

ExploreQGP workshop

Higher-order moments of the elliptic flow distribution in PbPb collisions at $\sqrt{s_{NN}} = 5.02$ TeV

Jovan Milošević

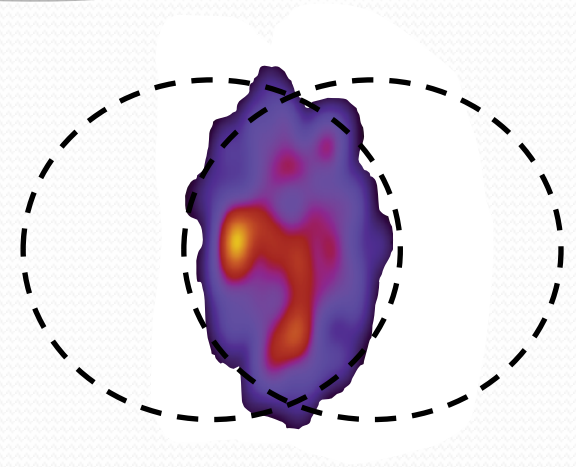
University of Belgrade

Vinča Institute of Nuclear Sciences, Serbia

on behalf of the CMS Collaboration

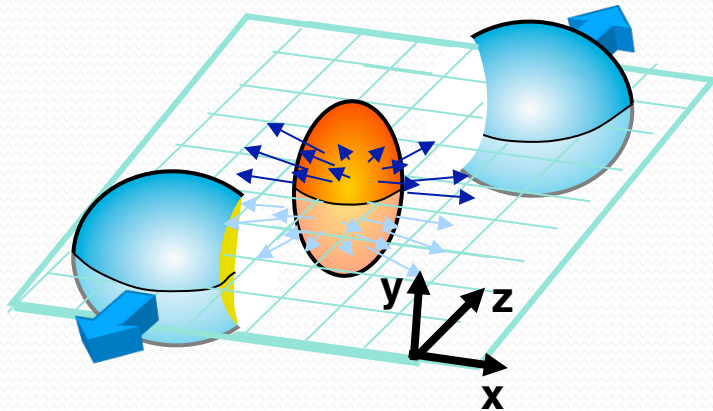


Outline

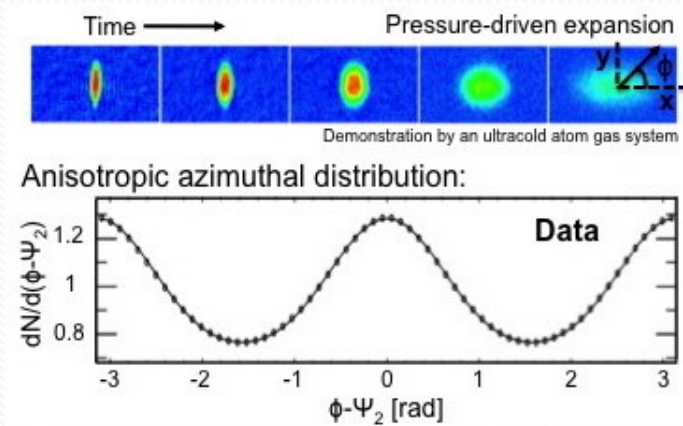


- ❖ v_2 magnitude - Introduction
- ❖ Motivation
- ❖ Collectivity in PbPb collisions – studied by Q-cumulants
- ❖ Centrality dependence of the hydrodynamic probes
- ❖ Measurement of central moments of the v_2 distribution
- ❖ Conclusions

Azimuthal anisotropy



Ψ_n (angle of n^{th} -order flow symmetry plane)



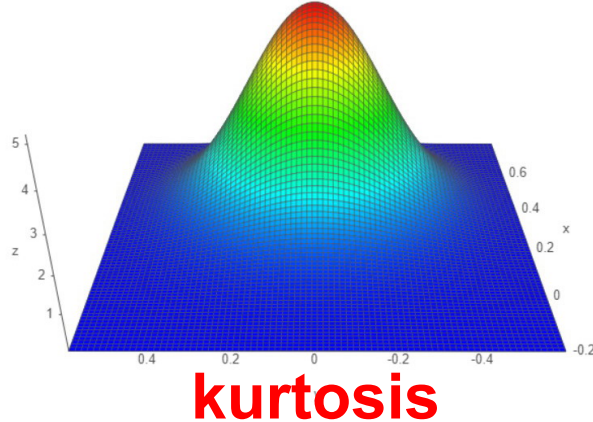
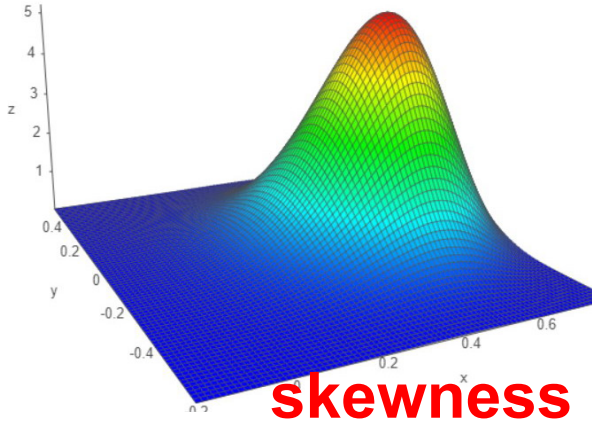
$$\frac{2\pi}{N} \frac{dN}{d\phi} = 1 + \sum_{n=1}^{\infty} 2v_n \cos[n(\phi - \Psi_n)]$$

v_n – **Fourier harmonics depend on**

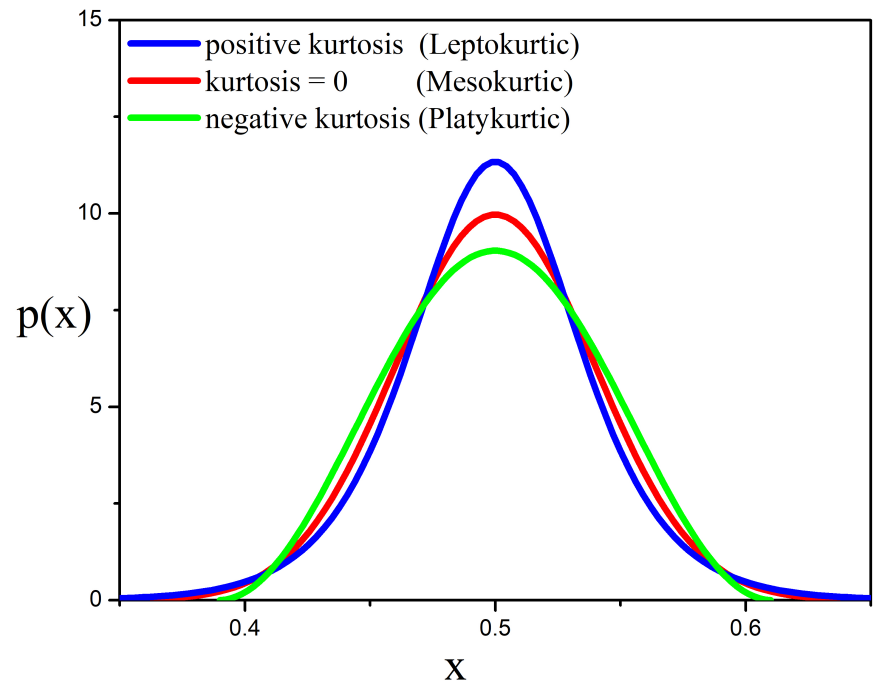
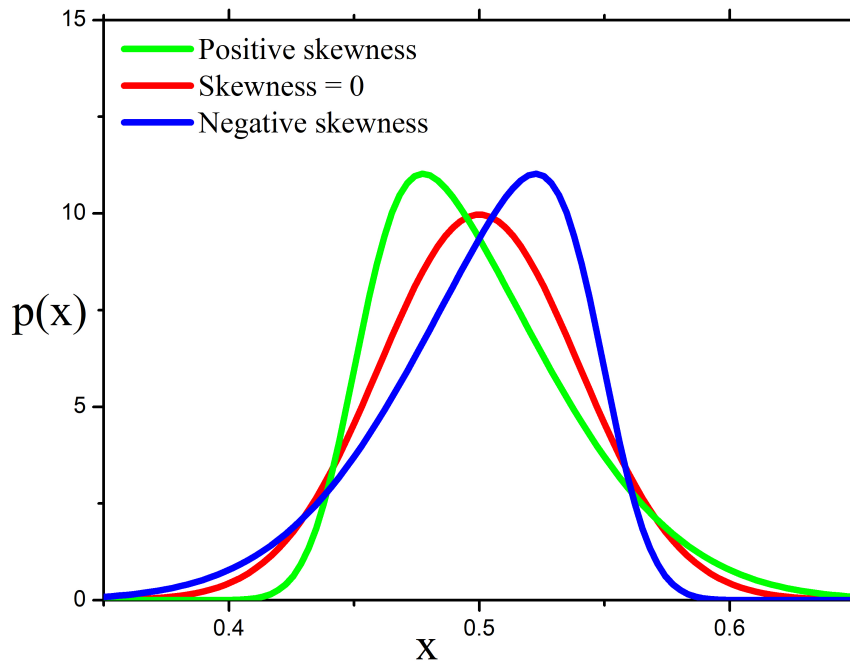
- initial state geometry
- initial state fluctuations
- medium transport properties (e.g. η/s)

$$v_n \equiv \langle \cos[n(\phi - \Psi_n)] \rangle$$

Shape of the v_2 distribution



- ❖ Non-Gaussianities are present in the early stage.
- ❖ Partially washed out during hydro expansion



Hydrodynamic probe as a motivation

[Phys. Rev. C **95** (2017) 014913]

Hydrodynamic probe:
$$\frac{v_2\{6\} - v_2\{8\}}{v_2\{4\} - v_2\{6\}} \approx \frac{1}{11} \approx 0.091$$



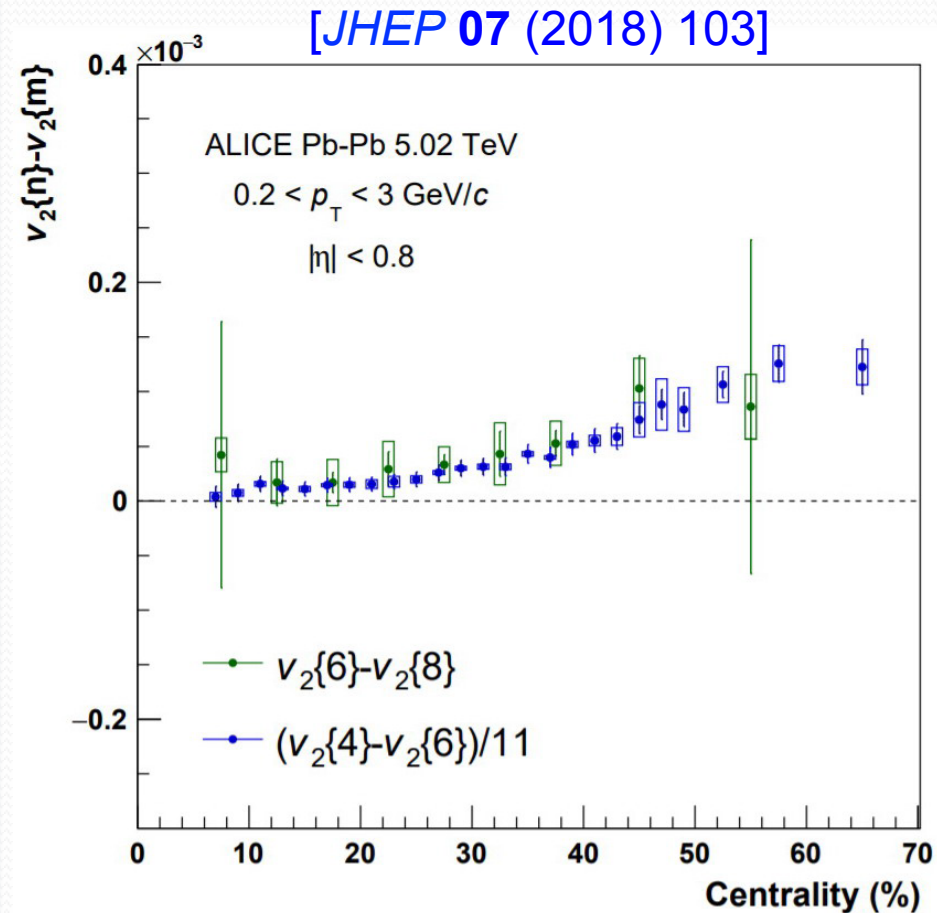
“Good for central collisions”, “Higher order expansion of the v_2 distribution are required”

[Phys. Lett. B **789** (2019) 643-665]

$0.143 \pm 0.008(\text{stat}) \pm 0.014(\text{syst})$ centrality: 20–25%
 $0.185 \pm 0.005(\text{stat}) \pm 0.012(\text{syst})$ centrality: 55–60%



“Higher order terms in a cumulant expansion of the v_2 distribution are required”



Precision was not satisfactory!

Q-cumulant method

Multi-particle correlations

$$\langle 2 \rangle \equiv \langle e^{in(\phi_1 - \phi_2)} \rangle \quad \langle 4 \rangle \equiv \langle e^{in(\phi_1 + \phi_2 - \phi_3 - \phi_4)} \rangle$$

Q-vector

$$Q_n = \sum_{i=1}^M e^{in\phi_i}$$

$$\langle 2 \rangle = \frac{|Q_n|^2 - M}{M(M-1)}$$

$$\langle 4 \rangle = \frac{|Q_n|^4 + |Q_{2n}|^2 - 2 \operatorname{Re}[Q_{2n} Q_n^* Q_n^*]}{M(M-1)(M-2)(M-3)} - 2 \frac{2(M-2)|Q_n|^2 - M(M-3)}{M(M-1)(M-2)(M-3)}$$



Averaging over all events

$$\langle\langle 2 \rangle\rangle \equiv \langle\langle e^{in(\phi_1 - \phi_2)} \rangle\rangle$$

$$\langle\langle 4 \rangle\rangle \equiv \langle\langle e^{in(\phi_1 + \phi_2 - \phi_3 - \phi_4)} \rangle\rangle$$

Ideal detector case

Q-cumulant method

[Phys. Rev. C 104 (2021) 034906]:

$$c_n \{2k\} = \langle\langle 2k \rangle\rangle - \sum_{m=1}^{k-1} \binom{k}{m} \binom{k-1}{m} \langle\langle 2m \rangle\rangle c_n \{2k-2m\}$$

$$c_n \{2\} = \langle\langle 2 \rangle\rangle$$

$$c_n \{4\} = \langle\langle 4 \rangle\rangle - 2\langle\langle 2 \rangle\rangle^2$$

General formulas
for any order



$$v_n \{2k\} = \sqrt[2k]{\frac{(2k)!}{2^{2k} (k!)^2} \left[\frac{d^{2k}}{dl^{2k}} \ln I_0(l) \right]_{l=0}^{-1}} c_n \{2k\}$$

$$v_n \{2\} = \sqrt{c_n \{2\}}$$

$$v_n \{4\} = \sqrt[4]{-c_n \{4\}}$$

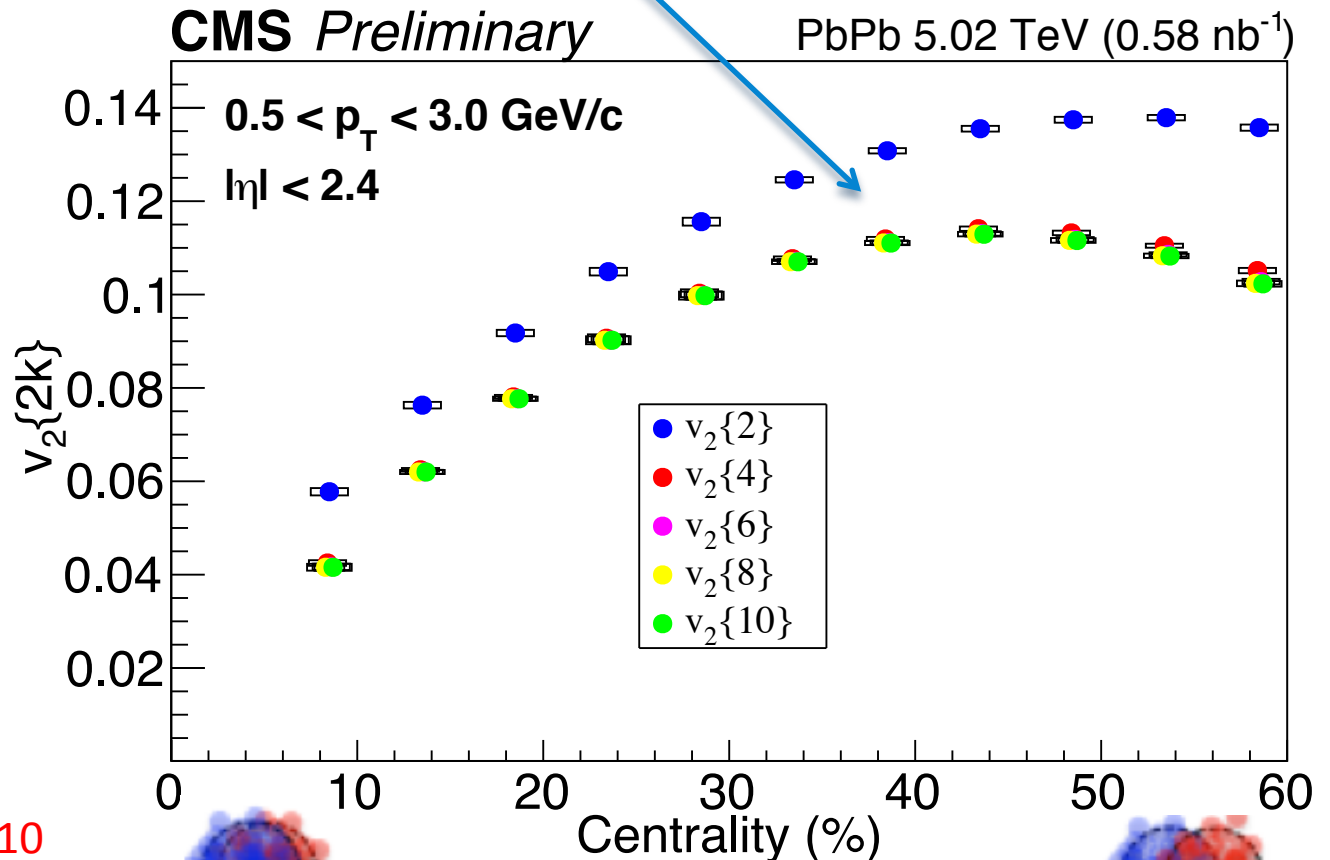
v_2 from Q-cumulants

- Flow fluctuations, σ_v , -> a gap between $v_2\{2\}$ and higher-order cumulants:
 $v_2\{2\}^2 = v_2\{2k\}^2 + 2\sigma_v^2$, for $(k>1)$
- Stat. uncertainties ~ 2 orders of magnitude smaller wrt syst. ones

Dominant source
syst. uncertainties:
variation on criteria
for tracks

fine splitting

$$v_2\{4\} \gtrsim v_2\{6\} \gtrsim v_2\{8\} \gtrsim v_2\{10\}$$

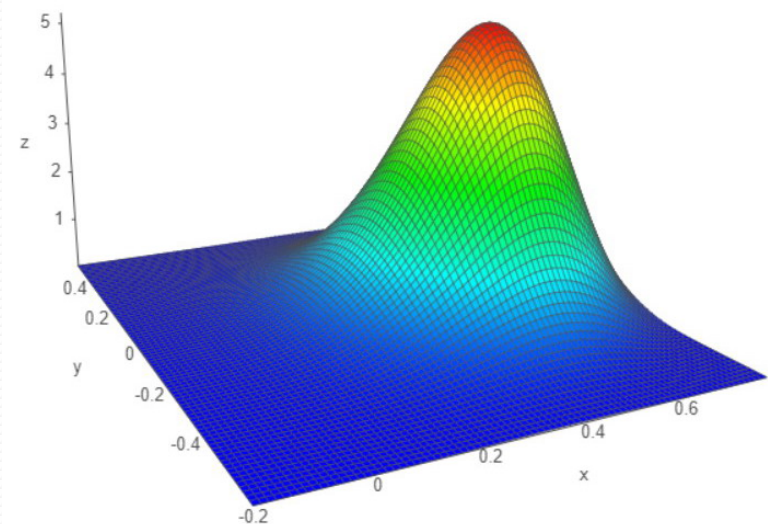


CMS-PAS-HIN-21-010

Expansion of hydro probes in central moments

$$h_1 = \frac{v_2\{6\} - v_2\{8\}}{v_2\{4\} - v_2\{6\}} \approx \frac{1}{11} - \frac{4\kappa_{40} + \frac{8(p_{50} + p_{32})}{\bar{v}_2}}{11 \left[2\bar{v}_2 s_{30} + 3(\kappa_{40} + \kappa_{22}) + \frac{3(p_{50} + 2p_{32} + p_{14}) - 2(\sigma_y^2 - \sigma_x^2)(5s_{30} - 6s_{12})}{2\bar{v}_2} \right]}$$

$$h_2 = \frac{v_2\{8\} - v_2\{10\}}{v_2\{6\} - v_2\{8\}} \approx \frac{3}{19} - \frac{88p_{50}}{95 \left[4\bar{v}_2^2 s_{30} - 2\bar{v}_2(\kappa_{40} - 3\kappa_{22}) - 13(p_{50} + 10p_{32} - 3p_{14}) - 2(\sigma_y^2 - \sigma_x^2)(5s_{30} - 6s_{12}) \right]}$$



$$\frac{v_2\{6\} - v_2\{8\}}{v_2\{4\} - v_2\{6\}} \approx \frac{1}{11} - \frac{1}{11} \frac{v_2\{4\}^2 - 12v_2\{6\}^2 + 11v_2\{8\}^2}{v_2\{4\}^2 - v_2\{6\}^2 + \frac{(\sigma_y^2 - \sigma_x^2)s_{30}}{3\bar{v}_2^3}}$$

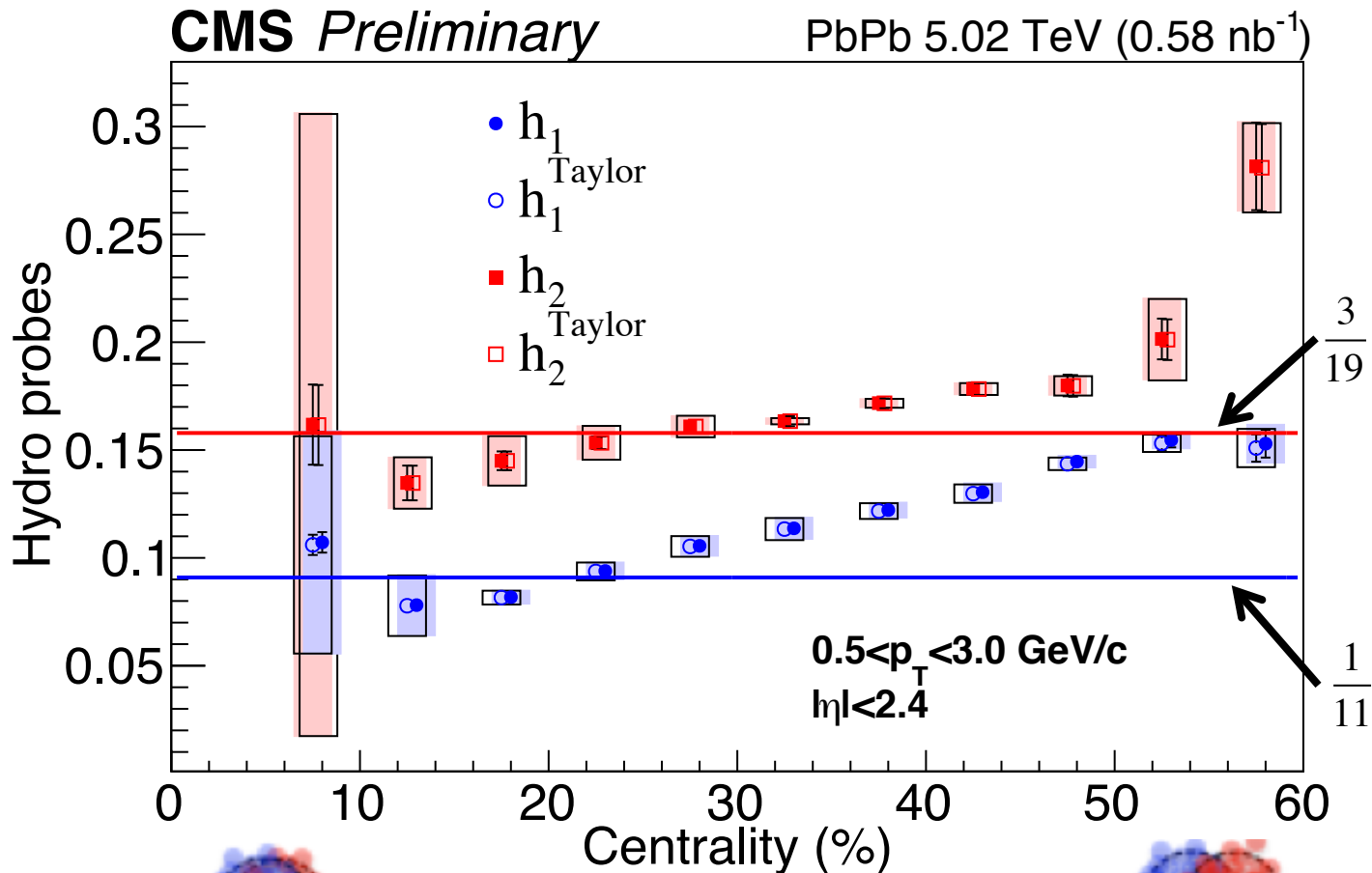
negligible

$$\frac{v_2\{8\} - v_2\{10\}}{v_2\{6\} - v_2\{8\}} \approx \frac{3}{19} - \frac{1}{19} \frac{3v_2\{6\}^2 - 22v_2\{8\}^2 + 19v_2\{10\}^2}{v_2\{6\}^2 - v_2\{8\}^2 + \frac{(\sigma_y^2 - \sigma_x^2)s_{30}}{33\bar{v}_2^3}}$$

Hydrodynamic probes

$$h_1 = \frac{v_2\{6\} - v_2\{8\}}{v_2\{4\} - v_2\{6\}} \approx h_1^{Taylor} = \frac{1}{11} - \frac{1}{11} \frac{v_2^2\{4\} - 12v_2^2\{6\} + 11v_2^2\{8\}}{v_2\{4\}^2 - v_2\{6\}^2}$$

$$h_2 = \frac{v_2\{8\} - v_2\{10\}}{v_2\{6\} - v_2\{8\}} \approx h_2^{Taylor} = \frac{3}{19} - \frac{1}{19} \frac{3v_2^2\{6\} - 22v_2^2\{8\} + 19v_2^2\{10\}}{v_2\{6\}^2 - v_2\{8\}^2}$$



CMS-PAS-HIN-21-010

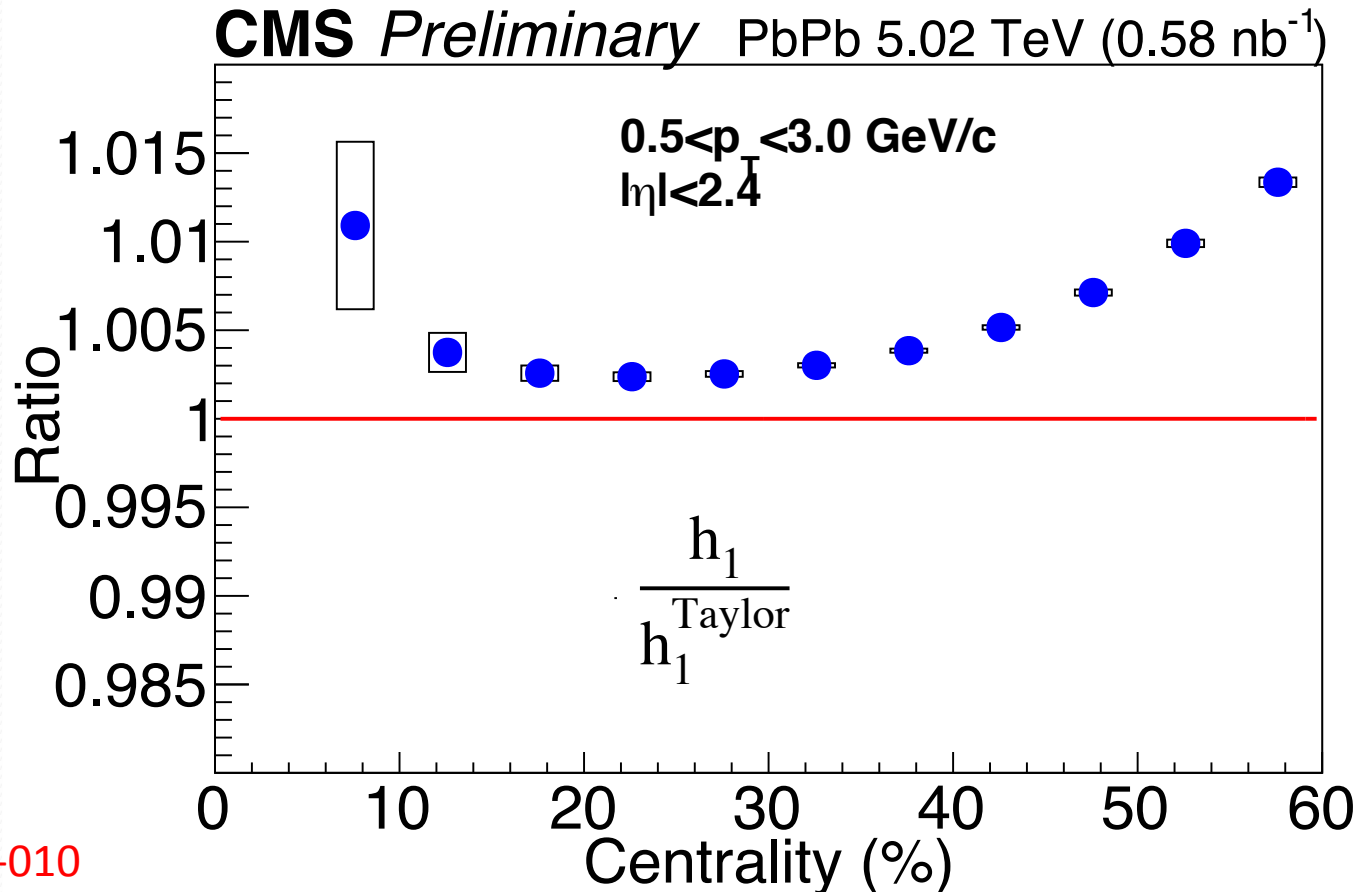
Higher-order moments necessary to describe data

Ratio between probe and its Taylor expansion

$$\frac{h_1}{h_1^{Taylor}} \approx \frac{\frac{v_2\{6\} - v_2\{8\}}{v_2\{4\} - v_2\{6\}}}{\frac{1}{11} - \frac{1}{11} \frac{v_2^2\{4\} - 12v_2^2\{6\} + 11v_2^2\{8\}}{v_2\{4\}^2 - v_2\{6\}^2}}$$

-Stat. uncertainties of the nominator and denominator are strongly correlated
 -Syst. uncertainties dominates
 -Term proportional to $(\sigma_y^2 - \sigma_x^2)$ is negligible in accordance with

PRC 95 (2017)
014913



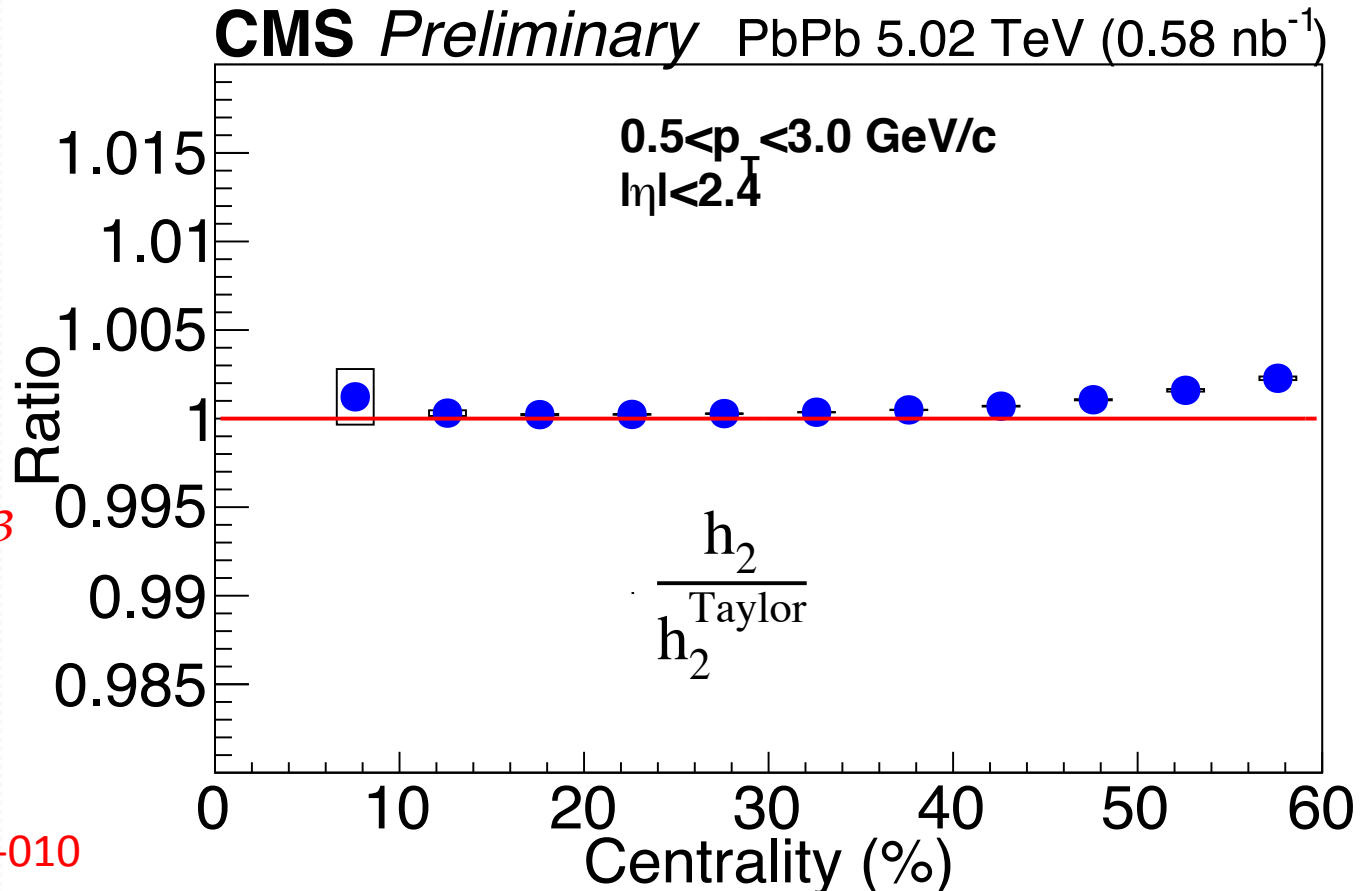
CMS-PAS-HIN-21-010

Ratio between the new probe and its Taylor expansion

$$\frac{h_2}{h_2^{Taylor}} \approx \frac{\frac{v_2\{8\} - v_2\{10\}}{v_2\{6\} - v_2\{8\}}}{\frac{3}{19} - \frac{1}{19} \frac{3v_2^2\{6\} - 22v_2^2\{8\} + 19v_2^2\{10\}}{v_2\{6\}^2 - v_2\{8\}^2}}$$

-Stat. uncertainties of the nominator and denominator are strongly correlated
 -Syst. uncertainties dominates
 -Term proportional to $(\sigma_y^2 - \sigma_x^2)$ is negligible
 in accordance with

PRC 95 (2017) 014913



CMS-PAS-HIN-21-010

Standardized & Cleaned moments

PRC 99 (2019) 014907

$$\gamma_1^{\text{exp}} = -2^{3/2} \frac{v_2 \{4\}^3 - v_2 \{6\}^3}{[v_2 \{2\}^2 - v_2 \{4\}^2]^{3/2}} \approx -2^{3/2} \frac{-s_{30} - O_N}{[2\sigma_x^2 + O_D]^{3/2}} \approx \frac{s_{30}}{\sigma_x^3} \equiv \gamma_1$$

$$\gamma_2^{\text{exp}} = -\frac{3 v_2 \{4\}^4 - 12 v_2 \{6\}^4 + 11 v_2 \{8\}^4}{2 [v_2 \{2\}^2 - v_2 \{4\}^2]^2} \approx \frac{\kappa_{40}}{\sigma_x^4} \equiv \gamma_2$$

Kurtosis

$$\gamma_3^{\text{exp}} = 6\sqrt{2} \frac{3v_2 \{6\}^5 - 22v_2 \{8\}^5 + 19v_2 \{10\}^5}{[v_2 \{2\}^2 - v_2 \{4\}^2]^{5/2}} \approx \frac{p_{50}}{\sigma_x^5} \equiv \gamma_3$$

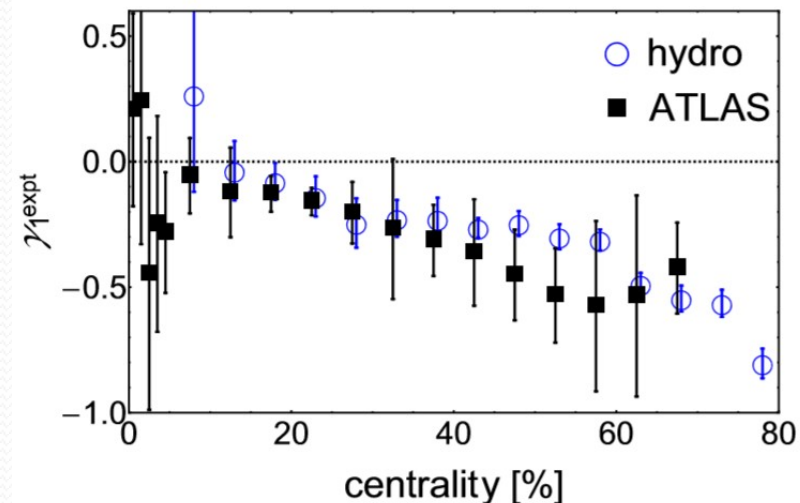
Superskewness

Conditions: $s_{12} \approx \frac{s_{30}}{3}$ $\kappa_{22} \approx \frac{\kappa_{40}}{3}$ $p_{32} \approx p_{14} \approx \frac{p_{50}}{5}$ Ell. pow. distr. param. $\varepsilon_0 \leq 0.15$

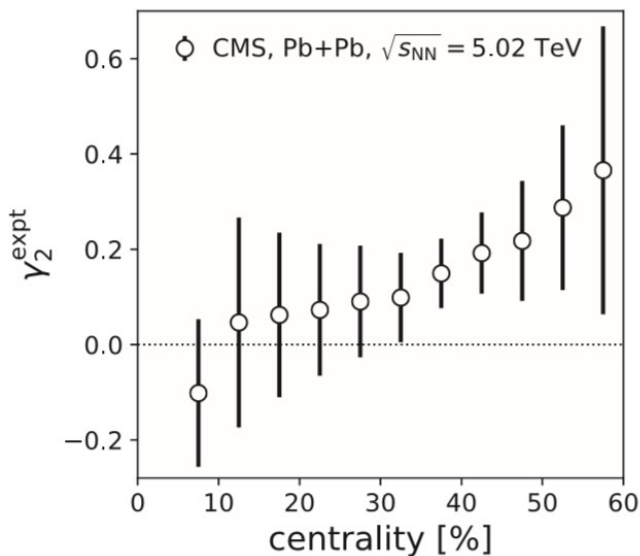
$$\gamma_{1,\text{corr}}^{\text{exp}} = -2^{3/2} \frac{187v_2 \{8\}^3 - 16v_2 \{6\}^3 - 171v_2 \{10\}^3}{[v_2 \{2\}^2 - 40v_2 \{6\}^2 + 495v_2 \{8\}^2 - 456v_2 \{10\}^2]^{3/2}}$$

Skewness, kurtosis and superskewness

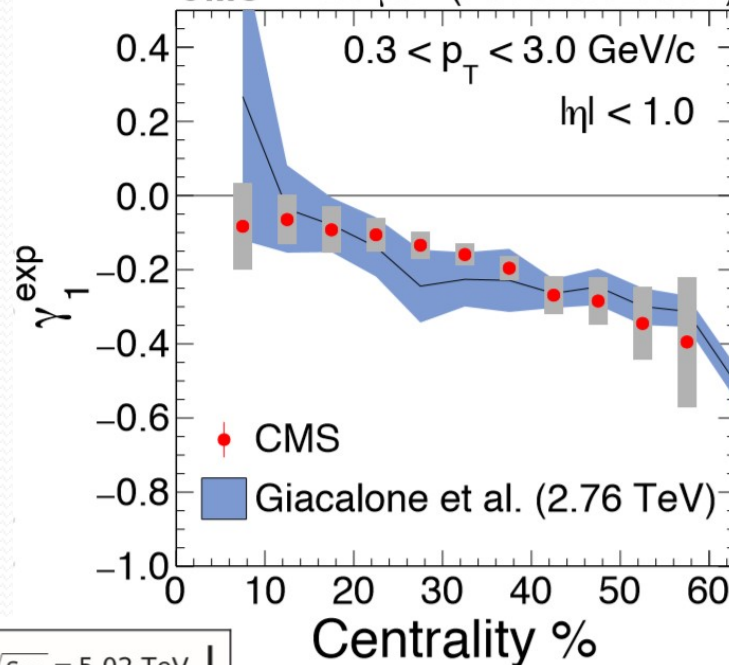
PHYSICAL REVIEW C 95, 014913 (2017)



Physical Review C 99 014907 (2019):

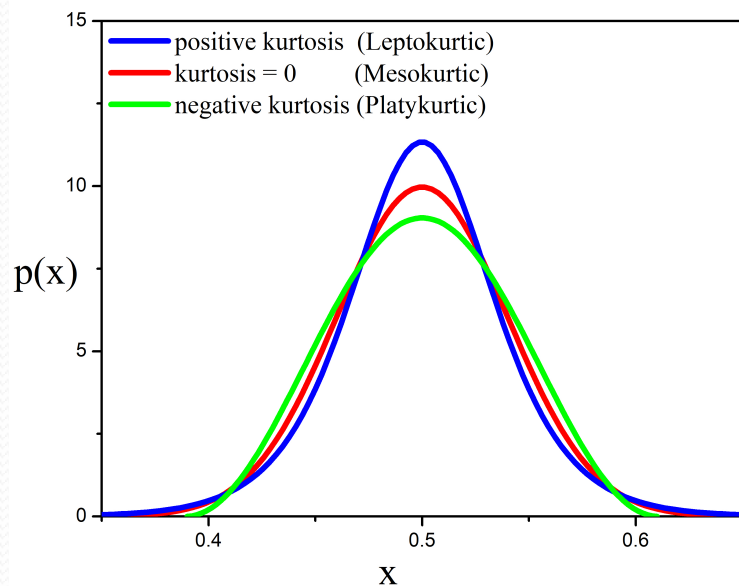
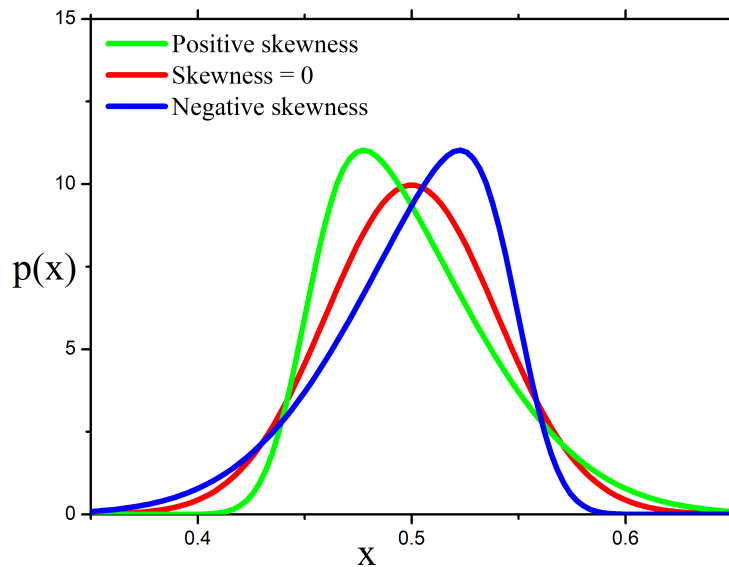


CMS $26 \mu\text{b}^{-1}$ (PbPb 5.02 TeV)

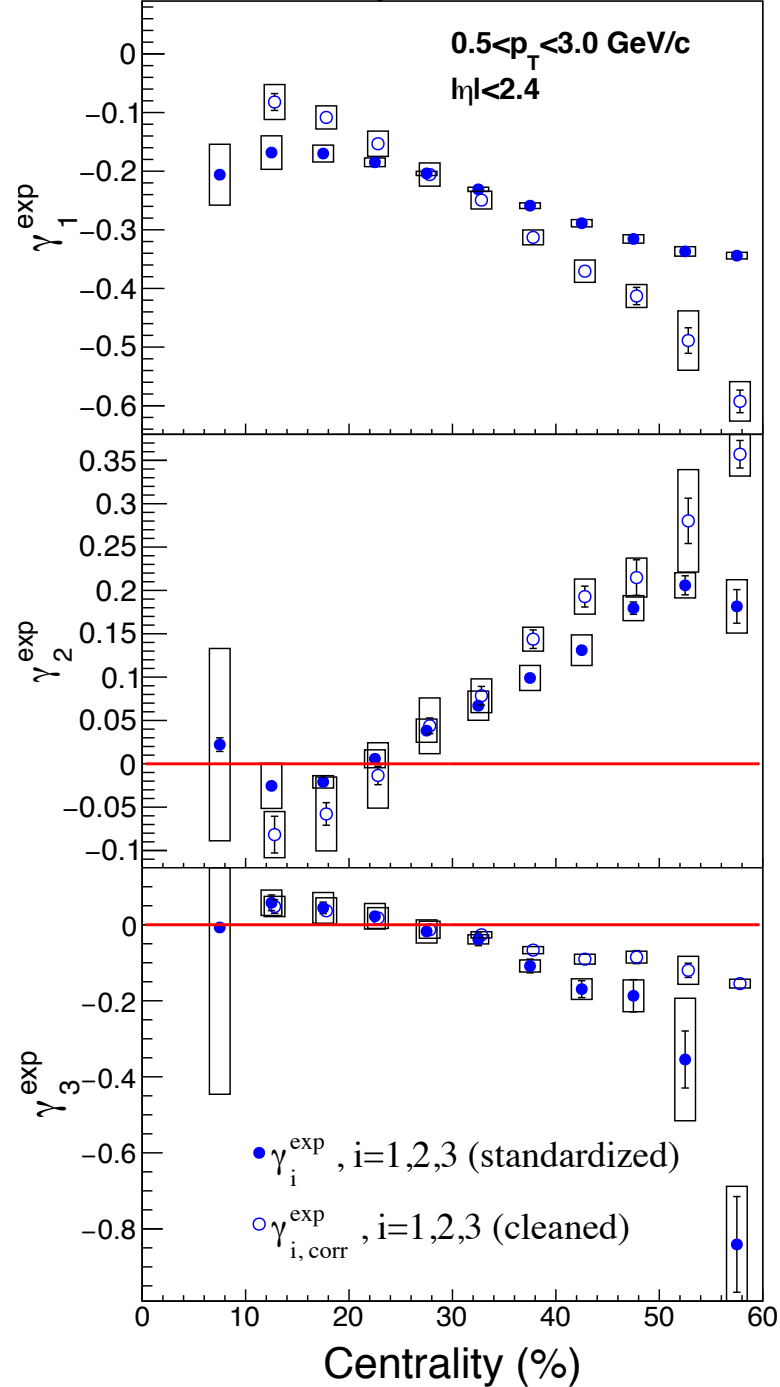


Phys. Lett. B 789 643 (2019):

Skewness, kurtosis and superskewness

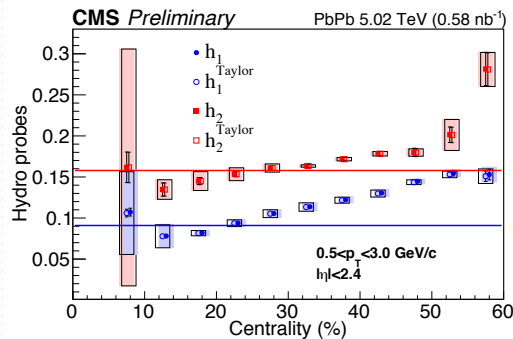


CMS Preliminary PbPb 5.02 TeV (0.58 nb⁻¹)

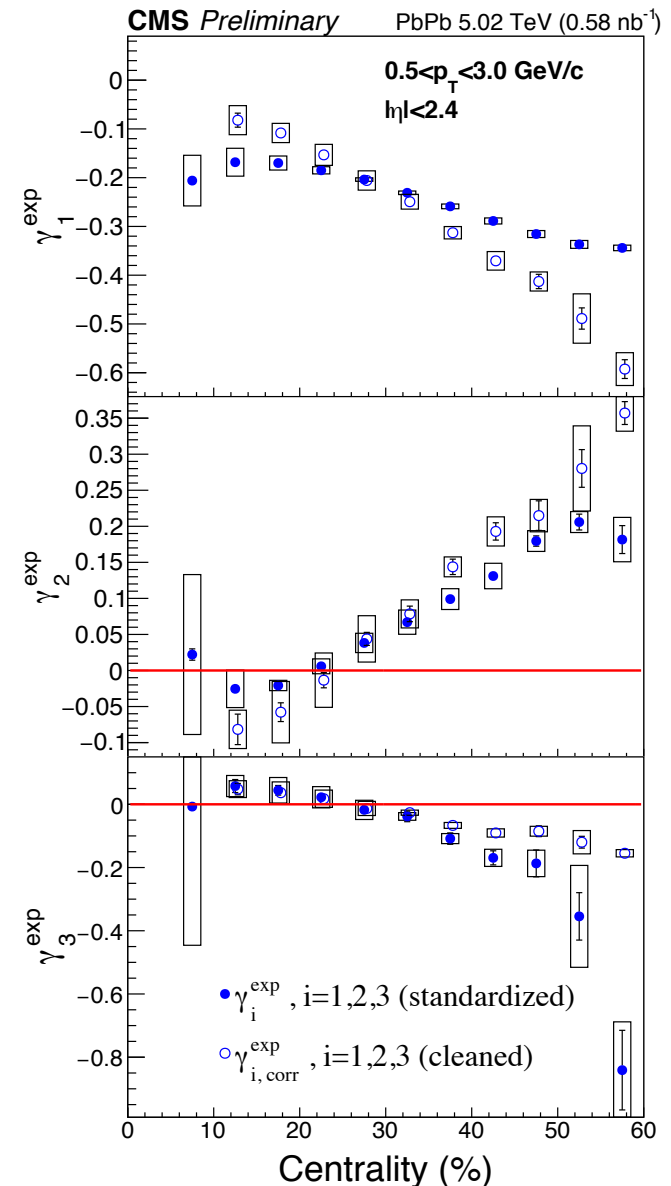


CMS-PAS-HIN-21-010

Conclusions



- ❖ Two hydrodynamic probes performed on CMS PbPb data at 5.02 TeV energy
- ❖ High precision measurement of skewness, kurtosis and superskewness of the v_2 distribution.
- ❖ These results can provide novel constraints on the initial state geometry used in hydrodynamic calculations of the medium expansion in high energy nuclear collisions





Backup

Ten-particle azimuthal angle correlation

- ◆ The size of formula increase with the order k
- ◆ Formula for the corresponding statistical uncertainties are even much bigger
- ◆ This is a kind of a disadvantage of the method because it is not easy to implement

$$c_n \{10\} = \langle\langle 10 \rangle\rangle - 25 \cdot \langle\langle 2 \rangle\rangle \langle\langle 8 \rangle\rangle - 100 \cdot \langle\langle 4 \rangle\rangle \langle\langle 6 \rangle\rangle + 400 \cdot \langle\langle 6 \rangle\rangle \langle\langle 2 \rangle\rangle^2 + 900 \cdot \langle\langle 2 \rangle\rangle \langle\langle 4 \rangle\rangle^2 - 3600 \cdot \langle\langle 4 \rangle\rangle \langle\langle 2 \rangle\rangle^3 + 2880 \cdot \langle\langle 2 \rangle\rangle^5$$

- ◆ For the first time $v_n \{10\}$:

$$v_n \{10\} = \sqrt[10]{\frac{1}{456} c_n \{10\}}$$

$$\begin{aligned} \langle 10 \rangle = & \frac{|Q_n|^{10} - 20\text{Re}[Q_{2n}|Q_n|^6 Q_n^* Q_n^*] + 100|Q_{2n}|^2 |Q_n|^6 + 30\text{Re}[Q_{2n} Q_{2n} |Q_n|^2 Q_n^* Q_n^* Q_n^* Q_n^*]}{M(M-1)(M-2)(M-3)(M-4)(M-5)(M-6)(M-7)(M-8)(M-9)} + \\ & + \frac{225|Q_{2n}|^4 |Q_n|^2 - 300\text{Re}[Q_{2n}|Q_{2n}|^2 |Q_n|^2 Q_n^* Q_n^*] + 40\text{Re}[Q_{3n}|Q_n|^4 Q_n^* Q_n^*]}{M(M-1)(M-2)(M-3)(M-4)(M-5)(M-6)(M-7)(M-8)(M-9)} + \\ & + \frac{600\text{Re}[Q_{3n} Q_n |Q_n|^2 Q_{2n}^* Q_n^*] - 400\text{Re}[Q_{3n}|Q_n|^4 Q_n^* Q_n^*] + 400|Q_{3n}|^2 |Q_n|^4}{M(M-1)(M-2)(M-3)(M-4)(M-5)(M-6)(M-7)(M-8)(M-9)} + \\ & + \frac{400\text{Re}[Q_{3n}|Q_{2n}|^2 Q_n^* Q_n^*] - 40\text{Re}[Q_{3n} Q_{2n} Q_n^* Q_n^* Q_n^* Q_n^*] - 600\text{Re}[Q_{3n}|Q_{2n}|^2 Q_n^* Q_n^*]}{M(M-1)(M-2)(M-3)(M-4)(M-5)(M-6)(M-7)(M-8)(M-9)} + \\ & + \frac{400|Q_{3n}|^2 |Q_{2n}|^2 - 800\text{Re}[|Q_{3n}|^2 Q_{2n} Q_n^* Q_n^*] - 60\text{Re}[Q_{4n}|Q_n|^2 Q_n^* Q_n^*]}{M(M-1)(M-2)(M-3)(M-4)(M-5)(M-6)(M-7)(M-8)(M-9)} + \\ & + \frac{600\text{Re}[Q_{4n}|Q_n|^2 Q_n^* Q_n^* Q_{2n}^*] - 900\text{Re}[Q_{4n}|Q_n|^2 Q_{2n}^* Q_n^*] - 1200\text{Re}[Q_{4n}|Q_n|^2 Q_n^* Q_{3n}^*]}{M(M-1)(M-2)(M-3)(M-4)(M-5)(M-6)(M-7)(M-8)(M-9)} + \\ & + \frac{1200\text{Re}[Q_{4n} Q_n Q_{2n}^* Q_{3n}^*] + 900|Q_{4n}|^2 |Q_n|^2 + 48\text{Re}[Q_{5n} Q_n^* Q_n^* Q_n^* Q_n^*]}{M(M-1)(M-2)(M-3)(M-4)(M-5)(M-6)(M-7)(M-8)(M-9)} + \\ & + \frac{720\text{Re}[Q_{5n} Q_n^* Q_{2n}^* Q_{2n}^*] - 480\text{Re}[Q_{5n} Q_n^* Q_n^* Q_{2n}^*] + 960\text{Re}[Q_{5n} Q_n^* Q_{3n}^*]}{M(M-1)(M-2)(M-3)(M-4)(M-5)(M-6)(M-7)(M-8)(M-9)} + \\ & + \frac{576|Q_{5n}|^2 - 960\text{Re}[Q_{5n} Q_{2n}^* Q_{3n}^*] - 1440\text{Re}[Q_{5n} Q_n^* Q_{4n}^*]}{M(M-1)(M-2)(M-3)(M-4)(M-5)(M-6)(M-7)(M-8)(M-9)} + \\ & + \frac{300\text{Re}[Q_{2n}|Q_n|^4 Q_n^* Q_n^*] - 25|Q_n|^{10} - 900|Q_{2n}|^2 |Q_n|^4 - 150\text{Re}[Q_{2n} Q_{2n} Q_n^* Q_n^* Q_n^*]}{M(M-1)(M-2)(M-3)(M-4)(M-5)(M-6)(M-7)(M-9)} + \\ & + \frac{900\text{Re}[Q_{2n}|Q_{2n}|^2 Q_n^* Q_n^*] - 225|Q_{2n}|^4 - 400\text{Re}[Q_{3n}|Q_n|^2 Q_n^* Q_n^*] + 2400\text{Re}[Q_{3n}|Q_n|^2 Q_n^* Q_{2n}^*]}{M(M-1)(M-2)(M-3)(M-4)(M-5)(M-6)(M-7)(M-9)} + \\ & + \frac{300\text{Re}[Q_{4n} Q_n^* Q_n^* Q_n^*] - 1200\text{Re}[Q_{3n} Q_n Q_{2n}^*] - 1600|Q_{3n}|^2 |Q_n|^2 - 1800\text{Re}[Q_{4n} Q_n^* Q_n^* Q_{2n}^*]}{M(M-1)(M-2)(M-3)(M-4)(M-5)(M-6)(M-7)(M-9)} + \\ & + \frac{900\text{Re}[Q_{4n} Q_{2n}^* Q_{2n}^*] + 2400\text{Re}[Q_{4n} Q_n^* Q_{3n}^*] - 900|Q_{4n}|^2}{M(M-1)(M-2)(M-3)(M-4)(M-5)(M-6)(M-7)(M-9)} + \\ & + \frac{200|Q_n|^6 - 1200\text{Re}[Q_{2n}|Q_n|^2 Q_n^* Q_n^*] + 1800|Q_{2n}|^2 |Q_n|^2}{M(M-1)(M-2)(M-3)(M-4)(M-5)(M-7)(M-8)} + \\ & + \frac{800\text{Re}[Q_{3n} Q_n^* Q_n^*] - 2400\text{Re}[Q_{3n} Q_n^* Q_n^*] + 800|Q_{3n}|^2}{M(M-1)(M-2)(M-3)(M-4)(M-5)(M-7)(M-8)} + \\ & + \frac{1200\text{Re}[Q_{2n} Q_n^* Q_n^*] - 600|Q_n|^4 - 600|Q_{2n}|^2}{M(M-1)(M-2)(M-3)(M-5)(M-6)(M-7)} + \\ & + \frac{600|Q_n|^2}{M(M-1)(M-3)(M-4)(M-5)(M-6)} - \frac{120}{(M-1)(M-2)(M-3)(M-4)(M-5)} \end{aligned}$$

Statistical uncertainty of the $v_n\{10\}$

- ◆ **Statistical uncertainties of the $v_n\{10\}$ cumulant is calculated analytically using the data [the same approach as in Phys. Rev. C 104 (2021) 034906, arXiv: 2104.00588]**

$$\begin{aligned}
 s^2[v_n\{10\}] \cdot 4560^2 (v_n\{10\})^{18} &= A^2 \sigma_{\langle\langle 2 \rangle\rangle}^2 + B^2 \sigma_{\langle\langle 4 \rangle\rangle}^2 \\
 &+ C^2 \sigma_{\langle\langle 6 \rangle\rangle}^2 + D^2 \sigma_{\langle\langle 8 \rangle\rangle}^2 + \sigma_{\langle\langle 10 \rangle\rangle}^2 + 2AB\sigma_{\langle\langle 2 \rangle\rangle, \langle\langle 4 \rangle\rangle} \\
 &+ 2AC\sigma_{\langle\langle 2 \rangle\rangle, \langle\langle 6 \rangle\rangle} + 2AD\sigma_{\langle\langle 2 \rangle\rangle, \langle\langle 8 \rangle\rangle} + 2A\sigma_{\langle\langle 2 \rangle\rangle, \langle\langle 10 \rangle\rangle} \\
 &+ 2BC\sigma_{\langle\langle 4 \rangle\rangle, \langle\langle 6 \rangle\rangle} + 2BD\sigma_{\langle\langle 4 \rangle\rangle, \langle\langle 8 \rangle\rangle} + 2B\sigma_{\langle\langle 4 \rangle\rangle, \langle\langle 10 \rangle\rangle} \\
 &+ 2CD\sigma_{\langle\langle 6 \rangle\rangle, \langle\langle 8 \rangle\rangle} + 2C\sigma_{\langle\langle 6 \rangle\rangle, \langle\langle 10 \rangle\rangle} + 2D\sigma_{\langle\langle 8 \rangle\rangle, \langle\langle 10 \rangle\rangle}
 \end{aligned}$$

$$\begin{aligned}
 A &= 14400 \langle\langle 2 \rangle\rangle^4 - 10800 \langle\langle 2 \rangle\rangle^2 \langle\langle 4 \rangle\rangle \\
 &+ 800 \langle\langle 6 \rangle\rangle \langle\langle 2 \rangle\rangle + 900 \langle\langle 4 \rangle\rangle^2 - 25 \langle\langle 8 \rangle\rangle \\
 B &= 1800 \langle\langle 4 \rangle\rangle \langle\langle 2 \rangle\rangle - 3600 \langle\langle 2 \rangle\rangle^3 - 100 \langle\langle 6 \rangle\rangle \\
 C &= 400 \langle\langle 2 \rangle\rangle^2 - 100 \langle\langle 4 \rangle\rangle \\
 D &= -25 \langle\langle 2 \rangle\rangle
 \end{aligned}$$

- ◆ **With variances**

$$\sigma_{\bar{x}_w}^2 = \frac{N \sum_{i=1}^N w_i^2 [x_i - \bar{x}_w]^2}{N-1 \left(\sum_{i=1}^N w_i \right)^2}$$

and covariances

- ← **weights have to be squared!**

$$\sigma_{\bar{x}_w, \bar{y}_w} = \frac{N \sum_{i=1}^N w_i^x w_i^y [x_i - \bar{x}_w][y_i - \bar{y}_w]}{N-1 \sum_{i=1}^N w_i^x \sum_{i=1}^N w_i^y}$$

The simplest case for the efficiency corrections within Q-cumulants

Formalism developed by A. Bilandzic

[Phys. Rev. C 83 (2011) 044913, and PhD thesis, Utrecht Uni. 2012]

Example of the simplest case of two particle correlation:

Without efficiency corrections

$$\langle 2 \rangle = \frac{|Q_n|^2 - M}{M(M-1)}$$

With efficiency corrections

$$\langle 2 \rangle = \frac{|Q_{n,1}|^2 - S_{1,2}}{S_{2,1} - S_{1,2}}$$

Where M is the number of the tracks, while

$$Q_n = \sum_{i=1}^M e^{in\phi_i}$$

$$S_{p,k} = \left[\sum_{i=1}^M w_i^k \right]^p$$

$$Q_{n,k} = \sum_{i=1}^M w_i^k e^{in\phi_i}$$

Taylor expansion of the hydrodynamics probe

Expansion up to 4th moment i.e. up to kurtosis

- ❖ Moments of the second order $\sigma_x^2 = \langle (v_x - \langle v_x \rangle)^2 \rangle$ $\sigma_y^2 = \langle v_y^2 \rangle$
- ❖ Moments of the third order $s_{30} = \langle (v_x - \langle v_x \rangle)^3 \rangle$ $s_{12} = \langle (v_x - \langle v_x \rangle) v_y^2 \rangle$
- ❖ Moments of the fourth order $\kappa_{40} = \langle (v_x - \langle v_x \rangle)^4 \rangle - 3\sigma_x^4$ $\kappa_{22} = \langle (v_x - \langle v_x \rangle)^2 v_y^2 \rangle - \sigma_x^2 \sigma_y^2$

$$v_2\{4\} \approx \bar{v}_2 + \frac{\sigma_y^2 - \sigma_x^2}{2\bar{v}_2} - \frac{s_{30} + s_{12}}{\bar{v}_2^2} - \frac{\kappa_{40} + 2\kappa_{22} + \kappa_{04}}{4\bar{v}_2^3} - \frac{5(\sigma_y^2 - \sigma_x^2)^2}{8\bar{v}_2^3} + \frac{(\sigma_y^2 - \sigma_x^2)(3s_{30} + 3s_{12})}{2\bar{v}_2^4}$$

$$v_2\{6\} \approx \bar{v}_2 + \frac{\sigma_y^2 - \sigma_x^2}{2\bar{v}_2} - \frac{2}{3} \frac{s_{30} + s_{12}}{\bar{v}_2^2} + \frac{\kappa_{40} - \kappa_{04}}{4\bar{v}_2^3} - \frac{5(\sigma_y^2 - \sigma_x^2)^2}{8\bar{v}_2^3} + \frac{p_{50} + 2p_{32} + p_{14}}{4\bar{v}_2^4} + \frac{(\sigma_y^2 - \sigma_x^2)(4s_{30} + 15s_{12})}{6\bar{v}_2^4}$$

$$v_2\{8\} \approx \bar{v}_2 + \frac{\sigma_y^2 - \sigma_x^2}{2\bar{v}_2} - \frac{7}{11} \frac{s_{30} + s_{12}}{\bar{v}_2^2} + \frac{31}{33} \frac{\kappa_{40} + 2\kappa_{22} - \kappa_{04}}{4\bar{v}_2^3} - \frac{5(\sigma_y^2 - \sigma_x^2)^2}{8\bar{v}_2^3} + \frac{5}{3} \frac{p_{50} + 2p_{32} + p_{14}}{11\bar{v}_2^4} + \frac{(\sigma_y^2 - \sigma_x^2)(13s_{30} + 57s_{12})}{22\bar{v}_2^4}$$

$$\frac{v_2\{6\} - v_2\{8\}}{v_2\{4\} - v_2\{6\}} \approx \frac{1}{11} \left(1 - \frac{4\kappa_{40} + \frac{8(p_{50} + p_{32})}{\bar{v}_2}}{2\bar{v}_2 s_{30} + 3(\kappa_{40} + \kappa_{22}) + \frac{3(p_{50} + 2p_{32} + p_{14}) - 2(\sigma_y^2 - \sigma_x^2)(5s_{30} - 6s_{12})}{2\bar{v}_2}} \right)$$

- ❖ The hydro probe is then

$$\frac{v_2\{6\} - v_2\{8\}}{v_2\{4\} - v_2\{6\}} = \frac{1}{11} - \frac{1}{11} \frac{v_2\{4\}^2 - 12v_2\{6\}^2 + 11v_2\{8\}^2}{v_2\{4\}^2 - v_2\{6\}^2 + \frac{(\sigma_y^2 - \sigma_x^2)s_{30}}{3\bar{v}_2^3}}$$

negligible



Taylor expansion of the new hydrodynamics probe

In this case, expansion up to 5th moment

❖ Moments of the fifth order $p_{50} = \langle (v_x - \langle v_x \rangle)^5 \rangle - 10\sigma_x^2 s_{30}$


$$p_{32} = \langle (v_x - \langle v_x \rangle)^3 v_y^2 \rangle - \sigma_y^2 s_{30} - 3\sigma_x^2 s_{12} \quad p_{14} = \langle (v_x - \langle v_x \rangle) v_y^4 \rangle - 6\sigma_y^2 s_{12}$$

$$v_2\{10\} \approx \bar{v}_2 + \frac{\sigma_y^2 - \sigma_x^2}{2\bar{v}_2} - \frac{12}{19} \frac{s_{30} + s_{12}}{\bar{v}_2^2} + \frac{53}{57} \frac{\kappa_{40} + \frac{4}{19} \kappa_{22} - \kappa_{04}}{4\bar{v}_2^3} - \frac{5(\sigma_y^2 - \sigma_x^2)^2}{8\bar{v}_2^3} + \frac{163}{60} \frac{p_{50}}{19\bar{v}_2^4} + \frac{47}{6} \frac{p_{32}}{19\bar{v}_2^4} + \frac{21}{4} \frac{p_{14}}{19\bar{v}_2^4} + \frac{(\sigma_y^2 - \sigma_x^2)(11s_{30} + \frac{99}{2}s_{12})}{19\bar{v}_2^4}$$

$$\frac{v_2\{8\} - v_2\{10\}}{v_2\{6\} - v_2\{8\}} \approx \frac{3}{19} - \frac{88p_{50}}{95 \left[4\bar{v}_2^2 s_{30} - 2\bar{v}_2 (\kappa_{40} - 3\kappa_{22}) - 13(p_{50} + 10p_{32} - 3p_{14}) - 2(\sigma^2 - \sigma_x^2)(5s_{30} - 6s_{32}) \right]}$$

❖ The new hydro probe is then given as

$$\frac{v_2\{8\} - v_2\{10\}}{v_2\{6\} - v_2\{8\}} = \frac{3}{19} - \frac{1}{19} \frac{3v_2\{6\}^2 - 22v_2\{8\}^2 + 19v_2\{10\}^2}{v_2\{6\}^2 - v_2\{8\}^2 + \frac{(\sigma_y^2 - \sigma_x^2)s_{30}}{33\bar{v}_2^3}}$$

negligible 

Cleaned moments

➤ Kurtosis:

$$\gamma_2^{\text{exp}} \equiv -\frac{3 v_2^4 \{4\} - 12 v_2^4 \{6\} + 11 v_2^4 \{8\}}{2 [v_2 \{2\}^2 - v_2 \{4\}^2]^2} \approx -\frac{3 \left[-\frac{8\kappa_{40}}{3} - O_N \right]}{2 [2\sigma_x^2 + O_D]^2} \approx \frac{\kappa_{40}}{\sigma_x^4} = \gamma_2$$

$$O_N = \frac{16(p_{50} + p_{32})}{3\bar{v}_2}$$

➤ Cleaned kurtosis:

$$\gamma_{2,\text{corr}}^{\text{exp}} \equiv -\frac{3 v_2^4 \{4\} + 24 v_2^4 \{6\} - 253 v_2^4 \{8\} + 228 v_2^4 \{10\}}{2 [v_2 \{2\}^2 - 40 v_2 \{6\}^2 + 495 v_2 \{8\}^2 - 456 v_2 \{10\}^2]^2} \approx -\frac{3 \left[-\frac{8\kappa_{40}}{3} \right]}{2 [2\sigma_x^2 + \bar{O}_D]^2} \approx \frac{\kappa_{40}}{\sigma_x^4} = \gamma_2$$

$$\bar{O}_D = -\frac{2(\sigma_y^2 - \sigma_x^2)(s_{30} - s_{12})}{\bar{v}_2^3} + \dots$$

➤ Superskewness:

$$\gamma_3^{\text{exp}} \equiv 6\sqrt{2} \frac{3v_2 \{6\}^5 - 22v_2 \{8\}^5 + 19v_2 \{10\}^5}{[v_2 \{2\}^2 - v_2 \{4\}^2]^{5/2}} \approx 6\sqrt{2} \frac{\frac{2}{3} p_{50}}{[2\sigma_x^2 + O_D]^{5/2}} \approx \frac{p_{50}}{\sigma_x^5} = \gamma_3$$

➤ Cleaned Superskewness:

$$\gamma_{3,\text{corr}}^{\text{exp}} = 6\sqrt{2} \frac{3v_2^5 \{6\} - 22v_2^5 \{8\} + 19v_2^5 \{10\}}{[v_2 \{2\}^2 - 40v_2 \{6\}^2 + 495v_2 \{8\}^2 - 456v_2 \{10\}^2]^{5/2}} \approx 6\sqrt{2} \frac{\frac{2}{3} p_{50}}{[2\sigma_x^2 + \bar{O}_D]^{5/2}} \approx \frac{p_{50}}{\sigma_x^5} = \gamma_3$$