Heavy quark diffusion coefficients in light of Gribov-Zwanziger action

Aritra Bandyopadhyay

UNIVERSITÄT HEIDELBERG ZUKUNFT SEIT 1386

Unterstützt von / Supported by Alexander von Humboldt Stiftung / Foundation

Based on Phys.Lett.B 838 (2023) 137714 [2210.03076 [hep-ph]]

In collaboration with S.Madni, A.Mukherjee and N.Haque

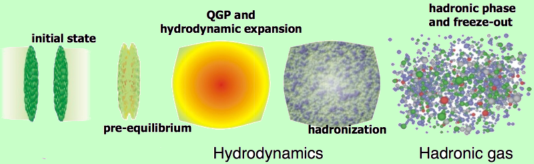
European Research Council Established by the European Commission

Talk prepared for ExploreQGP workshop

Outline

Outline

Heavy Quarks as QGP Signature



QGP Signatures
a) static properties
b) dynamic quantities.

Heavy Quarks

- **large mass** compared to T,
- **external** to the bulk medium.
- generated at the **early stage**
- experience drags & random **kicks**
- Theoretical inputs : **momentum diffusion coefficient**

} Less contamination

→ More information
→ **Langevin equations**
→ Momentum broadening, Energy loss

Static and Dynamic limit of HQ

- HQ momentum evolution according to the Langevin equations

$$\frac{dp_i}{dt} = \xi_i(t) - \eta_D p_i,$$

- Static limit $(M \gg T)$

$$\langle \xi_i(t)\xi_j(t')\rangle = \kappa \; \delta_{ij}\delta(t-t')$$

- Dynamic limit $(M \geq p \gg T)$

$$\langle \xi_i(t)\xi_j(t')\rangle = \kappa_{ij}(\mathbf{p})\delta(t-t')$$
$$\kappa_{ij}(\mathbf{p}) = \kappa_L(p)\hat{p}_i\hat{p}_j + \kappa_T(p)(\delta_{ij} - \hat{p}_i\hat{p}_j)$$

- We have worked within the static limit. Dynamic limit calculation is under progress.

Static and Dynamic limit of HQ

HQ momentum evolution according to the Langevin equations within static limit

$$\frac{dp_i}{dt} = \xi_i(t) - \eta_D p_i,$$

$$\langle \xi_i(t)\xi_j(t')\rangle = \kappa\delta_{ij}\delta(t-t'),$$

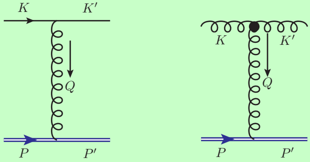
$$\langle x_i(t)x_j(t)\rangle = 2D_s t\delta_{ij},$$

$$\boxed{D_s = \frac{T}{M\eta_D} = \frac{2T^2}{\kappa}}$$

Outline

Heavy quark scattering within Hard Thermal Loop

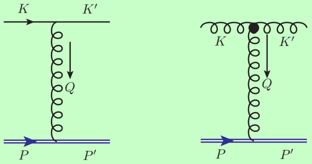
- $2 \leftrightarrow 2$ scattering : $qH \to qH$ (left) and $gH \to gH$ (right)



- Both the processes dominated by t-channel gluon exchange.
- In the static limit, Compton scattering is suppressed by factor $Q^2/PK \approx T/M$.
- $|\mathcal{M}|^2 \propto L_{\mu\nu}(P) M_{\alpha\beta}(K, K') G^{\mu\alpha}(Q) G^{\nu\beta}(Q)$
- Static limit, small energy transfer : $G(Q) \to 1/(q^2 + m_D^2)$
- $|\mathcal{M}|^2_{\rm qH} \propto M^2 k_0^2 \frac{1+\cos\theta_{kk'}}{(q^2+m_D^2)^2}$; $\quad |\mathcal{M}|^2_{\rm gH} \propto M^2 k_0^2 \frac{1+\cos^2\theta_{kk'}}{(q^2+m_D^2)^2}$

Heavy quark scattering within Hard Thermal Loop

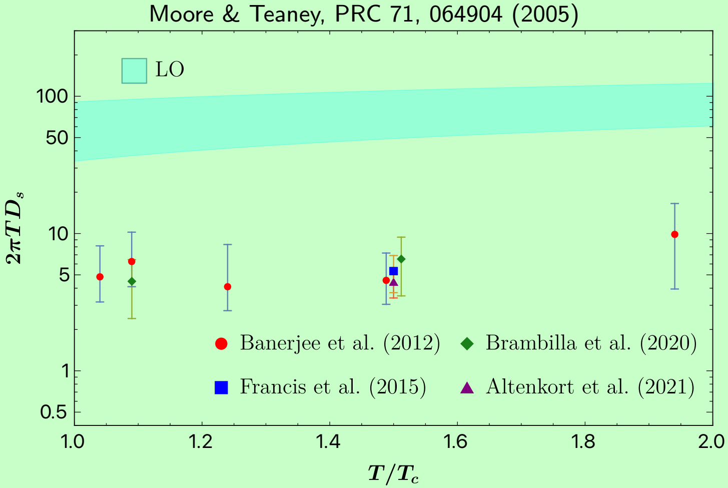
- $2 \leftrightarrow 2$ scattering : $qH \to qH$ (left) and $gH \to gH$ (right)



$$3\kappa = \frac{1}{16M^2}\int \frac{d^3\mathbf{k}}{(2\pi)^4 k k'}\int q^2 dq \int_{-1}^{1} d\cos\theta_{\mathbf{kq}}\Big[q^2\delta(k'-k)$$

$$\Big\{|\mathcal{M}|^2_{\mathrm{qH}}\; n_F(k)[1-n_F(k')] + \;|\mathcal{M}|^2_{\mathrm{gH}}\; n_B(k)[1+n_B(k')]\Big\}\Big]_{\mathbf{k}'=\mathbf{k}+\mathbf{q}}\,.$$

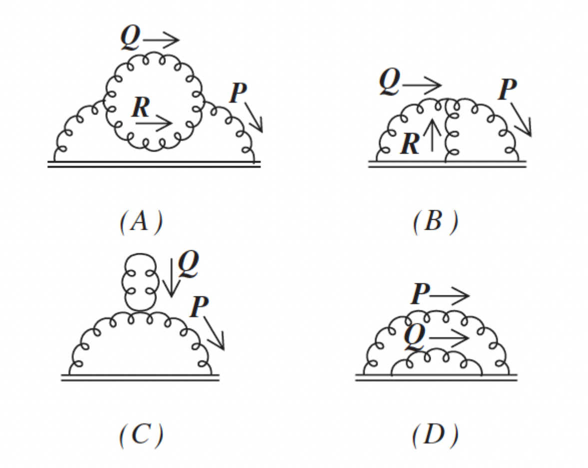
LO results



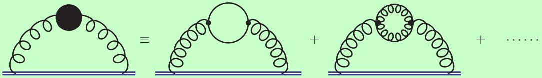
compared for pure glue, i.e. $N_f = 0$.

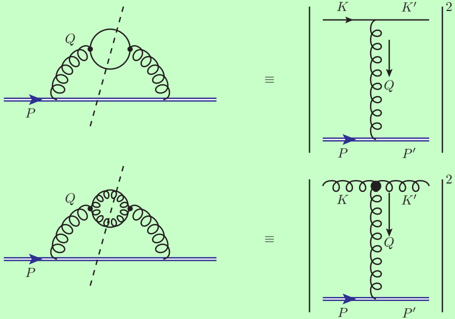
NLO results

Caron-Huot & Moore, PRL 100, 052301 (2008)



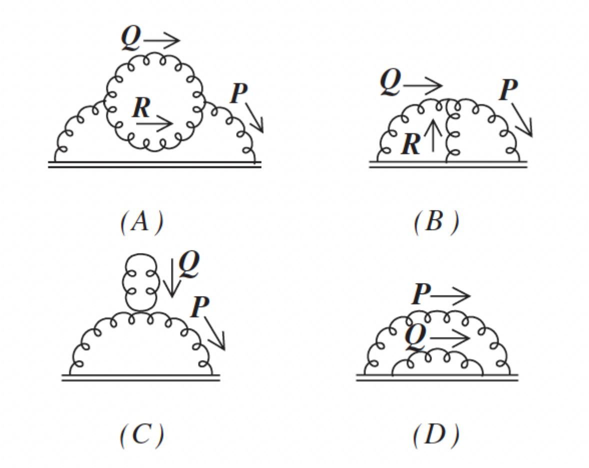
NLO results



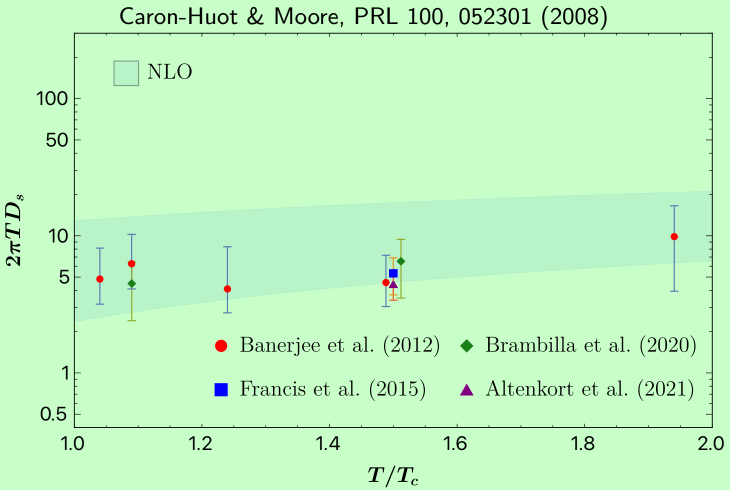


NLO results

Caron-Huot & Moore, PRL 100, 052301 (2008)



NLO results



compared for pure glue, i.e. $N_f = 0$.

Need for alternative results

- Even though NLO HTLpt result matches with the lattice QCD, the correction to LO is huge! $\rightarrow$ Indication of the poor convergence.
- Improvement is needed near T_c and for pure glue results.
- **Gribov-Zwanziger framework** showed improvements over 3-loop HTLpt for gluon thermodynamics, specifically near T_c.

  [K Fukushima, N Su, PRD 88, 076008 (2013)]
- Gribov prescription can be a promising setup to go forward!

Outline

Gribov-Zwanziger (GZ) framework

- The presence of a long-range force is required in QCD that confines colored objects.
- But the massless gluons that are supposed to transmit this force are absent from the physical spectrum.
- Apparent contradictory statements $\rightarrow$ Confinement Paradox $\rightarrow$ How does QCD incorporate confinement?
- addressed by Gribov and modified by Zwanziger $\rightarrow$ GZ framework.

Gribov-Zwanziger (GZ) framework

- Gribov demonstrated for the first time in 1978 that the gauge condition proposed by Faddeev and Popov is not ideal.
- Gribov showed that for certain scenarios, multiple copies of a gauge field can obey Landau gauge condition.

  If $A_\mu^{a'} = A_\mu^a + \mathcal{D}_\mu^{ab}\omega^b$, a vanishing $\partial_\mu \mathcal{D}_\mu^{ab}\omega^b$ generates Gribov copies $\rightarrow$ problem of overcounting.
- Faddeev-Popov operator :
  has no zero modes $\rightarrow$ first Gribov region.
  has its first zero mode $\rightarrow$ Gribov horizon.
- Gribov argued that overcounting problem will not be there if we limit the path integral up to the Gribov horizon.

Gribov-Zwanziger (GZ) framework

- GZ framework improves the infrared behaviour of QCD.
- GZ gluon propagator in covariant gauge

$$G^{\mu\alpha}(Q) = \left[\delta^{\mu\alpha} - (1-\xi)\frac{Q^\mu Q^\alpha}{Q^2}\right]\frac{Q^2}{Q^4+\gamma_G^4},$$

  $\xi \to$ gauge parameter.

- γ_G (Gribov parameter) $\to$ shifts the poles off the energy axis to an unphysical location $Q^2 = \pm i\gamma_G^2$, suggesting that the gluons are not physical excitations.
- Zwanziger (PRL94, 182301 (2005)) phenomenologically showed that a free gas of Gribov quasiparticles qualitatively captures the nonperturbative features of the lattice EoS.

Fixing γ_G - perturbative

- The Gribov mass parameter can be determined by the variational principle, leading to the following gap equation :

[K Fukushima & N Su, PRD 88 (2013), 076008]

$$\sum\!\!\!\!\!\!\!\int_P \frac{1}{P^4 + \gamma_G^4} = \frac{d}{(d-1)N_c g^2}$$

- Analytic form of γ_G in the limit $T \to \infty$,

$$\gamma_G(T) = \frac{d-1}{d}\frac{N_c}{4\sqrt{2}\pi} g^2(T)T,$$

where $g(T)$ is the one loop running coupling given by

$$\frac{g^2(T)}{4\pi} = \frac{6\pi}{(11N_c - 2N_f)\ln\left(\frac{\Lambda}{\Lambda_{\bar{M}S}}\right)}$$

Fixing γ_G - LQCD fitted coupling

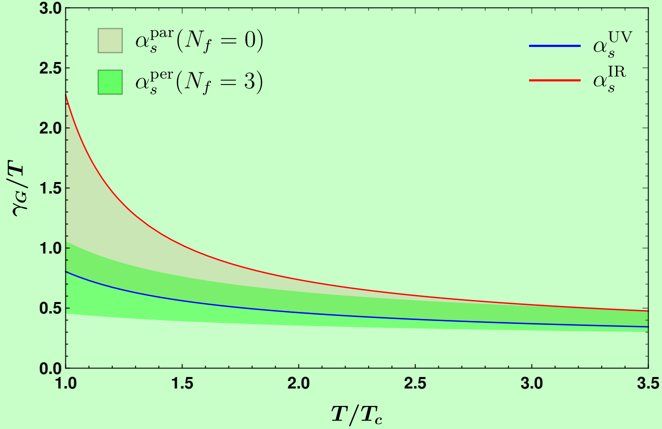
- The running coupling $g(T)$ can also be parameterised with a single parameter c as: [K Fukushima & N Su, PRD 88 (2013), 076008]

$$\alpha_s(T/T_c) \equiv \frac{g^2(T/T_c)}{4\pi} = \frac{6\pi}{11\ \mathrm{N_c}\ \ln[c(T/T_c)]}.$$

- The parameter c was calculated by fitting lattice QCD running coupling in the infrared and ultraviolet regime with $c_{IR} = 1.43$ for IR case and $c_{UV} = 2.97$ for UV case.

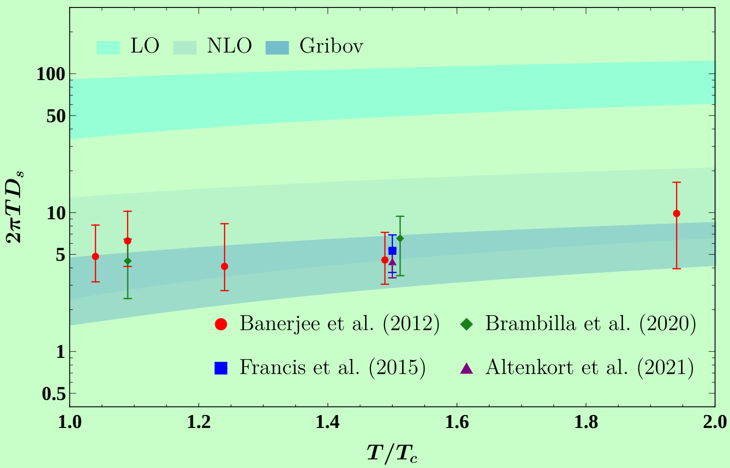
- The fitted parameter values correspond to the coupling data extracted from the large distance (IR) and the short distance (UV) behaviour of the heavy quark free energy.

Fixing γ_G - comparison

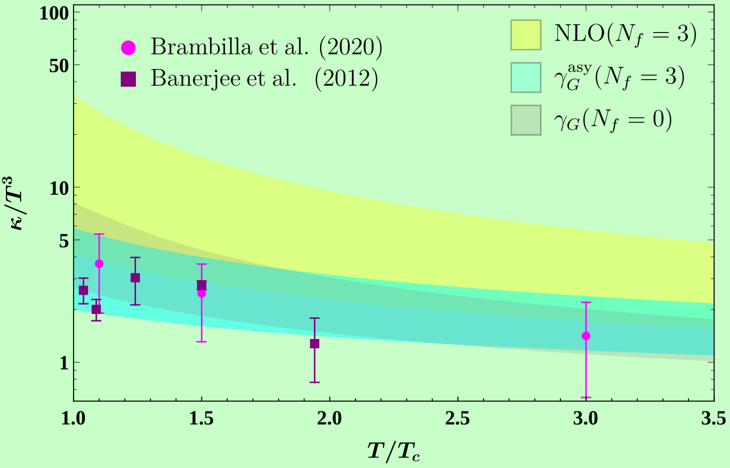


Outline

Spatial diffusion coefficients



Momentum diffusion coefficients



Summary

- We have discussed existing LO and NLO HTLpt results for heavy quark diffusion coefficient.
- We have discussed the motivation to include the Gribov Zwanziger framework in our calculation.
- Different procedures to fix the Gribov parameter γ_G have been explored.
- HQ diffusion coefficient (κ) within the static limit in light of GZ action shows reasonable improvements over the existing results.
- A natural next step is to go beyond the HQ static limit adopted in the present work.

Thanksgiving

Thank you for your kind attention.