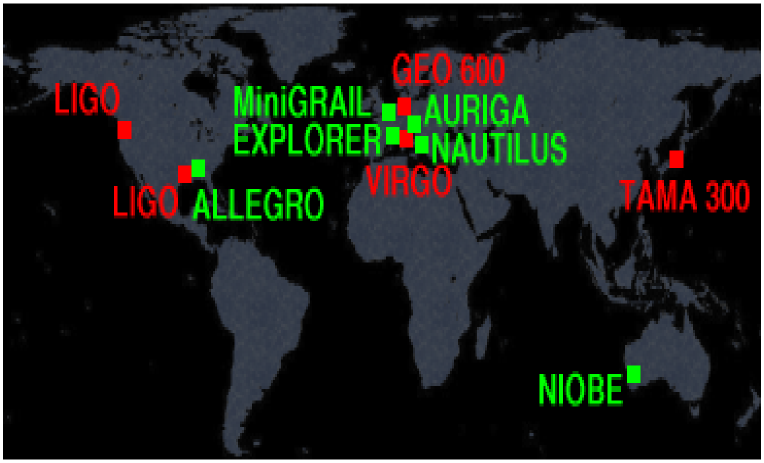
Због проблема које имају земаљски детектори, планира се и градња детектора у орбити око Земље (eLISA). Три сателита би формирала троугао при чему би свака страница била око 5 милиона километара. Тиме се добија добар вакуум, али и даље остаје проблем фотонског шума као и проблем са космичким зрачењем.

4.3. Високофреквентни детектори

Тренутно постоје два оперативна детектора који раде на горњој граници спектра $(10^{-7} - 10^{5}\text{Hz})$. Један је на Универзитету у Бирмингему (Енглеска) а други је на INFN-у у Ђенови (Италија). Трећи се гради на Универзитету у Чонкингу (Кина). Детектор у Бирмингему мери промене у стању поларизације микроталасног зрака који кружи по кругу пречника око 1 метра. Детектор у Ђенови је резонантна антена која се састоји од два спрегнута сферна суперпроводна хармонијска

Слика 4

VIRGO детектор



Слика 5

Распред детектора у свету

оцилатора пречника неколико центиметара. Осцилатори када нису спрегнути имају резонантне фреквенције које су скоро једнаке. Кинески детектор би требало да буде у стању да детектује таласе фреквенције реда 10 GHz.

5. Закључак

ОТР је у времену када је настала успела да објасни неке феномене који су били познати научницима попут скретања светлосних зрака у близини великих звезда и прецесију Меркуровог перихела. Свака "права" физичка теорија не објшњава само постојеће и познате феномене већ предвиђа и неке нове. Гравитациони таласи су један од тих феномена. Постојање гравитационих таласа теоријски је поткрепљено општом теоријом релативности јер следи из Ајнштајнових једначина гравитационог поља. Откриће бинарних пулсара (систем две неутронске звезде), који губе енергију потпуно у складу са предвиђањима ОТР, даје експериментални основ постојању гравитационих таласа. Пошто амплитуда таласа опада као са растојањем као $\sim 1/r$ ефекти гравитационог зрачења које се мере на Земљи су врло мали, па је и детекција прилично отежана. Граде се савремени детектори високе осетљивости који дају наду у коначну директну детекцију гравитационих таласа.

ДОДАТНА ЛИТЕРАТУРА

1. C. M. Misner, K. S. Thorne, J. A. Wheeler, *Gravitation*, W. H. Freeman and Co., San Francisco, 1973.
2. L.D. Landau, E.M. Lifshitz, *The Classical Theory of Fields*, Pergsmon Press, 1971.
3. http://en.wikipedia.org/wiki/Gravitational-wave observatory.

Subject **Re: RS2016**
From Ljubisa Nesic <nesiclj@junis.ni.ac.rs>
To Bojan Nikolic <bnikolic@ipb.ac.rs>, Ivan Dojcinovic <ivan.dojcinovic@ff.bg.ac.rs>
Date 2016-02-23 14:07

webmail

Dragi Bojane,
drago mi je sto si privhatio da drzis predavanje o talasima.
prosledjujem tvoj mail predsedniku DFS. Nadam se da ce ti dati odgovarajuca uputstva.

Ljubisa

On 2/23/2016 1:54 PM, Bojan Nikolic wrote:

> Dragi Ljubisa,
>
> hvala na pozivu. Sve je jasno sem jednog tehnickog detalja. Sta ja tacno treba da uradim da bih bio clan DFS? Kome treba da platim i koliko?
>
> Pozdrav,
> Bojan
>
> On 22 Feb 2016 17:05, rep.seminar@ff.bg.ac.rs wrote:
>
>> Postovani Bojane,
>> imam izuzetno zadovoljstvo da vas, ispred Strucnog odbora republickog seminara o nastavi fizike za 2016. godinu (http://www.dfs.rs/seminar2016/), pozovem da odrzite plenarno predavanje vezano za otkrice gravitacionih talasa.
>> Seminar ce biti odrzan na Zlatiboru od 12. do 14. maja a rok za pisanje rada je 14. mart. Radovi ce, nakon recenzije, biti objavljeni u 3. broju casopisa Nastava fizike koji smo pokrenuli prosle godine.
>> Na sajtu seminara mozete naci i upustvo za pisanje rada, odnosno odgovarajuci template a vi, kao predavac po pozivu, imate na raspolaganju do 10 strana. Ukoliko imate bilo kakav tehnicki problem slobodno se obratite. DFS ce snositi troskove vaseg puta i smestaja ali je potrebno da budete clan drustva.
>> Imajte u vidu da ce na seminaru biti takodje i jedno uvodno predavanje o OTR.
>>
>> Ljubisa Nesic
>> predsednik Komisije za seminare DFS

XXXIV Републички семинар о настави физике

Студентско одмаралиште "Ратко Митровић", Златибор, 12 - 14. мај 2016.

ПРОГРАМ

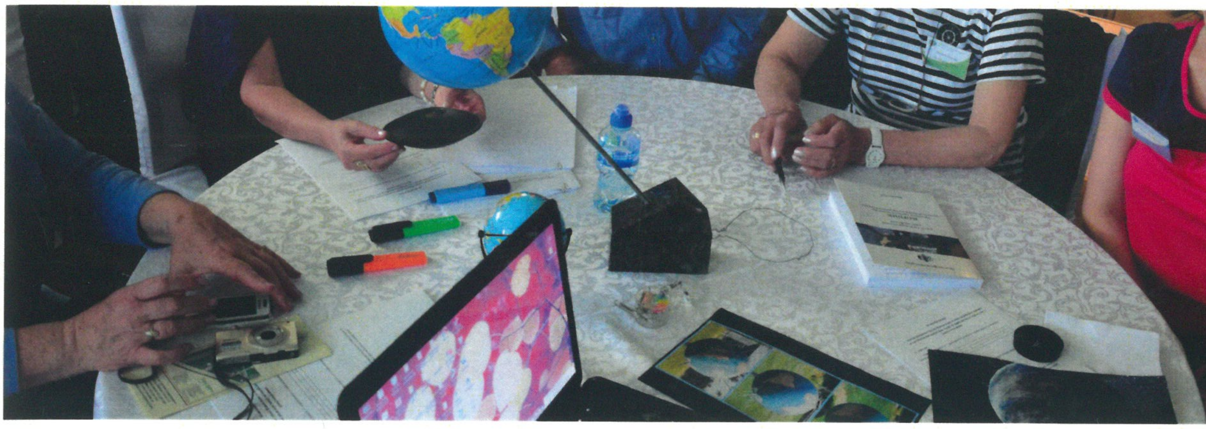
	Четвртак, 12. мај	
08.30-10.00	**Регистрација**	
09.00-09.45	**Састанак редакције часописа Настава физике**	
10.00-10.15	**Свечано отварање** **Председава: Љубиша Нешић** **Говорници: Иван Дојчиновић, Душанка Обадовић, Милан Распоповић**	
10.15-10.45	**И. Бикит, К. Бикит** *Нобелова награда за физику 2015. године: Физика неутрина*	
10.45-11.15	**Б. Драговић** *100 година Ајнштајнове теорије гравитације*	
11.15-11.45	**Кафе пауза;** ***Предаја пријава за радионице***	
	Председава: Мирјана Поповић-Божић	
11.45-12.15	**Б. Николић** *Гравитациони таласи - од теорије до директне детекције*	
12.15-12.35	***Љ. Нешић, Л. Раденковић*** *Часопис НАСТАВА ФИЗИКЕ и његов значај за методику наставе физике*	
12.35-12.55	**А. Жекић, М. Поповић-Божић, Б. Радиша, Б. Мисаиловић** *Прелиставајући и читајући међународне часописе у области истраживачког образовања у физици*	
12.55-13:15	**Т. Јовановић, Б. Јовановић** *Настава физике на медицинском факултету у Нишу од оснивања до данас*	
13.15-15.00	**Пауза за ручак**	
15.00-16.30	**Радионице - прва група** **I** ЦЕРН Мастеркласс, **(Ј. Милисављевић, И. Стојановић, С. Митић, Т. Марковић Топаловић, М. Радић, М. Ристановић)** **II** Научна визуелизација у школском простору и на паметном телефону, **(С. Булат, М. Давидовић, Љ. Иванчевић, М. Јоксимовић, Т. Марковић-Топаловић, М. Поповић-Божић, Б. Стојичић)** **III** Изабране лабораторијске вежбе из физике у гимназији **(М. Ковачевић, С. Ковачевић, А. Марковић, Д. Карајовић)** **IV** Како да искористим знања из наука у образовању за одрживи развој, **(С. Јокић, Љ. Јокић)**	
16.30-17.00	**Кафе пауза**	
17.00-18.30	**Округли сто (две теме по 45 минута свака):** **Тема 1: Популаризација физике (И. Дојчиновић, М. Степић, Т. Продановић, С. Ивковић)** **Тема 2: Реформа образовања и положај физике (Т. М. Топаловић, М. Ковачевић, О. Клисурић, А. Хрлец)**	
18.30-20.00	**Седница савета одељења за основно образовање ДФС**	**Седница савета одељења за средње образовање ДФС**

	Петак, 13. Мај
Време	**Председава: Маја Стојановић**
09.00-09.20	**М. Бошњак Степановић** *Приказ дисертације: Примена истраживачке методе у реализацији физичких садржају почетној настави природних наука*
09.20-09.40	***Г. Хајдуковић Јандрић*** *Приказ дисертације: Развој наставних инструкција у активној настави физике*
09.40-10.00	**Д. Радловић Чубрило** *Приказ дисертације: Ефекти примене мултимедије у настави физике у првом разреду средње стручне школе*
10.00-10.15	***М. Стојановић, М. Ковачевић, Љ. Костић*** *Приказ монографије «Поглавља методике наставе физике» Љубише Нешића*
10.15-10.30	***М. Поповић-Божић, А. Жекић*** *«Колегијално подучавање». Превод књиге «Peer Instruction» од Ерика Мазура*
10.30-11.00	**О. Зајков, Б. Митревски** *Физика и критичко мишљење*
11.00-11.30	**Кафе пауза**
	Председава: Душанка Обадовић
11.30-12.00	**Ф. Соколић** *Што је то свјетлост?*

ISSN 2406-2626

Број 3

НАСТАВА ФИЗИКЕ



Београд 2016.

Настава физике, број 3, 2016, стр 213 - 221

Гравитациони таласи – од теорије до директне детекције

Бојан Николић

Институт за физику, Универзитет у Београду, Прегревица 118, 11080 Земун

Апстракт. Пре једног века Алберт Ајнштајн формулисао је Општу теорију релативности (ОТР). Једна од последица Опште теорије релативности је постојање гравитационих таласа. У овом раду ћемо дати кратак теоријски преглед о (гравитационим) таласима, а значајну пажњу ћемо посветити свим сада могућим видовима детекције гравитационих таласа са акцентом на недавни успех – директну детекцију гравитационих таласа.

Кључне речи: гравитациони таласи, ОТР, директна детекција.

УВОД

Камен бачен у воду изазива појаву таласа на њеној површини, треперење гласних жица омогућава да чујемо саговорника, земљотреси изазивају појаву цунами таласа итд. Све наведено су примери *механичких* таласа. За простирање механичких таласа је потребна материјална средина. Поремећај настао на једном месту преноси се таласом кроз материјалну средину. Самим тим механички таласи се не простиру кроз вакуум. Положај честице средине у датом тренутку t у тачки $\vec{r}$ (колоквијално "поремећај") $u(\vec{r},t)$ задовољава хомогену таласну једначину

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}-\frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right)u(\vec{r},t)=0\,. \qquad (1)$$

Величина v представља брзину таласа у датој материјалној средини (није једнака брзини честице материјалне средине). Поремећај може бити ортогоналан на правац простирања таласа (трансверзални талас) или колинеаран са правцем простирања таласа (лонгитудиналан талас).

У другој половини 19. века енглески физичар Џејмс Кларк Максвел је, обједињавајући дотадашња експериментална сазнања, написао једначине електромагнетног поља познате у литератури као Максвелове једначине. Једноставна анализа тих једначина показује да у простору где нема наелектрисања и струја јачина електричног поља и јачина магнетног поља задовољавају хомогене таласне једначине. Простије речено, око простора у коме су задате расподеле наелектрисања и струја постоји електромагнетно (ЕМ) поље. Енергија ЕМ поља се преноси дуж правца који је ортогоналан на векторе јачине електричног и магнетног поља - ЕМ талас је трансверзалан. За разлику од механичких таласа, ЕМ таласи се простиру и кроз вакуум и то највећом брзином у природи $c = 3 \cdot 10^8$ m/s. Херцовим експериментом (1888) потврђено је постојање ЕМ таласа. Питање које су поставили научници тога времена тицало се средине кроз коју се ЕМ талас простире. По аналогији са механичким таласима морала је постојати нека средина која преноси таласе. Тада је уведен појам **етера**. Међутим, Мајкелсон-Морлијев експеримент као и многа унапређења овог експеримента потврдили су да је брзина светлости иста у свим правцима и да не зависи од избора референтног система - једноставније речено, етера, у облику како су га тада научници замишљали, нема. И тада (1905) се родила специјална теорија релативности (СТР).

СТР почива на два постулата. Први се тиче инварijнтности облика физичких закона у односу на избор инерцијалног система референце (то је већ био саставни део Галилејевог принципа релативности), док се другим постулатом потврђује експериментлна чињеница да је брзина светлости независна од избора инерцијалног система референце (Ајнштајн не спомиње експлицитно у својим радовима нити Мајкелсон-Морлијев експеримент нити друге сличне експерименте, али други постулат „признаје“ резултат тих експеримената).

У СТР се спомињу само инерцијални системи референце. Сам Ајнштајн није био задовољан и сматрао је да физички закони морају имати исти облик независно од избора референтног система, инерцијалног или неинерцијалног, тј. увидео је да у "причу" мора да укључи и гравитацију. Математичким језиком речено, какву год трансформацију координата да направимо закони физике морају очувати свој облик (строго математички речено инваријантност на дифеоморфизме [1,2]). И тако је дошао до Опште теорије релативности (ОТР).

Шта је уопште гравитација? По Њутновој теорији гравитација је **сила**. У Њутновој теорији гравитације **маса је извор гравитационог поља**. Свако друго тело одређене масе које се нађе у датом гравитационом пољу је изложено деловању привлачне силе. Њутн је дао аналитички облик за гравитациону интеракцију свом делу Математички принципи природне филозофије (*Philosophiae Naturalis Principia Mathematica*) које је први пут објављено 5. јула 1687. године.

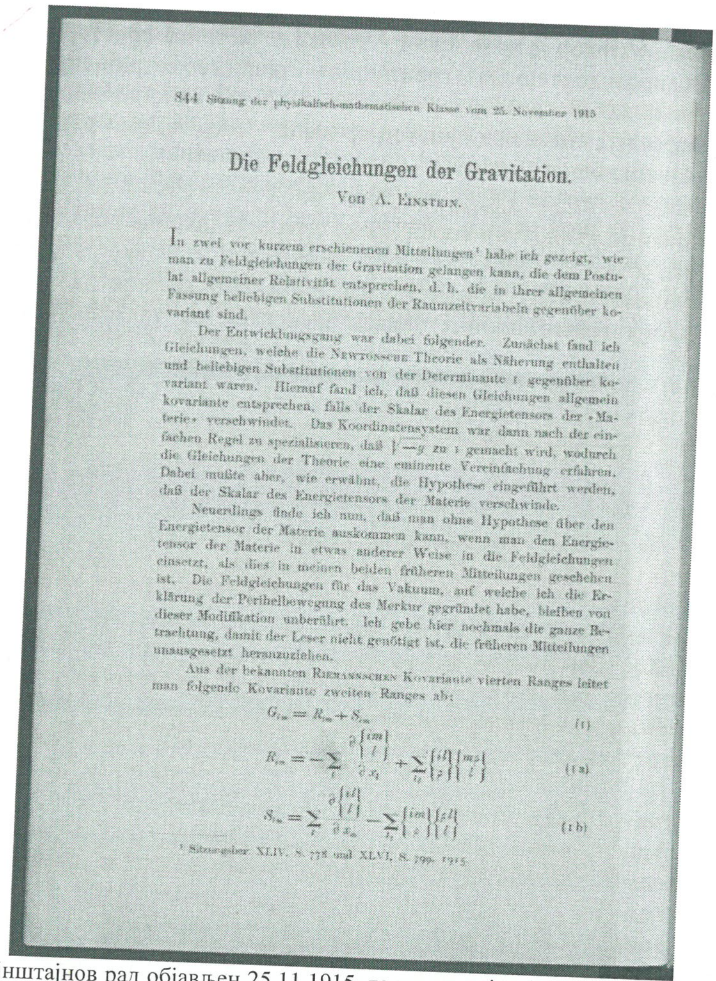
И по том питању није било никаквих квалитативних помака до почетка 20. века. А онда се појавио Алберт Ајнштајн, који је кроз СТР увео у физику обједињеност простора и времена у један просторно-временски континуум тј. време више није параметар већ координата, а са ОТР направио праву револуцију у разумевању гравитације као фундаментлне интеракције у природи.

Овде нећемо улазити у суптилне детаље извођења Ајнштајнових једначина за гравитационо поље. Ајнштајн је једначине извео користећи се законом одржања тензора енергије-импулса као и особинама неких геометријских величина. У савременој литератури извођење иде из одговарајућег дејства применом методе минимума дејства. Било како било, једначине за гравитационо поље су облика

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}, \qquad (2)$$

где је, најгрубље речено, на левој страни ГЕОМЕТРИЈА, а на десној страни МАТЕРИЈА. Ова једначина успоставља везу између геометрије простор-времена и материје која својим присуством "закривљује" тај простор-време. У Ајнштајновој слици гравитација није сила већ **геометрија простор-времена**.

Наравно, добра физичка теорија има особину да објашњава познате феномене и предвиђа неке нове. У оквиру Ајнштајнове теорије успешно је објашњена појава скретања светлосних зрака који пролазе близу Сунца, затим прецесија Меркуровог перихела. Теорија предвиђа постојање сингуларитета (основ за теорију Великог праска) као и црних рупа, за чије постојање постоје индиректни докази. Такође једна од последица ОТР је и постојање *гравитационих таласа*.



844 Sitzung der physikalisch-mathematischen Klasse vom 25. November 1915

Die Feldgleichungen der Gravitation.

Von A. Einstein.

In zwei vor kurzem erschienenen Mitteilungen[1] habe ich gezeigt, wie man zu Feldgleichungen der Gravitation gelangen kann, die dem Postulat allgemeiner Relativität entsprechen, d. h. die in ihrer allgemeinen Fassung beliebigen Substitutionen der Raumzeitvariabeln gegenüber kovariant sind.

Der Entwicklungsgang war dabei folgender. Zunächst fand ich Gleichungen, welche die Newtonsche Theorie als Näherung enthalten und beliebigen Substitutionen von der Determinante 1 gegenüber kovariant waren. Hierauf fand ich, daß diesen Gleichungen allgemein kovariante entsprechen, falls der Skalar des Energietensors der »Materie« verschwindet. Das Koordinatensystem war dann nach der einfachen Regel zu spezialisieren, daß $\sqrt{-g}$ zu 1 gemacht wird, wodurch die Gleichungen der Theorie eine eminente Vereinfachung erfahren. Dabei mußte aber, wie erwähnt, die Hypothese eingeführt werden, daß der Skalar des Energietensors der Materie verschwinde.

Neuerdings finde ich nun, daß man ohne Hypothese über den Energietensor der Materie auskommen kann, wenn man den Energietensor der Materie in etwas anderer Weise in die Feldgleichungen einsetzt, als dies in meinen beiden früheren Mitteilungen geschehen ist. Die Feldgleichungen für das Vakuum, auf welche ich die Erklärung der Perihelbewegung des Merkur gegründet habe, bleiben von dieser Modifikation unberührt. Ich gebe hier nochmals die ganze Betrachtung, damit der Leser nicht genötigt ist, die früheren Mitteilungen unausgesetzt heranzuziehen.

Aus der bekannten Riemannschen Kovariante vierten Ranges leitet man folgende Kovariante zweiten Ranges ab:

$$G_{im} = R_{im} + S_{im} \quad (1)$$

$$R_{im} = -\sum_{l} \frac{\partial \begin{Bmatrix} im \\ l \end{Bmatrix}}{\partial x_l} + \sum_{l\rho} \begin{Bmatrix} il \\ \rho \end{Bmatrix} \begin{Bmatrix} m\rho \\ l \end{Bmatrix} \quad (1a)$$

$$S_{im} = \sum_{l} \frac{\partial \begin{Bmatrix} il \\ l \end{Bmatrix}}{\partial x_m} - \sum_{l\rho} \begin{Bmatrix} im \\ \rho \end{Bmatrix} \begin{Bmatrix} \rho l \\ l \end{Bmatrix} \quad (1b)$$

[1] Sitzungsber. XLIV, S. 778 und XLVI, S. 799, 1915.

СЛИКА 1. Ајнштајнов рад објављен 25.11.1915. године у којем је заснована ОТР и изведена чувена једначина.

ГРАВИТАЦИОНИ ТАЛАСИ

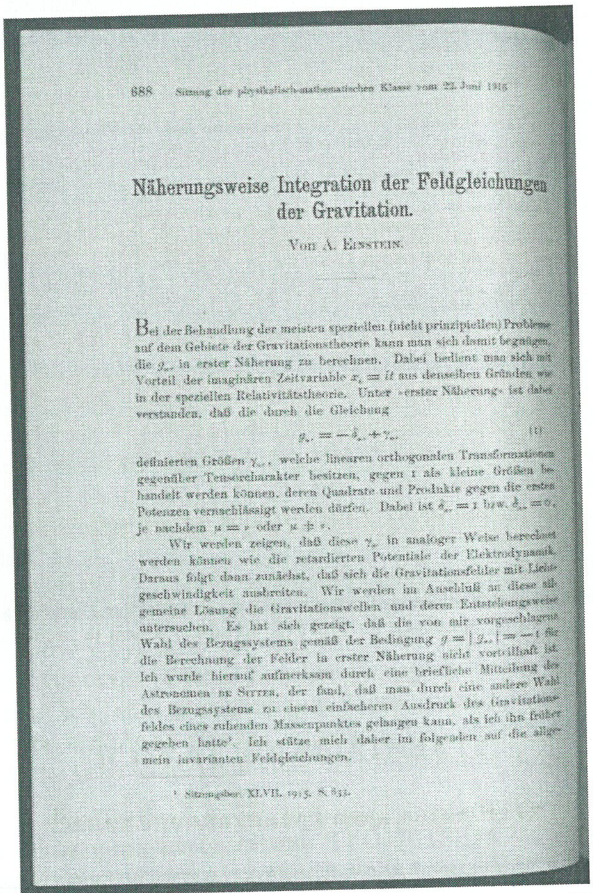
ОТР дозвољава гравитационе таласе

Математички доказ постојања гравитационих таласа у ОТР је врло једноставан. Уколико посматрамо Ајнштајнове једначине у празном простору далеко од маса, онда се испоставља да метрика простора задовољава таласну једначину. Добија се да су гравитациони таласи трансверзални таласи који се простиру брзином светлости у вакуму. У случају механичких таласа материјална средина се таласа, док у случају ЕМ таласа долази до таласања електричног и магнетног поља. Логично питање које се намеће код гравитационих таласа је шта се то таласа?

Формалан одговор је врло прост – таласа се метрика просторно-временског континуума. С обзиром да је по ОТР грвитација у ствари геометрија простор-времена онда је мало „физичкији“ одговор – таласа се сам просторно-временски континум. А како се манифестује таласање простор-времена? Посматрајмо растојање између две инфинитезимално блиске тачке

$$ds^2 = \sum_{\mu,\nu=0}^{3} g_{\mu\nu}(x)dx^\mu dx^\nu. \quad (3)$$

Уколико интегралимо квадратни корен десне стране једначине добићемо растојање између две тачке у закривљеном простору. Очигледно је да ако се метрика таласа онда се и растојање између тачака таласа. Видећемо касније да су све методе директне детекције гравитационих таласа засноване на таласању растојања тј. дужине.



688 Sitzung der physikalisch-mathematischen Klasse vom 22. Juni 1916

Näherungsweise Integration der Feldgleichungen der Gravitation.

Von A. Einstein.

Bei der Behandlung der meisten speziellen (nicht prinzipiellen) Probleme auf dem Gebiete der Gravitationstheorie kann man sich damit begnügen, die $g_{\mu\nu}$ in erster Näherung zu berechnen. Dabei bedient man sich mit Vorteil der imaginären Zeitvariable $x_4 = it$ aus denselben Gründen wie in der speziellen Relativitätstheorie. Unter »erster Näherung« ist dabei verstanden, daß die durch die Gleichung

$$g_{\mu\nu} = -\delta_{\mu\nu} + \gamma_{\mu\nu} \quad (1)$$

definierten Größen $\gamma_{\mu\nu}$, welche linearen orthogonalen Transformationen gegenüber Tensorcharakter besitzen, gegen 1 als kleine Größen behandelt werden können, deren Quadrate und Produkte gegen die ersten Potenzen vernachlässigt werden dürfen. Dabei ist $\delta_{\mu\nu} = 1$ bzw. $\delta_{\mu\nu} = 0$, je nachdem $\mu = \nu$ oder $\mu \neq \nu$.

Wir werden zeigen, daß diese $\gamma_{\mu\nu}$ in analoger Weise berechnet werden können wie die retardierten Potentiale der Elektrodynamik. Daraus folgt dann zunächst, daß sich die Gravitationsfelder mit Lichtgeschwindigkeit ausbreiten. Wir werden im Anschluß an diese allgemeine Lösung die Gravitationswellen und deren Entstehungsweise untersuchen. Es hat sich gezeigt, daß die von mir vorgeschlagene Wahl des Bezugssystems gemäß der Bedingung $g = |g_{\mu\nu}| = -1$ für die Berechnung der Felder in erster Näherung nicht vorteilhaft ist. Ich wurde hierauf aufmerksam durch eine briefliche Mitteilung des Astronomen de Sitter, der fand, daß man durch eine andere Wahl des Bezugssystems zu einem einfacheren Ausdruck des Gravitationsfeldes eines ruhenden Massenpunktes gelangen kann, als ich ihn früher gegeben hatte[1]. Ich stütze mich daher im folgenden auf die allgemein invarianten Feldgleichungen.

[1] Sitzungsber. XLVII, 1915, S. 831.

СЛИКА 2. *Приближно решавање једначина гравитационог поља* – математички доказ постојања гравитационих таласа (Ајнштајн, 22.06.1916. година).

ДЕТЕКЦИЈА ГРАВИТАЦИОНИХ ТАЛАСА

Пре него што пређемо на разматрање свих видова директне детекције као и анализе недавног директног мерења гравитационих таласа, потребно је рећи да је постојање гравитационих таласа индиректно потврђено 1993. године.

Године 1974. Расел Алан Халс и руководилац његове докторске тезе Џозеф Хутон Тејлор Јуниор су открили један бинарни пулсар који се састоји од пулсара (неутронске звезде) и пратеће звезде. Овај бинарни пулсар губи енергију на начин како и предвиђа ОТР па самим тим ово откриће је истовремено индиректни доказ постојања гравитационих таласа. За ово откриће Расел и Халс су добили Нобелову награду за физику 1993. године.

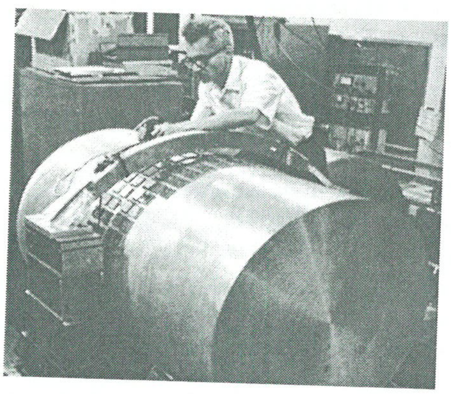
Детектори који се користе за директну детекцију гравитационих таласа деле се у три групе: механички, интерферометарски и високофреквентни детектори.

Механички детектори

<u>**Веберове шипке**</u>

Једноставан уређај за детекцију очекиваног таласног кретања је тзв. Веберова шипка - велика, чврста метална шипка изолована од спољашњих вибрација. Овај тип детектора је био први који је коришћен од стране конструктора Џозефа Вебера са Универзитета Мериленд. Он је чак тврдио да је детектовао гравитационе таласе, али су његови резултати доведени у сумњу због начина обраде података. Испоставило се на крају да је Веберова детекција гравитационих таласа фингирана због потреба финансирања пројекта.

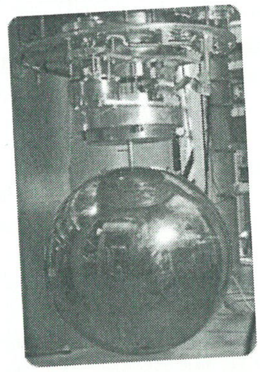
Принцип рада овог детектора је једноставан. Упадни гравитациони талас побуђује резонантно осциловање шипке, а шипка онда својим осциловањем појачава тај ефекат на детектабилни ниво. Савремене варијанте оваквих детектора су охлађене до екстремно ниских температура и опремљене квантним интерференционим уређајима за детекцију вибрација (на пример, ALLEGRO). Проблем са овим детекторима што се они могу користити само за врло јаке гравитационе таласе.



СЛИКА 3. Џозеф Вебер у својој лабораторији 1965. године

MiniGRAIL

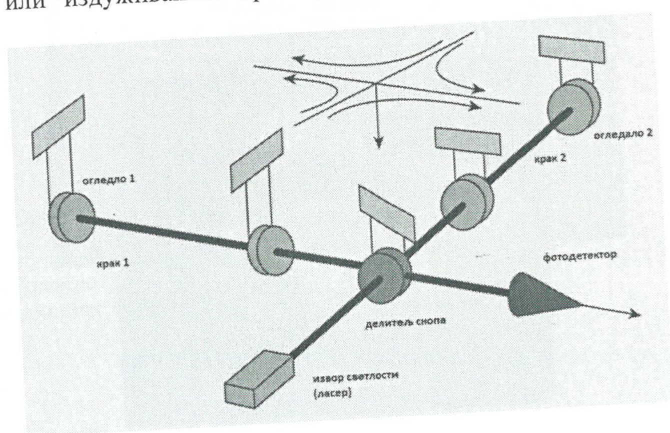
MiniGRAIL је антена за детекцију гравитационих таласа сферног облика. Ова антена се налази на Универзитету у Лајдену (Холандија), а састоји се од сфере масе 1150 килограма охлађене на температуру 20 mK. Облик сфере омогућава детекцију из свих праваца. Фреквенције које овај детектор најбоље "хвата" су интервалу 2-4 kHz, па је погодан за детекцију гравитационих таласа који настају у бинарним пулсарима и спајањем мањих црних рупа. Сличног типа је ултрахладни детектор AURIGA који се налази на INFN-у у Италији. Он се састоји од алуминијумског цилиндра дужине 3 метра који је охлађен на температуру реда величине mK.



СЛИКА 4. Детектор MiniGRAIL

Интерферометарски детектори

Ова група детектора користи ласерску интерферометрију за детекцију гравитационих таласа. Светлост крећући се кроз простор прати закривљење просторно-временског континума. Принцип рада ових детектора је да се измери ефекат интерференције ласерских зрака при чему је путна разлика настала "скраћивањем" или "издуживањем" простора због таласања.



СЛИКА 5. Принцип рада интерферометарског детектора

Данас постоје само интерферометри на Земљи. Тренутно најосетљивији интерферометарски детектор је LIGO (Laser Interferometer Gravitational Wave Observatory). LIGO има три детектора: један је у Ливингстону (држава Луизијана) а друга два су у Хенфорду (држава Вашингтон). Сви они се састоје од по два велика крака дужине 2-4 километра који су под правим углом. Ласерски зраци путују унутар кракова у цевима дијаметра 1 метар. Промене у дужини које ласерски зрак прелази услед проласка гравитационог таласа би у принципу требало да региструје детектор у виду неке (ласерске) интерференционе слике.

Интерферометарски детектори имају и своја ограничења. Прва од њих је шум који настаје као последица тога што ласерски извор производи фотоне у произвољним тренуцима. Ако уз то користимо и мало јачи ласер онда сами фотони својим импулсом могу да уздрмају детекторска огледала. Други проблем је проблем Брауновог кретања, а ни сеизмички шум се не може занемарити.

Због проблема које имају земаљски детектори, планира се и градња детектора у орбити око Земље (eLISA, пројекат започет децембра 2015. године). Три сателита би формирала троугао при чему би свака страница била око 5 милиона километара. Тиме се добија добар вакуум, али и даље остаје проблем фотонског шума као и проблем са космичким зрачењем.

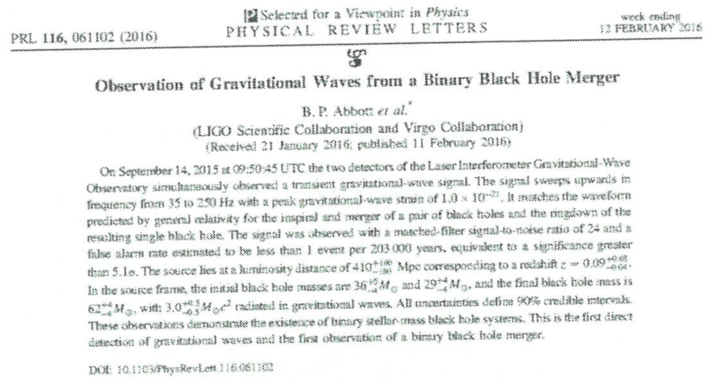
Високофреквентни детектори

Тренутно постоје два оперативна детектора који раде на горњој граници спектра (10^{-7}-10^{5} Hz). Један је на Универзитету у Бирмингему (Енглеска) а други је на INFN-у у Ђенови (Италија). Трећи се гради на Универзитету у Чонкингу (Кина). Детектор у Бирмингему мери промене у стању поларизације микроталасног зрака који кружи по кругу пречника око 1 метра. Детектор у Ђенови је резонантна антена која се састоји од два спрегнута сферна суперпроводна хармонијска оцилатора пречника неколико центиметара. Осцилатори када нису спрегнути имају резонантне фреквенције које су скоро једнаке. Кинески детектор би требало да буде у стању да детектује таласе фреквенције реда 10 GHz.

ДИРЕКТНА ДЕТЕКЦИЈА ГРВИТАЦИОНИХ ТАЛАСА

Група научника из две велике колаборације, LIGO и VIRGO, је објавила 11. фебруара 2016. године [3] да је обављена успешна директна детекција гравитационих таласа. Физички све се одиграло на америчком делу велике колаборације (LIGO).

Дана 14. септембра 2015. године детектори ове колаборације у Хенфорду (Вашингтон) и Ливингстону (Лујзијана) детектовали су гравитациони талас који је настао спајањем две црне рупе, једна масе 36 соларних маса а друга 29 соларних маса. Настала је црна рупа масе 62 соларне масе а 3 соларне масе су израчене у виду гравитационих таласа. Овакав резултат као и профили детектованих сигнала су (у границама грешке) у складу са предвиђањима ОТР. Овај експеримент је потврдио *постојање бинарних система црних рупа, омогућио директну детекцију гравитационих таласа и први детектовао спајање црних рупа*.


PRL 116, 061102 (2016) Selected for a Viewpoint in *Physics* PHYSICAL REVIEW LETTERS week ending 12 FEBRUARY 2016

Observation of Gravitational Waves from a Binary Black Hole Merger

B. P. Abbott *et al.*[*]

(LIGO Scientific Collaboration and Virgo Collaboration)

(Received 21 January 2016; published 11 February 2016)

On September 14, 2015 at 09:50:45 UTC the two detectors of the Laser Interferometer Gravitational-Wave Observatory simultaneously observed a transient gravitational-wave signal. The signal sweeps upwards in frequency from 35 to 250 Hz with a peak gravitational-wave strain of 1.0×10^{-21}. It matches the waveform predicted by general relativity for the inspiral and merger of a pair of black holes and the ringdown of the resulting single black hole. The signal was observed with a matched-filter signal-to-noise ratio of 24 and a false alarm rate estimated to be less than 1 event per 203 000 years, equivalent to a significance greater than 5.1σ. The source lies at a luminosity distance of 410^{+160}_{-180} Mpc corresponding to a redshift $z = 0.09^{+0.03}_{-0.04}$. In the source frame, the initial black hole masses are $36^{+5}_{-4} M_{\odot}$ and $29^{+4}_{-4} M_{\odot}$, and the final black hole mass is $62^{+4}_{-4} M_{\odot}$, with $3.0^{+0.5}_{-0.5} M_{\odot} c^2$ radiated in gravitational waves. All uncertainties define 90% credible intervals. These observations demonstrate the existence of binary stellar-mass black hole systems. This is the first direct detection of gravitational waves and the first observation of a binary black hole merger.

DOI: 10.1103/PhysRevLett.116.061102

СЛИКА 6. Апстракт рада [3] у коме је објављена директна детекција гравитационих таласа

Апаратура на којој је извршена детекција је унапређена верзија почетног LIGO детектора (AdvancedLIGO). Побољшања која су урађена првенствено се тичу повећања осетљивости сензора као и умањењу постојећих шумова.

Очекује се да детектори Advanced VIRGO, KAGRA као и могући трећи LIGO детектор у Индији дају додатну потврду овом открићу као и да подигну ниво прецизности и тачности мерења.

ЗАКЉУЧАК

ОТР је у времену када је настала (Први светски рат у пуном јеку!) успела да објасни неке феномене који су били познати научницима попут скретања светлосних зрака у близини великих звезда и прецесију Меркуровог перихела. Свака "права" физичка теорија не објашњава само постојеће и познате феномене већ предвиђа и неке нове. Гравитациони таласи су један од тих феномена. Постојање гравитационих таласа теоријски је поткрепљено Општом теоријом релативности јер следи из Ајнштајнових једначина гравитационог поља. Откриће бинарних пулсара (систем две неутронске звезде), који губе енергију потпуно у складу са предвиђањима ОТР, дало је експериментални основ постојању гравитационих таласа. Са изградњом интерферометарских детектора кренуло се у коначну потрагу за гравитационим таласима. Принципијелно није било препрека и све је било питање прецизности апаратуре. Коначно су у јесен 2015. године научници успели да детектују гравитационе таласе који су настали у једном врло интензивном догађају – судару црних рупа. Ово откриће даје наду да се могу детектовати гравитациони таласи настали после Великог праска што би у „неку руку“ био доказ да се тај Прасак стварно и десио.

ЛИТЕРАТУРА

1. C. M. Misner, K. S. Thorne, J. A. Wheeler, *Gravitation*, W. H. Freeman and Co., San Francisco, 1973.
2. L. D. Landau, E. M. Lifshitz, *The Classical Theory of Fields*, Pergsmon Press, 1971.

3. B. P. Abbott et al., *Observation of Gravitational Waves from a Binary Black Hole Merger*, Phys. Rev. Lett. **116** 061102 (2016).

Gravitational waves – from theory to direct detection

Bojan Nikolic

Institute of Physics, University of Belgrade, Pregrevica 118, Zemun

Abstract. A century ago Albert Einstein formulated General Theory of Relativity (GR). The existence of gravitational waves is one of the consequences of the GR. In this article we will give a short theoretical review about (gravitational) waves, and later we will dedicate the great attention to all known types of detection of the gravitational waves with accent on recent success – direct detection of gravitational waves.

Key words: gravitational waves, GR, direct detection.

Univerzitet u Beogradu
Fizički Fakultet

Master teza

Nekomutativnost i neasocijativnost zatvorene bozonske strune

Student: Danijel Obrić
Mentor: Dr Bojan Nikolić

Septembar, 2017

З А П И С Н И К

са X седнице Изборног и Наставно-научног већа одржане у среду 20. септембра 2017. године

Седници присуствује 42 члана Изборног и Наставно-научног већа.

Службено одсутни: проф. др Маја Бурић
проф. др Воја Радовановић
проф. др Бећко Касалица
доц. др Михајло Ваневић
доц. др Иван Виденовић

Оправдано одсутни: проф. др Владимир Милосављевић
проф. др Иванка Милошевић
проф. др Татјана Вуковић
проф. др Владан Вучковић
доц. др Саша Дмитровић
доц. др Немања Ковачевић
доц. др Драган Реџић
доц. др Никола Шишовић
Др Милош Скочић
Марјан Ћирковић
Нора Тркља
Светислав Мијатовић

Неоправдано одсутни: проф. др Илија Марић
доц. др Владимир Миљковић

Седница је започела у 11:10 часова одавањем поште минутом ћутања преминулом др Душану Михајловићу, некадашњем доценту и библиотекару Физичког факултета у пензији. Декан Факултета проф. др Јаблан Дојчиловић предложио је, затим, следећи

Д н е в н и р е д

1. Усвајање Записника са IX седнице Изборног и Наставно-научног већа Физичког факултета.

<u>Изборно веће</u>

2. Разматрање предлога катедара у вези са покретањем поступка за избор наставника Физичког факултета и то:
    a) Катедре за физику атома, молекула, јонизованих гасова, плазме и квантну оптику у вези са расписивањем конкурса за избор једног ванредног професора за ужу научну област Физика атома и молекула
    b) катедара Института за метеорологију у вези са расписивањем конкурса за избор једног доцента за ужу научну област Климатологија и примењена метеорологија
3. Усвајање Извештаја Комисије за избор наставника Физичког факултета и то:
    a) једног ванредног професора за ужу научну област Квантна и математичка физика
    b) једног доцента за ужу научну област Физика јонизованих гасова и плазме
    c) једног доцента за ужу научну област Настава физике
4. Покретање поступка за избор НОРЕ ТРКЉА у звање истраживач-сарадник
5. Усва јање Извештаја Комисије за избор у научно звање и то:
    a) др ВЛАДИМИРА СТОЈАНОВИЋА у звање виши научни сарадник
    b) др БРАНИСЛАВЕ МИСАИЛОВИЋ у звање научни сарадник

<u>Наставно-научно веће</u>

6. Одређивање Комисије за оцену испуњености услова и оправданост предложене теме за израду докторске дисертације за:

   a) ЈАСМИНУ АТИЋ, дипломираног инжењера електротехнике, која је пријавила докторску дисертацију под називом: „ТРАНСПОРТ ЕЛЕКТРОНА, РАЗВОЈ ЛАВИНА И ПРОПАГАЦИЈА СТРИМЕРА У ЈАКО ЕЛЕКТРОНЕГАТИВНИМ ГАСОВИМА"

7. Усвајање Извештаја Комисије за оцену испуњености услова и оправданост предложене теме за израду докторске дисертације и одређивање ментора за:
   a) НИКОЛУ ИВАНОВИЋА, дипломираног физичара, који је пријавио докторску дисертацију под називом: „ПРОУЧАВАЊЕ ОБЛИКА СПЕКТРАЛНИХ ЛИНИЈА Ne I И Ne II У ПРИКАТОДНОЈ ОБЛАСТИ АБНОРМАЛНОГ ТИЊАВОГ ПРАЖЊЕЊА"
   b) ВЕЉКА ЈАНКОВИЋА, дипломираног физичара, који је пријавио докторску дисертацију под називом: „EXCITON DYNAMICS AT PHOTOEXCITED ORGANIC HETEROJUNCTIONS" (Динамика екситона на органским хетороспојевима побуђеним светлошћу)
8. Усвајање Извештаја Комисије за преглед и оцену докторске дисертације и одређивање Комисије за одбрану дисертације за:
   a) АЛЕКСАНДРУ ДИМИТРИЕВСКУ, дипломираног физичара, која је предала докторску дисертацију под називом: „MEASUREMENT OF THE *W* BOSON MASS AND THE CALIBRATION OF THE MUON MOMENTUM WITH THE *ATLAS* DETECTOR" (Мерење масе W бозона и калибрација импулса миона на детектору ATLAS)
   b) ЈЕЛЕНУ ПЕШИЋ, дипломираног физичара, која је предала докторску дисертацију под називом: „INVESTIGATION OF SUPERCONDUCTIVITY IN GRAPHENE AND RELATED MATERIALS BASED ON Ab-INITIO" (Истраживање суперпроводности у графену и сличним материјалима коришћењем ab-initio метода)
   c) ГОРДАНУ МИЛУТИНОВИЋ-ДУМБЕЛОВИЋ, дипломираног физичара, која је предала докторску дисертацију под називом: „МЕТОДЕ МЕРЕЊА ОДНОСА ГРАНАЊА ХИГСОВОГ БОЗОНА У ПРОЦЕСИМА $H \rightarrow \mu^+\mu^-$ И $H \rightarrow ZZ^*$ НА 1.4 TeV НА БУДУЋЕМ ЛИНЕАРНОМ СУДАРАЧУ *CLIC*"
9. Усвајање пријављене теме за израду мастер рада, одређивање руководиоца и Комисије за одбрану рада за:
   a) ВЛАДАНА СИМИЋА, студента мастер студија физике, смер Општа физика, који је пријавио мастер рад под називом: "НУМЕРИЧКЕ СИМУЛАЦИЈЕ ПРОЦЕСА ХЛАЂЕЊА ГРАНУЛАРНОГ ГАСА"
   b) МИРЈАНУ РАКИЋЕВИЋ, студента мастер студија физике, смер Општа физика, која је пријавила мастер рад под називом: "ПРОУЧАВАЊЕ ДИЕЛЕКТРИЧНИХ И ОПТИЧКИХ КАРАКТЕРИСТИКА ПОЛИЕТИЛЕН ТЕРЕФТАЛАТ МЕМБРАНЕ МОДИФИКОВАНЕ ЈОНСКИМ СНОПОВИМА"
   c) НИКОЛУ ВУЈИЧИЋА, студента мастер студија метеорологије, који је пријавио мастер рад под називом: "КВАЗИ - РЕЗОНАНТНИ РОЗБИЈЕВИ ТАЛАСИ И ПОПЛАВЕ ТОКОМ МАЈА 2014 У СРБИЈИ"
   d) МИЛЕНУ ЛАЗАРЕВИЋ, студента мастер студија метеорологије, која је пријавила мастер рад под називом: "ТРАНСПОРТ САХАРСКОГ ПЕСКА ИЗНАД СРБИЈЕ ТОКОМ ПЕРИОДА 2012-2016"
   e) ДАНИЈЕЛА ОБРИЋА, студента мастер студија физике, смер Теоријска и експериментална физика, који је пријавио мастер рад под називом: "НЕКОМУТАТИВНОСТ И НЕАСОЦИЈАТИВНОСТ ЗАТВОРЕНЕ БОЗОНСКЕ СТРУНЕ"
   f) СТЕВАНА ЂУРЂЕВИЋА, студента мастер студија физике, смер Теоријска и експериментална физика, који је пријавио мастер рад под називом: "УТИЦАЈ ОРИЈЕНТАЦИЈЕ d-WAVE СУПЕРПРОВОДИНИКА НА ЏОЗЕФСОНОВУ СТРУЈУ У СПОЈЕВИМА СА НЕХОМОГЕНИМ ФЕРОМАГНЕТОМ"
   g) МИЛКУ ПОЛЕДИЦА, студента мастер студија физике, смер Теоријска и експериментална физика, која је пријавила мастер рад под називом: "ОДРЕЂИВАЊЕ НЕУТРАЛИЗАЦИОНИХ РАСТОЈАЊА ПРИ ИНТЕРАКЦИЈИ ВИШЕСТРУКО НАЕЛЕКТРИСАНИХ ЈОНА СА ПОВРШИНОМ ЧВРСТОГ ТЕЛА"
   h) НЕМАЊУ СТЕВИЋА, студента мастер студија физике, смер Примењена и компјутерска физика, који је пријавио мастер рад под називом: "ПРОЈЕКТОВАЊЕ И ИЗРАДА МИКРОКОНТРОЛЕРСКОГ ТЕРМОХИГРОМЕТРА "
10. Усвајање пријављене теме за израду дипломског рада, одређивање руководиоца и Комисије за одбрану рада за:
    a) АНДРИЈАНУ ЂОМЛИЈА, апсолвента физике, смер Теоријска и експериментална физика, која је пријавила дипломски рад под називом: "ОДРЕЂИВАЊЕ ПАРАМЕТАРА ЛАСЕРСКИ ИНДУКОВАНЕ ПЛАЗМЕ АНАЛИЗОМ ВРЕМЕНСКИ РАЗЛОЖЕНИХ СПЕКТРОСКОПСКИХ МЕРЕЊА"
11. Усвајање рецензије рукописа "Аномално ширење спектралних линија водоника у пражњењима" аутора др Николе Цветановића.
12. Давање сагласности Физичког факултета на ангажовање у настави и то:

a) проф. др Лазара Лазића за предмет Метеорологија и Моделовање загађења у атмосфери (на основним студијама Хемија животне средине)
b) проф. др Илије Марића за предмет Философија природних наука (на интегрисаним основним и мастер студијама Настава хемије)
c) проф. др Душана Поповића за предмет Физика (на интегисаним основним и мастер студијама Настава хемије)
d) доц. др Саве Галијаша за предмет Физика (на основним студијама Хемија и Хемија животне средине и интегрисаним основним и мастер студијама Настава хемије)
e) проф. др Владимира Милосављевића за предмет Одабрана поглавља физике (на основним студијама Хемија и Хемија животне средине и интегрисаним основним и мастер студијама Настава хемије)и Основи физике (на основним студијама Биохемије)
f) доц. др Славице Малетић за предмет Основи физике (на основним студијама Биохемије)
g) проф. др Братислава Обрадовића за предмет Унапређени оксидациони процеси (на докторским студијама)
h) Марјана Ћирковића за извођење наставе на Шумарском факултету Универзитета у Београду
i) др Браниславе Мисаиловић за извођење наставе из предмета Физика на Војној академији Универзитета одбране

13. Разматрање предлога катедара Института за метеорологију у вези са избором проф. др Иване Тошић за шефа Катедре за општу метеорологију.
14. Питања наставе, науке и финансија.
15. Захтеви за одобрење одсуства.
16. Усвајање извештаја са службених путовања.
17. Дописи и молбе упућене Наставно-научном већу.
18. Обавештења. Текућа питања. Питања и предлози.

Пошто је усвојен предложени Дневни ред, прешло се на

1. <u>тачку</u>

Усвојен је Записник са IX седнице Изборног и Наставно-научног већа Физичког факултета.

<u>Изборно веће</u>

2. <u>тачка</u>

На предлог катедара донета је одлука о покретању поступка за избор наставника Физичког факултета и то:

a) на предлог Катедре за физику атома, молекула, јонизованих гасова, плазме и квантну оптику донета је одлука о расписивању конкурса за избор једног ванредног професора за ужу научну област Физика атома и молекула
Комисија: *др Срђан Буквић, редовни професор ФФ*
др Владимир Милосављевић, редовни професор ФФ
др Братислав Маринковић, научни саветник ИФ

b) на предлог катедара Института за метеорологију донета је одлука о расписивању конкурса за избор једног доцента за ужу научну област Климатологија и примењена метеорологија
Комисија: *др Ивана Тошић, редовни професор ФФ*

др Владимир Ђурђевић, ванредни професор ФФ
др Мирослава Ункашевић, редовни професор ФФ

3. тачка

Поводом Извештаја Комисије за избор наставника Физичког факултета Изборно веће је донело следеће одлуке:

a) након јавног гласања у коме су учествовали редовни и ванредни професори Факултета, једногласно, са 29 гласова ЗА, др ТАТЈАНА ВУКОВИЋ је изабрана у звање ванредног професора за ужу научну област Квантна и математичка физика
b) након јавног гласања у коме су учествовали редовни и ванредни професори и доценти Факултета, једногласно, са 38 гласова ЗА, др НИКОЛА ШИШОВИЋ је изабран у звање доцента за ужу научну област Физика јонизованих гасова и плазме
c) након јавног гласања у коме су учествовали редовни и ванредни професори и доценти Факултета са 37 гласова ЗА и једним УЗДРЖАНИМ гласом, др САША ИВКОВИЋ је изабран у звање доцента за ужу научну област Настава физике

4. тачка

Изборно веће је донело одлуку о покретању поступка за избор НОРЕ ТРКЉА у звање истраживач-сарадник.

Комисија: *др Иван Дојчиновић, ванредни професор ФФ*
др Братислав Обрадовић, ванредни професор ФФ
др Милорад Кураица, редовни професор ФФ

5. тачка

Изборно веће је усвојило Извештај Комисије и донело одлуку о избору у научно звање и то:

a) након јавног гласања у коме су учествовали редовни и ванредни професори Факултета једногласно, са 29 гласова ЗА (од укупно 37 колико чини изборно тело), донета је одлука о избору др ВЛАДИМИРА СТОЈАНОВИЋА у звање виши научни сарадник
b) након јавног гласања у коме су учествовали редовни и ванредни професори и доценти Факултета једногласно, са 38 гласова ЗА (од укупно 53 колико чини изборно тело), донета је одлука о избору др БРАНИСЛАВЕ МИСАИЛОВИЋ у звање научни сарадник

Наставно-научно веће

6. тачка

Одређена је Комисија за оцену испуњености услова и оправданост предложене теме за израду докторске дисертације за:

a) ЈАСМИНУ АТИЋ, дипломираног инжењера електротехнике, која је пријавила докторску дисертацију под називом: „ТРАНСПОРТ ЕЛЕКТРОНА, РАЗВОЈ ЛАВИНА И ПРОПАГАЦИЈА СТРИМЕРА У ЈАКО ЕЛЕКТРОНЕГАТИВНИМ ГАСОВИМА"

Комисија: *др Саша Дујко, виши научни сарадник ИФ*
др Ђорђе Спасојевић, ванредни професор ФФ
др Срђан Буквић, редовни професор ФФ
др Зоран Љ. Петровић, научни саветник ИФ

7. тачка

Усвојен је Извештај Комисије за оцену испуњености услова и оправданост предложене теме за израду докторске дисертације и одређен ментор за:

a) НИКОЛУ ИВАНОВИЋА, дипломираног физичара, који је пријавио докторску дисертацију под називом: „ПРОУЧАВАЊЕ ОБЛИКА СПЕКТРАЛНИХ ЛИНИЈА Ne I И Ne II У ПРИКАТОДНОЈ ОБЛАСТИ АБНОРМАЛНОГ ТИЊАВОГ ПРАЖЊЕЊА"

Ментор: *др Никола Шишовић, доцент ФФ*

b) ВЕЉКА ЈАНКОВИЋА, дипломираног физичара, који је пријавио докторску дисертацију под називом: „EXCITON DYNAMICS AT PHOTOEXCITED ORGANIC HETEROJUNCTIONS" (Динамика екситона на органским хетороспојевима побуђеним светлошћу)

Ментор: *др Ненад Вукмировић, научни саветник ИФ*

8. тачка

Усвојен је Извештај Комисије за преглед и оцену докторске дисертације и одређена Комисија за одбрану дисертације за:

a) АЛЕКСАНДРУ ДИМИТРИЕВСКУ, дипломираног физичара, која је предала докторску дисертацију под називом: „MEASUREMENT OF THE *W* BOSON MASS AND THE CALIBRATION OF THE MUON MOMENTUM WITH THE *ATLAS* DETECTOR" (Мерење масе W бозона и калибрација импулса миона на детектору ATLAS)

Комисија: *др Ненад Врањеш, научни сарадник ИФ*
др Петар Аџић, редовни професор ФФ
др Маја Бурић, редовни професор ФФ
др Воја Радовановић, редовни професор ФФ
др Лидија Живковић, научни саветник ИФ
др Мартен Бонекамп, CEA Saclay, Париз (Француска)

b) ЈЕЛЕНУ ПЕШИЋ, дипломираног физичара, која је предала докторску дисертацију под називом: „INVESTIGATION OF SUPERCONDUCTIVITY IN GRAPHENE AND RELATED MATERIALS BASED ON Ab-INITIO" (Истраживање суперпроводности у графену и сличним материјалима коришћењем ab-initio метода)

Комисија: *др Радош Гајић, научни сарадник ИФ*
др Иванка Милошевић, редовни професор ФФ

др Милан Кнежевић, редовни професор ФФ
др Ђорђе Спасојевић, ванредни професор ФФ
др Зоран Поповић, научни саветник ИНН Винча
dr Kurt Hingerl, Johannes Kepler Univerzitet, Austrija

c) ГОРДАНУ МИЛУТИНОВИЋ-ДУМБЕЛОВИЋ, дипломираног физичара, која је предала докторску дисертацију под називом: „МЕТОДЕ МЕРЕЊА ОДНОСА ГРАНАЊА ХИГСОВОГ БОЗОНА У ПРОЦЕСИМА $H\rightarrow\mu+\mu-$ И $H\rightarrow ZZ*$ НА 1.4 TeV НА БУДУЋЕМ ЛИНЕАРНОМ СУДАРАЧУ *CLIC*"

Комисија: *др Иванка Божовић-Јелисавчић, научни саветник ИНН Винча*
др Воја Радовановић, редовни професор ФФ
др Јован Пузовић, ванредни професор ФФ

9. <u>тачка</u>

Усвојена је пријављена тема за израду мастер рада, одређен руководилац и Комисија за одбрану рада за:

a) ВЛАДАНА СИМИЋА, студента мастер студија физике, смер Општа физика, који је пријавио мастер рад под називом: "НУМЕРИЧКЕ СИМУЛАЦИЈЕ ПРОЦЕСА ХЛАЂЕЊА ГРАНУЛАРНОГ ГАСА"

Комисија: *др Слободан Врховац, научни саветник ИФ, руководилац рада*
др Андријана Жекић, ванредни професор ФФ
др Зорица Поповић, доцент ФФ

b) МИРЈАНУ РАКИЋЕВИЋ, студента мастер студија физике, смер Општа физика, која је пријавила мастер рад под називом: "ПРОУЧАВАЊЕ ДИЕЛЕКТРИЧНИХ И ОПТИЧКИХ КАРАКТЕРИСТИКА ПОЛИЕТИЛЕН ТЕРЕФТАЛАТ МЕМБРАНЕ МОДИФИКОВАНЕ ЈОНСКИМ СНОПОВИМА"

Комисија: *др Славица Малетић, доцент ФФ, руководилац рада*
др Душан Поповић, ванредни професор ФФ
др Драгана Церовић, научни сарадник ФФ

c) НИКОЛУ ВУЈИЧИЋА, студента мастер студија метеорологије, који је пријавио мастер рад под називом: "КВАЗИ - РЕЗОНАНТНИ РОЗБИЈЕВИ ТАЛАСИ И ПОПЛАВЕ ТОКОМ МАЈА 2014 У СРБИЈИ"

Комисија: *др Владимир Ђурђевић, доцент ФФ, руководилац рада*
др Катарина Вељовић, доцент ФФ
др Немања Ковачевић, доцент ФФ

d) МИЛЕНУ ЛАЗАРЕВИЋ, студента мастер студија метеорологије, која је пријавила мастер рад под називом: "ТРАНСПОРТ САХАРСКОГ ПЕСКА ИЗНАД СРБИЈЕ ТОКОМ ПЕРИОДА 2012-2016"

Комисија: *др Владимир Ђурђевић, доцент ФФ, руководилац рада*
др Лазар Лазић, редовни професор ФФ

др Катарина Вељовић, доцент ФФ

e) ДАНИЈЕЛА ОБРИЋА, студента мастер студија физике, смер Теоријска и експериментална физика, који је пријавио мастер рад под називом: "НЕКОМУТАТИВНОСТ И НЕАСОЦИЈАТИВНОСТ ЗАТВОРЕНЕ БОЗОНСКЕ СТРУНЕ"
Комисија: *др Бојан Николић, виши научни сарадник ИФ, руководилац рада*
др Воја Радовановић, редовни професор ФФ
др Душко Латас, доцент ФФ

f) СТЕВАНА ЂУРЂЕВИЋА, студента мастер студија физике, смер Теоријска и експериментална физика, који је пријавио мастер рад под називом: "УТИЦАЈ ОРИЈЕНТАЦИЈЕ d-WAVE СУПЕРПРОВОДИНИКА НА ЏОЗЕФСОНОВУ СТРУЈУ У СПОЈЕВИМА СА НЕХОМОГЕНИМ ФЕРОМАГНЕТОМ"
Комисија: *др Зорица Поповић, доцент ФФ, руководилац рада*
др Славица Малетић, доцент ФФ
др Божидар Николић, доцент ФФ
др Предраг Мирановић, редовни професор ПМФ Подгорица

g) МИЛКУ ПОЛЕДИЦА, студента мастер студија физике, смер Теоријска и експериментална физика, која је пријавила мастер рад под називом: "ОДРЕЂИВАЊЕ НЕУТРАЛИЗАЦИОНИХ РАСТОЈАЊА ПРИ ИНТЕРАКЦИЈИ ВИШЕСТРУКО НАЕЛЕКТРИСАНИХ ЈОНА СА ПОВРШИНОМ ЧВРСТОГ ТЕЛА"
Комисија: *др Сава Галијаш, доцент ФФ, руководилац рада*
др Славица Малетић, доцент ФФ
др Владимир Срећковић, виши научни сарадник ИФ

h) НЕМАЊУ СТЕВИЋА, студента мастер студија физике, смер Примењена и компјутерска физика, који је пријавио мастер рад под називом: "ПРОЈЕКТОВАЊЕ И ИЗРАДА МИКРОКОНТРОЛЕРСКОГ ТЕРМОХИГРОМЕТРА "
Комисија: *др Иван Белча, редовни професор ФФ, руководилац рада*
др Бећко Касалица, ванредни професор ФФ
др Ненад Тадић, истраживач сарадник ФФ

10. <u>тачка</u>

Усвојена је пријављена тема за израду мастер рада, одређен руководилац и Комисија за одбрану рада за:

a) АНДРИЈАНУ ЂОМЛИЈА, апсолвента физике, смер Теоријска и експериментална физика, која је пријавила дипломски рад под називом: "ОДРЕЂИВАЊЕ ПАРАМЕТАРА ЛАСЕРСКИ ИНДУКОВАНЕ ПЛАЗМЕ АНАЛИЗОМ ВРЕМЕНСКИ РАЗЛОЖЕНИХ СПЕКТРОСКОПСКИХ МЕРЕЊА"
Комисија: *др Срђан Буквић, редовни професор ФФ, руководилац рада*
др Никола Шишовић, доцент ФФ

др Милош Скочић, истраживач-сарадник ФФ

11. тачка

Наставно-научно веће је усвојило рецензију рукописа "Аномално ширење спектралних линија водоника у пражњењима" аутора др Николе Цветановића и прихватило предлог рецензената да се рукопис публикује као монографско научно дело.

12. тачка

Наставно-научно веће је ДАЛО САГЛАСНОСТ на ангажовање у настави и то:

a) проф. др Лазара Лазића за предмет Метеорологија и Моделовање загађења у атмосфери (на основним студијама Хемија животне средине) на Хемијском факултету Универзитета у Београду
b) проф. др Илије Марића за предмет Философија природних наука (на интегрисаним основним и мастер студијама Настава хемије) на Хемијском факултету Универзитета у Београду
c) проф. др Душана Поповића за предмет Физика (на интегрисаним основним и мастер студијама Настава хемије) на Хемијском факултету Универзитета у Београду
d) доц. др Саве Галијаша за предмет Физика (на основним студијама Хемија и Хемија животне средине и интегрисаним основним и мастер студијама Настава хемије) на Хемијском факултету Универзитета у Београду
e) проф. др Владимира Милосављевића за предмет Одабрана поглавља физике (на основним студијама Хемија и Хемија животне средине и интегрисаним основним и мастер студијама Настава хемије)и Основи физике (на основним студијама Биохемије) на Хемијском факултету Универзитета у Београду
f) доц. др Славице Малетић за предмет Основи физике (на основним студијама Биохемије) на Хемијском факултету Универзитета у Београду
g) проф. др Братислава Обрадовића за предмет Унапређени оксидациони процеси (на докторским студијама) на Хемијском факултету Универзитета у Београду
h) Марјана Ћирковића за извођење наставе на Шумарском факултету Универзитета у Београду
i) др Браниславе Мисаиловић за извођење наставе из предмета Физика на Војној академији Универзитета одбране

Такође, Наставно-научно веће је донело одлуку да пренесе овлашћење на декана Факултета да одобри ангажовања у настави на факултетима са којима имамо споразум о извођењу наставе за све захеве који пристигну накнадно, како наставни процес не би трпео због кашњења других факултета.

13. тачка

На предлог катедара Института за метеорологију донета је одлука да се проф. др Ивана Тошић изабере за шефа Катедре за општу метеорологију.

14. тачка

Питања наставе

Продекан за наставу, доц. др Славица Малетић, обавестила је чланове Већа да је тренутно у току III уписни рок за упис студената у I годину основних студија. Не рачунајући уписни рок који је у току и на коме се за упис пријавило 3 кандидата, ове године смо у I годину уписали укупно 95 студената, по смеровима:

Смер	Уписано Буџет+ Самоф.
Општа физика	14+0
Теоријска и експериментална	35+0
Примењена и компјутерска	21+2
Метеорологија	22+1
Укупно	92+3

Свечани пријем студената одржаће се у петак 29. септембра 2017. године у 12 сати у сали 60 у Цара Душана 13 и продекан је позвала наставнике и сараднике да му присуствују и представе се новим студентима Факултета.

Настава на основним студијама почиње у понедељак 2. октобра, а на мастер студијама две недеље касније, у понедељак 16. октобра.

Због уписа у наредну годину студија, продекан је подсетила наставнике да је све усмене испите потребно завршити до 30. септембра.

На конкурс за ангажовање у настави студената докторских и мастер студија пријавио се 31 кандидат. Сви пријављени кандидати испуњавају услове конкурса. У току идуће недеље ће бити заказан састанак шефова катедара и шефова смерова ради договора о ангажовању пријављених студената и њиховог распореда по предметима.

Питања финансија

Продекан за финансије проф. др Иван Белча обавестио је чланове Већа да ће упутити предлог Савету Факултета за усвајање ребаланса буџета Факултета за ову годину, с обзиром да смо имали непланиране трошкове (санација штете од поплаве, набавка рачунарске опреме ради отказивања постојеће и др).

Питања науке

У одсуству продекана за науку, декан је обавестио чланове Већа да је у току ре-акредитација Факултета као Научно-истраживачке организације.

Још увек нема вести из Министарства о термину расписивања новог конкурса за пројекте.

15. тачка

Наставно-научно веће је одобрило плаћено одсуство наставницима и сарадницима и то:

a) доц. др Ивану Виденовићу у периоду од 18. септембра до 6. октобра 2017. године ради студијског боравка у Међународној агенцији за атомску енергију у Бечу (Аустрија)
b) проф. др Петру Аџићу у периоду од 22. септембра до 10. октобра 2017. године ради присуствовања седницама Савета CERN-а у Женеви (Швајцарска)
c) проф. др Јовану Пузовићу у периоду од 4. до 15. октобра 2017. године ради рада на NA61/SHINE експерименту у Превесину (Француска)
d) Наставно-научно веће је одобрило и неплаћено одсуство др Милошу Бургеру у периоду од 1. октобра до краја текуће године, односно пројектног циклуса, ради наставка усавршавања на Универзитету у Мичигену (САД)

16. тачка

Проф. др Иван Дојчиновић обавестио је дописом чланове Већа о притисцима који се врше на њега како би се повукао са места председника Друштва физичара Србије.

Седница је завршена у 12:25 часова. Наредна седница Изборног и Наставно-научног већа планира се за 18. октобар.

Београд, 26.9.2017.

ДЕКАН ФИЗИЧКОГ ФАКУЛТЕТА
Проф. др Јаблан Дојчиловић, с.р.

З А П И С Н И К

са VIII седнице Изборног и Наставно-научног већа Физичког факултета
одржане у среду 31. маја 2023. године

Седници присуствује 39 чланова Изборног и Наставно-научног већа.

Службено одсутни:	проф. др Милорад Кураица проф. др Иванка Милошевић проф. др Братислав Обрадовић
Оправдано одсутни:	доц. др Александра Гочанин доц. др Сузана Путниковић доц. др Милош Скочић др Марјан Ћирковић
Неоправдано одсутни:	проф. др Зоран Борјан проф. др Предраг Миленовић доц. др Сава Галијаш доц. др Саша Ивковић доц. др Владимир Миљковић доц. др Драган Реџић

Декан Факултета, проф. др Иван Белча, отворио је седницу у 11.05 часова и предложио следећи

Д н е в н и   р е д

1. Усвајање Записника са VII седнице Изборног и Наставно-научног већа Физичког факултета.

Изборно веће

2. Покретање поступка за избор др ДРАГАНА ПРЕКРАТА у звање научни сарадник.
3. Усвајање Извештаја Комисије за избор ЛУКЕ РАЈАЧИЋА у звање истраживач-сарадник.

Наставно-научно веће

4. Усвајање Извештаја Комисије за преглед и оцену докторске дисертације и одређивање Комисије за одбрану дисертације за:
   a) АНУ ВРАНИЋ (2017/8006), мастер физичара, која је предала докторску дисертацију под називом: "EVOLVING COMPLEX NETWORKS: STRUCTURE AND DYNAMICS" (Растуће комплексне мреже: структура и динамика), урађену под менторством др Марије Митровић Данкулов, вишег научног сарадника Института за физику
   b) ДАНИЈЕЛА ОБРИЋА (8013/2018), мастер физичара, који је предао докторску дисертацију под називом: "T-DUALIZATION OF BOSONIC STRING AND TYPE IIB SUPERSTRING IN PRESENCE OF COORDINATE DEPENDENT BACKGROUND FIELDS" (Т-дуализација бозонске струне и тип IIB суперструне у присуству координатно зависних позадинских поља), урађену под менторством др Бојана Николића, вишег научног сарадника Института за физику
   c) ТИЈАНУ РАДЕНКОВИЋ (2017/8009), мастер физичара, која је предала докторску дисертацију под називом: "ВИШЕ ГРАДИЈЕНТНЕ ТЕОРИЈЕ И КВАНТНА ГРАВИТАЦИЈА", урађену под менторством др Марка Војновића, вишег научног сарадника Института за физику
5. Усвајање пријављене теме за израду мастер рада, одређивање ментора и Комисије за одбрану рада за:
   a) САЛИХИЈА МУХАМАДА, студента мастер студија смера Примењена и компјутерска физика, који је пријавио мастер рад под називом: "ФОТОКАТАЛИТИЧКЕ ПРИМЕНЕ ОКСИДНИХ СЛОЈЕВА ФОРМИРАНИХ ПЛАЗМЕНОМ ЕЛЕКТРОЛИТИЧКОМ ОКСИДАЦИЈОМ НИОБИЈУМА" (Photocalalytic applications of oxide coatings formed by plasma electrolytic oxidation of niobium)
   b) АНУ РАНЂЕЛОВ, студента мастер студија смера Теоријска и експериментална физика, која је пријавила мастер рад под називом: "ГЕНЕРИСАЊЕ КВАНТНО КОРЕЛИСАНИХ ПАРОВА ФОТОНА ПУТЕМ СПОНТАНЕ ПАРАМЕТАРСКЕ КОНВЕРЗИЈЕ ФОТОНА У НЕЛИНЕАРНОМ КРИСТАЛУ"

c) НИКОЛУ САВИЋА, студента мастер студија смера Теоријска и експериментална физика, који је пријавио мастер рад под називом: "КЛАСИЧНИ И КВАНТНИ ХАОС У РАСЕЈАЊУ ВИСОКО ЕКСЦИТОВАНИХ СТРУНА"
d) ТИЈАНУ ТАДИЋ, студента мастер студија смера Примењена и компјутерска физика, која је пријавила мастер рад под називом: "АНАЛИЗА СИМУЛАЦИЈА РАЗВИЈАНИХ КОРИШЋЕЊЕМ ПЛАТФОРМЕ myPhysicsLab"
e) НАТАШУ СИМИЋ, студента мастер студија смера Примењена и компјутерска физика, која је пријавила мастер рад под називом: "КОРИШЋЕЊЕ УЛТРАБРЗЕ ВИДЕО КАМЕРЕ У ПРАЋЕЊУ И АНАЛИЗИ ДИНАМИКЕ ФИЗИЧКИХ ПРОЦЕСА"

6. Давање сагласности на избор др ДРАГАНА ПРЕКРАТА у звање асистента са докторатом за ужу научну област Општа физика на Фармацеутском факултету Универзитета у Београду
7. Давање сагласности на ангажовање наставника на другој институцији и то:
   a) проф. др Драгане Вујовић за извођење наставе из предмета Ваздухопловна метеорологија (45+0+1) у зимском семестру школске 2023/24 године на Основним академским студијама Војне академије Министарства одбране Републике Србије
   b) доц. др Браниславе Вучетић за извођење вежби из предмета Физика А (0+60+20) у зимском семестру школске 2023/24 године на Основним академским студијама Војне академије Министарства одбране Републике Србије
   c) Никодина Недића за извођење вежби из предмета Физика А (0+60+20) у зимском семестру школске 2023/24 године на Основним академским студијама Војне академије Министарства одбране Републике Србије
   d) доц. др Зорана Поповића за извођење наставе из предмета Програмирање у физици 1 и 2 и Квантна механика 1 и 2 у школској 2023/24 години на Природно-математичком факултета Универзитета у Бањој Луци (БиХ, Република Српска)
8. Одређивање жирија за доделу Годишње награде за научни рад младом истраживачу, жребом из редова редовних професора Физичког факултета.
9. Усвајање докумената потребних за реакредитацију Центра за квантну теоријску физику као центра изврсности и то:
   a) Плана развоја Центра за квантну теоријску физику
   b) Плана развоја научно-истраживачког подмлатка Центра
10. Питања наставе, науке и финансија.
11. Захтеви за одобрење одсуства.
12. Усвајање извештаја са службених путовања.
13. Дописи и молбе упућене Наставно-научном већу.
14. Обавештења. Текућа питања. Питања и предлози.

Пошто је усвојен предложени Дневни ред, прешло се на

1. <u>тачку</u>

Усвојен је Записник са VII седнице Изборног и Наставно-научног већа Физичког факултета.

<u>Изборно веће</u>

2. <u>тачка</u>

Покренут је поступак за избор др ДРАГАНА ПРЕКРАТА у звање научни сарадник.

Комисија: *др Маја Бурић, редовни професор ФФ*
др Никола Коњик, доцент ФФ
др Марко Војиновић, виши научни сарадник ИФ

3. тачка

Усвојен је Извештај Комисије за избор ЛУКЕ РАЈАЧИЋА у звање истраживач-сарадник.

Наставно-научно веће

4. тачка

Усвојен је Извештај Комисије за преглед и оцену докторске дисертације и одређена Комисија за одбрану дисертације за:

a) АНУ ВРАНИЋ (2017/8006), мастер физичара, која је предала докторску дисертацију под називом: "EVOLVING COMPLEX NETWORKS: STRUCTURE AND DYNAMICS" (Растуће комплексне мреже: структура и динамика), урађену под менторством др Марије Митровић Данкулов, вишег научног сарадника Института за физику
Комисија: *др Сунчица Елезовић Хаџић, редовни професор ФФ*
др Светислав Мијатовић, доцент ФФ
др Антун Балаж, научни саветник ИФ

b) ДАНИЈЕЛА ОБРИЋА (8013/2018), мастер физичара, који је предао докторску дисертацију под називом: "T-DUALIZATION OF BOSONIC STRING AND TYPE IIB SUPERSTRING IN PRESENCE OF COORDINATE DEPENDENT BACKGROUND FIELDS" (Т-дуализација бозонске струне и тип IIB суперструне у пристуству координатно зависних позадинских поља), урађену под менторством др Бојана Николића, вишег научног сарадника Института за физику
Комисија: *др Маја Бурић, редовни професор ФФ*
др Воја Радовановић, редовни професор ФФ
др Бранислав Цветковић, научни саветник ИФ

c) ТИЈАНУ РАДЕНКОВИЋ (2017/8009), мастер физичара, која је предала докторску дисертацију под називом: "ВИШЕ ГРАДИЈЕНТНЕ ТЕОРИЈЕ И КВАНТНА ГРАВИТАЦИЈА", урађену под менторством др Марка Војиновића, вишег научног сарадника Института за физику
Комисија: *др Маја Бурић, редовни професор ФФ*
др Воја Радовановић, редовни професор ФФ
др Бранислав Цветковић, научни саветник ИФ

5. тачка

Усвојена је пријављена тема за израду мастер рада, одређен ментор и Комисијае за одбрану рада за:

a) САЛИХИЈА МУХАМАДА, студента мастер студија смера Примењена и компјутерска физика, који је пријавио мастер рад под називом: "ФОТОКАТАЛИТИЧКЕ ПРИМЕНЕ ОКСИДНИХ СЛОЈЕВА ФОРМИРАНИХ ПЛАЗМЕНОМ ЕЛЕКТРОЛИТИЧКОМ ОКСИДАЦИЈОМ НИОБИЈУМА" (Photocalalytic applications of oxide coatings formed by plasma electrolytic oxidation of niobium)
Комисија: *др Стеван Стојадиновић, редовни професор ФФ, ментор*
др Марија Петковић Беназзоуз, научни сарадник ФФ
др Ненад Радић, научни саветник ИХТМ

b) АНУ РАНЂЕЛОВ, студента мастер студија смера Теоријска и експериментална физика, која је пријавила мастер рад под називом: "ГЕНЕРИСАЊЕ КВАНТНО КОРЕЛИСАНИХ ПАРОВА ФОТОНА ПУТЕМ СПОНТАНЕ ПАРАМЕТАРСКЕ КОНВЕРЗИЈЕ ФОТОНА У НЕЛИНЕАРНОМ КРИСТАЛУ"
Комисија: *др Александра Гочанин, доцент ФФ, метор*
др Душан Арсеновић, научни саветник ИФ, ментор
др Милорад Кураица, редовни професор ФФ
др Предраг Ранитовић, научни саветник ФФ

c) НИКОЛУ САВИЋА, студента мастер студија смера Теоријска и експериментална физика, који је пријавио мастер рад под називом: "КЛАСИЧНИ И КВАНТНИ ХАОС У РАСЕЈАЊУ ВИСОКО ЕКСЦИТОВАНИХ СТРУНА"
Комисија: *др Марија Димитријевић Ћирић, редовни професор ФФ, ментор*
др Михаило Чубровић, научни сарадник ИФ, ментор
др Драгољуб Гочанин, доцент ФФ

d) ТИЈАНУ ТАДИЋ, студента мастер студија смера Примењена и компјутерска физика, која је пријавила мастер рад под називом: "АНАЛИЗА СИМУЛАЦИЈА РАЗВИЈАНИХ КОРИШЋЕЊЕМ ПЛАТФОРМЕ myPhysicsLab"
Комисија: *др Зоран Николић, редовни професор ФФ, ментор*
др Едиб Добарџић, ванредни професор ФФ
др Ненад Тадић, доцент ФФ

e) НАТАШУ СИМИЋ, студента мастер студија смера Примењена и компјутерска физика, која је пријавила мастер рад под називом: "КОРИШЋЕЊЕ УЛТРАБРЗЕ ВИДЕО КАМЕРЕ У ПРАЋЕЊУ И АНАЛИЗИ ДИНАМИКЕ ФИЗИЧКИХ ПРОЦЕСА"
Комисија: *др Зоран Николић, редовни професор ФФ, ментор*
др Иван Петронијевић, научни сарадник ФФ
др Небојша Поткоњак, научни сарадник ИНН Винча

6. <u>тачка</u>

Наставно-научно веће је ДАЛО САГЛАСНОСТ на избор др ДРАГАНА ПРЕКРАТА у звање асистента са докторатом за ужу научну област Општа физика на Фармацеутском факултету Универзитета у Београду.

7. <u>тачка</u>

Наставно-научно веће је ДАЛО САГЛАСНОСТ на ангажовање наставника на другој институцији и то:

a) проф. др Драгане Вујовић за извођење наставе из предмета Ваздухоплова метеорологија (45+0+1) у зимском семестру школске 2023/24 године на Основним академским студијама Војне академије Министарства одбране Републике Србије
b) доц. др Браниславе Вучетић за извођење вежби из предмета Физика А (0+60+20) у зимском семестру школске 2023/24 године на Основним академским студијама Војне академије Министарства одбране Републике Србије
c) Никодина Недића за извођење вежби из предмета Физика А (0+60+20) у зимском семестру школске 2023/24 године на Основним академским студијама Војне академије Министарства одбране Републике Србије

d) доц. др Зорана Поповића за извођење наставе из предмета Програмирање у физици 1 и 2 и Квантна механика 1 и 2 у школској 2023/24 години на Природно-математичком факултета Универзитета у Бањој Луци (БиХ, Република Српска)

8. <u>тачка</u>

Жребом из редова редовних професора Физичког факултета одређен је жири за доделу Годишње награде за научни рад младом истраживачу:

- проф. др Марија Димитријевић Ћирић
- проф. др Владимир Милосављевић
- проф. др Ђорђе Спасојевић
- резервни члан жирија проф. др Горан Попарић

9. <u>тачка</u>

Наставно-научно веће је усвојило документа потребна за реакредитацију Центра за квантну теоријску физику као центра изврсности и то:

a) План развоја Центра за квантну теоријску физику
b) План развоја научно-истраживачког подмлатка Центра

10. <u>тачка</u>

Питања наставе

Продекан за наставу, доц. др Зорица Поповић, обавестила је чланове Већа да је расписан Конкурс за упис студената у прву годину студија школске 2023/24 године. Наставно-научно веће је, на предлог продекана, именовало комисије за упис и то:

Комисија за израду задатака за пријемни испит из математике:

- проф. др Татјана Вуковић
- проф. др Божидар Николић
- доц. др Светислав Мијатовић

Комисија за израду задатака за пријемни испит из физике:

- доц. др Зоран Поповић
- др Горан Сретеновић

Комисија за жалбе кандидата на ранг листу:

- проф. др Андријана Жекић
- проф. др Воја Радовановић
- проф. др Иван Белча
- проф. др Ивана Тошић
- доц. др Зорица Поповић

Представник истраживача на Наставно-научном већу, студент докторских студија Душан Ђорђевић изнео је примедбе на рад Комисије за такмичења Друштва физичара Србије. После дискусије у којој је учествовало више чланова Већа, донет је закључак да Факултет треба више да се укључи у рад комисија за такмичење ДФС-а, с обзиром да на такмичења иду ђаци који су у великом броју наши будући студенти. Наставно-научно веће је затим дало предлог Комисије за задатке у саставу:

- проф. др Воја Радовановић

- проф. др Божидар Николић
- проф. др Зоран Николић
- проф. др Горан Попарић
- проф. др Милош Вићић
- проф. др Братислав Обрадовић
- Душан Ђорђевић и
- Ђорђе Богдановић.

11. <u>тачка</u>

Наставно-научно веће је одобрило одсуства следећим наставницима и сарадницима:

a) Ириди Лазић у периоду од 31. маја до 17. јуна 2023. године ради учешћа на летњој школи FERS High Level Course on Data Science and Machine Learning for Climate Research која се одржава у Беринору (Италија)
b) Лазару Марковићу у периоду од 1. јуна до 31. августа 2023. године ради студијског боравка на Универзитету у Торину (Италија)
c) проф. др Марији Димитријевић Ћирић, доц. др Николи Коњику и Ђорђу Богдановићу у периоду од 5. до 10. јуна 2023. године ради стручне посете Институту Руђер Бошковић у Загребу (Хрватска)
d) проф. др Андријани Жекић и Милици Милојевић у периоду од 14. до 16. јуна 2023. године ради учешћа на 28. Конференцији Српског кристалографског друштва, која се одржава у Чачку (Србија)
e) доц. др Драгољубу Гочанину, доц. др Александри Гочанин, Ивани Стојиљковић и Душану Ђорђевићу у периоду од 18. јуна до 1. јула 2023. године ради студијског боравка на Институту за квантну информацију у Бечу (Аустрија)
f) проф. др Ивану Дојчиновићу од 19. до 23. јуна 2023. године ради учешћа на 14th Serbian Conference on Spectral Line Shapes in Astrophysics која се одржава у Бајиној Башти (Србија)
g) Дарку Савићу у периоду од 27. до 29. јуна 2023. године ради учешћа на конференцији International Conference on Hydro Climate Extremes and Society, која се одржава у Новом Саду (Србија)
h) Ириди Лазић у периоду од 27. до 29. јуна 2023. године ради учешћа на конференцији International Conference on Hydro Climate Extremes and Society, која се одржава у Новом Саду (Србија)
i) Милица Тошић у периоду од 27. до 29. јуна 2023. године ради учешћа на конференцији International Conference on Hydro Climate Extremes and Society, која се одржава у Новом Саду (Србија)
j) проф. др Иванки Милошевић у периоду од 30. јуна до 9. јула 2023. године ради учешћа на International Conferences and Exhibition on Nanotechnologies, Organic Electronics & Nanomedicine која се одржава у Солуну (Грчка)
k) Милици Тошић у периоду од 27. августа до 3. септембра 2023. године ради учешћа на летњој школи Mediterranean Machine Learning Summer School која се одржава у Солуну (Грчка)

l) проф. др Милораду Кураици и проф. др Братиславу Обрадовићу у периоду од 15. до 24. септембра 2023. године ради учешћа на конференцији 16th International Conference "Gas Discharge Plasma and Their Applications" - GDPA 2023, која се одржава у Уфи (Русија)
m) проф. др Марији Димитријевић Ћирић у периоду од 16. до 23. септембра 2023. године ради учешћа на конференцији Workshop on Noncommutative and generalized geometry in string theory, gauge theory and related physical models која се одржава на Крфу (Грчка)
n) проф. др Славици Малетић у периоду од 20. до 23. септембра 2023. године ради учешћа на 26. конгресу СЦТМ који се одржава у Орхриду (Македонија)
o) проф. др Владимиру Милосављевићу у периоду од 20. октобра до 15. новембра 2023. године ради учешћа на конференцији 69. AVC - International Symposium and Exhibition која се одржава у Портланду (САД)

Седница је завршена у 11.50 часова.

Београд, 14.6.2023.

ДЕКАН ФИЗИЧКОГ ФАКУЛТЕТА
Проф. др Иван Белча, с.р.

UNIVERSITY OF BELGRADE
FACULTY OF PHYSICS

Danijel Obrić

T-dualization of bosonic string and type IIB superstring in presence of coordinate dependent background fields

Doctoral Dissertation

Belgrade, 2022.

UNIVERZITET U BEOGRADU
FIZIČKI FAKULTET

Danijel Obrić

T-dualizacija bozonske strune i tip IIB superstrune u prisustvu koorinatno zavisnih pozadinskih polja

Doktorska disertacija

Beograd, 2022.

Supervisor:

- Prof. dr Bojan Nikolić, associate research professor, Institute of Physics, University of Belgrade

Advisory Committee:

- Prof. dr Maja Burić full professor, Faculty of Physics, University of Belgrade
- Prof. dr Branislav Cvetković, full research professor, Institute of Physics, University of Belgrade
- Prof. dr Voja Radovanović, full professor, Faculty of Physics, University of Belgrade

Defense date:

Acknowledgment

While this PhD thesis is written by one author its existence would not be possible without assistance, both scientific and emotional, of small army of people. At the forefront of this group is my supervisor dr Bojan Nikolić who introduced me to the topics of string theory and non-commutativity. I would like to thank him for all the support he has provided over the years, for all discussions we had and lastly for understanding and patience he showed.

I would also like to give thanks to all professors from both Faculty of physics and Institute of physics who, during my schooling, were instrumental in shaping of my understanding of physical phenomena. It is sufficient to say that without them, this whole project would be terminated at first equation.

Lastly, but not the least, I would like to thank my family for their emotional support and for always pushing me in direction of completion of this thesis.

T-dualization of bosonic string and type IIB superstring in presence of coordinate dependent background fields

Abstract

Topic of this disertation is examination of non-commutative and non-associative properties that emerge in context of closed string theory. This examination will be carried out on two distinct models. One where we work with bosonic string and other where we work with type IIB superstring. Furthermore, both of these models will be analyzed in presence of coordinate dependent background fields. Subjecting these models to T-dualization we will be able to obtain both T-dual theories and transformation laws that connect coordinates of starting theory with T-dual one. Utilizing transformation laws and commutative relations of starting theory we will be able to deduce non-commutative properties of T-dual theories. Method for obtaining T-duality will be based on Buscher procedure and its extensions. Main idea of Buscher procedure lies in localization of translational symmetry by replacing partial derivatives and coordinates that appear in action with covariant derivatives and invariant coordinates. This substitution inevitably introduces additional degrees of freedom which are encoded in gauge fields. By elimination of newly introduced degrees of freedom with method of Lagrange multipliers and subsequently finding equations of motion for gauge fields we obtain transformation laws. Inserting these laws into the action we will obtain T-dual theory.

In examination of bosonic string theory, we will work with $3D$ space where Kalb-Ramond background field will have infinitesimal linear dependence on one coordinate, z coordinate. Dualization will be carried along two distinct chains, one where coordinate that appears in background fields will be dualized last and other where it will be dualized first. By comparing these two approaches we will be able do discern what are necessary components for emergence of non-commutative properties.

Second part of thesis will be concerned with T-duality of type II superstring that propagates in linearly coordinate dependent Ramond-Ramond field. Unlike previous case, this theory possesses both bosonic and fermionic coordinates, however background field will only depend on bosonic part. T-duality will first focus only on bosonic part and later we will also incorporate fermionic part. We will also present alternative chain of duality where first we dualize fermionic coordinates and later bosonic ones. It will be shown that both chains produce same non-commutative relations. Finally, at the end of the thesis, we will also make analysis of same case when we have more general Ramond-Ramond field.

Key words: String theory, non-commutativity, non-associativity, Buscher procedure
Scientific area: Physics
Scientific subfield: High energy theoretical physics

T-dualizacija bozonske strune i tip IIB superstrune u prisustvu koorinatno zavisnih pozadinskih polja

Sažetak

Tema ove disertacije je bazirana na izučavanju nekomutativnih i neasocijativnih osobina koje se javljaju u kontekstu teorije struna. Ovo izučavanje će biti obavljeno na dva različita modela. Prvi model sa kojim ćemo raditi je model bozonske strune dok je drugi model za tip IIB superstrunu. Oba modela će biti analizirana u prisustvu koordinatno zavisnih pozadinskih polja. Podvrgavanjem ovih modela T-dualizaciji bićemo u stanju da dobijemo T-dualne teorije i zakone transfromacija koji povezuju koordinate početnih i T-dualnih teorija. Korišćenjem datih zakona transformacije, kao i komutativnih osobina početnih teorija bićemo u stanju da dedukujemo nekomutativne osobine T-dualnih teorija. Metoda za dobijanje T-dualnosti je bazirana na Bušerovoj proceduri i njenim proširenjima. Glavna ideja Bušerove procedure leži u lokalizaciji translacione simetrije, gde mi zamenjujemo parcijalne izvode i koordinate koje se javljaju u dejstvu sa kovarijantnim izvodima i invarijantnim koordinatama. Ova smena sa sobom povlači i uvođenje dodatnih stepeni slobode koji su kodirani preko kalibracionih polja. Eliminacijom novih stepeni slobode preko metode Lagranževih množitelja a zatim pronalaženjem jednačina kretanja za kalibraciona polja, dobijamo zakone transformacija između koordinata. Ubacivanjem ovih zakona transformacija u dejstvo dobijamo T-dualnu teoriju.

U proučavanje bozonske teorije struna, radićemo sa $3D$ prostorom gde uzimamo da Kalb-Ramondovo pozadinsko polje ima infinitezimalu linearnu koordinatnu zavisnost od samo jedne koordinate, z koordinate. Dualizacija će biti sprovedena duž dva različita lanca, jedan gde tek na kraju dualizujemo duž koordinate koja se javlja u pozadinskom polju a druge gde ovu koordinatu prvu dualizujemo. Poređenjem ova dva pristupa bićemo u stanju da zaključimo koji su sastojci neophodni za javljanje nekomutativnih osobina.

Drugi deo disertacije tiče se T-dualizacije tip II superstrune koja se kréće u linearno koordinatno zavisnom Ramond-Ramon polju. Za razliku od plošlig slučaja, ova teorija poseduje i bozonske i fermionske koordinate, doduše pozadinska polja zavise samo od bozonskih koordinata. T-dualizacija će se prvo fokusirati samo na bozonski deo, nakon toga ćemo uključiti i fermionske koordinate. Kao i u prošlom slučaju, predstavićemo i jos jedan alternativan lanaz dualizacije, lanac gde prvo dualizujemo fermionske a zatim bozonske koordinate. Pokazaćemo da oba lanca vode do istih nekomutativnih relacija. Konačno, na kraju disertacije, izvršićemo analizu iste teorije ali sa opštijim slučajem Ramond-Ramond polja.

Ključne reči: Teroija struna, nekomutativnost, neasocijativnost, Bušerova procedura
Naučna oblast: Fizika
Uža naučna oblast: Teorijska fizika visokih energija

Contents

На основу члана 29 Закона о општем управном поступку («Службени гласник РС» број 18/2016 и 95/2018), и члана 149 Статута Универзитета у Београду - Физичког факултета, по захтеву ДАНИЈЕЛА ОБРИЋА, мастер физичара, издаје се следеће

У В Е Р Е Њ Е

ДАНИЈЕЛ ОБРИЋ, мастер физичар, дана 18. септембра 2023. године, одбранио је докторску дисертацију под називом

"T-DUALIZATION OF BOSONIC STRING AND TYPE IIB SUPERSTRING IN PRESENCE OF COORDINATE DEPENDENT BACKGROUND FIELDS" (Т-дуализација бозонске струне и тип IIB суперструне у пристуству координатно зависних позадинских поља)

пред Комисијом Универзитета у Београду - Физичког факултета и тиме испунио све услове за промоцију у ДОКТОРА НАУКА – ФИЗИЧКЕ НАУКЕ.

Уверење се издаје на лични захтев, а служи ради регулисања права из радног односа и важи до промоције, односно добијања докторске дипломе.

Уверење је ослобођено плаћања таксе.

ДЕКАН ФИЗИЧКОГ ФАКУЛТЕТА

Проф. др Иван Белча

З А П И С Н И К

са VIII седнице Изборног и Наставно-научног већа одржане у среду 23. јуна 2021. године

Седници присуствује 43 чланова Изборног и Наставно-научног већа.

Оправдано одсутни: проф. др Марија Димитријевић Ћирић
проф. др Драгана Вујовић
проф. др Владимир Ђурђевић
проф. др Предраг Миленовић
доц. др Весна Ковачевић
доц. др Сузана Путниковић
доц. др Драгољуб Гочанин
др Биљана Николић

Неоправдано одсутни: проф. др Милорад Кураица
проф. др Мићо Митровић
проф. др Растко Василић
доц. др Владимир Миљковић
доц. др Зоран Поповић

Декан Факултета проф. др Иван Белча отворио је седницу у 11:15 часова и предложио следећи

Д н е в н и р е д

1. Усвајање Записника са VII седнице Изборног и Наставно-научног већа Физичког факултета.

Изборно веће

2. Разматрање предлога Катедре за квантну и математичку физику у вези са избором проф. др Часлава Брукнера у звање *гостујући професор*
3. Усвајање Извештаја комисије за избор наставника Физичког факултета и то:
   a) једног ванредног професора са до 30% радног времена за ужу научну област Физика кондензоване материје једног ванредног професора са до 30% радног времена за ужу научну област Статистичка физика

Наставно-научно веће

4. Усвајање Извештаја Комисије за оцену испуњености услова и оправданост предложене теме за израду докторске дисертације и одређивање ментора за:
   a) АНУ ВРАНИЋ, мастер физичара, која је пријавила докторску дисертацију под називом „EVOLVING COMPLEX NETWORKS: STRUCTURE AND DYNAMICS“ (Растуће комплексне мреже: структура и динамика)
   b) БОЈАНУ БОКИЋ, мастер физичара, која је пријавила докторску дисертацију под називом: „ДИНАМИКА ПРОСТИРАЊА ЕИРИЈЕВИХ СНОПОВА У ФОТОРЕФРАКТИВНИМ СРЕДИНАМА“
5. Одређивање Комисије за преглед и оцену докторске дисертације за:
   a) ТАТЈАНУ МАРКОВИЋ - ТОПАЛОВИЋ (8011/2016), дипломираног физичара, која је пријавила докторску дисертацију под називом: „ЕДУКАЦИОНА ВИЗУЕЛИЗАЦИЈА ФИЗИЧКИХ ФЕНОМЕНА У ПАРКУ НАУКЕ И ШКОЛСКОМ ПРОСТОРУ" под менторством др Андријане Жекић, редовног професора Физичког факултета
   b) МАРЈАНА ЋИРКОВИЋА (8011/2013), мастер физичара, који је пријавио докторску дисертацију под називом: "ПРОДУКЦИЈА K_s^0 МЕЗОНА У НЕЕЛАСТИЧНИМ p+p СУДАРИМА НА ЕНЕРГИЈИ ОД 158 GeV МЕРЕНА NA61/SHINE ДЕТЕКТОРОМ НА SPS-У У CERN-У" под менторством др Јована Пузовића, редовног професора Физичког факултета
6. Усвајање пријављене теме за израду мастер рада, одређивање руководиоца и Комисије за одбрану рада за:

a) ЂУРЂИЈУ ЈЕЛЕНКОВИЋ, мастер студента физике, смер Примењена и компјутерска физика, која је пријавила мастер рад под називом: "МЕРЕЊЕ КОЕФИЦИЈЕНТА АПСОРПЦИЈЕ ЗВУКА МАТЕРИЈАЛА КОЈИ СЕ КОРИСТЕ У ПОЗОРИШНИМ И КОНЦЕРТНИМ ДВОРАНАМА"

b) СТЕФАНА ЂОРЂЕВИЋА, мастер студента физике, смер Теоријска и експериментална физика, који је пријавио мастер рад под називом: „ЕНТРОПИЈА ХОКИНГОВОГ ЗРАЧЕЊА У 2D ДИЛАТОНСКОЈ ГРАВИТАЦИЈИ"

c) АНАСТАСИЈУ МИЛАДИНОВИЋ, мастер студента физике, смер Примењена и компјутерска физика, која је пријавила мастер рад под називом: "СЕМИ-СТОХАСТИЧКИ МОДЕЛ ЕЛАСТИЧНОГ РАСЕЈАЊА ЕЛЕКТРОНА НА АТОМУ ВОДОНИКА"

d) ЈОВАНУ ЈЕЛИЋ, мастер студента физике, смер Примењена и компјутерска физика, која је пријавила мастер рад под називом: "РАЗВОЈ ЕКСПЕРИМЕНТАЛНЕ ПОСТАВКЕ ЗА ДЕТЕКЦИЈУ ПОЈЕДИНАЧНИХ МОЛЕКУЛА ПРИМЕНОМ ФЛУОРЕСЦЕНТНЕ КОРЕЛАЦИОНЕ СПЕКТРОСКОПИЈЕ"

e) АЛЕКСУ ДЕНЧЕВСКОГ, мастер студента физике, смер Примењена и компјутерска физика, који је пријавио мастер рад под називом: "КАРАКТЕРИЗАЦИЈА И СИНХРОНИЗАЦИЈА ПОБУДНОГ, УПРАВЉАЧКОГ И ДЕТЕКЦИОНОГ СИСТЕМА МИКРОСКОПА СА СТРУКТУРИСАНИМ ПРОСВЕТЉАВАЊЕМ"

f) ДРАГАНУ РУЛИЋ, мастер студента физике, смер Општа физика, која је пријавила мастер рад под називом: "УПОРЕДНА АНАЛИЗА ПАРАМЕТАРА ФОТОЛУМЕНИСЦЕНТНИХ СИГНАЛА ВОДЕНИХ РАСТВОРА"

g) ЈОВАНА ЈАЊИЋА, мастер студента физике, смер Теоријска и експериментална физика, који је пријавио мастер рад под називом: "ТИП II-B СУПЕРСТРУНА - ЕФЕКТИВНА ТЕОРИЈА, НЕКОМУТАТИВНОСТ И Т-ДУАЛНОСТ"

h) ЈЕЛЕНУ СТОШИЋ, мастер студента физике, смер Општа физика, која је пријавила мастер рад под називом: "ПОЈМОВИ МАСЕ, ГРАВИТАЦИОНЕ СИЛЕ, СИЛЕ ЗЕМЉИНЕ ТЕЖЕ И ТЕЖИНЕ У УЏБЕНИЧКОЈ ЛИТЕРАТУРИ"

i) ВАСИЛИЈА МАТИЋА, мастер студента физике, смер Теоријска и експериментална физика, који је пријавио мастер рад под називом: "ЈЕДНОСТАВАН МОДЕЛ АУТОЛОКАЛИЗАЦИЈЕ ВИБРОНСКИХ ПОБУЂЕЊА У АЛФА-ХЕЛИКОИДАЛНИМ БИОМОЛЕКУЛИМА"

j) РАТОМИРА САВИЋА, мастер студента физике, смер Општа физика, који је пријавио мастер рад под називом: "УЛОГА ПЛАТФОРМИ ЗА УПРАВЉАЊЕ УЧЕЊЕМ У ШКОЛСКОЈ НАСТАВИ ФИЗИКЕ"

k) ЕМИЛИЈУ ПОПОВИЋ, мастер студента физике, смер Општа физика, који је пријавио мастер рад под називом: "ПРИМЕРИ И АНАЛИЗА ИЗАЗОВА У НАСТАВИ ФИЗИКЕ НА ДАЉИНУ"

l) ЂОРЂА БОГДАНОВИЋА, мастер студента физике, смер Теоријска и експериментална физика, који је пријавио мастер рад под називом: "*L* БЕСКОНАЧНО АЛГЕБРА И *BRST* КВАНТИЗАЦИЈА"

m) МИШУ ТОМАНА, мастер студента физике, смер Теоријска и експериментална физика, који је пријавио мастер рад под називом: " *L* БЕСКОНАЧНО АЛГЕБРЕ ЕЛЕКТРОДИНАМИКЕ НА КОМУТАТИВНОМ И НЕКОМУТАТИВНОМ ПРОСТОРУ"

n) САЊУ ЈОВАНОВИЋ, мастер студента метеорологије, која је пријавила мастер рад под називом: "АНАЛИЗА СИНОПТИЧКОГ СЛУЧАЈА ПРОЛАСКА ИНТЕНЗИВНОГ ЦИКЛОНА ПРЕКО ЕВРОПЕ У ПЕРИОДУ ОД 3. ДО 7. ФЕБРУАРА 2020. ГОДИНЕ"

o) АЛЕКСАНДРУ СТЕВКОВ, мастер студента метеорологије, који је пријавио мастер рад под називом: "ПРОСТОРНО-ВРЕМЕНСКА АНАЛИЗА СУША У СЕВЕРНОЈ МАКЕДОНИЈИ ПРИМЕНОМ РАЗЛИЧИТИХ ИНДЕКСА ЗА МОНИТОРИНГ СУШЕ"

p) ХАНУ ШИФ, мастер студента физике, смер Теоријска и експериментална физика, која је пријавила мастер рад под називом: "БЕНД КОРЕПРЕЗЕНТАЦИЈЕ И ТОПОЛОШКЕ ФАЗЕ КВАЗИ-ЈЕДНОДИМЕНЗИОНАЛНИХ СИСТЕМА СА МАГНЕТНИМ УРЕЂЕЊЕМ"

q) ИВАНУ СТОЈИЉКОВИЋ, мастер студента физике, смер Теоријска и експериментална физика, која је пријавила мастер рад под називом: "ИМПЛЕМЕНТАЦИЈА ПРОТОКОЛА ЗА ДЕТЕКЦИЈУ КВАНТНИХ КОРЕЛАЦИЈА У ПРОГРАМСКОМ ПАКЕТУ *QISKIT*"

r) ДУШАНА ЂОРЂЕВИЋА, мастер студента физике, смер Теоријска и експериментална физика, који је пријавио мастер рад под називом: "NONCOMMUTATIVE FIVE - DIMENSIONAL CHERN - SIMONS GRAVITY",Nekomutativna petodimenziona Черн-Сајмонсова гравитација

s) АЛЕКСАНДРУ ТРНАВАЦ, мастер студента физике, смер Примењена и компјутерска физика, која је пријавила мастер рад под називом: "МОДЕЛОВАЊЕ ИНТЕРАКЦИЈА У НЕФИЗИЧКИМ СИСТЕМИМА - АНАЛИЗА ГРАВИТАЦИОНОГ МОДЕЛА У МЕЂУНАРОДНОЈ ТРГОВИНИ"

t) МД РАШАДУЛ ИСЛАМ, мастер студента физике, смер Примењена и компјутерска физика, који је пријавио мастер рад под називом: "DEVELOPMENT OF OPTIMIZED SOLUTIONS FOR BIG DATA ANALYTICS ON THE INTERNET USING THE PYTHON PANDAS LIBRARY" (РАЗВОЈ ОПТИМИЗОВАНИХ РЕШЕЊА ЗА АНАЛИЗУ ВЕЛИКИХ КОЛЕКЦИЈА ПОДАТАКА НА ИНТЕРНЕТУ КОРИШЋЕЊЕМ *PYTHON PANDAS* БИБЛИОТЕКЕ)

u) ЈОВАНУ ПЕТКОВИЋ, мастер студента физике, смер Примењена и компјутерска физика, која је пријавила мастер рад под називом: "ЕЛЕКТРИЧНА КАРАКТЕРИЗАЦИЈА И ЕМИСИОНИ СПЕКТРИ ДИЕЛЕКТРИЧНОГ БАРИЈЕРНОГ ПРАЖЊЕЊА СА СЕГМЕНТИРАНОМ ЕЛЕКТРОДОМ"

v) НЕДУ БАБУЦИЋ, мастер студента физике, смер Примењена и компјутерска физика, која је пријавила мастер рад под називом: "УНАПРЕЂЕЊЕ ИНТЕРФЕРОМЕТРИЈЕ МЕТОДОМ СОПСТВЕНОГ МЕШАЊА СИГНАЛА ЛАСЕРСКЕ ДИОДЕ"

w) ЈЕЛЕНУ МАРКОВИЋ, мастер студента физике, смер Примењена и компјутерска физика, која је пријавила мастер рад под називом: "НУМЕРИЧКА АНАЛИЗА РАДИО-ФРЕКВЕНТНИХ НАПОНСКИХ СИГНАЛА ЗА МЕРЕЊЕ СНАГЕ ПРЕДАТЕ ПЛАЗМИ У ФРЕКВЕНТНОМ И ВРЕМЕНСКОМ ДОМЕНУ"

x) АЛЕКСАНДРА ЗЕЧЕВИЋА, мастер студента метеорологије, који је пријавио мастер рад под називом: „ПРОЈЕКЦИЈА ПРОИЗВОДЊЕ ЕЛЕКТРИЧНЕ ЕНЕРГИЈЕ У ВЈЕТРОПАРКУ КРНОВО (НИКШИЋ) НА ОСНОВУ АНАЛИЗЕ И ПРОГНОЗЕ МЕТЕОРОЛОШКИХ ВЕЛИЧИНА"

y) КАТАРИНУ ЛОЈАНИЦУ, мастер студента физике, смер Општа физика, која је пријавила мастер рад под називом: „ВОЛУМЕТРИЈСКА АНАЛИЗА ЗАСНОВАНА НА МЕРЕЊУ ВРЕМЕНА РЕВЕРБЕРАЦИЈЕ ЗВУЧНИХ ТАЛАСА“

z) СТЕФАНА ЈОКУ, мастер студента физике, смер Теоријска и експериментална физика, који је пријавио мастер рад под називом: "ЕНТРОПИЈА ЦРНИХ РУПА У 3D ГРАВИТАЦИЈИ"

7. Усвајање пријављене теме за израду дипломског рада, одређивање руководиоца и Комисије за одбрану рада за:
   a) ДРАГАНУ СТАНИМИРОВИЋ, апсолвента физике, смер Општа физика, која је пријавила дипломски рад под називом: "ПРИМЕНА САВРЕМЕНИХ НАСТАВНИХ МЕТОДА У ОБРАДИ НАСТАВНЕ ТЕМЕ *ПРИТИСАК*"
8. Давање сагласности на ангажовање наставника и сарадника и то:
   a) др ДЕЈАНА ЈАНЦА, ванредног професора Физичког факултета за држање наставе из предмета *Вазухопловна метеорологија* на Војној академији Министарства одбране РС
   b) др БРАНИСЛАВЕ ВУЧЕТИЋ, доцента Физичког факултета, за држање наставе из предмета *Физика А* на Војној академији Министарства одбране РС
   c) НИКОДИНА НЕДИЋА, истраживача Физичког факултета, за држање вежби из предмета *Физика А* на Војној академији Министарства одбране РС
   d) др СРЂАНА БУКВИЋА, редовног профсора Физичког факултета, за држање наставе из предмета *Методе мјерења* на Природно-математичком факултету Универзитета у Бањој Луци
   e) др ЗОРАНА ПОПОВИЋА, доцента Физичког факултета, за држање наставе из предмета *Програмирање у физици 1* и *2, Квантна механика 1* и *2* на Природно-математичком факултету Универзитета у Бањој Луци
   f) др АЛЕКСАНДРЕ ДИМИЋ, доцента Физичког факултета, за држање наставе из предмета Биофотоника на Докторским студијама Универзитета у Београду
9. Усвајање рецензије рукописа "Атмосферски електрицитет" аутора доц. др Немање Ковачевића.
10. Избор представника Факултета у Већу групација природно-математичких наука из реда редовних професора.
11. Питања наставе, науке и финансија.
12. Захтеви за одобрење одсуства.
13. Усвајање извештаја са службених путовања.
14. Дописи и молбе упућене Наставно-научном већу.
15. Обавештења. Текућа питања. Питања и предлози.

Пошто је усвојен предложени Дневни ред, прешло се на

1. <u>тачку</u>

Усвојен је Записник са VII седнице Изборног и Наставно-научног већа Физичког факултета.

<u>Изборно веће</u>

2. <u>тачка</u>

Усвојен је предлог Катедре за квантну и математичку физику у вези са избором проф. др Часлава Брукнера у звање *гостујући професор*

Комисија: *др Иванка Милошевић, редовни професор ФФ*
др Татјана Вуковић, редовни професор ФФ
др Милан Дамњановић, редовни професор ФФ у пензији

3. <u>тачка</u>

Изборно веће је усвојило Извештај комисије за избор наставника Физичког факултета и то:

a) после краће дискусије и јавног гласања, једногласно, са 26 гласова ЗА од укупно 33 колико чини изборно тело, донета је одлука о избору др ДАРКА ТАНАСКОВИЋА у звање ванредног професора са до 30% радног времена за ужу научну област Физика кондензоване материје

b) после краће дискусије и јавног гласања, једногласно, са 26 гласова ЗА од укупно 33 колико чини изборно тело, донета је одлука о избору др МИЛИЦЕ МИЛОВАНОВИЋ у звање ванредног професора са до 30% радног времена за ужу научну област Статистичка физика

Наставно-научно веће

4. тачка

Усвојен је Извештај Комисије за оцену испуњености услова и оправданост предложене теме за израду докторске дисертације и одређен ментор за:

a) АНУ ВРАНИЋ, мастер физичара, која је пријавила докторску дисертацију под називом „EVOLVING COMPLEX NETWORKS: STRUCTURE AND DYNAMICS“ (Растуће комплексне мреже: структура и динамика)
Ментор: др Марија Митровић Данкулов, виши научни сарадник ИФ

b) БОЈАНУ БОКИЋ, мастер физичара, која је пријавила докторску дисертацију под називом: „ДИНАМИКА ПРОСТИРАЊА ЕИРИЈЕВИХ СНОПОВА У ФОТОРЕФРАКТИВНИМ СРЕДИНАМА“
Ментор: др Бранислав Јеленковић, научни саветник ИФ у пензији

5. тачка

Одређена је Комисија за преглед и оцену докторске дисертације за:

a) ТАТЈАНУ МАРКОВИЋ - ТОПАЛОВИЋ (8011/2016), дипломираног физичара, која је пријавила докторску дисертацију под називом: „ЕДУКАЦИОНА ВИЗУЕЛИЗАЦИЈА ФИЗИЧКИХ ФЕНОМЕНА У ПАРКУ НАУКЕ И ШКОЛСКОМ ПРОСТОРУ" под менторством др Андријане Жекић, редовног професора Физичког факултета
Комисија: др Мићо Митровић, редовни професор ФФ
др Јаблан Дојчиловић, редовни професор ФФ у пензији
др Милена Давидовић, доцент Грађевинског факултета

b) МАРЈАНА ЋИРКОВИЋА (8011/2013), мастер физичара, који је пријавио докторску дисертацију под називом: "ПРОДУКЦИЈА K_S^0 МЕЗОНА У НЕЕЛАСТИЧНИМ p+p СУДАРИМА НА ЕНЕРГИЈИ ОД 158 GeV МЕРЕНА NA61/SHINE ДЕТЕКТОРОМ НА SPS-У У CERN-У" под менторством др Јована Пузовића, редовног професора Физичког факултета
Комисија: др Јован Пузовић, редовни професор ФФ
др Предраг Миленовић, ванредни професор ФФ
др Татјана Шуша, научни саветник Института Руђер Бошковић

6. тачка

Усвојена је пријављена тема за израду мастер рада, одређен руководилац и Комисија за одбрану рада за:

a) ЂУРЂИЈУ ЈЕЛЕНКОВИЋ, мастер студента физике, смер Примењена и компјутерска физика, која је пријавила мастер рад под називом: "МЕРЕЊЕ КОЕФИЦИЈЕНТА АПСОРПЦИЈЕ ЗВУКА МАТЕРИЈАЛА КОЈИ СЕ КОРИСТЕ У ПОЗОРИШНИМ И КОНЦЕРТНИМ ДВОРАНАМА"

Комисија: др Горан Попарић, редовни професор ФФ, руководилац рада
др Бећко Касалица, редовно професор ФФ
др Мирјана Војновић, научни сарадник ФФ

b) СТЕФАНА ЂОРЂЕВИЋА, мастер студента физике, смер Теоријска и експериментална физика, који је пријавио мастер рад под називом: „ЕНТРОПИЈА ХОКИНГОВОГ ЗРАЧЕЊА У 2D ДИЛАТОНСКОЈ ГРАВИТАЦИЈИ“

Комисија: др Воја Радовановић, редовни професор ФФ, руководилац рада
др Маја Бурић, редовни професор ФФ
др Драгољуб Гочанин, доцент ФФ
др Александра Димић, доцент ФФ

c) АНАСТАСИЈУ МИЛАДИНОВИЋ, мастер студента физике, смер Примењена и компјутерска физика, која је пријавила мастер рад под називом: "СЕМИ-СТОХАСТИЧКИ МОДЕЛ ЕЛАСТИЧНОГ РАСЕЈАЊА ЕЛЕКТРОНА НА АТОМУ ВОДОНИКА"

Комисија: др Горан Попарић, редовни професор ФФ, руководилац рада
др Сава Галијаш, доцент ФФ
др Мирјана Војновић, научни сарадник ФФ

d) ЈОВАНУ ЈЕЛИЋ, мастер студента физике, смер Примењена и компјутерска физика, која је пријавила мастер рад под називом: "РАЗВОЈ ЕКСПЕРИМЕНТАЛНЕ ПОСТАВКЕ ЗА ДЕТЕКЦИЈУ ПОЈЕДИНАЧНИХ МОЛЕКУЛА ПРИМЕНОМ ФЛУОРЕСЦЕНТНЕ КОРЕЛАЦИОНЕ СПЕКТРОСКОПИЈЕ"

Комисија: др Александар Крмпот, виши научни сарадник ИФ, руководилац рада
др Братислав Обрадовић, редовни професор ФФ
др Милорад Кураица, редовни професор ФФ

e) АЛЕКСУ ДЕНЧЕВСКОГ, мастер студента физике, смер Примењена и компјутерска физика, који је пријавио мастер рад под називом: "КАРАКТЕРИЗАЦИЈА И СИНХРОНИЗАЦИЈА ПОБУДНОГ, УПРАВЉАЧКОГ И ДЕТЕКЦИОНОГ СИСТЕМА МИКРОСКОПА СА СТРУКТУРИСАНИМ ПРОСВЕТЉАВАЊЕМ"

Комисија: др Михаило Рабасовић, научни сарадник ИФ, руководилац рада
др Иван Белча, редовни професор ФФ
др Стеван Стојадиновић, редовни професор ФФ

f) ДРАГАНУ РУЛИЋ, мастер студента физике, смер Општа физика, која је пријавила мастер рад под називом: "УПОРЕДНА АНАЛИЗА ПАРАМЕТАРА ФОТОЛУМЕНИСЦЕНТНИХ СИГНАЛА ВОДЕНИХ РАСТВОРА"

Комисија: др Владимир Милосављевић, редовни професор ФФ, руководилац рада
др Горан Попарић, редовни професор ФФ
др Душан Поповић, ванредни професор ФФ

g) ЈОВАНА ЈАЊИЋА, мастер студента физике, смер Теоријска и експериментална физика, који је пријавио мастер рад под називом: "ТИП II-B СУПЕРСТРУНА - ЕФЕКТИВНА ТЕОРИЈА, НЕКОМУТАТИВНОСТ И Т-ДУАЛНОСТ"

Комисија: др Бојан Николић, виши научни сарадник ИФ, руководилац рада
др Воја Радовановић, редовни професор ФФ
др Марија Димитријевић, редовни професор ФФ

h) ЈЕЛЕНУ СТОШИЋ, мастер студента физике, смер Општа физика, која је пријавила мастер рад под називом: "ПОЈМОВИ МАСЕ, ГРАВИТАЦИОНЕ СИЛЕ, СИЛЕ ЗЕМЉИНЕ ТЕЖЕ И ТЕЖИНЕ У УЏБЕНИЧКОЈ ЛИТЕРАТУРИ"

Комисија: *др Мићо Митровић, редовни професор ФФ, руководилац рада*
др Андријана Жекић, редовни професор ФФ
Нора Тркља Боца, истраживач-сарадник ФФ

i) ВАСИЛИЈА МАТИЋА, мастер студента физике, смер Теоријска и експериментална физика, који је пријавио мастер рад под називом: "ЈЕДНОСТАВАН МОДЕЛ АУТОЛОКАЛИЗАЦИЈЕ ВИБРОНСКИХ ПОБУЂЕЊА У АЛФА-ХЕЛИКОИДАЛНИМ БИОМОЛЕКУЛИМА"

Комисија: *др Далибор Чевизовић, в. н. сарадник ИНН Винча, руководилац рада*
др Ђорђе Спасојевић, редовни професор ФФ
др Зорица Поповић, доцент ФФ

j) РАТОМИРА САВИЋА, мастер студента физике, смер Општа физика, који је пријавио мастер рад под називом: "УЛОГА ПЛАТФОРМИ ЗА УПРАВЉАЊЕ УЧЕЊЕМ У ШКОЛСКОЈ НАСТАВИ ФИЗИКЕ"

Комисија: *др Андријана Жекић, редовни професор ФФ, руководилац рада*
др Горан Попарић, редовни професор ФФ
др Саша Ивковић, доцент ФФ

k) ЕМИЛИЈУ ПОПОВИЋ, мастер студента физике, смер Општа физика, који је пријавио мастер рад под називом: "ПРИМЕРИ И АНАЛИЗА ИЗАЗОВА У НАСТАВИ ФИЗИКЕ НА ДАЉИНУ"

Комисија: *др Андријана Жекић, редовни професор ФФ, руководилац рада*
др Горан Попарић, редовни професор ФФ
др Саша Ивковић, доцент ФФ

l) ЂОРЂА БОГДАНОВИЋА, мастер студента физике, смер Теоријска и експериментална физика, који је пријавио мастер рад под називом: "*L* БЕСКОНАЧНО АЛГЕБРА И *BRST* КВАНТИЗАЦИЈА"

Комисија: *др Воја Радовановић, редовни професор ФФ, руководилац рада*
др Марија Димитријевић Ћирић, редовни професор ФФ
др Никола Коњик, доцент ФФ

m) МИШУ ТОМАНА, мастер студента физике, смер Теоријска и експериментална физика, који је пријавио мастер рад под називом: " *L* БЕСКОНАЧНО АЛГЕБРЕ ЕЛЕКТРОДИНАМИКЕ НА КОМУТАТИВНОМ И НЕКОМУТАТИВНОМ ПРОСТОРУ"

Комисија: *др Марија Димитријевић Ћирић, ред. проф. ФФ, руководилац рада*
др Воја Радовановић, редовни професор ФФ
др Никола Коњик, доцент ФФ

n) САЊУ ЈОВАНОВИЋ, мастер студента метеорологије, која је пријавила мастер рад под називом: "АНАЛИЗА СИНОПТИЧКОГ СЛУЧАЈА ПРОЛАСКА ИНТЕНЗИВНОГ ЦИКЛОНА ПРЕКО ЕВРОПЕ У ПЕРИОДУ ОД 3. ДО 7. ФЕБРУАРА 2020. ГОДИНЕ"

Комисија: *др Катарина Вељовић Корачин, доцент ФФ, руководилац рада*
др Немања Ковачевић, доцент ФФ
др Лазар Лазић, редовни професор ФФ у пензији

o) АЛЕКСАНДРУ СТЕВКОВ, мастер студента метеорологије, који је пријавио мастер рад под називом: "ПРОСТОРНО-ВРЕМЕНСКА АНАЛИЗА СУША У СЕВЕРНОЈ МАКЕДОНИЈИ ПРИМЕНОМ РАЗЛИЧИТИХ ИНДЕКСА ЗА МОНИТОРИНГ СУШЕ"
Комисија: *др Ивана Тошић, редовни професор ФФ, руководилац рада*
др Владимир Ђурђевић, ванредни професор ФФ
др Драгана Вујовић, ванредни професор ФФ

p) ХАНУ ШИФ, мастер студента физике, смер Теоријска и експериментална физика, која је пријавила мастер рад под називом: "БЕНД КОРЕПРЕЗЕНТАЦИЈЕ И ТОПОЛОШКЕ ФАЗЕ КВАЗИ-ЈЕДНОДИМЕНЗИОНАЛНИХ СИСТЕМА СА МАГНЕТНИМ УРЕЂЕЊЕМ"
Комисија: *др Саша Дмитровић, доцент ФФ, руководилац рада*
др Татјана Вуковић, редовни професор ФФ
др Наташа Лазић, научни сарадник ФФ

q) ИВАНУ СТОЈИЉКОВИЋ, мастер студента физике, смер Теоријска и експериментална физика, која је пријавила мастер рад под називом: "ИМПЛЕМЕНТАЦИЈА ПРОТОКОЛА ЗА ДЕТЕКЦИЈУ КВАНТНИХ КОРЕЛАЦИЈА У ПРОГРАМСКОМ ПАКЕТУ *QISKIT*"
Комисија: *др Александра Димић, доцент ФФ, руководилац рада*
др Иванка Милошевић, редовни професор ФФ
др Татјана Вуковић, редовни професор ФФ
др Саша Дмитровић, доцент ФФ

r) ДУШАНА ЂОРЂЕВИЋА, мастер студента физике, смер Теоријска и експериментална физика, који је пријавио мастер рад под називом: "NONCOMMUTATIVE FIVE - DIMENSIONAL CHERN - SIMONS GRAVITY",Nekomutativna petodimenziona Черн-Сајмонсова гравитација
Комисија: *др Драгољуб Гочанин, доцент ФФ, руководилац рада*
др Воја Радовановић, редовни професор ФФ
др Марија Димитријевић Ћирић, редовни професор ФФ

s) АЛЕКСАНДРУ ТРНАВАЦ, мастер студента физике, смер Примењена и компјутерска физика, која је пријавила мастер рад под називом: "МОДЕЛОВАЊЕ ИНТЕРАКЦИЈА У НЕФИЗИЧКИМ СИСТЕМИМА - АНАЛИЗА ГРАВИТАЦИОНОГ МОДЕЛА У МЕЂУНАРОДНОЈ ТРГОВИНИ"
Комисија: *др Зоран Николић, ванредни професор ФФ, руководилац рада*
др Едиб Добарџић, ванредни професор ФФ
др Јасна Атанасијевић, ванредни професор ПМФ Нови Сад

t) МД РАШАДУЛ ИСЛАМ, мастер студента физике, смер Примењена и компјутерска физика, који је пријавио мастер рад под називом: "DEVELOPMENT OF OPTIMIZED SOLUTIONS FOR BIG DATA ANALYTICS ON THE INTERNET USING THE PYTHON PANDAS LIBRARY" (РАЗВОЈ ОПТИМИЗОВАНИХ РЕШЕЊА ЗА АНАЛИЗУ ВЕЛИКИХ КОЛЕКЦИЈА ПОДАТАКА НА ИНТЕРНЕТУ КОРИШЋЕЊЕМ *PYTHON PANDAS* БИБЛИОТЕКЕ)
Комисија: *др Зоран Николић, ванредни професор ФФ, руководилац рада*
др Едиб Добарџић, ванредни професор ФФ
др Милош Вићић, редовни професор ФФ

u) ЈОВАНУ ПЕТКОВИЋ, мастер студента физике, смер Примењена и компјутерска физика, која је пријавила мастер рад под називом: "ЕЛЕКТРИЧНА КАРАКТЕРИЗАЦИЈА И ЕМИСИОНИ СПЕКТРИ ДИЕЛЕКТРИЧНОГ БАРИЈЕРНОГ ПРАЖЊЕЊА СА СЕГМЕНТИРАНОМ ЕЛЕКТРОДОМ"

Комисија: др Никола Шкоро, виши научни сарадник ИФ, руководилац рада
др Стеван Стојадиновић, редовни професор ФФ
др Горан Сретеновић, научни сарадник ФФ

v) НЕДУ БАБУЦИЋ, мастер студента физике, смер Примењена и компјутерска физика, која је пријавила мастер рад под називом: "УНАПРЕЂЕЊЕ ИНТЕРФЕРОМЕТРИЈЕ МЕТОДОМ СОПСТВЕНОГ МЕШАЊА СИГНАЛА ЛАСЕРСКЕ ДИОДЕ"
Комисија: др Ненад Сакан, научни сарадник ИФ, руководилац рада
др Милорад Кураица, редовни професор ФФ
др Братислав Обрадовић, редовни професор ФФ

w) ЈЕЛЕНУ МАРКОВИЋ, мастер студента физике, смер Примењена и компјутерска физика, која је пријавила мастер рад под називом: "НУМЕРИЧКА АНАЛИЗА РАДИО-ФРЕКВЕНТНИХ НАПОНСКИХ СИГНАЛА ЗА МЕРЕЊЕ СНАГЕ ПРЕДАТЕ ПЛАЗМИ У ФРЕКВЕНТНОМ И ВРЕМЕНСКОМ ДОМЕНУ"
Комисија: др Невена Пуач, научни саветник ИФ, руководилац рада
др Горан Попарић, редовни професор ФФ
др Стеван Стојадиновић, редовни професор ФФ

x) АЛЕКСАНДРА ЗЕЧЕВИЋА, мастер студента метеорологије, који је пријавио мастер рад под називом: „ПРОЈЕКЦИЈА ПРОИЗВОДЊЕ ЕЛЕКТРИЧНЕ ЕНЕРГИЈЕ У ВЈЕТРОПАРКУ КРНОВО (НИКШИЋ) НА ОСНОВУ АНАЛИЗЕ И ПРОГНОЗЕ МЕТЕОРОЛОШКИХ ВЕЛИЧИНА"
Комисија: др Драгана Вујовић, ванредни професор ФФ, руководилац рада
др Лазар Лазић, редовни професор ФФ, у пензији
др Дејан Јанц, ванредни професор ФФ

y) КАТАРИНУ ЛОЈАНИЦУ, мастер студента физике, смер Општа физика, која је пријавила мастер рад под називом: „ВОЛУМЕТРИЈСКА АНАЛИЗА ЗАСНОВАНА НА МЕРЕЊУ ВРЕМЕНА РЕВЕРБЕРАЦИЈЕ ЗВУЧНИХ ТАЛАСА“
Комисија: др Зоран Николић, ванредни професор ФФ, руководилац рада
др Бећко Касалица, редовни професор ФФ
др Зоран Поповић, доцент ФФ

z) СТЕФАНА ЈОКУ, мастер студента физике, смер Теоријска и експериментална физика, који је пријавио мастер рад под називом: "ЕНТРОПИЈА ЦРНИХ РУПА У 3D ГРАВИТАЦИЈИ"
Комисија: др Бранислав Цветковић, научни саветник ИФ, руководилац рада
др Душко Латас, доцент ФФ
др Драгољуб Гочанин, доцент ФФ

7. тачка

Усвојена је пријављена тема за израду дипломског рада, одређен руководилац и Комисија за одбрану рада за:

a) ДРАГАНУ СТАНИМИРОВИЋ, апсолвента физике, смер Општа физика, која је пријавила дипломски рад под називом: "ПРИМЕНА САВРЕМЕНИХ НАСТАВНИХ МЕТОДА У ОБРАДИ НАСТАВНЕ ТЕМЕ *ПРИТИСАК*"
Комисија: др Горан Попарић, редовни професор ФФ, руководилац рада
др Андријана Жекић, редовни професор ФФ
др Мирјана Војновић, научни сарадник ФФ

8. тачка

Дата је сагласност на ангажовање наставника и сарадника и то:

a) др ДЕЈАНА ЈАНЦА, ванредног професора Физичког факултета за држање наставе из предмета *Вазухопловна метеорологија* на Војној академији Министарства одбране РС
b) др БРАНИСЛАВЕ ВУЧЕТИЋ, доцента Физичког факултета, за држање наставе из предмета *Физика А* на Војној академији Министарства одбране РС
c) НИКОДИНА НЕДИЋА, истраживача Физичког факултета, за држање вежби из предмета *Физика А* на Војној академији Министарства одбране РС
d) др СРЂАНА БУКВИЋА, редовног профсора Физичког факултета, за држање наставе из предмета *Методе мјерења* на Природно-математичком факултету Универзитета у Бањој Луци
e) др ЗОРАНА ПОПОВИЋА, доцента Физичког факултета, за држање наставе из предмета *Програмирање у физици 1* и *2, Квантна механика 1* и *2* на Природно-математичком факултету Универзитета у Бањој Луци
f) др АЛЕКСАНДРЕ ДИМИЋ, доцента Физичког факултета, за држање наставе из предмета Биофотоника на Докторским студијама Универзитета у Београду

9. тачка

Усвојена је рецензија рукописа "Атмосферски електрицитет" аутора доц. др Немање Ковачевића.

10. тачка

Изабрани су представници Факултета у Већу групација природно-математичких наука из реда редовних професора.

Представници: проф. др Андријана Жекић
проф. др Братислав Обрадовић

11. тачка

Питања наставе

Продекан за наставу доц. др Славица Малетић обавестила је чланове већа да данас почиње пријава кандидата за упис у прву годину студија. Пријава траје до петка 25.6, а пријемни испити се полажу у понедељак 28. и уторак 29. јула.

Продекан је замолила шефове катедара да јој пошаљу распоред наставника и сарадника по предметима за јесењи семестар.

На предлог продекана Наставно-научно веће је расписало конкурс за ангажовање у настави студената докторских и мастер студија. Услови су исти као и ранијих година. Потребан је просек оцена са студија најмање 8.5, и највише две поновљене године.

Наставно-научно веће је оформило комисију за студенте по старом програму који се пребацују на наставни план по Болоњи:

- проф. др Андријана Жекић
- проф. др Владан Вучковић
- проф. др Горан Попарић
- проф. др Воја Радовановић
- доц. др Зорица Поповић

Питања науке

Продекан за науку проф. др Стеван Стојадиновић обавестио је чланове Већа да ће се идуће среде одржати састанак Колегијума докторских студија.

Питања финансија

Продекан за финансије проф. др Горан Попарић подсетио је чланове Већа да нам предстоји плаћање поновне акредитације Факултета. Ради нешто неповољније финансијске ситуације, варијабила за часова је враћена на редовних 5%.

<u>12. тачка</u>

Наставно-научно веће је усвојило плаћена одсуства следећим наставницим и сарадницима:

a) др Иванки Милошевић у периоду од 05. 07. до 10. 07. 2021. године ради учешћа на међународном научном скупу *NANOTEXNOLOGY 2021* који се одржава у Солуну, Грчка.

b) др Зорици Поповић у периоду од 24. 06. до 29. 06. 2021. године ради посете Природно математичком факултету Универзитета Црне Горе у Подгорици која се реализује у оквиру Билатералног пројекта Србија-Црна Гора.

c) Кристини Мојсиловић од 1. 07. до 30. 07. 2021. године боравак као гостујући истраживач на Војном институту технологије (Wojskowa Akademia Techniczna, Wydzialu Nowych Technologii i Chemii односно Military University of Technology, Faculty of Advanced Technologies & Chemistry) у Варшави, Пољска.

d) Јелени Пајовић у периоду од 19. 07. до 19. 10. 2021. године ради посете Универзитету у Колораду, Болдер, САД.

<u>13. тачка</u>

Наставно-научно веће је усвојило извештај са службеног путовања проф. др Зорана Николића који је у периоду од 3. до 6. јуна 2021. године учествовао на међународној конференцији X International Conference on Social and Technological Development STED 2021, која је одржана у Требињу, Босна и Херцеговина.

<u>14. тачка</u>

Секретар Факултета Лелица Вуковић Радош замолила је чланове Већа да је убудуће на време обавесте о неплаћеном одсуству, зато што се све промене у радном односу морају спроводити кроз ЦРОСО регистар, а било какво ретроактивно уписивање промене није могуће.

Лелица Вуковић Радош је, такође обавестила чланове Већа да је члан Савета Факултета проф. др Милорад Кураица дао оставку на чланство. Уместо њега је потребно изабрати новог члана Савета из редова наставника Факултета. Наставно-научно веће је донело одлуку да тај избор остави за септембар када ће бити потребно бирати и члана Савета уместо доц. др Зорице Поповић, која ће преузети дужност продекана за наставу.

Седница је завршена у 11:52 часова.

Београд, 29.6.2021. год.

ДЕКАН ФИЗИЧКОГ ФАКУЛТЕТА
Проф. др Иван Белча, с.р.

Univerzitet u Beogradu
Fizički fakultet

Master rad

Tip IIB superstruna - efektivna teorija, nekomutativnost i T-dualnost

Student: Jovan Janjić (7013/2020)

Mentor: Bojan Nikolić

Ovom prilikom želim da izrazim zahvalnost mom mentoru Bojanu Nikoliću, čije su mi strpljenje i korisni komentari bili od velike pomoći prilikom mog upuštanja u izazovnu oblast kakva je teorija struna. Takođe, želim da izrazim veliku zahvalnost mojoj porodici, bez čije nesebične podrške ovaj rad i završetak mojih studija ne bi bili mogući.

Jovan Janjić
Beograd, 26.9.2023.

UNIVERZITET U BEOGRADU
FIZIČKI FAKULTET

MASTER RAD

T-DUALNOST NA TORUSU PREKO KOMPLEKSNIH PARAMETARA

Student: Milivoje Jojić
Mentor: dr Bojan Nikolić

Beograd, 2015.

З А П И С Н И К

са XI седнице Наставно-научног већа одржане у среду 30. септембра 2015.

Седници присуствује 49 чланова Наставно-научног већа

Службено одсутни: проф. др Милорад Кураица
проф. др Обрадовић Братислав
Мирослав Поповић

Оправдано одсутни: проф. др Срђан Буквић
проф. др Стеван Ђениже
проф. др Владимир Милосављевић
проф. др Владан Вучковић
мр Саша Ивковић
Марјан Ћирковић
Биљана Николић

Декан Факултета проф. др Јаблан Дојчиловић отворио је седницу у 13:15 часова и предложио следећи

Д н е в н и р е д

1. Усвајање Записника са X седнице Изборног и Наставно-научног већа.
2. Избор 11 чланова Савета Физичког факултета из редова наставника за мандатни период 2015-2018 година
3. Одређивање Комисије за оцену испуњености услова и оправданост предложене теме за израду докторске дисертације за:
   a) БЛАНКУ ШКИПИНА, дипломираног физичара, која је пријавила докторску дисертацију под називом: „ПОВРШИНСКЕ ФОТОДИЕЛЕКТРИЧНЕ ОСОБИНЕ ПОЛИМЕРНИХ КОМПОЗИТА“
4. Усвајање Извештаја Комисије за оцену испуњености услова и оправданост предложене теме за израду докторске дисертације и одређивање ментора за:
   a) МИЛОША ДРАЖИЋА, дипломираног физичара, који је пријавио докторску дисертацију под називом: „ТЕОРИЈА ЕЛЕКТРОНСКОГ ТРАНСПОРТА КРОЗ КВАНТНЕ ТАЧКЕ И МОЛЕКУЛЕ“
5. Усвајање пријављене теме за израду мастер рада, одређивање руководиоца и Комисије за одбрану рада за:
   a) МИЛИВОЈА ЈОЈИЋА, студента мастер студија физике, смер Теоријска и експериментална физика, који је пријавио мастер рад под називом: „Т-ДУАЛИЗАЦИЈА НА ТОРУСУ ПРЕКО КОМПЛЕКСНИХ ПАРАМЕТАРА“
6. Усвајање пријављене теме за израду дипломског рада, одређивање руководиоца и Комисије за одбрану рада за:
   a) ИВАНА ГОРГИЕВСКОГ, апсолвента метеорологије, који је пријавио дипломски рад под називом: „КАРАКТЕРИСТИКА ЛЕТЊИХ И ЗИМСКИХ ОЛУЈА“
7. Разматрање захтева Хемијског факултета у вези са давањем сагласности на ангажовање наставника и сарадника Физичког факултета за школску 2015/2016 годину и то:
   a) проф. др Лазара Лазића за предмет Метеорологија и Моделовање загађења у атмосфери (на основним студијама Хемија животне средине)
   b) проф. др Илије Марића за предмет Философија природних наука (на интегрисаним основним и мастер студијама Настава хемије)
   c) проф. др Душана Поповића за предмет Физика (на интегрисаним основним и мастер студијама Настава хемије)
   d) доц. др Саве Галијаша за предмет Физика (на основним студијама Хемија и Хемија животне средине и интегрисаним основним и мастер студијама Настава хемије)
   e) проф. др Владимира Милосављевића за предмет Одабрана поглавља физике (на основним студијама Хемија и Хемија животне средине и интегрисаним основним и мастер студијама Настава хемије)и Основи физике (на основним студијама Биохемије)

   f) доц. др Славице Малетић за предмет Основи физике (на основним студијама Биохемије) и Физика (на основним студијама Хемија и Хемија животне средине и интегрисаним основним и мастер студијама Настава хемије)
   g) доц. др Зорице Поповић за предмет Физика (на основним студијама Хемија и Хемија животне средине и интегрисаним основним и мастер студијама Настава хемије)
   h) проф. др Братислава Обрадовића за предмет Унапређени оксидациони процеси (на докторским студијама)
8. Разматрање захтева Биолошког факултета у вези са давањем сагласности на ангажовање проф. др Илије Марића за извођење наставе из предмета Философија природних наука на основним академским студијама Биологија.
9. Избор шефова смерова основних и мастер студија.
10. Питања наставе, науке и финансија.
11. Захтеви за одобрење одсуства.
12. Усвајање извештаја са службених путовања.
13. Дописи и молбе упућене Наставно-научном већу.
14. Обавештења. Текућа питања. Питања и предлози.

Пошто је усвојен предложени Дневни ред, прешло се на

1. <u>тачку</u>

Усвојена је примедба на Записник са претходне седнице коју је изнео проф. др Јован Пузовић. Примедба се односила на тачку 11, став 4 у који треба додати реч „кандидат“, те треба да гласи „кандидат за редовног професора мора имати педагошко искуство у високошколској установи“ (уместо „редовни професор мора имати педагошко искуство у високошколској установи“).

2. <u>тачка</u>

Поводом избора 11 чланова Савета Физичког факултета из редова наставника за мандатни период 2015-2018 година, изабрана је Изборна комисија у саставу:
- проф. др Дејан Јанц
- доц. др Зоран Поповић
- Весна Ковачевић

Савет Физичког факултета из редова запослених на Факултету ће чинити још 2 представника административно-техничког особља и 2 представника истраживача. Они ће своје представнике изабрати на посебним скуповима.

Након што су чланови Већа дали своје предлоге, утврђена је листа кандидата за чланове Савета и то:

1. Буквић Срђан
2. Бурић Маја
3. Вељовић Катарина
4. Вићић Милош
5. Вујовић Драгана
6. Вуковић Татјана
7. Вучковић Владан
8. Димитријевић-Ћирић Марија

9. Дмитровић Саша
10. Дојчиновић Иван
11. Жекић Андријана
12. Касалица Бећко
13. Милошевић Иванка
14. Митровић Мићо
15. Обрадовић Братислав
16. Попарић Горан
17. Поповић Душан
18. Поповић Зорица

Након тога се приступило тајном гласању. Гласало је укупно 47 чланова Већа, чиме је задовољен услов за двотрећинском већином. Комисија је пребројала гласове, те је декан прочитао списак 11 кандидата који су добили највише гласова и који ће бити чланови Савета Физичког факултета за мандатни период 2015-2018 година:

1. Вељовић Катарина
2. Вујовић Драгана
3. Попарић Горан
4. Поповић Зорица
5. Поповић Душан
6. Вучковић Владан
7. Бурић Маја
8. Дојчиновић Иван
9. Жекић Андријана
10. Касалица Бећко
11. Митровић Мићо

3. <u>тачка</u>

Одређена је Комисија за оцену испуњености услова и оправданост предложене теме за израду докторске дисертације за:

а) БЛАНКУ ШКИПИНА, дипломираног физичара, која је пријавила докторску дисертацију под називом: „ПОВРШИНСКЕ ФОТОДИЕЛЕКТРИЧНЕ ОСОБИНЕ ПОЛИМЕРНИХ КОМПОЗИТА“
Комисија: *др Душко Дудић, научни сарадник ИНН Винча*
др Јаблан Дојчиловић, редовни професор ФФ
др Душан Поповић, ванредни професор ФФ
др Драгана Церовић, научни сарадник Висока текстилна струк. школа

4. <u>тачка</u>

Усвојен је Извештај Комисије за оцену испуњености услова и оправданост предложене теме за израду докторске дисертације и одређен ментор за:

a) МИЛОША ДРАЖИЋА, дипломираног физичара, који је пријавио докторску дисертацију под називом: „ТЕОРИЈА ЕЛЕКТРОНСКОГ ТРАНСПОРТА КРОЗ КВАНТНЕ ТАЧКЕ И МОЛЕКУЛЕ“
Ментор: *др Виктор Церовски, виши научни сарадник ИФ*

5. тачка

Усвојена је пријављена тема за израду мастер рада, одређен руководилац и Комисија за одбрану рада за:

a) МИЛИВОЈА ЈОЈИЋА, студента мастер студија физике, смер Теоријска и експериментална физика, који је пријавио мастер рад под називом: „Т-ДУАЛИЗАЦИЈА НА ТОРУСУ ПРЕКО КОМПЛЕКСНИХ ПАРАМЕТАРА“
Комисија: *др Бојан Николић, научни сарадник ИФ, руководилац рада*
др Воја Радовановић, редовни професор ФФ
др Маја Бурић, редовни професор ФФ
Драгољуб Гочанин, истраживач

6. тачка

Усвојена је пријављена тема за израду дипломског рада, одређен руководилац и Комисија за одбрану рада за:

a) ИВАНА ГОРГИЕВСКОГ, апсолвента метеорологије, који је пријавио дипломски рад под називом: „КАРАКТЕРИСТИКА ЛЕТЊИХ И ЗИМСКИХ ОЛУЈА“
Комисија: *др Млађен Ћурић, редовни професор ФФ, руководилац рада*
др Дејан Јанц, ванредни професор ФФ
др Катарина Вељовић, доцент ФФ

7. тачка

На захтев Хемијског факултета Наставно-научно веће је ДАЛО САГЛАСНСТ на ангажовање наставника и сарадника Физичког факултета за школску 2015/2016 годину и то:

a) проф. др Лазара Лазића за предмет Метеорологија и Моделовање загађења у атмосфери (на основним студијама Хемија животне средине)
b) проф. др Илије Марића за предмет Философија природних наука (на интегрисаним основним и мастер студијама Настава хемије)
c) проф. др Душана Поповића за предмет Физика (на интегрисаним основним и мастер студијама Настава хемије)
d) доц. др Саве Галијаша за предмет Физика (на основним студијама Хемија и Хемија животне средине и интегрисаним основним и мастер студијама Настава хемије)
e) проф. др Владимира Милосављевића за предмет Одабрана поглавља физике (на основним студијама Хемија и Хемија животне средине и интегрисаним основним и мастер студијама Настава хемије)и Основи физике (на основним студијама Биохемије)

f) доц. др Славице Малетић за предмет Основи физике (на основним студијама Биохемије) и Физика (на основним студијама Хемија и Хемија животне средине и интегрисаним основним и мастер студијама Настава хемије)
g) доц. др Зорице Поповић за предмет Физика (на основним студијама Хемија и Хемија животне средине и интегрисаним основним и мастер студијама Настава хемије)
h) проф. др Братислава Обрадовића за предмет Унапређени оксидациони процеси (на докторским студијама)

8. <u>тачка</u>

На захтев Биолошког факултета Наставно-научно веће је ДАЛО САГЛАСНОСТ на ангажовање проф. др Илије Марића за извођење наставе из предмета Философија природних наука на основним академским студијама Биологија.

9. <u>тачка</u>

Наставно-научно веће је за руководиоце смерова основних и мастер студија именовало:
- проф. др Јаблана Дојчиловића за руководиоца смера Општа физика
- проф. др Воју Радовановића за руководиоца смера Теоријска и експериментална физика
- проф. др Ивана Белчу за руководиоца смера Примењена и компјутерска физика
- проф. др Лазара Лазића за руководиоца смера Метеорологија

10. <u>тачка</u>

Наставно-научно веће разматрало је списак кандидата пријављених на конкурс за ангажовање студената докторских и мастер студија за извођење дела наставе на основним студијама. Продекан за наставу проф. др Иван Дојчиновић предочио је члановима Већа списак пријављених кандидата од којих је један број кандидата предложио и предметни наставник за извођење наставе из свог предмета, док је један број пријављених кандидат нераспоређен по предметима. Према условима конкурса, потребно је да кандидат има просечну оцену на основним студијама најмање 8.5 и да дужина његових студија не прелази шест година. По овом питању развила се дужа дискусија, с обзиром да четири кандидата које су предложили предметни наставници не испуњавају услове конкурса. Након расправе је донета одлука да ти кандидати не могу бити ангажовани, те је Наставно-научно веће усвојило следећи списак кандидата и њихово ангажовање по предметима:

Р.б.	Име и презиме	Предмет	Година уписа	Година дипломирања	Просечна оцена и смер
1.	Марко Миливојевић	- Механика - Квантна механика I и II - Симетрије у физици	2008.	2012.	9,63 Б
2.	Нора Тркља	- Физика атома - Примењена спектроскопија	2008.	2012.	9,37 Б

		- Лабораторија физике I и II			
3.	Горан Сретеновић	- Лабораторији физике 3 и 4 - Физика (за хемичаре)	2002.	2006.	9,15 А
4.	Милош Скочић	- Лабораторија физике 3 и 4 - Физика јонизованих гасова - Обрада резултата мерења	2005.	2010.	8,53 Ц
5.	Милош Бургер	- Увод у информационе системе	2004.	2010.	9,00 Ц
6.	Бранислава Мисаиловић	- Физика у школи 1 - Физика у школи 2	2005.	2010.	8,71 Ц
7.	Јелена Пајовић	- Физика чврстог стања А, Б, Ц - Лабораторија физике I и II	2007.	2011.	9,85 Б
8.	Биљана Радиша	- Савремена физика I - Методика наставе (ФФХ)	2008.	2012.	9,60 А
9.	Светислав Мијатовић	- Лабораторија физике I и II - Физичка механика - Молекуларна физика и ТД	2009.	2013.	10,00 Б
10.	Петар Бокан	- Лабораторија физике I и II	2008.	2013.	8,98 Б
11.	Драгољуб Гочанин	- Статистичка физика I и II	2009.	2013.	9,84 Б
12.	Вељко Јанковић	- Квантна статистичка физика	2009.	2013.	9,97 Б
13.	Срђан Ставрић	- Теорија кондензованог стања	2009.	2013.	9,92 Б
14.	Драгутин Јовковић	- Лабораторија физике I и II	2009.	2014.	9,13 Б
15.	Стефан Мијин	- Теоријска физика плазме	2011.	2015.	9,97 Б
17.	Катарина Милетић	- Лабораторија физике I и II	2010.	2014.	9,52 Ц
18.	Илија Иванишевић	- Методи математичке физике - Основи математичке физике	2010.	2014.	9,66 Б
19.	Данило Николић	- Основи електродинамике	2011.	2015.	10,00 Б
20.	Ненад Тадић	- Електроника	2006.	2011.	9,07 Ц
21.	Никола Коњик	- Електродинамика I и II - Теорија елементарних честица	2009.	2013.	9,66 Б
22.	Немања Ковачевић	- Метеоролошка мерења	2000.	2005.	9,31 М
23.	Сузана Путниковић	- Општа метеорологија 1 - Климатологија	2006.	2010.	9,73 М
26.	Владимир Чубровић	- Физика (за хемичаре)	2004.	2010.	9,28 Ц

Нераспоређени кандидати:

Р.б.	Име и презиме	Предмет	Година уписа	Година дипломирања	Просечна оцена
1.	Александра Димић		2010.	2014.	10,00 Б
2.	Виолета Станковић		2009.	2014.	8,69 Ц
3.	Милош Вујовић		2008.	2014.	8,62 М
4.	Ана Ђулаковић		2010.	2013.	8,79 А
5.	Ивана Дугалић		2011.	2014.	9,59 А
6.	Марија Јанковић		2010.	2015.	9,95 Б
7.	Јелена Костић		2011.	2014.	9,19 А
8.	Вукашин Милошевић		2011.	2015.	10,00 Б
9.	Јана Петровић		2011.	2014.	9,42 А
10.	Јелена Репић		2011.	2014.	9,19 А
11.	Инес Скоко		2010.	2014.	9,57 М
12.	Весна Стојанац			2012.	8,84 А

Кандидати чије ангажовање није одобрено:

Р.б.	Име и презиме	Предмет	Година уписа	Година дипломирања	Просечна оцена и смер
1.	Милица Васиљевић	- Лабораторија физике 3 и 4	2008.	2014.	8,21 Нови Сад
2.	Иван Крстић	- Физика (за хемичаре)	2003.	2010.	9,11 Ц
3.	Иван Петронијевић	- Физика (за хемичаре)	2001.	2010.	8,06 Ц
4.	Филип Маринковић	- Физика (за хемичаре)	2004.	2010.	8,00 Ц

Након одлуке о кандидатима који ће бити ангажовани у делу наставе на основним студијама, предметни наставник на курсу Физика за студенте хемије, проф. др Душан Поповић, обавестио је Наставно-научно веће да више не жели да изводи наставу на том предмету. Декан је професора Поповића упутио да своју намеру у писменој форми достави Катедри за општи курс физике на првој години студија.

Продекан за наставу проф. др Иван Дојчиновић обавестио је чланове Већа да је на I годину основних студија школске 2015/2016 године уписано 169 студената. У оквиру уписне квоте је уписано 165 студената чиме су попуњена сва места, а 4 студента мимо квоте су уписана по другим основама. Проф. др Иван Дојчиновић је затим подсетио чланове Већа, да је, после 6 година, њему ово последњи дан на месту продекана за наставу, а од нове школске године 1. октобра дужност продекана преузима доц. др Славица Малетић. Наставно-научно веће се проф. др Ивану Дојчиновићу захвалило аплаузом.

11. тачка

Наставно-научно веће је одобрило плаћено одсуство проф. др Душану Поповићу у периоду од 6. до 10. новембра 2015. године ради учешћа на конференцији „6th International Conference on Nanotechnology“ која се одржава у Риму (Италија).

Наставно-научно веће НИЈЕ ОДОБРИЛО продужетак неплаћеног одсуства Мирославу Поповићу, који се се од јануара 2013. године налази на усавршавању на Калифорнијском универзитету у Берклију (САД).

Седница је завршена у 13:30 часова.

Београд, 15.10.2015.

ДЕКАН ФИЗИЧКОГ ФАКУЛТЕТА

Проф. др Јаблан Дојчиловић

З А П И С Н И К

са X седнице Изборног и Наставно-научног већа одржане у среду 11. септембра 2019. године

Седници присуствује 41 члан Изборног и Наставно-научног већа.

Службено одсутни:	проф. др Предраг Миленовић проф. др Андријана Жекић проф. др Марија Димитријевић-Ћирић Марјан Ћирковић
Оправдано одсутни:	проф. др Ђорђе Спасојевић проф. др Зоран Борјан доц. др Сава Галијаш проф. др Маја Бурић доц. др Драган Реџић др Биљана Николић
Неоправдано одсутни:	проф. др Милорад Кураица проф. др Мићо Митровић проф. др Драгана Вујовић проф. др Владан Вучковић проф. др Илија Марић доц. др Саша Ивковић доц. др Владимир Миљковић Филип Маринковић Марко Миливојевић

Декан Факултета проф. др Иван Белча отворио је седницу у 11:15 часова и предложио следећи

Д н е в н и   р е д

1. Усвајање Записника са IX седнице Изборног и Наставно-научног већа Физичког факултета.

<u>Изборно веће</u>

2. Разматрање предлога Катедре за физику језгра и честица о покретању поступка за избор једног доцента за ужу научну област Физика честица и поља.
3. Усвајање Извештаја Комисије за избор наставника Физичког факултета и то:
   a) једног редовног професора за ужу научну област Примењена физика
   b) једног ванредног професора за ужу научну област Физика облака
4. Усвајање Извештаја Комисије за избор у научно звање и то:
   a) др ТОМИСЛАВЕ ВУКИЋЕВИЋ у звање научни саветник,
   b) др АЛЕКСАНДРА ЋИРИЋА у звање научни сарадник.

<u>Наставно-научно веће</u>

5. Одређивање Комисије за преглед и оцену докторске дисертације за:
   a) АЛЕКСАНДРУ ДИМИЋ, дипломираног физичара, која је предала докторску дисертацију под називом: "DETECTION OF QUANTUM CORRELATIONS" (Детекција квантних корелација)
   b) ДРАГОЉУБА ГОЧАНИНА, дипломираног физичара, који је предао докторску дисертацију под називом: "FIELD THEORY IN SO(2,3)$_*$ MODEL OF NONCOMMUTATIVE GRAVITY" (Теорија поља у SO(2,3)$_*$ моделу некомутативне гравитације)

   c) СРЂАНА СТАВРИЋА, дипломираног физичара, који је предао докторску дисертацију под називом: "FIRST-PRINCIPLES STUDY OF THE STRUCTURAL AND ELECTRONIC PROPERTIES OF METALS ADSORBED ON TWO-DIMENSIONAL MATERIALS" (Ab initio истраживање структурних и електронских особина метала адсорбованих на дводимензионалним материјалима)
   d) ДУШАНА ВУДРАГОВИЋА, дипломираног физичара, који је предао докторску дисертацију под називом: " FARADAY WAVES IN ULTRACOLD DIPOLAR BOSE GASES" (Фарадејеви таласи у ултрахладним диполним Бозе гасовима)

6. Усвајање извештаја Комисије за преглед и оцену докторске дисертације и одређивање Комисије за одбрану дисертације за:
   a) ВЛАДИМИРА ВЕЉИЋА, дипломираног физичара, који је предао докторску дисертацију под називом: „QUANTUM KINETIC THEORY FOR ULTRACOLD DIPOLAR FERMI GASES“ (наслов на српском језику: „КВАНТНА КИНЕТИЧКА ТЕОРИЈА ЗА УЛТРАХЛАДНЕ ДИПОЛНЕ ФЕРМИ ГАСОВЕ“)
   b) МАРКА МИЛИВОЈЕВИЋА, дипломираног физичара, који је предао докторску дисертацију под називом: „SPIN-ORBIT INTERACTION IN LOW DIMENSIONAL SYSTEMS: SYMMETRY BASED APPROACH” (наслов на српском језику: „СПИН-ОРБИТ ИНТЕРАКЦИЈА У НИСКОДИМЕНЗИОНАЛНИМ СИСТЕМИМА: СИМЕТРИЈСКИ ПРИСТУП)
   c) НИКОЛУ КОЊИКА, дипломираног физичара, који је предао докторску дисертацију под називом: „ФЕНОМЕНОЛОГИЈА НЕКОМУТАТИВНОГ СКАЛАРНОГ ПОЉА У РАВНОМ И ЗАКРИВЉЕНОМ ПРОСТОРУ“
   d) ИВАНЕ ВУКАШИНОВИЋ, дипломираног физичара, која је предала докторску дисертацију под називом: "ЗАВИСНОСТ ДИСТРИБУЦИЈЕ РАДИОНУКЛИДА U- И Th- СЕРИЈЕ ^{40}K И ^{137}Cs ОД ФИЗИЧКО-ХЕМИЈСКИХ ОСОБИНА ЗЕМЉИШТА У СИСТЕМУ ЗЕМЉИШТЕ-БИЉКА"

7. Усвајање пријављене теме за израду дипломског рада, одређивање руководиоца и Комисије за одбрану рада за:
   a) ЈЕЛЕНУ ТАСИЋ, апсолвента физика, смера Општа физика, која је пријавила дипломски рад под називом: „КОМПАРАТИВНЕ МЕТОДЕ МОДЕЛИРАЊА СЛОБОДНОГ ПАДА И ВУЧНЕ СИЛЕ – ЕКСПЕРИМЕНТИ У ФИЗИЦИ“

8. Усвајање пријављене теме за израду мастер рада, одређивање руководиоца и Комисије за одбрану рада за:
   a) ДАНИЛА РАКОЊЦА, студента мастер студија смера Теоријска и експериментална физика, који је пријавио мастер рад под називом: „ТРОДИМЕНЗИОНА ПОЕНКАРЕОВА ГРАДИЈЕНТНА ТЕОРИЈА: НАРУШЕЊЕ ПАРНОСТИ У *ADS* СЕКТОРУ“
   b) ПЕТРА МИТРИЋА, студента мастер студија смера Теоријска и експериментална физика, који је пријавио мастер рад под називом: „КАНОНСКА СТРУКТУРА ТЕЛЕПАРАЛЕЛНЕ ФОРМУЛАЦИЈЕ ОПШТЕ ТЕОРИЈЕ РЕЛАТИВНОСТИ“
   c) ВЕЉКА МАКСИМОВИЋА, студента мастер студија смера Теоријска и експериментална физика, који је пријавио мастер рад под називом: „МОГУЋНОСТИ ЗА УНАПРЕЂЕЊЕ МЕРЕЊА МАСЕ *W* БОЗОНА НА ЕКСПЕРИМЕНТУ АТЛАС”
   d) СУЗАНА МИЛАДИЋ, студента мастер студија смера Теоријска и експериментална физика, која је пријавила мастер рад под називом: „СПИНСКА РЕЗОНАНЦА И РЕЛАКСАЦИЈА У КВАНТНИМ ТАЧКАМА УНУТАР *InSb* НАНОЖИЦА“
   e) НЕМАЊЕ СИМОВИЋА, студента мастер студија смера Теоријска и експериментална физика, који је пријавио мастер рад под називом: „НЕКОМУТАТИВНОСТ ОТВОРЕНЕ БОЗОНСКЕ СТРУНЕ У ПРИСУСТВУ КОНСТАНТНИХ ПОЗАДИНСКИХ ПОЉА“
   f) ЛУКУ РАЈАЧИЋА, студента мастер студија смера Теоријска и експериментална физика, који је пријавио мастер рад под називом: "ИНТЕРАКТИВНИ КИНЕТИЧКИ МОДЕЛ ПРИКАТОДНЕ ОБЛАСТИ АБНОРМАЛНОГ ТИЊАВОГ ПРАЖЊЕЊА У НЕОНУ СА РАВНОМ КАТОДОМ"
   g) ЛУКУ БЛАГОЈЕВИЋА, студента мастер студија смера Теоријска и експериментална физика, који је пријавио мастер рад под називом: "PRODUCTION AND DISTRIBUTION OF KNOWLEDGE: AN AGENT BASED MODEL" (Продукција и дистрибуција знања: модел заснован на агентима)
   h) МИЛИЦУ ЂЕКИЋ, студента мастер студија физике, смер Општа физика, која је пријавила мастер рад под називом: "ПРОУЧАВАЊЕ УТИЦАЈА БОМБАРДОВАЊА ЈОНИМА N^{5+} НА СПЕКТРАЛНЕ КАРАКТЕРИСТИКЕ ПОЛИМЕРА ПОЛИЕТИЛЕНТЕРЕФТАЛАТА"
   i) ЈЕЛЕНУ ПЕТРОВИЋ, студента мастер студија метеорологије, које је пријавила мастер рад под називом: "ПРОГНОЗА ИНДЕКСА ЗАЛЕЂИВАЊА ВАЗДУХОПЛОВА У ЛЕТУ"

9. Усвајање извештаја о рецензији рукописа "Даљинска мерења у метеорологији", аутора др Немање Ковачевића.
10. Питања наставе, науке и финансија.
11. Захтеви за одобрење одсуства.
12. Усвајање извештаја са службених путовања.
13. Дописи и молбе упућене Наставно-научном већу.
14. Обавештења. Текућа питања. Питања и предлози.

Пошто је усвојен предложени Дневни ред, прешло се на

1. тачку

Усвојен је Записник са IX седнице Изборног и Наставно-научног већа Физичког факултета.

Изборно веће

2. тачка

Изборно веће је подржало предлог Катедре за физику језгра и честица и донело одлуку о расписивању конкурса за избор једног доцента за ужу научну област Физика честица и поља.

Комисија: др Маја Бурић, редовни професор ФФ
др Воја Радовановић, редовни професор ФФ
др Бранислав Цветковић, научни саветник ИФ

3. тачка

Усвојен је Извештај Комисије и донета одлука о избору наставника Физичког факултета и то:

a) након тајног гласања у коме су учествовали редовни професори Факултета, са 15 гласова ЗА и 2 УЗДРЖАНА, утврђен је предлог за избор др МИЛОША ВИЋИЋА у звање редовног професора за ужу научну област Примењена физика
b) након гласања у коме су учествовали редовни и ванредни професори Факултета, једногласно, са 27 гласова ЗА утврђен је предлог за избор др ВЛАДАНА ВУЧКОВИЋА у звање ванредног професора за ужу научну област Физика облака

4. тачка

Усвојен је Извештај Комисије за избор у научно звање и то:

a) након гласања у коме су учествовали редовни професори Физичког факултета, једногласно је, са 17 гласова ЗА (од укупно 21 колико чини изборно тело) утврђен је предлог за избор др ТОМИСЛАВЕ ВУКИЋЕВИЋ у звање научни саветник
b) након гласања у коме су учествовали редовни професори, ванредни професори и доценти, Изборно веће је једногласно, са 36 гласова ЗА (од укупно 52 колико чини

изборно тело), утврђен предлог за избор др АЛЕКСАНДРА ЋИРИЋА у звање научни сарадник.

5. тачка

Одређена је Комисија за преглед и оцену докторске дисертације за:

a) АЛЕКСАНДРУ ДИМИЋ, дипломираног физичара, која је предала докторску дисертацију под називом: "DETECTION OF QUANTUM CORRELATIONS" (Детекција квантних корелација)
Комисија: др Боривоје Дакић, доцент ФФ Универзитета у Бечу
др Иванка Милошевић, редовни професор ФФ
др Татјана Вуковић, ванредни професор ФФ
др Антун Балаж, научни саветник ИФ
др Милан Дамњановић, редовни професор ФФ

b) ДРАГОЉУБА ГОЧАНИНА, дипломираног физичара, који је предао докторску дисертацију под називом: "FIELD THEORY IN SO(2,3)$_*$ MODEL OF NONCOMMUTATIVE GRAVITY" (Теорија поља у SO(2,3)$_*$ моделу некомутативне гравитације)
Комисија: др Воја Радовановић, редовни професор ФФ
др Марија Димитријевић-Ћирић, ванредни професор ФФ
др Бранислав Цветковић, научни саветник ИФ

c) СРЂАНА СТАВРИЋА, дипломираног физичара, који је предао докторску дисертацију под називом: "FIRST-PRINCIPLES STUDY OF THE STRUCTURAL AND ELECTRONIC PROPERTIES OF METALS ADSORBED ON TWO-DIMENSIONAL MATERIALS" (Ab initio истраживање структурних и електронских особина метала адсорбованих на дводимензионалним материјалима)
Комисија: др Жељко Шљиванчанин, научни саветник ИНН Винча
др Татјана Вуковић, ванредни професор ФФ
др Иванка Милошевић, редовни професор ФФ

d) ДУШАНА ВУДРАГОВИЋА, дипломираног физичара, који је предао докторску дисертацију под називом: " FARADAY WAVES IN ULTRACOLD DIPOLAR BOSE GASES" (Фарадејеви таласи у ултрахладним диполним Бозе гасовима)
Комисија: др Антун Балаж, научни саветник ИФ
др Ивана Васић, виши научни сарадник ИФ
др Милан Дамњановић, редовни професор ФФ
др Милан Кнежевић, редовни професор ФФ

5. тачка

Усвојен је извештај Комисије за преглед и оцену докторске дисертације и одређена Комисија за одбрану дисертације за:

a) ВЛАДИМИРА ВЕЉИЋА, дипломираног физичара, који је предао докторску дисертацију под називом: „QUANTUM KINETIC THEORY FOR ULTRACOLD DIPOLAR FERMI GASES“ (наслов на српском језику: „КВАНТНА КИНЕТИЧКА ТЕОРИЈА ЗА УЛТРАХЛАДНЕ ДИПОЛНЕ ФЕРМИ ГАСОВЕ“)
Комисија: др Антун Балаж, научни саветник ИФ
др Ивана Васић, виши научни сарадник
др Милан Дамњановић, редовни професор ФФ
др Милан Кнежевић, редовни професор ФФ

b) МАРКА МИЛИВОЈЕВИЋА, дипломираног физичара, који је предао докторску дисертацију под називом: „SPIN-ORBIT INTERACTION IN LOW DIMENSIONAL SYSTEMS: SYMMETRY BASED APPROACH” (наслов на српском језику: „СПИН-ОРБИТ ИНТЕРАКЦИЈА У НИСКОДИМЕНЗИОНАЛНИМ СИСТЕМИМА: СИМЕТРИЈСКИ ПРИСТУП)
Комисија: др Татјана Вуковић, ванредни професор ФФ
др Милан Дамњановић, редовни професор ФФ
др Владимир Дамљановић, виши научни сарадник ИФ
др Саша Дмитровић, доцент ФФ
др Наташа Лазић, научни сарадник ФФ

c) НИКОЛУ КОЊИКА, дипломираног физичара, који је предао докторску дисертацију под називом: „ФЕНОМЕНОЛОГИЈА НЕКОМУТАТИВНОГ СКАЛАРНОГ ПОЉА У РАВНОМ И ЗАКРИВЉЕНОМ ПРОСТОРУ“
Комисија: др Воја Радовановић, редовни професор ФФ
др Марија Димитријевић-Ћирић, ванредни професор ФФ
др Анђело Самсаров, научни сарадник Инст. Р. Бошковић Загреб
др Душко Латас, доцент ФФ
др Бранислав Цветковић, научни саветник ИФ

d) ИВАНУ ВУКАШИНОВИЋ, дипломираног физичара, која је предала докторску дисертацију под називом: "ЗАВИСНОСТ ДИСТРИБУЦИЈЕ РАДИОНУКЛИДА U- И Th-СЕРИЈЕ ^{40}K И ^{137}Cs ОД ФИЗИЧКО-ХЕМИЈСКИХ ОСОБИНА ЗЕМЉИШТА У СИСТЕМУ ЗЕМЉИШТЕ-БИЉКА"
Комсија: др Драгана Тодоровић, научни саветник ИНН Винча
др Јован Пузовић, редовни професор ФФ
др Милош Вићић, ванредни професор ФФ

7. <u>тачка</u>

Усвојена је пријављена тема за израду дипломског рада, одређен руководилац и Комисија за одбрану рада за:

a) ЈЕЛЕНУ ТАСИЋ, апсолвента физика, смера Општа физика, која је пријавила дипломски рад под називом: „КОМПАРАТИВНЕ МЕТОДЕ МОДЕЛИРАЊА СЛОБОДНОГ ПАДА И ВУЧНЕ СИЛЕ – ЕКСПЕРИМЕНТИ У ФИЗИЦИ“

Комисија: др Мићо Митровић, редовни професор ФФ, руководилац рада
др Андријана Жекић, ванредни професор ФФ
др Бранислава Мисаиловић, научни сарадник ФФ

8. тачка

Усвајена је пријављена тема за израду мастер рада, одређен руководилац и Комисија за одбрану рада за:

a) ДАНИЛА РАКОЊЦА, студента мастер студија смера Теоријска и експериментална физика, који је пријавио мастер рад под називом: „ТРОДИМЕНЗИОНА ПОЕНКАРЕОВА ГРАДИЈЕНТНА ТЕОРИЈА: НАРУШЕЊЕ ПАРНОСТИ У *ADS* СЕКТОРУ“
Комисија: др Бранислав Цветковић, научни саветник ИФ, руководилац рада
др Марија Димитријевић-Ћирић, ванредни професор ФФ
др Душко Латас, доцент ФФ

b) ПЕТРА МИТРИЋА, студента мастер студија смера Теоријска и експериментална физика, који је пријавио мастер рад под називом: „КАНОНСКА СТРУКТУРА ТЕЛЕПАРАЛЕЛНЕ ФОРМУЛАЦИЈЕ ОПШТЕ ТЕОРИЈЕ РЕЛАТИВНОСТИ“
Комисија: др Бранислав Цветковић, научни саветник ИФ, руководилац рада
др Душко Латас, доцент ФФ
др Воја Радовановић, редовни професор ФФ

c) ВЕЉКА МАКСИМОВИЋА, студента мастер студија смера Теоријска и експериментална физика, који је пријавио мастер рад под називом: „МОГУЋНОСТИ ЗА УНАПРЕЂЕЊЕ МЕРЕЊА МАСЕ *W* БОЗОНА НА ЕКСПЕРИМЕНТУ АТЛАС”
Комисија: др Ненад Врањеш, виши научни сарадник ИФ, руководилац рада
др Петар Аџић, редовни професор ФФ у пензији
др Душко Латас, доцент ФФ

d) СУЗАНА МИЛАДИЋ, студента мастер студија смера Теоријска и експериментална физика, која је пријавила мастер рад под називом: „СПИНСКА РЕЗОНАНЦА И РЕЛАКСАЦИЈА У КВАНТНИМ ТАЧКАМА УНУТАР *InSb* НАНОЖИЦА“
Комисија: др Едиб Добарџић, ванредни професор ФФ, руководилац рада
др Зоран Поповић, доцент ФФ
Марко Миливојевић, истраживач сарадник ФФ

e) НЕМАЊЕ СИМОВИЋА, студента мастер студија смера Теоријска и експериментална физика, који је пријавио мастер рад под називом: „НЕКОМУТАТИВНОСТ ОТВОРЕНЕ БОЗОНСКЕ СТРУНЕ У ПРИСУСТВУ КОНСТАНТНИХ ПОЗАДИНСКИХ ПОЉА“
Комисија: др Бојан Николић, виши научни сарадник ИФ, руководилац рада
др Воја Радовановић, редовни професор ФФ
др Марија Димитријевић-Ћирић, ванредни професор ФФ

f) ЛУКУ РАЈАЧИЋА, студента мастер студија смера Теоријска и експериментална физика, који је пријавио мастер рад под називом: "ИНТЕРАКТИВНИ КИНЕТИЧКИ МОДЕЛ ПРИКАТОДНЕ ОБЛАСТИ АБНОРМАЛНОГ ТИЊАВОГ ПРАЖЊЕЊА У НЕОНУ СА РАВНОМ КАТОДОМ"

Комисија: др Ђорђе Спасојевић, редовни професор ФФ, руководилац рада
др Срђан Буквић, редовни професор ФФ
др Никола Ивановић, доцент Пољопривредног факулета

g) ЛУКУ БЛАГОЈЕВИЋА, студента мастер студија смера Теоријска и експериментална физика, који је пријавио мастер рад под називом: "PRODUCTION AND DISTRIBUTION OF KNOWLEDGE: AN AGENT BASED MODEL" (Продукција и дистрибуција знања: модел заснован на агентима)

Комисија: др Александра Алорић, научни сарадник ИФ, руководилац рада
др Ђорђе Спасојевић, редовни професор ФФ
др Зорица Поповић, доцент ФФ

h) МИЛИЦУ ЂЕКИЋ, студента мастер студија физике, смер Општа физика, која је пријавила мастер рад под називом: "ПРОУЧАВАЊЕ УТИЦАЈА БОМБАРДОВАЊА ЈОНИМА N^{5+} НА СПЕКТРАЛНЕ КАРАКТЕРИСТИКЕ ПОЛИМЕРА ПОЛИЕТИЛЕНТЕРЕФТАЛАТА"

Комисија: др Славица Малетић, доцент ФФ, руководилац рада
др Драгана Церовић, виши научни сарадник ФФ
др Јаблан Дојчиловић, редовни професор ФФ

i) ЈЕЛЕНУ ПЕТРОВИЋ, студента мастер студија метеорологије, које је пријавила мастер рад под називом: "ПРОГНОЗА ИНДЕКСА ЗАЛЕЂИВАЊА ВАЗДУХОПЛОВА У ЛЕТУ"

Комисија: др Драгана Вујовић, ванредни професор ФФ, руководилац рада
др Лазар Лазић, редовни професор ФФ
др Владан Вучковић, ванредни професор ФФ

9. <u>тачка</u>

Усвојен је Извештај комисије о рецензији рукописа „Даљинска мерења у метеорологији", аутора др Немање Ковачевића.

<u>10. тачка</u>

Питања наставе

Продекан за наставу, доц. др Славица Малетић, обавестила је чланове Већа да настава за све године почиње у уторак 1. октобра, када ће се одржати и свечани пријем бруцоша. Да би студенти стигли да упишу наредну годину студија, сви испити се морају завршити до 20. септембра. Из истог разлога продекан је замолила наставнике да на време архивирају записнике са испита, а затим их обавезно одштампане и потписане предају судентској служби.

Продекан је члановима Већа предложила календар за наредну годину студија у којој су оба семестра скраћена на по 13 недеља. Настава је скраћена по препоруци Универзитета, а према налогу Министарства просвете, пошто се у лето 2020. године у Београду одржавају Европске летње универзитетске игре. Ради смештаја учесника Универзијаде у јулу месецу ће бити исељени студенски домови, па семестар, као и јунски и јулски испитни рок морају да се заврше раније.

После краће дискусије, са 1 гласом ПРОТИВ, 5 УЗДРЖАНИХ и 35 гласова ЗА, усвојен је календар за наредну школску годину.

Наставно-научно веће усвојило је списак студената докторских и мастер студија који су се пријавили на конкурс за ангажовање у настави.

Ради одласка у пензију проф. др Илије Марића, од идуће школске године предмет Философија природних наука укида се као изборни предмет на основним студијама. Проф. др Илија Марић биће ангажован за наставу на мастер студијама.

Доц. др Славица Малетић обавестила је чланове Већа да се у другом уписном року за упис студената у I годину основних студија на Факултет уписало још 17 студената. Сенат ће ових дана одлучивати о расписивању трећег уписног рока, с обзиром да је остало доста места за упис на више факултета.

Питања науке

На предлог продекана за науку проф. др Стевана Стојадиновића, оформљена је Комисија за упис на докторске студије и то:

- *проф. др Воја Радовановић*
- *проф. др Горан Попарић*
- *проф. др Ивана Тошић*

Поводом дописа проф. др Татјане Вуковић, доц. др Саше Дмитровића, доц. др Зорана Поповића и др Јовице Јововића, оформљена је Комисија за израду правилника о распоређивању средстава стечених на основу пројеката које ће финансирати Фонд за науку Републике Србије.

Комисија: *проф. др Стеван Стојадиновић*
проф. др Горан Попарић
проф. др Татјана Вуковић
Лелица Вуковић-Радош

13. <u>тачка</u>

Наставно-научно веће је одобрило плаћена одсуства следећим наставницима и сарадницима:

a) проф. др Мићи Митровићу у периоду од 10.09. до 12.09.2019. године, ради учествовања на Међународној конференцији Collaborative Conference on Crystal Growth (3CG 2019), у Милану, Италија.
b) др Бранислави Мисаиловић у периоду од 10.09. до 12.09.2019. године, ради учествовања на Међународној конференцији „Collaborative Conference on Crystal Growth (3CG 2019)", у Милану, Италија.
c) проф. др Владимиру Ђурђевићу

- у периоду од 16.09. до 20.09.2019. године, ради учешћа на радионици нумеричког моделирања атмосфере „SEE Network" пројекта, у Загребу, Хрватска;
- у периоду од 6.10. до 11.10.2019. године, ради учешћа на конфренцији „Interdisciplinary Endeavor in Technology: Energy Nanoscale, Environment", у Сплиту, Хрватска;
- у периоду од 13.10. до 15.10.2019. године, ради учешћа на састанку H2020 пројекта Терефика, у Паризу, Француска.

d) Николи Коњику, Драгољубу Гочанину и Александри Димић у периоду од 16.09. до 26.09.2019. године, ради учешћа на конференцији „Humboldt Kolleg Frontiers in Physics: From the Electroweak to the Planck Scales" и „Workshop on Quantum Geometry, Field Theory and Gravity", на Крфу, Грчка

e) проф. др Милораду Кураици и проф. др Братиславу Обрадовићу
- у периоду од 18.09. до 23.09. 2019. године, ради учешћа на 25th Congress of SCTM у Охриду, Северна Македонија.
- у периоду од 26.09. до 05.10.2019. године ради посете Институту за физику Националне академије Белорусије у Минску.

f) проф. др Стевану Стојадиновићу и проф. др Растку Василићу, у периоду од 20.09. до 19.10.2019. године ради одласка на службени пут у оквиру пројекта FUNCOAT из програма Хоризонт 2020, у Ахен, Немачка.

g) проф. др Ивану Виденовићу у периоду од 20.10. до 26.10.2019. године ради учешћа на састанку Regional Workshop on Nuclear Safety and Security Education, у оквиру регионалног пројекта техничке сарадње Међународне агенције за атомску енергију RER/0/043 Enhancing Capacity Building Activities in the European Nuclear and Radiation Safety Organiyations for the Safe Operation of Facilities, у Атини, Грчка.

Наставно-научно веће је условно одобрило проф. др Зорану Борјану коришћење једне слободне школске године од 1. октобра 2019. до 30. септембра 2020. године, уколико катедре на којима професор држи наставу одреде одговарајућу замену.

14. тачка

Усвојен је предлог Катедре за Општу и Динамичку метеорологију и изабран проф. др Лазар Лазић за члана Колегијума докторских студија.

15. тачка

Жребом су одређени чланови жирија за доделу Годишње награде Физичког факултета за научни рад младом истраживачу и то:

- *проф. др Јован Пузовић*
- *проф. др Мићо Митровић*
- *проф. др Иванка Милошевић*

16. <u>тачка</u>

Поводом молбе за сагласност за учешће Физичког факултета на програму мастер студија Оптика и биофотоника за биомедицину на Универзитету у Београду, развила се дискусија у којој је учествовало више чланова Већа. Приступило се и гласању, приликом чега је утврђено да је седница изгубила кворум потребан за одлучивање, те је одлука о овом питању одложена.

Седница је завршена у 13 часова.

Београд, 20.9.2019.

ДЕКАН ФИЗИЧКОГ ФАКУЛТЕТА
Проф. др Иван Белча, с.р.

Master rad

Nekomutativnost koordinata na Dp-brani u prisustvu konstantnih pozadinskih polja

Fizički fakultet Univerzitet u Beogradu

Mentor: dr Bojan Nikolić

Student: Nemanja Simović

ЗЕМУНСКА ГИМНАЗИЈА

ЗЕМУНСКА ГИМНАЗИЈА

тел/факс: 011/261-6915; 011/316-2680; адреса: Градски парк бр.1
Деловодни број: 18/62 mail:office@zemunskagimnazija.edu.rs
Датум: 01.09.2020. године web: www.zemunskagimnazija.edu.rs

На основу члана 126. а у вези члана 158. Закона о основама система образовања и васпитања („Службени гласник РС ” 88/2017, 27/2018 – др. закон, 10/2019 и 6/2020) и сагласности декана Физичког факултета Универзитета у Београду, бр. ________ од ________.20__. године,

1) Земунска гимназија, са седиштем у улици Градски парк 1 Земун (у даљем тексту: Школа), коју заступа директор школе Милош Бјелановић и

2) Бојан Николић, доктор физике, из Београда са станом у улици Валтазара Богишића бр. 4 , општина Савски венац, (у даљем тексту: Извршилац посла), закључили су:

УГОВОР О ИЗВОЂЕЊУ НАСТАВЕ

Члан 1.

Школа уступа, а Извршилац посла преузима посао који се састоји у извођењу наставе за предмет: Рачунски практикум I и Рачунски практикум II за ученике са посебним способностима за физику у Земунској гимназији у школској 2020/2021. години и то **за 4 (2+2) часа** недељно у периоду од 01.09.2020. године до краја наставне године, што укупно износи 30 % радног времена месечно.

Члан 2.

Школа има обавезу да за обављени посао исплати све порезе и доприносе у складу са законом и другим општим актима.
Школа ће износ нето накнаде за обављен посао уплатити на текући рачун извршиоца посла

Члан 3.

Извршилац послова обавезује се да послове обавља савесно и одговорно.
Извођењем послова наставе Извршилац посла не стиче својство запосленог у школи а право на накнаду за обављени рад стиче на основу извештаја о одржаним часовима наставе.
Извршилац посла учествује у раду стручних органа без права одлучивања, осим у раду одељењског већа.

Члан 4.

Извршилац посла је дужан да ступи на рад 01.09.2020. године.

Члан 5.

Уговор престаје да важи и пре истека рока на који је закључен у следећим случајевима:

- споразумом између Школе и Извршиоца посла,
- отказом уговора од стране Школе или Извршиоца посла,
- у другим случајевима утврђеним законом.

Члан 6.

Школа може отказати уговор:

- ако Извршилац посла несавесно, нестручно и неблаговремено обавља послове из овог уговора,
- ако Извршилац посла не поштује радну дисциплину.

Члан 7.

Овај уговор сачињен је у 3 (три) истоветна примерака, (1) један примерак за Извршиоца посла а (2) два за Школу.

ИЗВРШИЛАЦ ПОСЛА — ДИРЕКТОР ШКОЛЕ

Бојан Николић — Милош Бјелановић

ЗЕМУНСКА ГИМНАЗИЈА

ЗЕМУНСКА ГИМНАЗИЈА

тел/факс: 011/7455-852; 011/316-7226; адреса: Градски парк бр.1
Деловодни број: 18/47-48 mail:office@zemunskagimnazija.edu.rs
Датум: 01.09.2023. године web: www.zemunskagimnazija.edu.rs

На основу члана 126. а у вези члана 158. Закона о основама система образовања и васпитања („Службени гласник РС " 88/2017, 27/2018 – др. закон, 10/2019, 27/2018 – др. закон, 6/2020 и 129/2021) и сагласности Института за физику у Земуну, бр. ________ од ________.20____. године,

1) Земунска гимназија, са седиштем у улици Градски парк 1 Земун (у даљем тексту: Школа), коју заступа директор школе Милош Бјелановић и

2) Бојан Николић, доктор физике, из Београда са станом у улици Валтазара Богишића бр. 4 , општина Савски венац, (у даљем тексту: Извршилац посла), закључили су:

УГОВОР О ИЗВОЂЕЊУ НАСТАВЕ

Члан 1.

Школа уступа, а Извршилац посла преузима посао који се састоји у извођењу наставе за предмет: Рачунски практикум I и Рачунски практикум II за ученике са посебним способностима за физику у Земунској гимназији у школској 2023/2024. години и то **за 4 (2+2) часа** недељно у периоду од 01.09.2023. године до краја наставне године, што укупно износи 30 % радног времена месечно.

Члан 2.

Школа има обавезу да за обављени посао исплати све порезе и доприносе у складу са законом и другим општим актима.
Школа ће износ нето накнаде за обављен посао уплатити на текући рачун извршиоца посла

Члан 3.

Извршилац послова обавезује се да послове обавља савесно и одговорно.
Извођењем послова наставе Извршилац посла не стиче својство запосленог у школи а право на накнаду за обављени рад стиче на основу извештаја о одржаним часовима наставе.
Извршилац посла учествује у раду стручних органа без права одлучивања, осим у раду одељењског већа.

Члан 4.

Извршилац посла је дужан да ступи на рад 01.09.2023. године.

Члан 5.

Уговор престаје да важи и пре истека рока на који је закључен у следећим случајевима:
- споразумом између Школе и Извршиоца посла,
- отказом уговора од стране Школе или Извршиоца посла,
- у другим случајевима утврђеним законом.

Члан 6.

Школа може отказати уговор:
- ако Извршилац посла несавесно, нестручно и неблаговремено обавља послове из овог уговора,
- ако Извршилац посла не поштује радну дисциплину.

Члан 7.

Овај уговор сачињен је у 3 (три) истоветна примерака, (1) један примерак за Извршиоца посла а (2) два за Школу.

ИЗВРШИЛАЦ ПОСЛА | ДИРЕКТОР ШКОЛЕ

Бојан Николић | Милош Бјелановић

Табела 5.1 Спецификација предмета на студијском програму докторских студија

Назив предмета: Теорија струна
Наставник или наставници: Бојан Николић
Статус предмета: изборни
Број ЕСПБ: 15
Услов: Квантна теорија градијентних поља, Суперсиметрија
Циљ предмета Разумевање основа теорије струна, као теорије која претендује на уједињење свих интеракција.
Исход предмета Студенти су припремљени за истраживачки рад у овој области.
Садржај предмета *Теоријска настава* 1. Бозонска струна. Дејство. - Канонска квантизација. - Квантизација на светлосном конусу. 2. Конформна теорија поља и интеракција струна. - BRST квантизација . -Позадинска поља. -Вертексни оператори. 3. Ramond-Neveu-Schwarz струне (са суперсиметријом светске површи) 4, Green-Schwarz струне (са суперсиметријом простор-времена). 5. Т-дуалност . -Dp-бране. 6. ТипI,типII,хетеротичке теорије струна. -М-теорија. *Практична настава* Студенти решавају самостално домаће задатке уз контролу наставника
Препоручена литература **1.** K. Becker, M Becker, J. H. Schwarz, String theory and M_theory, A modern introduction, Cambridge Univerzity press, UK, 2007. **2.** M. B. Green, J. H. Schwarz, E. Witten, Superstring theory, (vol. I i II) Cambridge Univerzity press, UK, 1987. **3.** B. Zwiebach, A first course in string theory, Cambridge University press, 2004.
Број часова активне наставе	Теоријска настава: 5	Практична настава:
Методе извођења наставе Предавања, консултације,израда домаћих задатака.
Оцена знања (максимални број поена 100) **Писмени испит 40%** **Усмени испит 40 %** **Домаћи задаци 10 %** **Семинари 10 %**
Начин провере знања могу бити различити : писмени испит, усмени испит, семинар, израда домаћих задатака
*максимална дужна 1 страница А4 формата

Table 5.1 Specification of subjects in the doctoral studies study program

Name of the subject: String theory		
Teacher(s): Bojan Nikolic		
Status of the subject: optional		
Number of ЕСПБ points: 15		
Condition: Quantum theory of gauge fields, Supersymmetry		
Goal of the subject **Understanding foundations of string theory as theory which unifies all known interactions**		
Outcome of the subject **Students are prepared for research work in the domain of string theory**		
Content of the subject *Theoretical lectures* *1. Bosonic string. Action. Canonical and light-cone quantization* *2. Conformal field theory and string interactions. Background fields. Vertex operators. BRST quantization* *3. RNS superstring* *4. GS superstring* *5. T-duality. Dp-branes* *6. Type I, type II, heterotic string. M-theory* *Practical lectures* *Solving computational problems in string theory*		
Recommended literature **1.** K. Becker, M Becker, J. H. Schwarz, String theory and M_theory, A modern introduction, Cambridge Univerzity press, UK, 2007. **2.** M. B. Green, J. H. Schwarz, E. Witten, Superstring theory, (vol. I i II) Cambridge Univerzity press, UK, 1987. **3.** B. Zwiebach, A first course in string theory, Cambridge University press, 2004.		
Number of active classes	Theory: 5	Practice:

Methods of delivering lectures: written examination, oral examination, seminar, homework
Evaluation of knowledge (maximum number of points 100) **written examination 40 %, oral examination 40 %, seminar 10 %, homework 10 %**
Weays of testing the knowledge may vary: (written tests, oral exam, project presentation, seminars ets......
*maximum length 1 A4 page

УНИВЕРЗИТЕТ У БЕОГРАДУ
ИНСТИТУТ ЗА ФИЗИКУ | БЕОГРАД
ИНСТИТУТ ОД НАЦИОНАЛНОГ ЗНАЧАЈА ЗА РЕПУБЛИКУ СРБИЈУ

Прегревица 118, 11080 Земун - Београд, Република Србија
Телефон: +381 11 3713000, Факс: +381 11 3162190, www.ipb.ac.rs
ПИБ: 100105980, Матични број: 07018029, Текући рачун: 205-66984-23

УНИВЕРЗИТЕТ У БЕОГРАДУ
ИНСТИТУТ ЗА ФИЗИКУ | БЕОГРАД
ИНСТИТУТ ОД НАЦИОНАЛНОГ ЗНАЧАЈА ЗА РЕПУБЛИКУ СРБИЈУ
www.ipb.ac.rs
Број 0801-113/1
Датум 26 JAN 2024

ПОТВРДА О РУКОВОЂЕЊУ ПРОЈЕКТНИМ ЗАДАТКОМ

Овим потврђујем да је виши научни сарадник **др Бојан Николић**, ангажован у Групи за гравитацију, честице и поља Института за физику у Београду, у оквиру пројекта "Квантна гравитација преко виших гејџ теорија" (QGHG-2021) из програма ИДЕЈЕ Фонда за науку Републике Србије руководи пројектним задатком под називом "Истраживање аспеката класичне теорије гравитације" (радни пакет 1 пројекта) почев од 01.01.2023. године. Планиран завршетак projeтног задатка и целог пројекта је 31.12.2024. године. На овом пројектном задатку су ангажовани следећи истраживачи: др Бојан Николић, др Игор Салом, др Марко Војиновић, др Данијел Обрић и др Тијана Раденковић.

др Марко Војиновић
виши научни сарадник
Института за физику у Београду
и руководилац пројекта QGHG-2021 из
програма ИДЕЈЕ Фонда за науку Републике Србије

Потврда о руковођењу потпројектом

Овим потврђујем да др Бојан Николић (за кога се покреће реизбор у звање виши научни сарадник) у оквиру Групе за гравитацију, честице и поља Института за физику Универзитета у Београду, односно у оквиру пројекта ОН 171031 "*Физичке импликације модификованог простор-времена*" руководи темом "*Т-дуализација отворене и затворене (супер)струне*". На поменутом потпројекту су ангажовани истраживачи: др Бојан Николић, др Бранислав Саздовић, др Љубица Давидовић и студент Илија Иванишевић.

Др Бојан Николић је био представник пројекта ОН 171031 у Научном савету Института за физику Универзитета у Београду у периоду 2011.-2013. године, а у току целог трајања пројекта тј. од 2011. године организационо и административно води део истраживања која се обављају у Институту за физику.

Београд, 3. јул 2018.

Maja Burić

Руководилац пројекта ОН 171031

Проф. др Маја Бурић

Subject Thank you - let us know how we can improve the reviewing process
From Foundations of Physics (FOOP) <em@editorialmanager.com>
Sender <em.foop.0.646389.496db679@editorialmanager.com>
To Bojan Nicolic <bnikolic@ipb.ac.rs>
Reply-To Foundations of Physics (FOOP) <karthika.navukkarasu@springernature.com>
Date 2019-07-06 10:50

Dear Dr Nicolic,

Thank you very much for your review of manuscript FOOP-D-19-00285, "A MEETING with WIGNER".

We greatly appreciate your assistance.

With kind regards,

Fedde Benedictus
Managing Editor
Foundations of Physics

We really value your feedback! Please spend 1 minute to tell us about your experience of reviewing - click https://springernature.eu.qualtrics.com/jfe/form/SV_cNPY5OM4ZC3PkON?J=10701

Nikolić, Bojan - Author de | Bojan Nikolic - Google Ak | ORCID for Researchers - | Bojan Nikolic (0000-0001 | Bojan Nikolic (0000-0001

https://orcid.org/0000-0001-5206-5859

https://orcid.org/
0000-0001-5206-5859

Other IDs

Scopus Author ID: 16234167200
ResearcherID: N-3634-2018

Name
Bojan Nikolic

Activities Expand all

Employment (1) Sort

Education and qualifications (1) Sort

Works (27) Sort

Peer review (1 review for 1 publication/grant) Sort

Review activity for **Foundations of physics.** (1)

journal, Springer Nature
ISSN: 0015-9018

Review date: 2019 **Type: review** **Role: reviewer** Show less detail View

Review identifier(s)
SOURCE-WORK-ID: f114f221-e24f-41d1-ba1f-132c643b8d60

Convening organization
Springer Nature (New York, US)

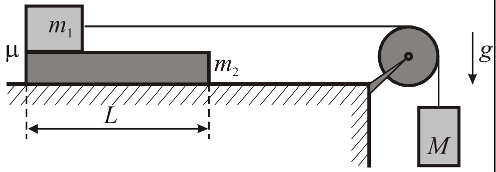
Source: Springer Nature @ Editorial Manager

II РАЗРЕД
Група П

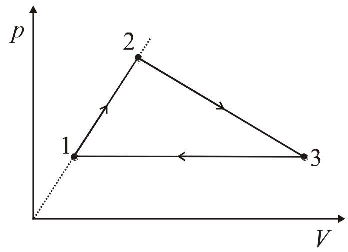
Друштво Физичара Србије
Министарство Просвете и Науке Републике Србије
ЗАДАЦИ

СЕНТА
21.04.2012.

1. На дугачком глатком хоризонталном столу лежи даска масе $m_2 = 2\,\text{kg}$ и дужине $L = 1\,\text{m}$. На левом крају даске постављено је тело масе $m_1 = 1\,\text{kg}$. Коефицијент трења између даске и тела једнак је μ. Тело m_1 везано је лаком нерастегљивом нити са тегом $M = 1\,\text{kg}$ преко глатког лаког котура (види слику). Дати систем почне да се креће. а) При којим вредностима коефицијента трења μ ће тело m_1 и даска m_2 да се крећу као једна целина (без проклизавања)? б) Израчунајте минималну вредност коефицијента трења μ_{min} при којој је могуће кретање без проклизавања. в) За $\mu = \mu_{\text{min}}/2$ израчунајте укупно време t од почетка кретања које је тело m_1 провело на дасци? Узети да је $g = 10\,\text{m/s}^2$. **(20 п)**



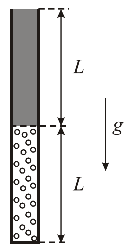
2. Један мол хелијума као идеалног гаса врши кружни циклус приказан на pV дијаграму на слици. Тај циклус се састоји из два дела у којима постоји линеарна зависност притиска p од запремине V, и једне изобаре. Познато је да је на изобари 3–1 над гасом извршен рад A_{31} $(A_{31} > 0)$, а при томе је температура гаса смањена α пута. Стања 2 и 3 припадају истој изотерми. Тачке 1 и 2 припадају правој која пролази кроз координатни почетак.
а) Изведите општи израз зависности укупног рада A гаса у циклусу од рада A_{31}; б) Пет независних експеримената је извршено са истом количином хелијума и истим циклусом у којима је за различите вредности A_{31} израчунаван укупан рад A гаса у циклусу, и добијени су следећи резултати:



A_{31}/[J]	200	300	400	500	600
A/[J]	105	145	200	255	295

Користећи дату табелу нацртајте график $A = f(A_{31})$; в) на основу резултата под а) и б) израчунајте температуре T_1 у тачки 1 за сваки појединачни експеримент, знајући да је $\delta A_{31} = \Delta A_{31}/A_{31} = 5\% = 0{,}05$. Универзална гасна константа износи $R = 8{,}3\ \text{J/(molK)}$. **(25 п)**

3. Доњи крај вертикалне стаклене цеви дужине $2L$ је затопљен, док је горњи отворен ка атмосфери. У доњој половини цеви налази се идеални гас на температури T_0, док се у горњој половини налази жива (види слику). а) Ако цев почнемо лагано да загревамо, до које минималне температуре треба загрејати гас у цеви да би он истиснуо живу у потпуности? б) Ако цев почнемо лагано да хладимо, изведите општи израз промене температуре гаса у цеви. Атмосферски притисак p_0 је дат у милиметрима живиног стуба $(H_0 = L)$. **(15 п)**



4. У суду који мирује запремине $V = 30\,\text{dm}^3$, и који има чврсте и савршено топлотно непропусне зидове налазе се ваздух при нормалним условима $(p_0 = 10^5\ \text{Pa}, T_0 = 273\,\text{K})$ и вода масе $m_{\text{в}} = 9\ \text{g}$. У једном тренутку суд почне да се креће транслаторно брзином v. После успостављања топлотне равнотеже ваздух у суду има релативну влажност $\varphi = 50\%$. Израчунајте коликом брзином v се креће суд. Узети да су: специфична топлота испаравања воде $\lambda = 2{,}2 \times 10^6\ \text{J/kg}$, специфични топлотни капацитет воде $c = 4200\,\text{J/(kg}\cdot\text{K)}$, притисак засићене водене паре при нормалним условима $p = 600\,\text{Pa}$, специфична топлотна капацитативност ваздуха при константној запремини $c_V = 720\,\text{J/(kg}\cdot\text{K)}$, средња моларна маса ваздуха $M = 0{,}029\,\text{kg/mol}$, моларна маса воде $M_{H_2O} = 0{,}018\,\text{kg/mol}$, универзална гасна константа $R = 8{,}31\,\text{J/(mol}\cdot\text{K)}$. Притисак засићене водене паре при температури од $t = 100\,°\text{C}$ износи

$p = p_0 = 10^5$ Pa. Релативна влажност ваздуха φ се дефинише као однос масе водене паре $m_п$ у некој запремини и масе коју би имала засићена водена пара $m_п^{зас}$ у истој тој запремини: $\varphi = m_п / m_п^{зас}$. **(20 п)**

5. На растојању d од хоризонталне, бесконачне равне, танке и равномерно наелектрисане плоче, брзина електрона је износила v_0, при чему је правац брзине заклапао угао α са вертикалом. Плоча је наелектрисана негативним наелектрисањем, при чему је површинска густина наелектрисања (по апсолутној вредности) $\sigma = \mathrm{const}$.
а) Претпостављајући да је електрично поље довољно јако да електрон сво време остаје са исте стране наелектрисане плоче, наћи најмање растојање $d_{\min}$ електрона од плоче током кретања. Колико времена τ протекне од почетка кретања електрона до тачке минималног растојања од плоче?
б) Ако је електрично поље такво да путања електрона тангира наелектрисану плочу, наћи координате те тачке.
в) Нека је електрично поље такво да електрон пролази кроз плочу. Колики је угао β (мерен у односу на вертикалу као и α) под којим електрон улеће у област испод наелектрисане плоче. Колика је брзина електрона (који се налази испод плоче) на растојању h од плоче?
Задатак решити без примене закона одржања енергије. Силу теже као и утицај електрона на прерасподелу наелектрисања на плочи занемарити. **(20 п)**

* * * * * * * * * *

ПОМОЋ

Електрично поље бесконачне, површински равномерно наелектрисане плоче постоји са обе њене стране. Интензитет вектора јачине електричног поља око плоче једнак је $E = \sigma/(2\varepsilon_0)$, где је σ површинска густина наелектрисања, а ε_0 диелектрична пропустљивост вакуума. На пробно позитивно наелектрисање (са једне или друге стране равни) делује одбојна сила ако је плоча позитивно наелектрисана, а привлачна ако је плоча негативно наелектрисана. Сила око плоче делује увек дуж правца нормале на плочу.

Неке основне тригонометријске једнакости и формуле:

$\sin^2\alpha + \cos^2\alpha = 1$	$\sin^2(\alpha/2) = (1-\cos\alpha)/2$
$\sin\alpha = 1/\sqrt{1+\mathrm{ctg}^2\alpha}$	$\cos^2(\alpha/2) = (1+\cos\alpha)/2$
$\cos\alpha = 1/\sqrt{1+\mathrm{tg}^2\alpha}$	$\sin 2\alpha = 2\sin\alpha\cos\alpha$
$\mathrm{tg}\,2\alpha = 2\mathrm{tg}\alpha/(1-\mathrm{tg}^2\alpha)$	$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$
$\sin(\alpha\pm\beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$	$\cos(\alpha\pm\beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

Парабола $\left(1-\frac{x}{2a}\right)\left(1+\frac{x}{a}\right)$ има нуле у $x_1 = 2a$, $x_2 = -a$, а минимум за $x = \frac{1}{2}a$.

* * * * * * * * * *

Задатке припремили: *др Сања Тошић,* Институт за физику, Београд
др Бојан Николић, Институт за физику, Београд
Рецензент: *др Драган Д. Маркушев,* Институт за физику, Београд
Председник Комисије за такмичење ДФС: *др Александар Крмпот,* Институт за физику, Београд

Р1. а) Означимо силу затезања нити са T а силу трења са $F_{тр}$. Напишимо други Њутнов закон за сва три тела:

$$m_1a_1 = T - F_{тр}\,,\ m_2a_2 = F_{тр}\,,\ Ma_1 = Mg - T\,. \ \textbf{(3 п)}$$

Из ових једначина следи да су:

$$a_1 = \frac{Mg - F_{тр}}{m_1 + M} \ \text{ и } \ a_2 = \frac{F_{тр}}{m_2}\,. \ \textbf{(2 п)}$$

Приликом кретања без проклизавања важи да је $a_1 = a_2$ **(1 п)**, па је:

$$\frac{Mg - F_{тр}}{m_1 + M} = \frac{F_{тр}}{m_2} \ \text{ тј. } \ F_{тр} = \frac{Mm_2}{m_1 + m_2 + M}g\,. \ \textbf{(2 п)}$$

При томе сила трења не прелази вредност $\mu m_1 g$. Значи, за кретање без проклизавања је

$$F_{тр} = \frac{Mm_2}{m_1 + m_2 + M}g \leq \mu m_1 g\,, \ \textbf{(2 п)}$$

одакле је

$$\boxed{\mu \geq \frac{Mm_2}{m_1(m_1 + m_2 + M)}.} \ \textbf{(2 п)}$$

б) Минимална вредност коефицијента трења μ_{min} при којој је могуће кретање без проклизавања, на основу претходног резултата, износи

$$\boxed{\mu_{min} = \frac{Mm_2}{m_1(m_1 + m_2 + M)} = \frac{1}{2}.} \ \textbf{(2 п)}$$

За све вредности $\mu \geq \mu_{min}$ нема проклизавања, док ће за $\mu < \mu_{min}$ тело m_1 клизити по дасци.

в) На основу претходне анализе видимо да се за $\mu < \mu_{min}$ тело m_1 и даска m_2 крећу различитим убрзањима $(a_1 \neq a_2)$. У том случају су

$$a_1 = \frac{M - \mu m_1}{m_1 + M}g \ \text{ и } \ a_2 = \frac{\mu m_1}{m_2}g\,. \ \textbf{(2 п)}$$

Релативно убрзање $a_{рел}$ износи $(\mu = \mu_{min}/2 = 1/4)$:

$$a_{рел} = a_1 - a_2 = \left(\frac{M - \mu m_1}{m_1 + M} - \frac{\mu m_1}{m_2}\right)g = \frac{g}{4}\,. \ \textbf{(2 п)}$$

Време клизања по дасци налазимо из формуле

$$L = a_{\text{рел}} \frac{t^2}{2}, \text{ па је } \boxed{t = \sqrt{\frac{2L}{a_{\text{рел}}}} = 0{,}9\,\text{s}.} \textbf{ (2 п)}$$

* * *

P2. а) Ако температуру хелијума у тачки 1 означимо са T_1, онда је по услову задатка $T_3 = \alpha T_1$. Рад A_{31} извршен над гасом дуж изобаре износи

$$A_{31} = p_1(V_3 - V_1) = \nu R(T_3 - T_1) = \nu R T_1(\alpha - 1). \textbf{ (1 п)}$$

Одатле је, за $\nu = 1$,

$$T_1 = \frac{A_{31}}{R(\alpha - 1)}. \textbf{ (1 п)}$$

Укупан рад гаса у циклусу добија се као површина троугла

$$A = \frac{1}{2}(p_2 - p_1)(V_3 - V_1). \textbf{ (1 п)}$$

За изобарски процес важи да је $V \sim T$, па можемо да пишемо да је

$$V_3 - V_1 = V_1(\alpha - 1). \textbf{ (1 п)}$$

Пошто тачке 1 и 2 леже на правој која пролази кроз координатни почетак, онда је

$$p_2 = \frac{V_2}{V_1} p_1, \textbf{ (1 п)}$$

док тачке 2 и 3 припадају истој изотерми па је

$$p_2 V_2 = p_3 V_3. \textbf{ (1 п)}$$

Искористивши последње три једначине добијамо, уз $p_3 = p_1$, да је

$$p_2 = \sqrt{\frac{V_3}{V_1}} p_1 = \sqrt{\alpha} p_1. \textbf{ (1 п)}$$

Сада можемо да напишемо да је, за $\nu = 1$,

$$\boxed{A = \frac{1}{2} p_1 V_1 (\sqrt{\alpha} - 1)(\alpha - 1) = \frac{1}{2} R T_1 (\sqrt{\alpha} - 1)(\alpha - 1) = \frac{(\sqrt{\alpha} - 1)}{2} A_{31}.} \textbf{ (2 п)}$$

Последња једначина представља општи израз зависности укупног рада A гаса у циклусу од рада

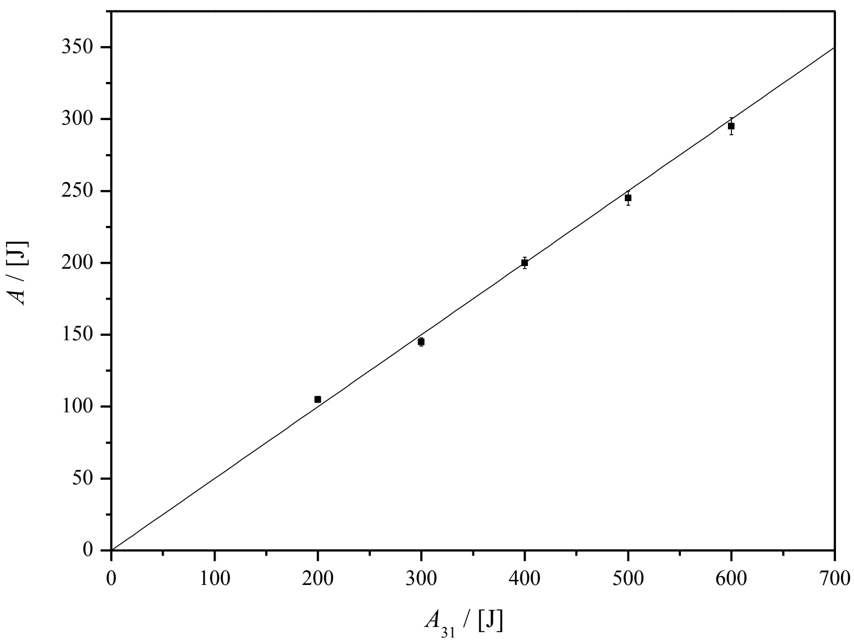
A_{31} дуж изобаре, и видимо да се може изразити у облику линеарне функције $A = f(A_{31}) = kA_{31}$, где је k коефицијент правца ове функције, који износи

$$k = \frac{(\sqrt{\alpha} - 1)}{2}.$$ **(1 п)**

б) На основу дате табеле у поставци и услова задатка $\delta A_{31} = \Delta A_{31} / A_{31} = 2\% = 0{,}02$ добијамо да је $\delta A = \Delta A / A = 2\% = 0{,}02$, па график цртамо помоћу табеле

A_{31} /[J]	200±4	300±6	400±8	500±10	600±12
A /[J]	105±2	145±3	200±4	245±5	295±6

(1 п)



(4 п)

Када нацртамо поменути график, узмемо две тачке са праве и то тако да прву тачку узимамо између прве две **(1 п)** A(250 J, 125 J), а другу између последње две експерименталне тачке **(1 п)** B(550 J, 275 J). На основу њих добијамо коефицијент правца праве

$$k = \frac{A_{\mathrm{B}} - A_{\mathrm{A}}}{A_{31\mathrm{B}} - A_{31\mathrm{A}}} = \frac{(275 - 125)}{(550 - 250)} = \frac{150}{300} = \frac{1}{2}.$$ **(1 п)**

Грешка коефицијента правца износи

$$\Delta k = k\left(\frac{\Delta A_{\mathrm{A}} + \Delta A_{\mathrm{B}}}{A_{\mathrm{B}} - A_{\mathrm{A}}} + \frac{\Delta A_{31\mathrm{A}} + \Delta A_{31\mathrm{B}}}{A_{31\mathrm{B}} - A_{31\mathrm{A}}}\right) = \frac{1}{2}\left(\frac{2+6}{150} + \frac{4+12}{300}\right) = 0{,}05.$$ **(1 п)**

Сада је

$$\boxed{k = (5{,}0 \pm 0{,}5)\times 10^{-1}.}\ \textbf{(1 п)}$$

Сада се, на основу $k = \frac{\left(\sqrt{\alpha}-1\right)}{2}$ може израчунати α као

$$k = \frac{\left(\sqrt{\alpha}-1\right)}{2} \rightarrow \ \alpha = (2k+1)^2 = 4\,,\ \textbf{(1 п)}$$

чија је грешка једнака

$$\frac{\Delta\alpha}{\alpha} = 2\frac{\Delta(2k+1)}{2k+1} \rightarrow \Delta\alpha = 4\alpha\frac{\Delta k}{2k+1} = 0{,}4\,,\ \textbf{(1 п)}$$

па је на крају

$$\boxed{\alpha = (4{,}0 \pm 0{,}4).}\ \textbf{(1 п)}$$

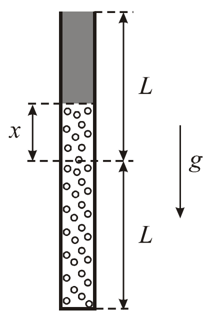
Сада се, на основу $T_1 = \frac{A_{31}}{R(\alpha-1)}$ и $\Delta T_1 = T_1\left(\frac{\Delta A_{31}}{A_{31}} + \frac{\Delta\alpha}{(\alpha-1)}\right)$ **(1 п)** допуни табела са траженим температурама:

A_{31}/[J]	200±4	300±6	400±8	500±10	600±12
A/[J]	105±2	145±3	200±4	245±5	295±6
T_1/[K]	8±1	12±2	16±2	20±3	24±4

(1 п)

* * *

Р3. а) Претпоставимо да је гас у цеви истиснуо живу за дужину x (види слику). Нађимо општи израз зависности температуре T гаса у цеви од почетне температуре T_0 за овај процес. Из једначине стања идеалног гаса следи да за почетни и крајњи положај нивоа живе имамо једнакост



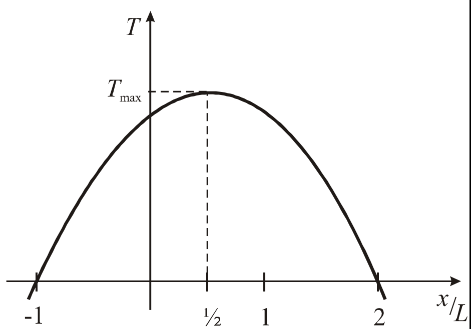
$$\frac{(p_0 + \rho g L)SL}{T_0} = \frac{[p_0 + \rho g(L-x)]S(L+x)}{T}\,,\ \textbf{(3 п)}$$

што уз дефиницију атмосферског притиска $p_0 = \rho g H_0 = \rho g L$ даје

$$\frac{2L^2}{T_0} = \frac{(2L-x)(L+x)}{T}\,.\ \textbf{(1 п)}$$

Одавде следи да је

$$T = T_0\left(1-\frac{x}{2L}\right)\left(1+\frac{x}{L}\right).\ \textbf{(1 п)}$$



Функција $T = f(x)$ има облик параболе као на слици. Парабола има максимум тачно на средини између њених корена $x_1 = 2L$ и $x_2 = -L$, а то је вредност $x = L/2$. За ту вредност x добијамо да је

$$T_{\max} = T_0\left(1 - \frac{L/2}{2L}\right)\left(1 + \frac{L/2}{L}\right) = \frac{9}{8}T_0 .$$ **(2 п)**

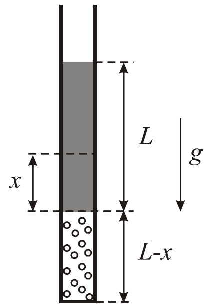
Област између $x/L = 0$ и $x/L = 1/2$ представља област стабилних равнотежних стања живе **(1 п)**. Ниво живе се у тој области подиже загревањем гаса (повећањем температуре). Уколико у неком тренутку у овој области пре достизања $T_{\max}$ прекинемо загревање, ниво живе ће престати да се креће. Ако после тога наставимо са загревањем гаса и ниво живе ће наставити да се подиже све дотле док гас не достигне $T_{\max}$, када равнотежа живе постаје нестабилна, и жива комплетно излази из цеви (област неравнотежних стања живе) **(1 п)**. Значи тражена минимална температура $T_{\min}$ при којој треба загрејати гас да би он у потпуности истиснуо живу из цеви износи

$$\boxed{T_{\min} = T_{\max} = \frac{9}{8}T_0 .}$$ **(2 п)**

Стања која су описана датом параболом за област $x/L > 1/2$ не разматрамо јер нису физички оправдана тј. описују парадоксалне ситуације: све тачке параболе у тој области одговарале би *спуштању* нивоа живе приликом *загревања* гаса!

б) Уколико гас у цеви почнемо да хладимо из почетног стања T_0, ниво живе ће почети да се спушта, али ће притисак гаса бити исти (изобарски процес), па из једначине стања идеалног гаса следи да је

$$\frac{(p_0 + \rho g L)SL}{T_0} = \frac{(p_0 + \rho g L)S(L - x)}{T},$$ **(2 п)**



што даје

$$\boxed{T = T_0\left(1 - \frac{x}{L}\right),}$$ **(2 п)**

тј. зависност положаја нивоа живе у цеви од температуре је линеарна.

* * *

Р4. У почетном тренутку маса водене паре $m_п$ је много мања у односу на масу воде $m_в$ у суду:

$$m_п = \frac{M_{H_2O}pV}{RT_0} = 0{,}15\,\mathrm{g} << m_в = 9\,\mathrm{g},$$ **(1 п)**

па можемо сматрати да се сва вода у почетном тренутку налазила у течном стању. По покретању суда и успостављеној температури у њему нема више од $m_п = 9\,\mathrm{g}$ водене паре при влажности од $\varphi = 50\% = 0{,}5$. Да би при тој температури вода била засићена потребно је да у тој запремини буде $m_п^{зас} = m_п/\varphi = 18\,\mathrm{g}$ водене паре тј. 1 мол. Познато је да вода при атмосферском притиску ври при температури од $t = 100\ °\mathrm{C}$, што значи да је притисак засићене водене паре при температури од $T_1 = 373\ \mathrm{K}$ једнак $p_0 = 10^5\ \mathrm{Pa}$. Из једначине стања идеалног гаса следи да 1 мол водене паре при притиску од $10^5\ \mathrm{Pa}$ и температури од $373\ \mathrm{K}$ заузима запремину од

$$V_1 = \frac{RT_1}{p_0} \approx 31\,\mathrm{dm}^3 = V\,,\ \mathbf{(1\ п)}$$

па можемо да закључимо да је у суду успостављена температура $T_1 = 373\ \mathrm{K}$ **(2 п)**.
Из услова задатка јасно је да се запремина суда не мења, и да су зидови суда топлотно непропусни тј. енергија коју поседују ваздух и вода не губи се разменом топлоте са околином. Ради једноставности посматрајмо суд из референтног система који се креће брзином v. У том систему наш суд је на почетку имао брзину $-v$ а затим је нагло стао. Ваздух и вода су имали, као целина, кинетичку и унутрашњу енергију. По заустављању суда укупна енергија воде и ваздуха остаје иста, али се кинетичка енергија воде и ваздуха, као целине, претворила у допунску унутрашњу енергију. Из једначине стања идеалног гаса маса ваздуха m у суду износи

$$m = \frac{Mp_0V}{RT_0} \approx 39{,}6\,\mathrm{g}\,,\ \mathbf{(1\ п)}$$

па је укупна маса воде и ваздуха у суду $m_{\text{в}} + m \approx 48{,}6\,\mathrm{g}$. Укупна промена унутрашње енергије система последица је загревања ваздуха, загревања воде и њеног испаравања. Ако израчунамо сваки појединачни допринос тој промени имамо: За загревање ваздуха до 100 °C потребна је количина топлоте

$$Q_1 = mc_V\left(T_1 - T_0\right) \approx 2850\,\mathrm{J}\,.\ \mathbf{(3\ п)}$$

Промена унутрашње енергије воде зависи само од почетног и крајњег стања система а не од природе процеса пре том преласку, па можемо сматрати да смо воду прво загрејали до $t =$ 100 °C, а затим је превели у пару. За загревање воде потребно је утрошити количину топлоте

$$Q_2 = m_{\text{в}}c\left(T_1 - T_0\right) \approx 3780\,\mathrm{J}\,.\ \mathbf{(3\ п)}$$

За стварање водене паре потребна је количина топлоте

$$Q_3 = m\lambda \approx 19800\,\mathrm{J}\,.\ \mathbf{(3\ п)}$$

Укупна промена унутрашње енергије посматраног система једнака је кинетичкој енергији коју су имали вода и ваздух током кретања:

$$\frac{\left(m + m_{\text{в}}\right)v^2}{2} = \Delta U = Q_1 + Q_1 + Q_1\,,\ \mathbf{(3\ п)}$$

па је тражена брзина суда

$$\boxed{v = \sqrt{2\cdot\frac{Q_1 + Q_1 + Q_1}{m + m_{\text{в}}}} = \sqrt{2\cdot\frac{26430\,\mathrm{J}}{48{,}6\,\mathrm{g}}} \approx 10^3\,\frac{\mathrm{m}}{\mathrm{s}}.}\ \mathbf{(3\ п)}$$

* * *

P5. Електрично поље бесконачно наелектрисане плоче је по интензитету једнако $E = \sigma/(2\varepsilon_0)$, где је ε_0 диелектрична пропустљивост вакуума. У нашем случају је плоча негативно наелектрисана, а последица тога је да се електрон по правцу ортогоналном на плочу изнад ње креће успорено, а испод убрзано са убрзањем интензитета

$$a = \frac{F}{m} = \frac{eE}{m} = \frac{e\sigma}{2m\varepsilon_0}. \text{ (2 п)}$$

Кретање по правцу паралелном са плочом је кретање са константном брзином **(1 п)**. Поставимо координатни систем тако да је x оса паралелна са плочом, y оса ортогонална на њу и усмерена ка плочи а координатни почетакје у полазној тачци електрона.

а) Компоненте брзине електрона су

$$v_x = v_0 \sin\alpha = \text{const.} \quad \text{и} \quad v_y^2 = v_0^2 \cos^2\alpha - 2ay. \text{ (2 п)}$$

У тачки где је електрон најближи плочи важи $v_y = 0$ **(1 п)**. Одатле добијамо да је y координата те тачке $Y = \frac{m v_0^2 \varepsilon_0 \cos^2\alpha}{e\sigma}$. Минимално растојање које се тражи је

$$d_{\min} = d - Y. \text{ (2 п)}$$

С друге стране знамо да је $v_y = v_0 \cos\alpha - at$, па из услова $v_y = 0$ имамо да је тражено време

$$\tau = \frac{v_0 \cos\alpha}{a} = \frac{2 m v_0 \varepsilon_0 \cos\alpha}{e\sigma}. \text{ (1 п)}$$

б) У овом случају важи $Y = d$, а x координата тачке додира је

$$X = v_0 \tau \sin\alpha = \frac{m v_0^2 \varepsilon_0 \sin 2\alpha}{e\sigma}. \text{ (2 п)}$$

в) Једначине кретања електрона изнад плоче су

$$x = v_0 t \sin\alpha \quad \text{и} \quad y = v_0 t \cos\alpha - \frac{1}{2} a t^2. \text{ (1 п)}$$

Када електрон пролази кроз плочу имамо $x = X$ и $y = d$. **(1 п)**

Други услов даје два решења за временске тренутке проласка електрона кроз плочу

$$t_{\pm} = \frac{v_0 \cos\alpha \pm \sqrt{v_0^2 \cos^2\alpha - 2ad}}{a} = \tau \pm \frac{\sqrt{v_0^2 \cos^2\alpha - 2ad}}{a}.$$

Пошто је очигледно $t_+ > \tau > t_-$ физичког смисла има само краће време јер оно одговара пролазу кроз плочу, тако да је тражено време $t = t_-$ **(2 п)**. Компоненте брзине у тој тачци су

$$v_x = v_0 \sin\alpha \quad \text{и} \quad v_y = \sqrt{v_0^2 \cos^2\alpha - 2ad}.$$ **(1 п)**

Интензитет брзине је $v = \sqrt{v_x^2 + v_y^2} = \sqrt{v_0^2 - 2ad},$ па је угао β дат једначином

$$\sin\beta = \frac{v_x}{v} = \frac{v_0 \sin\alpha}{\sqrt{v_0^2 - 2ad}}.$$ **(2 п)**

Даље се електрон креће дуж y правца убрзано са убрзањем a, а константном брзином дуж x осе. На растојању h испод плоче, по услову задатка, брзина је

$$v = \sqrt{v_0^2 + 2a(h - d)}.$$ **(2 п)**

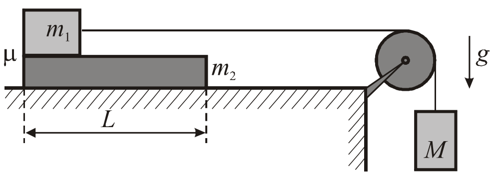
50. РЕПУБЛИЧКО ТАКМИЧЕЊЕ ИЗ ФИЗИКЕ
УЧЕНИКА СРЕДЊИХ ШКОЛА ШКОЛСКЕ 2011/2012. ГОДИНЕ.

II РАЗРЕД
Група О и С

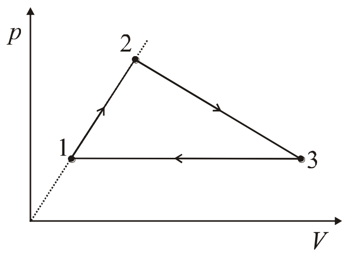
Друштво Физичара Србије
Министарство Просвете и Науке Републике Србије
ЗАДАЦИ

СЕНТА
21.04.2012.

1. На дугачком глатком хоризонталном столу лежи даска масе $m_2 = 2\,\mathrm{kg}$ и дужине $L = 1\,\mathrm{m}$. На левом крају даске постављено је тело масе $m_1 = 1\,\mathrm{kg}$. Коефицијент трења између даске и тела једнак је μ. Тело m_1 везано је лаком нерастегљивом нити са тегом $M = 1\,\mathrm{kg}$ преко глатког лаког котура (види слику). Дати систем почне да се креће. а) При којим вредностима коефицијента трења μ ће тело m_1 и даска m_2 да се крећу као једна целина (без проклизавања)? б) Израчунајте минималну вредност коефицијента трења $\mu_{\min}$ при којој је могуће кретање без проклизавања. в) За $\mu = \mu_{\min}/2$ израчунајте укупно време t од почетка кретања које је тело m_1 провело на дасци? Узети да је $g = 10\,\mathrm{m/s^2}$. **(20 п)**



2. Један мол хелијума као идеалног гаса врши кружни циклус приказан на pV дијаграму на слици. Тај циклус се састоји из два дела у којима постоји линеарна зависност притиска p од запремине V, и једне изобаре. Познато је да је на изобари 3–1 над гасом извршен рад A_{31} $(A_{31} > 0)$, а при томе је температура гаса смањена α пута. Стања 2 и 3 припадају истој изотерми. Тачке 1 и 2 припадају правој која пролази кроз координатни почетак. а) Изведите општи израз зависности укупног рада A гаса у циклусу од рада A_{31}; б) Пет независних експеримената је извршено са истом количином хелијума и истим циклусом у којима је за различите вредности A_{31} израчунаван укупан рад A гаса у циклусу, и добијени су следећи резултати:



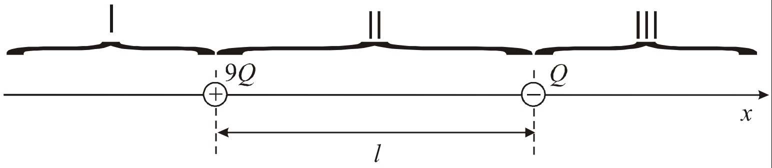
A_{31} /[J]	200	300	400	500	600
A /[J]	105	145	200	255	295

Користећи дату табелу нацртајте график $A = f(A_{31})$; в) на основу резултата под а) и б) израчунајте температуре T_1 у тачки 1 за сваки појединачни експеримент, знајући да је $\delta A_{31} = \Delta A_{31} / A_{31} = 5\% = 0{,}05$. Универзална гасна константа износи $R = 8{,}3\,\mathrm{J/(mol\,K)}$. **(25 п)**

3. Гвоздена кугла пречника $d = 8\,\mathrm{cm}$ извади се из воде која кључа и постави на површину дебеле ледене плоче температуре $t_1 = 0\,°\mathrm{C}$. До које ће дубине кугла утонути у лед? Претпоставити да кугла предаје топлоту само леду испод себе и да тоне праволинијски дуж нормале на површину ледене плоче (површина ледене плоче је нормална на правац дејства гравитационог поља Земље). Занемарити загревање воде и топлотну проводљивост леда и воде. Притисак околног ваздуха је $p = 1013\,\mathrm{mbar}$, густине гвожђа и леда су $\rho = 7800\,\mathrm{kg/m^3}$ и $\rho_0 = 917\,\mathrm{kg/m^3}$ респективно, специфична топлота гвожђа је $c = 460\,\mathrm{J/(kg\cdot K)}$ и топлота топљења леда $q = 0{,}33\,\mathrm{MJ/kg}$. Коментарисати добијени резултат у контексту промене потенцијалне енергије куглице. **(15 п)**

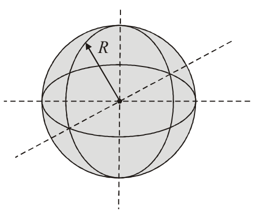
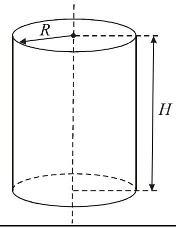
4. Из округлог отвора истиче вертикалан млаз воде. На једном месту пречник хоризонталног попречног пресека млаза износи $d = 2{,}0\,\mathrm{mm}$ а на другом, које је за $l = 2\,\mathrm{cm}$ ниже, пречник попречног пресека истог млаза је $n = 1{,}5$ пута мањи. Изведите општи израз и израчунајте бројну вредност величине $Q = \Delta V / \Delta t$ – запремине воде која за једну секунду истекне кроз поменути отвор. Узети да је $g = 10\,\mathrm{m/s^2}$, густина воде $\rho = 1\,\mathrm{g/cm^3}$, а коефицијент површинског напона воде $\gamma = 73\,\mathrm{mN/m}$. **(20 п)**

5. Два тачкаста наелектрисања $9Q$ и $-Q$ фиксирамо дуж x- осе на међусобном растојању $l = 50\,\mathrm{cm}$ тако да чине изоловани систем, као на слици. Треће наелектрисање Q_1 може да се креће слободно дуж x- осе у свакој од приказаних области I, II или III. а) Израчунати тачан положај наелектрисања Q_1 на x- оси у приказаним областима, у коме ће баш Q_1 бити у равнотежи. б) При којој вредности предзнака наелектрисања Q_1 (+ или –) ће његова равнотежа бити стабилна? (Равнотежу ћемо звати стабилном уколико при малом померају наелектрисања Q_1 из равнотежног положаја дође до појаве сила које ће тежити да то наелектрисање врате у равнотежни положај). **(20 п)**



* * * * * * * * * *

ПОМОЋ

Запремина кугле износи $V = \frac{4}{3}\pi R^3$ где је R полупречник кугле.	Запремина цилиндра износи $V = \pi R^2 H$ где је R полупречник основе цилиндра, а H је његова висина.
	

Задатке припремили: *др Сања Тошић,* Институт за физику, Београд
др Бојан Николић, Институт за физику, Београд
Рецензент: *др Драган Д. Маркушев,* Институт за физику, Београд
Председник Комисије за такмичење ДФС: *др Александар Крмпот,* Институт за физику, Београд

Р1. а) Означимо силу затезања нити са T а силу трења са $F_{\text{тр}}$. Напишимо други Њутнов закон за сва три тела:

$$m_1 a_1 = T - F_{\text{тр}}, \; m_2 a_2 = F_{\text{тр}}, \; M a_1 = Mg - T. \; \textbf{(3 п)}$$

Из ових једначина следи да су:

$$a_1 = \frac{Mg - F_{\text{тр}}}{m_1 + M} \text{ и } a_2 = \frac{F_{\text{тр}}}{m_2}. \; \textbf{(2 п)}$$

Приликом кретања без проклизавања важи да је $a_1 = a_2$ **(1 п)**, па је:

$$\frac{Mg - F_{\text{тр}}}{m_1 + M} = \frac{F_{\text{тр}}}{m_2} \text{ тј. } F_{\text{тр}} = \frac{Mm_2}{m_1 + m_2 + M} g. \; \textbf{(2 п)}$$

При томе сила трења не прелази вредност $\mu m_1 g$. Значи, за кретање без проклизавања је

$$F_{\text{тр}} = \frac{Mm_2}{m_1 + m_2 + M} g \leq \mu m_1 g, \; \textbf{(2 п)}$$

одакле је

$$\boxed{\mu \geq \frac{Mm_2}{m_1(m_1 + m_2 + M)}.} \; \textbf{(2 п)}$$

б) Минимална вредност коефицијента трења $\mu_{\min}$ при којој је могуће кретање без проклизавања, на основу претходног резултата, износи

$$\boxed{\mu_{\min} = \frac{Mm_2}{m_1(m_1 + m_2 + M)} = \frac{1}{2}.} \; \textbf{(2 п)}$$

За све вредности $\mu \geq \mu_{\min}$ нема проклизавања, док ће за $\mu < \mu_{\min}$ тело m_1 клизити по дасци.

в) На основу претходне анализе видимо да се за $\mu < \mu_{\min}$ тело m_1 и даска m_2 крећу различитим убрзањима $(a_1 \neq a_2)$. У том случају су

$$a_1 = \frac{M - \mu m_1}{m_1 + M} g \text{ и } a_2 = \frac{\mu m_1}{m_2} g. \; \textbf{(2 п)}$$

Релативно убрзање $a_{\text{рел}}$ износи $(\mu = \mu_{\min}/2 = 1/4)$:

$$a_{\text{рел}} = a_1 - a_2 = \left(\frac{M - \mu m_1}{m_1 + M} - \frac{\mu m_1}{m_2}\right) g = \frac{g}{4}.$$ **(2 п)**

Време клизања по дасци налазимо из формуле

$$L = a_{\text{рел}} \frac{t^2}{2}, \text{ па је } \boxed{t = \sqrt{\frac{2L}{a_{\text{рел}}}} = 0{,}9\,\text{s}.}$$ **(2 п)**

* * *

P2. а) Ако температуру хелијума у тачки 1 означимо са T_1, онда је по услову задатка $T_3 = \alpha T_1$. Рад A_{31} извршен над гасом дуж изобаре износи

$$A_{31} = p_1(V_3 - V_1) = \nu R(T_3 - T_1) = \nu R T_1(\alpha - 1).$$ **(1 п)**

Одатле је, за $\nu = 1$,

$$T_1 = \frac{A_{31}}{R(\alpha - 1)}.$$ **(1 п)**

Укупан рад гаса у циклусу добија се као површина троугла

$$A = \frac{1}{2}(p_2 - p_1)(V_3 - V_1).$$ **(1 п)**

За изобарски процес важи да је $V \sim T$, па можемо да пишемо да је

$$V_3 - V_1 = V_1(\alpha - 1).$$ **(1 п)**

Пошто тачке 1 и 2 леже на правој која пролази кроз координатни почетак, онда је

$$p_2 = \frac{V_2}{V_1} p_1,$$ **(1 п)**

док тачке 2 и 3 припадају истој изотерми па је

$$p_2 V_2 = p_3 V_3.$$ **(1 п)**

Искористивши последње три једначине добијамо, уз $p_3 = p_1$, да је

$$p_2 = \sqrt{\frac{V_3}{V_1}} p_1 = \sqrt{\alpha}\, p_1.$$ **(1 п)**

Сада можемо да напишемо да је, за $\nu = 1$,

$$\boxed{A = \frac{1}{2} p_1 V_1 (\sqrt{\alpha} - 1)(\alpha - 1) = \frac{1}{2} R T_1 (\sqrt{\alpha} - 1)(\alpha - 1) = \frac{(\sqrt{\alpha} - 1)}{2} A_{31}.}$$ **(2 п)**

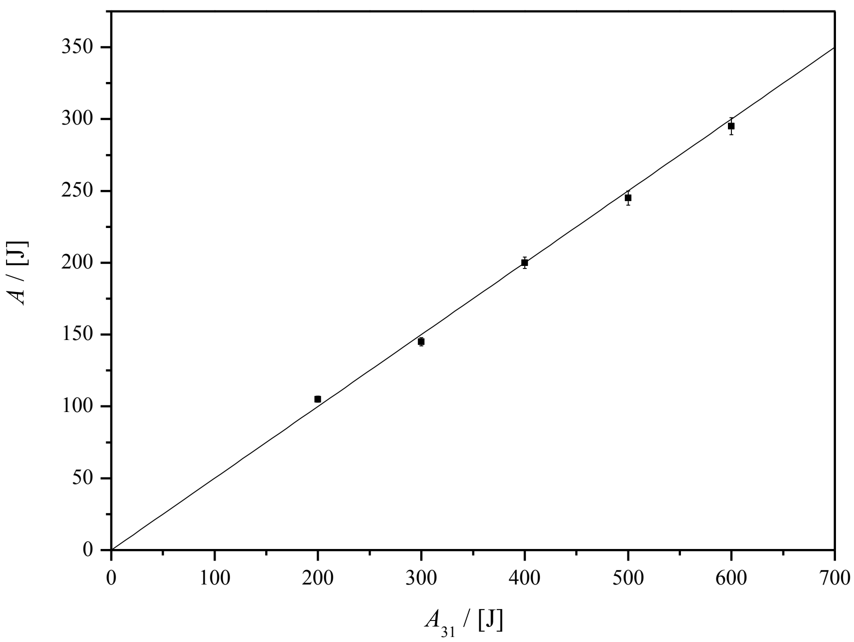
Последња једначина представља општи израз зависности укупног рада A гаса у циклусу од рада A_{31} дуж изобаре, и видимо да се може изразити у облику линеарне функције $A = f\left(A_{31}\right) = kA_{31}$, где је k коефицијент правца ове функције, који износи

$$k = \frac{\left(\sqrt{\alpha} - 1\right)}{2}.$$ **(1 п)**

б) На основу дате табеле у поставци и услова задатка $\delta A_{31} = \Delta A_{31} / A_{31} = 2\% = 0{,}02$ добијамо да је $\delta A = \Delta A / A = 2\% = 0{,}02$, па график цртамо помоћу табеле

A_{31} /[J]	200±4	300±6	400±8	500±10	600±12
A /[J]	105±2	145±3	200±4	245±5	295±6

(1 п)



(4 п)

Када нацртамо поменути график, узмемо две тачке са праве и то тако да прву тачку узимамо између прве две **(1 п)** A(250 J, 125 J), а другу између последње две експерименталне тачке **(1 п)** B(550 J, 275 J). На основу њих добијамо коефицијент правца праве

$$k = \frac{A_{\mathrm{B}} - A_{\mathrm{A}}}{A_{31\mathrm{B}} - A_{31\mathrm{A}}} = \frac{(275 - 125)}{(550 - 250)} = \frac{150}{300} = \frac{1}{2}.$$ **(1 п)**

Грешка коефицијента правца износи

$$\Delta k = k\left(\frac{\Delta A_{\mathrm{A}} + \Delta A_{\mathrm{B}}}{A_{\mathrm{B}} - A_{\mathrm{A}}} + \frac{\Delta A_{31\mathrm{A}} + \Delta A_{31\mathrm{B}}}{A_{31\mathrm{B}} - A_{31\mathrm{A}}}\right) = \frac{1}{2}\left(\frac{2+6}{150} + \frac{4+12}{300}\right) = 0{,}05.$$ **(1 п)**

Сада је

$$\boxed{k = (5{,}0 \pm 0{,}5)\times 10^{-1}.}\ \textbf{(1 п)}$$

Сада се, на основу $k = \frac{(\sqrt{\alpha}-1)}{2}$ може израчунати α као

$$k = \frac{(\sqrt{\alpha}-1)}{2} \rightarrow \alpha = (2k+1)^2 = 4,\ \textbf{(1 п)}$$

чија је грешка једнака

$$\frac{\Delta\alpha}{\alpha} = 2\frac{\Delta(2k+1)}{2k+1} \rightarrow \Delta\alpha = 4\alpha\frac{\Delta k}{2k+1} = 0{,}4,\ \textbf{(1 п)}$$

па је на крају

$$\boxed{\alpha = (4{,}0 \pm 0{,}4).}\ \textbf{(1 п)}$$

Сада се, на основу $T_1 = \frac{A_{31}}{R(\alpha-1)}$ и $\Delta T_1 = T_1\left(\frac{\Delta A_{31}}{A_{31}} + \frac{\Delta\alpha}{(\alpha-1)}\right)$ **(1 п)** допуни табела са траженим температурама:

A_{31} /[J]	200±4	300±6	400±8	500±10	600±12
A /[J]	105±2	145±3	200±4	245±5	295±6
T_1 /[K]	8±1	12±2	16±2	20±3	24±4

(1 п)

* * *

Р3. По услову задатка $(p = 1013\,\text{mbar})$, температура кугле после вађења из воде је $t_2 = 100\,°\text{C}$. **(1 п)** После стављања на лед, кугла почиње да га топи. Топећи лед кугла истовремено кроз њега и пропада. Пропадање кроз лед престаје када се температура кугле изједначи са температуром леда t_1. **(1 п)** Маса гвоздене кугле износи

$$m_к = \rho V_к = \frac{1}{6}\rho d^3\pi.\ \textbf{(1 п)}$$

Запремина истопљеног леда $V_л$ добија се као збир запремине цилиндра висине $h - \frac{d}{2}$ и пречника основе d (h је коначна дубина пропадања) и запремине полулопте пречника d,

$$V_л = \frac{d^2\pi}{4}\left(h - \frac{d}{6}\right).\ \textbf{(2 п)}$$

Маса истопљеног леда износи

$$m_л = \rho_0 V_л. \text{ (2 п)}$$

По условима задатка, сва топлота која се ослобађа хлађењем кугле троши се на топљење леда. Из закона одржања енергије

$$m_л q = m_к c\Delta t\text{, (3 п)}$$

где је $\Delta t = 100\,°\mathrm{C}$, добија се коначна дубина продирања кугле

$$h = \frac{d}{6}\left(1 + \frac{4\rho c\Delta t}{\rho_0 q}\right). \text{ (2 п)}$$

Заменом бројних вредности добија се да је

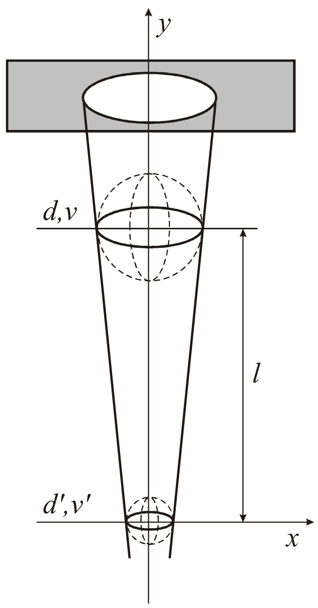
$$h \approx 7{,}7\,\mathrm{cm}\text{. (1 п)}$$

Коментар - упоредити ред величине промене потенцијалне енергије кугле $m_к gh$ са енергијом хлађења кугле $m_к c\Delta t$. Величина gh је реда величине јединице (у J/kg) док је $c\Delta t \sim 10^4$ тј. $c\Delta t >> gh$. Због тога је промена потенцијалне енергије $m_к gh$ занемарена у закону одржања енергије (једначини топлотног биланса) **(2 п).**

* * *

P4. Захваљујући деловању сила површинског напона, течност која истиче кроз отвор тежи да смањи своју слободну површину па се због тога попречни пресек млаза смањује. На основу слике и једначине континуитета можемо да напишемо да је, уз $d' = d/n$,

$$Sv = S'v' \Rightarrow \frac{\pi d^2}{4}v = \frac{\pi}{4}\left(\frac{d}{n}\right)^2 v' \Rightarrow v' = n^2 v. \text{ (3 п)}$$



Посматрајмо горњи пресек млаза (d, v). Притисак p у њему је већи од атмосферског p_0 и износи:

$$p = p_0 + \frac{4\gamma}{d}. \text{ (3 п)}$$

Аналогно томе притисак у доњем пресеку млаза (d', v') износи:

$$p = p_0 + \frac{4n\gamma}{d}. \text{ (3 п)}$$

На основу Бернулијеве једначине за ова два пресека можемо да напишемо да је

$$\frac{p_0 + \frac{4\gamma}{d}}{\rho} + \frac{1}{2}v^2 + gl = \frac{p_0 + \frac{4n\gamma}{d}}{\rho} + \frac{1}{2}n^4 v^2. \quad \textbf{(3 п)}$$

Из последње једначине се сређивањем добија да је

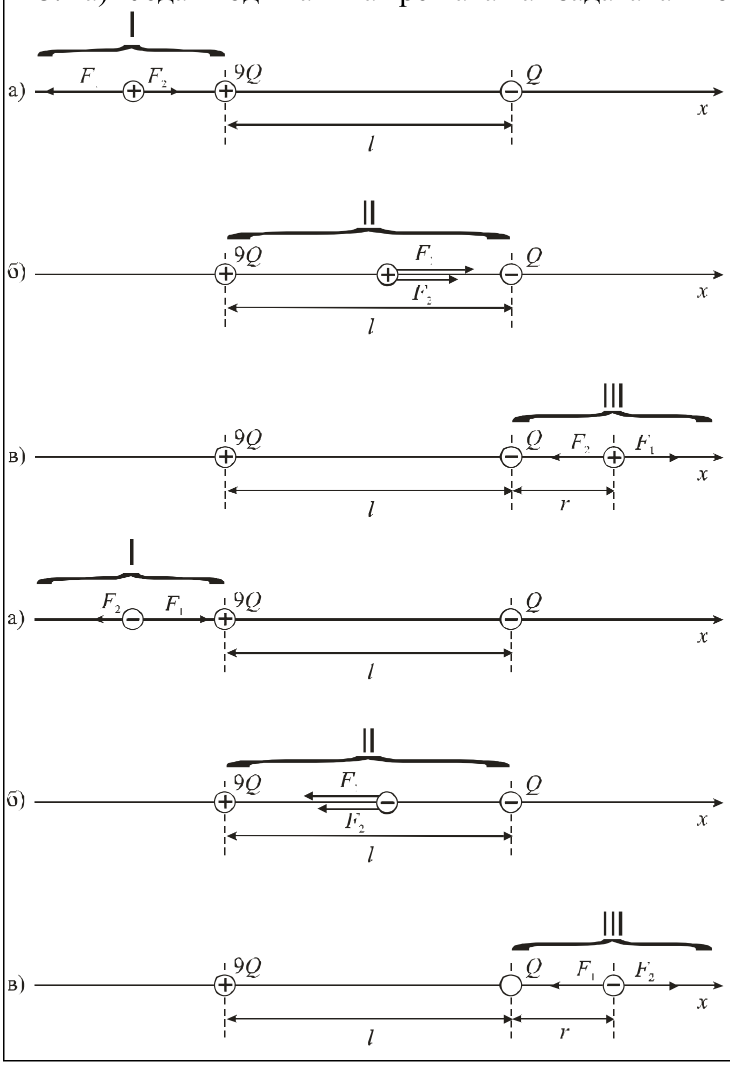
$$v = \sqrt{\frac{2gl - \frac{8\gamma}{\rho d}(n-1)}{n^4\text{-}1}}. \quad \textbf{(3 п)}$$

Запремина воде $Q = \Delta V / \Delta t$ која за једну секунду истекне кроз поменути отвор може се добити као:

$$\Delta V = S\Delta y = \frac{\pi d^2}{4} v\Delta t \Rightarrow \boxed{Q = \frac{\Delta V}{\Delta t} = \frac{\pi d^2}{4} v = \frac{\pi d^2}{4}\sqrt{\frac{2gl - \frac{8\gamma}{\rho d}(n-1)}{n^4\text{-}1}} = 0{,}8\,\frac{\text{cm}^3}{\text{s}}.} \quad \textbf{(5 п)}$$

* * *

P5. а) Један од начина решавања задатака може бити и следећи: претпоставимо да је наелектрисање Q_1 позитивно. У области I (на слици под а)) на наелектрисање Q_1 дуж x - осе делују две супротно усмерене силе: $\mathbf{F}_1$ и $\mathbf{F}_2$. Сила $\mathbf{F}_1$ потиче од стране наелектрисања $9Q$, и у било којој тачки ове области та сила ће бити већа од силе $\mathbf{F}_2$ која потиче од наелектрисања $-Q$, јер је (веће по интензитету) наелектрисање $9Q$ увек ближе наелектрисању Q_1 него наелектрисање $-Q$. Због тога је *положај равнотеже Q_1 у области I немогућ*. **(2 п)**



У области II (на слици под б)) обе силе $\mathbf{F}_1$ и $\mathbf{F}_2$ усмерене су дуж x - осе у свакој тачки ове области на исту страну: ка наелектрисању $-Q$. Због тога је *положај равнотеже Q_1 у области II немогућ*. **(2 п)**

У области III (на слици под в)) обе силе $\mathbf{F}_1$ и $\mathbf{F}_2$ су супротно усмерене дуж x - осе, као и у области I, али је сада (мање по интензитету) наелектрисање $-Q$ увек ближе Q_1 него наелектрисање $9Q$. То значи да је могуће наћи такву тачку на x - оси у којој ће силе $\mathbf{F}_1$ и $\mathbf{F}_2$ бити једнаке по интензитету: $|\mathbf{F}_1| = |-\mathbf{F}_2|$. Због тога је *положај равнотеже* Q_1 *у области III могућ*. **(2 п)**

Ако међусобно растојање између Q_1 и $-Q$ означимо са r, онда ће растојање између $9Q$ и Q_1 бити $l+r$. Једнакост сила нам даје

$$\frac{1}{4\pi\varepsilon_0}\frac{9QQ_1}{(l+r)^2}=\frac{1}{4\pi\varepsilon_0}\frac{QQ_1}{r^2},\ \textbf{(1 п)}$$

што после скраћивања даје

$$l+r=\pm 3r,\ \textbf{(1 п)}$$

одакле је

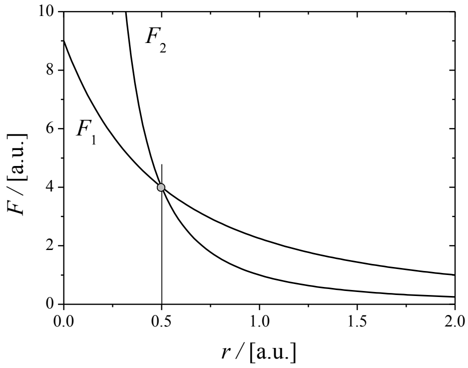
$$r_1=+\frac{l}{2}\ \text{и}\ r_2=-\frac{l}{4}.\ \textbf{(1 п)}$$

Корен r_2 не задовољава физички услов задатка (у тој тачки силе $\mathbf{F}_1$ и $\mathbf{F}_2$ јесу једнаке по интензитету, али су усмерене на исту страну) **(1 п).** Значи Q_1 је у равнотежном положају када је $r=r_1$. **(1п)**

Истоветна анализа само са претпоставком да је Q_1 негативно доводи до истог резултата (види слику). **(3 п)**

б) Да би одредили предзнак наелектрисања Q_1 при којем ће равнотежа бити стабилна, анализираћемо померај Q_1 из равнотежног положаја у два случаја, када је: (i) Q_1 позитивно (ii) Q_1 негативно.

(i) Уколико је Q_1 *позитивно*, његов *померај улево* доводи до пораста сила F_1 и F_2. Сила F_1 расте спорије (наелектрисање $9Q$ је увек даље од $-Q$). Следи да је F_2 по интензитету већа од F_1, и на наелектрисање Q_1 ће деловати *резултујућа сила* која је такође *усмерена улево*. Под дејством резултујуће силе наелектрисање Q_1 ће се *удаљавати* од равнотежног положаја. Исто ће се десити и ако Q_1 померимо *удесно*. Сила F_2 ће слабити брже од силе F_1 па је *резултујућа сила усмерена удесно*, а наелектрисање Q_1 ће се, под дејством те резултујуће силе, *удаљавати од равнотежног положаја*. Ова анализа нам указује на то да је у случају *позитивног предзнака* наелектрисања Q_1 *равнотежа у области III нестабилна*. **(3 п)**



(ii) Уколико је Q_1 негативно, његов *померај улево* доводи до пораста сила F_1 и F_2. Сила F_1 расте спорије (наелектрисање $9Q$ је увек даље од $-Q$). Следи да је F_2 по интензитету већа од F_1, и на наелектрисање Q_1 ће деловати *резултујућа сила* која је такође *усмерена удесно*. Под дејством резултујуће силе наелектрисање Q_1 ће се *враћати* ка равнотежном положају. Исто ће се десити и ако Q_1 померимо *удесно*. Сила F_2 ће слабити брже од силе F_1 па је *резултујућа сила усмерена улево*, а наелектрисање Q_1 ће се, под дејством те резултујуће силе, *враћати ка равнотежном положају*. Ова анализа нам указује на то да је *у случају негативног предзнака* наелектрисања Q_1 *равнотежа стабилна*. **(3 п)**

Интензитет наелектрисања Q_1 је за ову анализу небитан.

* * *

Subject **Re: RS2016**
From Ljubisa Nesic <nesiclj@junis.ni.ac.rs>
To Bojan Nikolic <bnikolic@ipb.ac.rs>, Ivan Dojcinovic <ivan.dojcinovic@ff.bg.ac.rs>
Date 2016-02-23 14:07

webmail

Dragi Bojane,
drago mi je sto si privhatio da drzis predavanje o talasima.
prosledjujem tvoj mail predsedniku DFS. Nadam se da ce ti dati odgovarajuca uputstva.

Ljubisa

On 2/23/2016 1:54 PM, Bojan Nikolic wrote:

> Dragi Ljubisa,
>
> hvala na pozivu. Sve je jasno sem jednog tehnickog detalja. Sta ja tacno treba da uradim da bih bio clan DFS? Kome treba da platim i koliko?
>
> Pozdrav,
> Bojan
>
> On 22 Feb 2016 17:05, rep.seminar@ff.bg.ac.rs wrote:
>
>> Postovani Bojane,
>> imam izuzetno zadovoljstvo da vas, ispred Strucnog odbora republickog seminara o nastavi fizike za 2016. godinu (http://www.dfs.rs/seminar2016/), pozovem da odrzite plenarno predavanje vezano za otkrice gravitacionih talasa.
>> Seminar ce biti odrzan na Zlatiboru od 12. do 14. maja a rok za pisanje rada je 14. mart. Radovi ce, nakon recenzije, biti objavljeni u 3. broju casopisa Nastava fizike koji smo pokrenuli prosle godine.
>> Na sajtu seminara mozete naci i upustvo za pisanje rada, odnosno odgovarajuci template a vi, kao predavac po pozivu, imate na raspolaganju do 10 strana. Ukoliko imate bilo kakav tehnicki problem slobodno se obratite. DFS ce snositi troskove vaseg puta i smestaja ali je potrebno da budete clan drustva.
>> Imajte u vidu da ce na seminaru biti takodje i jedno uvodno predavanje o OTR.
>>
>> Ljubisa Nesic
>> predsednik Komisije za seminare DFS

XXXIV Републички семинар о настави физике

Студентско одмаралиште "Ратко Митровић", Златибор, 12 - 14. мај 2016.

ПРОГРАМ

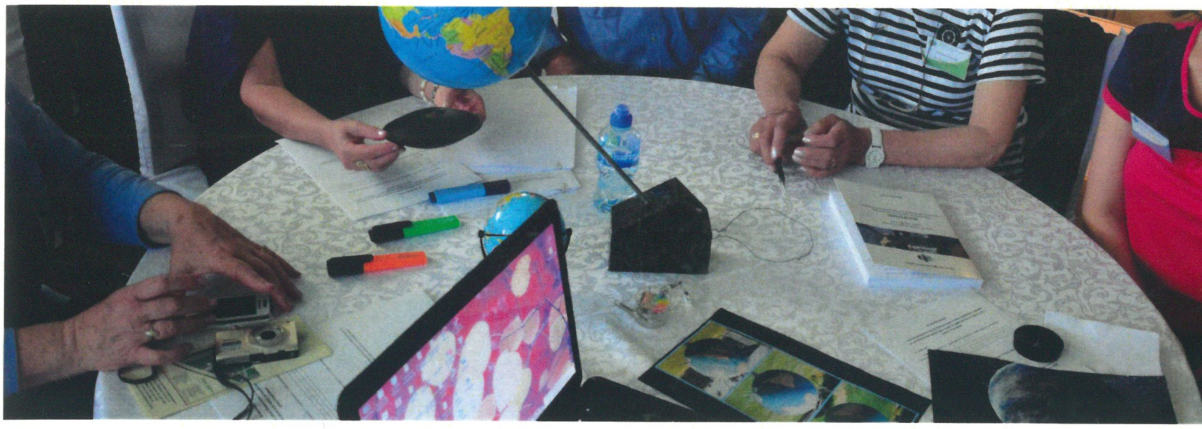
	Четвртак, 12. мај	
08.30-10.00	**Регистрација**	
09.00-09.45	**Састанак редакције часописа Настава физике**	
10.00-10.15	**Свечано отварање** **Председава: Љубиша Нешић** **Говорници: Иван Дојчиновић, Душанка Обадовић, Милан Распоповић**	
10.15-10.45	**И. Бикит, К. Бикит** *Нобелова награда за физику 2015. године: Физика неутрина*	
10.45-11.15	**Б. Драговић** *100 година Ајнштајнове теорије гравитације*	
11.15-11.45	Кафе пауза; ***Предаја пријава за радионице***	
	Председава: Мирјана Поповић-Божић	
11.45-12.15	**Б. Николић** *Гравитациони таласи - од теорије до директне детекције*	
12.15-12.35	***Љ. Нешић, Л. Раденковић*** *Часопис НАСТАВА ФИЗИКЕ и његов значај за методику наставе физике*	
12.35-12.55	**А. Жекић, М. Поповић-Божић, Б. Радиша, Б. Мисаиловић** *Прелиставајући и читајући међународне часописе у области истраживачког образовања у физици*	
12.55-13:15	**Т. Јовановић, Б. Јовановић** *Настава физике на медицинском факултету у Нишу од оснивања до данас*	
13.15-15.00	**Пауза за ручак**	
15.00-16.30	**Радионице - прва група** **I** ЦЕРН Мастерклас, **(Ј. Милисављевић, И. Стојановић, С. Митић, Т. Марковић Топаловић, М. Радић, М. Ристановић)** **II** Научна визуелизација у школском простору и на паметном телефону, **(С. Булат, М. Давидовић, Љ. Иванчевић, М. Јоксимовић, Т. Марковић-Топаловић, М. Поповић-Божић, Б. Стојичић)** **III** Изабране лабораторијске вежбе из физике у гимназији **(М. Ковачевић, С. Ковачевић, А. Марковић, Д. Карајовић)** **IV** Како да искористим знања из наука у образовању за одрживи развој, **(С. Јокић, Љ. Јокић)**	
16.30-17.00	**Кафе пауза**	
17.00-18.30	**Округли сто (две теме по 45 минута свака):** **Тема 1: Популаризација физике (И. Дојчиновић, М. Степић, Т. Продановић, С. Ивковић)** **Тема 2: Реформа образовања и положај физике (Т. М. Топаловић, М. Ковачевић, О. Клисурић, А. Хрлец)**	
18.30-20.00	**Седница савета одељења за основно образовање ДФС**	**Седница савета одељења за средње образовање ДФС**

	Петак, 13. Мај
Време	**Председава: Маја Стојановић**
09.00-09.20	**М. Бошњак Степановић** *Приказ дисертације: Примена истраживачке методе у реализацији физичких садржају почетној настави природних наука*
09.20-09.40	***Г. Хајдуковић Јандрић*** *Приказ дисертације: Развој наставних инструкција у активној настави физике*
09.40-10.00	**Д. Радловић Чубрило** *Приказ дисертације: Ефекти примене мултимедије у настави физике у првом разреду средње стручне школе*
10.00-10.15	***М. Стојановић, М. Ковачевић, Љ. Костић*** *Приказ монографије «Поглавља методике наставе физике» Љубише Нешића*
10.15-10.30	***М. Поповић-Божић, А. Жекић*** *«Колегијално подучавање». Превод књиге «Peer Instruction» од Ерика Мазура*
10.30-11.00	**О. Зајков, Б. Митревски** *Физика и критичко мишљење*
11.00-11.30	**Кафе пауза**
	Председава: Душанка Обадовић
11.30-12.00	**Ф. Соколић** *Што је то свјетлост?*

ISSN 2406-2626

Број 3

НАСТАВА ФИЗИКЕ



Београд 2016.

Настава физике, број 3, 2016, стр 213 - 221

Стручни рад

Гравитациони таласи – од теорије до директне детекције

Бојан Николић

Институт за физику, Универзитет у Београду, Прегревица 118, 11080 Земун

Апстракт. Пре једног века Алберт Ајнштајн формулисао је Општу теорију релативности (ОТР). Једна од последица Опште теорије релативности је постојање гравитационих таласа. У овом раду ћемо дати кратак теоријски преглед о (гравитационим) таласима, а значајну пажњу ћемо посветити свим сада могућим видовима детекције гравитационих таласа са акцентом на недавни успех – директну детекцију гравитационих таласа.

Кључне речи: гравитациони таласи, ОТР, директна детекција.

УВОД

Камен бачен у воду изазива појаву таласа на њеној површини, треперење гласних жица омогућава да чујемо саговорника, земљотреси изазивају појаву цунами таласа итд. Све наведено су примери *механичких* таласа. За простирање механичких таласа је потребна материјална средина. Поремећај настао на једном месту преноси се таласом кроз материјалну средину. Самим тим механички таласи се не простиру кроз вакуум. Положај честице средине у датом тренутку t у тачки $\vec{r}$ (колоквијално "поремећај") $u(\vec{r},t)$ задовољава хомогену таласну једначину

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}-\frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right)u(\vec{r},t)=0\,. \qquad (1)$$

Величина v представља брзину таласа у датој материјалној средини (није једнака брзини честице материјалне средине). Поремећај може бити ортогоналан на правац простирања таласа (трансверзални талас) или колинеаран са правцем простирања таласа (лонгитудиналан талас).

У другој половини 19. века енглески физичар Џејмс Кларк Мексвел је, обједињавајући дотадашња експериментална сазнања, написао једначине електромагнетног поља познате у литератури као Мексвелове једначине. Једноставна анализа тих једначина показује да у простору где нема наелектрисања и струја јачина електричног поља и јачина магнетног поља задовољавају хомогене таласне једначине. Простије речено, око простора у коме су задате расподеле наелектрисања и струја постоји електромагнетно (ЕМ) поље. Енергија ЕМ поља се преноси дуж правца који је ортогоналан на векторе јачине електричног и магнетног поља - ЕМ талас је трансверзалан. За разлику од механичких таласа, ЕМ таласи се простиру и кроз вакуум и то највећом брзином у природи $c = 3\cdot 10^8$ m/s. Херцовим експериментом (1888) потврђено је постојање ЕМ таласа. Питање које су поставили научници тога времена тицало се средине кроз коју се ЕМ талас простире. По аналогији са механичким таласима морала је постојати нека средина која преноси таласе. Тада је уведен појам **етера**. Међутим, Мајкелсон-Морлијев експеримент као и многа унапређења овог експеримента потврдили су да је брзина светлости иста у свим правцима и да не зависи од избора референтног система - једноставније речено, етера, у облику како су га тада научници замишљали, нема. И тада (1905) се родила специјална теорија релативности (СТР).

СТР почива на два постулата. Први се тиче инваријнтности облика физичких закона у односу на избор инерцијалног система референце (то је већ био саставни део Галилејевог принципа релативности), док се другим постулатом потврђује експериментлна чињеница да је брзина светлости независна од избора инерцијалног система референце (Ајнштајн не спомиње експлицитно у својим радовима нити Мајкелсон-Морлијев експеримент нити друге сличне експерименте, али други постулат „признаје“ резултат тих експеримената).

У СТР се спомињу само инерцијални системи референце. Сам Ајнштајн није био задовољан и сматрао је да физички закони морају имати исти облик независно од избора референтног система, инерцијалног или неинерцијалног, тј. увидео је да у "причу" мора да укључи и гравитацију. Математичким језиком речено, какву год трансформацију координата да направимо закони физике морају очувати свој облик (строго математички речено инваријантност на дифеоморфизме [1,2]). И тако је дошао до Опште теорије релативности (ОТР).

Шта је уопште гравитација? По Њутновој теорији гравитација је **сила**. У Њутновој теорији гравитације **маса је извор гравитационог поља**. Свако друго тело одређене масе које се нађе у датом гравитационом пољу је изложено деловању привлачне силе. Њутн је дао аналитички облик за гравитациону интеракцију свом делу Математички принципи природне филозофије (*Philosophiae Naturalis Principia Mathematica*) које је први пут објављено 5. јула 1687. године.

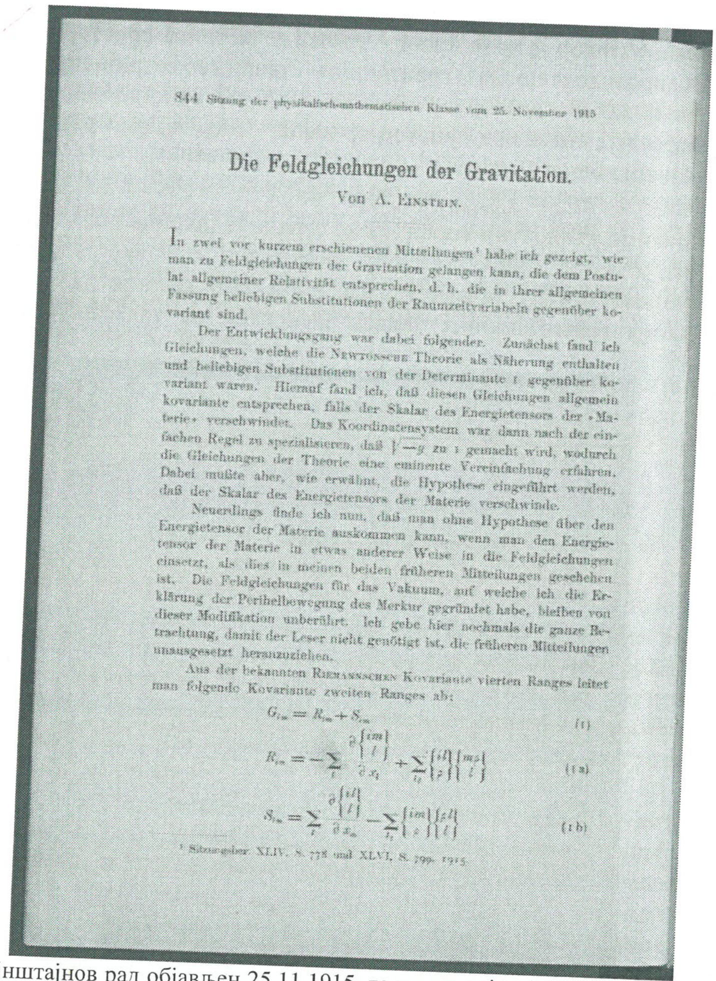
И по том питању није било никаквих квалитативних помака до почетка 20. века. А онда се појавио Алберт Ајнштајн, који је кроз СТР увео у физику обједињеност простора и времена у један просторно-временски континуум тј. време више није параметар већ координата, а са ОТР направио праву револуцију у разумевању гравитације као фундаментлне интеракције у природи.

Овде нећемо улазити у суптилне детаље извођења Ајнштајнових једначина за гравитационо поље. Ајнштајн је једначине извео користећи се законом одржања тензора енергије-импулса као и особинама неких геометријских величина. У савременој литератури извођење иде из одговарајућег дејства применом методе минимума дејства. Било како било, једначине за гравитационо поље су облика

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}, \qquad (2)$$

где је, најгрубље речено, на левој страни ГЕОМЕТРИЈА, а на десној страни МАТЕРИЈА. Ова једначина успоставља везу између геометрије простор-времена и материје која својим присуством "закривљује" тај простор-време. У Ајнштајновој слици гравитација није сила већ **геометрија простор-времена**.

Наравно, добра физичка теорија има особину да објашњава познате феномене и предвиђа неке нове. У оквиру Ајнштајнове теорије успешно је објашњена појава скретања светлосних зрака који пролазе близу Сунца, затим прецесија Меркуровог перихела. Теорија предвиђа постојање сингуларитета (основ за теорију Великог праска) као и црних рупа, за чије постојање постоје индиректни докази. Такође једна од последица ОТР је и постојање *гравитационих таласа*.



844 Sitzung der physikalisch-mathematischen Klasse vom 25. November 1915

Die Feldgleichungen der Gravitation.

Von A. Einstein.

In zwei vor kurzem erschienenen Mitteilungen[1] habe ich gezeigt, wie man zu Feldgleichungen der Gravitation gelangen kann, die dem Postulat allgemeiner Relativität entsprechen, d. h. die in ihrer allgemeinen Fassung beliebigen Substitutionen der Raumzeitvariabeln gegenüber kovariant sind.

Der Entwicklungsgang war dabei folgender. Zunächst fand ich Gleichungen, welche die Newtonsche Theorie als Näherung enthalten und beliebigen Substitutionen von der Determinante 1 gegenüber kovariant waren. Hierauf fand ich, daß diesen Gleichungen allgemein kovariante entsprechen, falls der Skalar des Energietensors der »Materie« verschwindet. Das Koordinatensystem war dann nach der einfachen Regel zu spezialisieren, daß $\sqrt{-g}$ zu 1 gemacht wird, wodurch die Gleichungen der Theorie eine eminente Vereinfachung erfahren. Dabei mußte aber, wie erwähnt, die Hypothese eingeführt werden, daß der Skalar des Energietensors der Materie verschwinde.

Neuerdings finde ich nun, daß man ohne Hypothese über den Energietensor der Materie auskommen kann, wenn man den Energietensor der Materie in etwas anderer Weise in die Feldgleichungen einsetzt, als dies in meinen beiden früheren Mitteilungen geschehen ist. Die Feldgleichungen für das Vakuum, auf welche ich die Erklärung der Perihelbewegung des Merkur gegründet habe, bleiben von dieser Modifikation unberührt. Ich gebe hier nochmals die ganze Betrachtung, damit der Leser nicht genötigt ist, die früheren Mitteilungen unausgesetzt heranzuziehen.

Aus der bekannten Riemannschen Kovariante vierten Ranges leitet man folgende Kovariante zweiten Ranges ab:

$$G_{im} = R_{im} + S_{im} \qquad (1)$$

$$R_{im} = -\sum_{l} \frac{\partial \begin{Bmatrix} im \\ l \end{Bmatrix}}{\partial x_l} + \sum_{l\rho} \begin{Bmatrix} il \\ \rho \end{Bmatrix} \begin{Bmatrix} m\rho \\ l \end{Bmatrix} \qquad (1a)$$

$$S_{im} = \sum_{l} \frac{\partial \begin{Bmatrix} il \\ l \end{Bmatrix}}{\partial x_m} - \sum_{l\rho} \begin{Bmatrix} im \\ \rho \end{Bmatrix} \begin{Bmatrix} \rho l \\ l \end{Bmatrix} \qquad (1b)$$

[1] Sitzungsber. XLIV, S. 778 und XLVI, S. 799, 1915.

СЛИКА 1. Ајнштајнов рад објављен 25.11.1915. године у којем је заснована ОТР и изведена чувена једначина.

ГРАВИТАЦИОНИ ТАЛАСИ

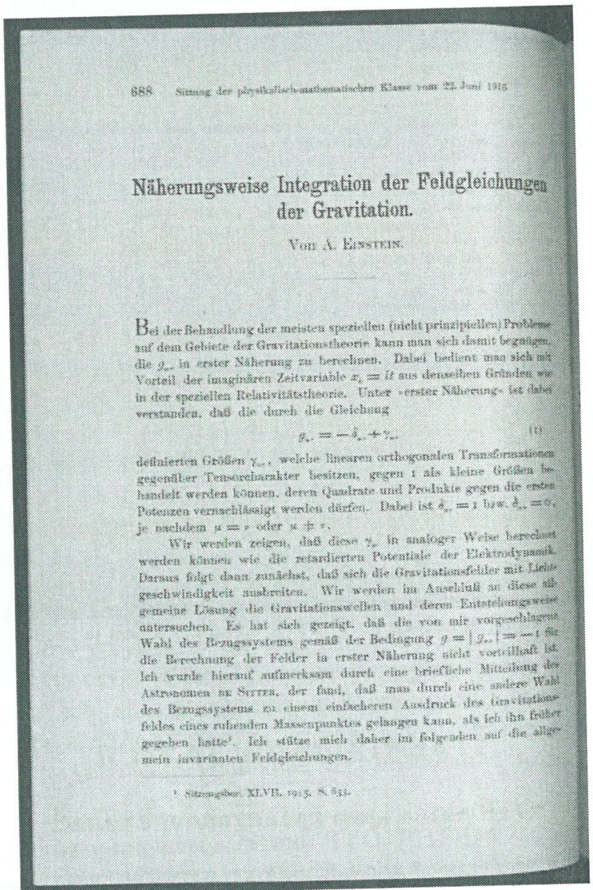
ОТР дозвољава гравитационе таласе

Математички доказ постојања гравитационих таласа у ОТР је врло једноставан. Уколико посматрамо Ајнштајнове једначине у празном простору далеко од маса, онда се испоставља да метрика простора задовољава таласну једначину. Добија се да су гравитациони таласи трансверзални таласи који се простиру брзином светлости у вакуму. У случају механичких таласа материјална средина се таласа, док у случају ЕМ таласа долази до таласања електричног и магнетног поља. Логично питање које се намеће код гравитационих таласа је шта се то таласа?

Формалан одговор је врло прост – таласа се метрика просторно-временског континума. С обзиром да је по ОТР грвитација у ствари геометрија простор-времена онда је мало „физичкији“ одговор – таласа се сам просторно-временски континум. А како се манифестује таласање простор-времена? Посматрајмо растојање између две инфинитезимално блиске тачке

$$ds^2 = \sum_{\mu,\nu=0}^{3} g_{\mu\nu}(x)dx^\mu dx^\nu. \quad (3)$$

Уколико интегралимо квадратни корен десне стране једначине добићемо растојање између две тачке у закривљеном простору. Очигледно је да ако се метрика таласа онда се и растојање између тачака таласа. Видећемо касније да су све методе директне детекције гравитационих таласа засноване на таласању растојања тј. дужине.


688 Sitzung der physikalisch-mathematischen Klasse vom 22. Juni 1916

Näherungsweise Integration der Feldgleichungen der Gravitation.

Von A. Einstein.

Bei der Behandlung der meisten speziellen (nicht prinzipiellen) Probleme auf dem Gebiete der Gravitationstheorie kann man sich damit begnügen, die $g_{\mu\nu}$ in erster Näherung zu berechnen. Dabei bedient man sich mit Vorteil der imaginären Zeitvariable $x_4 = it$ aus denselben Gründen wie in der speziellen Relativitätstheorie. Unter »erster Näherung« ist dabei verstanden, daß die durch die Gleichung

$$g_{\mu\nu} = -\delta_{\mu\nu} + \gamma_{\mu\nu} \quad (1)$$

definierten Größen $\gamma_{\mu\nu}$, welche linearen orthogonalen Transformationen gegenüber Tensorcharakter besitzen, gegen 1 als kleine Größen behandelt werden können, deren Quadrate und Produkte gegen die ersten Potenzen vernachlässigt werden dürfen. Dabei ist $\delta_{\mu\nu} = 1$ bzw. $\delta_{\mu\nu} = 0$, je nachdem $\mu = \nu$ oder $\mu \neq \nu$.

Wir werden zeigen, daß diese $\gamma_{\mu\nu}$ in analoger Weise berechnet werden können wie die retardierten Potentiale der Elektrodynamik. Daraus folgt dann zunächst, daß sich die Gravitationsfelder mit Lichtgeschwindigkeit ausbreiten. Wir werden im Anschluß an diese allgemeine Lösung die Gravitationswellen und deren Entstehungsweise untersuchen. Es hat sich gezeigt, daß die von mir vorgeschlagene Wahl des Bezugssystems gemäß der Bedingung $g = |g_{\mu\nu}| = -1$ für die Berechnung der Felder in erster Näherung nicht vorteilhaft ist. Ich wurde hierauf aufmerksam durch eine briefliche Mitteilung des Astronomen de Sitter, der fand, daß man durch eine andere Wahl des Bezugssystems zu einem einfacheren Ausdruck des Gravitationsfeldes eines ruhenden Massenpunktes gelangen kann, als ich ihn früher gegeben hatte[1]. Ich stütze mich daher im folgenden auf die allgemein invarianten Feldgleichungen.

[1] Sitzungsber. XLVII, 1915, S. 833.

СЛИКА 2. *Приближно решавање једначина гравитационог поља* – математички доказ постојања гравитационих таласа (Ајнштајн, 22.06.1916. година).

ДЕТЕКЦИЈА ГРАВИТАЦИОНИХ ТАЛАСА

Пре него што пређемо на разматрање свих видова директне детекције као и анализе недавног директног мерења гравитационих таласа, потребно је рећи да је постојање гравитационих таласа индиректно потврђено 1993. године.

Године 1974. Расел Алан Халс и руководилац његове докторске тезе Џозеф Хутон Тејлор Јуниор су открили један бинарни пулсар који се састоји од пулсара (неутронске звезде) и пратеће звезде. Овај бинарни пулсар губи енергију на начин како и предвиђа ОТР па самим тим ово откриће је истовремено индиректни доказ постојања гравитационих таласа. За ово откриће Расел и Халс су добили Нобелову награду за физику 1993. године.

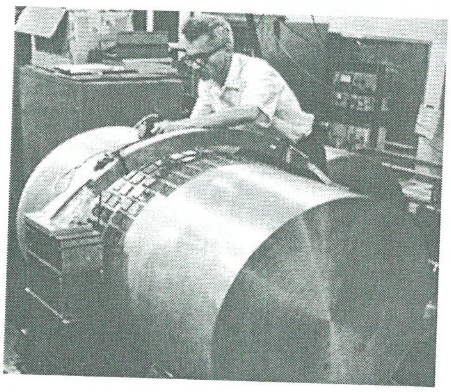
Детектори који се користе за директну детекцију гравитационих таласа деле се у три групе: механички, интерферометарски и високофреквентни детектори.

Механички детектори

__Веберове шипке__

Једноставан уређај за детекцију очекиваног таласног кретања је тзв. Веберова шипка - велика, чврста метална шипка изолована од спољашњих вибрација. Овај тип детектора је био први који је коришћен од стране конструктора Џозефа Вебера са Универзитета Мериленд. Он је чак тврдио да је детектовао гравитационе таласе, али су његови резултати доведени у сумњу због начина обраде података. Испоставило се на крају да је Веберова детекција гравитационих таласа фингирана због потреба финансирања пројекта.

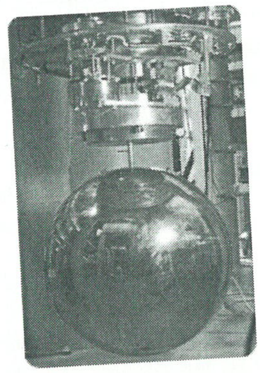
Принцип рада овог детектора је једноставан. Упадни гравитациони талас побуђује резонантно осциловање шипке, а шипка онда својим осциловањем појачава тај ефекат на детектабилни ниво. Савремене варијанте оваквих детектора су охлађене до екстремно ниских температура и опремљене квантним интерференционим уређајима за детекцију вибрација (на пример, ALLEGRO). Проблем са овим детекторима што се они могу користити само за врло јаке гравитационе таласе.



СЛИКА 3. Џозеф Вебер у својој лабораторији 1965. године

MiniGRAIL

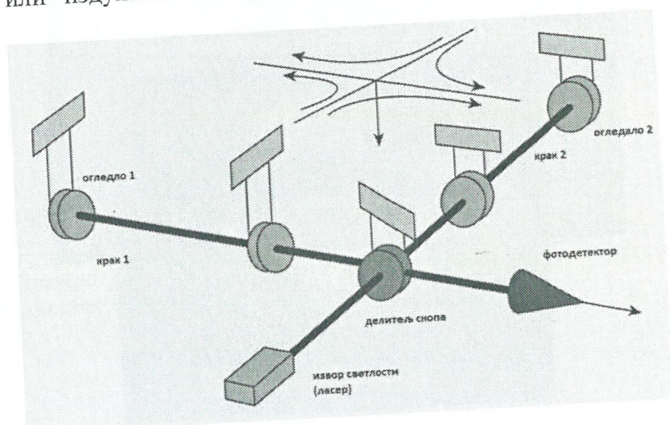
MiniGRAIL је антена за детекцију гравитационих таласа сферног облика. Ова антена се налази на Универзитету у Лајдену (Холандија), а састоји се од сфере масе 1150 килограма охлађене на температуру 20 mK. Облик сфере омогућава детекцију из свих праваца. Фреквенције које овај детектор најбоље "хвата" су интервалу 2-4 kHz, па је погодан за детекцију гравитационих таласа који настају у бинарним пулсарима и спајањем мањих црних рупа. Сличног типа је ултрахладни детектор AURIGA који се налази на INFN-у у Италији. Он се састоји од алуминијумског цилиндра дужине 3 метра који је охлађен на температуру реда величине mK.



СЛИКА 4. Детектор MiniGRAIL

Интерферометарски детектори

Ова група детектора користи ласерску интерферометрију за детекцију гравитационих таласа. Светлост крећући се кроз простор прати закривљење просторно-временског континума. Принцип рада ових детектора је да се измери ефекат интерференције ласерских зрака при чему је путна разлика настала "скраћивањем" или "издуживањем" простора због таласања.



СЛИКА 5. Принцип рада интерферометарског детектора

Данас постоје само интерферометри на Земљи. Тренутно најосетљивији интерферометарски детектор је LIGO (Laser Interferometer Gravitational Wave Observatory). LIGO има три детектора: један је у Ливингстону (држава Луизијана) а друга два су у Хенфорду (држава Вашингтон). Сви они се састоје од по два велика крака дужине 2-4 километра који су под правим углом. Ласерски зраци путују унутар кракова у цевима дијаметра 1 метар. Промене у дужини које ласерски зрак прелази услед проласка гравитационог таласа би у принципу требало да региструје детектор у виду неке (ласерске) интерференционе слике.

Интерферометарски детектори имају и своја ограничења. Прва од њих је шум који настаје као последица тога што ласерски извор производи фотоне у произвољним тренуцима. Ако уз то користимо и мало јачи ласер онда сами фотони својим импулсом могу да уздрмају детекторска огледала. Други проблем је проблем Брауновог кретања, а ни сеизмички шум се не може занемарити.

Због проблема које имају земаљски детектори, планира се и градња детектора у орбити око Земље (eLISA, пројекат започет децембра 2015. године). Три сателита би формирала троугао при чему би свака страница била око 5 милиона километара. Тиме се добија добар вакуум, али и даље остаје проблем фотонског шума као и проблем са космичким зрачењем.

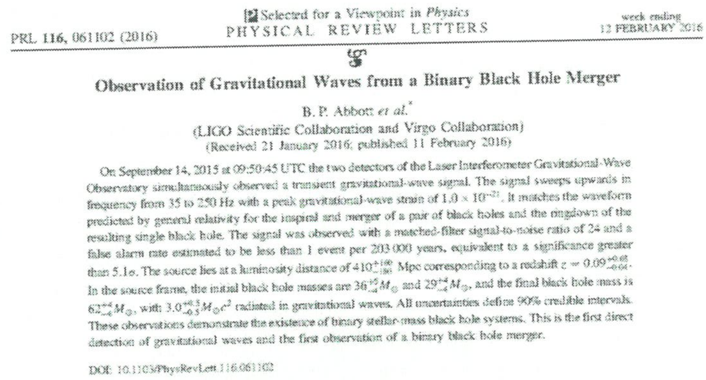
Високофреквентни детектори

Тренутно постоје два оперативна детектора који раде на горњој граници спектра (10^{-7}-10^{5} Hz). Један је на Универзитету у Бирмингему (Енглеска) а други је на INFN-у у Ђенови (Италија). Трећи се гради на Универзитету у Чонкингу (Кина). Детектор у Бирмингему мери промене у стању поларизације микроталасног зрака који кружи по кругу пречника око 1 метра. Детектор у Ђенови је резонантна антена која се састоји од два спрегнута сферна суперпроводна хармонијска оцилатора пречника неколико центиметара. Осцилатори када нису спрегнути имају резонантне фреквенције које су скоро једнаке. Кинески детектор би требало да буде у стању да детектује таласе фреквенције реда 10 GHz.

ДИРЕКТНА ДЕТЕКЦИЈА ГРВИТАЦИОНИХ ТАЛАСА

Група научника из две велике колаборације, LIGO и VIRGO, је објавила 11. фебруара 2016. године [3] да је обављена успешна директна детекција гравитационих таласа. Физички све се одиграло на америчком делу велике колаборације (LIGO).

Дана 14. септембра 2015. године детектори ове колаборације у Хенфорду (Вашингтон) и Ливингстону (Лујзијана) детектовали су гравитациони талас који је настао спајањем две црне рупе, једна масе 36 соларних маса а друга 29 соларних маса. Настала је црна рупа масе 62 соларне масе а 3 соларне масе су израчене у виду гравитационих таласа. Овакав резултат као и профили детектованих сигнала су (у границама грешке) у складу са предвиђањима ОТР. Овај експеримент је потврдио *постојање бинарних система црних рупа, омогућио директну детекцију гравитационих таласа и први детектовао спајање црних рупа*.



PRL 116, 061102 (2016) — Selected for a Viewpoint in *Physics* — PHYSICAL REVIEW LETTERS — week ending 12 FEBRUARY 2016

Observation of Gravitational Waves from a Binary Black Hole Merger

B. P. Abbott *et al.**

(LIGO Scientific Collaboration and Virgo Collaboration)

(Received 21 January 2016; published 11 February 2016)

On September 14, 2015 at 09:50:45 UTC the two detectors of the Laser Interferometer Gravitational-Wave Observatory simultaneously observed a transient gravitational-wave signal. The signal sweeps upwards in frequency from 35 to 250 Hz with a peak gravitational-wave strain of 1.0×10^{-21}. It matches the waveform predicted by general relativity for the inspiral and merger of a pair of black holes and the ringdown of the resulting single black hole. The signal was observed with a matched-filter signal-to-noise ratio of 24 and a false alarm rate estimated to be less than 1 event per 203 000 years, equivalent to a significance greater than 5.1σ. The source lies at a luminosity distance of 410^{+160}_{-180} Mpc corresponding to a redshift $z = 0.09^{+0.03}_{-0.04}$. In the source frame, the initial black hole masses are $36^{+5}_{-4} M_{\odot}$ and $29^{+4}_{-4} M_{\odot}$, and the final black hole mass is $62^{+4}_{-4} M_{\odot}$, with $3.0^{+0.5}_{-0.5} M_{\odot} c^2$ radiated in gravitational waves. All uncertainties define 90% credible intervals. These observations demonstrate the existence of binary stellar-mass black hole systems. This is the first direct detection of gravitational waves and the first observation of a binary black hole merger.

DOI: 10.1103/PhysRevLett.116.061102

СЛИКА 6. Апстракт рада [3] у коме је објављена директна детекција гравитационих таласа

Апаратура на којој је извршена детекција је унапређена верзија почетног LIGO детектора (AdvancedLIGO). Побољшања која су урађена првенствено се тичу повећања осетљивости сензора као и умањењу постојећих шумова.

Очекује се да детектори Advanced VIRGO, KAGRA као и могући трећи LIGO детектор у Индији дају додатну потврду овом открићу као и да подигну ниво прецизности и тачности мерења.

ЗАКЉУЧАК

ОТР је у времену када је настала (Први светски рат у пуном јеку!) успела да објасни неке феномене који су били познати научницима попут скретања светлосних зрака у близини великих звезда и прецесију Меркуровог перихела. Свака "права" физичка теорија не објашњава само постојеће и познате феномене већ предвиђа и неке нове. Гравитациони таласи су један од тих феномена. Постојање гравитационих таласа теоријски је поткрепљено Општом теоријом релативности јер следи из Ајнштајнових једначина гравитационог поља. Откриће бинарних пулсара (систем две неутронске звезде), који губе енергију потпуно у складу са предвиђањима ОТР, дало је експериментални основ постојању гравитационих таласа. Са изградњом интерферометарских детектора кренуло се у коначну потрагу за гравитационим таласима. Принципијелно није било препрека и све је било питање прецизности апаратуре. Коначно су у јесен 2015. године научници успели да детектују гравитационе таласе који су настали у једном врло интензивном догађају – судару црних рупа. Ово откриће даје наду да се могу детектовати гравитациони таласи настали после Великог праска што би у „неку руку“ био доказ да се тај Прасак стварно и десио.

ЛИТЕРАТУРА

1. C. M. Misner, K. S. Thorne, J. A. Wheeler, *Gravitation*, W. H. Freeman and Co., San Francisco, 1973.
2. L. D. Landau, E. M. Lifshitz, *The Classical Theory of Fields*, Pergsmon Press, 1971.

3. B. P. Abbott et al., *Observation of Gravitational Waves from a Binary Black Hole Merger*, Phys. Rev. Lett. **116** 061102 (2016).

Gravitational waves – from theory to direct detection

Bojan Nikolic

Institute of Physics, University of Belgrade, Pregrevica 118, Zemun

Abstract. A century ago Albert Einstein formulated General Theory of Relativity (GR). The existence of gravitational waves is one of the consequences of the GR. In this article we will give a short theoretical review about (gravitational) waves, and later we will dedicate the great attention to all known types of detection of the gravitational waves with accent on recent success – direct detection of gravitational waves.

Key words: gravitational waves, GR, direct detection.

Subject **Publikation of your recent paper in Fortschritte der Physik**
From Dieter Luest <dieter.luest@lmu.de>
To <bnikolic@ipb.ac.rs>, <dobric@ipb.ac.rs>
Date 2018-01-29 10:02

Dear Dr. Nikolic and dear Dr. Obric,

I am just reading your interesting paper "Noncommutativity and nonassociativity of closed bosonic string .." (arXiv:1801.08772).
Would you be interested to publish this paper in "Fortschritte der Physik" ?
As editor of the journal I would be very happy to publish this paper and I can promise very speedy publication.

For your information, you can go to the web-side of the journal to see, which papers are currently published in the journal:

http://onlinelibrary.wiley.com/journal/10.1002/(ISSN)1521-3978/earlyview

Best regards, Dieter Lüst

****
* Prof. Dieter Luest
* Max-Planck-Institute for Physics *
* Foehringer Ring 6 *
* D-80805 Muenchen; Germany *
* Phone: +49-89-32354 282 *
* Fax: +49-89-32354 501 *
* E-mail: luest@mppmu.mpg.de *
* wwwth.mppmu.mpg.de/members/luest/ *
*
* and
*
* Ludwig-Maximilians-Universitaet *
* Arnold-Sommerfeld-Center *
* Theresienstrasse 37 *
* D-80333 Muenchen, Germany *
* Phone: +49-89-2180 4372
* Fax: +49-89-2180 4186 *
* E-mail: luest@theorie.physik.uni-muenchen.de
* www.theorie.physik.uni-muenchen.de/~luest/ *
* *
****

ЗАДУЖБИНА ИЛИЈЕ М. КОЛАРЦА

основана 1878.

ЦЕНТАР ЗА ПРЕДАВАЧКУ ДЕЛАТНОСТ

Циклус предавања

КВАНТНА ГРАВИТАЦИЈА – СВЕТИ ГРАЛ САВРЕМЕНЕ ФИЗИКЕ

1. Зашто квантна механика

Др Игор Салом

26. септембар 2023. у 18.00

2. Зашто општа релативност

Др Данијел Обрић

3. октобар 2023. у 18.00

3. Зашто теорија поља

Др Бојан Николић

10. октобар 2023. у 18.00

4. Зашто поља у кривом простору

Др Марко Војиновић

17. октобар 2023. у 18.00

5. Зашто квантна гравитација

Др Тијана Раденковић

24. октобар 2023. у 18.00

Мала сала Коларчеве задужбине

Циклус је реализован у сарадњи са пројектом „Квантна гравитација из виших гејџ теорија“ (QGHG-2021), број 7745968 програма ИДЕЈЕ Фонда за науку

тел. 2637-609, 2638-472; факс: 3031-711
www.kolarac.rs; e-mail: predavanja@kolarac.rs

Програме подржавају **Секретаријат за културу Скупштине града Београда и Министарство за науку Републике Србије**

LMU LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN

ARNOLD SOMMERFELD CENTER FOR THEORETICAL PHYSICS

Dr. Michael Haack
Theresienstr. 37
80333 München
Germany
Telefon: 089/2180-4377
Telefax: 089/2180-4186
michael.haack@lmu.de

Dr. Bojan Nikolic
Institute of Physics
Pregrevica 118
11080 Belgrade
Serbia

January 17, 2012

Invitation for Dr. Bojan Nikolic

To whom it may concern

It is my pleasure to invite Dr. Bojan Nikolic from the Institute of Physics, Belgrade, to visit the Arnold Sommerfeld Center for Theoretical Physics (ASC) at the LMU in Munich.

Dr. Nikolic is being invited to do research within Prof. Lüst's group for mathematical physics and string theory. It is agreed that Dr. Nikolic will visit the ASC in the period of six months from July 1 to December 24, 2012, provided his local expenses in Munich, as well as the required health insurance, will be covered by the home institute in Belgrade.

If you need any further information, please do not hesitate to contact me.

Sincerely,

Michael Haack
Scientific Manager of the Arnold Sommerfeld Center

млади**физичар**

Часопис за све пријатеље физике и фудбала
Издавач Друштво физичара Србије
Школска година 2005/06
http://mf.dfs.org.yu
Цена: 175 дин

103

Тема броја **ФИЗИКА И ФУДБАЛ**

ISSN 0351-5575
9 770351 557003

младифизичар

Садржај 103

Примена физике на фудбалском терену

Млади физичар у овом броју борави у предсократској Грчкој. Двадесет векова пре заснивања модерне физике и 150 година пре настанка Аристотеловог списа “Физика” по коме је наука о природи добила име.

младифизичар
Часопис за све пријатеље физике

Издавач
Друштво физичара Србије
Прегревица 118, 11080 Београд-Земун
тел: 011/3160-260 лок. 166, Факс: 011/3162-190
mail: mf@dfs.org.yu
web: http://mf.dfs.org.yu

За издавача:
проф. др Илија Савић, Председник Друштва физичара Србије

Главни уредник и арт директор
Душко Латас

Заменици уредника
Бранислав Цветковић и Зорица Пајовић

Уредник web издања
Антун Балаж

Редакција
Слободан Бубњевић, Марко Војиновић, Никола Веселиновић, Ненад Вукмировић, Марија Димитријевић, Драгана Илић, Владимир Јовановић, Славица Малетић, Катарина Матић, Предраг Миленовић, Марија Митровић, Бојан Николић, Новица Пауновић, Зоран Риствојевић, Игор Салом, Владимир Шамара, Милован Шуваков

Спољни сарадници
проф. др Јарослав Лабат, проф. др Дарко Капор, др Александар Богојевић, Наташа Каделбург, Наташа Чалуковић, Славиша Станковић, Дејан Крунић, Слободанка Раковић, Игор Ђорић, Милан Жежељ, Огњен Илић, Добросав Обрадовић, Коста Ристић, Димитрије Малетић, Феђа Хаџић, Јелена Станковић, Милија Јовичић

Претплата
Весна Вучић (011)3160-260/143

Штампа
САВПО, Стара Пазова

ISSN 0351-5575
COBISS.SR-ID 53980423

Претходни уредници "Младог физичара":
1976/77 Ђорђе Басарић и Слободан Жегарац; 1977/78 Душан Ристановић и Драшко Грујић; 1978/79-1981/82 Љубо Ристовски и Душан Коледин; 1982/83 Душан Коледин, Драгана Поповић и Јаблан Дојчиловић; 1983/84-1986/87 Драшко Грујић; 1991/92-1993/94 Јаблан Дојчиловић; 1994/95-1996/97 Томислав Петровић; 1997/98 Александар Стаматовић; 1998/99 Душан Арсеновић; 1998/99-2003/04 Драган Маркушев

"Понекад видите прелепе људе без мозга. Понекад имате ружне људе који су интелигентни, попут научника. Наш терен је управо такав. На први поглед делује срамотно, али се лопта по њему креће нормалном брзином." Овако је тренер Челсија Хозе Муриньо одговорио челницима Барселоне, који су имали примедбе на стање у коме се налази травната подлога на Станфорд Брицу, стадиону Челсија, непосредно пред четвртфинални меч овогодишње Лиге шампиона. С друге стране, аналитичари телевизијског магазина фудбалске Лиге шампиона израчунали су да је чак 90% победника Лиге или Купа шампиона у последњих 20 година носило дресове скроз или делимично црвене боје. Остатак су освојили тимови са потпуно или делимично белим дресовима. Физичари би рекли да успех екипе у Лиги шампиона зависи од таласне дужине светлости коју емитује материјал од којег је начињен дрес, а и сами тренери виде да фудбалски терен личи на научнике. Кад се томе дода да ће 2006. година бити обележена Светским фудбалским првенством у Немачкој, као и сећањем на нашег сународника који нас је најбоље представљао у свету својом изузетном игром главом, било је јасно да морамо мало детаљније да проучимо везе између фудбала и физике. Као резултат тога, настао је овај број Младог физичара - на задовољство свих пријатеља физике и фудбала.

Dusko Latas

Највећа галаксија је централна галаксија галактичког грозда Абел 2029 удаљеног 1070 милиона светлосних година у правцу сазвежђа Девице. Ова галаксија има пречник од 5.6 милиона светлосних година што је 80 пута више од пречника наше галаксије, док јој је маса сразмерно томе вероватно пола милиона пута већа и вероватно садржи 100 милиона милијарди звезда. Ако се неко још увек пита има ли живота у свемиру, овде ће га сигурно наћи. Можда је Џорџ Лукас имао у виду баш ову " galaxy far, far away "...

Најсветлији објект у свемиру је квазар APM08279+5255 откривен марта 1998. године у сазвежђу Стрелца. Процењено је да је његова сјајност између 4 и 5 милиона милијарди пута већа од сјајности Сунца или као 50000 просечних галаксија. Астрофизичари верују да су квазари супермасивне црне рупе које гутају колосалне количине материје и делимично је претварају у енергију. Ово је једини познати начин на који би се могла објаснити екстремна сјајност квазара. Овај квазар би морао да прогута читаву једну планету величине Сатурна сваке секунде и претвори је у енергију да би могао да има оволику снагу.

Најсјајнија звезда у нашој галаксији је LBV 1806-20 удаљена 45000 светлосних година од земље, и има сјајност која је процењена на 5 до 40 милиона сјајности Сунца. Маса јој је најмање 150 већа од Сунчеве а пречник најмање 200 пута већи. Ова звезда је толико топла да већину зрачења заправо израчи у ултраљубичастом и делимично чак у рентгенском делу спектра. И поред своје енормне масе, због оволике потрошње енергије ова звезда има веома кратак животни век, реда милион година, што је око 10000 пута мање од процењеног животног века нашег Сунца.

Највиша планина у Сунчевом систему је Олимпус Монс на Марсу. Висина његовог врха од подножја (обзиром да се на Марсу наравно не може дефинисати "надморска" висина) износи 25 км, што је скоро три пута више од Монт Евереста. Но и поред велике висине има веома благ нагиб својих падина обзиром да му је пречник преко двадесет пута већи од висине. Ова планина је чисто вулканског порекла, а оволику висину омогућава нижа марсовска гравитација, 37% земаљске. Оволика планина не би могла постојати на Земљи, јер би се урушила под сопственом тежином, тачније услед огромног притиска у подножју би почела да "тече", слично глечерима.

Најближа црна рупа (која је позната) је удаљена "свега" 1600 светлосних година од Земље и позната је под ознаком V4641 Sgr. Открили су је астрономи са Масачусетског технолошког института јануара 2000. Црна рупа сама по себи јасно не може бити виђена, и једино може бити откривена на основу понашања материје у њеној близини, нпр. ако има звезду пратиоца и ако има довољно материје коју апсорбује при чему се обично јавља јак одлив зрачења, нпр. рентгентског. Сасвим је извесно да постоје и црне рупе које су нам ближе али које су усамљене и као такве немогуће за детекцију било којом познатом техником.

Најсјајнија супернова модерних времена је SN 1987A која је 1987. године експлодирала у Великом Магелановом Облаку, на удаљености од 164000 светлосних година. Иако су већина претходно виђених супернова, нпр. супернове из 1054. или 1604. године, биле знатно сјајније обзиром да су се налазиле у нашој Галаксији, ова супернова је прва супернова од открића телескопа која се појавила у непосредној близини (Велики Магеланов Облак је мала сателитска галаксија наше Галаксије). Све остале супернове су виђене у удаљеним галаксијама. На врхунцу сјајности имала је звездану магнитуду 3 и могла се јасно видети голим оком. Супернове су главни извор свих елемената тежих од гвожђа. Огроман број атома наших тела је некада давно настао у центру супернова, тако да смо сви ми на неки начин "звездана деца".

Рекорде припремио: Новица Пауновић

Физика и фудбал

СВЕ НАС СПАЈА ФУДБАЛ

млади**физичар**

98

Часопис за све пријатеље физике и новца
Издавач Друштво физичара Србије
Школска година 2004/05
http://mf.dfs.org.yu

ISSN 0351-5575
9 770351 557003

Тема броја ФИЗИКА И НОВАЦ

младифизичар

Садржај 98

млади**физичар** Часопис за све пријатеље физике
98 Откријте занимљиву физику

Издавач
ДРУШТВО ФИЗИЧАРА СРБИЈЕ
Прегревица 118
11080 Београд-Земун
Тел: 011/3160-260 лок. 166
Факс: 011/3162-190
Mail: mf@dfs.org.yu
Web: http://mf.dfs.org.yu

Главни и одговорни уредник
Душко ЛАТАС

Заменици уредника
Бранислав ЦВЕТКОВИЋ и Зорица ПАЈОВИЋ

Уредник интернет издања
Антун БАЛАЖ

Редакција
Слободан БУБЊЕВИЋ, Марко ВОЈИНОВИЋ, Ненад ВУКМИРОВИЋ, Драгана ИЛИЋ, Владимир ЈОВАНОВИЋ, Катарина МАТИЋ, Предраг МИЛЕНОВИЋ, Марија МИТРОВИЋ, Бојан НИКОЛИЋ, Новица ПАУНОВИЋ, Зоран РИСТИВОЈЕВИЋ, Славица СПАСОВИЋ, Владимир ШАМАРА, Милован ШУВАКОВ
Спољни сарадници
проф. др Јарослав ЛАБАТ, проф. др Дарко КАПОР, проф. др Милан ДИМИТРИЈЕВИЋ, проф. др Вукота БАБОВИЋ, др Александар БОГОЈЕВИЋ, др Борко ВУЈИЧИЋ, др Горан ЂОРЂЕВИЋ, др Љубиша НЕШИЋ, Наташа КАДЕЛБУРГ, Наташа ЧАЛУКОВИЋ, Славиша СТАНКОВИЋ, Дејан КРУНИЋ, Милан ЖЕЖЕЉ, Огњен ИЛИЋ, Никола КЕКИЋ, Добросав ОБРАДОВИЋ, Коста РИСТИЋ

Претплата
Весна ВУЧИЋ (011)3160-260/143

YU ISSN 0351-5575

За издавача:
проф. др Илија САВИЋ, Председник Друштва физичара Србије

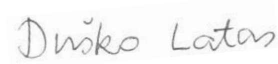
Претходни уредници “Младог физичара”:
1976/77 Ђорђе БАСАРИЋ и Слободан ЖЕГАРАЦ; 1977/78 Душан РИСТАНОВИЋ и Драшко ГРУЈИЋ; 1978/79-1981/82 Љубо РИСТОВСКИ и Душан КОЛЕДИН; 1982/83 Душан КОЛЕДИН, Драгана ПОПОВИЋ и Јаблан ДОЈЧИЛОВИЋ; 1983/84-1986/87 Драшко ГРУЈИЋ; 1991/92-1993/94 Јаблан ДОЈЧИЛОВИЋ; 1994/95-1996/97 Томислав ПЕТРОВИЋ; 1997/98 Александар СТАМАТОВИЋ; 1998/99 Душан АРСЕНОВИЋ; 1998/99-2003/04 Драган МАРКУШЕВ

Часопис је ослобођен пореза на промет на основу мишљења Министарства просвете Републике Србије бр. 443-00-14/2000-01 од 29. марта 2000.

Сваки пут кад платите додатни час физике или купите карту за биоскоп ви користите новац. Кад мало порастете, почећете и да га зарађујете. Тада ћете схватити да је новац врло важан. А шта је заправо новац? Да ли сте се икад питали како се праве новчанице и како се спречава њихово кривотворење? Ко су људи чији се портрети налазе на њима? Пажљивији међу вама су сигурно приметили да на новчаницама има и формула. Шта оне значе?

Припремајући овај броја “Младог физичара” трудили смо се да одговоримо на ова питања. Верујем да ћете уживати читајући садржај који смо спремили и да ће вас то обогатити. Нећете сазнати којих седам бројева ће бити извучени у следећем колу лотоа и прикупити гомилу новца, ваше богатство ће бити другачије. А какво је оно - откријте сами.

Кад смо већ код новца, морам да вас обавестимо да ће Млади физичар убудуће бити мало скупљи. Сви ви који сте се раније претплатили добићете на време своје примерке, само за нове претплатнике важе нове цене. Ипак, морате призанати да један примерак још увек кошта мање од биоскопске карте. А кад смо већ код биоскопа, вероватно сте гледали “Анђеле 2”. Да ли сте приметили ко је на крају први пољубио Софију? Па наравно, наш редовни читаоц - Марко!



100 година од Ајнштајнових открића

Ове године се навршава тачно сто година од чувене, и по мишљењу многих, за физику чудесне 1905. Тада је Алберт Ајнштајн објавио три научна рада која ће, како се касније испоставило, променити поглед на свет и изазвати револуцију у науци. Уједињене нације су због тих радова прогласиле ову годину за светску годину физике, коју се у неким земљама назива и Ајнштајновом годином.

Први Ајнштајнов рад је био о фотоелектричном ефекту у коме је доказано да се светлост може понашати као сноп честица са дискретним енергијама. У овом раду је уведен појам кванта енергије као дискретне "порције" енергије која се може предати физичком објетку. Други рад се тицао Брауновог кретања и нудио је експерименталну потврду за теорију о топлоти. Трећи рад је увео специјалну теорију релативности и уздрмао темеље, до тада опште прихваћене, Њутнове механике у коју се безрезервно веровало. За рад о фотоелектричном ефекту Ајнштајн је награђен Нобеловом наградом 1921. године.

Широм света се ова година се обележава на различите начине.

Немачка је 19. јануара започела обележавање. Том приликом је немачки канцелар Герхард Шредер у Берлину отворио низ конгреса, изложби и других манифестација чији је циљ да Ајнштајново дело и његов лик приближи народу. Иако је Ајнштајн 1933. године, због доласка нациста на власт, напустио заувек своју родну земљу, Немачка жели да на овај начин ода почаст "слободном духу, миротворцу, грађанину света и визионару".

Почетак године Ајнштајна званично је обележен и у Великој Британији. Свечаност је одржана у Музеју науке у Лондону уз премијерно извођење салта на бициклу и другим акробацијама. Ову вратоломију је извео Бен Валас, британски шампион у акробатској вожњи бициклом, а осмислила ју је Хелен Церски, физичарка са универзитета у Кембрицу. Она је за ову прилику направила компјутерски модел да би пројектовала салто. Названа Ајнштајнов скок, вратоломија има за циљ да максимално помери границе онога што људи могу да ураде на бициклу. Бицикл је симболично изабран да означи почетак године Ајнштајна у Британији и годину физике у свету, зато што је научник тврдио да је до своје теорије дошао возећи бицикл. Током године у Великој Британији одржаће се на стотине манифестација са играчким тачкама, филмовима и поезијом, да би младима од 11 до 14 година била приближена генијална открића Ал-

GRAVITY and STRING THEORY NEW IDEAS FOR UNSOLVED PROBLEMS III

SEPTEMBER 7-9 2018
ZLATIBOR, SERBIA

Organization

The school is organized by:

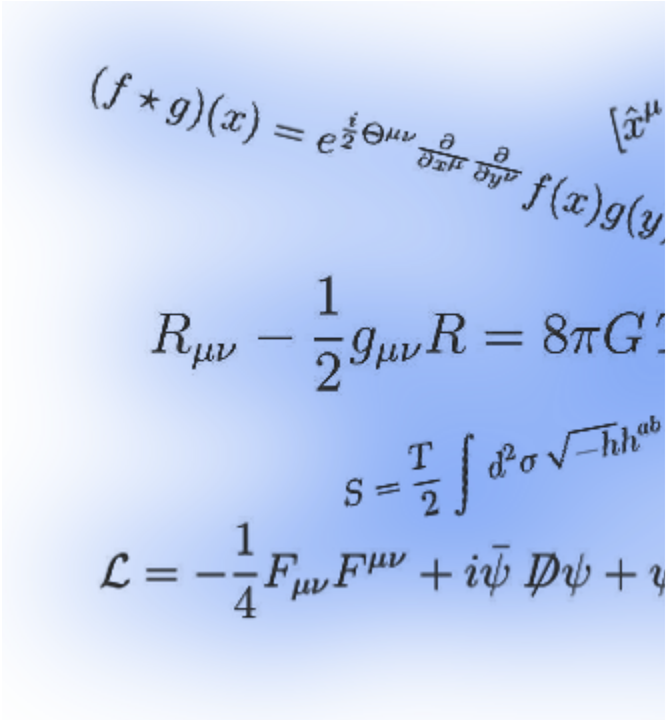
- Group for Gravitation, Particles and Fields
- Institute of Physics Belgrade
- Faculty of Physics, University of Belgrade

Scientific Committee:

(to be announced)

Local Organizing Committee:

- Ljubica Davidović Institute of Physics Belgrade
- Bojan Nikolić Institute of Physics Belgrade
- Marko Vojinović Institute of Physics Belgrade



SEMINARS EVENTS CONTACT LINKS

WELCOME RESEARCH PEOPLE PUBLICATIONS

GPF Gravity II Workshop

Presentations and slides:

- Milutin Blagojević
- Maja Burić
- Branislav Cvetković
- Ljubica Davidović
- Duško Latas
- Luka Nenadović
- Bojan Nikolić
- Voja Radovanović
- Igor Salom
- Marko Vojinović

7th MATHEMATICAL PHYSICS MEETING: Summer School and Conference on Modern Mathematical Physics

9 - 19 September 2012, Belgrade, Serbia

Main page

General information

Programme

Committees

Lecturers/speakers

Participants

Registration

Accommodation

Poster

Previous meetings

Sponsors

Travel/visa

Proceedings

Photos

Committees

International Advisory Committee

Ignatios Antoniadis (Geneve, Switzerland)
Irina Arefeva (Moscow, Russia)
Loriano Bonora (Trieste, Italy)
Lars Brink (Gothenburg, Sweden)
Maja Buric (Belgrade, Serbia)
Dieter Luest (Munich, Germany) (www1, www2)
Miodrag Mateljevic (Belgrade, Serbia)
Viatcheslav Mukhanov (Munich, Germany)
Hermann Nicolai (Potsdam, Germany)
Anatoly Nikitin (Kyiv, Ukraine)
Emil Nissimov (Sofia, Bulgaria)
Matej Pavsic (Ljubljana, Slovenia)
Stevan Pilipovic (Novi Sad, Serbia)
Branislav Sazdovic (Belgrade, Serbia)
Dejan Stojkovic (Buffalo, USA)
Vasiliy Vladimirov (Moscow, Russia)
Mihai Visinescu (Bucharest, Romania)

International Organizing Committee

Goran Djordjevic (Nis, Serbia)
Vladimir Dobrev (Sofia, Bulgaria)
Branko Dragovich (Belgrade, Serbia)
Anatol Odzijewicz (Bialystok, Poland)
Zoran Rakic (Belgrade, Serbia)
Igor Volovich (Moscow, Russia)
George Zoupanos (Athens, Greece)

Local Organizing Committee

Branko Dragovich (Chairman, Inst. of Phys., Belgrade)
Igor Salom (Vice Chairman, Inst. of Phys., Belgrade)
Branislav Cvetković (Inst. of Phys., Belgrade)

Ljubica Davidović (Inst. of Phys., Belgrade)
Bojan Nikolić (Inst. of Phys., Belgrade)
Dragan Savić (Inst. of Phys., Belgrade)
Marko Vojinović (Inst. of Phys., Belgrade)

Committees

International Advisory Committee

Loriano Bonora (Trieste, Italy)
Maja Buric (Belgrade, Serbia)
Martin Cederwall (Goteborg, Sweden)
Dieter Luest (Munich, Germany) (www1, www2)
Vladimir Dragovic (Belgrade, Serbia)
Miodrag Mateljevic (Belgrade, Serbia)
Hermann Nicolai (Potsdam, Germany)
Matej Pavsic (Ljubljana, Slovenia)
Branislav Sazdovic (Belgrade, Serbia)
Dmitry Vasilievich Shirkov (Dubna, Russia)
Djordje Sijacki (Belgrade, Serbia)
Ivan Todorov (Sofia, Bulgaria)
Francesco Toppan (Rio de Janeiro, Brasil)
Josip Trampetic (Zagreb, Croatia)
Mihai Visinescu (Bucharest, Romania)
Vasiliy Vladimirov (Moscow, Russia)
Rade Zivaljevic (Belgrade, Serbia)

International Organizing Committee

Irina Arefeva (Moscow, Russia)
Marija Dimitrijevic (Belgrade, Serbia)
Goran Djordjevic (Nis, Serbia)
Vladimir Dobrev (Sofia, Bulgaria)
Branko Dragovich (Belgrade, Serbia)
Anatol Odzijewicz (Bialystok, Poland)
Zoran Rakic (Belgrade, Serbia)
Igor Volovich (Moscow, Russia)
George Zoupanos (Athens, Greece)

Local Organizing Committee

Branko Dragovich (Chairman, Inst. of Phys., Belgrade)
Branislav Cvetkovic (Vice Chairman, Inst. of Phys., Belgrade)
Ljubica Davidovic (Inst. of Phys., Belgrade)
Dejan Jokovic (Inst. of Phys., Belgrade)
Bojan Nikolic (Inst. of Phys., Belgrade)
Igor Salom (Inst. of Phys., Belgrade)
Dragan Savic (Inst. of Phys., Belgrade)

Committees

International Advisory Committee

Loriano Bonora (Trieste, Italy)
Martin Cederwall (Goteborg, Sweden)
Alexandre T. Filippov (Dubna, Russia)
Harald Grosse (Vienna, Austria)
Nemanja Kaloper (Davis, USA)
Petr Kulish (St. Petersburg, Russia)
John Madore (Paris, France)
Gradimir Milovanovic (Nis, Serbia)
Viatcheslav Mukhanov (Munich, Germany)
Hermann Nicolai (Potsdam, Germany)
Anatol Odzijewicz (Bialystok, Poland)
Voja Radovanovic (Belgrade, Serbia)
Seifallah Randjbar-Daemi (Trieste, Italy)
Branislav Sazdovic (Belgrade, Serbia)
Djordje Sijacki (Belgrade, Serbia)
Ivan Todorov (Sofia, Bulgaria)
Josip Trampetic (Zagreb, Croatia)
Mihai Visinescu (Bucharest, Romania)
Vasiliy Vladimirov (Moscow, Russia)
Rade Zivaljevic (Belgrade, Serbia)
Stanislaw Woronowitz (Warsaw, Poland)

International Organizing Committee

Irina Arefeva (Moscow, Russia)
Maja Buric (Belgrade, Serbia)
Vladimir Dobrev (Sofia, Bulgaria)
Branko Dragovich (Belgrade, Serbia)
Dieter Luest (Munich, Germany) (www1, www2)
Zoran Rakic (Belgrade, Serbia)
Igor Volovich (Moscow, Russia)
George Zoupanos (Athens, Greece)

Local Organizing Committee

Branko Dragovich (Chairman, Inst. of Phys., Belgrade)
Sanja Cirkovic (Inst. of Phys., Belgrade)
Branislav Cvetkovic (Inst. of Phys., Belgrade)
Dusko Latas (Fac. of Phys., Belgrade)
Bojan Nikolic (Inst. of Phys., Belgrade)
Igor Salom (Inst. of Phys., Belgrade)
Dragan Savic (Inst. of Phys., Belgrade)
Marko Vojinovic (Inst. of Phys., Belgrade)

Committees

International Advisory Committee

Arefeva Irina (Moscow, Russia)
Bokan Neda (Fac. of Math., Belgrade, Serbia)
Bonora Loriano (Trieste, Italy)
Buric Maja (Fac. of Phys., Belgrade, Serbia)
Dobrev Vladimir (Sofia, Bulgaria)
Doplicher Sergio (Rome, Italy)
Ferrara Sergio (CERN, Geneva, Switzerland)
Henneaux Marc (Brussels, Belgium)
Kac Victor (MIT, Boston, USA)
Kaloper Nemanja (Davis, USA)
Manin Yuri (Bonn, Germany)
Meljanac Stjepan (IRB, Zagreb, Croatia)
Mukhanov Viatcheslav (Munich, Germany)
Nicolai Hermann (Golm, Germany)
Randjbar-Daemi Seifallah (ICTP, Trieste, Italy)
Sazdovic Branislav (Inst. of Phys., Belgrade, Serbia)
Todorov Ivan (Sofia, Bulgaria)
Vladimirov Vasiliy (Moscow, Russia)
Volovich Igor (Moscow, Russia)
Woronowicz Stanislaw (Warsaw, Poland)
Zivaljevic Rade (Math. Inst., Belgrade, Serbia)
Zoupanos George (Athens, Greece)

Local Organizing Committee

Branko Dragovich (Chairman, Inst. of Phys., Belgrade)
Sanja Cirkovic (Inst. of Phys., Belgrade)
Alexandra Dragovich (Moscow, Russia)
Dejan Jokovic (Inst. of Phys., Belgrade)
Bojan Nikolic (Inst. of Phys., Belgrade)
Voja Radovanovic (Fac. of Phys., Belgrade)
Zoran Rakic (Fac. of Math., Belgrade)
Igor Salom (Inst. of Phys., Belgrade)
Dragan Savic (Inst. of Phys., Belgrade)
Srdjan Vukmirovic (Fac. of Math., Belgrade)

Co-Directors of the School

Branko Dragovich (Belgrade)
Dieter Luest (Munich) (www1, www2)
Viatcheslav Mukhanov (Munich, Germany)
Julius Wess (Munich) (www1, www2)

Secretary of the School

Branislav Jurco (Munich)
Frank Meyer (Munich)

Advances in Theoretical and Mathematical Physics

Info Current issue All issues Search

Adv. Theor. Math. Phys.

← Previous issue Next issue →

Volume 14, Number 1

January 2010

Select/deselect all Export citations

Select/deselect all Export citations

Top

Browse Search About Researchers Librarians Publishers
Accessibility Help Contact RSS Log in

Site feedback