RELATIVISTIC MAGNETOHYDRODYNAMICS FOR SPIN POLARIZED MEDIA

Radoslaw Ryblewski The H. Niewodniczański Institute of Nuclear Physics Polish Academy of Sciences, Kraków, Poland

In collaboration with: Samapan Bhadury (IFT UJ, Kraków), Wojciech Florkowski (IFT UJ, Kraków), **Amaresh Jaiswal** (NISER Bhubaneswar), **Avdhesh Kumar** (IOP Academia Sinica Taipei)

based on: *Phys. Lett. B* 814 136096 (2021); *Phys. Rev. D* 103, 014030 (2021); *Phys. Rev. Lett* 129, 192301 (2022)

> EXPLORING QUARK-GLUON PLASMA THROUGH SOFT AND HARD PROBES, 29-31 MAY 2023, SANU, BELGRADE, SERBIA





THE HENRYK NIEWODNICZAŃSKI **INSTITUTE OF NUCLEAR PHYSICS POLISH ACADEMY OF SCIENCES**



Μοτινατιον

QGP EVOLVES HYDRODYNAMICALLY



- Behaves like a fluid - Low viscosity

Well established properties of the produced QCD matter: hydrodynamics applicable inclusion of dissipative effects required

1113-1117 (2019) 15, Bass, Nature Phys. . S Bernhard, J. Moreland, figu



NON-CENTRAL HEAVY-ION COLLISIONS



Non-central collisions are interesting: - Large initial orbital angular momentum

- Large magnetic field



F. Becattini, F. Piccinini, J. Rizzo, PRC 77 (2008) 024906

A. Bzdak and, V. Skokov, PLB 710 (2012) 171-174

SPIN POLARIZATION DUE TO GLOBAL ORBITAL ANGULAR MOMENTUM

Part of the angular momentum can be transferred from the orbital to the spin part

Liang ZT, Wang XN. PRL 94:102301 (2005) Betz B, Gyulassy M, Torrieri G. PRC 76:044901 (2007) Gao JH, et al. PRC 77:044902 (2008) Becattini F, Piccinini F, et al. J. Phys. G 35:054001 (2008)

$m{J}_{ ext{init}} = m{L}_{ ext{init}} = m{L}_{ ext{final}} + m{S}_{ ext{final}}$

Emitted **particles are expected to be polarized** along the fireball's global angular momentum.



figure: R. Ryblewski

Measurement of Λ and $\bar{\Lambda}$ global spin polarization



6

SPIN POLARIZATION IN EQUILIBRATED QGP — SPIN-THERMAL APPROACH

In thermodynamic equilibrium one can establish a link between spin and vorticity

Becattini F, Chandra V, Del Zanna L, Grossi E. AP 338:32 (2013) F. Becattini, L. Csernai, and D. J. Wang, PRC 88, 034905 (2013) Fang R, Pang L,Wang Q,Wang X. PRC 94:024904 (2016) F. Becattini, I. Karpenko, M. Lisa, I. Upsal, and S. Voloshin PRC 95, 054902 (2017)

$$S^{\mu}(p) = -\frac{1}{8m} \epsilon^{\mu\rho\sigma\tau} p_{\tau} \frac{\int d\Sigma_{\lambda} p^{\lambda} n_F \left(1 - n_F\right) \varpi_{\rho\sigma}}{\int d\Sigma_{\lambda} p^{\lambda} n_F}$$
$$\varpi_{\mu\nu} = -\frac{1}{2} \left(\partial_{\mu} \beta_{\nu} - \partial_{\nu} \beta_{\mu}\right) \qquad \beta^{\mu} = \frac{u^{\mu}}{T}$$

Spin is enslaved to thermal vorticity

Very attractive: Allows to extract polarization at the freeze-out hypersurface in any model which provides u^{μ} , *T* and μ .



(from the MADAI

MEASUREMENT VS SPIN-THERMAL APPROACH: GLOBAL POLARIZATION

Global polarization data supports the spin-thermal approach

Signal is robust and agrees well with predictions of transport and hydrodynamic models

Azimuthal modulation is not captured







UrQMD+vHLLE: I. Karpenko, F. Becattini, EPJC 77, 213 (2017) AMPT: H. Li, L. Pang, Q. Wang, and X. Xia, PRC 96, 054908 (2017)



LONGITUDINAL (BEAM-AXIS) POLARIZATION



Flow structures in the plane transverse to beam (jet, ebe fluctuations etc.) may generate longitudinal polarization



LONGITUDINAL POLARIZATION — 'SPIN SIGN' PUZZLE



T. Niida, NPA 982 (2019) 511514



LONGITUDINAL POLARIZATION — 'SPIN SIGN' PUZZLE



T. Niida, NPA 982 (2019) 511514



thermal model with projected vorticity $\omega_{\mu\nu} = \varpi_{\alpha\beta} \overline{\Delta}^{\alpha}_{\mu} \overline{\Delta}^{\beta}_{\nu}$

W.Florkowski, A. Kumar, A. Mazeliauskas, R.R., [1904.00002]



3D VH + AMPT IC with *T*-vorticity $\omega_{\mu\nu}^{(T)} = -\frac{1}{2} \left[\partial_{\mu} (Tu_{\nu}) - \partial_{\nu} (Tu_{\mu}) \right]$ H-Z Wu, L-G Pang, X-G Huang, Q. Wang [1906.09385]



11



LONGITUDINAL POLARIZATION — 'SPIN SIGN' PUZZLE



T. Niida, NPA 982 (2019) 511514





12



FLUID DYNAMICS OF SPIN?!

Spin-thermal approach does not capture properly phenomena seen in experiment.

Nonequilibrium dynamics of spin is suggested.

If spin polarization is trully hydrodynamic quantity it should not be enslaved to thermal vorticity.

W. Florkowski, B. Friman, A. Jaiswal, E. Speranza, PRC 97 (4) (2018) 041901





Fluid dynamics with spin





INCORPORATING SPIN IN HYDRODYNAMICS

CONSERVATION OF ANGULAR MOMENTUM AND SPIN CHEMICAL POTENTIAL

Conservation of charge (baryon number, electric charge, ...)

$$\partial_{\mu}\widehat{N}^{\mu}(x) = 0$$
 (1 equa

Conservation of energy and momentum

ation/charge)



equations)



The conservation of angular momentum implies introduction of new hydrodynamic variables — spin chemical potential

$$x) = \widehat{T}_C^{\beta\alpha}(x) - \widehat{T}_C^{\alpha\beta}(x)$$





CONSERVATION OF ANGULAR MOMENTUM AND SPIN CHEMICAL POTENTIAL

If the **energy-momentum tensor is symmetric** the spin tensor is conserved

W. Florkowski, B. Friman, A. Jaiswal, E. Speranza, PRC 97 (4) (2018) 041901 W. Florkowski, B. Friman, A. Jaiswal, R. R., E. Speranza, PRD 97 (2018) 116017 F. Becattini, W. Florkowski, E. Speranza, PLB 789 (2019) 419-425 W. Florkowski, A. Kumar, R. R., Prog. Part. Nucl. Phys. 108 (2019) 103709

$$\partial_{\mu}T^{\mu\nu}=0,$$

What are the constitutive relations which enter equations of motion?

$$T^{\mu\nu} = T^{\mu\nu}[\beta, \omega, \xi], \quad S^{\mu,\lambda\nu} =$$

Fluid dynamics with spin should tell how the spin chemical potential evolves but not its origin — need for modeling of initial conditions!

$$\partial_{\lambda}S^{\lambda,\mu\nu} = 0, \quad \partial_{\mu}N^{\mu} = 0$$

 $= S^{\mu,\lambda\nu}[\beta,\omega,\xi], \quad N^{\mu} = N^{\mu}[\beta,\omega,\xi]$

To include spin in kinetic theory, we start from the **Wigner function (WF)** that bridges the gap between QFT and RKT

$$\mathcal{W}_{\alpha\beta} = \frac{1}{4} \left(\mathcal{F} + i\gamma^{5}\mathcal{P} + \gamma^{\mu}\mathcal{V}_{\mu} + \gamma^{5}\gamma^{\mu}\mathcal{A}_{\mu} + \frac{1}{2}\Sigma^{\mu\nu}\mathcal{S}_{\mu\nu} \right)_{\alpha\beta}$$

- $\mathcal{F} \rightarrow \text{scalar component},$
- $\mathcal{P} \rightarrow \text{pseudoscalar component},$
- $\mathcal{V}_{\mu} \rightarrow$ vector component,
- $\mathcal{A}_{\mu} \rightarrow \text{axial vector component},$
- $S_{\mu\nu} \rightarrow \text{tensor component.}$

For spin-1/2 particles the Wigner function satisfies the **quantum kinetic equation**

$$\left[\gamma \cdot \left(p + \frac{i}{2}\partial\right) - m\right] \mathcal{W}_{\alpha\beta} = \mathcal{C}\left[\mathcal{W}_{\alpha\beta}\right]$$

[Xin-Li Sheng, PhD Thesis (2019), N. Weickgenannt et al, PRL 127 (2021) 5, 052301, PRD 100, 056018 (2019).]

$$\Sigma^{\mu\nu} = i\gamma^{[\mu}\gamma^{\nu]}$$

From the leading and next-to-leading orders of the semi-classical expansion of the WF in powers of h, one obtains two independent kinetic equations for the scalar and axial-vector components

	Scalar Component
Kin. Eq.	$k^{\mu}\partial_{\mu}\mathcal{F}(x,k)=\mathcal{C}_{\mathcal{F}}$
RTA	$C_{\mathcal{F}} = \frac{(k \cdot u)}{\tau_{\text{eq}}} \Big[\mathcal{F}_{\text{eq}}(x, k) - \mathcal{F}(x, k) \Big]$
Dist. fn.	$\mathcal{F}^{\pm}(x,k) = 2m \int_{p,s} f^{\pm}(x,p,s) \delta^{(4)}(x)$
	$\int_{s}(\cdots) ightarrow ($

[S.B., W. Florkowski, A. Jaiswal, A. Kumar and, R. Ryblewski, PLB 814 (2021) 136096, PRD 103 (2021) 1, 014030]

Axial Component

$$k^{\mu}\partial_{\mu}\mathcal{A}^{\nu}(x,k) = \mathcal{C}^{\nu}_{\mathcal{A}}$$

)]
 $C^{\nu}_{\mathcal{A}} = \frac{(k \cdot u)}{\tau_{eq}} \left[\mathcal{A}^{\nu}_{eq}(x,k) - \mathcal{A}^{\nu}(x,k) \right]$
 $O(k \mp p) \left[\mathcal{A}^{\mu}_{\pm}(x,k) = 2m \int_{p,s} s^{\mu} f^{\pm}(x,p,s) \, \delta^{(4)}(k \mp p) \right]$

$$(m/\pi\mathfrak{s})\int d^4s\delta(s\cdot s+\mathfrak{s}^2)$$
 $\mathfrak{g}^2=\frac{1}{2}(1+\frac{1}{2})$

CLASSICAL APPROACH TO SPIN HYDRODYNAMICS

In the classical treatments of particles with spin-1/2 one introduces internal angular momentum tensor of particles [M. Mathisson, APPB 6 (1937) 163–2900]

$$s^{lphaeta} = rac{1}{m} \epsilon^{lphaeta\gamma\delta} p_{\gamma} s_{\delta}.$$

 $s^{\alpha\beta}$ is antisymmetric *i.e.* $s^{\alpha\beta} = -s^{\beta\alpha}$ and satisfies Frenkel (or Weyssenhoff) $p_{\alpha}s^{\alpha\beta}=0.$

The spin four vector can be obtained by above equation,

$$s^{lpha} = rac{1}{2m} \epsilon^{lpha eta \gamma \delta}$$

In particle rest frame (PRF) where $p^{\mu} = (m, 0, 0, 0)$, $s^{\alpha} = (0, \mathbf{s}_*)$ with the length of spin vector given by $-s^2 = -s^{\alpha}s_{\alpha} = |\mathbf{s}_*|^2 = \hat{\mathbf{s}}^2 = \frac{1}{2}(1+\frac{1}{2}) = \frac{3}{4}$.

$$p_eta s_{\gamma\delta}$$



M.Mathissor



J. Weyssenhoff

momentum of the particles

[Suttorp, de Groot, Nuovo Cimento A (1965-1970), van Weert Thesis (1970)]

$$p^{\mu}\partial_{\mu}^{(x)}f^{\pm} + m\mathcal{F}^{\mu}\partial_{\mu}^{(p)}f^{\pm} + m\mathcal{S}^{\mu\nu}\partial_{\mu\nu}^{(s)}f^{\pm} = \mathcal{C}[f^{\pm}]$$
$$\partial_{\mu}^{(x)} \equiv \frac{\partial}{\partial x^{\mu}}, \qquad \partial_{\mu}^{(p)} \equiv \frac{\partial}{\partial p^{\mu}}, \qquad \partial_{\mu\nu}^{(s)} \equiv \frac{\partial}{\partial s^{\mu\nu}}, \qquad \mathcal{F}^{\alpha} \equiv \frac{dp^{\alpha}}{d\tau} \qquad \mathcal{S}^{\alpha\beta} \equiv \frac{ds^{\alpha\beta}}{d\tau}$$

where

$$p^{\mu}\partial_{\mu}^{(x)}f^{\pm} + m\mathcal{F}^{\mu}\partial_{\mu}^{(p)}f^{\pm} + m\mathcal{S}^{\mu\nu}\partial_{\mu\nu}^{(s)}f^{\pm} = \mathcal{C}[f^{\pm}]$$
$$\partial_{\mu}^{(x)} \equiv \frac{\partial}{\partial x^{\mu}}, \qquad \partial_{\mu}^{(p)} \equiv \frac{\partial}{\partial p^{\mu}}, \qquad \partial_{\mu\nu}^{(s)} \equiv \frac{\partial}{\partial s^{\mu\nu}}, \qquad \mathcal{F}^{\alpha} \equiv \frac{dp^{\alpha}}{d\tau} \qquad \mathcal{S}^{\alpha\beta} \equiv \frac{ds^{\alpha\beta}}{d\tau}$$

Using the Frenkel condition, one can derive the force (Lorentz and Mathisson) and torque term [Suttorp, de Groot, Nuovo Cimento A (1965-1970), van Weert Thesis (1970), Nora W. et al. PRD 100 (2019) 5, 056018]

$$\mathcal{F}^{\alpha} = \frac{\mathfrak{q}}{m} F^{\alpha\beta} p_{\beta} + \frac{1}{2} \left(\partial^{\alpha} F^{\beta\gamma} \right) m_{\beta\gamma}$$

where magnetic dipole moment is

The distribution function in the extended phase-space is a function of spacetime, momentum, and internal angular

Following Liouville theorem the kinetic equation governing the evolution of the distribution function can be written as

$$\mathcal{S}^{\alpha\beta} = 2 F^{\gamma[\alpha} m^{\beta]}_{\ \gamma} - \frac{1}{m^2} \left(\chi - \frac{\mathfrak{q}}{m} \right) F_{\phi\gamma} s^{\phi[\alpha} p^{\beta]} p^{\gamma}$$

 $m^{\alpha\beta} = \chi \, s^{\alpha\beta}$



In the limit of **infinite conductivity**, field strength tensor is

 $F^{\mu\nu} \to B^{\mu\nu}$

 $u_{\mu}B^{\mu} = \mathbf{0}$

If the medium if magnetizable, then the **Maxwell's equations** are given by [Balakin, Grav.Cosmol. 13 (2007) 163-177; Hehl and, Obukhov, Phys. Lett. A 311, 277 (2003)]

> $\partial_{\mu}H^{\mu\nu} = J^{\nu},$ $\left(\widetilde{F}^{\mu\nu}=\right)$

$$\dot{} = \epsilon^{\mu\nu\alpha\beta} \, u_\alpha \, B_\beta$$

 $B_{\mu}B^{\mu} \leq 0$

$$\partial_{\mu}\widetilde{F}^{\mu\nu} = \mathbf{0},$$
$$= \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta} \Big)$$

 $H^{\mu\nu} = F^{\mu\nu} + M^{\mu\nu}$

The particle current, energy-momentum tensor, and spin tensor of the fluid can be expressed as [S.B., W. Florkowski, A. Jaiswal, A. Kumar and, R. Ryblewski, PLB 814 (2021) 136096, PRD 103, 014030 (2021)]

 $N^{\mu} = \int_{p,s} p^{\mu} \left(f^{+} - f^{-} \right),$ $T_f^{\mu\nu} = \int_{p,s} p^{\mu} p^{\nu} \left(f^+ + f^- \right),$ $S^{\lambda,\mu\nu} = \int_{n-s} p^{\lambda} s^{\mu\nu} \left(f^+ + f^- \right)$

while the **polarization-magnetization tensor** is

 $M^{\alpha\beta} = m$

$$\int_{p,s} m^{\alpha\beta} \left(f^+ - f^- \right)$$

Assuming that the microscopic interactions preserve conservation laws one requires

 $J_{p,s}$ $\int_{p,s}$ $J_{p,s}$

Zeroth, first, and 'spin' moment of the kinetic equation (in absence of the torque term) then lead to

 $\partial_{\nu}T_{\star}$

$$N^{\mu} = nu^{\mu} + n^{\mu} \qquad \qquad J^{\mu}_{\rm f} = \mathfrak{q} N^{\mu}$$

$$\mathcal{C}[f] = 0,$$

$$p^{\mu}\mathcal{C}[f] = 0,$$

$$s^{\mu\nu}\mathcal{C}\left[f
ight]=0$$

 $\partial_{\mu}N^{\mu} = \mathbf{0}$

$$f_{f}^{\mu\nu} = F^{\mu}_{\ \alpha}J^{\alpha}_{f} + \frac{1}{2}\left(\partial^{\mu}F^{\nu\alpha}\right)M_{\nu\alpha}$$

 $\partial_{\lambda} S^{\lambda,\mu\nu} = \mathbf{0}$

$$T_{\rm f}^{\mu\nu} = \epsilon u^{\mu}u^{\nu} - \left(P + \Pi\right)\Delta^{\mu\nu} + \pi^{\mu\nu}$$

Using RTA kinetic equation we can write the first-order gradient correction as

$$f^{\pm}(x, p, s) = f^{\pm}_{eq}(x, p, s) + \delta f^{\pm}(x, p, s).$$

The equilbrium distribution function has the form (we use the small polarization limit)

$$f_{\rm eq} = \frac{1}{1 + \exp\left[\beta(u \cdot p) - \xi - \frac{1}{2}\omega : s\right]}$$

$$\delta f_{(1)}^{\pm} = -\mathcal{D} f_{\text{eq}}^{\pm},$$
$$\mathcal{D} = \frac{\tau_R}{(u \cdot p)} \left(p^{\alpha} \frac{\partial}{\partial x^{\alpha}} + \mathcal{F}^{\alpha} \frac{\partial}{\partial p^{\alpha}} \right)$$

$$f_{eq} = f_0 + \frac{1}{2} (\omega : s) f_0 \tilde{f}_0,$$
$$f_0 \equiv \{1 + \exp \left[\beta(u \cdot p) - \xi\right]\}^{-1} \qquad \tilde{f}_0 \equiv 1 - f_0$$

$$\begin{split} \Pi &= -\frac{\Delta_{\alpha\beta}}{3} \int_{p,s} p^{\alpha} p^{\beta} \left(\delta f^{+} + \delta f^{-}\right) \\ \pi^{\mu\nu} &= \Delta_{\alpha\beta}^{\mu\nu} \int_{p,s} p^{\alpha} p^{\beta} \left(\delta f^{+} + \delta f^{-}\right) \\ n^{\mu} &= \Delta_{\alpha}^{\mu} \int_{p,s} p^{\alpha} \left(\delta f^{+} - \delta f^{-}\right) \\ \delta S^{\lambda,\mu\nu} &= \int_{p,s} p^{\lambda} s^{\mu\nu} \left(\delta f^{+} + \delta f^{-}\right) \end{split}$$

The expressions for dissipative currents in terms of the nonequilibrium correction to the distribution function are

Equilibrium polarization-magnetization tensor is

$$M_{\rm eq}^{\mu\nu} = a_1(T,\mu)$$

In global equilibrium, spin polarization tensor corresponds to rotation of the fluid.

$$\omega^{\mu
u}|_{
m geq} \propto \varpi^{\mu
u} = \left(\partial^{\mu}\beta^{
u} - \partial^{
u}\beta^{\mu}
ight)/2$$

We conclude that **rotation of the fluid produces magnetization**, which is precisely the physics of **Barnett effect**.

 $\omega^{\mu\nu} + a_2(T,\mu) u^{[\mu} u_{\gamma} \omega^{\nu]\gamma}$



figure: Journal of the Physical Society of Japan 90, 081003 (2021)

Using the spin matching condition we obtain the evolution equation for the spin polarization tensor

$$\dot{\omega}^{\mu\nu} = \mathcal{D}_{\Pi}^{\mu\nu} \theta + \mathcal{D}_{n}^{\mu\nu\gamma} (\nabla_{\gamma}\xi) + \mathcal{D}_{a}^{\mu\nu\gamma} \dot{u}_{\gamma} + \mathcal{D}_{\pi}^{\mu\nu\rho\kappa} \sigma_{\rho\kappa} + \mathcal{D}_{\Omega}^{\mu\nu\rho\kappa} \Omega_{\rho\kappa} + \mathcal{D}_{\Sigma}^{\mu\nu\phi\rho\kappa} (\nabla_{\phi}\omega_{\rho\kappa})$$

$$\Omega_{\mu\nu} \equiv (\partial_{\mu}u_{\nu} - \partial_{\nu}u_{\mu})/2$$

We observe that the above equation contains information about the **connection between** evolution of spin polarization tensor and fluid vorticity.

 $\mathcal{D}^{\mu\nu\rho\kappa}_{\Omega}$ vanishes in absence of electromagnetic field which leads us to conclude that the **conversion between spin-polarization and vorticity proceeds via coupling with electromagnetic field**.

$$\begin{split} X &= \tau_{\text{eq}} \Big[\beta_{X\Pi} \,\theta + \beta_{Xn}^{\alpha} \left(\nabla_{\alpha} \xi \right) + \beta_{Xa}^{\alpha} \dot{u}_{\alpha} + \beta_{X\pi}^{\alpha\beta} \sigma_{\alpha\beta} \\ &+ \beta_{X\Omega}^{\alpha\beta} \Omega_{\alpha\beta} + \beta_{XF}^{\alpha\beta} \left(\nabla_{\alpha} B_{\beta} \right) + \beta_{X\Sigma}^{\alpha\beta\gamma} \left(\nabla_{\alpha} \omega_{\beta\gamma} \right) \Big], \end{split}$$

where

 $X \equiv n^{\mu}, I$

Demanding that the divergence of the above entropy current is positive definite we identify which first-order gradient terms are dissipative

$$\Pi = -\zeta\theta, \quad n^{\mu} = \kappa^{\mu\alpha} \left(\nabla_{\alpha} \xi \right), \quad \pi^{\mu\nu} = \eta^{\mu\nu\alpha\beta} \sigma_{\alpha\beta},$$
$$\delta S^{\mu,\alpha\beta} = \Sigma^{\mu\alpha\beta\lambda\gamma\rho} \left(\nabla_{\lambda} \omega_{\gamma\rho} \right).$$

The expressions for **dissipative currents** in terms of the nonequilibrium corrections to the distribution function are

$$\Pi, \ \pi^{\mu\nu}, \ \delta S^{\lambda,\mu\nu}$$



with spin in the limit of small polarization.

We demonstrated that multiple transport coefficients, dissipative as well as non-dissipative, are present.

We showed that our framework naturally leads to the **emergence of the relativistic analog of Barnett effect**.

We show that the **coupling between the magnetic field and spin polarization appears at gradient order**.

CONCLUSIONS

- We presented the first kinetic theory formulation of **relativistic dissipative nonresistive magnetohydrodynamics**
- Simulation based on our unified framework has the potential of explaining the difference of Λ and anti- Λ polarization.

THANK YOU FOR YOUR ATTENTION.