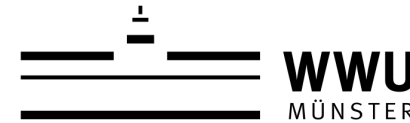


The post-mortem QGP analysis: the statistical hadronization model for heavy quarks

A. Andronic - University of Münster



-
- The statistical model and the thermal fits
 - The charm quarks
 - The beauty quarks

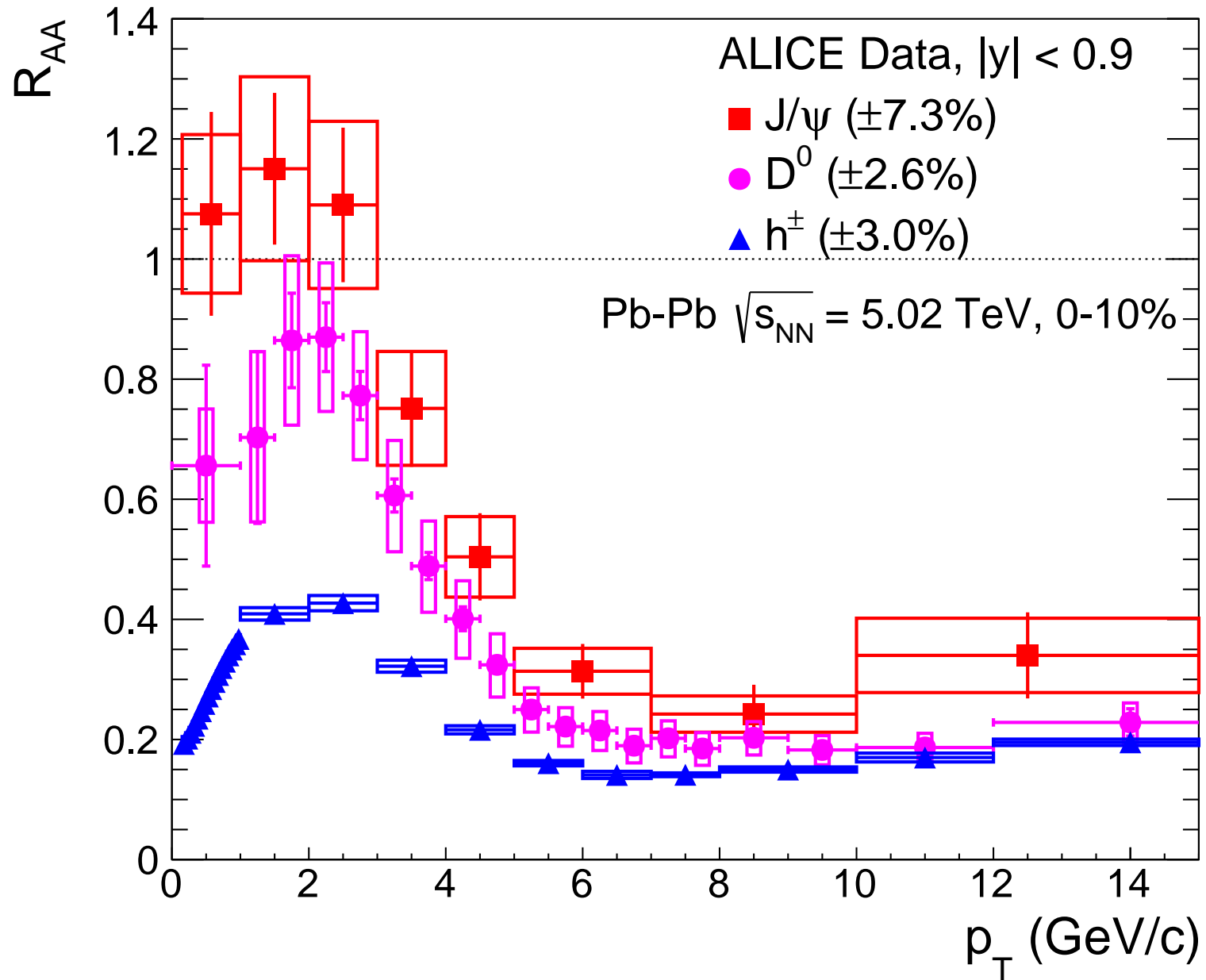
Andronic, Braun-Munzinger, Redlich, Stachel, [Nature 561 \(2018\) 321](#)

...+ Köhler, Mazeliauskas, Vislavicius, [JHEP 07 \(2021\) 035](#)

Charm data at the LHC: in perspective

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The statistical (thermal) model

grand canonical partition function for specie (hadron) i :

$$\ln Z_i = \frac{V g_i}{2\pi^2} \int_0^\infty \pm p^2 dp \ln[1 \pm \exp(-(E_i - \mu_i)/T)]$$

$g_i = (2J_i + 1)$ spin degeneracy factor; T temperature;

$E_i = \sqrt{p^2 + m_i^2}$ total energy; (+) for fermions (-) for bosons

$\mu_i = \mu_B B_i + \mu_{I_3} I_{3i} + \mu_S S_i + \mu_C C_i$ chemical potentials

μ ensure conservation (on average) of quantum numbers, fixed by “initial conditions”

i) isospin: $\sum_i n_i I_{3i} / \sum_i n_i B_i = I_3^{tot} / N_B^{tot}$, $N_B^{tot} \sim \mu_B$

I_3^{tot} , N_B^{tot} isospin and baryon number of the system (=0 at high energies)

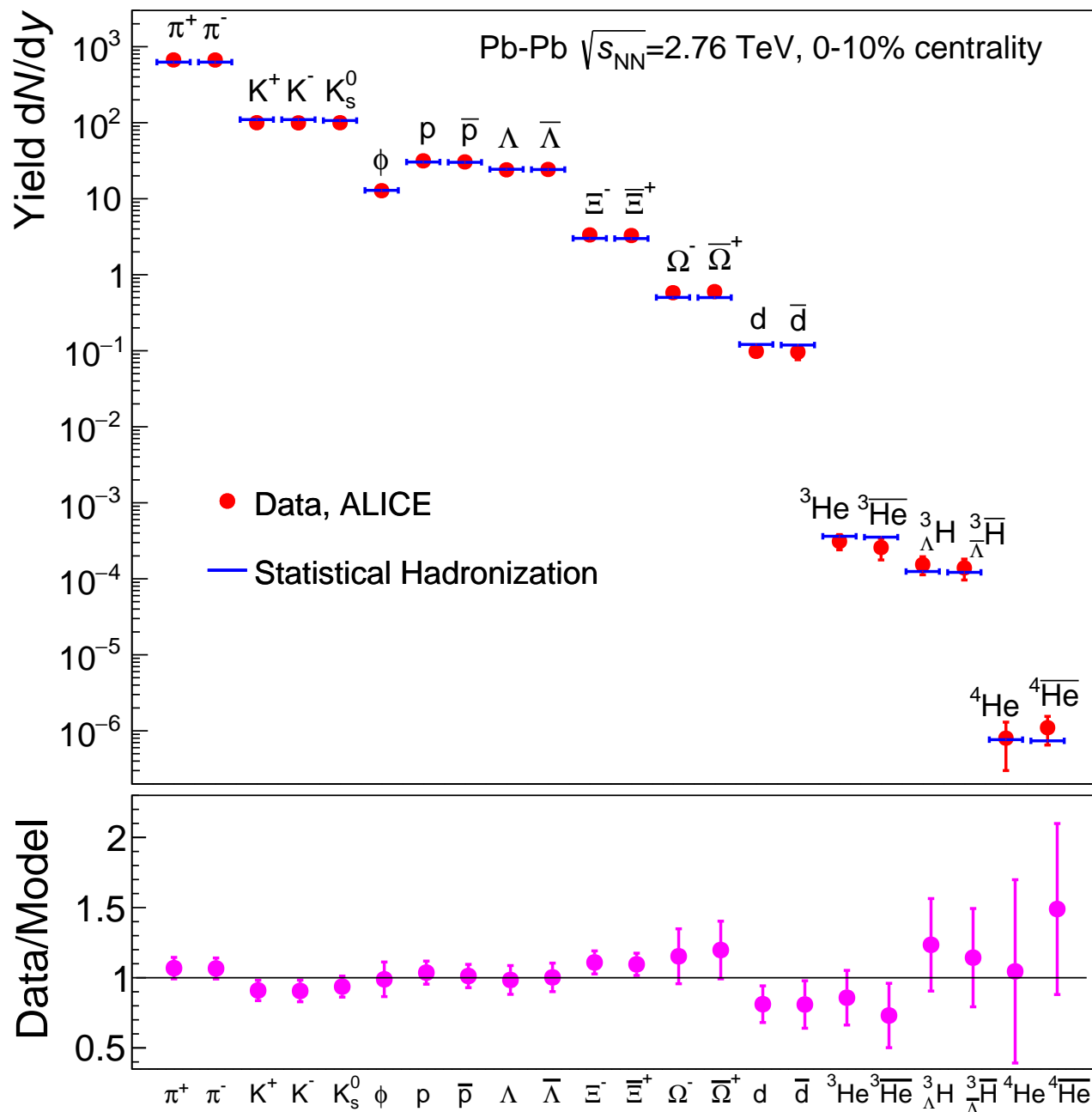
ii) strangeness: $\sum_i n_i S_i = 0$

iii) charm: $\sum_i n_i C_i = 0$.

Thermal fit – LHC, Pb–Pb, 0-10%

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matter and antimatter produced in equal amounts

$$T_{CF} = 156.6 \pm 1.7 \text{ MeV}$$

$$\mu_B = 0.7 \pm 3.8 \text{ MeV}$$

$$V_{\Delta y=1} = 4175 \pm 380 \text{ fm}^3$$

$$\chi^2/N_{df} = 16.7/19$$

S-matrix treatment

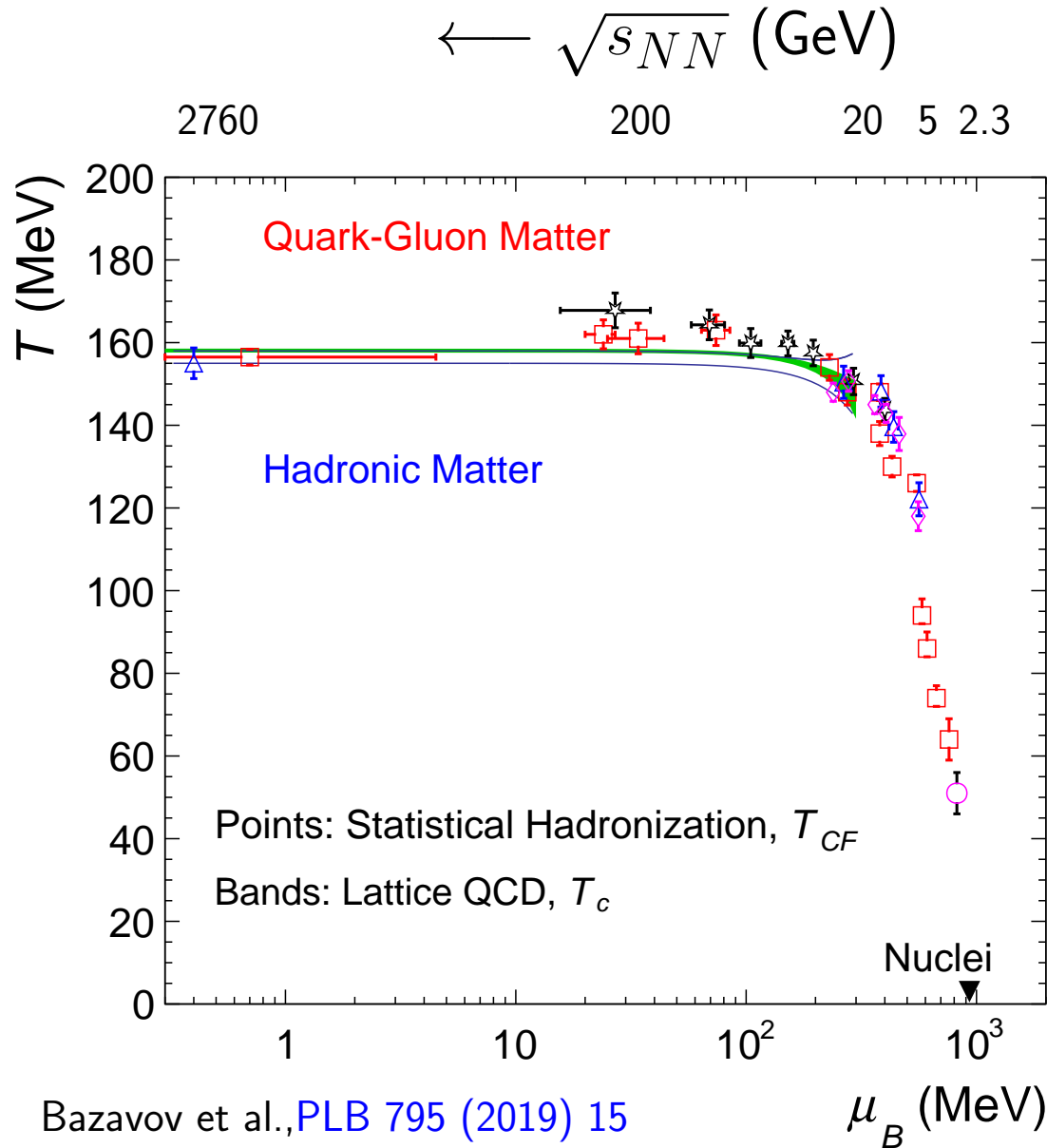
remarkably, loosely-bound objects are also well described (${}^3_{\Lambda}\text{H}$ with 25% B.R.)

hadronization as bags of quarks and gluons?

The phase diagram of QCD

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at LHC, remarkable “coincidence” with Lattice QCD results

at LHC ($\mu_B \simeq 0$): purely-produced (anti)matter ($m = E/c^2$), as in the Early Universe

$\mu_B > 0$: more matter, from “remnants” of the colliding nuclei

$\mu_B \gtrsim 400$ MeV: *the critical point awaiting discovery*

(RHIC BES / FAIR)

Bazavov et al., [PLB 795 \(2019\) 15](#)

Borsanyi et al., [PLB 370 \(2014\) 99](#)

see refs. in [Nature 561 \(2018\) 321](#)

points: independent analyses of same data → “model/code uncert.” are small

SHM for charm (SHMc)

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pQCD production, "throw in": $N_{c\bar{c}} = 9.6 \rightarrow g_c = 30.1$ ($I_1/I_0 = 0.974$)

LHC, central collisions

assume:

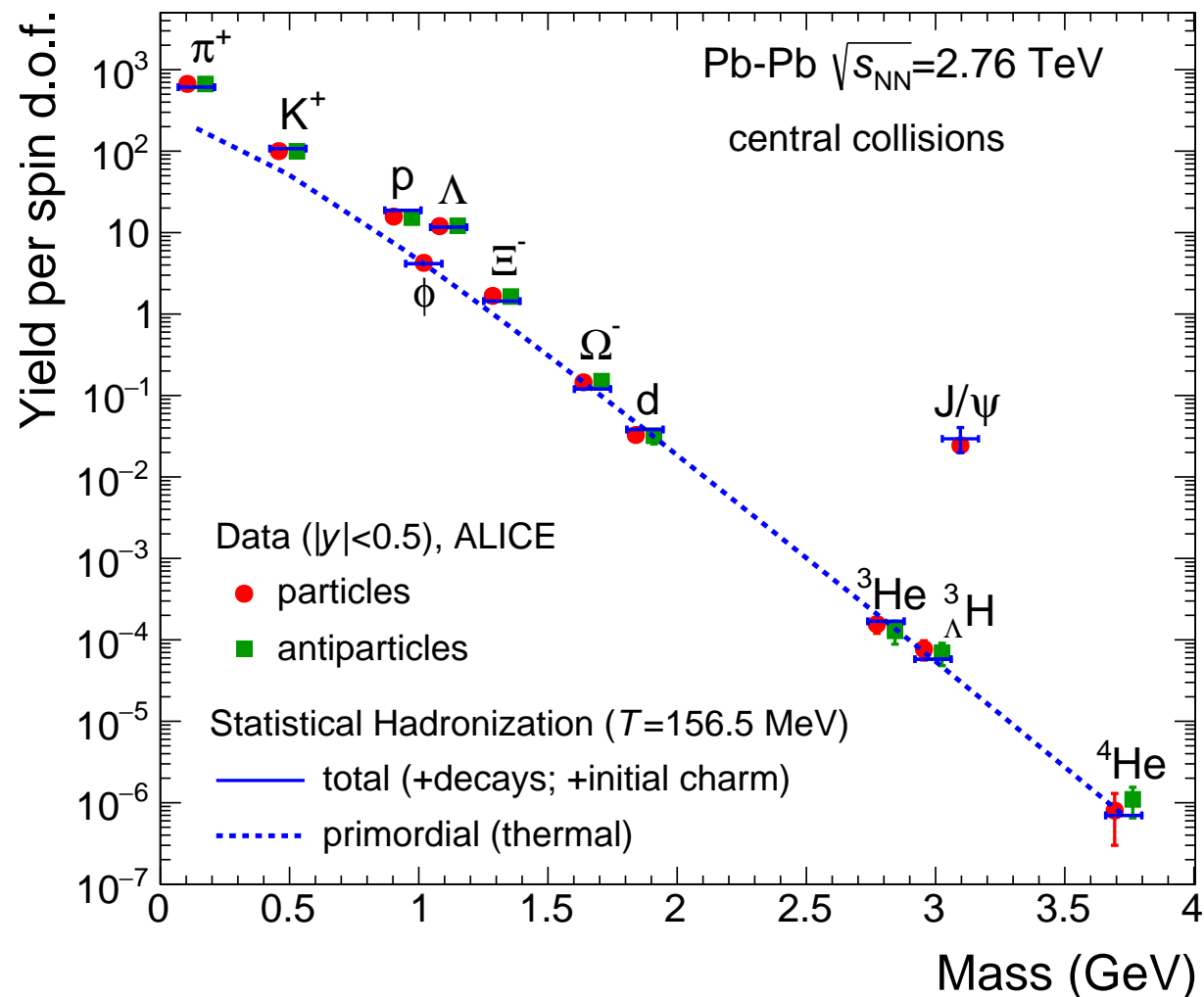
- full thermalization of c, \bar{c}
("mobility" in $V \simeq 4000 \text{ fm}^3$)

- full color screening
(Matsui-Satz)

Braun-Munzinger, Stachel, [PLB 490 \(2000\) 196](#)

Model predicts all charm
chemistry ($\psi(2S), X(3872)$)

π, K^\pm, K^0 from charm included in the thermal fit
(0.7%, 2.9%, 3.1% for $T=156.5 \text{ MeV}$)



[PLB 797 \(2019\) 134836](#)

SHMc: method and inputs

Braun-Munzinger, Stachel, [PLB 490 \(2000\) 196](#), [NPA 690 \(2001\) 119](#)

- Thermal model calculation (grand canonical) T, μ_B : $\rightarrow n_X^{th}$
- $N_{c\bar{c}}^{dir} = \frac{1}{2}g_c V (\sum_i n_{D_i}^{th} + n_{\Lambda_i}^{th}) + g_c^2 V (\sum_i n_{\psi_i}^{th} + n_{\chi_i}^{th})$
- $N_{c\bar{c}} \ll 1 \rightarrow$ Canonical (Cleymans, Redlich, Suhonen, Z. Phys. C51 (1991) 137):

Gorenstein, Kostyuk, Stöcker, Greiner, [PLB 509 \(2001\) 277](#)

$$N_{c\bar{c}}^{dir} = \frac{1}{2}g_c N_{oc}^{th} \frac{I_1(g_c N_{oc}^{th})}{I_0(g_c N_{oc}^{th})} + g_c^2 N_{c\bar{c}}^{th} \rightarrow g_c(N_{part}) \text{ (charm fugacity)}$$

Outcome: $N_D = g_c V n_D^{th} I_1/I_0 + N_D^{corona}$, $N_{J/\psi} = g_c^2 V n_{J/\psi}^{th} + N_{J/\psi}^{corona}$

Inputs: $T, \mu_B, V_{\Delta y=1} (= (dN_{ch}^{exp}/dy)/n_{ch}^{th}), N_{c\bar{c}}^{dir}$ (exp. or pQCD)

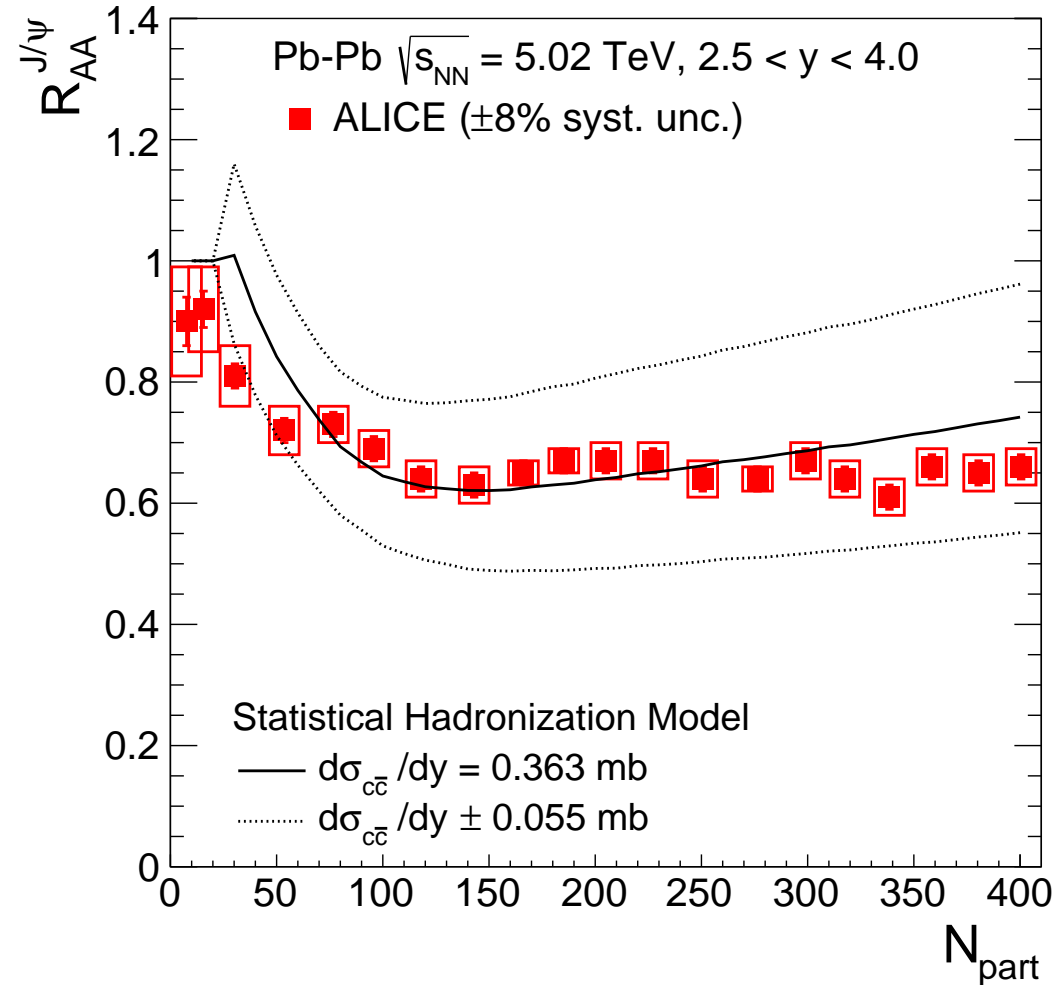
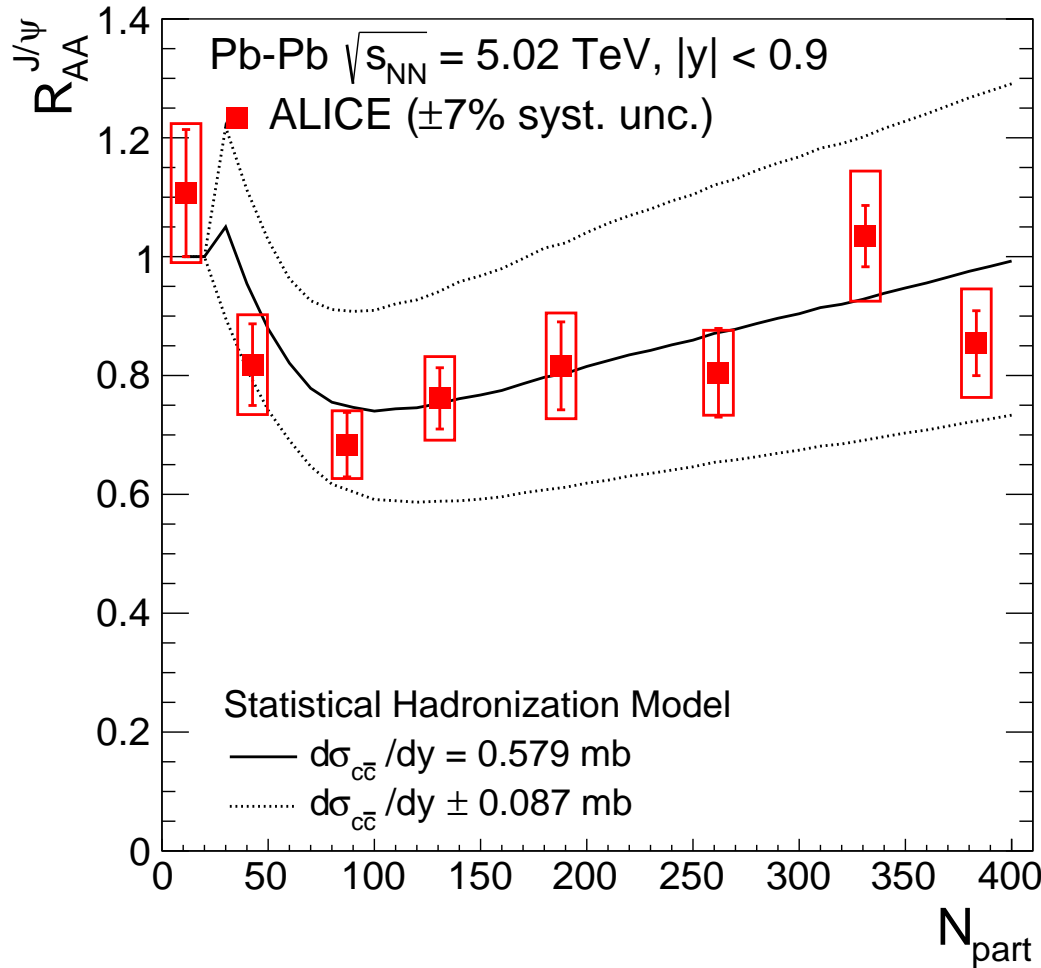
Assumed minimal volume for QGP: $V_{QGP}^{min} = 200 \text{ fm}^3$

SHMc and charmonium data at the LHC

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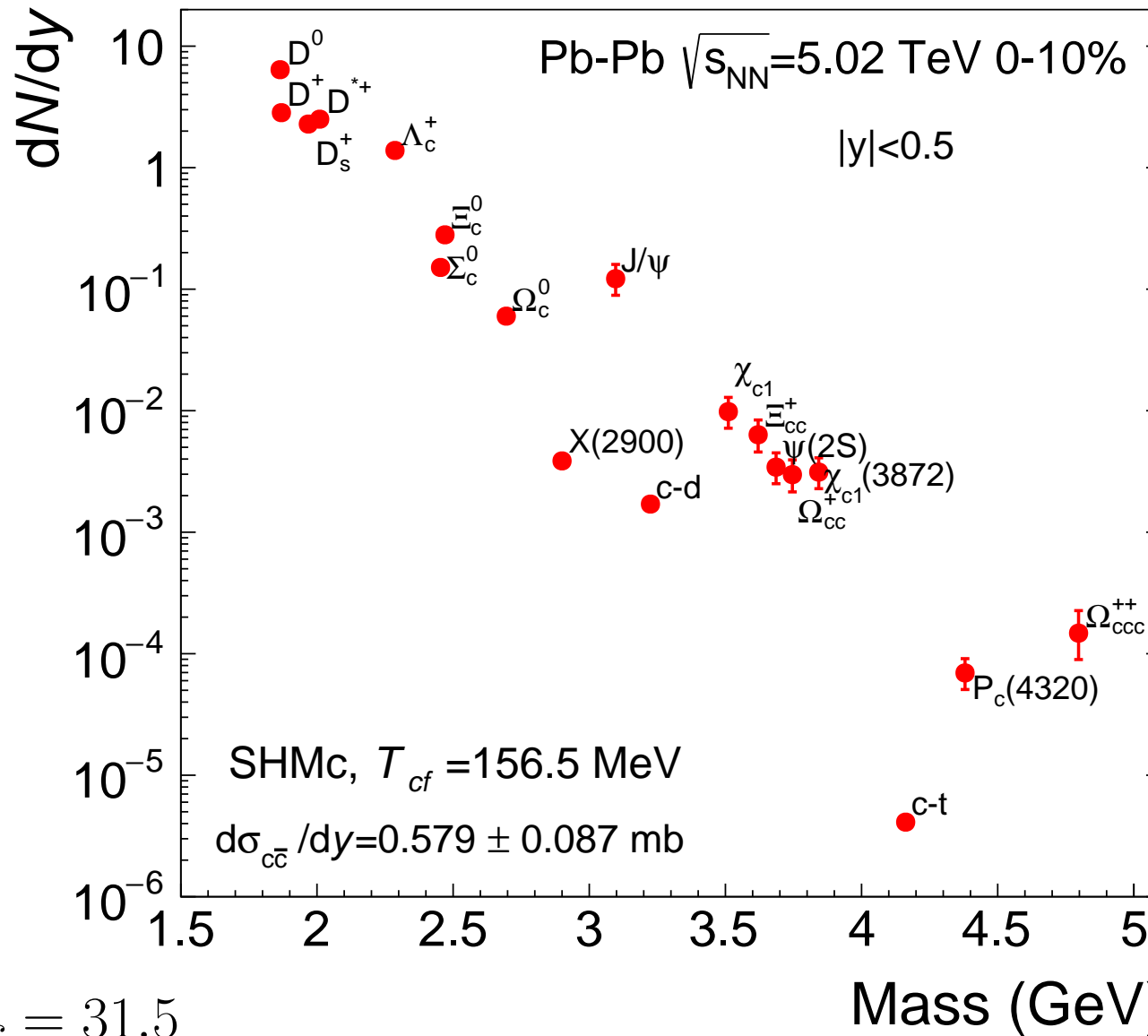
full thermalization of c quarks in QGP, hadronization at chemical freeze-out



$d\sigma_{c\bar{c}}/dy$ via normalization to D^0 in Pb-Pb 0-10%, ALICE, [arXiv:2110.09420](https://arxiv.org/abs/2110.09420)

$dN/dy = 6.82 \pm 1.03$ ($|y| < 0.5$; FONLL for $y=2.5-4$; assuming hadronization fractions in data as in SHMc)

SHMc: the full charm zoo



$$\frac{dN_{c\bar{c}}}{dy} = 13.8$$

$$\rightarrow g_c = 31.5$$

$$T_{cc}^+ \simeq 0.9 \cdot \chi_{c1}(3872)$$

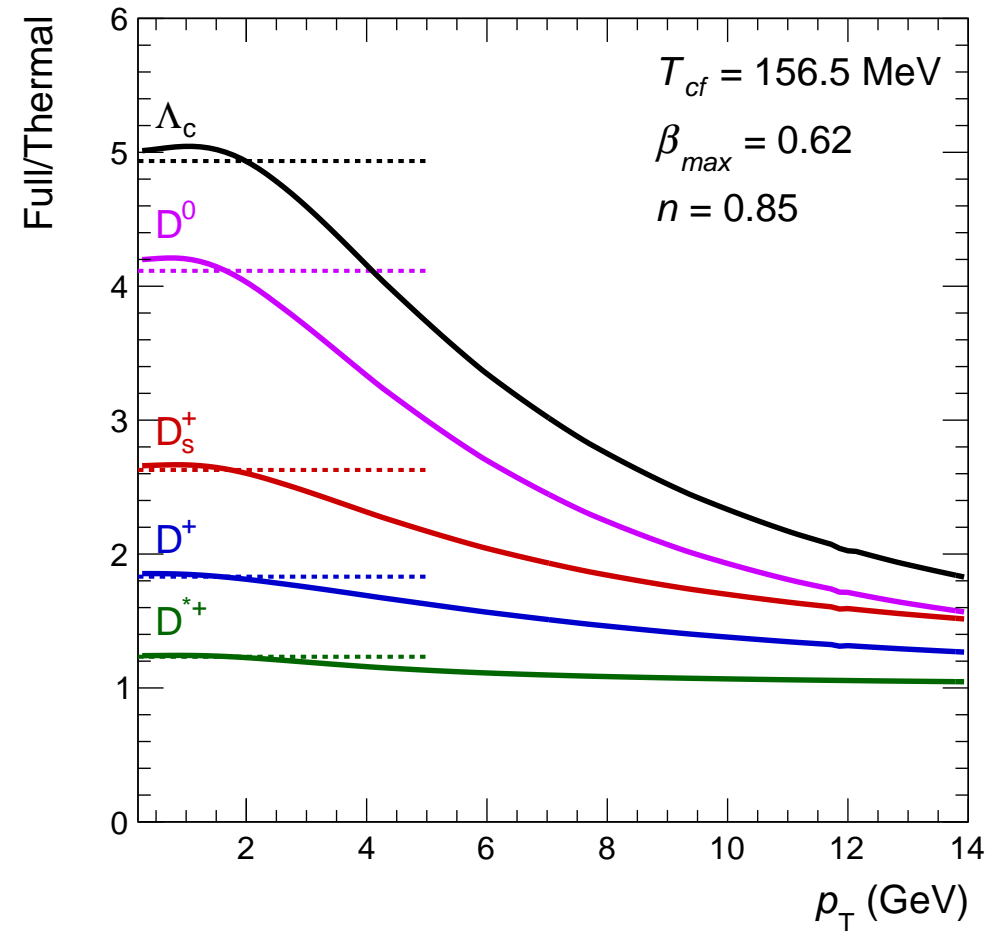
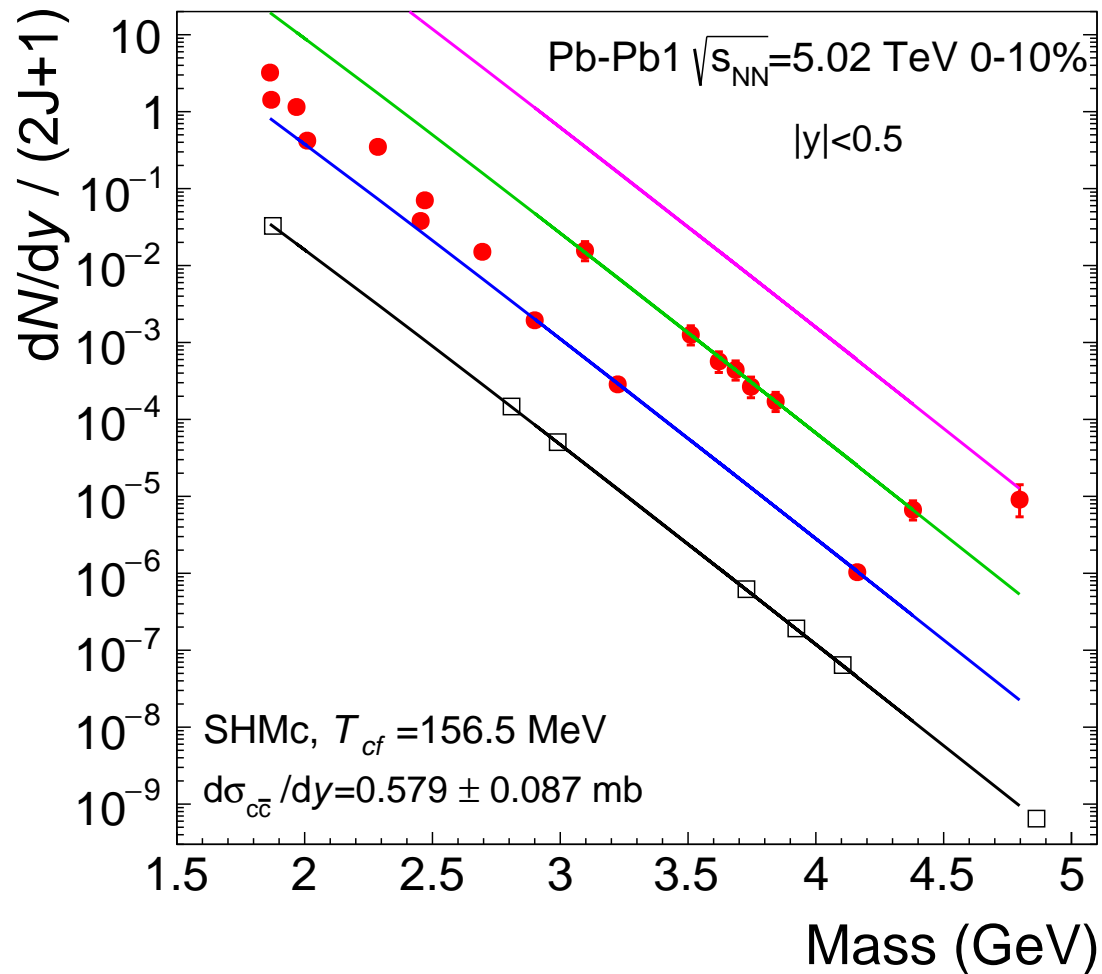
$$X(6900) \sim 10^{-8}$$

The power of the model: predicting the full suite of charmed hadrons

Full charm predictions for the LHC

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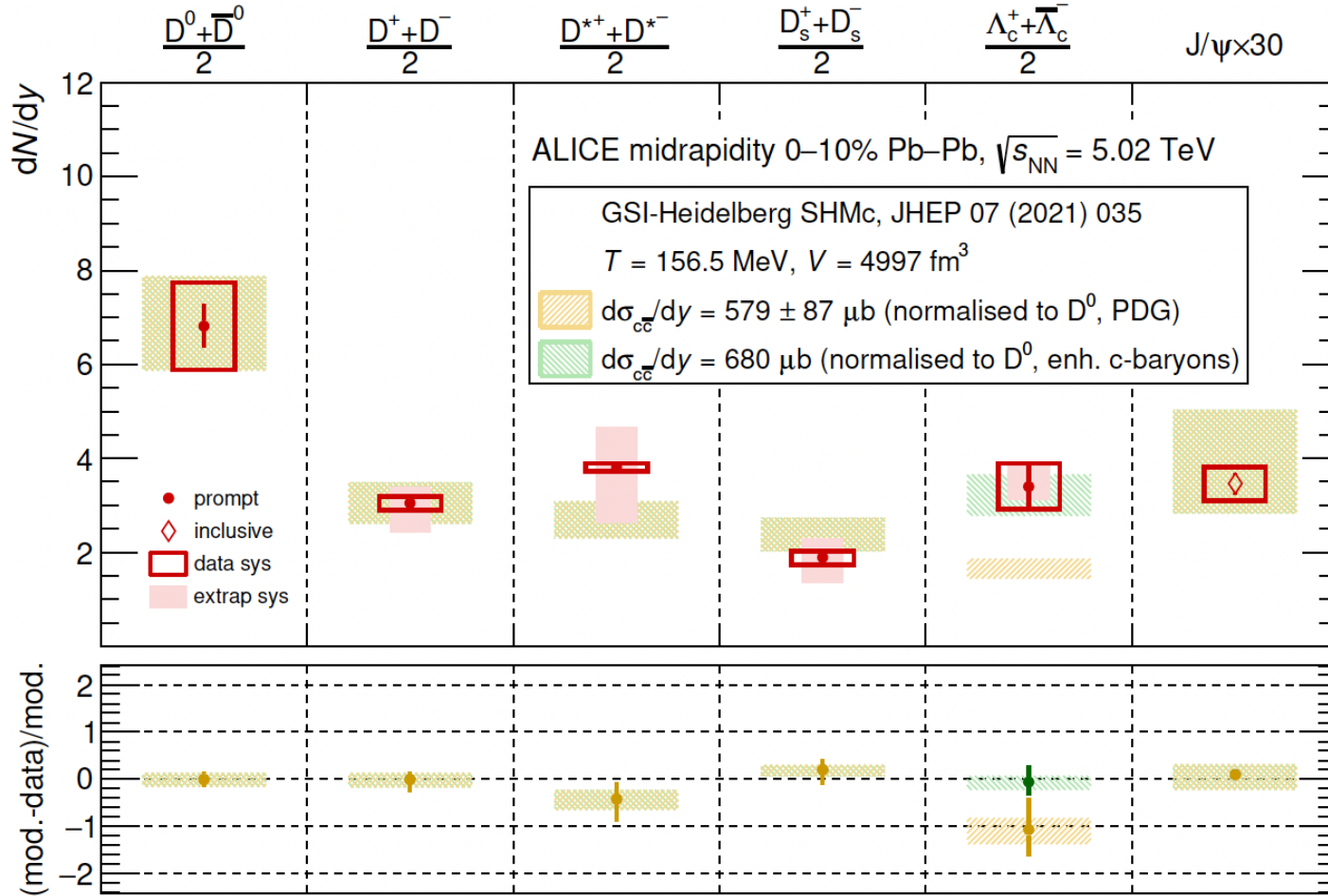
Charm-hadron spectrum as in PDG: 55 c-mesons, 74 c-baryons (part.+antipart.)

...large, but may not be complete

Charm data and SHMc model

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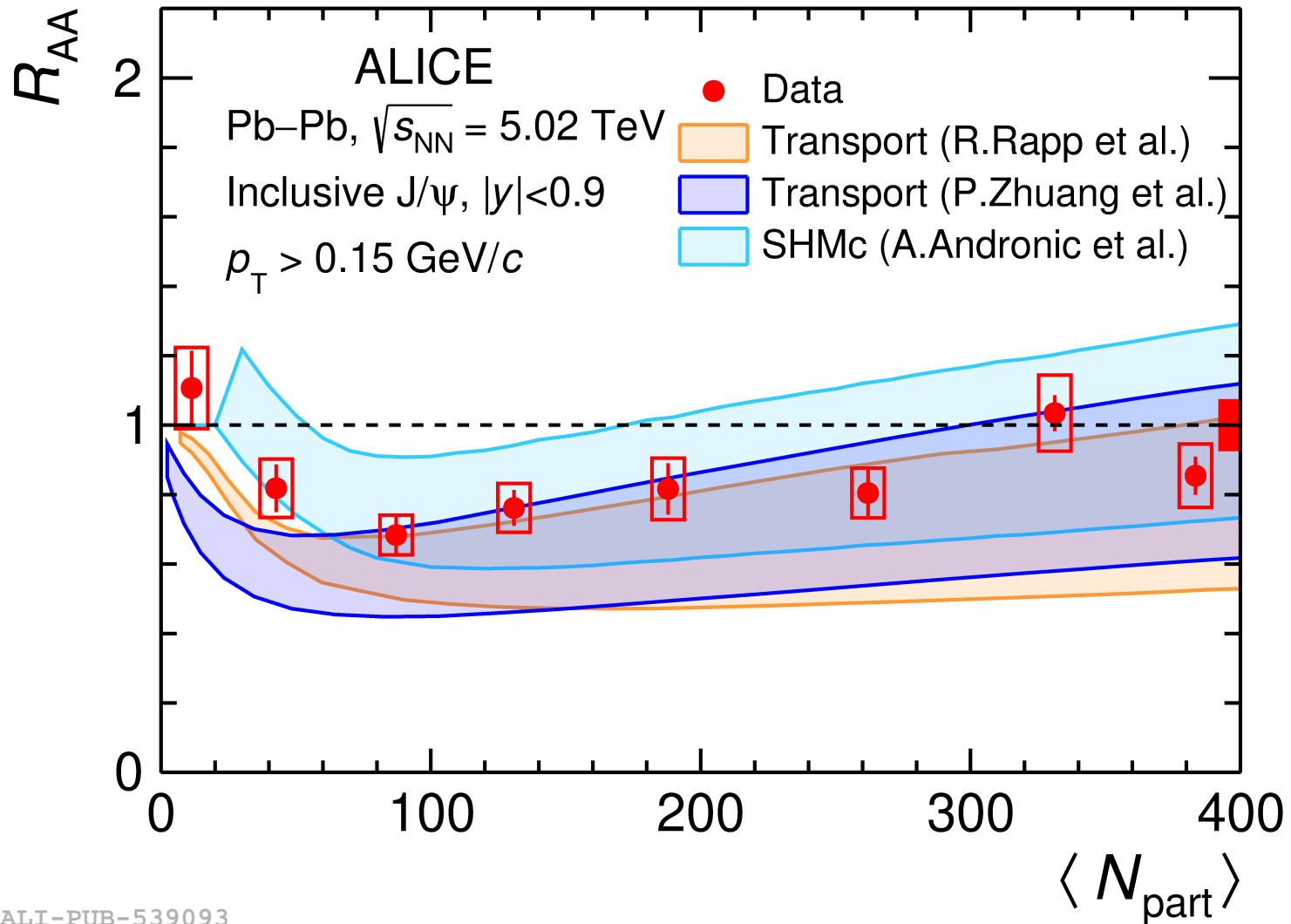
ALICE, [arXiv:2212.04348](https://arxiv.org/abs/2212.04348)

Enh. c-baryons: *tripled* the excited charm-baryon states, and $d\sigma_{c\bar{c}}/dy$: +19%

RQM: He, Rapp, [PLB 795 \(2019\) 117](https://arxiv.org/abs/1905.07541); LQCD, Bazavov et al., [PLB 737 \(2014\) 210](https://arxiv.org/abs/1403.7093)

leaves the mesonic sector unaffected, for the commensurately larger $\sigma_{c\bar{c}}$

SHMc vs. transport models



ALI-PUB-539093

ALICE, [arXiv:2303.13361](https://arxiv.org/abs/2303.13361)

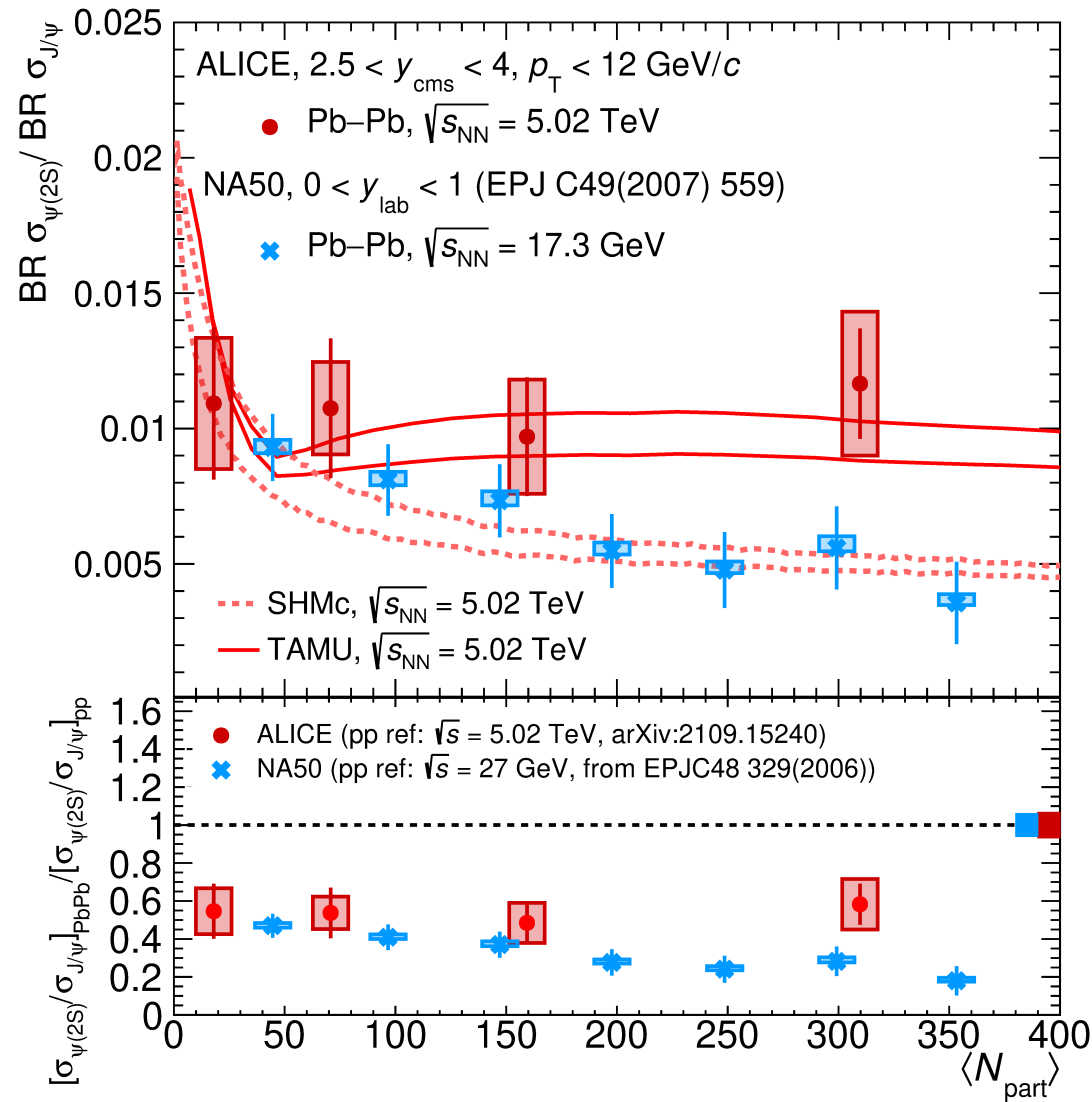
SHMc: $d\sigma_{c\bar{c}}/dy$ via normalization to D^0 in Pb–Pb 0-10%, ALICE, [arXiv:2110.09420](https://arxiv.org/abs/2110.09420)

$dN/dy = 6.82 \pm 1.03$ ($|y| < 0.5$; FONLL for $y=2.5-4$; assuming hadronization fractions in data as in SHMc)

$\psi(2S)/J/\psi$ at the LHC (and SPS)

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ALI-PUB-528400

In SHMc uncertainty only due to nuclear-corona
 ($\sigma_{c\bar{c}}$ cancels out completely)

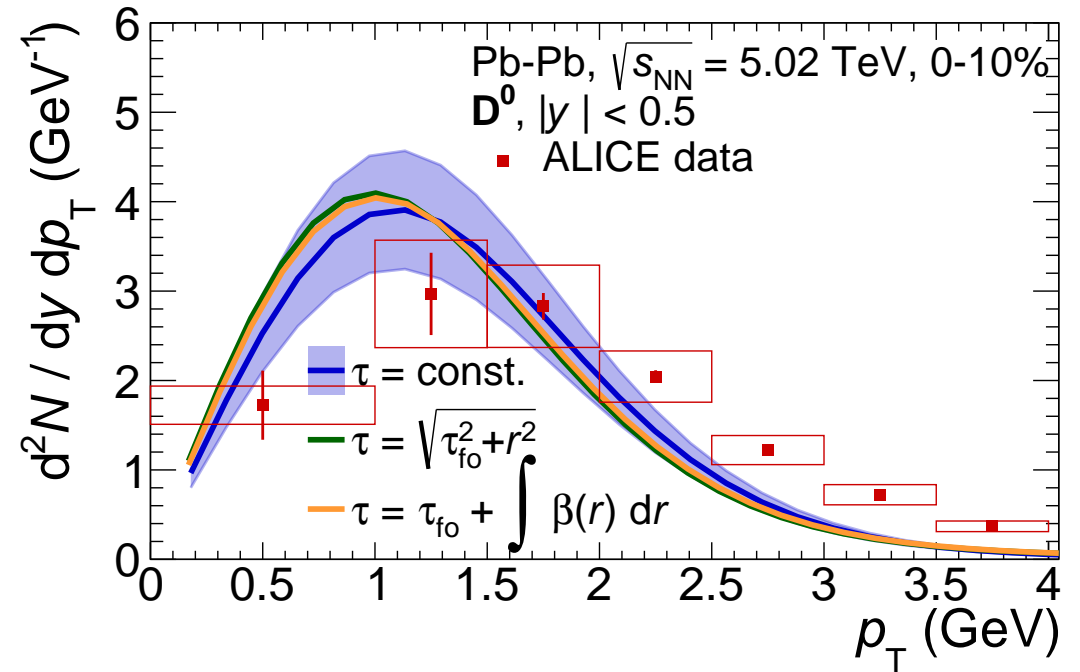
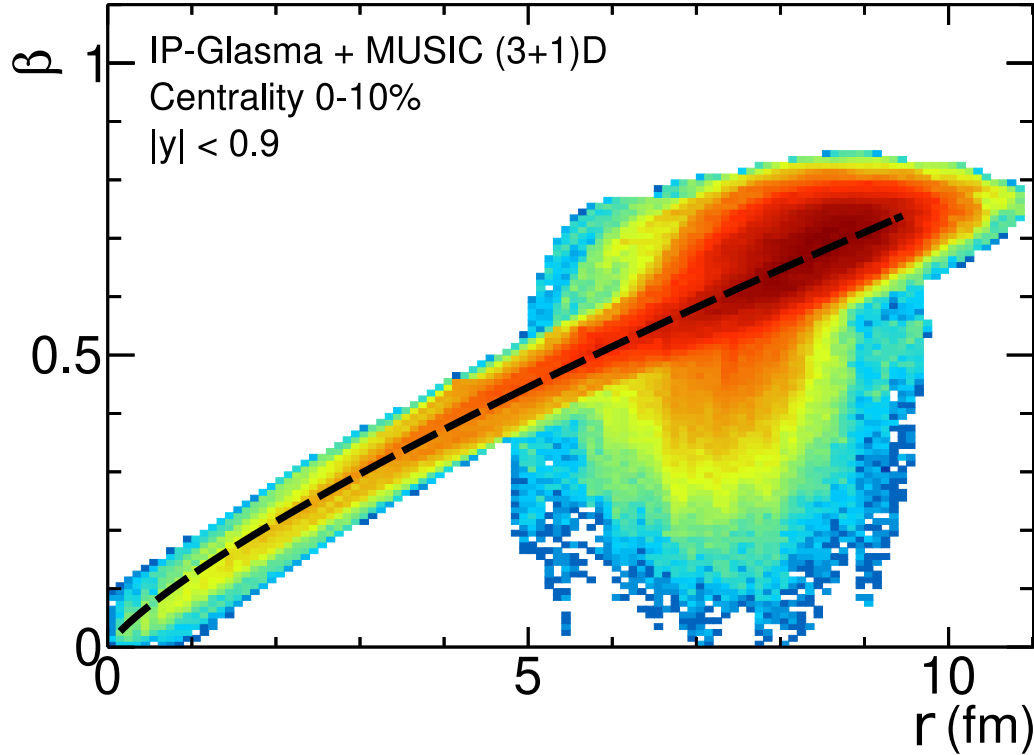
ALICE, [arXiv:2210.08893](https://arxiv.org/abs/2210.08893)

SHMc: p_T dependence

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full hydrodynamic flow (MUSIC(3+1)D, IP-Glasma; parametrized via blast-wave



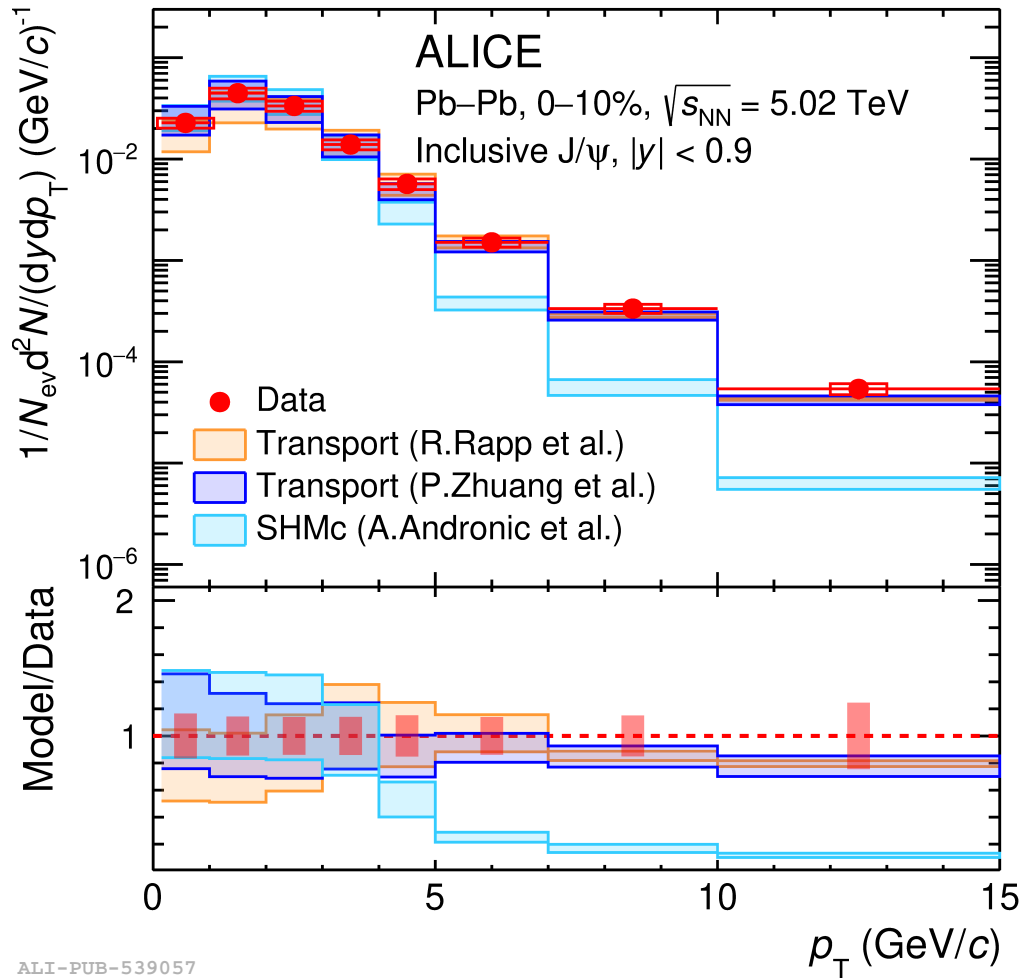
$$\frac{d^2N}{2\pi p_T dp_T dy} = \frac{2J+1}{(2\pi)^3} \int d\sigma_\mu p^\mu f(p) = \frac{2J+1}{(2\pi)^3} \int_0^{r_m} dr \tau(r) r \left[K_1^{\text{eq}} - \frac{\partial \tau}{\partial r} K_2^{\text{eq}} \right]$$

$$K_1^{\text{eq}}(p_T, u^r) = 4\pi m_T I_0 \left(\frac{p_T u^r}{T} \right) K_1 \left(\frac{m_T u^r}{T} \right), \quad K_2^{\text{eq}}(p_T, u^r) = 4\pi p_T I_1 \left(\frac{p_T u^r}{T} \right) K_0 \left(\frac{m_T u^r}{T} \right)$$

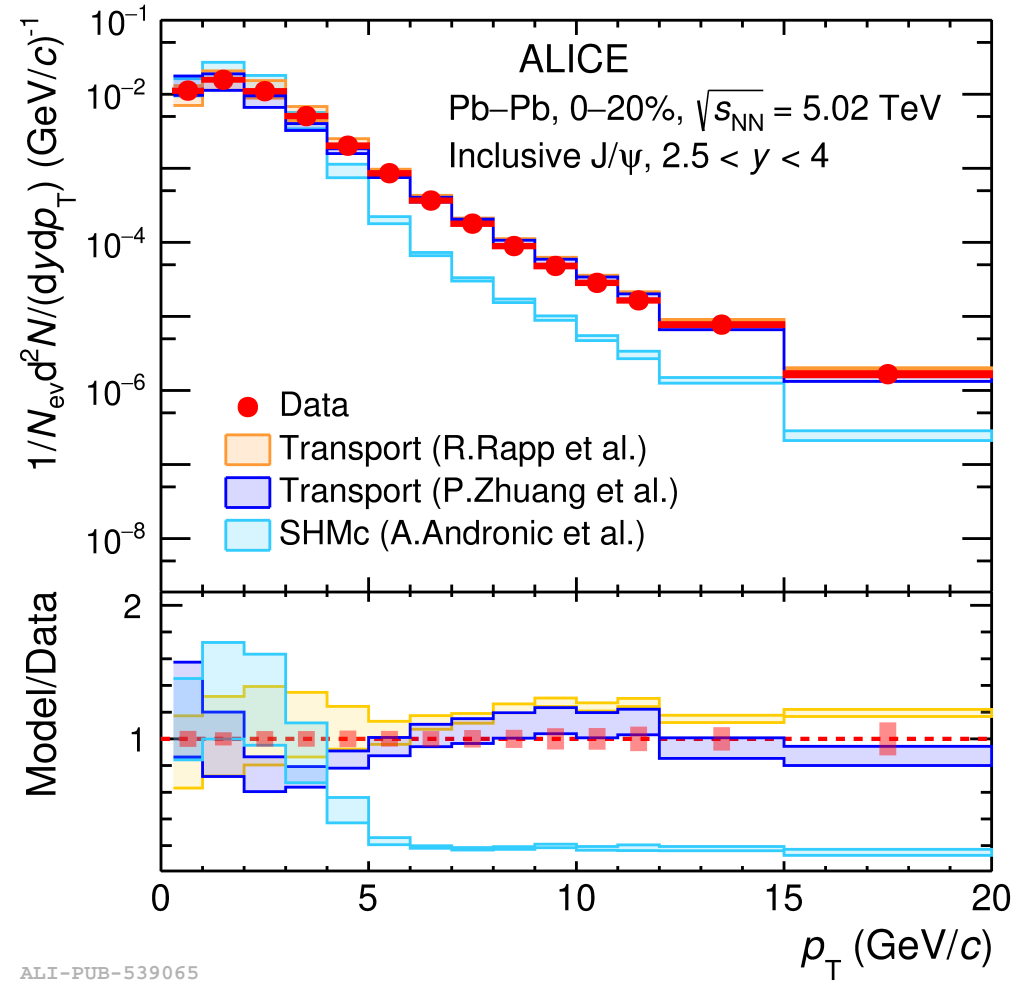
SHMc and charmonium data at the LHC

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ALI-PUB-539057



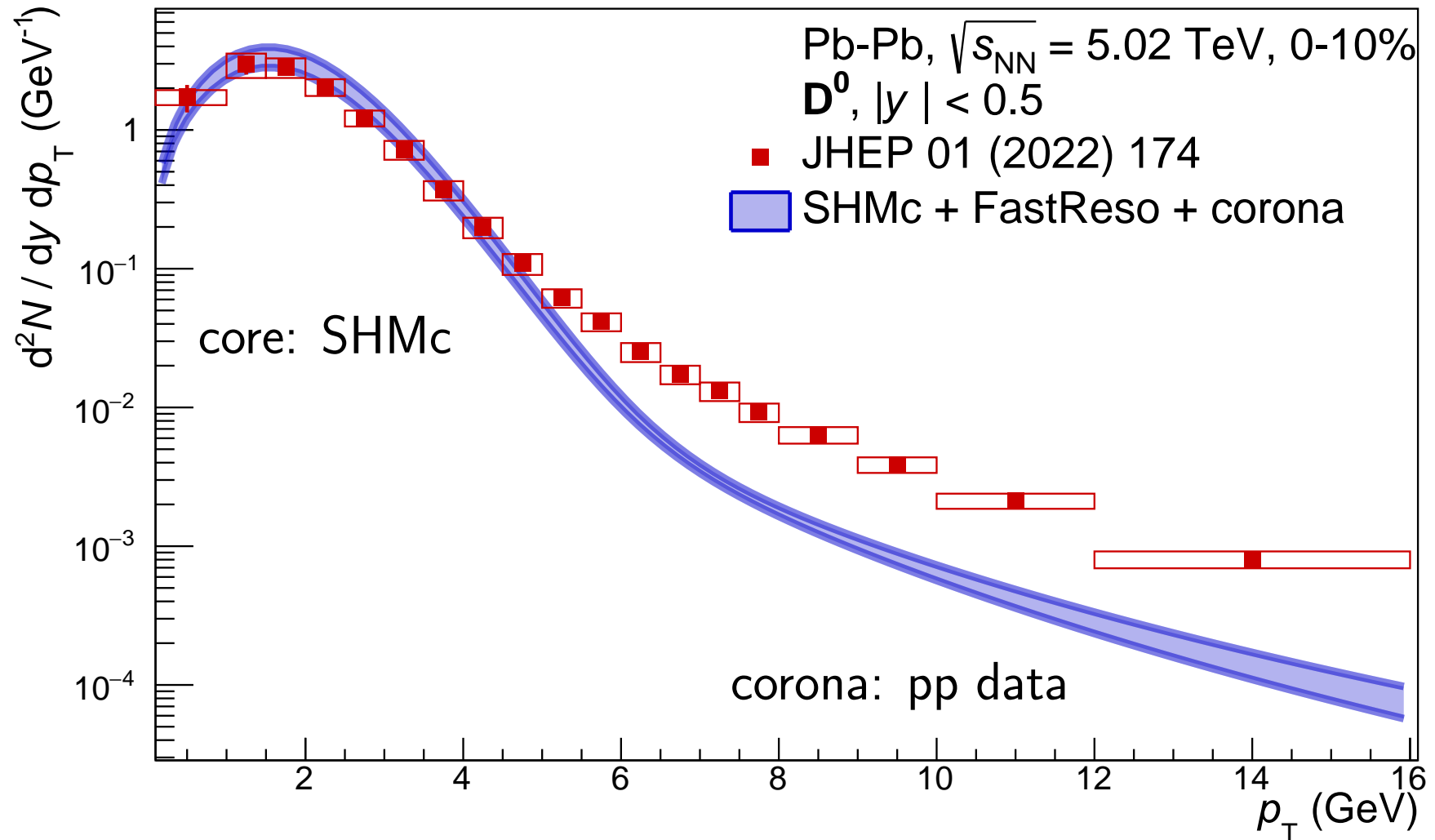
ALI-PUB-539065

ALICE, [arXiv:2303.13361](https://arxiv.org/abs/2303.13361)

SHMc: p_T dependence

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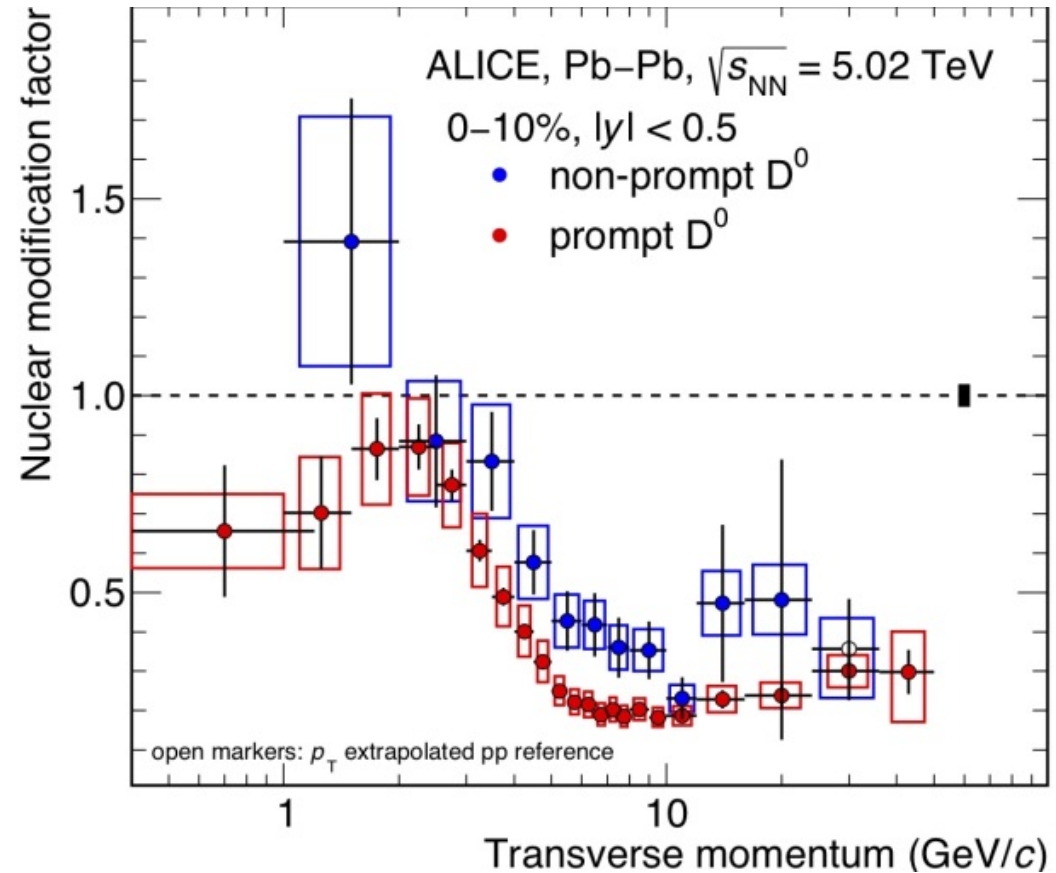
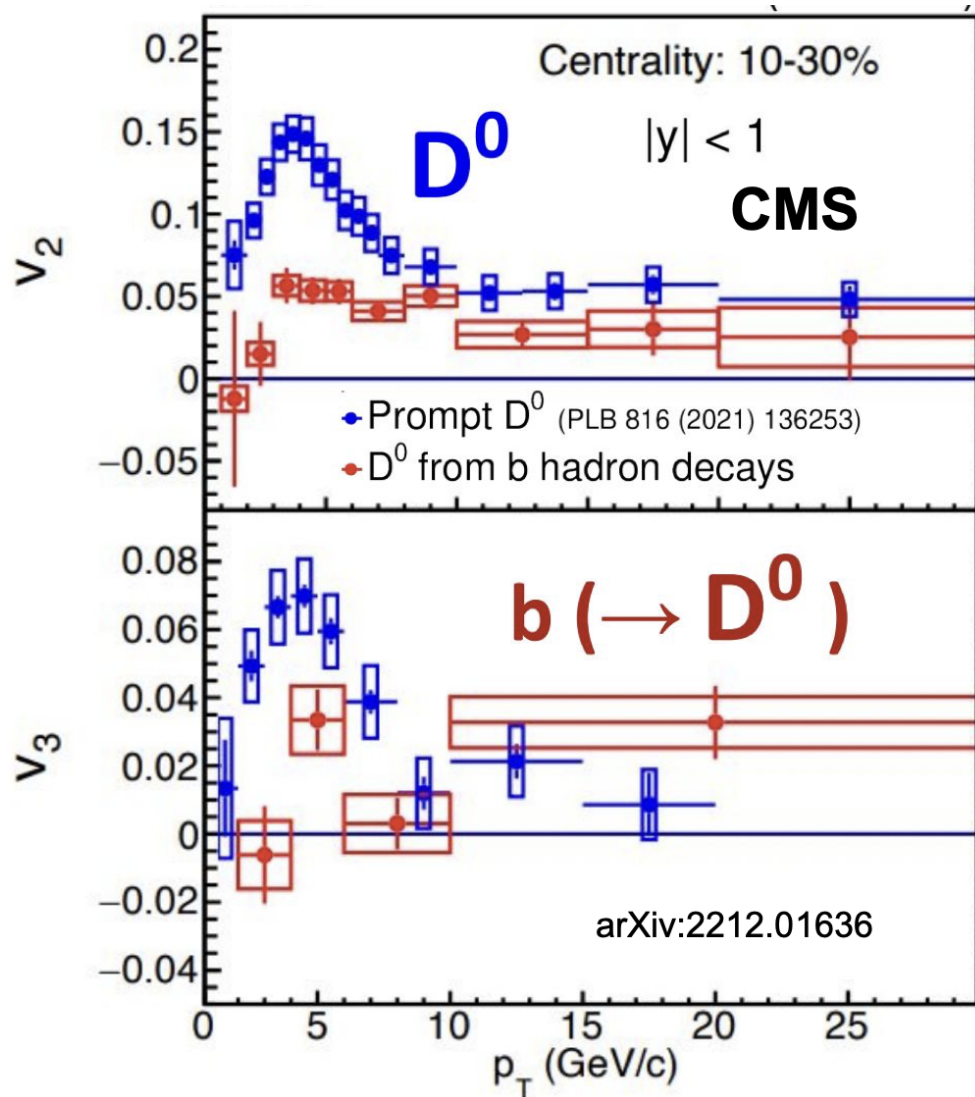


SHMc: low p_T ; high p_T : only nuclear-corona contribution (incl. uncert.)

Beauty quark thermalization?

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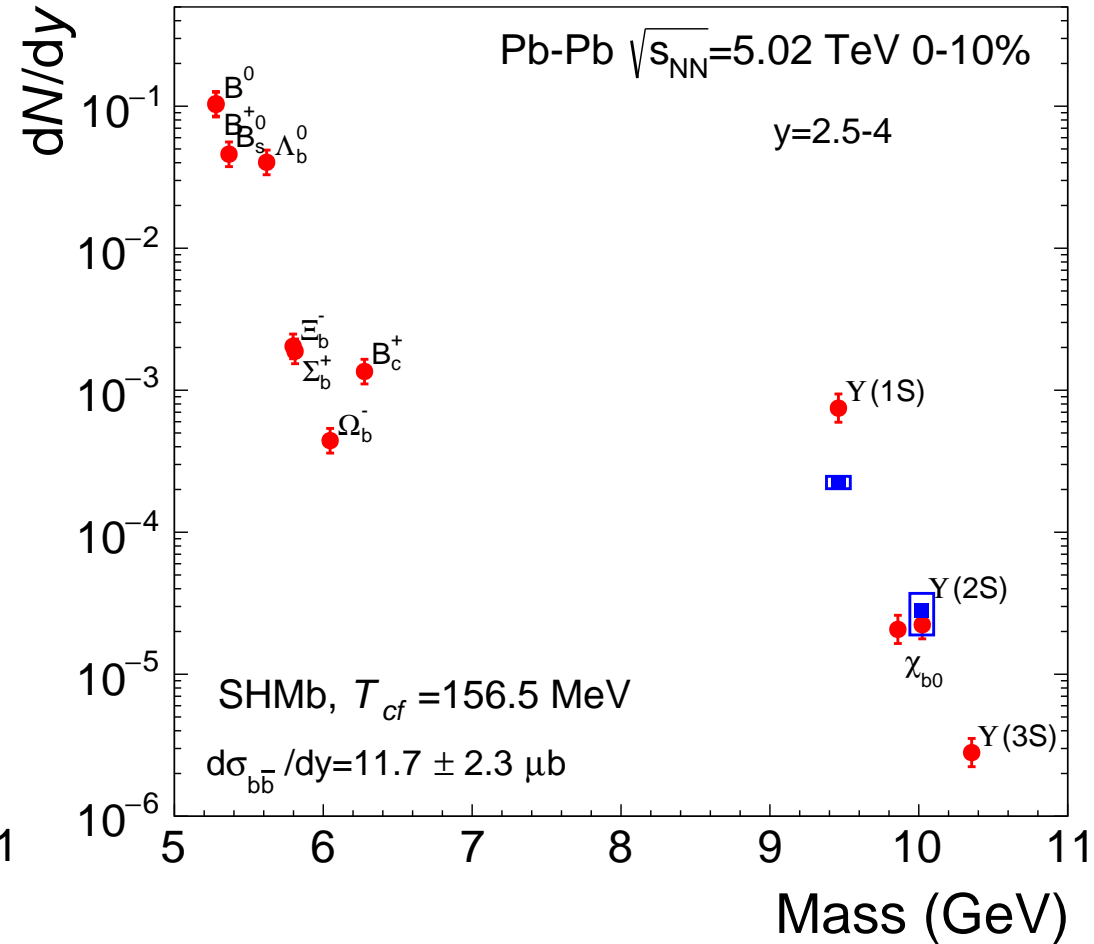
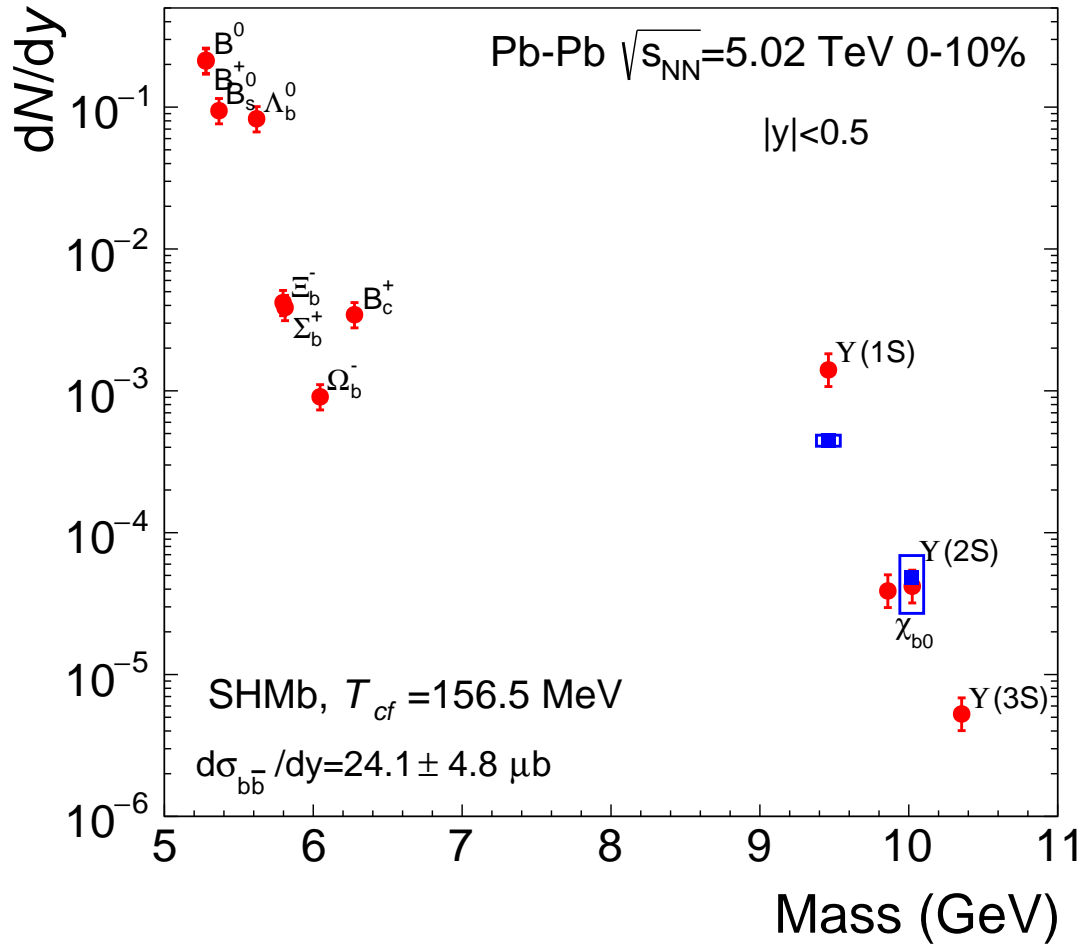


ALICE, [arXiv:2202.00815](https://arxiv.org/abs/2202.00815), ATLAS

definitely strong flow but clearly less strong than for charm (CMS, QM'22, HIN-21-003)

...and a strong coupling with the medium (less energy loss than charm, $p_T \simeq 10$ GeV/c)

The limiting case: full beauty thermalization



$$g_b = 1.05 \cdot 10^9 \quad \left(\frac{dN_{bb^-}}{dy} = 0.57 \right)$$

$$B_c : 3.44 \cdot 10^{-3}$$

$$g_b = 0.86 \cdot 10^9 \quad \left(\frac{dN_{bb^-}}{dy} = 0.28 \right)$$

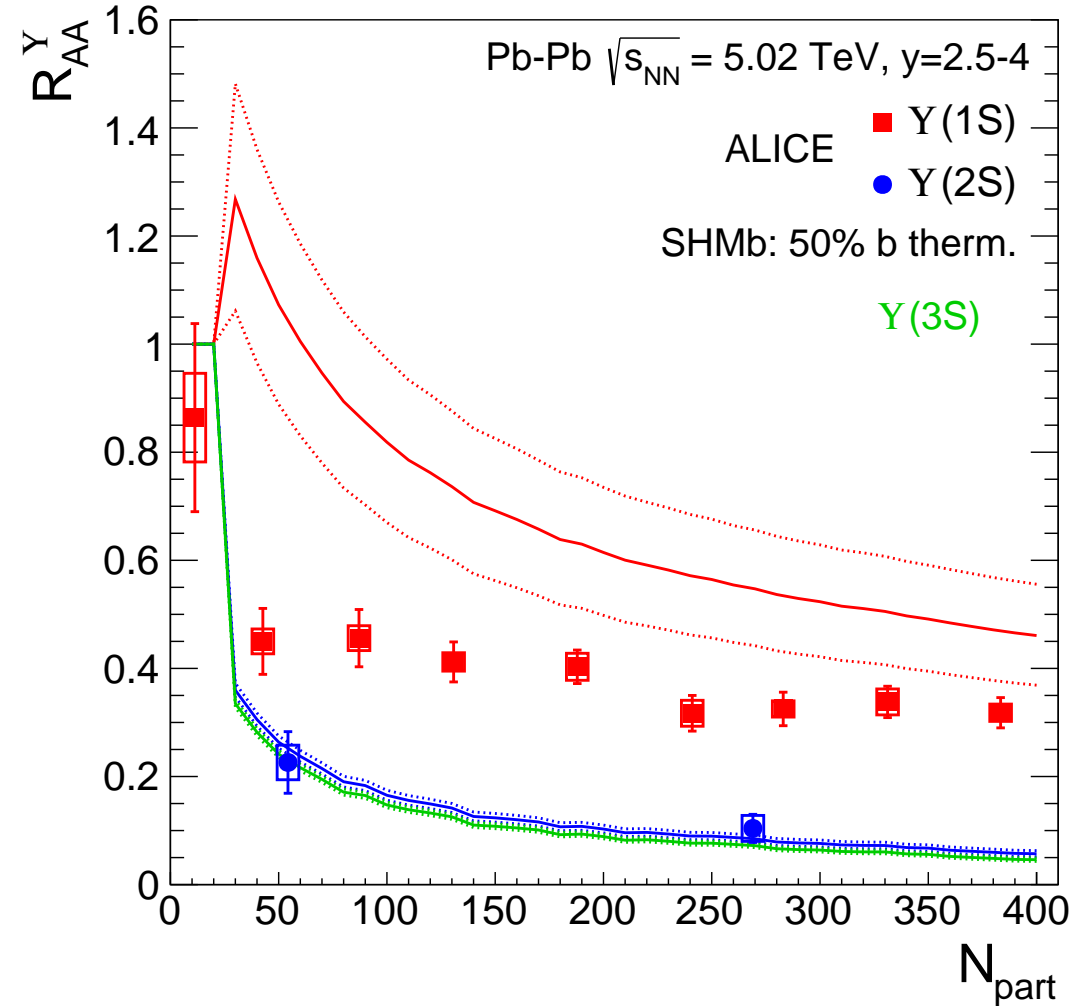
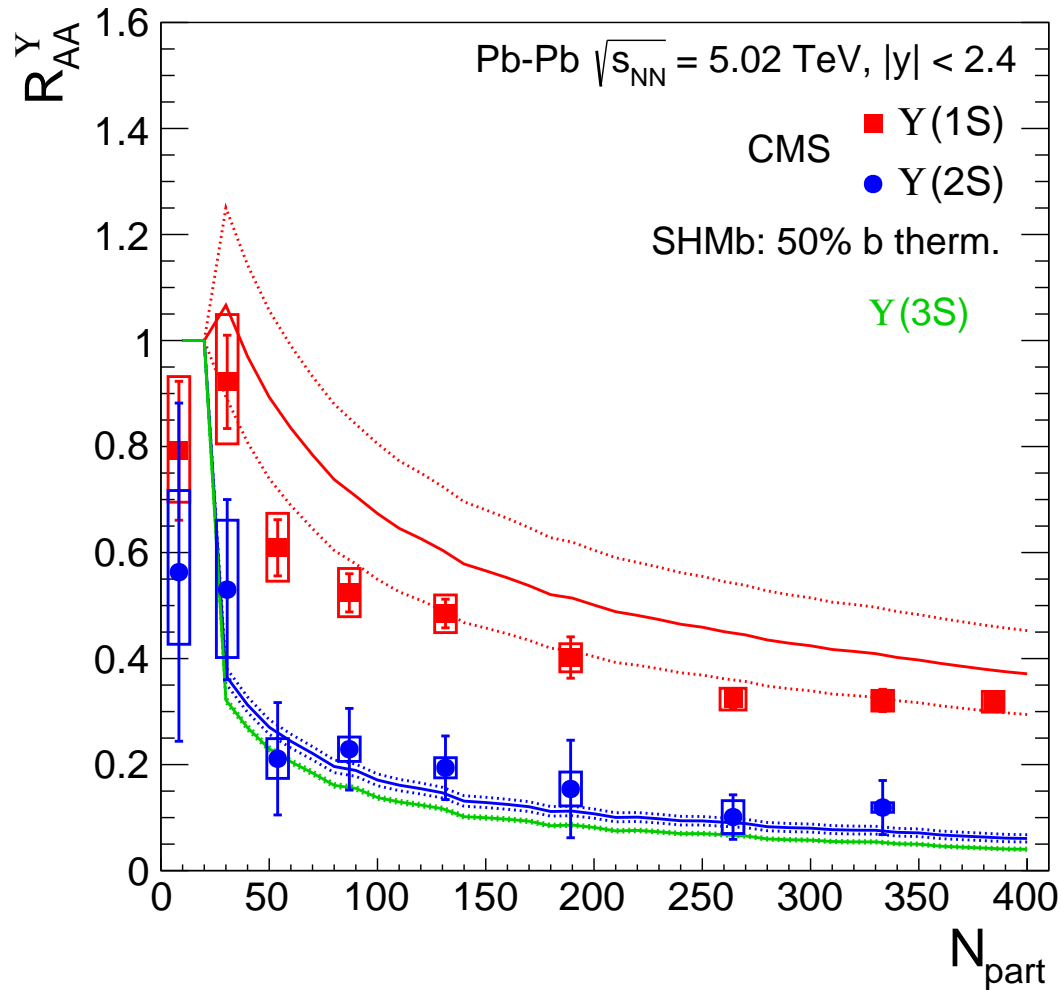
$$B_c : 1.36 \cdot 10^{-3}$$

Blue: Υ data (CMS, ALICE): calc. based on R_{AA} and pp (would be nice to include in publications dN/dy)

R_{AA} , 50% $b\bar{b}$ thermalized

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CMS, PRL 120 (2018) 142301

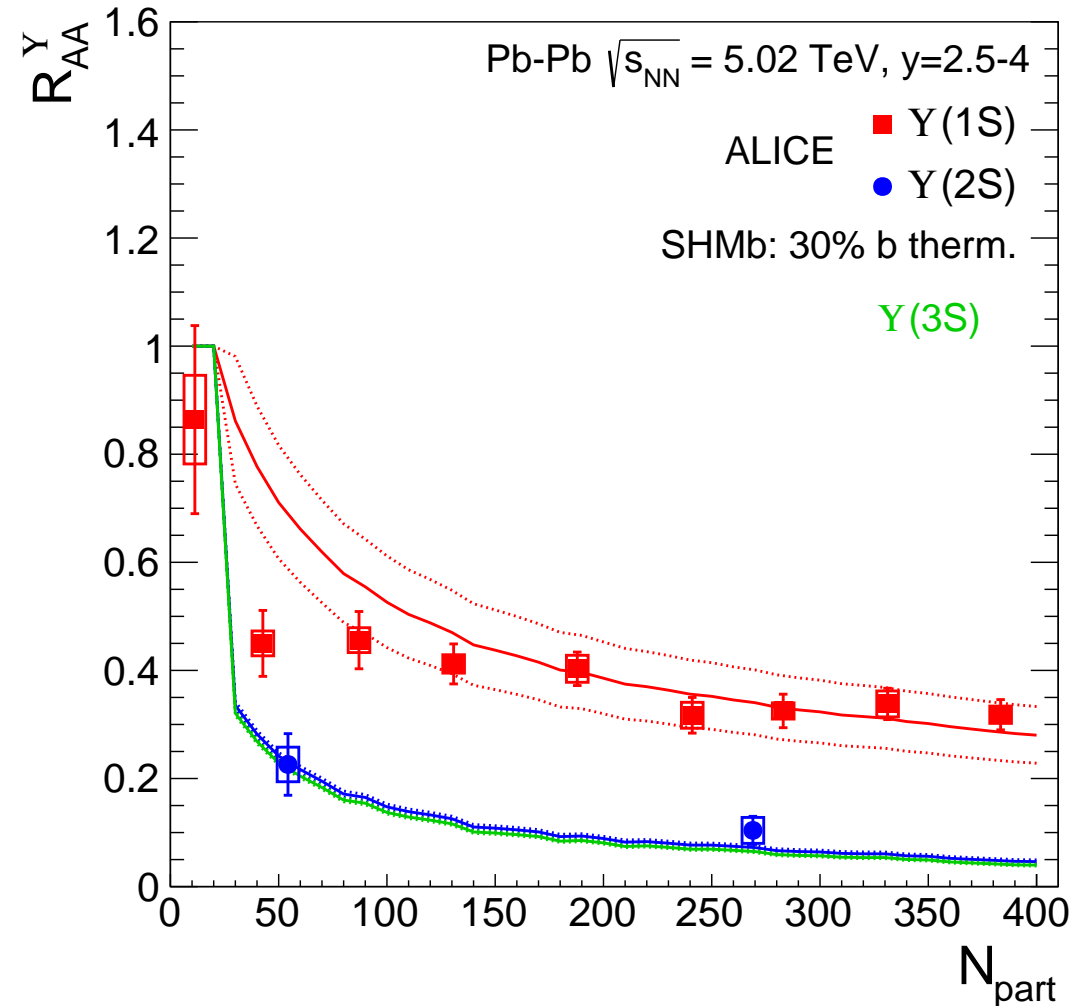
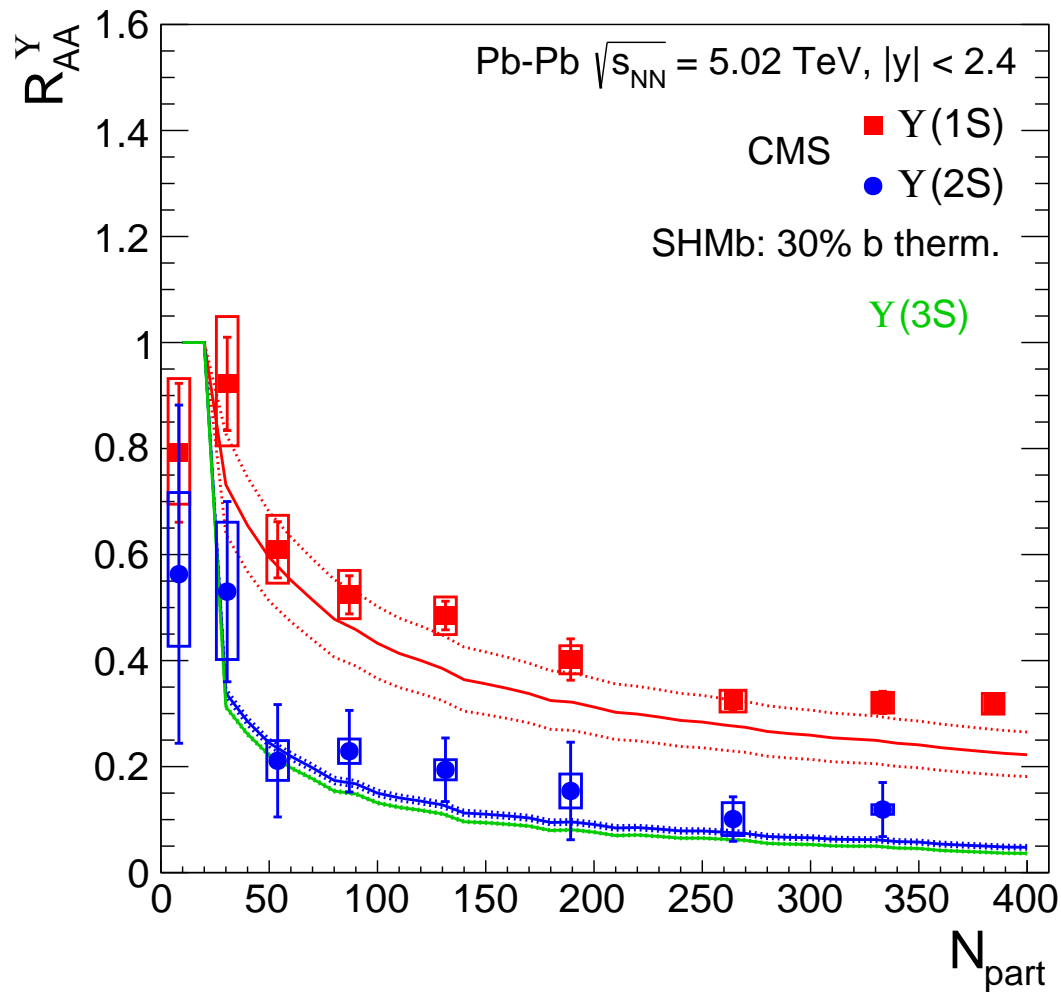
ALICE, PLB 822 (2021) 136579

What does non-thermalized beauty produce? (no room for it in SHMb)

R_{AA} , 30% $b\bar{b}$ thermalized

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CMS, PRL 120 (2018) 142301

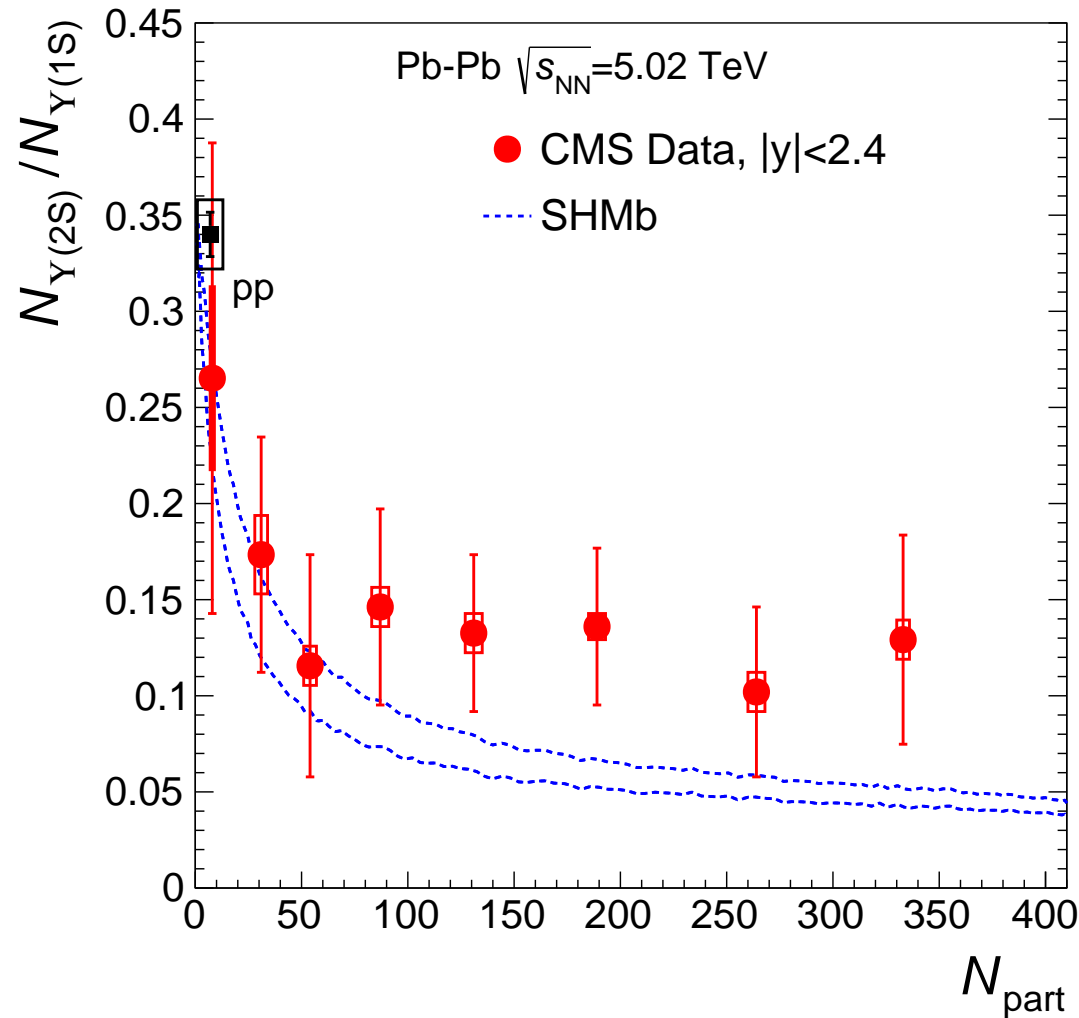
ALICE, PLB 822 (2021) 136579

What does non-thermalized beauty produce? (no room for it in SHMb)

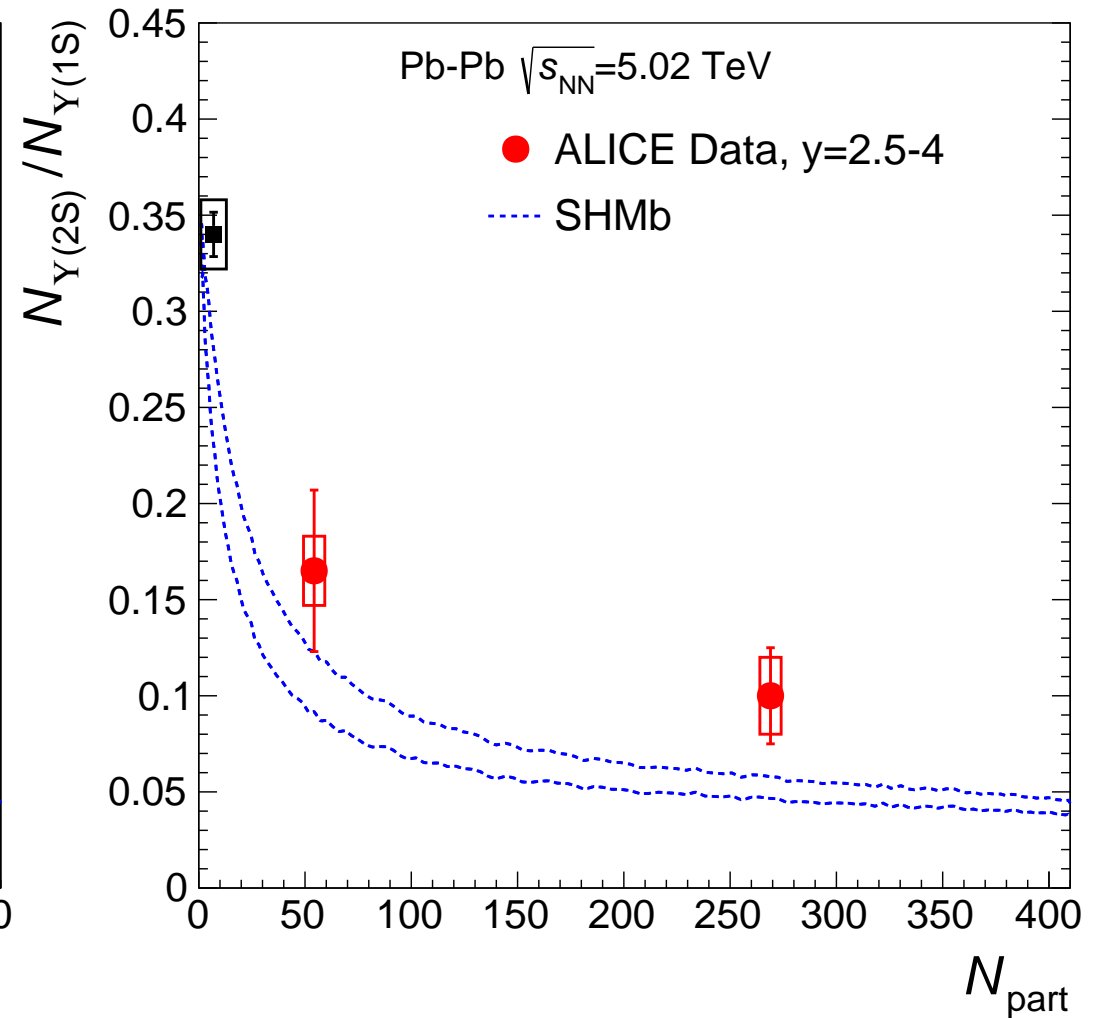
$\Upsilon(2S)/\Upsilon(1S)$ ratio (100% b thermalization)

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CMS, [PRL 120 \(2018\) 142301](#)



ALICE, [PLB 822 \(2021\) 136579](#)

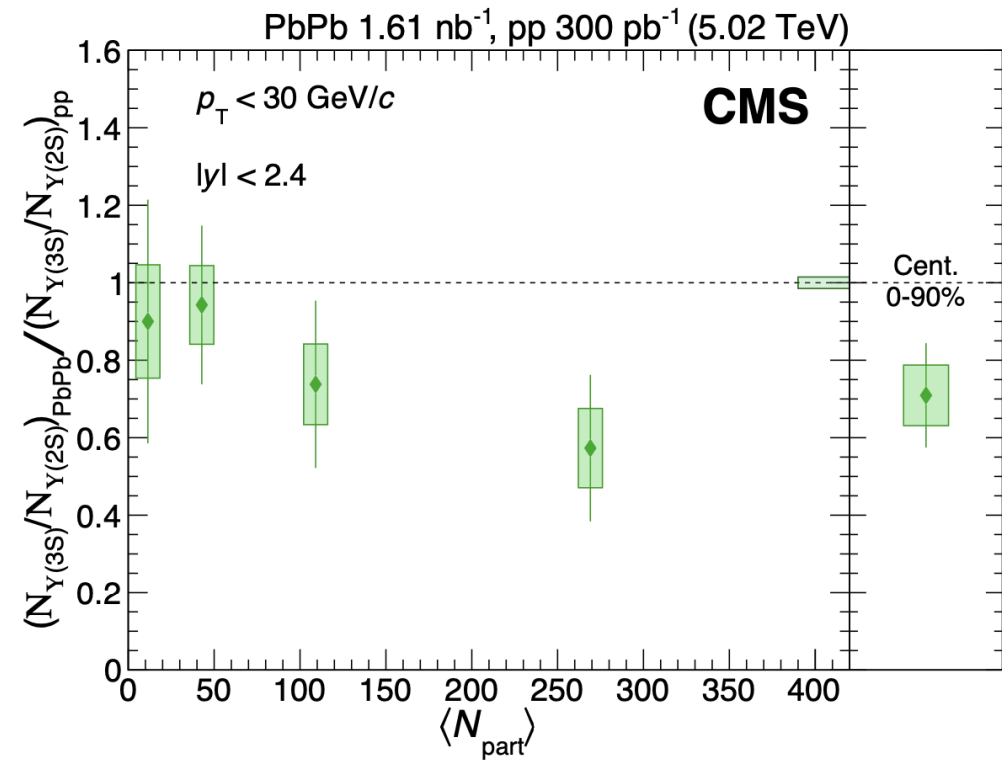
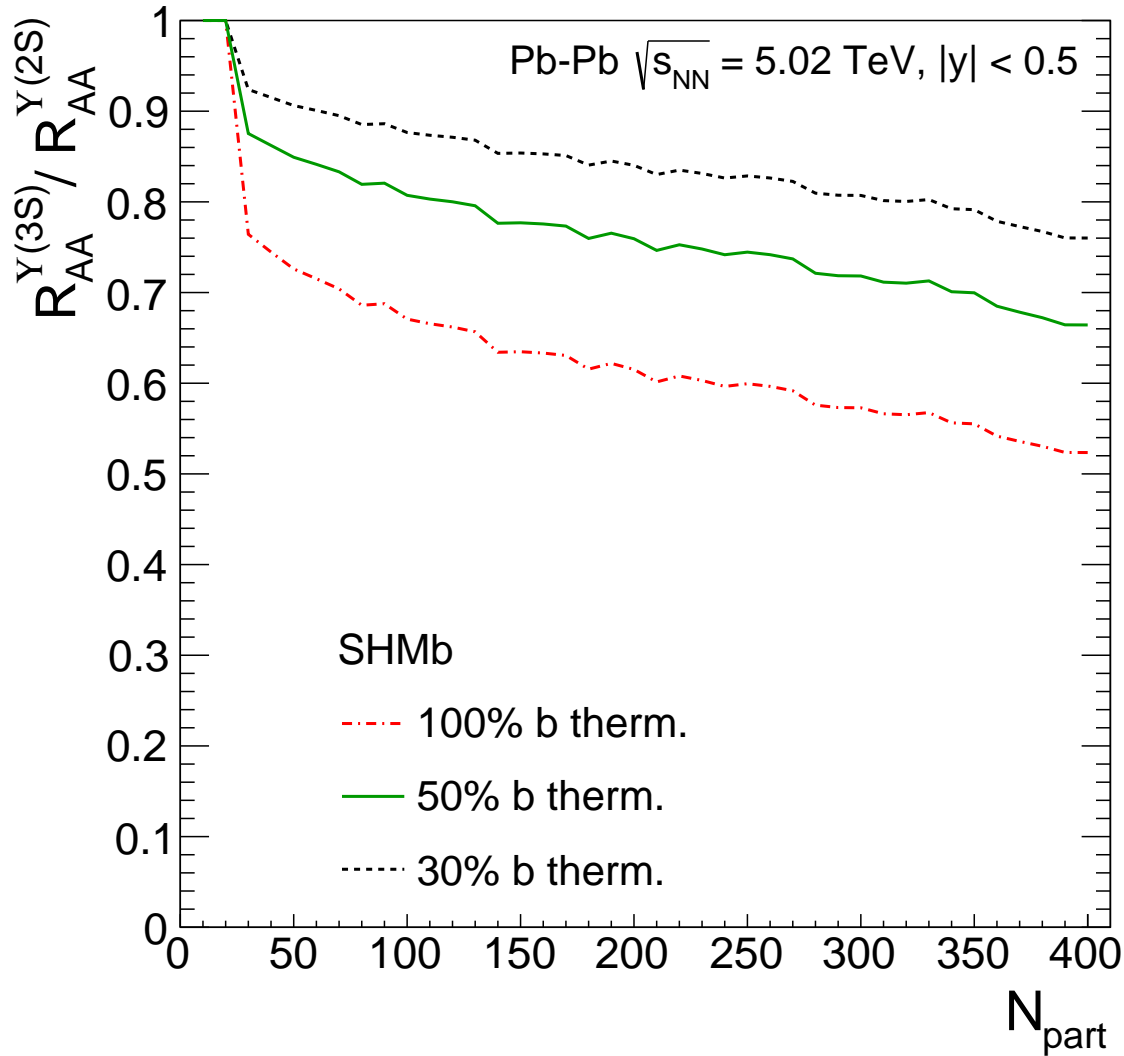
ALICE pp: $\Upsilon(2S)/\Upsilon(1S) = 0.5 \pm 0.1$, [arXiv:2109.15240](#)

SHMb uncert.: nuclear-corona (fraction)

$\Upsilon(3S)/\Upsilon(2S) R_{AA}$ ratio

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CMS, [arXiv:2303.17026](https://arxiv.org/abs/2303.17026)

The $\Upsilon(3S)/\Upsilon(2S) R_{AA}$ ratio is quite sensitive to the degree of b thermalization

Summary / Conclusions: charm

In the (our) statistical hadronization model:

- The hadronization is a rapid process in which all quark flavors take part concurrently
- All charmonium and open charm states are generated exclusively at hadronization (chemical freeze-out) ...full color screening

The model is very successful in reproducing the J/ψ and open charm data
A handle for hadronization T with a mass scale well above T

"The competition":

the kinetic model, continuous J/ψ destruction and (re)generation in QGP

(only up to 2/3 of the J/ψ yield (LHC, central collisions) originates from deconfined c and \bar{c} quarks)

Discriminating the two pictures implies providing an answer to fundamental questions related to the fate of hadrons in a hot deconfined medium.

A precision ($\pm 10\%$) measurement of $d\sigma_{c\bar{c}}/dy$ in Pb-Pb (Au-Au) collisions needed for a stringent test
(within reach with the upgraded detectors at the LHC and RHIC)

Summary / Conclusions: beauty

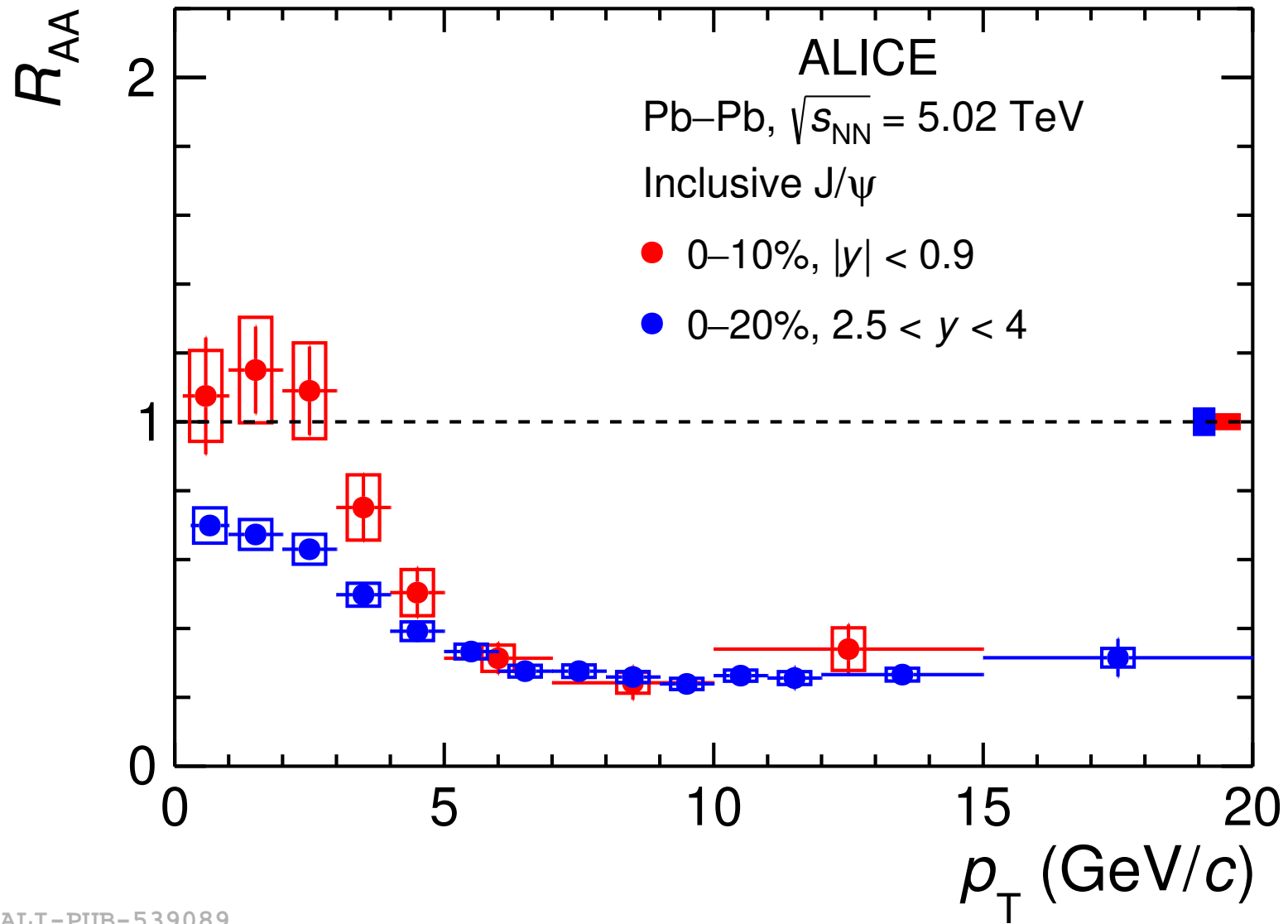
- Full beauty thermalization seems not realized in nature
 - ...with 30-50% of beauty quarks fully thermalized we can explain the Υ data
 - ...but this fraction is (significantly) dependent on the b-hadron spectrum
- What does non/partially-thermalized beauty produce?
 - no Υ because strong coupling with the medium destroys the $b\bar{b}$ correlation?
 - ...related: is there non-screened bottomonium at all? (...or maybe just $\Upsilon(1S)$?)
- Another difficulty: $R_{AA}^{Y(1S)}(p_T)$ is flat (we would predict a bump), v_2 is small

similar to Reygers et al., [PRC 101 \(2020\) 064905](#)

forthcoming LHC data will (hopefully) clarify these questions (...in a while)

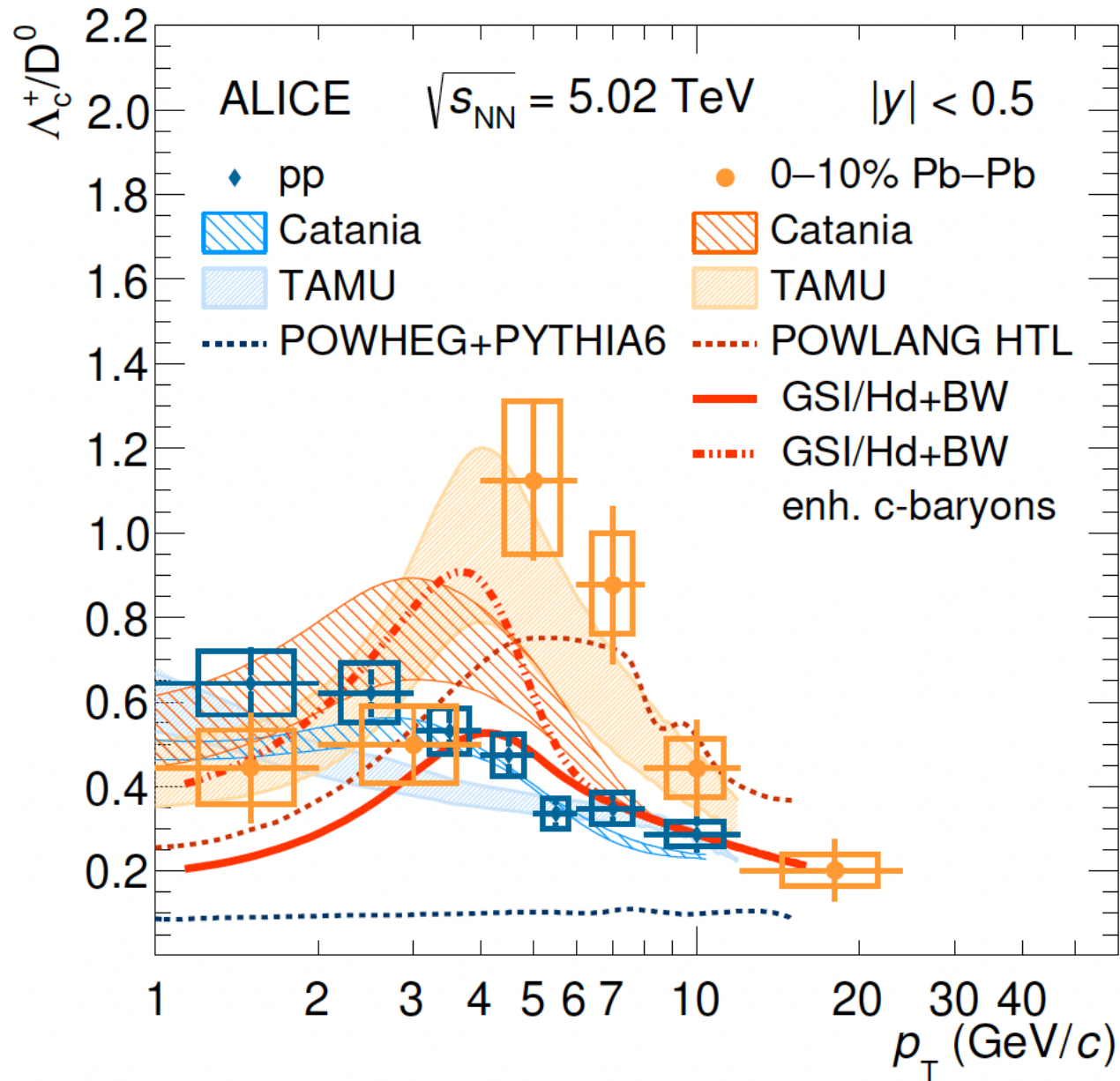
inspite (because:) of its simplicity SHM provides for all more sophisticated models a meaningful (powerful) limit to check (worth even if not fully realized in nature:)

Additional material



ALI-PUB-539089

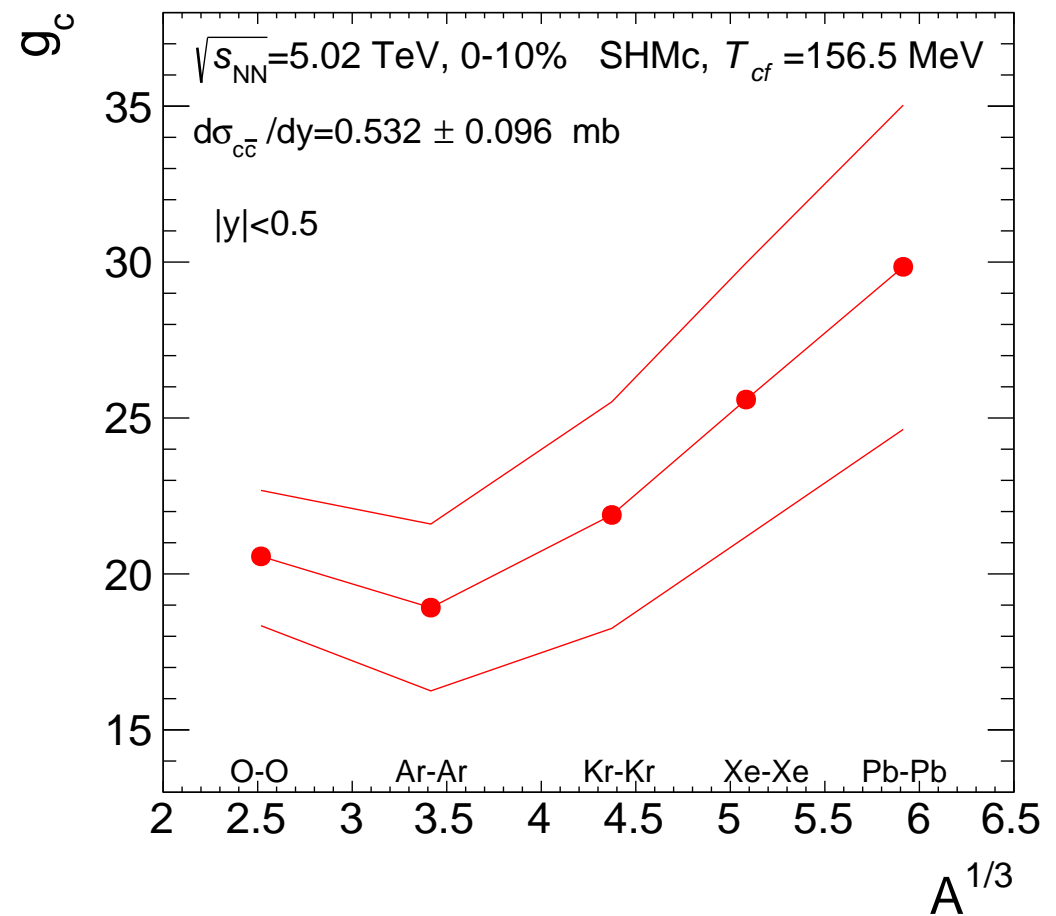
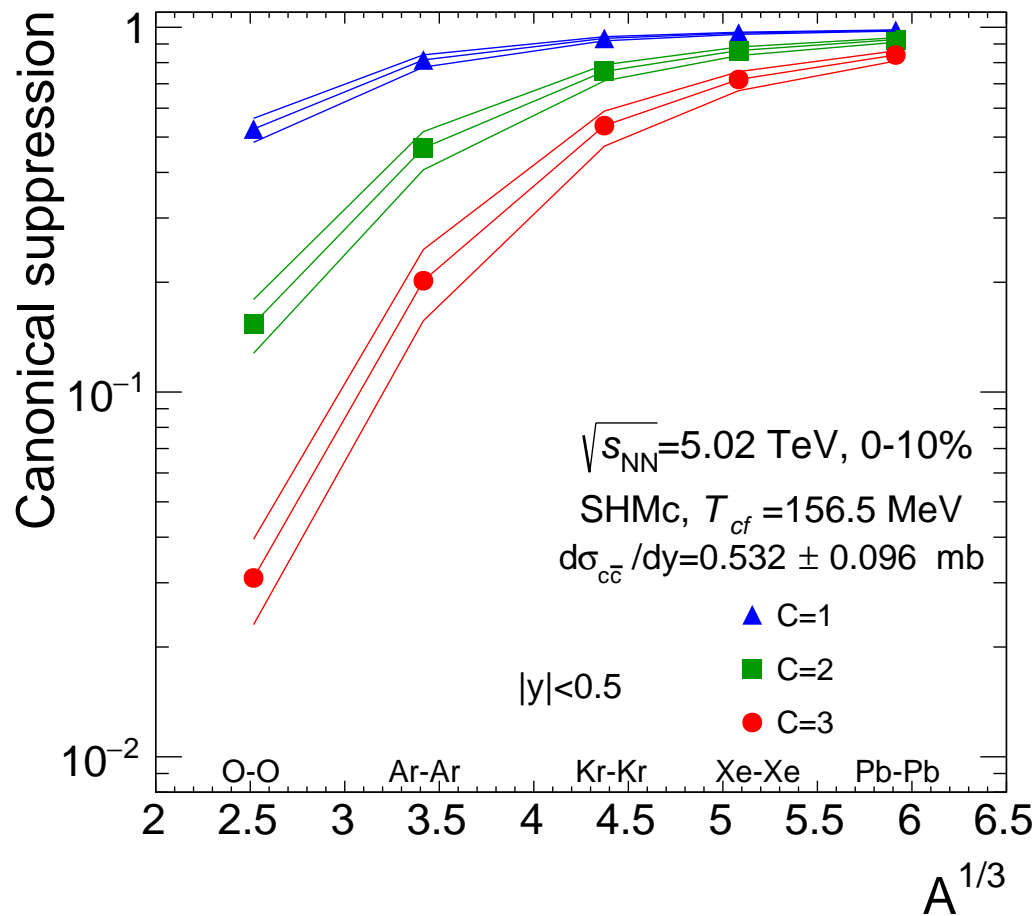
Open charm data vs. models at the LHC



SHMc: system-size dependence (central, 0-10%)

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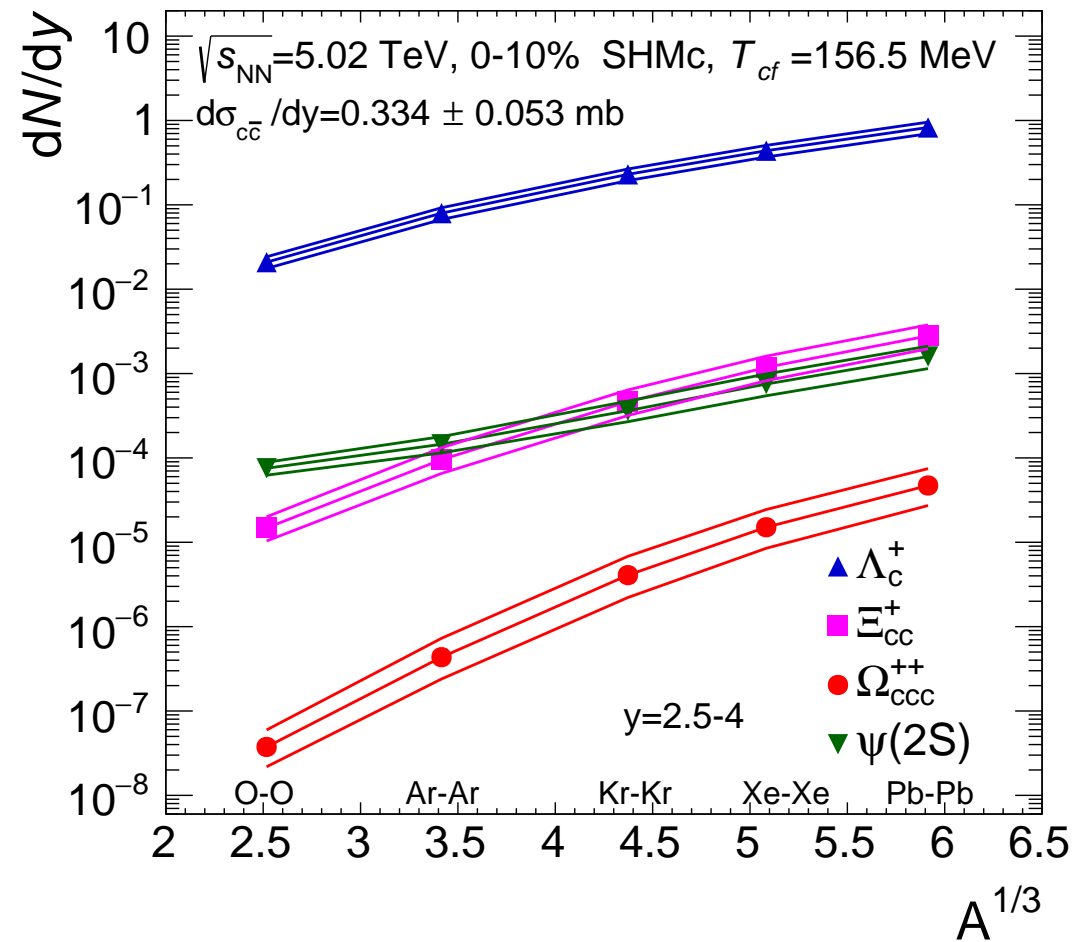
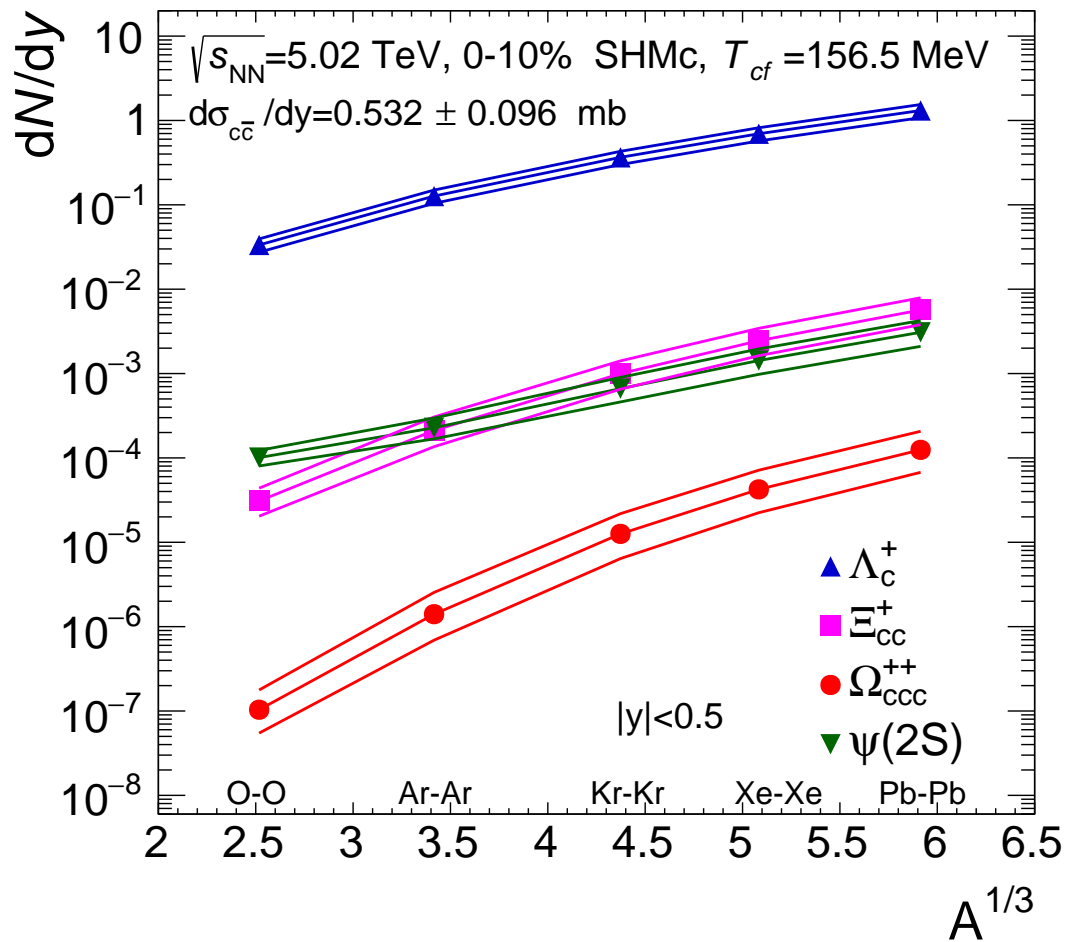


Strong canonical suppression for light systems (for multi-charm hadrons)

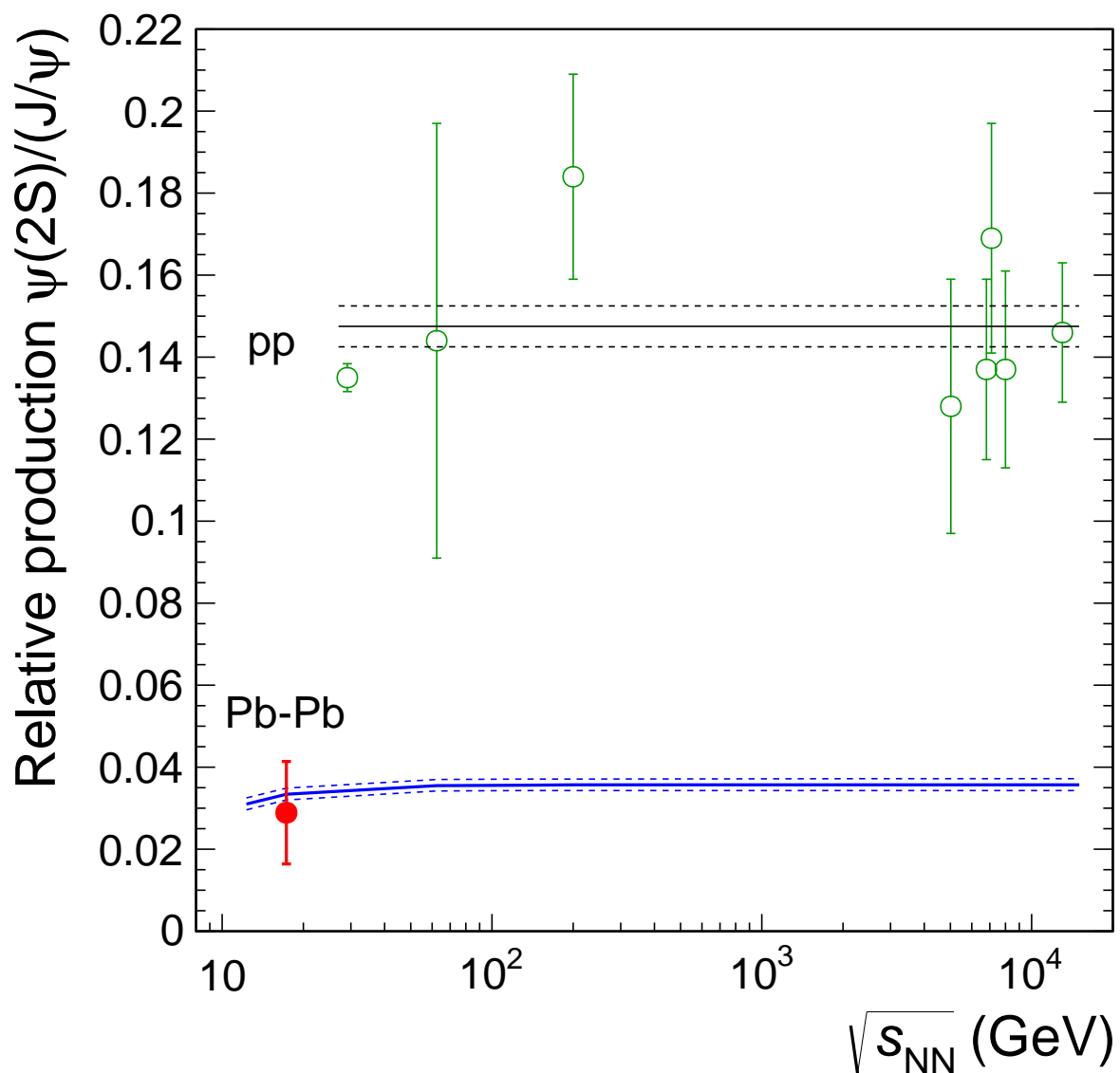
SHMc: system-size dependence (central, 0-10%)

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IF charm thermalizes (fully) also at lower energies



...SHMc is easy to be extended to lower energies

AA, Braun-Munzinger, Redlich, Stachel, [PLB 659 \(2008\) 149](#)

..litmus test: $\psi(2S)$ ($+v_2$, R_{AA})

SHMc works (was proposed) at SPS

Braun-Munzinger, Stachel, [PLB 490 \(2000\) 196](#)

for D, stat. hadronization is a simpler act, may be at work in pp and in e^+e^-

[PLB 678 \(2009\) 350](#)

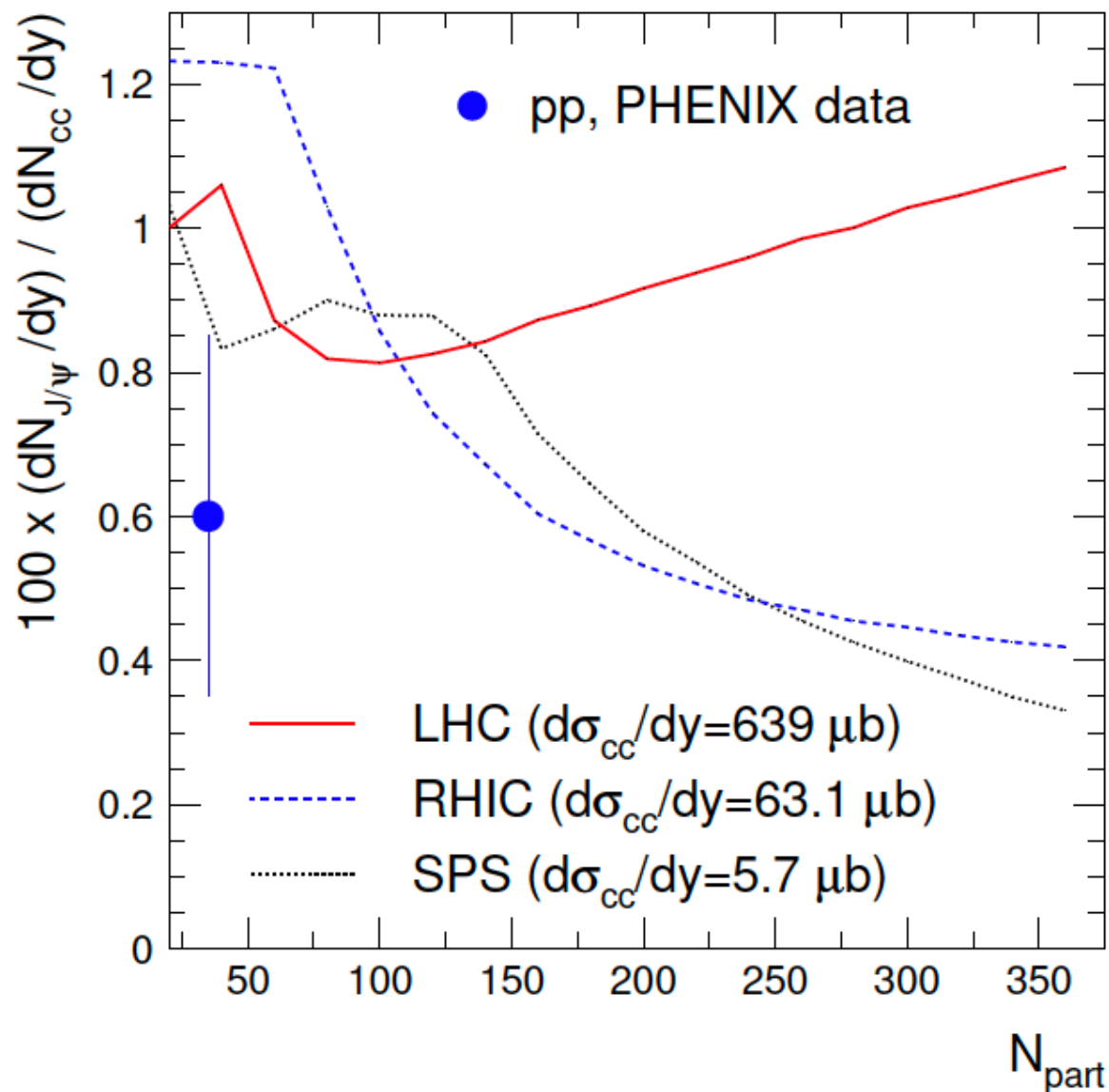
The measurement in Pb-Pb at LHC is a central goal for Run 3,4 ([YR, WG5 HL-LHC](#))

J/ψ production relative to charm

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...an observable with similar features as R_{AA}



NPA 789 (2007) 334

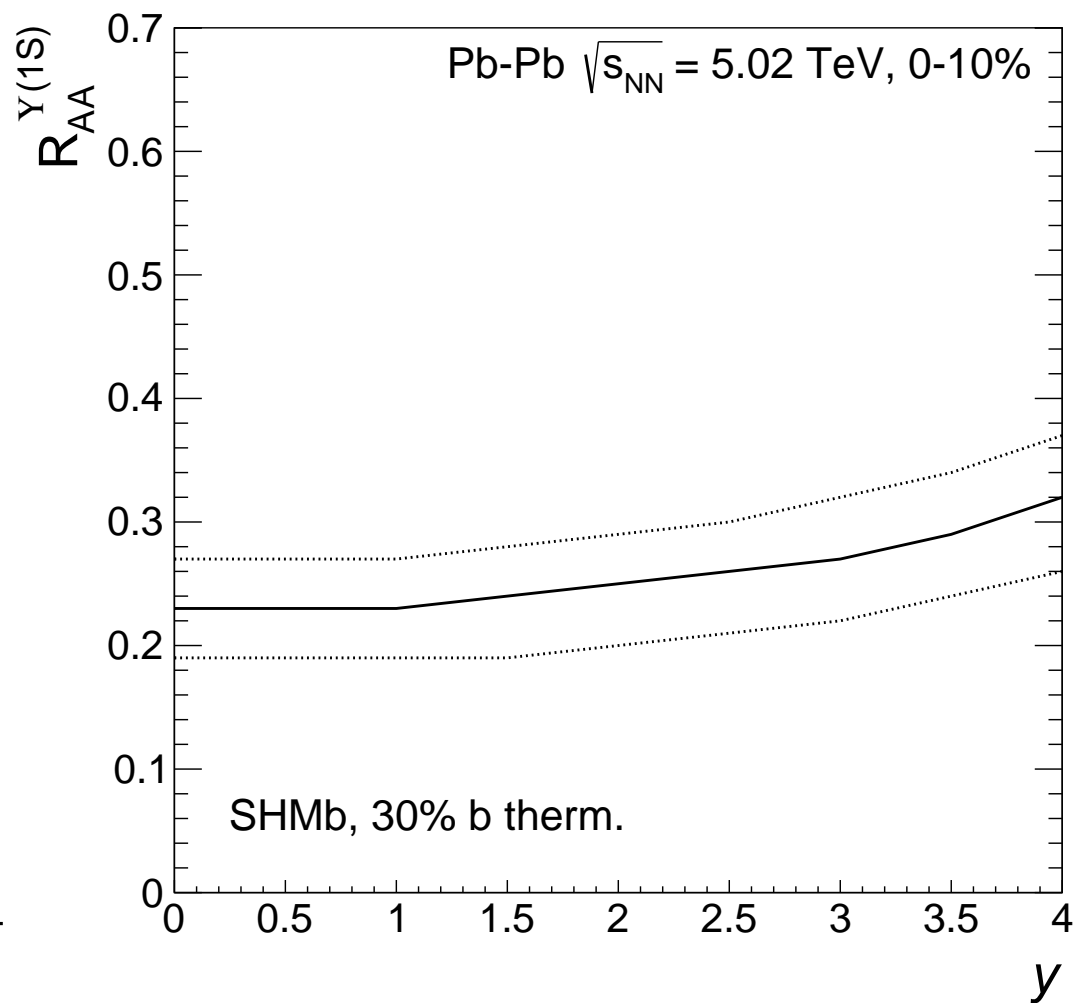
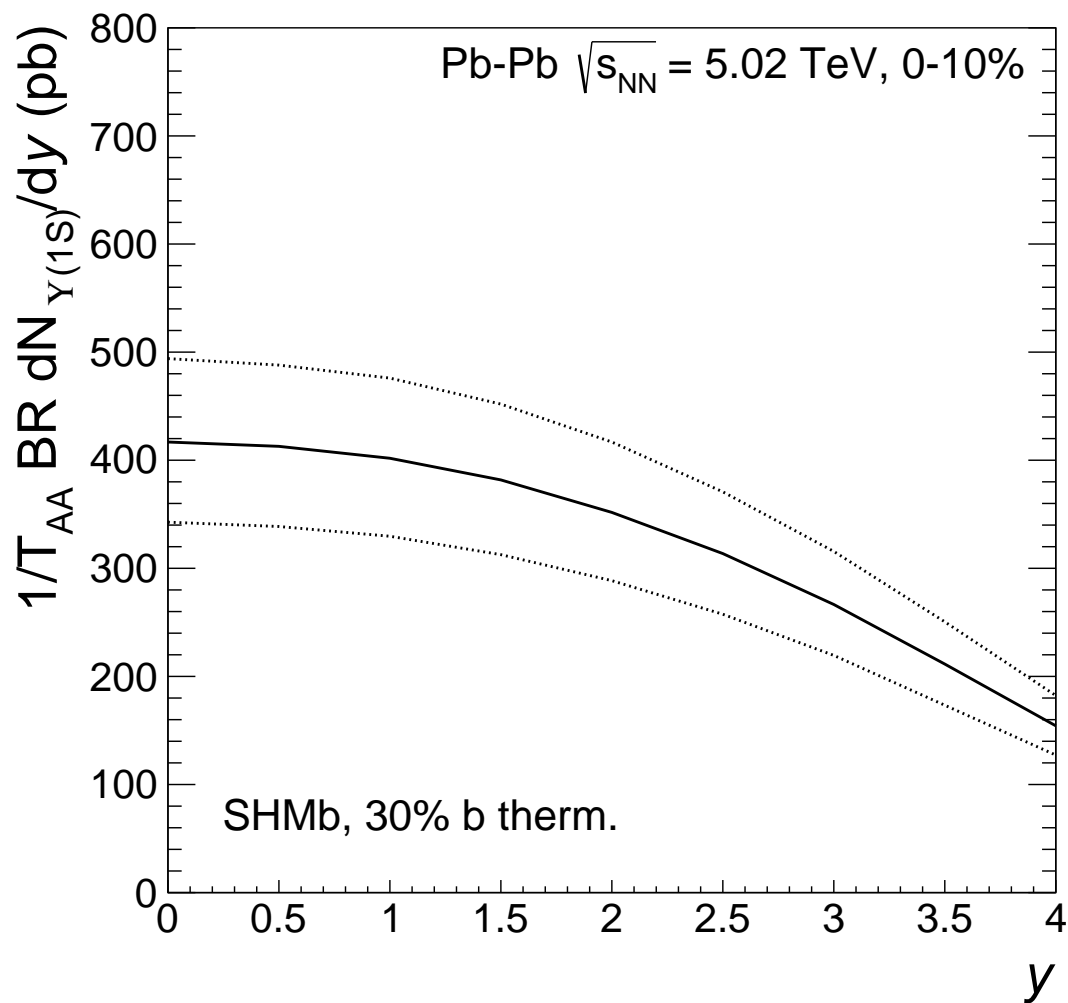
(see also: Satz, *Adv.HEP* 2013 (2013) 242918)

- similar values at RHIC and SPS
...with differences in fine details
...determined by canonical suppression of open charm
same features in RHIC BES data
- enhancement-like at LHC
can. suppr. lifted, quadratic term dominant

Rapidity dependence $\Upsilon(1S)$, 30% $b\bar{b}$ thermalized

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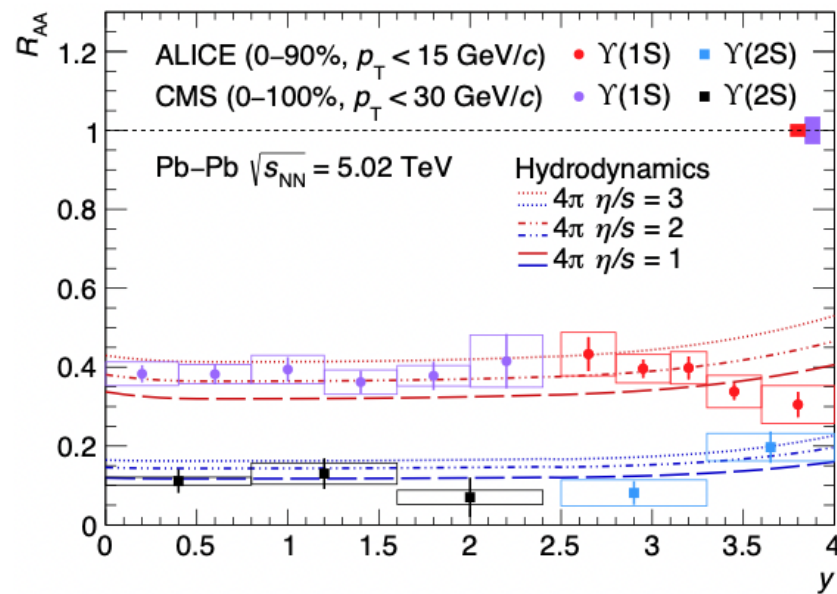
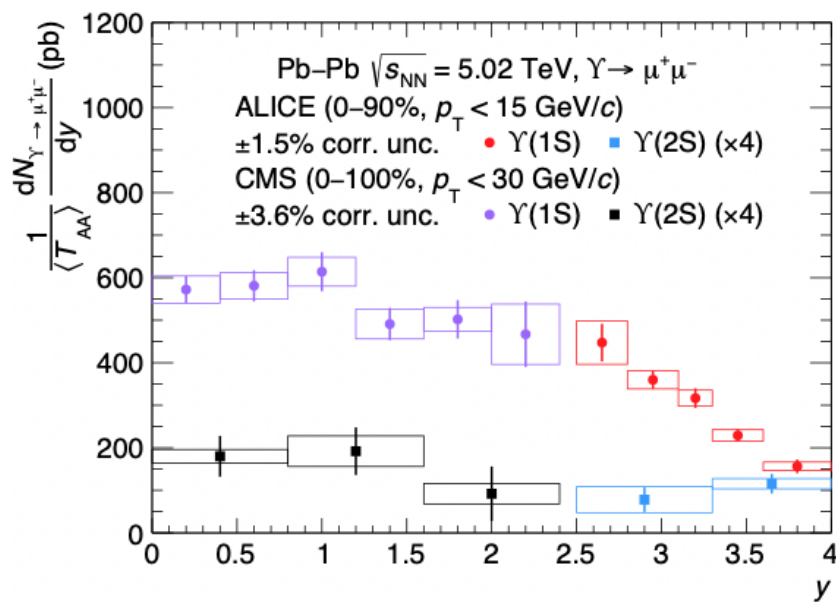
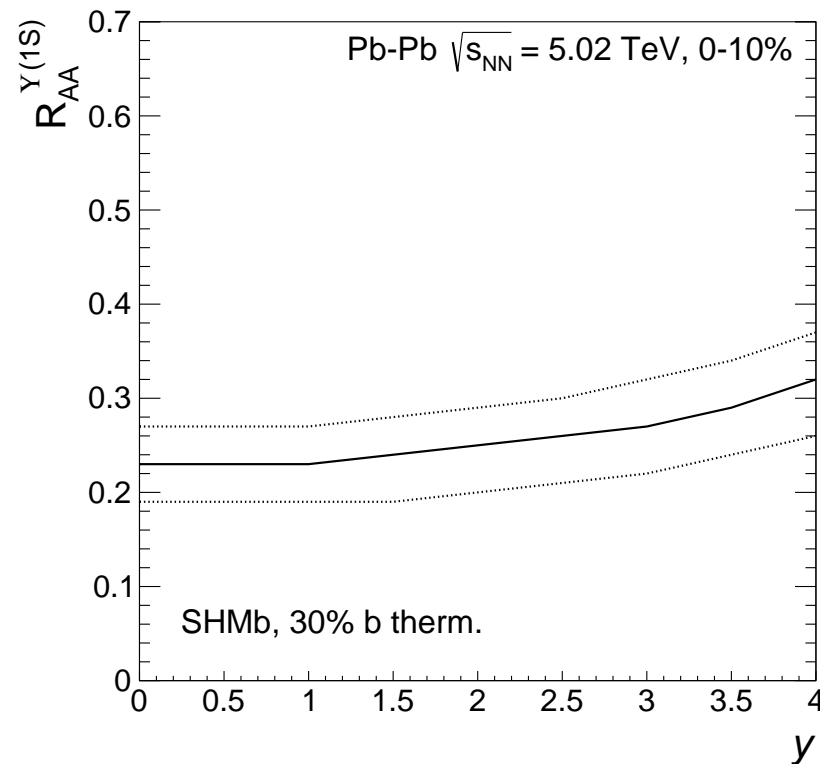
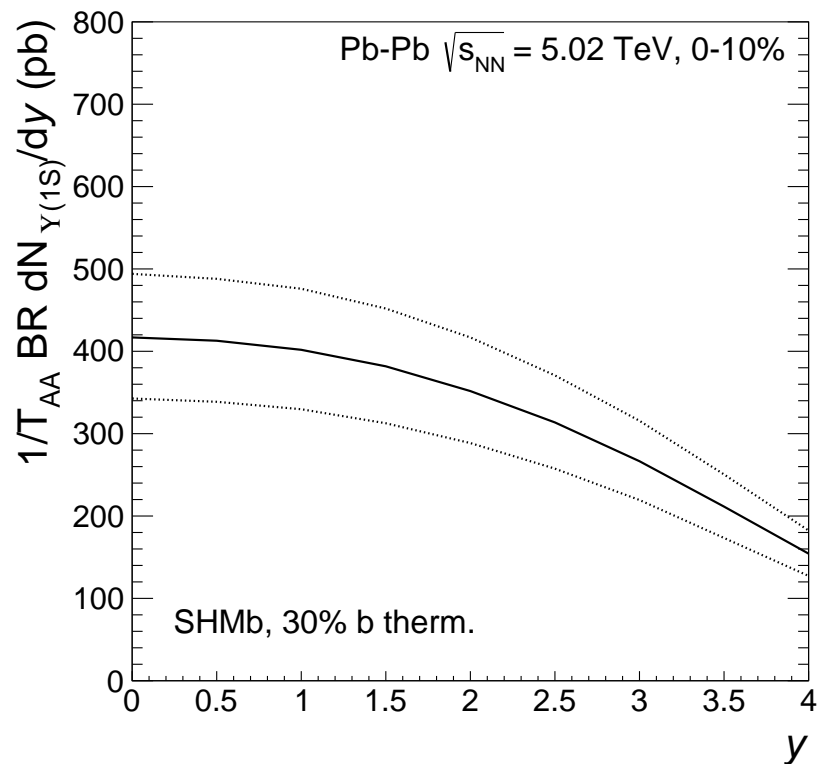


Data available only centrality-integrated ...is 0-10% (or 0-20%) doable?

Rapidity dependence $\Upsilon(1S)$, 30% $b\bar{b}$ thermalized

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SHMb parameters

NB: none is a free parameter, except the 30% b thermalization fraction

