RELATIVISTIC MAGNETOHYDRODYNAMICS FOR SPIN POLARIZED MEDIA

based on: *Phys. Lett. B* 814 136096 (2021); *Phys. Rev. D* 103, 014030 (2021); *Phys. Rev. Lett* 129, 192301 (2022)

> *EXPLORING QUARK-GLUON PLASMA THROUGH SOFT AND HARD PROBES*, 29-31 MAY 2023, SANU, BELGRADE, SERBIA

THE HENRYK NIEWODNICZAŃSKI **INSTITUTE OF NUCLEAR PHYSICS** POLISH ACADEMY OF SCIENCES

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MOTIVATION

QGP EVOLVES HYDRODYNAMICALLY

Well established properties of the produced QCD matter: - Behaves like a fluid \blacksquare hydrodynamics applicable - Low viscosity **Exercise Service** inclusion of dissipative effects required

 $1113 - 117$ (2019) figure: J. Bernhard, J. Moreland, S. Bass, Nature Phys. 15, 1113–1117 (2019)15, Bass, Nature Phys. \vec{S} Bernhard, J. Moreland, **Ire:** figu

NON-CENTRAL HEAVY-ION COLLISIONS

Non-central collisions are interesting: - Large initial orbital angular momentum

- -
- Large magnetic field
	-

F. Becattini, F. Piccinini, J. Rizzo, PRC 77 (2008) 024906

A. Bzdak and, V. Skokov, PLB 710 (2012) 171-174

SPIN POLARIZATION DUE TO GLOBAL ORBITAL ANGULAR MOMENTUM

Part of the **angular momentum can be transferred from the orbital to the spin part**

figure: R. Ryblewski

Emitted **particles are expected to be polarized** along the fireball's global angular momentum.

Liang ZT, Wang XN. PRL 94:102301 (2005) Betz B, Gyulassy M, Torrieri G. PRC 76:044901 (2007) Gao JH, et al. PRC 77:044902 (2008) Becattini F, Piccinini F, et al. J. Phys. G 35:054001 (2008)

$\bm{J}_{\mathrm{init}} \, = \bm{L}_{\mathrm{init}} \, = \bm{L}_{\mathrm{final}} \, + \bm{S}_{\mathrm{final}}$

MEASUREMENT OF Λ **AND** Λ¯ **GLOBAL SPIN POLARIZATION**

SPIN POLARIZATION IN EQUILIBRATED QGP — SPIN-THERMAL APPROACH

(from the MADAI

In thermodynamic equilibrium one can establish a link between spin and vorticity

Very attractive: Allows to extract polarization at the freeze-out hypersurface in any model which provides u^{μ} , T and μ .

$$
S^{\mu}(p) = -\frac{1}{8m} \epsilon^{\mu\rho\sigma\tau} p_{\tau} \frac{\int d\Sigma_{\lambda} p^{\lambda} n_{F} (1 - n_{F}) \overline{\omega}_{\rho\sigma}}{\int d\Sigma_{\lambda} p^{\lambda} n_{F}}
$$

Becattini F, Chandra V, Del Zanna L, Grossi E. AP 338:32 (2013) F. Becattini, L. Csernai, and D. J. Wang, PRC 88, 034905 (2013) Fang R, Pang L,Wang Q,Wang X. PRC 94:024904 (2016) F. Becattini, I. Karpenko, M. Lisa, I. Upsal, and S. Voloshin PRC 95, 054902 (2017)

Spin is enslaved to thermal vorticity

MEASUREMENT VS SPIN-THERMAL APPROACH: GLOBAL POLARIZATION

UrQMD+vHLLE: I. Karpenko, F. Becattini, EPJC 77, 213 (2017) AMPT: H. Li, L. Pang, Q. Wang, and X. Xia, PRC 96, 054908 (2017)

Global polarization data supports the spin-thermal approach

Signal is robust and agrees well with predictions of transport and hydrodynamic models

Azimuthal modulation is not captured

Flow structures in the plane transverse to beam (jet, ebe fluctuations etc.) may generate longitudinal polarization

LONGITUDINAL (BEAM-AXIS) POLARIZATION

LONGITUDINAL POLARIZATION — 'SPIN SIGN' PUZZLE

T. Niida, NPA 982 (2019) 511514

LONGITUDINAL POLARIZATION — 'SPIN SIGN' PUZZLE

T. Niida, NPA 982 (2019) 511514

thermal model with projected vorticity $\omega_{\mu\nu} = \varpi_{\alpha\beta} \overline{\Delta}^{\alpha}_{\mu} \overline{\Delta}^{\beta}_{\nu}$

W.Florkowski, A. Kumar, A. Mazeliauskas, R.R., [1904.00002]

3D VH + AMPT IC with T-vorticity $\omega_{\mu\nu}^{(T)} = -\frac{1}{2} \left[\partial_{\mu} (T u_{\nu}) - \partial_{\nu} (T u_{\mu}) \right]$ H-Z Wu, L-G Pang, X-G Huang, Q. Wang [1906.09385]

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LONGITUDINAL POLARIZATION — 'SPIN SIGN' PUZZLE

T. Niida, NPA 982 (2019) 511514

FLUID DYNAMICS OF SPIN?!

Spin-thermal approach does not capture properly phenomena seen in experiment.

W. Florkowski, B. Friman, A. Jaiswal, E. Speranza, PRC 97 (4) (2018) 041901

INCORPORATING SPIN IN HYDRODYNAMICS

CONSERVATION OF ANGULAR MOMENTUM AND SPIN CHEMICAL POTENTIAL

Conservation of charge (baryon number, electric charge, ...)

$$
\partial_{\mu} \widehat{N}^{\mu}(x) = 0 \qquad \text{(1 equa)}
$$

Conservation of energy and momentum

$$
\begin{aligned}\n\widehat{J}_{C}^{\mu,\alpha\beta}(x) &= \frac{x^{\alpha}\widehat{T}_{C}^{\mu\beta}(x) - x^{\beta}\widehat{T}_{C}^{\mu\alpha}(x) + \widehat{S}_{C}^{\mu,\alpha\beta}(x)}{\widehat{\iota}_{C}^{\mu,\alpha\beta}(x)} & \partial_{\mu}\widehat{T}_{C}^{\mu\alpha}(x) = 0 \qquad \text{(4 equations)} \\
& \text{Conservation of total angular momentum} \\
\partial_{\mu}\widehat{J}_{C}^{\mu,\alpha\beta}(x) &= 0 \qquad \Rightarrow \qquad \partial_{\mu}\widehat{S}_{C}^{\mu,\alpha\beta}(x) = \widehat{T}_{C}^{\beta\alpha}(x) - \widehat{T}_{C}^{\alpha\beta}(x)\n\end{aligned}
$$

ation/charge)

$$
\sum_{\mu\nu}\equiv T\omega_{\mu\nu}
$$

The conservation of angular momentum implies introduction of new hydrodynamic variables — spin chemical potential

If the **energy-momentum tensor is symmetric** the spin tensor is conserved

$$
\partial_{\mu}T^{\mu\nu}=0, \quad \partial_{\lambda}S^{\lambda,\mu\nu}=0, \quad \partial_{\mu}N^{\mu}=0
$$

 $[\beta, \omega, \xi], \quad S^{\mu, \lambda \nu} = S^{\mu, \lambda \nu}[\beta, \omega, \xi], \quad N^{\mu} = N^{\mu}[\beta, \omega, \xi]$

$$
T^{\mu\nu}=T^{\mu\nu}[\beta,\omega,\xi],\quad S^{\mu,\lambda\nu}=
$$

What are the constitutive relations which enter equations of motion?

W. Florkowski, B. Friman, A. Jaiswal, E. Speranza, PRC 97 (4) (2018) 041901 W. Florkowski, B. Friman, A. Jaiswal, R. R., E. Speranza, PRD 97 (2018) 116017 F. Becattini, W. Florkowski, E. Speranza, PLB 789 (2019) 419-425 W. Florkowski, A. Kumar, R. R., Prog. Part. Nucl. Phys. 108 (2019) 103709

$$
\partial_{\mu}T^{\mu\nu}=0,
$$

Fluid dynamics with spin should tell how the spin chemical potential evolves but not its origin — need for modeling of initial conditions!

CONSERVATION OF ANGULAR MOMENTUM AND SPIN CHEMICAL POTENTIAL

RELATIVISTIC KINETIC THEORY WITH SPIN

To include spin in kinetic theory, we start from the **Wigner function (WF)** that bridges the gap between QFT and RKT

$$
\mathcal{W}_{\alpha\beta}=\frac{1}{4}\left(\mathcal{F}+i\gamma^5\mathcal{P}+\gamma^{\mu}\mathcal{V}_{\mu}+\gamma^5\gamma^{\mu}\mathcal{A}_{\mu}+\frac{1}{2}\varSigma^{\mu\nu}\mathcal{S}_{\mu\nu}\right)_{\alpha\beta}
$$

- $\mathcal{F} \rightarrow$ scalar component,
- $P \rightarrow$ pseudoscalar component,
- $\mathcal{V}_{\mu} \rightarrow$ vector component,
- $\mathcal{A}_{\mu} \rightarrow$ axial vector component,
- $S_{\mu\nu} \rightarrow$ tensor component.

For spin-1/2 particles the Wigner function satisfies the **quantum kinetic equation**

$$
\left[\gamma \cdot \left(p + \frac{i}{2}\partial\right) - m\right] \mathcal{W}_{\alpha\beta} = \mathcal{C} \left[\mathcal{W}_{\alpha\beta}\right]
$$

[Xin-Li Sheng, PhD Thesis (2019), N. Weickgenannt et al, PRL 127 (2021) 5, 052301, PRD 100, 056018 (2019).]

$$
\Sigma^{\mu\nu}=i\gamma^{[\mu}\gamma^{\nu]}
$$

From the leading and next-to-leading orders of the **semi-classical expansion of the WF** in powers of *h*, one obtains **two independent kinetic equations for the scalar and axial-vector components**

[S.B., W. Florkowski, A. Jaiswal, A. Kumar and, R. Ryblewski, PLB 814 (2021) 136096, PRD 103 (2021) 1, 014030]

$$
A xial Component
$$
\n
$$
k^{\mu} \partial_{\mu} A^{\nu}(x, k) = C_{\mathcal{A}}^{\nu}
$$
\n
$$
C_{\mathcal{A}}^{\nu} = \frac{(k \cdot u)}{\tau_{eq}} \left[A_{eq}^{\nu}(x, k) - A^{\nu}(x, k) \right]
$$
\n
$$
(k \mp p) A_{\pm}^{\mu}(x, k) = 2m \int_{p, s} s^{\mu} f^{\pm}(x, p, s) \delta^{(4)}(k \mp p)
$$

$$
(m/\pi s) \int d^4 s \delta(s \cdot s + s^2) \qquad \qquad \mathbf{S}^2 = \frac{1}{2} (1 + \frac{1}{2})
$$

RELATIVISTIC KINETIC THEORY WITH SPIN

CLASSICAL APPROACH TO SPIN HYDRODYNAMICS

In the classical treatments of particles with spin-1/2 one introduces internal angular momentum tensor of particles [M. Mathisson, APPB 6 (1937) 163-2900]

$$
s^{\alpha\beta}=\frac{1}{m}\epsilon^{\alpha\beta\gamma\delta}p_{\gamma}s_{\delta}.
$$

 $s^{\alpha\beta}$ is antisymmetric *i.e.* $s^{\alpha\beta} = -s^{\beta\alpha}$ and satisfies Frenkel (or Weyssenhoff) $p_{\alpha} s^{\alpha\beta} = 0.$

The spin four vector can be obtained by above equation,

$$
s^{\alpha} = \frac{1}{2m} \epsilon^{\alpha \beta \gamma \delta}
$$

In particle rest frame (PRF) where $p^{\mu} = (m, 0, 0, 0)$, $s^{\alpha} = (0, s_*)$ with the length of spin vector given by $-s^2 = -s^{\alpha} s_{\alpha} = |\mathbf{s}_*|^2 = \hat{s}^2 = \frac{1}{2} (1 + \frac{1}{2}) = \frac{3}{4}$.

-
-

$$
\boldsymbol{\mathop{\mathcal{D}_{\beta}}\nolimits} \mathop{\mathcal{S}_{\gamma}}\nolimits_{\delta}
$$

M.Mathissor

J. Weyssenhof

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Following Liouville theorem the **kinetic equation** governing the evolution of the distribution function can be written as

$$
\mathcal{S}^{\alpha\beta}=2\,F^{\gamma[\alpha}\,m^{\beta]}\gamma-\frac{1}{m^2}\Big(\!\chi-\frac{\mathfrak{q}}{m}\!\Big)F_{\phi\gamma}\,s^{\phi[\alpha}\,p^{\beta]}p^{\gamma}
$$

 $m^{\alpha\beta} = \chi s^{\alpha\beta}$

where

$$
p^{\mu}\partial_{\mu}^{(x)}f^{\pm} + m\mathcal{F}^{\mu}\partial_{\mu}^{(p)}f^{\pm} + m\mathcal{S}^{\mu\nu}\partial_{\mu\nu}^{(s)}f^{\pm} = \mathcal{C}[f^{\pm}]
$$

$$
\partial_{\mu}^{(x)} \equiv \frac{\partial}{\partial x^{\mu}}, \qquad \partial_{\mu}^{(p)} \equiv \frac{\partial}{\partial p^{\mu}}, \qquad \partial_{\mu\nu}^{(s)} \equiv \frac{\partial}{\partial s^{\mu\nu}}, \qquad \mathcal{F}^{\alpha} \equiv \frac{dp^{\alpha}}{d\tau} \qquad \mathcal{S}^{\alpha\beta} \equiv \frac{ds^{\alpha\beta}}{d\tau}
$$

Using the Frenkel condition, one can derive the **force (Lorentz and Mathisson)** and **torque** term [Suttorp, de Groot, Nuovo Cimento A (1965-1970), van Weert Thesis (1970), Nora W. et al. PRD 100 (2019) 5, 056018]

$$
\mathcal{F}^{\alpha} = \frac{\mathfrak{q}}{m} F^{\alpha \beta} p_{\beta} + \frac{1}{2} \left(\partial^{\alpha} F^{\beta \gamma} \right) m_{\beta \gamma}
$$

The **distribution function** in the **extended phase-space** is a function of **spacetime**, **momentum**, and **internal angular**

$$
(x,p,s)
$$

 f^{\pm}

momentum of the particles

[Suttorp, de Groot, Nuovo Cimento A (1965-1970), van Weert Thesis (1970)]

$$
p^{\mu}\partial_{\mu}^{(x)}f^{\pm} + m\mathcal{F}^{\mu}\partial_{\mu}^{(p)}f^{\pm} + m\mathcal{S}^{\mu\nu}\partial_{\mu\nu}^{(s)}f^{\pm} = \mathcal{C}[f^{\pm}]
$$

$$
\partial_{\mu}^{(x)} \equiv \frac{\partial}{\partial x^{\mu}}, \qquad \partial_{\mu}^{(p)} \equiv \frac{\partial}{\partial p^{\mu}}, \qquad \partial_{\mu\nu}^{(s)} \equiv \frac{\partial}{\partial s^{\mu\nu}}, \qquad \mathcal{F}^{\alpha} \equiv \frac{dp^{\alpha}}{d\tau} \qquad \qquad \mathcal{S}^{\alpha\beta} \equiv \frac{ds^{\alpha\beta}}{d\tau}
$$

where **magnetic dipole moment** is

RELATIVISTIC KINETIC THEORY WITH SPIN

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RELATIVISTIC KINETIC THEORY WITH SPIN

 If the medium if magnetizable, then the **Maxwell's equations** are given by[Balakin, Grav.Cosmol. 13 (2007) 163-177; Hehl and, Obukhov, Phys. Lett. A 311, 277 (2003)]

 $\partial_{\mu}H^{\mu\nu}=J^{\nu},$

 $\int \widetilde{F}^{\mu\nu}$ =

 $H^{\mu\nu} = F^{\mu\nu} + M^{\mu\nu}$

$$
^{\prime }=\epsilon ^{\mu \nu \alpha \beta }\,u_{\alpha }\,B_{\beta }
$$

 $B_\mu B^\mu \leq 0$

$$
\partial_{\mu}\widetilde{F}^{\mu\nu} = 0,
$$

$$
= \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}
$$

In the limit of **infinite conductivity**, field strength tensor is

 $F^{\mu\nu} \rightarrow B^{\mu\nu}$

 $u_\mu B^\mu = 0$

The **particle current**, **energy-momentum tensor**, and **spin tensor** of the fluid can be expressed as

[S.B., W. Florkowski, A. Jaiswal, A. Kumar and, R. Ryblewski, PLB 814 (2021) 136096, PRD 103, 014030 (2021)]

 $N^{\mu}=\int_{p,s}$ $T_f^{\mu\nu} = \int_{p,s}$
 $S^{\lambda,\mu\nu} = \int$

while the **polarization-magnetization tensor** is

 $M^{\alpha\beta} = m$

$$
p^{\mu} (f^{+} - f^{-}),
$$

$$
p^{\mu} p^{\nu} (f^{+} + f^{-}),
$$

$$
\int_{p,s} p^{\lambda} s^{\mu\nu} (f^{+} + f^{-})
$$

$$
\int_{p,s} \, m^{\alpha\beta} \left(f^+ - f^- \right)
$$

RELATIVISTIC KINETIC THEORY WITH SPIN

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$$
T_{\rm f}^{\mu\nu} = \epsilon u^{\mu} u^{\nu} - (P + \Pi) \Delta^{\mu\nu} + \pi^{\mu\nu}
$$

Assuming that the microscopic interactions **preserve conservation laws** one requires

 $J_{p,s}$ $\int_{\rho,s}$

Zeroth, first, and 'spin' moment of the kinetic equation (**in absence of the torque term**) then lead to

 $\partial_{\mu}N$

 $\partial_{\nu}T$

 $\partial_{\lambda}S^{\lambda,\mu\nu}=0$

$$
N^{\mu}=nu^{\mu}+n^{\mu} \qquad J^{\mu}_{\mathrm{f}}=\mathfrak{q}N^{\mu}
$$

$$
\mathcal{C}[f] = 0,
$$

\n
$$
\rho^{\mu} \mathcal{C}[f] = 0,
$$

\n
$$
\rho^{\mu} \mathcal{C}[f] = 0,
$$

$$
\mathsf{s}^{\mu\nu}\mathcal{C}\left[\mathsf{f}\right]=0
$$

$$
\text{V}^\mu=0
$$

$$
f^{\mu\nu} = F^{\mu}_{\ \alpha} J^{\alpha}_{f} + \frac{1}{2} \left(\partial^{\mu} F^{\nu\alpha} \right) M_{\nu\alpha}
$$

RELATIVISTIC MHD WITH SPIN

The **equilbrium distribution function** has the form (we use the small polarization limit)

$$
f_{\rm eq} = \frac{1}{1+\exp\left[\beta(u\!\cdot\! p)-\xi-\frac{1}{2}\,\omega:s\right]}.
$$

$$
\delta f_{(1)}^{\pm} = -\mathcal{D} f_{\text{eq}}^{\pm},
$$

$$
\mathcal{D} = \frac{\tau_R}{(u \cdot p)} \left(p^{\alpha} \frac{\partial}{\partial x^{\alpha}} + \mathcal{F}^{\alpha} \frac{\partial}{\partial p^{\alpha}} \right)
$$

$$
f_{\text{eq}} = f_0 + \frac{1}{2} (\omega : s) f_0 \tilde{f}_0,
$$

$$
f_0 \equiv \left\{ 1 + \exp \left[\beta (u \cdot p) - \xi \right] \right\}^{-1} \qquad \tilde{f}_0 \equiv 1 - f_0
$$

RELATIVISTIC MHD WITH SPIN

Using RTA kinetic equation we can write the **first-order gradient correction** as

$$
f^{\pm}(x,p,s) = f_{\text{eq}}^{\pm}(x,p,s) + \delta f^{\pm}(x,p,s).
$$

The expressions for dissipative currents in terms of the nonequilibrium correction to the distribution function are

RELATIVISTIC MHD WITH SPIN

$$
\Pi = -\frac{\Delta_{\alpha\beta}}{3} \int_{p,s} p^{\alpha} p^{\beta} (\delta f^{+} + \delta f^{-})
$$

$$
\pi^{\mu\nu} = \Delta^{\mu\nu}_{\alpha\beta} \int_{p,s} p^{\alpha} p^{\beta} (\delta f^{+} + \delta f^{-})
$$

$$
n^{\mu} = \Delta^{\mu}_{\alpha} \int_{p,s} p^{\alpha} (\delta f^{+} - \delta f^{-})
$$

$$
\delta S^{\lambda,\mu\nu} = \int_{p,s} p^{\lambda} s^{\mu\nu} (\delta f^{+} + \delta f^{-})
$$

Equilibrium polarization-magnetization tensor is

$$
M_{\text{eq}}^{\mu\nu} = a_1(T,\mu)
$$

RELATIVISTIC MHD WITH SPIN

We conclude that **rotation of the fluid produces magnetization**, which is precisely the physics of **Barnett effect**. **Figure: Journal of the Physical Society of Japan 90, 081003 (2021)**

 $\omega^{\mu\nu}+a_{\rm\scriptscriptstyle 2}(T,\mu)\,u^{[\mu}u_{\gamma}\omega^{\nu]\gamma}$

In global equilibrium, spin polarization tensor corresponds to rotation of the fluid.

$$
\omega^{\mu\nu}|_{\text{geq}\alpha} \propto \varpi^{\mu\nu} = \left(\partial^\mu\beta^\nu - \partial^\nu\beta^\mu\right)/2
$$

RELATIVISTIC MHD WITH SPIN

Using the spin matching condition we obtain the **evolution equation for the spin polarization tensor**

$$
\begin{aligned}\n\dot{\omega}^{\mu\nu} &= \mathcal{D}_{\Pi}^{\mu\nu} \theta + \mathcal{D}_{\mathbf{n}}^{\mu\nu\gamma} \left(\nabla_{\gamma} \xi \right) + \mathcal{D}_{\mathbf{a}}^{\mu\nu\gamma} \dot{u}_{\gamma} + \mathcal{D}_{\pi}^{\mu\nu\rho\kappa} \sigma_{\rho\kappa} + \mathcal{D}_{\Omega}^{\mu\nu\rho\kappa} \Omega_{\rho\kappa} + \mathcal{D}_{\Sigma}^{\mu\nu\phi\rho\kappa} \left(\nabla_{\phi} \omega_{\rho\kappa} \right)\n\end{aligned}
$$
\n
$$
\Omega_{\mu\nu} \equiv \left(\partial_{\mu} u_{\nu} - \partial_{\nu} u_{\mu} \right) / 2
$$

We observe that the above equation contains information about the **connection between evolution of spin polarization tensor and fluid vorticity.**

 $\mathcal{D}_\Omega^{\mu\nu\rho\kappa}$ vanishes in absence of electromagnetic field which leads us to conclude that the **conversion between spin-polarization and vorticity proceeds via coupling with electromagnetic field**.

RELATIVISTIC MHD WITH SPIN

$$
X = \tau_{eq} \left[\beta_{X\Pi} \theta + \beta_{X\Pi}^{\alpha} (\nabla_{\alpha} \xi) + \beta_{X\Pi}^{\alpha} \dot{u}_{\alpha} + \beta_{X\Pi}^{\alpha\beta} \sigma_{\alpha\beta} + \beta_{X\Omega}^{\alpha\beta} \Omega_{\alpha\beta} + \beta_{XF}^{\alpha\beta} (\nabla_{\alpha} B_{\beta}) + \beta_{X\Sigma}^{\alpha\beta\gamma} (\nabla_{\alpha} \omega_{\beta\gamma}) \right],
$$

The expressions for **dissipative currents** in terms of the nonequilibrium corrections to the distribution function are

$$
\varPi,\ \pi^{\mu\nu},\ \delta S^{\lambda,\mu\nu}
$$

where

 $X \equiv n^{\mu}, I$

Demanding that the divergence of the above entropy current is positive definite we identify which first-order gradient terms are dissipative

$$
\Pi = -\zeta \theta, \quad n^{\mu} = \kappa^{\mu \alpha} (\nabla_{\alpha} \xi), \quad \pi^{\mu \nu} = \eta^{\mu \nu \alpha \beta} \sigma_{\alpha \beta},
$$

$$
\delta S^{\mu, \alpha \beta} = \Sigma^{\mu \alpha \beta \lambda \gamma \rho} (\nabla_{\lambda} \omega_{\gamma \rho}).
$$

CONCLUSIONS

with spin in the limit of small polarization.

We demonstrated that multiple transport coefficients, dissipative as well as non-dissipative, are present.

- We presented the first kinetic theory formulation of **relativistic dissipative nonresistive magnetohydrodynamics**
	-
	-
	-
- Simulation based on our unified framework has the potential of explaining the difference of Λ and anti-Λ polarization.

We showed that our framework naturally leads to the **emergence of the relativistic analog of Barnett effect**.

We show that the **coupling between the magnetic field and spin polarization appears at gradient order**.

THANK YOU FOR YOUR ATTENTION.