Light-nuclei production and QCD critical point

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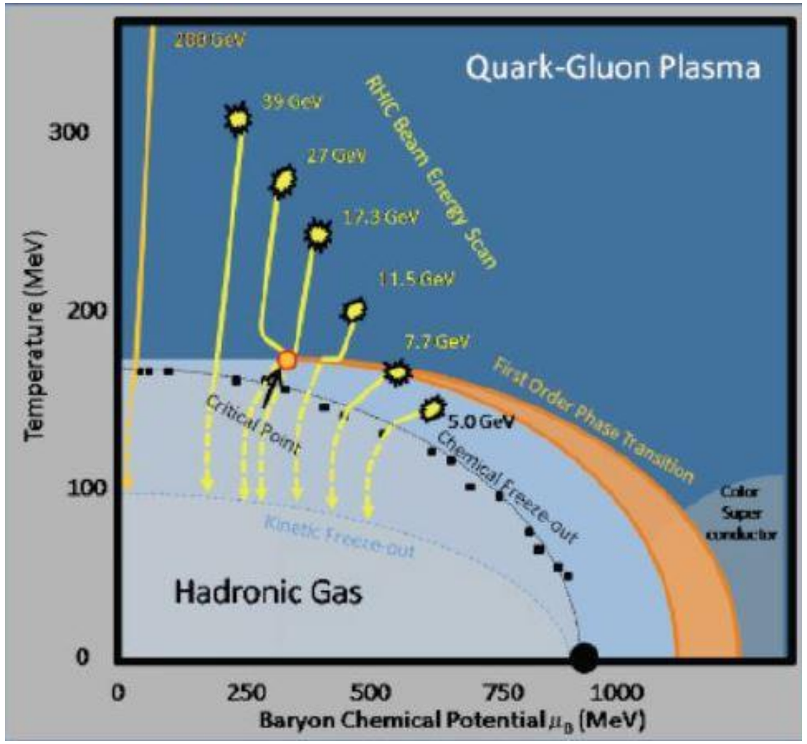
QCD phase diagram

- **Lattice QCD** (small μ_B finite T):
  - Crossover
- **Effective models** (large μ_B)
  - 1st order phase trans.

→ Critical point

- Lattice QCD: sign problem at large μ_B
- Effective models: parameters dependent

→ **Heavy-ion collisions :**

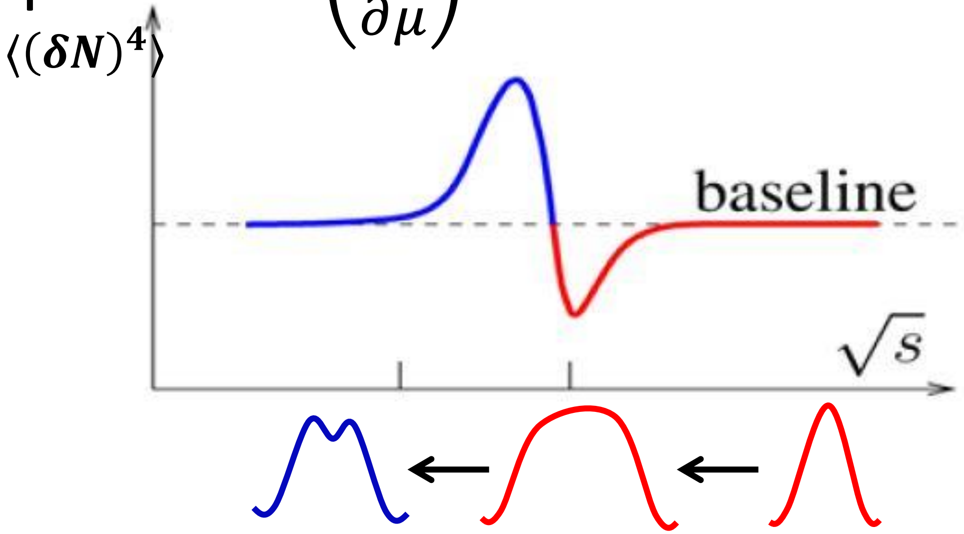


  - tuning $\sqrt{s_{NN}}$,mapping $T-\mu$ phase diagram: RHIC(BES),NICA,FAIR,J_PARC….

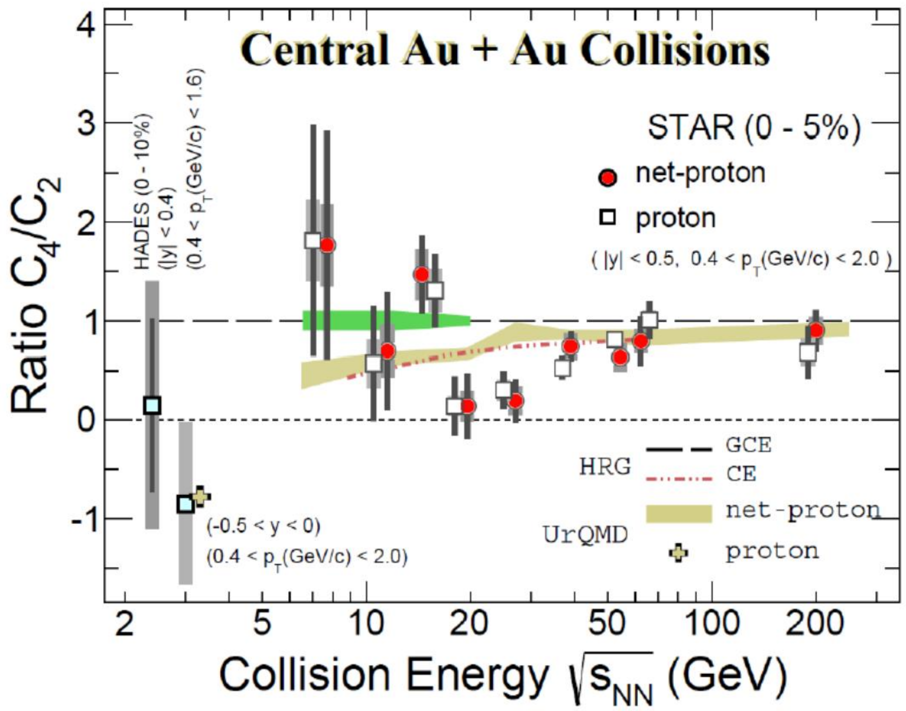
Net-proton fluctuations near critical point

- Characteristic feature of critical point:
  - long range correlation
  - large fluctuations
- **Non-monotonicity** of Net-Proton Cumulant

$\sim$ susceptibilities $\left(\frac{\partial}{\partial \mu}\right)^n \ln Z$



M.Stephanov, PRL 107,052301

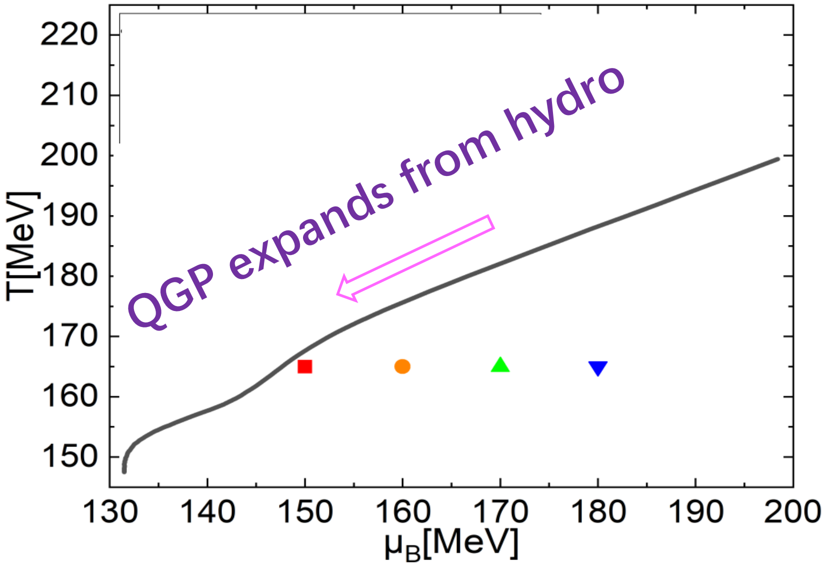


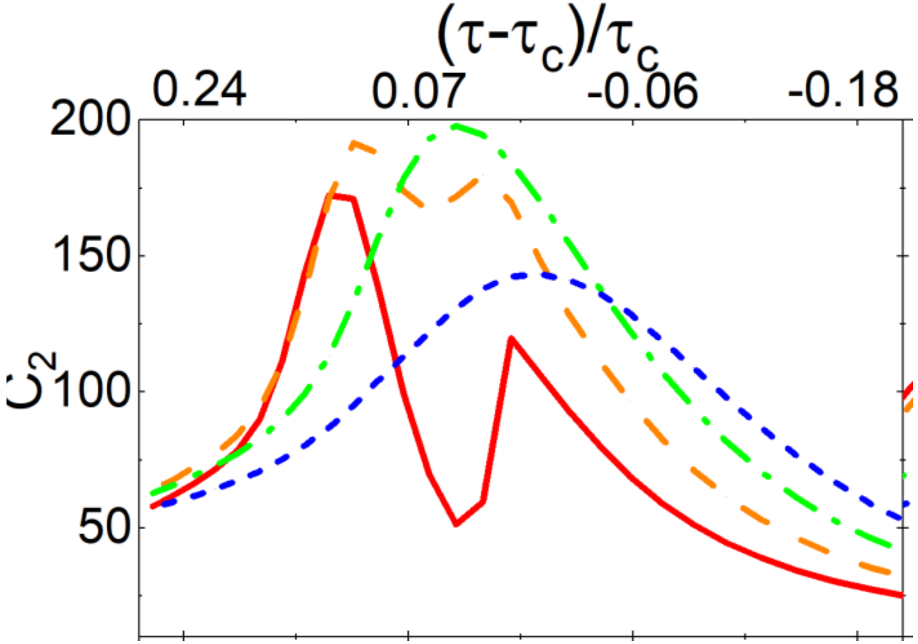
STAR, PRL 126,092301
STAR,PRL 128,202303

Fluctuations is non-trivial in expanding QGP

S.Tang, SW, H.Song, 2303.15017

- Hydro background cools down => Critical Slowing Down.
- Critical slowing down effects suppress the fluctuations
- Fireball closer to critical point, Larger fluctuations but **larger suppression**



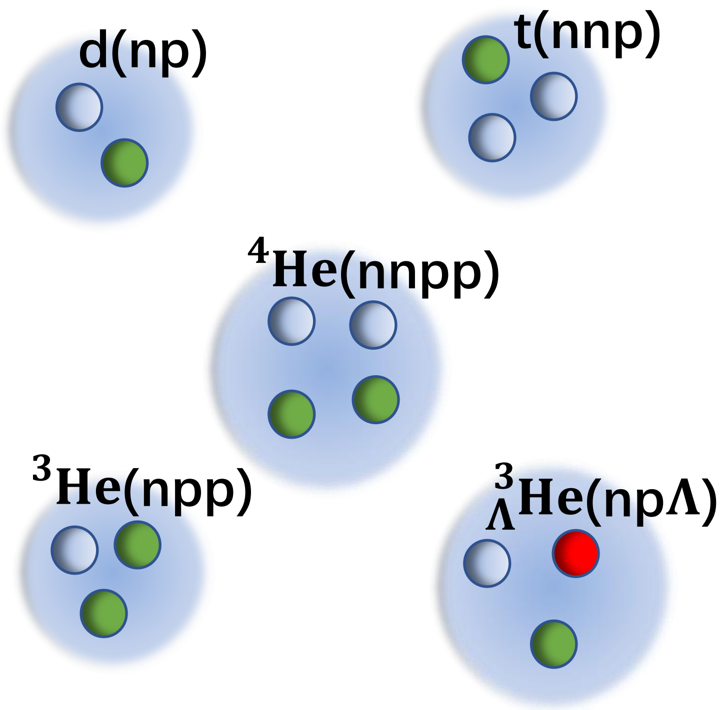


$C_2 \sim \xi^2$

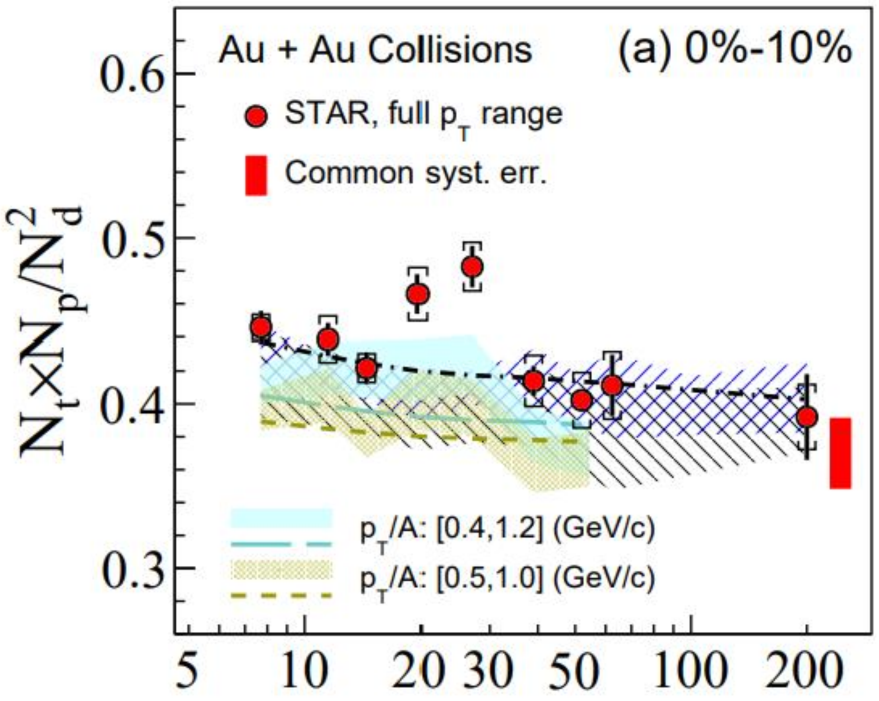
$\tau_{relax} \sim \xi^z, z = 3$

Other observable: Light Nuclei?

Light Nuclei Production

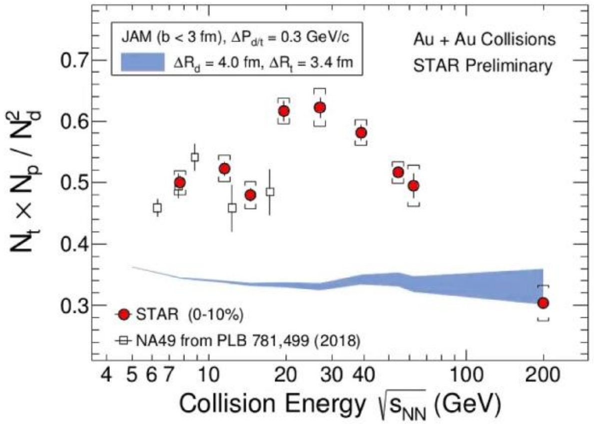


STAR Collaboration, PRL 130.202301

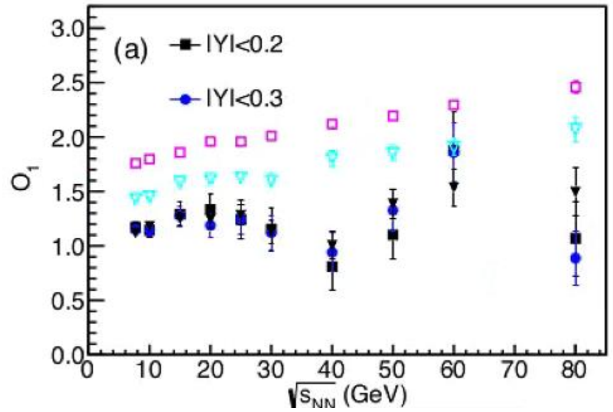


- Light nuclei produced at late stage of heavy-ion collisions
- **Non-monotonic behavior** also been observed

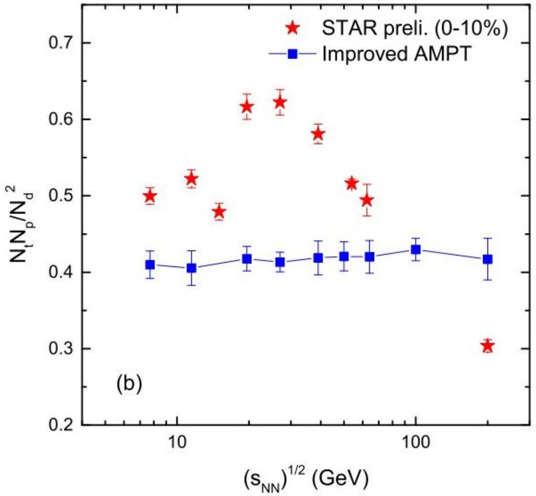
Dynamical models on Light-Nuclei



Hui Liu et al., PLB (2020)

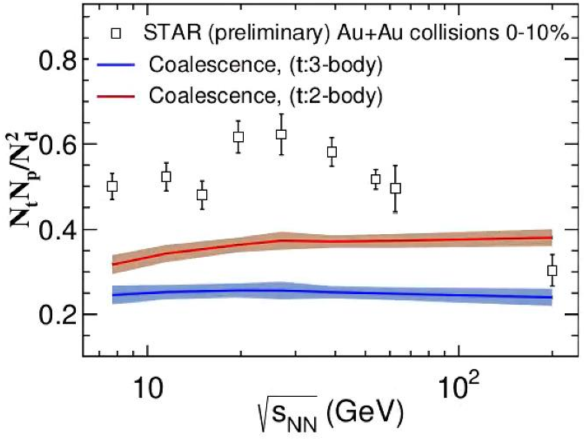


X.Deng et al., PLB (2020)

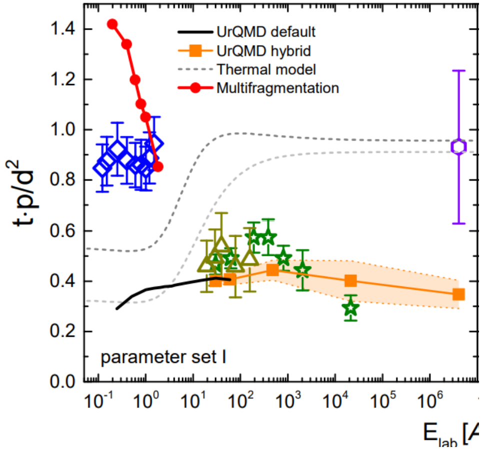


K.Sun et al., PRC (2021) ….

And others….



W. Zhao et al., PRC (2018)



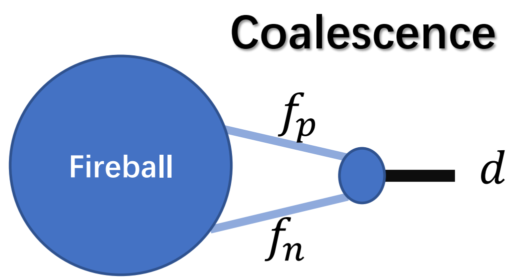
P.Hillmann et al., 2109.05972

Can light nuclei detect critical effects?

- **Light-nuclei production:**

  phase-space, nucleons interaction

  Fireball size R, homogeneity length l



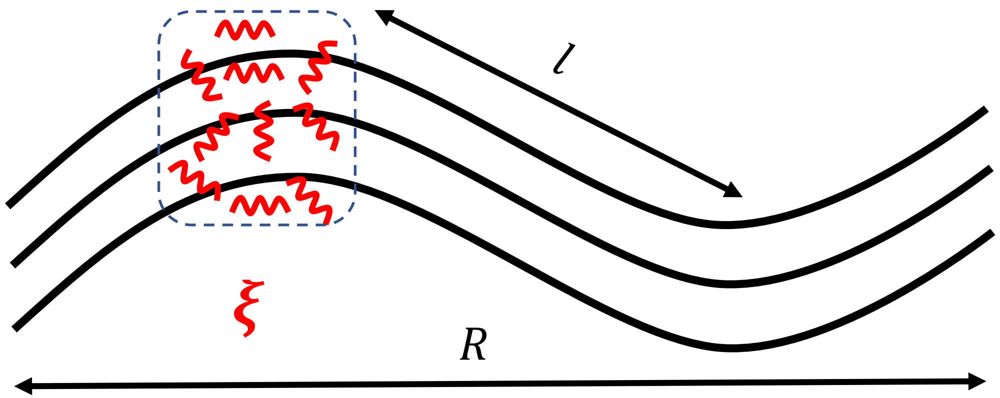
- **Homogeneity:**

  Nucleons close to each other in r space have similar momentum p

  =>Homogeneity length $l \sim 1/\partial_\mu u^\mu$

  R.Scheibl, U.Heinz, PRC 59, 1585

- **When not so close to critical point:**
  - Fireball size R, homogeneity length $l \gg \xi$
  - **Background is large**, comparing critical signal

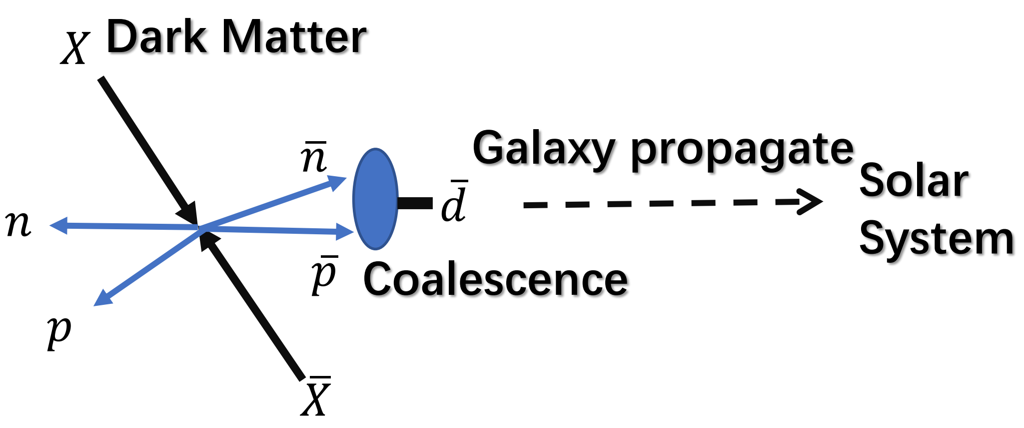


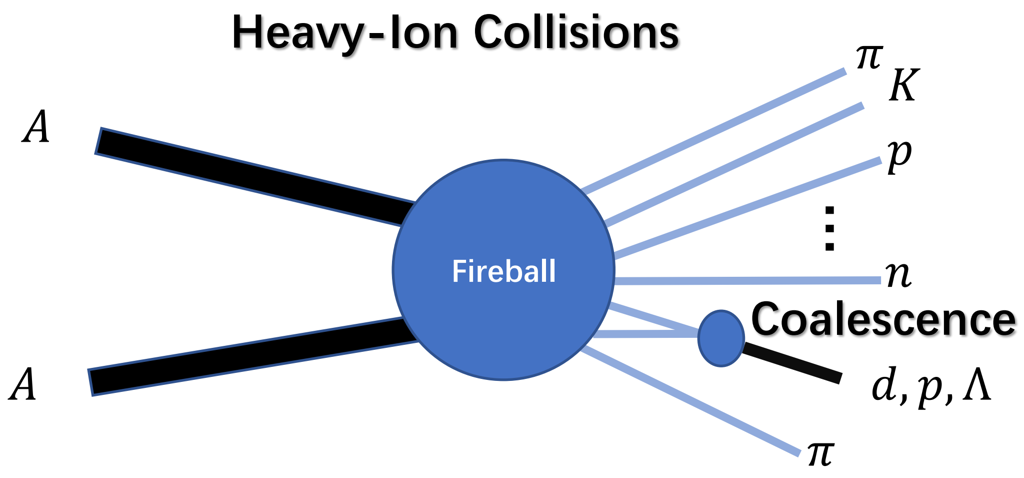
R : Fireball size
l:homogeneity length
ξ: correlation length

Light Nuclei Yield Ratio (Background+~~Critical~~):

Suppress the background

Coalescence is widely used model





Anti Light nuclei as Indirect detection of Dark Matter

See N.Fornengo et al., JCAP 09 (2013) 031 for review

Coalescence in Heavy-Ion Collisions

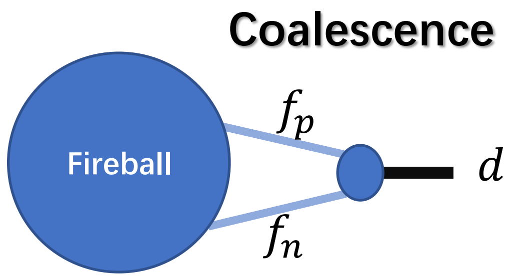
- quark + quark -> hardon
- S quark -> Lambda polarization
- nucleon + nucleon -> light nuclei

R.J.Fries et al., PRC 68.044902
L.-W.Chen et al., PRC 68.017601
X.-L. Sheng et al., PRD 102. 056013

Coalescence model

SW, K.Murase, S.Tang, H.Song, Phys.Rev.C.106.034905

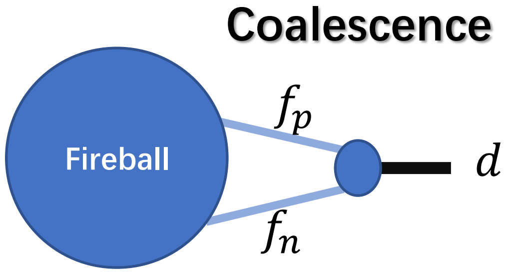
$$N_A = g_A \int \left[\prod_i^A d^3\boldsymbol{r}_i d^3\boldsymbol{p}_i f(\boldsymbol{r}_i, \boldsymbol{p}_i) \right] W_A(\{\boldsymbol{r}_i, \boldsymbol{p}_i\}_{i=1}^A)$$



Coalescence model

SW, K.Murase, S.Tang, H.Song, Phys.Rev.C.106.034905

$N_A = g_A \int$ [Phase space density] [Wigner function]



- Wigner func.(probability to produce the light nuclei):

Only depends on the relative distance in phase space $x_p - x_n$ NOT $(x_p + x_n)/2$

Examples of phase-space density

SW, K.Murase, S.Tang, H.Song, Phys.Rev.C.106.034905

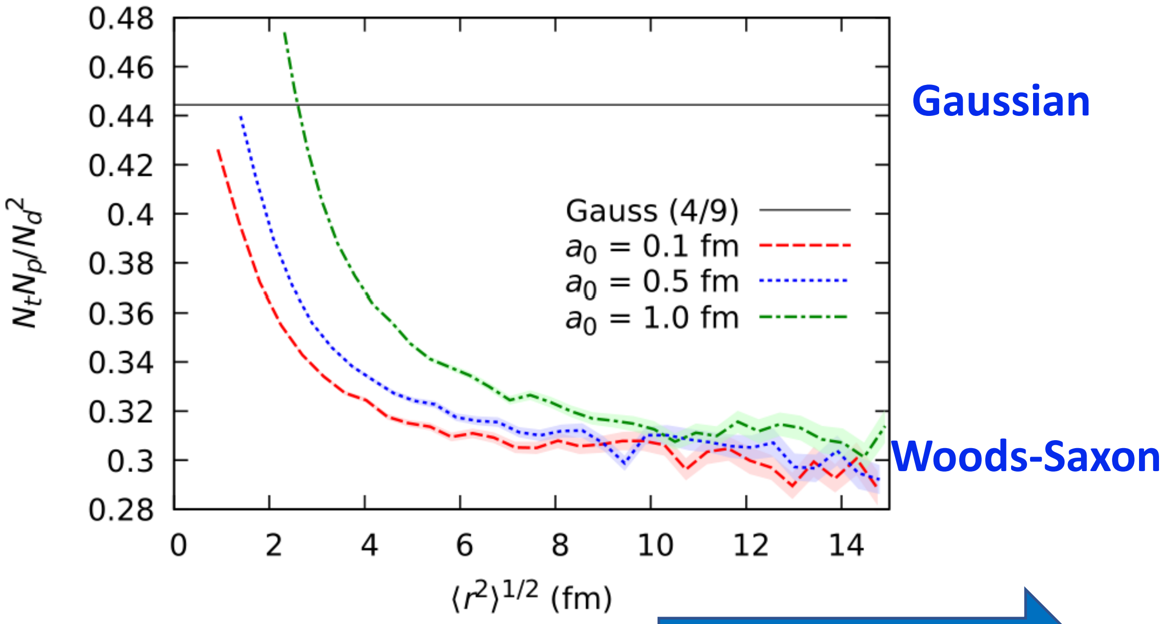
Example 1: Gaussian

$$f(\boldsymbol{r},\boldsymbol{p}) = \frac{\rho_0}{(2\pi mT)^{3/2}} \exp\left(-\frac{\boldsymbol{r}^2}{2R_s^2}\right) \exp\left(-\frac{\boldsymbol{p}^2}{2mT}\right)$$

$$N_A = g_A \int \left[\prod_i^A d^3\boldsymbol{r}_i d^3\boldsymbol{p}_i f(\boldsymbol{r}_i,\boldsymbol{p}_i)\right] W_A(\{\boldsymbol{r}_i,\boldsymbol{p}_i\}_{i=1}^A)$$

Example 2: Woods-Saxon

$$f(\boldsymbol{r},\boldsymbol{p}) = \frac{\rho_{\rm WS}}{1+\exp\frac{r-R_0}{a_0}} \cdot \frac{1}{(2\pi mT)^{3/2}} \exp\left(-\frac{\boldsymbol{p}^2}{2mT}\right)$$

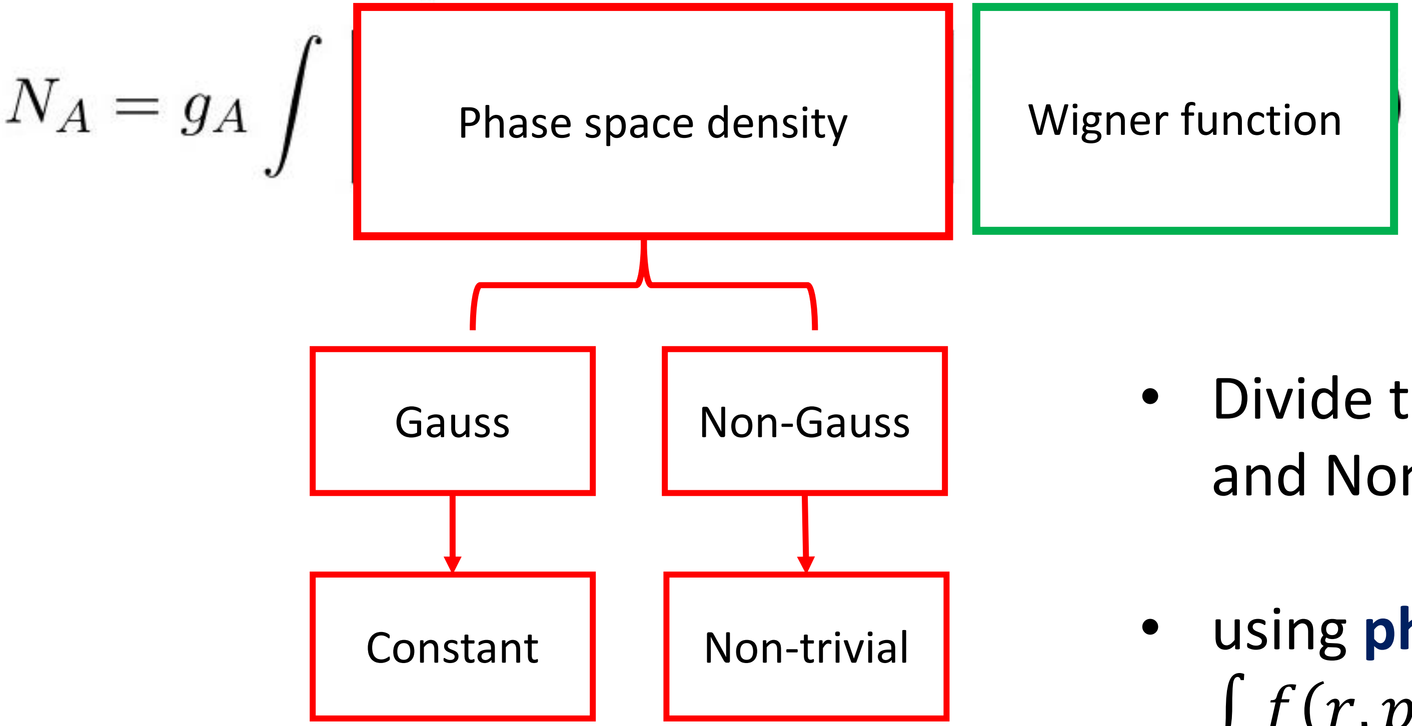


Broadening dist

Gaussian form distribution of nucleon phase-space is trivial

Light-nuclei yield (Background)

SW, K.Murase, S.Tang, H.Song, Phys.Rev.C.106.034905



$N_d : A = 2$
$N_t : A = 3$
$N_{4He} : A = 4$

- Divide the distribution into Gaussian and Non-Gaussian component
- using **phase-space cumulant** $\langle r^n p^m \rangle \sim \int f(r,p) r^n p^m$

Light-nuclei yield (Background)

SW, K.Murase, S.Tang, H.Song, Phys.Rev.C.106.034905

$N_A = g_A \int$ [Phase space density] [Wigner function]

Phase space density → Gauss / Non-Gauss

$$N_d : A = 2$$
$$N_t : A = 3$$
$$N_{4He} : A = 4$$

$$N_A = g_A N_p \left[\frac{8N_p}{\sqrt{\det(C_2 + cosnt.)}} \right]^{A-1} [1 + O(\{C_\alpha\}_{|\alpha| \geq 3})]$$

$$\langle r^n p^m \rangle \sim \int f(r,p) r^n p^m$$
$$C_2 : n + m = 2$$

N_d, N_t, N_{4He} have similar behavior in case of Gaussian phase-space density

Similar result with: R.Scheibl, U.Heinz, PRC 59, 1585; K.Blum, M.Takimoto,PRC 99,044913

Phase-space cumulant in light nuclei

SW, K.Murase, S.Tang, H.Song, Phys.Rev.C.106.034905

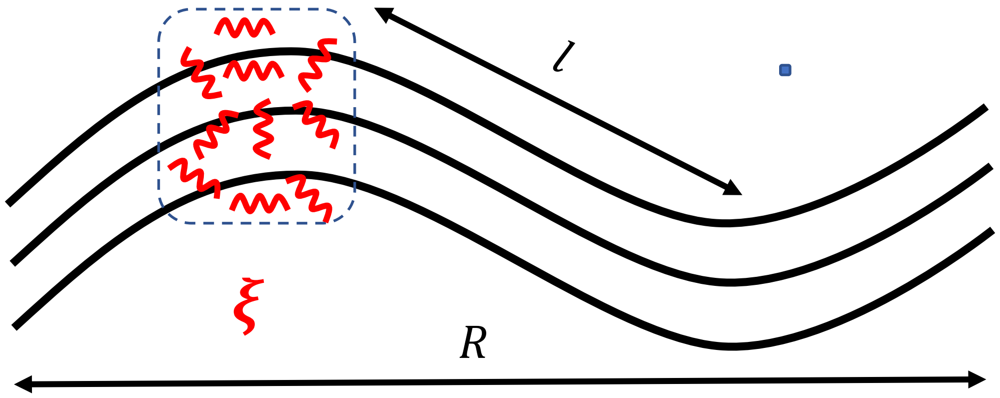
$$N_A = g_A N_p \left[\frac{8N_p}{\sqrt{\det(C_2 + cosnt.)}} \right]^{A-1} \left[1 + O(\{C_\alpha\}_{|\alpha|\geq 3})\right]$$

phase-space cumulant $\langle r^n p^m \rangle \sim \int f(r,p) r^n p^m$

2nd phase-space cumulant

$$C_2 = 2\begin{pmatrix} \#\langle rr^T \rangle & \langle rp^T \rangle \\ \langle pr^T \rangle & \#\langle pp^T \rangle \end{pmatrix}$$

$$\sim 2\begin{pmatrix} \#R_{fireball}^2 & \#l_{homoge} \\ \#l_{homoge} & \#T_{fo} \end{pmatrix}$$



$\boldsymbol{R}$: Fireball size
$\boldsymbol{l}$:homogeneity length
ξ: correlation length

Relevant scales in light-nuclei yield N_A : Fireball size $R_{fireball}$, homogeneity length l_{homoge} and freeze-out temperature T_{fo}

Example: Anisotropic flow (Blast-Wave)

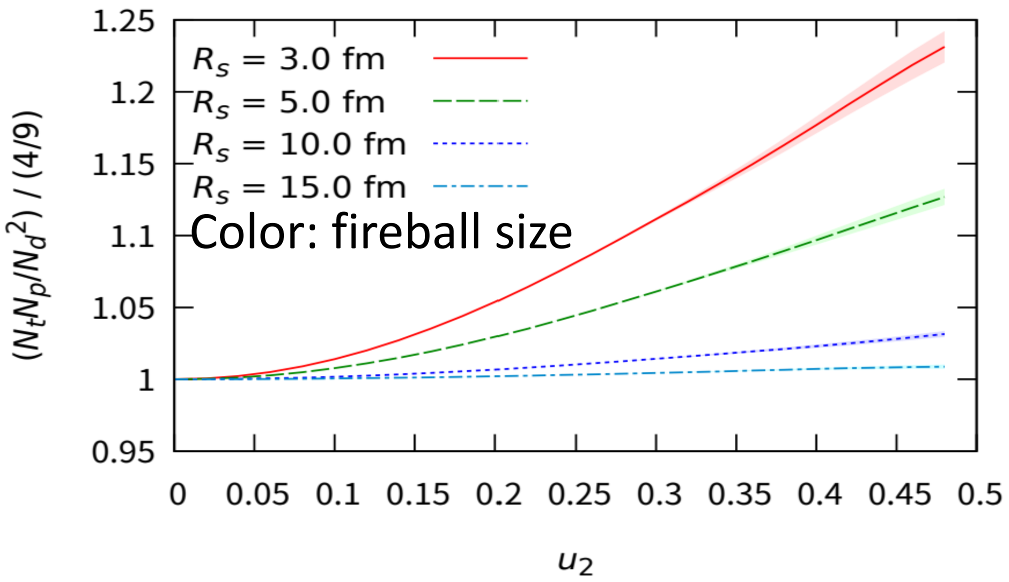
SW, K.Murase, S.Tang, H.Song, Phys.Rev.C.106.034905

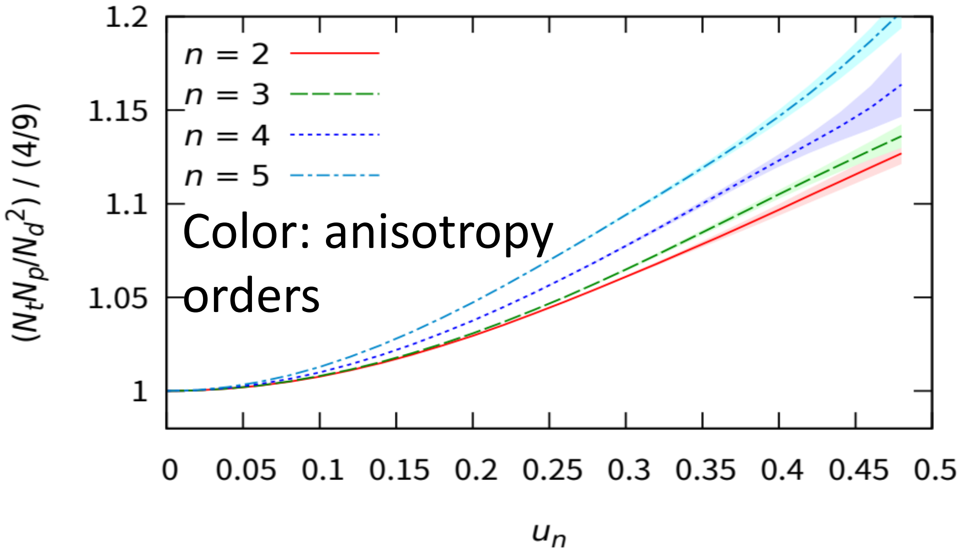
phase-space distribution

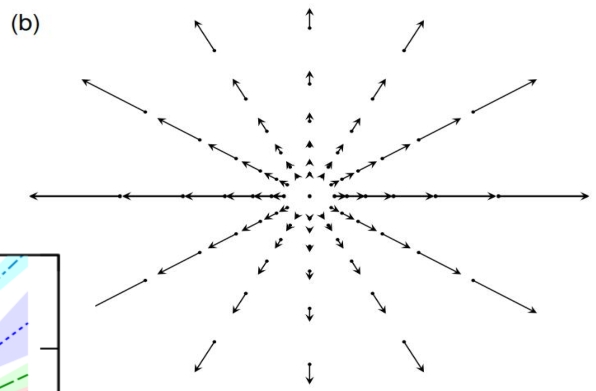
Blast-wave flow P.Huovinen et al, PLB 503, 58(2001)

Anisotropy param.

$$f(\boldsymbol{r},\boldsymbol{p}) = \frac{\rho_0}{(2\pi m T)^{3/2}} e^{-\frac{r^2}{2R_s^2}} \exp\left(-\frac{m}{2T}\left[\frac{\boldsymbol{p}}{m} - \boldsymbol{v}(\boldsymbol{r})\right]^2\right) \qquad \boldsymbol{v}(\boldsymbol{r}) = \frac{1}{R_s}(r_x, r_y, 0)^{\mathrm{T}}(1 + 2u_2 \cos 2\phi_s)$$







Profile for n=2

Anisotropic effects is negligible when fireball size is large

Momentum anisotropy increase the ratio

Light-nuclei yield (Background)

SW, K.Murase, S.Tang, H.Song, Phys.Rev.C.106.034905

$$N_A = g_A \int$$ [Phase space density] [Wigner function]

Phase space density → Gauss / Non-Gauss

$$N_A = g_A N_p \left[\frac{8N_p}{\sqrt{\det(C_2 + cosnt.)}} \right]^{A-1} [1 + O(\{C_\alpha\}_{|\alpha| \geq 3})]$$

$N_d : A = 2$

$N_t : A = 3$

$N_{4He} : A = 4$

N_d, N_t, N_{4He} have similar behavior in case of Gaussian phase-space density

Light Nuclei Ratio Near QCD Critical Point:

(Background+Critical)

Critical fluctuations δf in light nuclei

SW, K.Murase, S.Zhao, H.Song, in preparation

Introduce critical fluctuations δf

$$N_A \sim \left\langle (f_0 + \delta f)^A \right\rangle_\sigma \sim f_0^A + \left\langle (\delta f)^2 \right\rangle_\sigma^{\beta_2} + \left\langle (\delta f)^3 \right\rangle_\sigma^{\beta_3} + \left\langle (\delta f)^4 \right\rangle_\sigma^{\beta_4} + \cdots$$

2-point correlator

3-point correlator

4-point correlator

- N_A: includes contribution from $\mathbf{2, 3, \ldots}$ ***A*-point critical correlator**
- **Contribution hierarchy:** $f_0^A \gg \left\langle (\delta f)^2 \right\rangle_\sigma^{\beta_2} \gg \left\langle (\delta f)^3 \right\rangle_\sigma^{\beta_3} \gg \cdots \gg \left\langle (\delta f)^A \right\rangle_\sigma^{\beta_A}$

Light nuclei yield: Background+Critical

SW, K.Murase, S.Zhao, H.Song, in preparation

$N_A = g_A \int$ [Phase space density: Background+Critical] [Wigner func]

Scales of R, l

Scale of ξ

$$N_A = g_A N_p \left[\frac{8N_p}{\sqrt{\det(C_2 + cosnt.)}}\right]^{A-1} [1 + \#(2pt.)^{\#} + \#(3pt.)^{\#} + \#(4pt.)^{\#}]$$

Bkg   Cri

N_A share a analogous structure $N_A \propto [\ldots]^{A-1}[Bkg + Cri]$ => Construct ratios of N_A suppress Bkg and highlight Cri

$\widetilde{\boldsymbol{R}}(\boldsymbol{A}, \boldsymbol{B})$ =Ratio($\boldsymbol{N_t}, \boldsymbol{N_d}$)-statistical factor $\sim \mathcal{O}(\xi)$

$\widetilde{\boldsymbol{R}}(\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C})$ =Ratio($\boldsymbol{N_t}, \boldsymbol{N_d}$)-#Ratio($\boldsymbol{N_t}, \boldsymbol{N_d}, \boldsymbol{N_{4He}}$) $\sim \mathcal{O}(\xi)$

Example: near critical regime

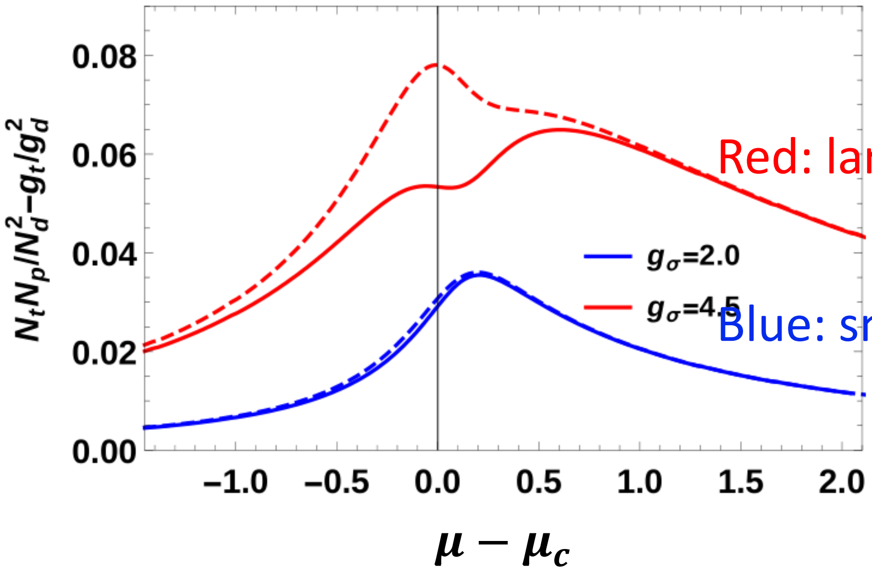
SW, K.Murase, S.Zhao, H.Song, in preparation

Ratio 1

Ratio(N_t, N_d)-statistical factor
~2pt - 3pt - (2pt)^2

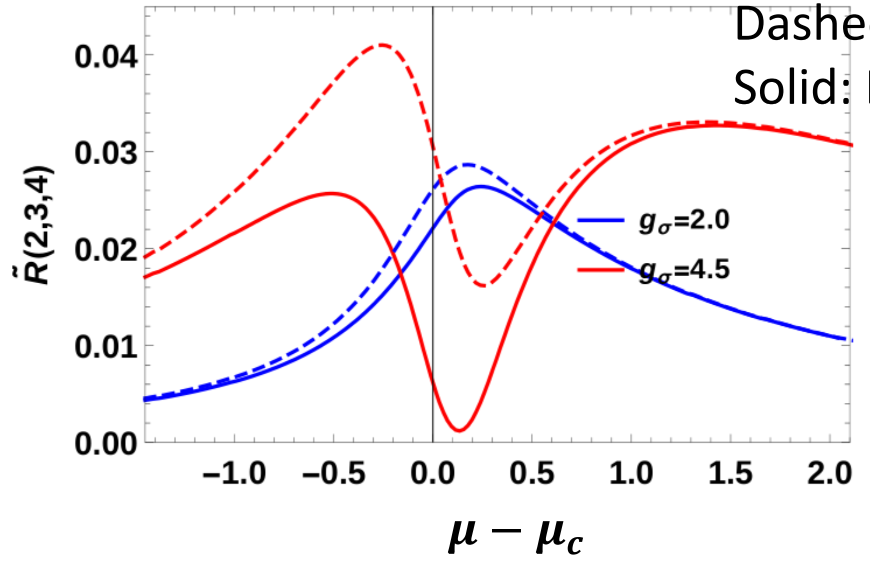
Ratio 2

Ratio(N_t, N_d)-Ratio(N_t, N_d, N_{4He})
~2pt -4 (2pt)^2



Red: large critical signal

Blue: small critical signal



Dashed: Above T_c
Solid: Below T_c

Light nuclei ratios have a peak near critical point μ_c, also have double peak because of (2pt.)^2. when the critical effect is large

Conclusion and Outlook

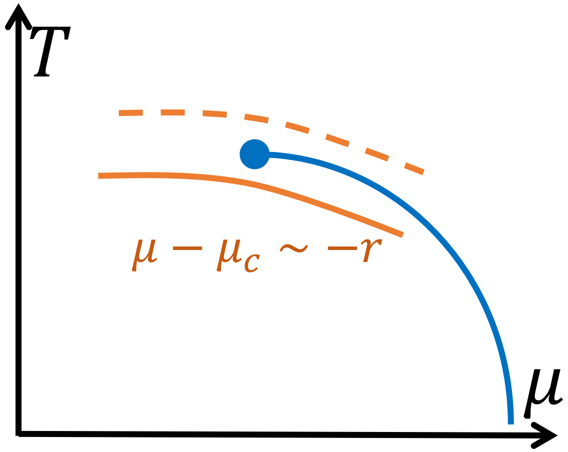
- N_d, N_t, N_{4He} depends on fireball size, homogeneity length, freeze temperature in analogous way when nucleon distribution close to Gaussian, because Wigner function depends on relative distance
- Construct the ratios to suppress the background effects
- Long range correlation results a peak, and the square of 2-point correlation induces a double peak

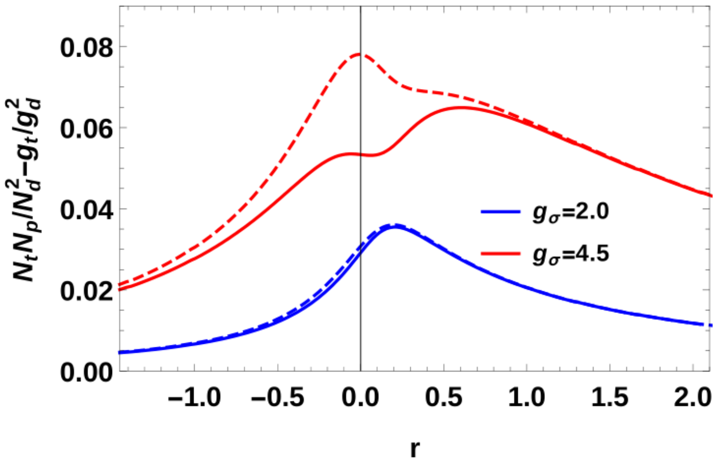
- Non-critical EbyE in light-nuclei: K.Murase, ATHIC2023

Backup

Example: in the Ising critical regime

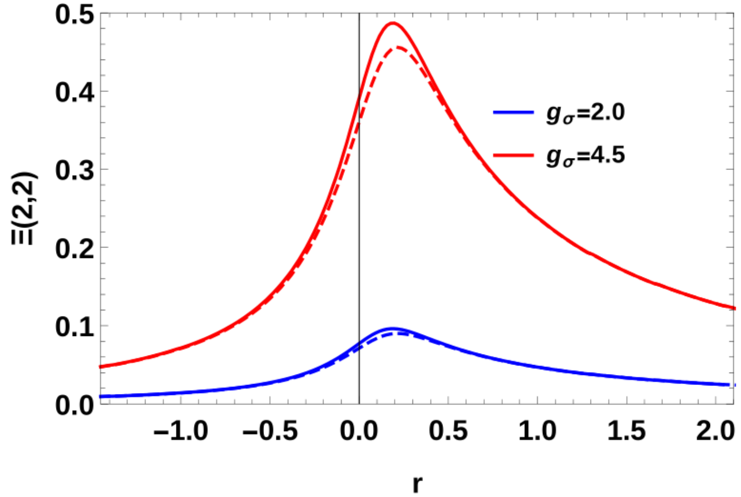
SW, K.Murase, S.Zhao, H.Song, in preparation

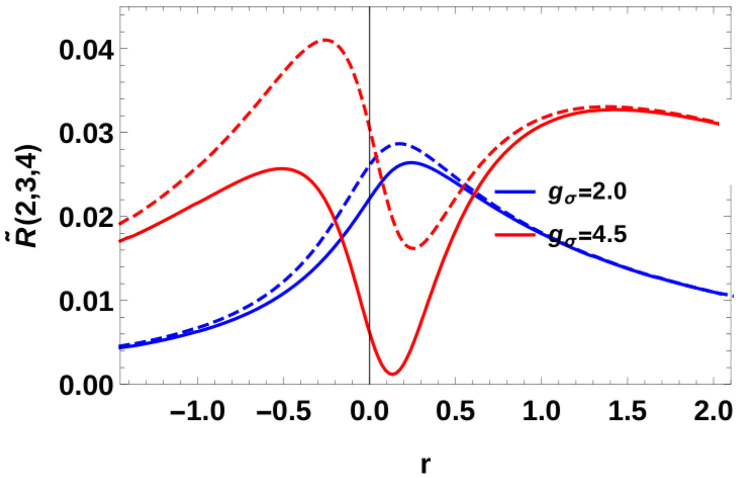




Ratio(N_t, N_d)-statistical factor

~2pt - 3pt - (2pt)^2





Ratio(N_t, N_d)-Ratio(N_t, N_d, N_{4He})

~2pt -4 (2pt)^2