

FEMTOSCOPY WITH LÉVY DISTRIBUTIONS FROM SPS TO LHC

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EXPLORING QGP WORKSHOP 2023, BELGRADE





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CONTENTS OF THIS TALK

- Basics of femtoscopy and Lévy sources
- First thorough Lévy HBT analysis in AA by PHENIX
- Recent phenomenological updates
- Recent experimental results
- Summary and outlook

LEVY HBT EXPERIMENT PHENOMENOLOGY NEW RESULTS



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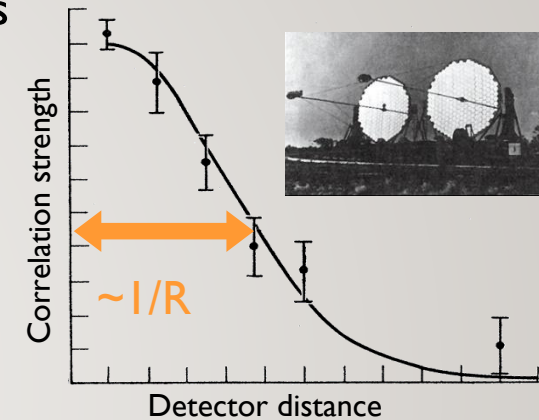
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FEMTOSCOPY IN HIGH ENERGY PHYSICS

- R. Hanbury Brown, R. Q. Twiss - observing Sirius with radio telescopes
 - Intensity correlations vs detector distance \Rightarrow source size
 - Measure the sizes of apparently point-like sources!
- Goldhaber et al: applicable in high energy physics
- Understanding: Glauber, Fano, Baym, ...
Phys. Rev. Lett. 10, 84; Rev. Mod. Phys. 78 1267, ...



- Momentum correlation $C(q)$ related to source $S(r)$

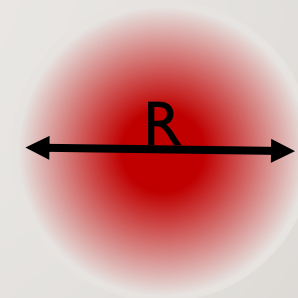
$$C(q) \cong 1 + \left| \int S(r) e^{iqr} dr \right|^2$$

(under some assumptions)

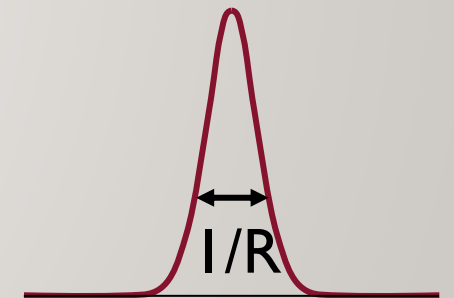
- Also with distance distribution $D(r)$:

$$C(q) \cong 1 + \int D(r) e^{iqr} dr$$

- Neglected: pair reconstruction, final state interactions, multi-particle correlations, coherence, ...



source function $S(r)$



correlation funct. $C(q)$

- Only way to map out source space-time geometry on femtometer scale!



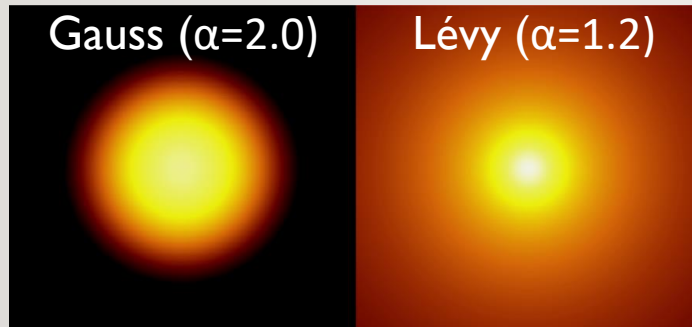
LÉVY DISTRIBUTIONS IN HEAVY ION PHYSICS

- Central limit theorem (**diffusion**) and thermodynamics lead to Gaussians
- Measurements suggest phenomena beyond Gaussian distribution

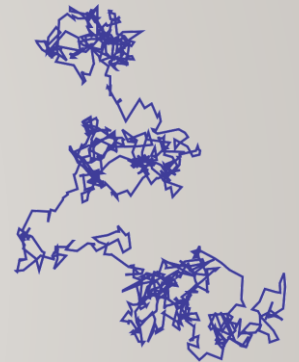
• Lévy-stable distribution:

$$\mathcal{L}(\alpha, R; r) = (2\pi)^{-3} \int d^3q e^{iqr} e^{-\frac{1}{2}|qR|^\alpha}$$

- From generalized central limit theorem, power-law tail $\sim r^{-(1+\alpha)}$
- Special cases: $\alpha = 2$ Gaussian, $\alpha = 1$ Cauchy



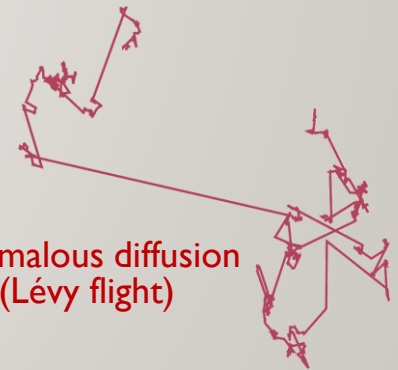
Normal diffusion



• Shape of the correlation functions with Lévy source:

- $C_2(q) = 1 + \lambda \cdot e^{-|qR|^\alpha}$; $\alpha = 2$: Gaussian; $\alpha = 1$: exponential
Csörgő, Hegyi, Zajc, Eur.Phys.J. C36 (2004) 67-78

Anomalous diffusion (Lévy flight)



• A possible reason for Lévy source: **anomalous diffusion**, many others





WHY DOES LÉVY APPEAR, WHY IS IT IMPORTANT?

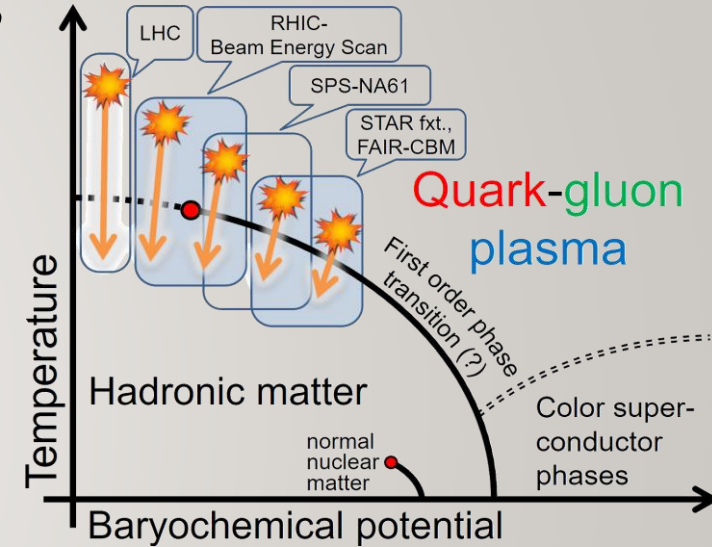
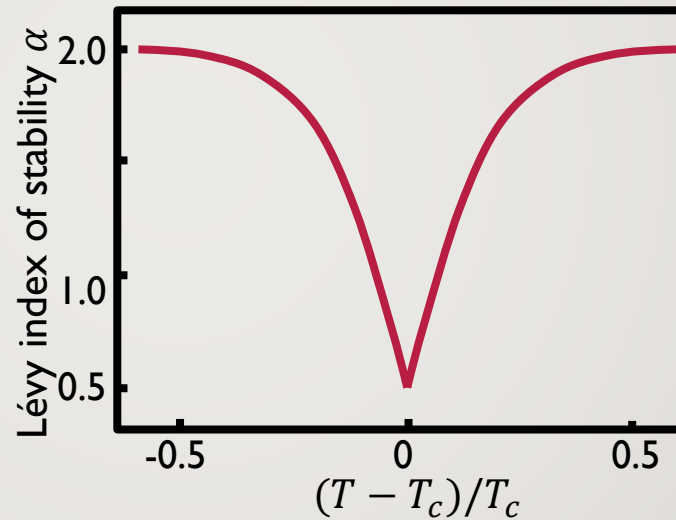
- A more comprehensive list of possible reasons:
 - Jet fragmentation (Csörgő, Hegyi, Novák, Zajc, Acta Phys.Polon. B36 (2005) 329-337)
 - See talk by Yacine M-T. yesterday
 - Critical phenomena (Csörgő, Hegyi, Novák, Zajc, AIP Conf.Proc. 828 (2006) no.1, 525-532)
 - Direction averaging and non-sphericity (Cimerman et al., Phys.Part.Nucl. 51 (2020) 282)
 - Event averaging (Cimerman et al., Phys.Part.Nucl. 51 (2020) 282)
 - Resonance decays (Csanád, Csörgő, Nagy, Braz.J.Phys. 37 (2007) 1002; Kincses, Stefaniak, Csanád, Entropy 24 (2022) 308)
 - Hadronic rescattering, Lévy flight (Braz.J.Phys. 37 (2007) 1002; Entropy 24 (2022) 308)
- Importance of utilizing Lévy sources:
 - Measuring α and R
 - Order of quark-hadron transition, critical point search, understanding source dynamics
 - Measuring λ also requires correct shape assumption
 - In-medium mass modification, coherent pion production





LÉVY INDEX AS A CRITICAL EXPONENT?

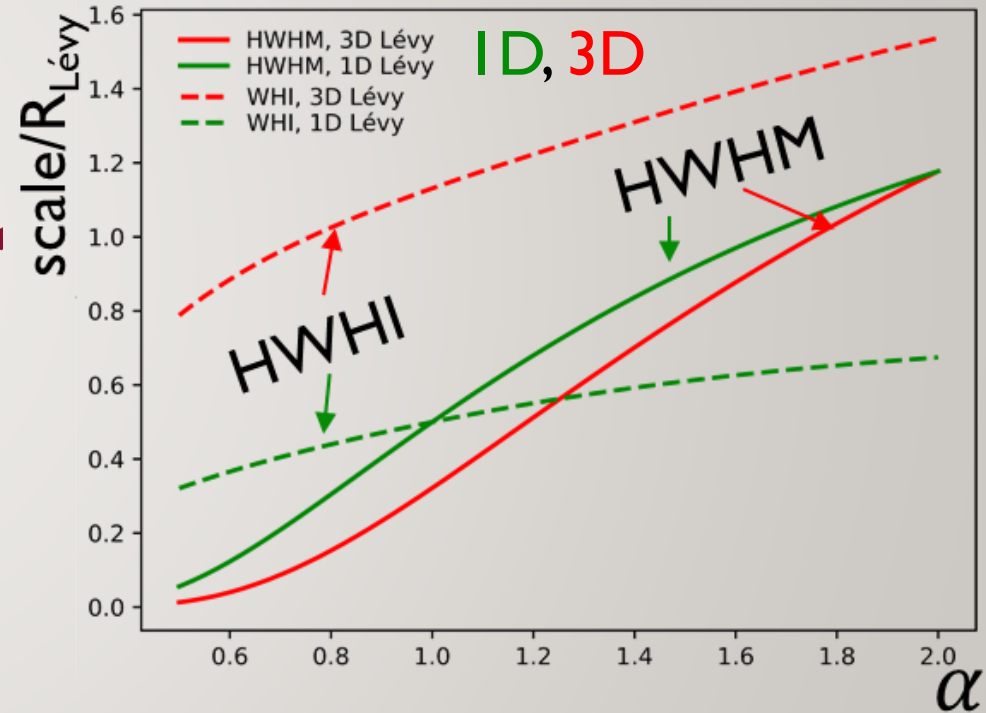
- Critical spatial correlation: $\sim r^{-(d-2+\eta)}$; Lévy source: $\sim r^{-(1+\alpha)}$; $\alpha \Leftrightarrow \eta?$
Csörgő, Hegyi, Zajc, Eur.Phys.J. C36 (2004) 67
- QCD universality class \leftrightarrow 3D Ising
Halasz et al., Phys.Rev.D58 (1998) 096007
Stephanov et al., Phys.Rev.Lett.81 (1998) 4816
- At the critical point:
 - Random field 3D Ising: $\eta = 0.50 \pm 0.05$
Rieger, Phys.Rev.B52 (1995) 6659
 - 3D Ising: $\eta = 0.03631(3)$
El-Showk et al., J.Stat.Phys.157 (4-5): 869
- Motivation for precise Lévy HBT!
- Change in $\alpha_{\text{Lévy}}$ proximity of CEP?
- Finite size/time & non-equilibrium effects \rightarrow what does power-law tail mean?





WHAT IS THE TRUE SIZE OF THE SOURCE?

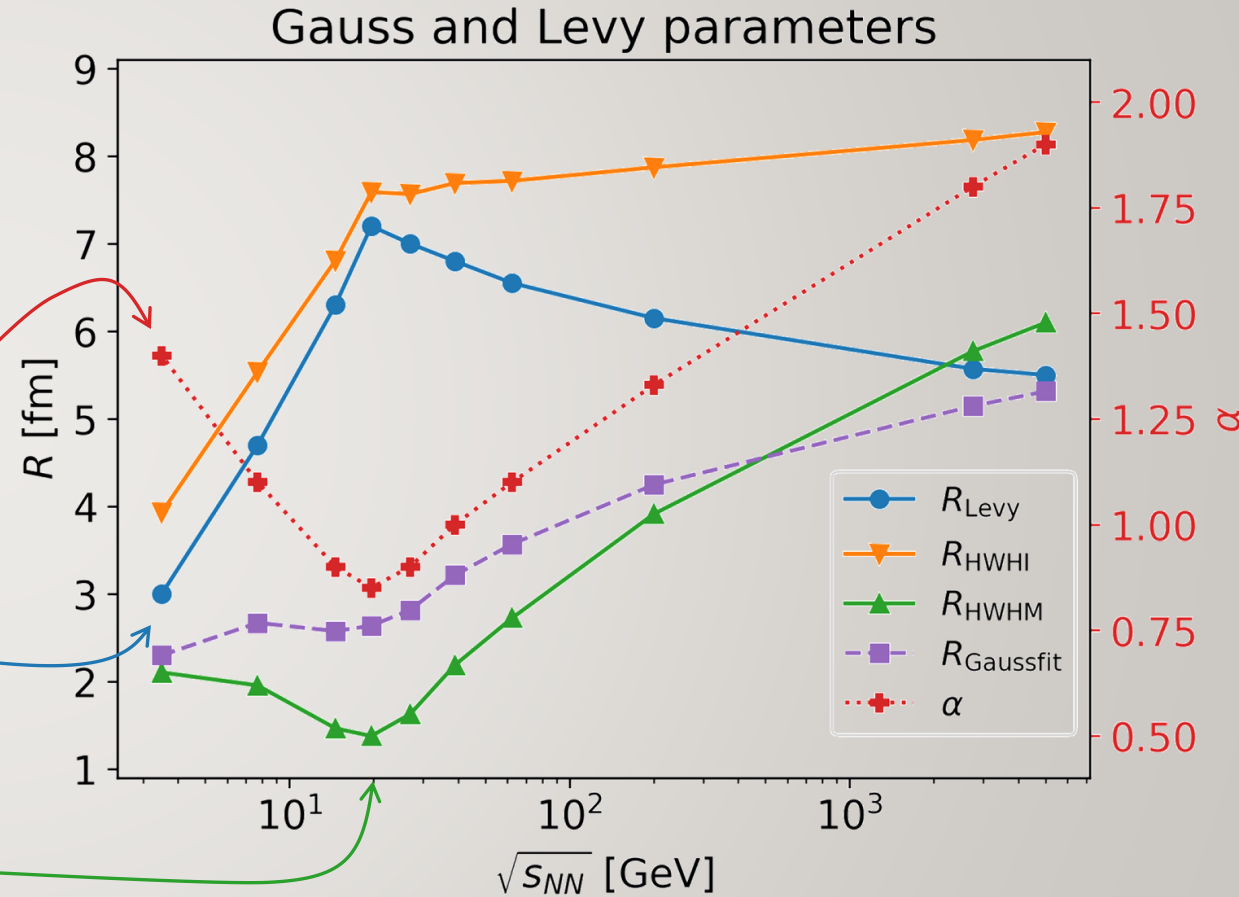
- No tail if $\alpha = 2$, power law if $\alpha < 2$; tail depends on α
- If $S(r)$ Lévy, $D(r)$ Lévy with same α and $R \rightarrow 2^{1/\alpha} R$
- In principle, $RMS = \infty$ if $\alpha < 2$, practice: depends on cutoff
- What do Gaussian HBT radii mean?
- Alternative measures:
 - Width at half integral
 - HWHI, on the plot: - - - - -
 - Width at half max
 - HWHM, on the plot: ———
 - Scale (normalized by R) nontrivially depends on α





SOURCE SIZE MEASURES AROUND THE CRITICAL POINT?

- Lévy source parameters: R_{Levy}, α
- R_{Gaussfit} : $C(Q; R_{\text{Levy}}, \alpha)$ fitted with $\alpha = 2$ fixed
- R_{HWHM} : half width at half maximum
- R_{HWHI} : half width at half integral
- **Simulated scenario:**
 - minimum in α vs. S_{NN}
 - maximum in R_{Levy} vs. S_{NN}
- **Observation:**
 - R_{Gaussfit} : approximately monotonic increase
 - minimum in R_{HWHM} !
 - Trend change in R_{HWHI} !





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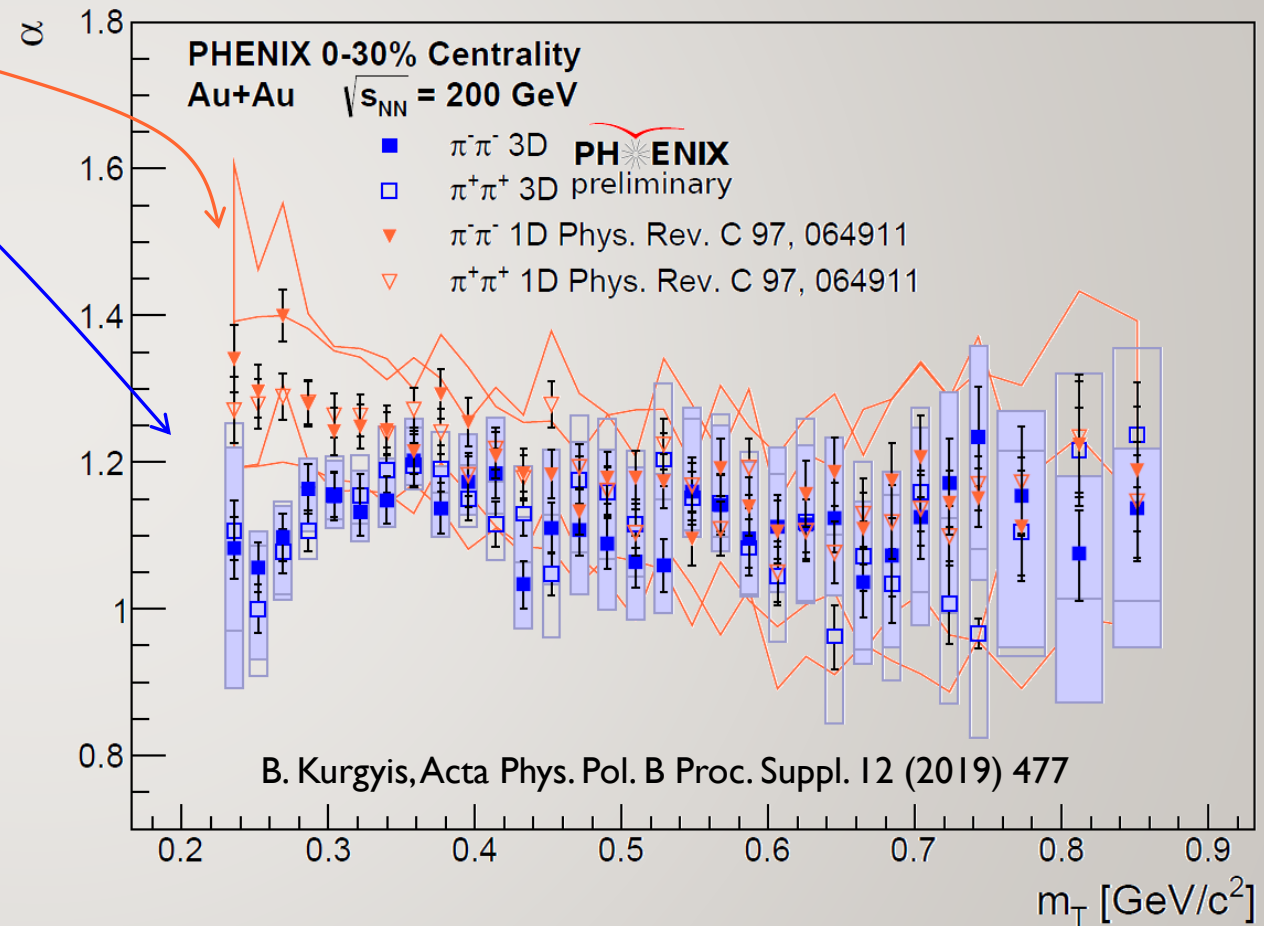
LEVY HBT EXPERIMENT PHENOMENOLOGY NEW RESULTS



LÉVY EXPONENT VERSUS TRANSVERSE MASS, 1D AND 3D

- Lévy exponent α in 3D close to 1D result
- On average still far from 2
- Observable differences at low m_T
- Maybe due to lack of spherical symmetry?
- Coulomb effect for non-spherical sources?
 - Approximation possible
B. Kurgyis et al., arXiv:2007.10173
 - If spherical in LCMS, radius in PCMS:

$$R_{PCMS} = \sqrt{\frac{1-2\beta_T^2/3}{1-\beta_T^2}} \cdot R_{LCMS}$$

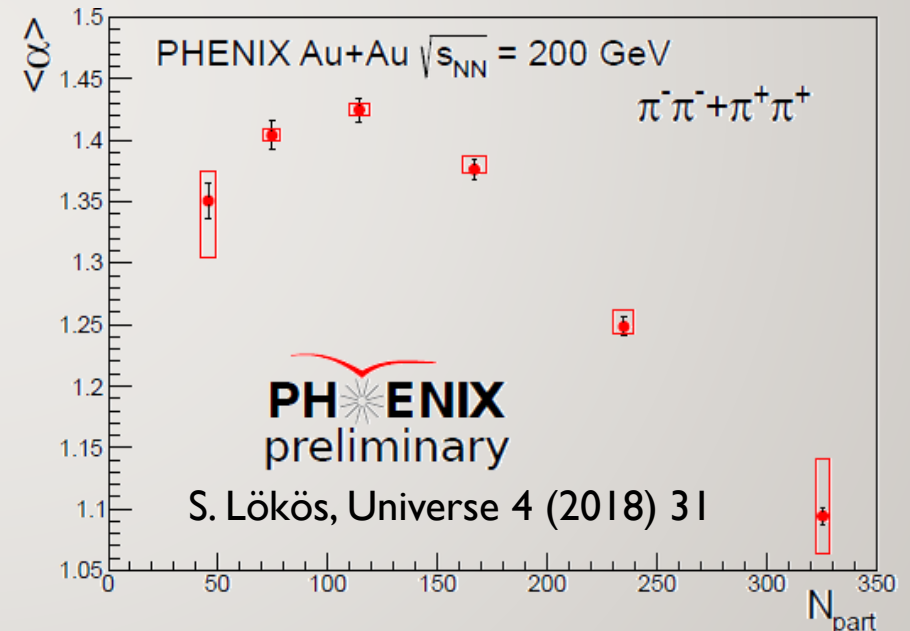
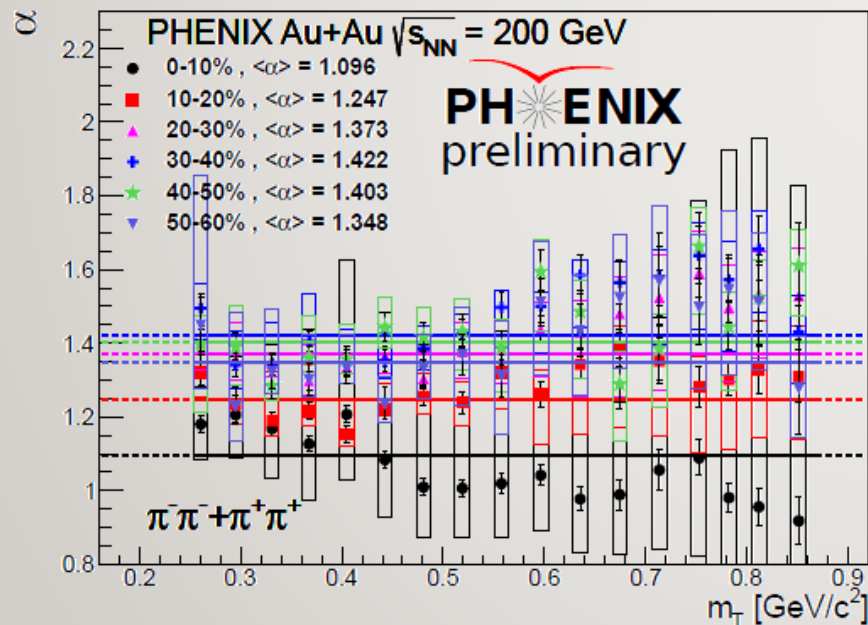




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ANALYZING THE CENTRALITY DEPENDENCE

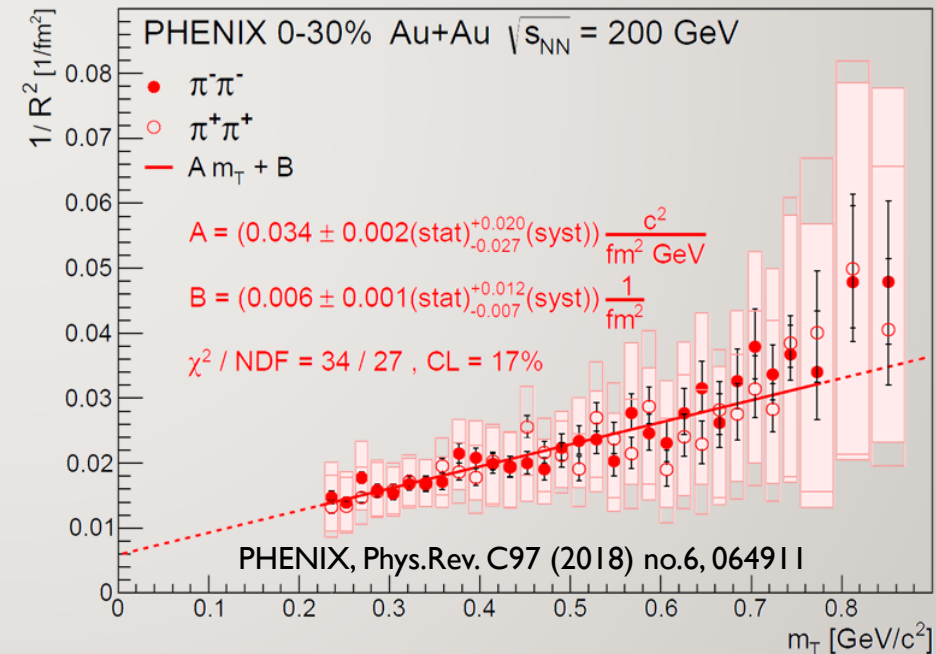
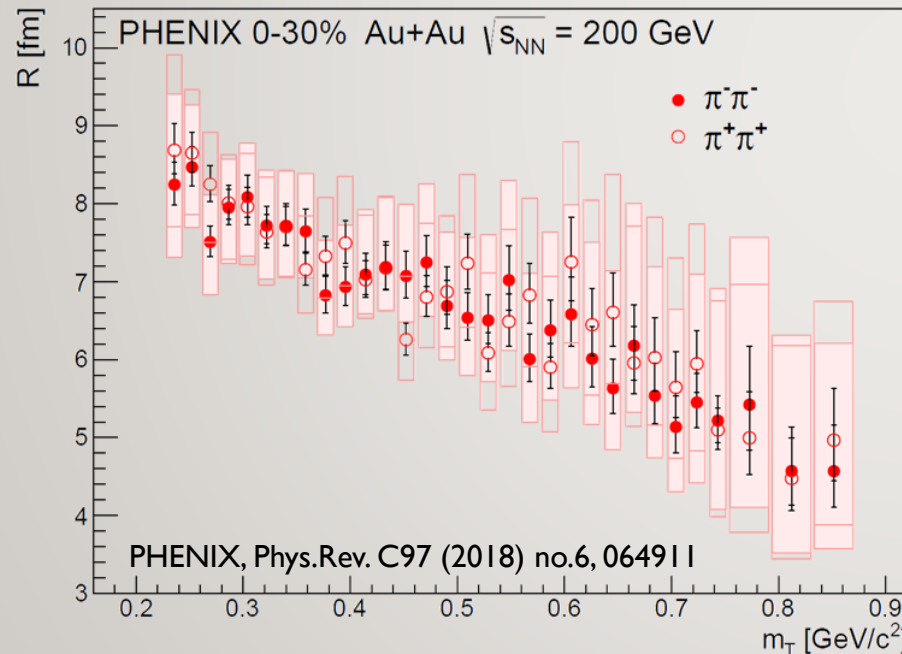
- α vs m_T : Slightly non-monotonic, averaging still possible
- $\langle \alpha \rangle$ vs N_{part} : Slightly non-monotonic, strong decreasing for large N_{part}
- No clear interpretation or understanding of this trend, need theory comparison
- Final data and publication in under collaboration review in PHENIX





LÉVY SCALE PARAMETER R AT RHIC

- Similar decreasing trend as Gaussian HBT radii, but it is not an RMS!
 - RMS of a Lévy source: in principle infinity, obtained value depends on cutoff
- What do model calculations, simulations say about this?
- Hydro behavior ($1/R^2 \sim m_T$) not invalid; but: **predicted for Gaussian case only!**

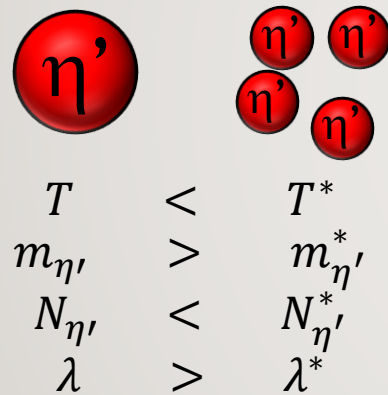




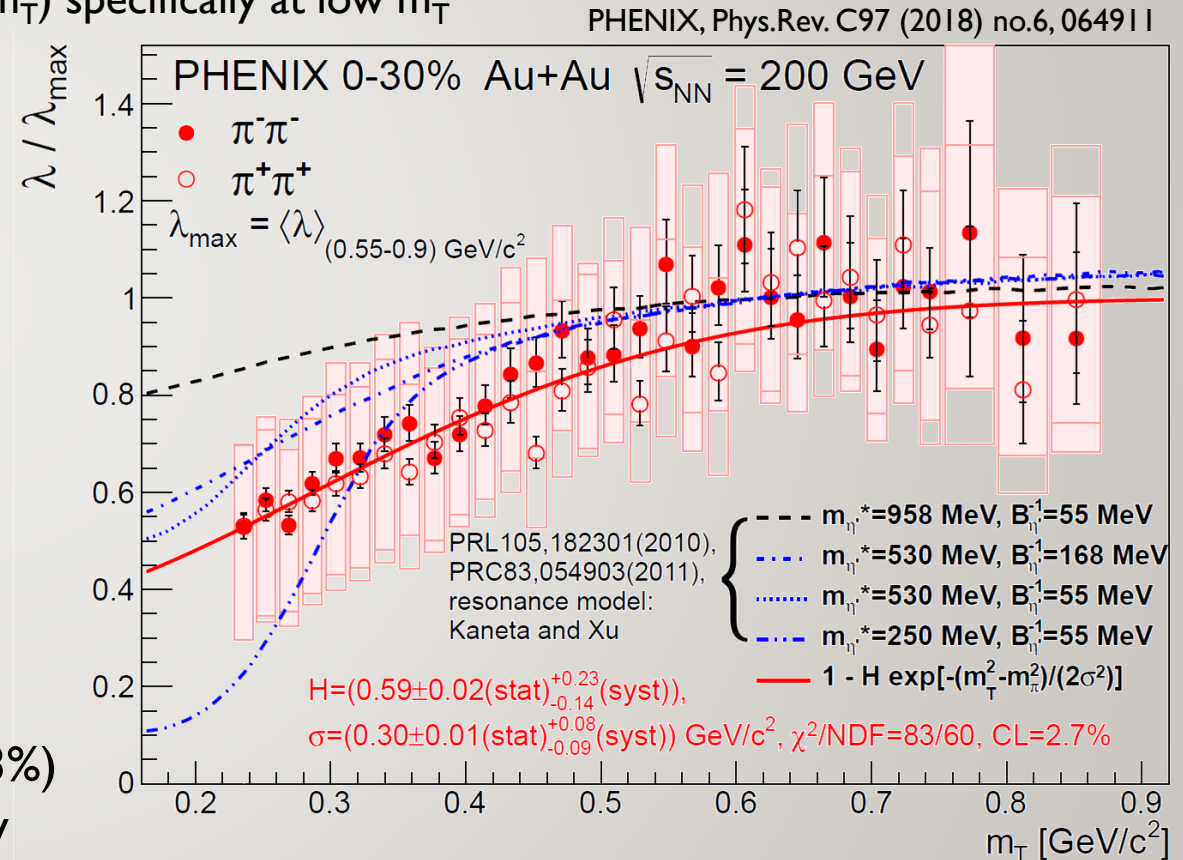
CORRELATION STRENGTH λ : IN-MEDIUM MASS?

- Connection to chiral restoration
 - Decreased η' mass \rightarrow more η' produced \rightarrow more decay pions $\rightarrow \lambda$ decreases
 - Kinematics: $\eta' \rightarrow \pi\pi\pi\pi$ with low $m_T \rightarrow$ decreased $\lambda(m_T)$ specifically at low m_T
 - Dependence on in-medium η' mass?

Kapusta, Kharzeev, McLerran, PRD53 (1996) 5028
 Vance, Csörgő, Kharzeev, PRL 81 (1998) 2205
 Csörgő, Vértesi, Sziklai, PRL105 (2010) 182301



- Results not incompatible with this
- Recall: 3D results similar to 1D
- Would need direct check with photons ($\eta' \rightarrow \gamma\gamma$, 2.3%)
- Centrality dependent analysis in collaboration review





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EVENT BY EVENT SHAPE ANALYSIS WITH EPOS

- EPOS model: parton-based Gribov-Regge theory (PBGRT)
 - K.Werner et al., PRC82 (2010) 044904, PRC89 (2014) 064903, ...
 - Core-Corona division, viscous hydro evolution (vHLLE), hadronic cascades (UrQMD)
- $\sqrt{s_{NN}} = 200$ GeV Au+Au collisions generated by EPOS359

- Pair distribution calculated: $D(\mathbf{r}_{LCMS}) = \int d\Omega dt D(t, \mathbf{r}_x, \mathbf{r}_y, \mathbf{r}_z)$
angle-averaged radial source distribution of like-sign pion pairs

$$r_{LCMS} = \sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z_{LCMS})^2}; \Delta z_{LCMS} = \Delta z - \frac{\beta(\Delta t)}{\sqrt{1 - \beta^2}}; \beta = \frac{p_{z,1} + p_{z,2}}{E_1 + E_2}$$

- Investigated cases:
 - CORE, primordial pions only
 - CORE, decay products included
 - CORE+CORONA+UrQMD, primordial pions only
 - CORE+CORONA+UrQMD, decay products included

Kincses, Stefaniak, Csanád, Entropy 24 (2022) 308 [arXiv:2201.07962]





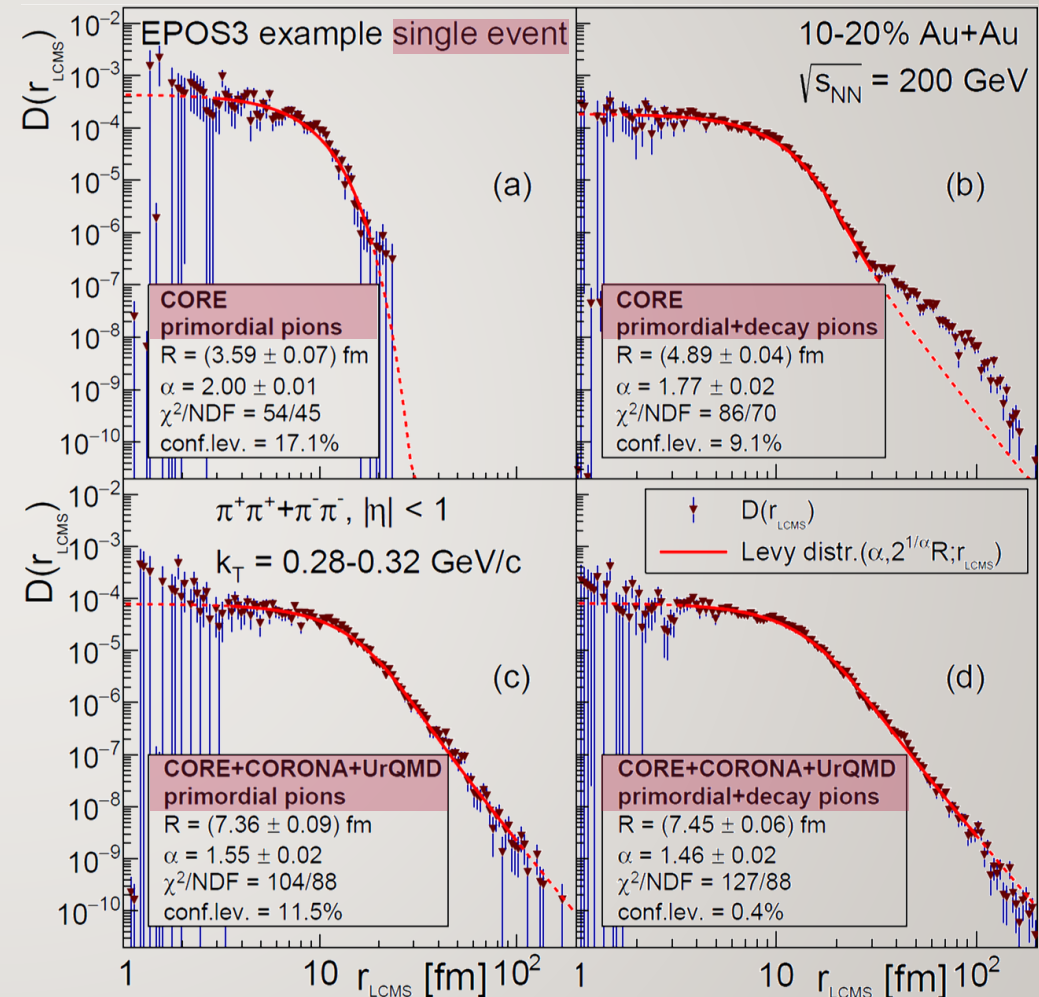
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VARIOUS PARTICLE SETS COMPARED

- CORE, primordial pions
 - Gaussian source
- CORE + decays
 - power-law structures
- CORE+CORONA+UrQMD
 - Lévy-shape
- CORE+CORONA+UrQMD+decays
 - Lévy-shape
- Important: Lévy appears in all single events!

Kincses et al., Entropy 24 (2022) 308

New 2.76 TeV PbPb analysis: arXiv:2212.02980

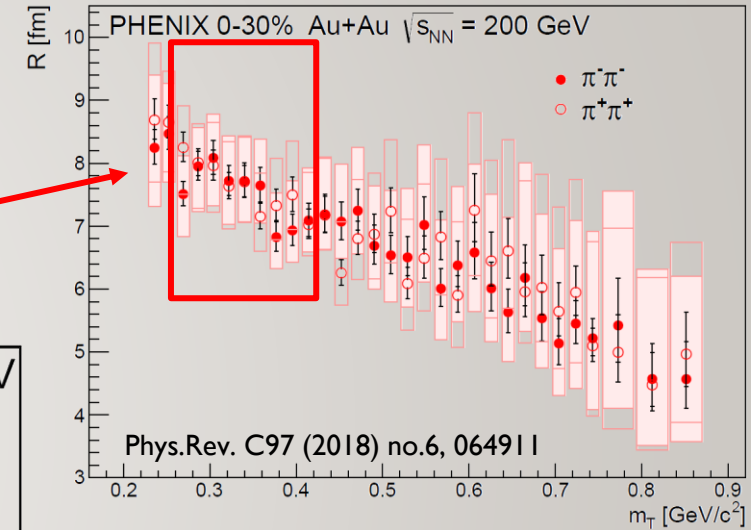
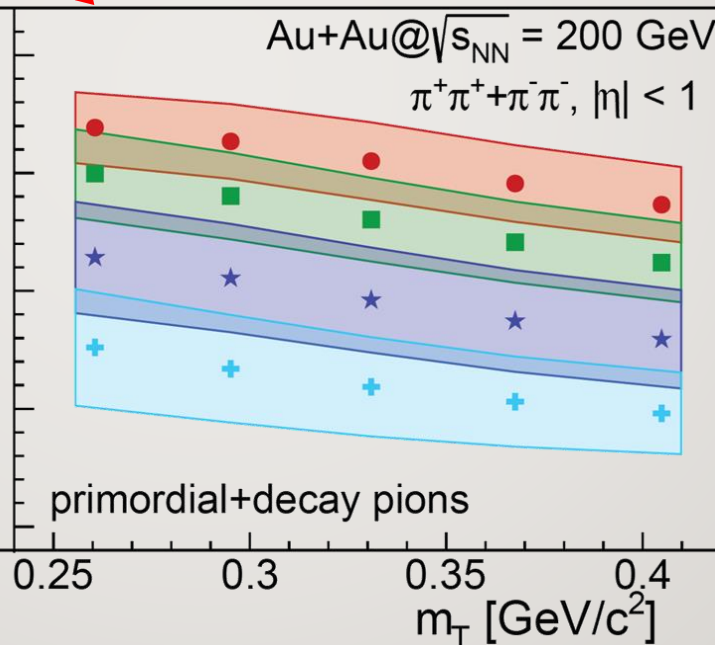
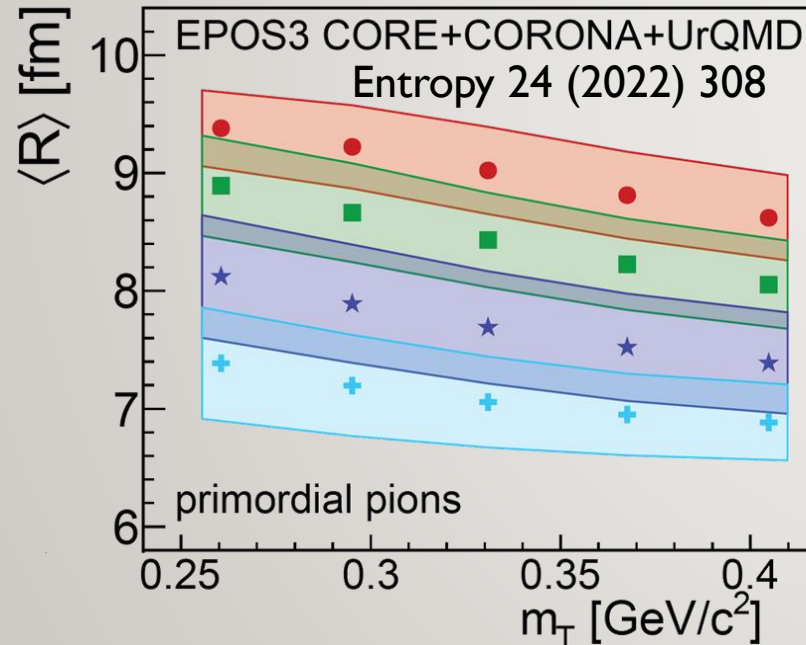


LEVY HBT EXPERIMENT PHENOMENOLOGY NEW RESULTS



AVERAGE LÉVY SCALE R VS TRANSVERSE MASS

- $\langle R \rangle$ as a function of m_T and centrality
 - Clear dependence on both
 - Distribution width displayed as uncertainty band
- Trends, magnitudes like data
- With decays: slightly higher $\langle R \rangle$ values

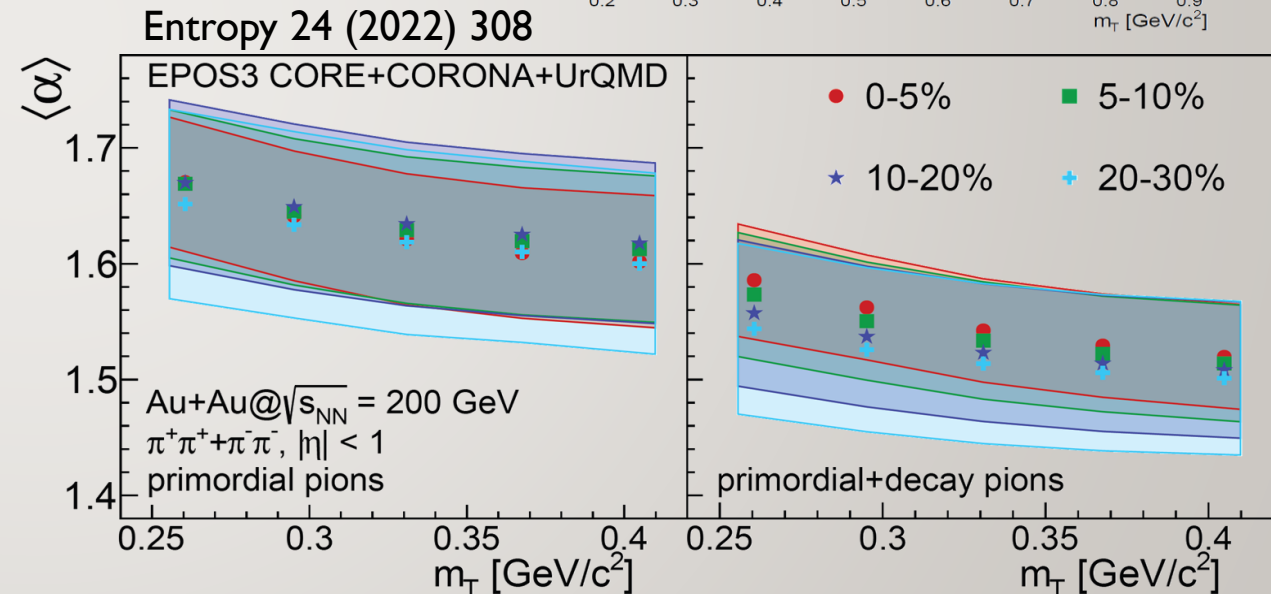
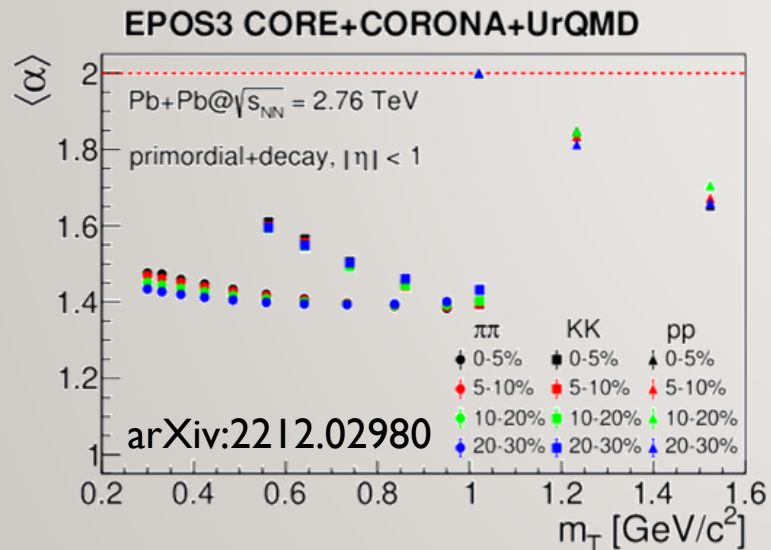
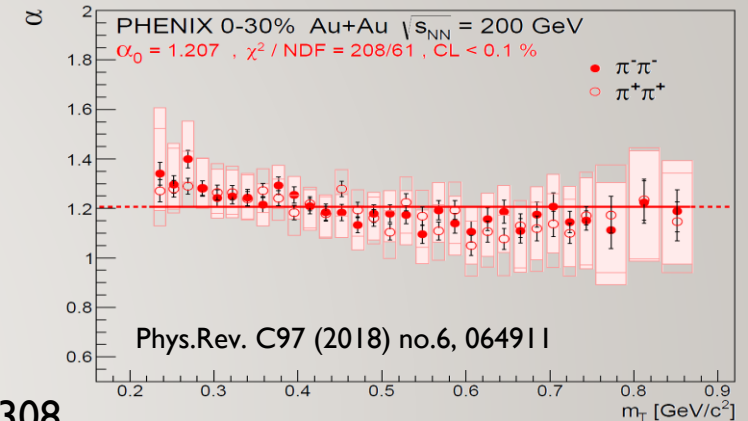


- 0-5%
- 5-10%
- ★ 10-20%
- + 20-30%



AVERAGE LÉVY EXPONENT VS TRANSVERSE MASS

- $\langle \alpha \rangle$ as a function of m_T and centrality
 - Small m_T and centrality dependence
- Both with and without decays: larger than data
- New manuscript at 2.76 TeV [2212.02980]: particle type dependence as well





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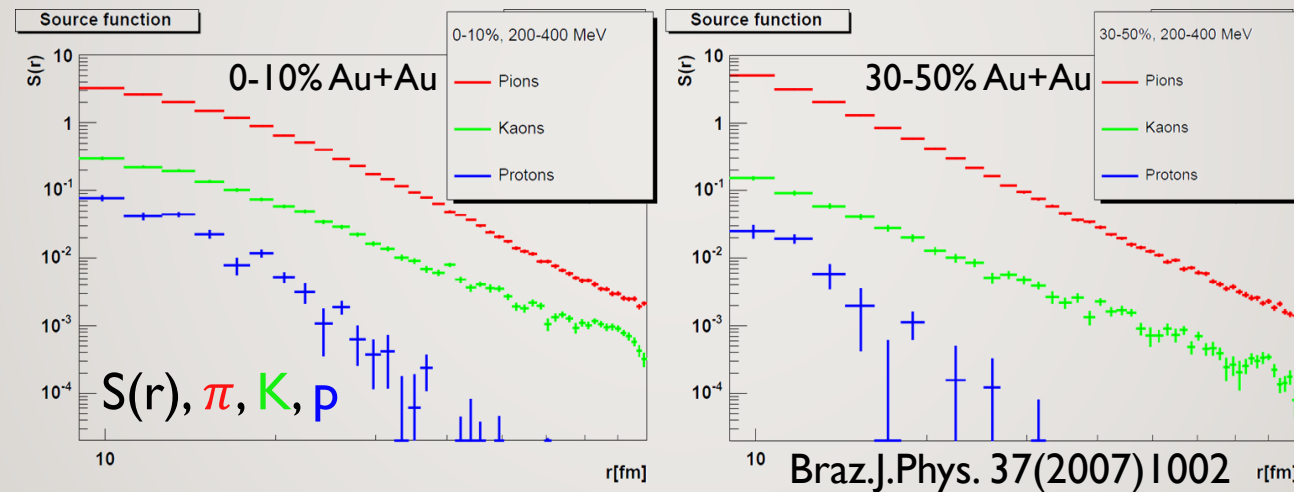
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THE IMPORTANCE OF A KAON ANALYSIS

- Kaons: smaller cross-section, larger mean free path
- Mean free path increases more during a time-step \rightarrow heavier power-law tail?
- Prediction for π , K , p based on Humanic's Resonance Model (HRM): anomalous diffusion due to rescattering
Humanic, Int.J.Mod.Phys. E15 (2006) 197 [nucl-th/0510049]
Csanád, Csörgő, Nagy, Braz.J.Phys. 37 (2007) 1002 [hep-ph/0702032]



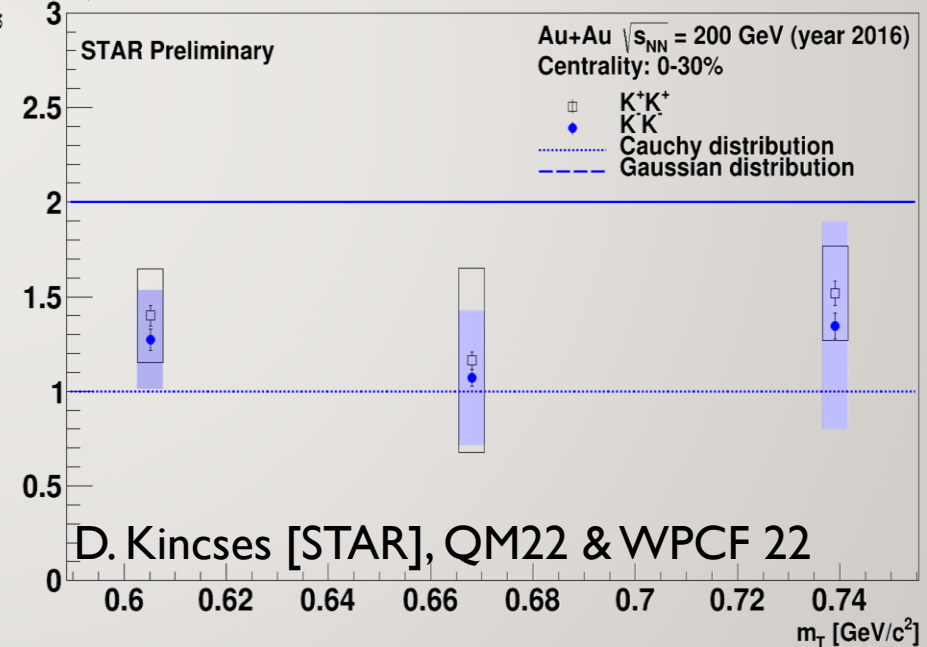
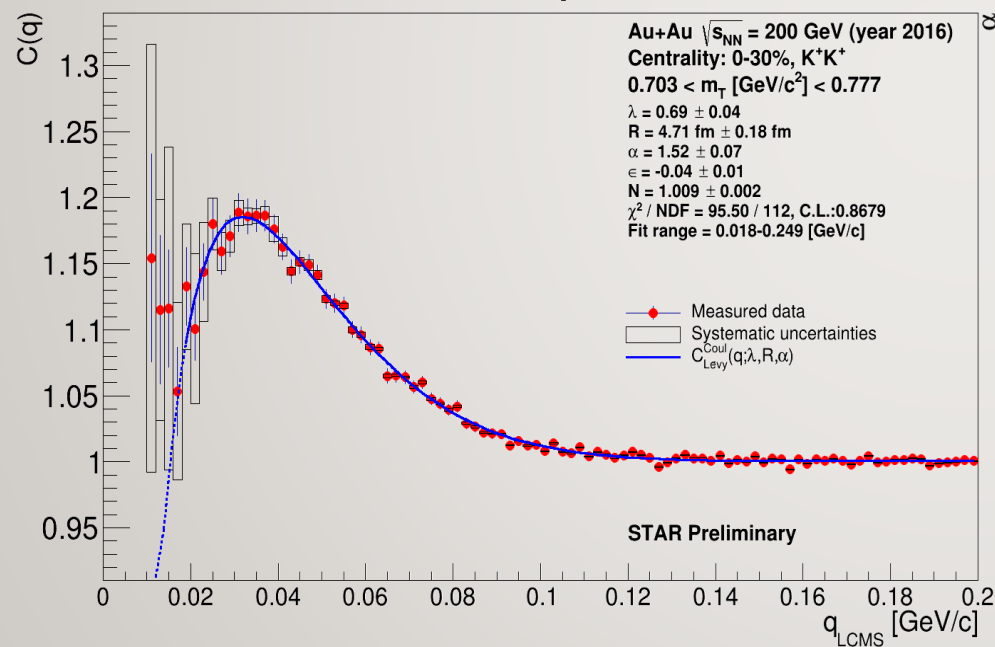
- Kaon HBT radii: m_T scaling or its violation for Lévy scale R ?
- Prediction: $\alpha(p) > \alpha(\pi) > \alpha(K)$



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KAON ANALYSIS AT STAR

- Data successfully described by Lévy fits
- Lévy-stability parameter α between 1 and 2
- Kaon and pion source of same shape at the same m_T ?
- Unlike anomalous diffusion expectation of $\alpha(K) < \alpha(\pi)$

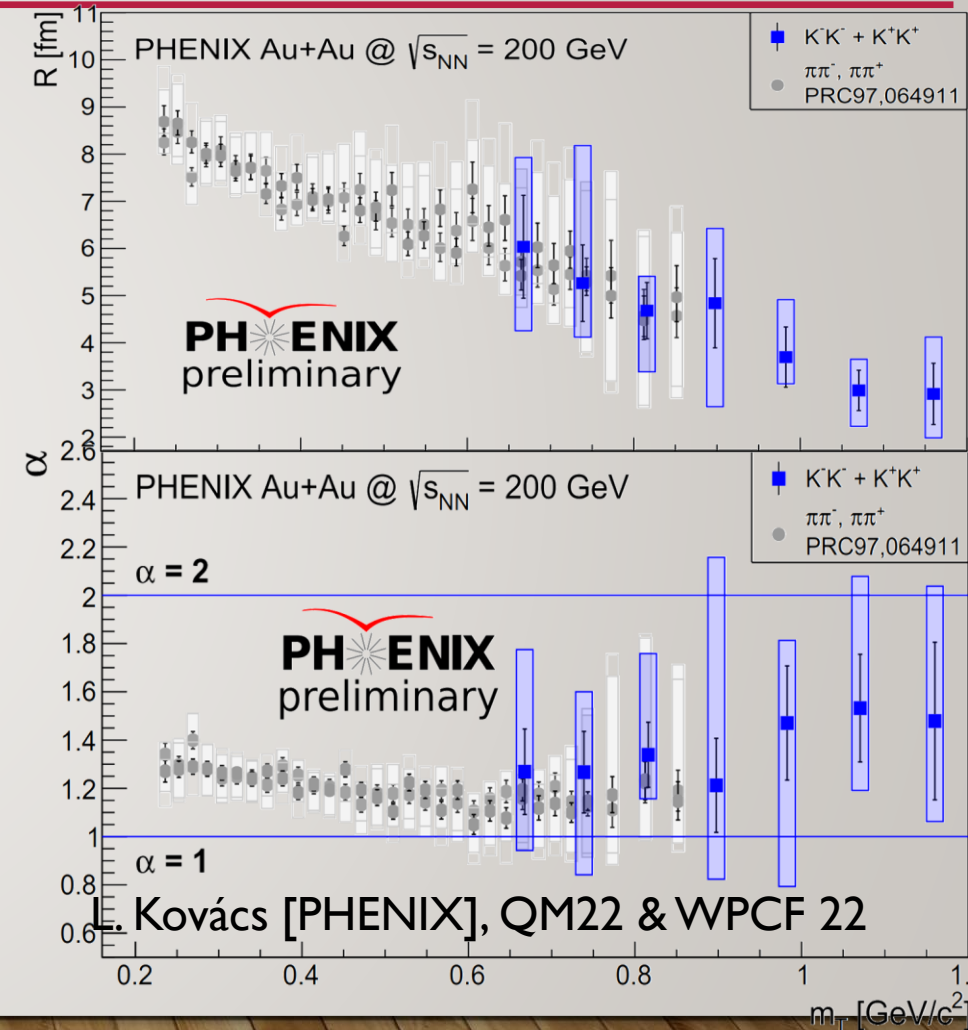


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KAON ANALYSIS AT PHENIX

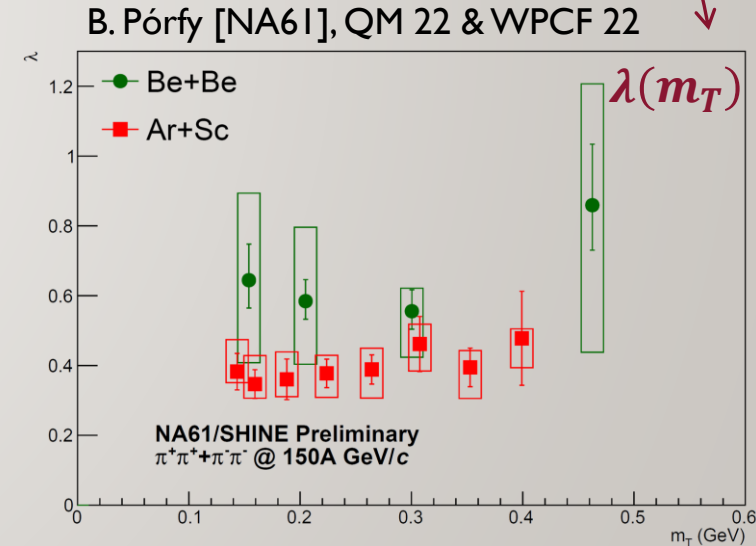
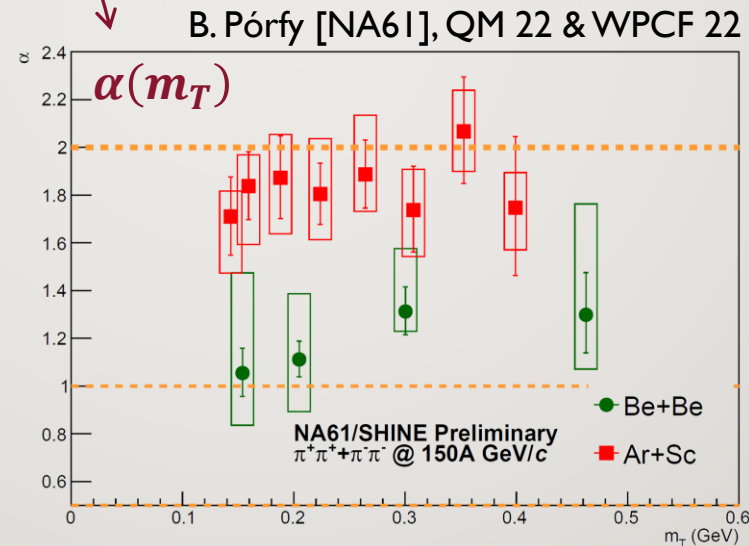
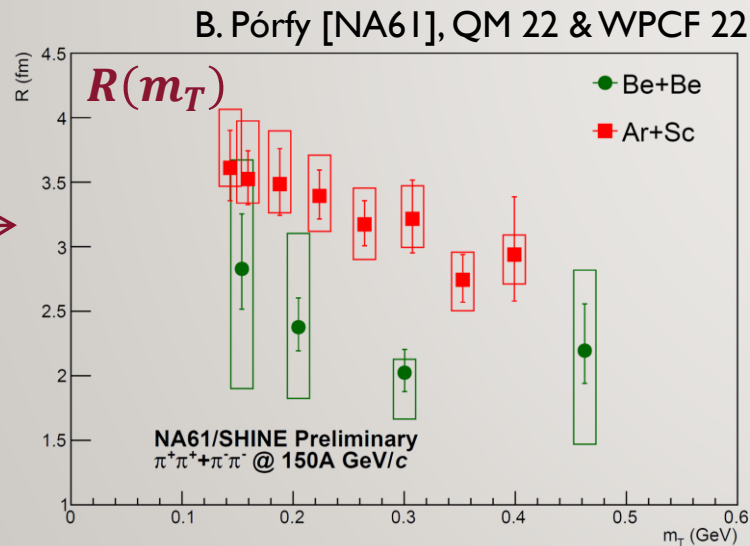
- More detailed analysis performed at PHENIX
- Kaon and pion data seem compatible at the same m_T
- Lévy scale R shows hydro type of scaling with m_T
 - R depending on m_T but not on particle type separately
- $\alpha(K) \geq \alpha(\pi)$, but anomalous diffusion suggests opposite
- Dominant mechanism creating Lévy source?
 - Not only rescattering?
 - Anomalous hydro at the sQGP stage?





PION ANALYSIS AT SPS NA61/SHINE

- Lévy scale R of Ar+Sc [preliminary for QM22] and Be+Be [final publication, arXiv:2302.04593]:
 - Compatible with initial geometry factor 1.6 between Ar+Sc and Be+Be
 - Decrease with m_T due to transverse flow?
- No m_T dependence in λ , in contrast to RHIC result – can be turned off?
- Lévy index α : significant difference

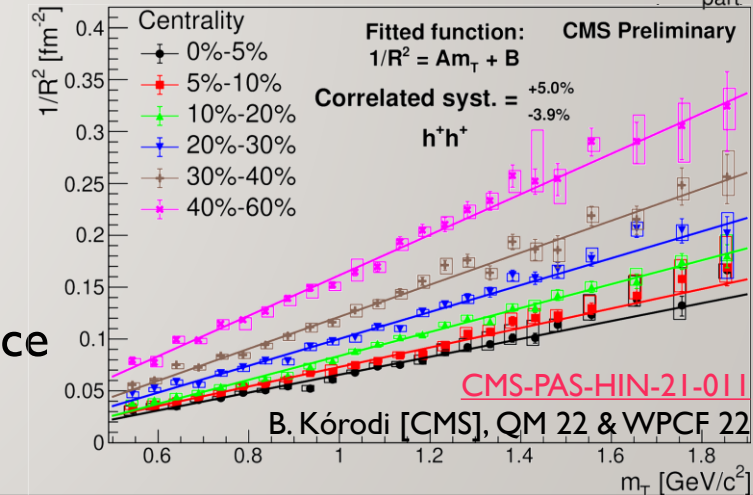
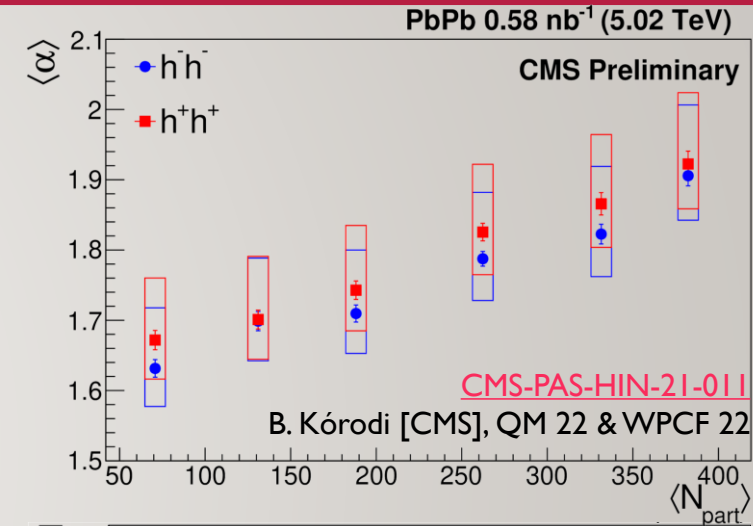




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CHARGED HADRON ANALYSIS IN 5 TEV PB+PB

- Lévy index α measured:
 - Far from Cauchy
 - Not exactly Gaussian
 - Closer to Gaussian for large N_{part} , **unlike RHIC**
- Lévy scale R : hydro scaling confirmed
 - In every centrality class
 - Despite non-Gaussianity
 - Hubble coefficient can be extracted: 0.12-0.18 c/fm , smaller than at RHIC
- Correlation strength λ also analyzed
- Low- Q deviation cross-checked with Monte-Carlo: two-track acceptance
- CMS result about to be final

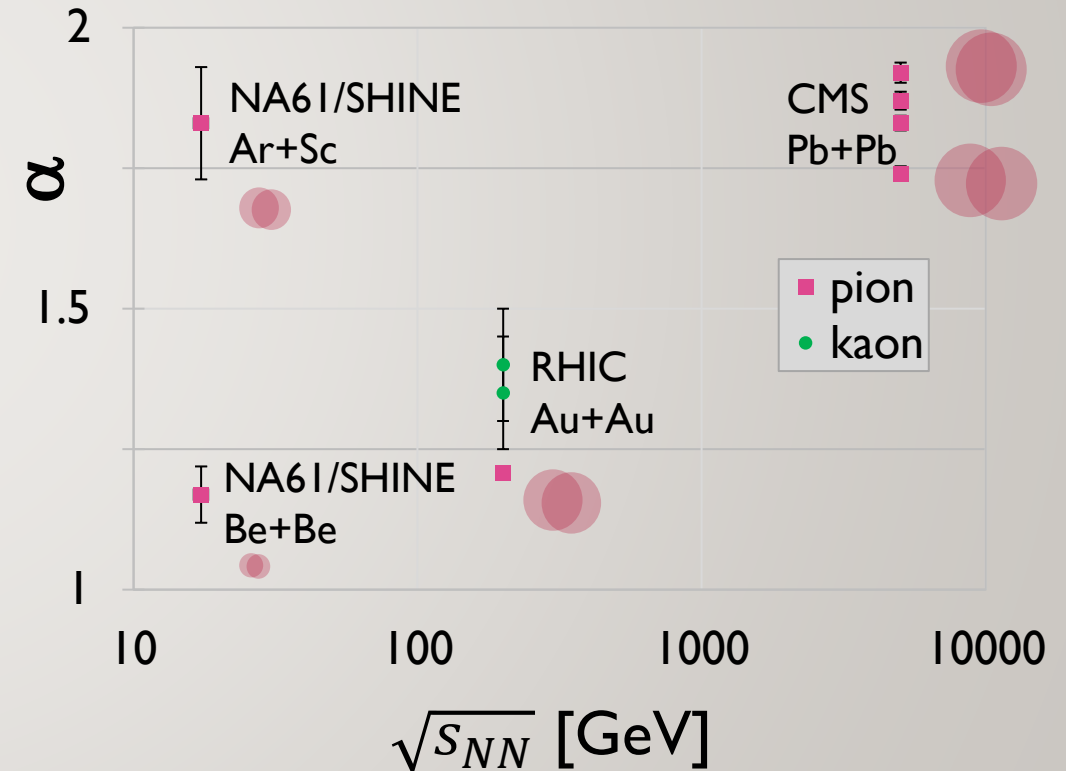
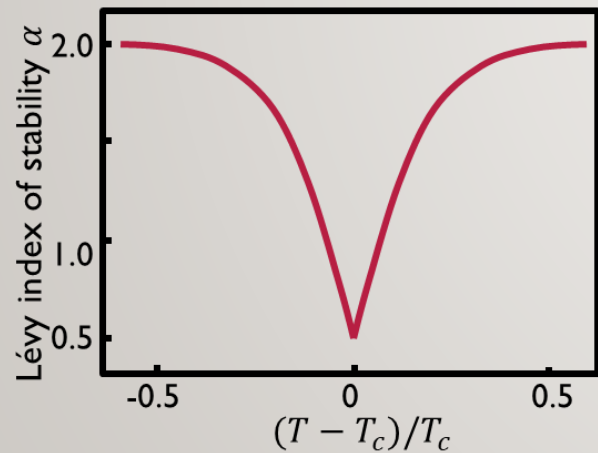


LEVY HBT EXPERIMENT PHENOMENOLOGY NEW RESULTS



STABILITY PARAMETER α FROM SPS TO LHC

- Different values for small (Be+Be) & medium (Ar+Sc) systems at SPS
 - Also true for PbPb and pp at LHC? (pp: $\alpha = 1$ assumed so far)
- Medium and large systems: non-monotonic trend
- Compare to expectation cartoon based on Csörgő, Hegyi, Zajc, Eur.Phys.J. C36 (2004) 67





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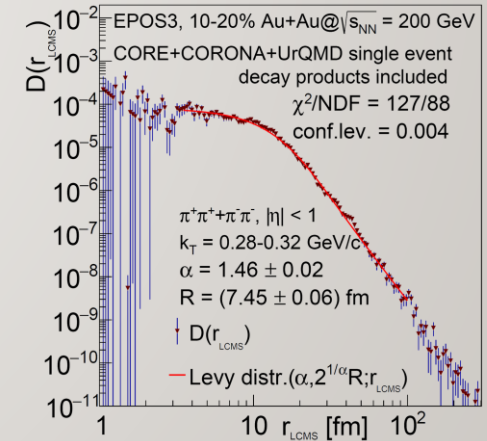
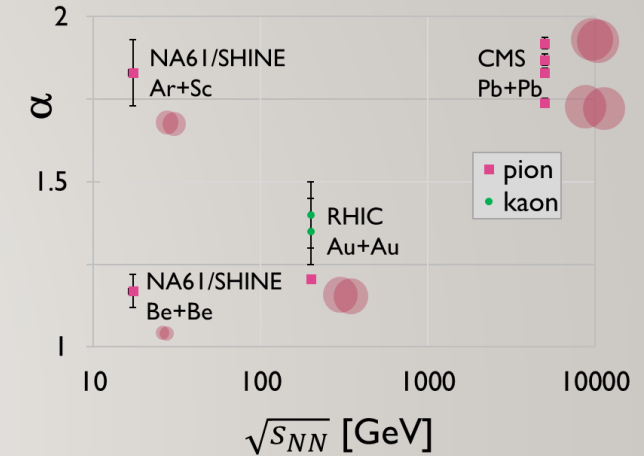
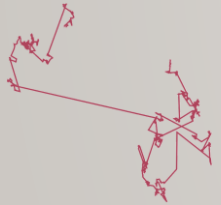
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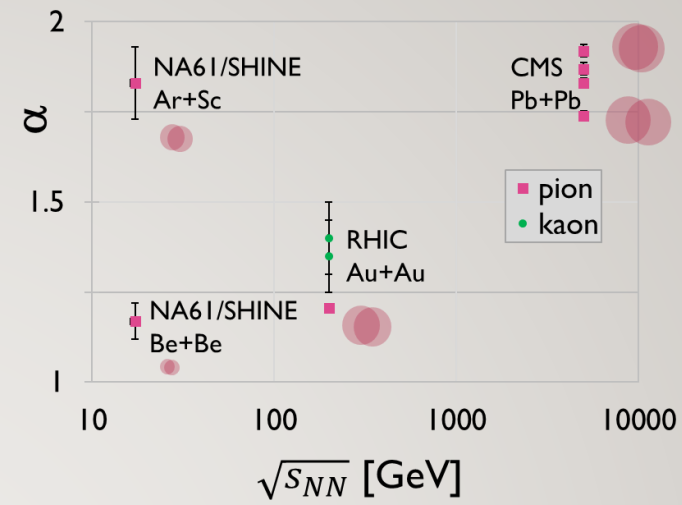
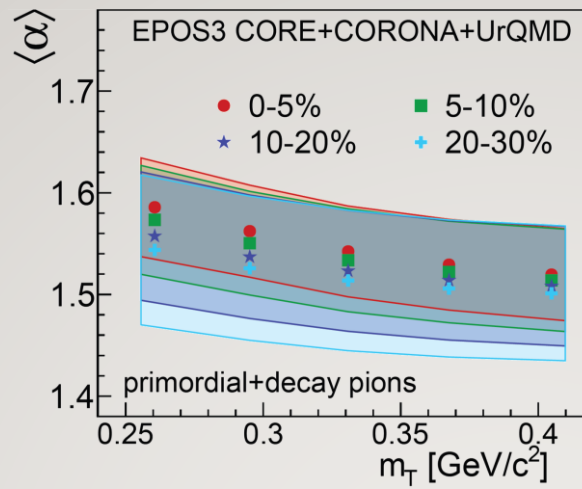
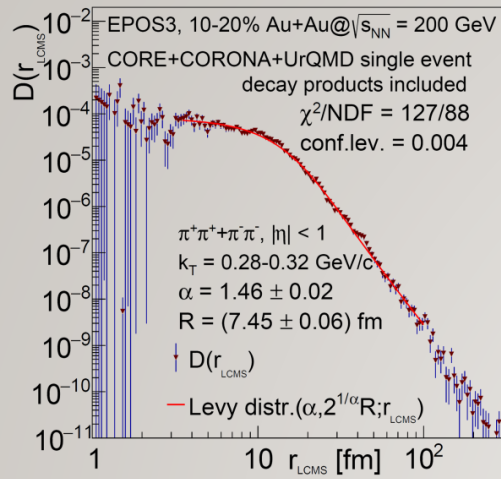
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CONCLUSIONS AND OUTLOOK

- Lévy sources from SPS to RHIC and LHC
 - **Lévy α** : between 1 and 2, increases with $\sqrt{s_{NN}}$?
 - Contrary to expectations, $\alpha(K) \geq \alpha(\pi)$
 - **Lévy R** : hydro scaling, despite not Gaussian
 - **Lévy λ** : signs of η' in-medium mass modification
- Possible reasons:
 - Jet fragmentation \rightarrow not dominant in AA collisions
 - **Critical phenomena** \rightarrow maybe at lowest RHIC energies and SPS
 - Directional averaging \rightarrow source is (approx.) spherical in LCMS, 3D cross-check done
 - Event averaging \rightarrow event-by-event simulations show Lévy
 - **Resonance decays** \rightarrow part of the reason, not enough alone
 - **Hadronic rescattering, Lévy flight** $\rightarrow \alpha(K) \geq \alpha(\pi)$ puzzling
- Questions to be answered:
 - When measuring α , what effects need to be considered?
 - Can there be anomalous diffusion in the quark stage?
 - What is the role of finite size and finite time?





THANK YOU FOR YOUR ATTENTION

And if you are interested in these topics:

<http://zimanyischool.kfki.hu/23/>

<https://indico.cern.ch/e/ismd23>



ZIMÁNYI SCHOOL 2023



A. Gáspár: Calculate the Entropy XIV

23rd ZIMÁNYI SCHOOL
WINTER WORKSHOP
ON HEAVY ION PHYSICS

December 4-8, 2023
Budapest, Hungary



József Zimányi (1931 - 2006)



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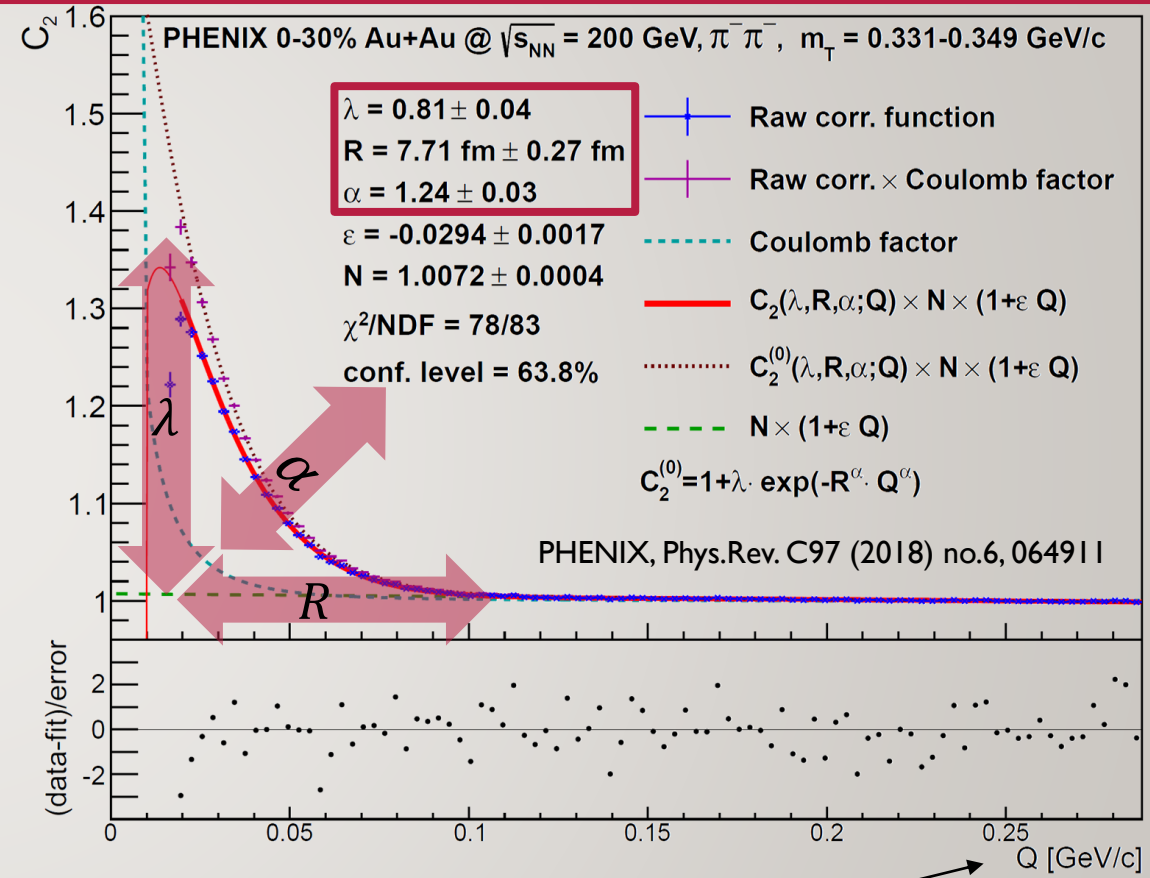
BACKUP





EXAMPLE $C_2(Q_{LCMS})$ CORRELATION FUNCTION

- Correlation function: spherical in **LCMS**
 - ID measurement possible
 - Done in several m_T bins
- Fit with calculation based on Lévy distribution
- Only converging fits with good confidence level should be accepted
- Physical parameters:
 R, λ, α measured versus pair m_T

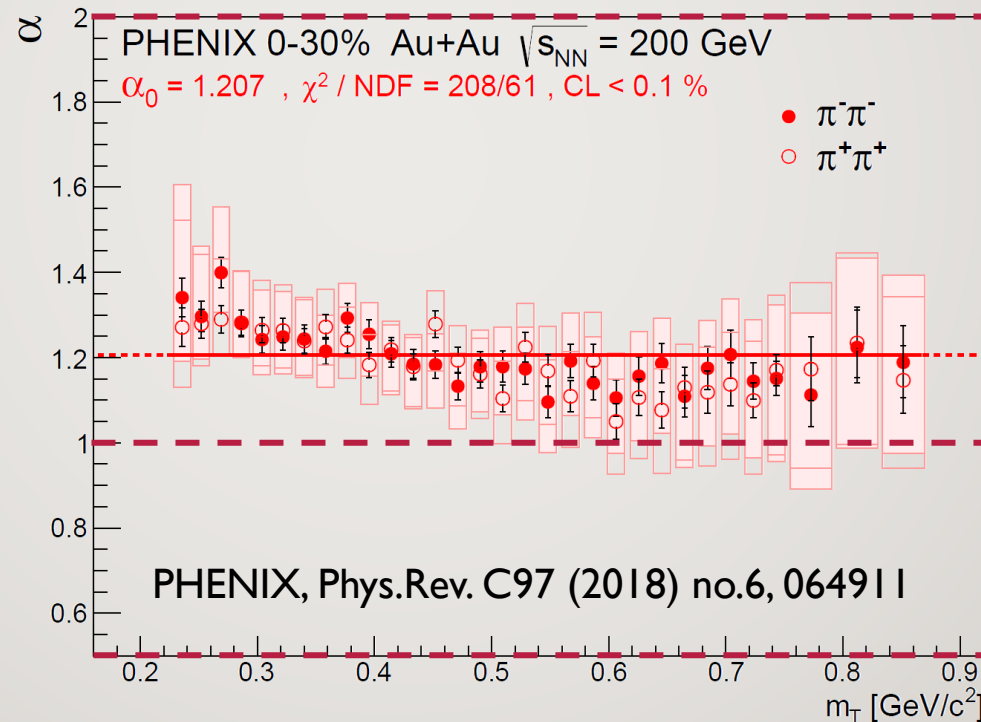
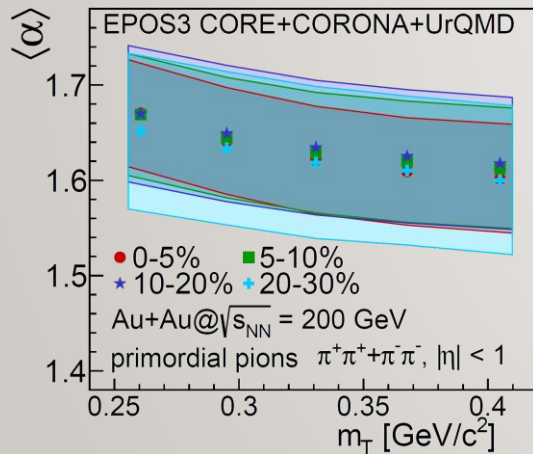


Q: ID momentum difference in
 Longitudinally CoMoving System (LCMS)



LÉVY EXPONENT α IN 200 GEV AU+AU AT RHIC

- Measured value far from Gaussian ($\alpha = 2$), inconsistent with expo. ($\alpha = 1$)
- Far from random field 3D Ising value at CEP ($\alpha = 0.5$)
- Approximately constant (at least within systematic uncertainties)
- What do models and calculations say?
- EPOS evt-by-evt analysis:



$\alpha = 2.0$ (Gauss)

$\alpha = 1.0$ (Cauchy)

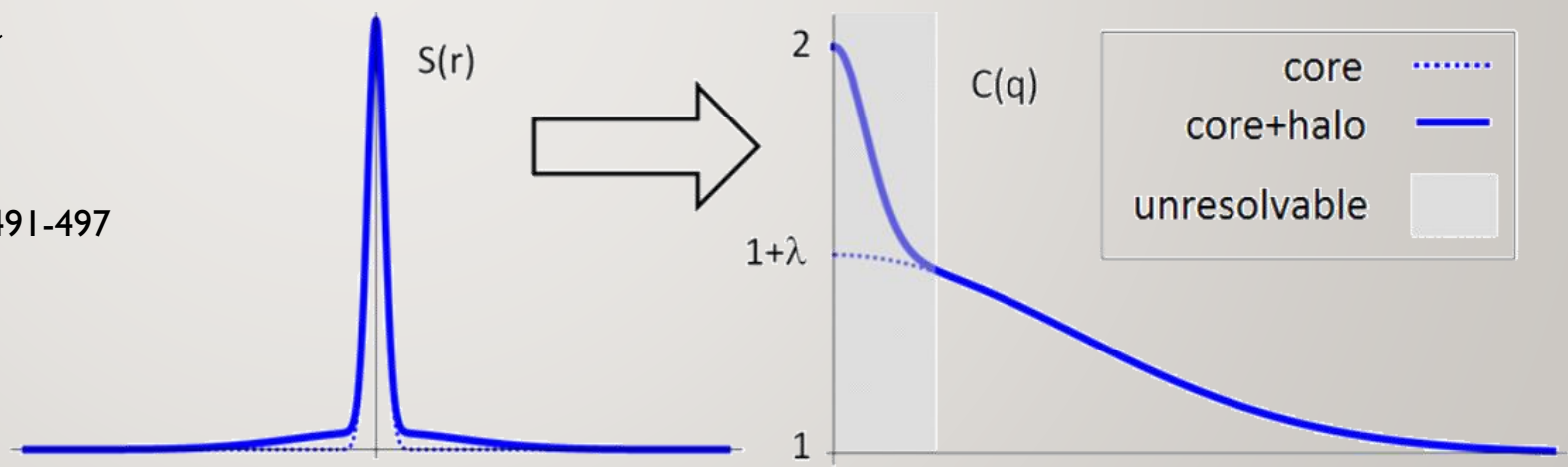
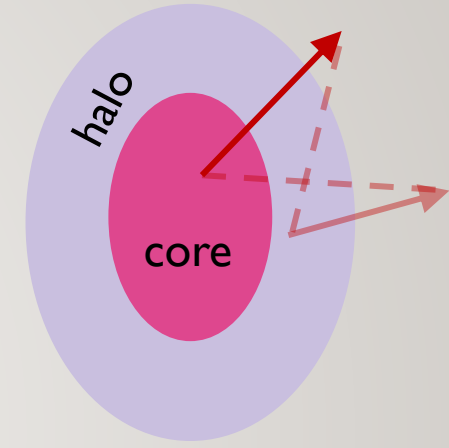
$\alpha = 0.5$ (random field Ising CEP)



CORRELATION STRENGTH λ : CORE/HALO

- Two-component core+halo source
 - Core: hydrodynamically expanding, thermal medium
 - Halo: long lived resonances ($\gtrsim 10$ fm/c, $\omega, \eta, \eta', K_0^S, \dots$)
 - Unresolvable experimentally
 - Define $f_C = N_{\text{core}}/N_{\text{total}}$
- True $q \rightarrow 0$ limit: $C(0) = 2$
- Apparently $C(q \rightarrow 0) \rightarrow 1 + \lambda$
- $\lambda(m_T) = f_C^2(m_T)$

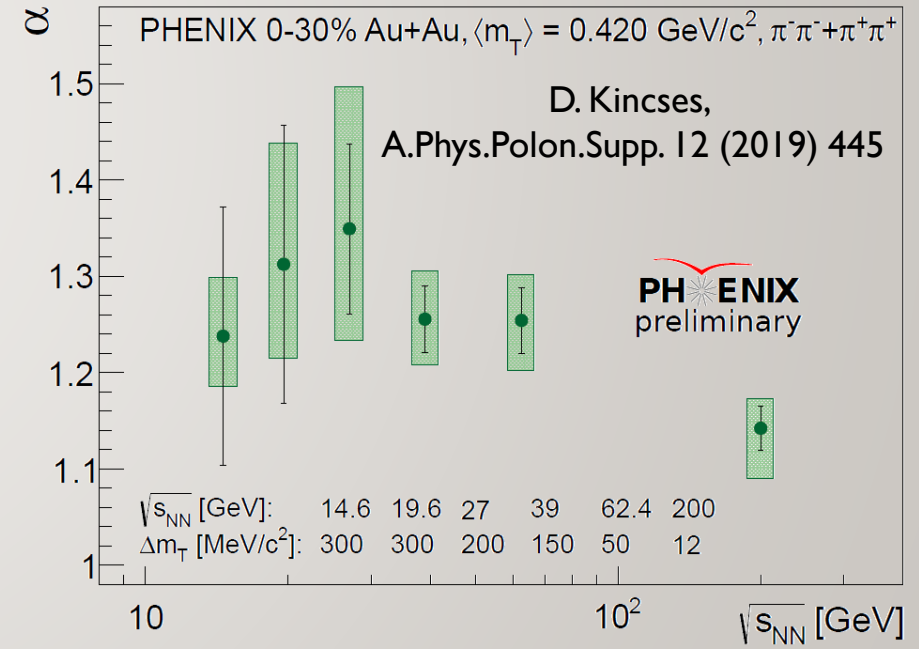
Bolz et al, Phys.Rev. D47 (1993) 3860-3870;
Csörgő, Lörstad, Zimányi, Z.Phys. C71 (1996) 491-497





COLLISION ENERGY DEPENDENCE

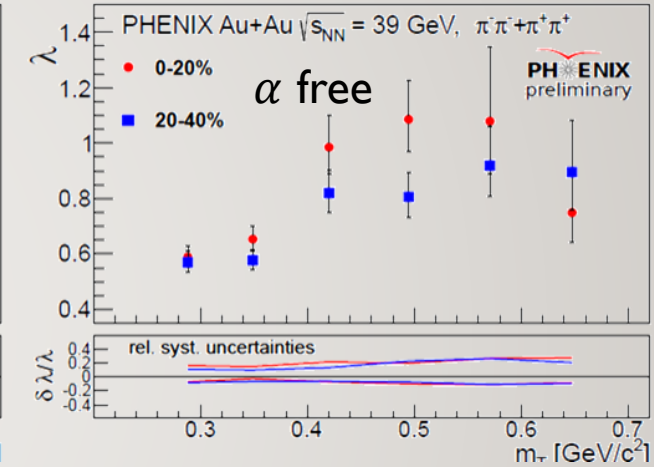
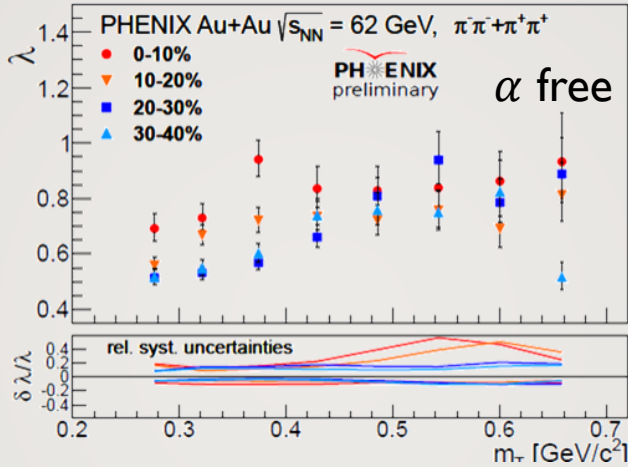
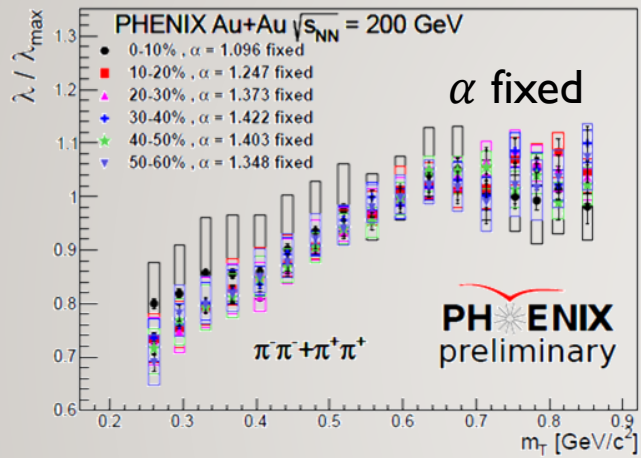
- $\langle \alpha \rangle$ approximately monotonic versus $\sqrt{s_{NN}}$
 - No clear interpretation or understanding of this trend
 - With widely varying m_T interval (due to statistics) \rightarrow may influence outcome
 - Important w.r.t. shape averaging interpretation of $\alpha \neq 2$
- Lévy exponent α still far from conjectured CEP limit of 0.5
 - Very much dependent on m_T bin width
 - Working on final results...





HOLE IN $\lambda(m_T)$: ALL MEASUREMENTS AT RHIC

- Hole apparent for $\sqrt{s_{NN}} \geq 39$ GeV, \sim independently of centrality



- Due to reduced η' mass?
- Sign for chiral restoration?
- To be cross-checked with photons, dileptons, etc.
- Working on finalized PHENIX results



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COHERENCE WITH THREE-PION LÉVY HBT

- Recall: two particle correlation strength $\lambda = f_C^2$ where $f_C = N_{\text{core}}/N_{\text{total}}$
- Generalization for higher order correlations: $\lambda_2 = f_C^2, \lambda_3 = 2f_C^3 + 3f_C^2$
- If there is partial coherence (p_C):

$$\lambda_2 = f_C^2 [(1 - p_C)^2 + 2p_C(1 - p_C)]$$

$$\lambda_3 = 2f_C^3 [(1 - p_C)^3 + 3p_C(1 - p_C)^2] + 3f_C^2 [(1 - p_C)^2 + 2p_C(1 - p_C)]$$

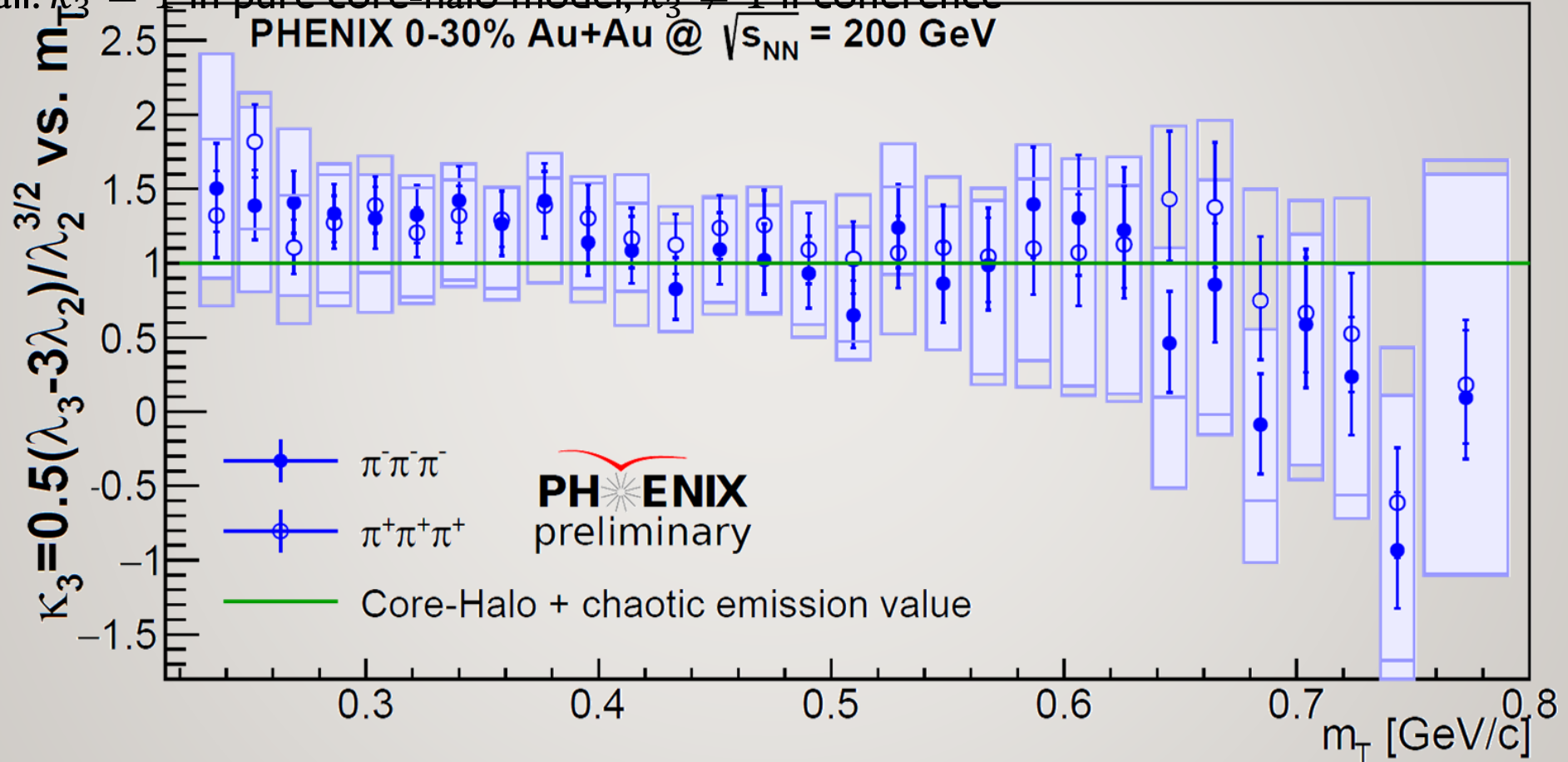
- Introduce core-halo independent parameter $\kappa_3 = \frac{\lambda_3 - 3\lambda_2}{2\sqrt{\lambda_2}^3}$
 - does not depend on f_C
 - $\kappa_3 = 1$ if no coherence
- Finite meson sizes?
 - Gavrilik, SIGMA 2 (2006) 074 [hep-ph/0512357]
- Phase shift (a la Aharonov-Bohm) in hadron gas?
 - Random fields create random phase shift, on average distorts Bose-Einstein correlations
Csanád et al., Gribov-90 (2021) 261-273 [arXiv:2007.07167]

LEVY HBT EXPERIMENT PHENOMENOLOGY NEW RESULTS



TEST OF CORE-HALO MODEL / COHERENCE

- Recall: $\kappa_3 = 1$ in pure core-halo model, $\kappa_3 \neq 1$ if coherence

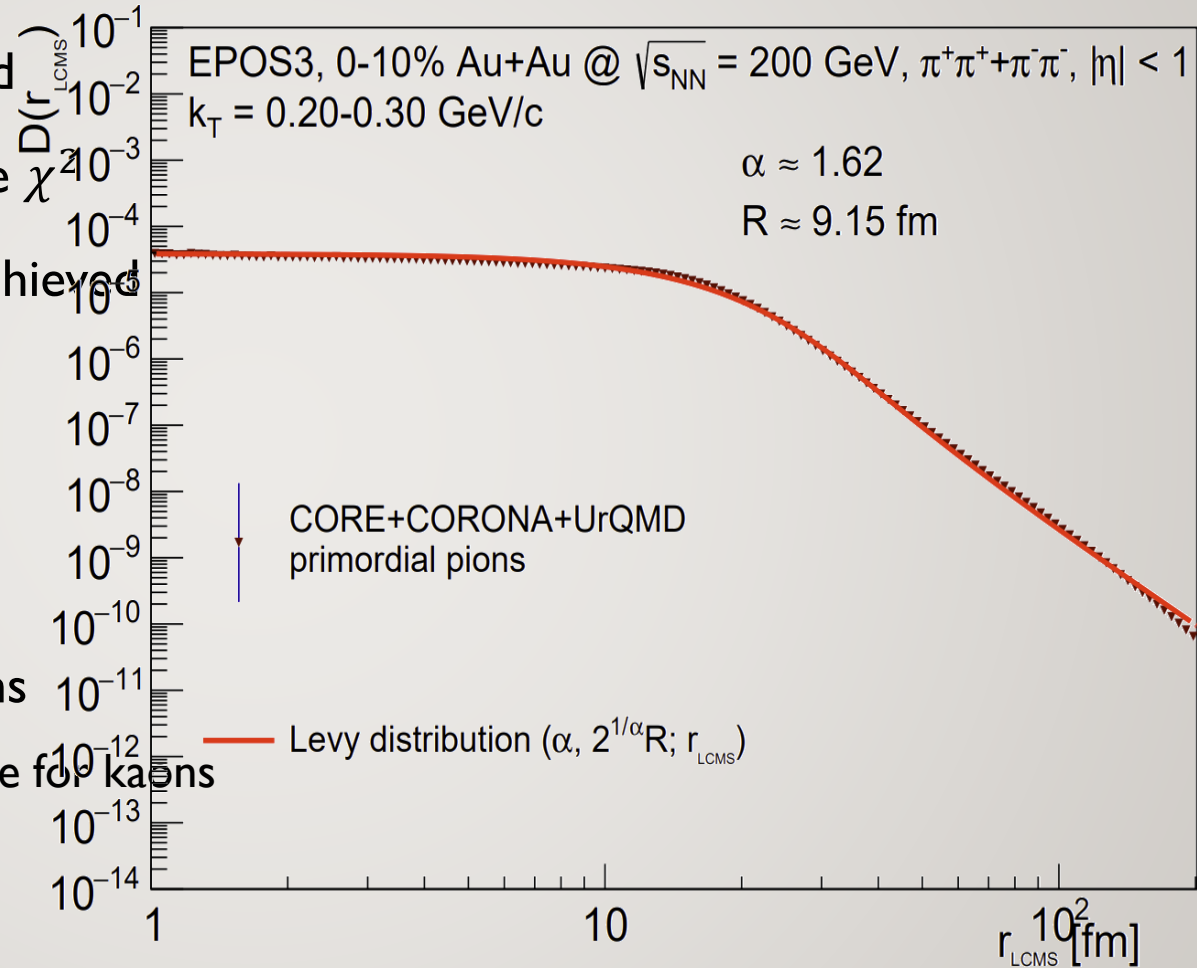


WPCF 19, B. Kurgyis, Phys. Part. Nuclei 51 (2020) 263-266



ROLE OF EVENT AVERAGING?

- Event-averaged source also analyzed
- Not perfectly Lévy shape, very large χ^2
- Nevertheless: similar parameters achieved
 - Event averaged:
 $\alpha \approx 1.62, R \approx 9.15 \text{ fm}$
 - Event-by-event:
 $\alpha \approx 1.66, R \approx 8.96 \text{ fm}$
- More reasonable approach for kaons
 - No event-by-event analysis possible for kaons





SOURCE OR PAIR DISTRIBUTION?

- Under some circumstances (thermal emission, no interactions, ...):

$$C_2(q, K) = \int S\left(r_1, K + \frac{q}{2}\right) S\left(r_2, K - \frac{q}{2}\right) |\Psi_2(r_1, r_2)|^2 dr_1 dr_2 \\ \cong 1 + \left| \int S(r, K) e^{iqr} dr \right|^2$$

- Let us introduce the spatial pair distribution:

$$D(r, K) = \int S\left(\rho + \frac{r}{2}, K\right) S\left(\rho - \frac{r}{2}, K\right) d\rho$$

- Then the Bose-Einstein correlation function becomes:

$$C_2(q, K) \cong \int D(r, K) |\Psi_2(r)|^2 dr = 1 + \int D(r, K) e^{iqr} dr$$

- **Bose-Einstein correlations measure spatial pair distributions!**
- Coulomb and strong Final State Interactions? Under control for Lévy sources

Csanad, Lökös, Nagy, Phys. Part. Nuclei 51 (2020) 238 [arXiv:1910.02231]

Kincses, Nagy, Csanad Phys. Rev. C 102, 064912 (2020) [arXiv:1912.01381]



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INTERACTIONS: THE COULOMB-EFFECT

- Plane-wave result, based on $|\Psi_2^{(0)}(r)|^2 = 1 + e^{iqr}$:

$$C_2(q, K) \cong \int D(r, K) |\Psi_2^{(0)}(r)|^2 dr = 1 + \int D(r, K) e^{iqr} dr$$

- If there is interaction:

$$\Psi_2^{(0)}(r) \rightarrow \Psi_2^{(\text{int})}(r_1, r_2)$$

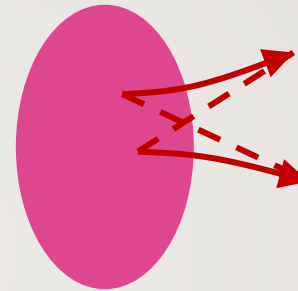
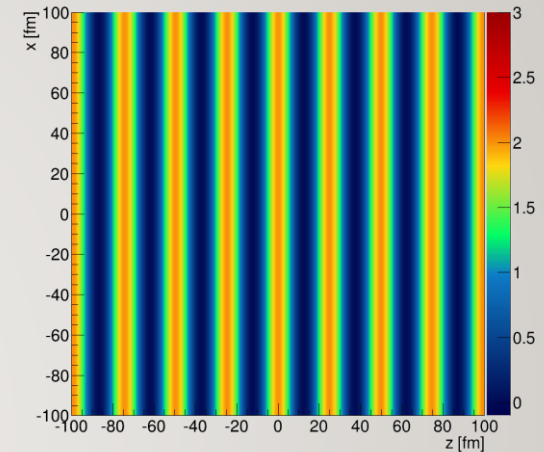
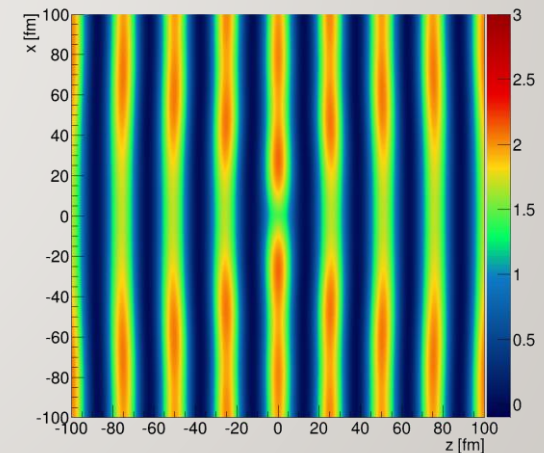
- For Coulomb:

$$|\Psi_2^{(C)}(r)|^2 = \frac{\pi\eta}{e^{2\pi\eta}-1} \cdot (\text{complicated hypergeometric expression})$$

- Direct fit with this, or the usual iterative Coulomb-correction:

$$C_{\text{Bose-Einstein}}(q)K(q), \text{ where } K(q) = \frac{\int D(r, K) |\Psi_2^{(C)}(r)|^2 dr}{\int D(r, K) |\Psi_2^{(0)}(r)|^2 dr}$$

- Complication: need for integrating power-law tails**
- In this analysis: assuming spherical source
- Parametrization possible Csanád, Lökös, Nagy, Phys.Part.Nucl. 51 (2020) 238

plane wave $|\psi|^2$, $k=25\text{MeV}$ Coulomb w.f. $|\psi|^2$, $k=25\text{MeV}$ 

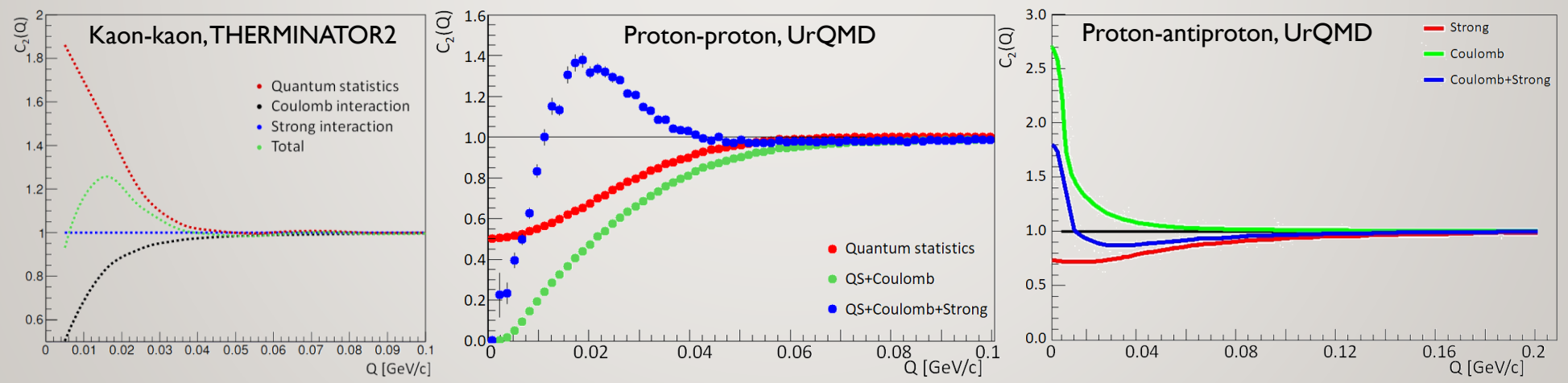


ROLE OF THE STRONG INTERACTION

- In case of other interactions or not identical bosons, the formula still works:

$$C_2(q, K) \cong \int D(r, K) |\Psi_2(r)|^2 dr$$

- Pair wave function determines $D \leftrightarrow C_2$ connection
- Mesons, baryons: strong interaction; fermions: anticorrelation
- Non-identical pairs: interaction modifies wave function

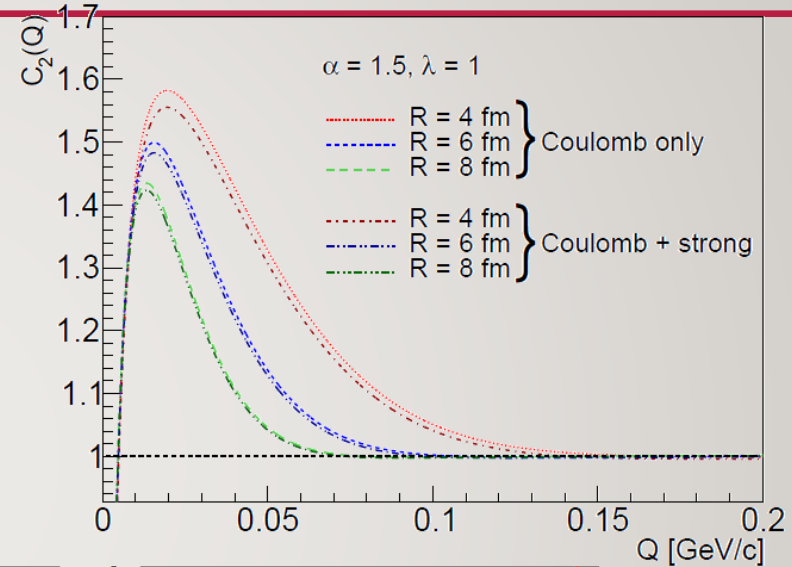


From e.g. H. Zbroszczyk's talk at Zimányi School 2019

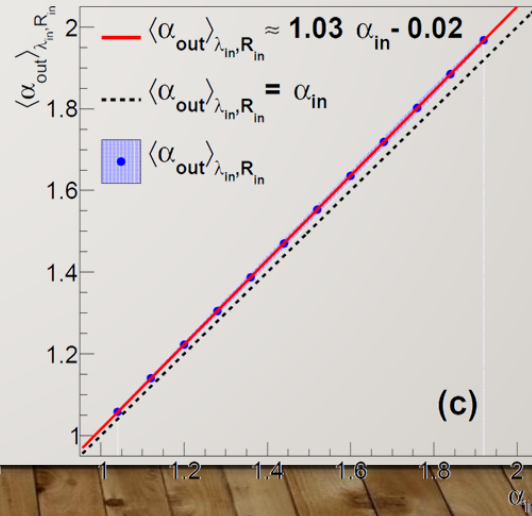
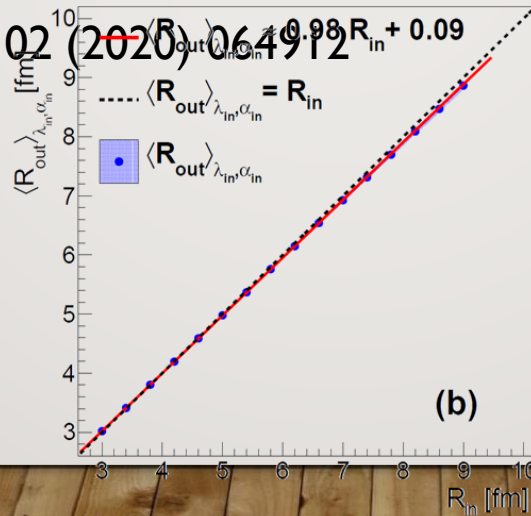
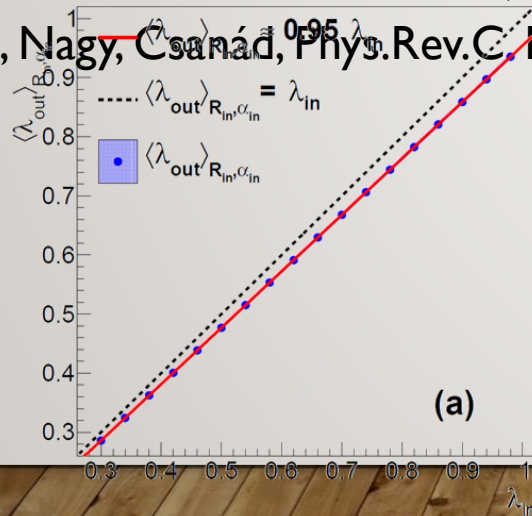


STRONG INTERACTION FOR PION PAIRS

- Additional potential appearing
- Possible handling: strong phase shift, Modify s-wave component in wave func.
R. Lednicky, Phys. Part. Nucl.40, 307 (2009)
- Small difference in case of pions
- Few percent modification in λ, α



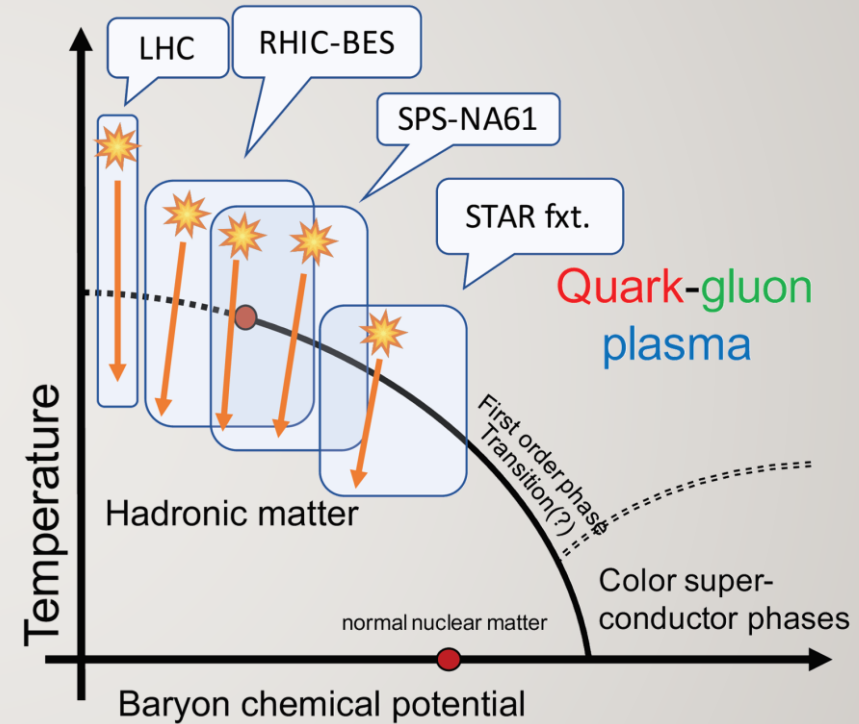
Kincses, Nagy, Csanád, Phys.Rev.C 102 (2020) 064912





HBT MEASUREMENTS AND THE PHASE DIAGRAM

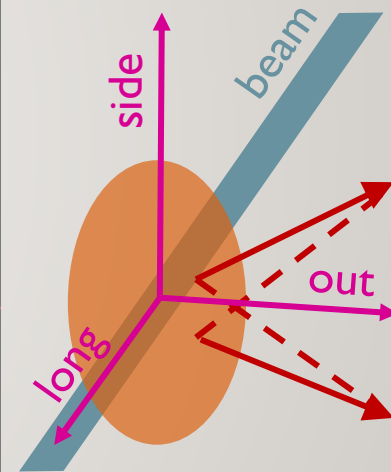
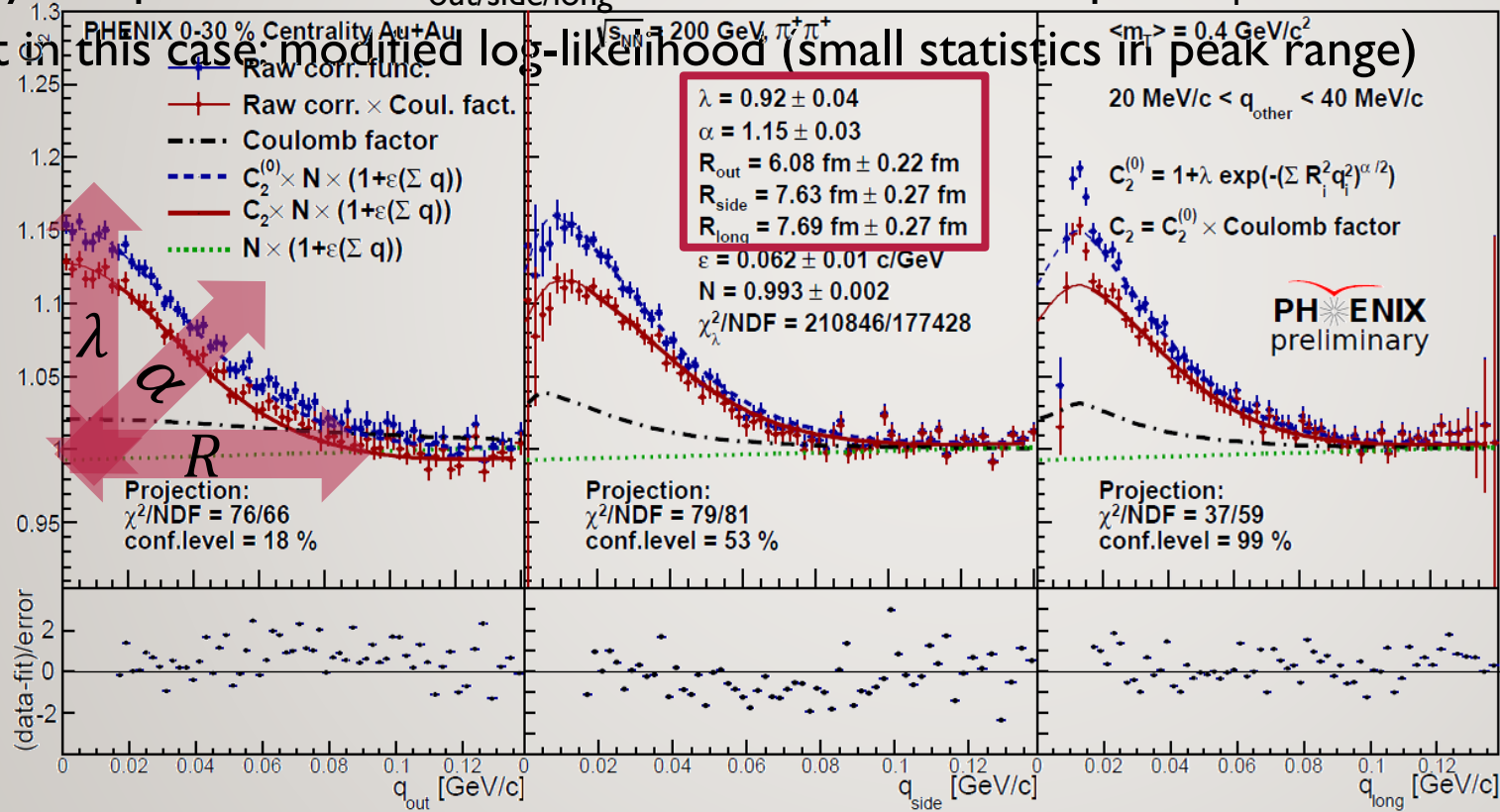
- LHC: measurement at CMS
 - 2-5 ATeV energy, p+p & Pb+Pb
- RHIC: measurement at PHENIX+STAR
 - 10-200 AGeV energy, Au+Au
- SPS: measurement at NA61
 - 17 AGeV energy, Be+Be
- Phase diagram can be investigated





A CROSS-CHECK: 3D LÉVY FEMTOSCOPY

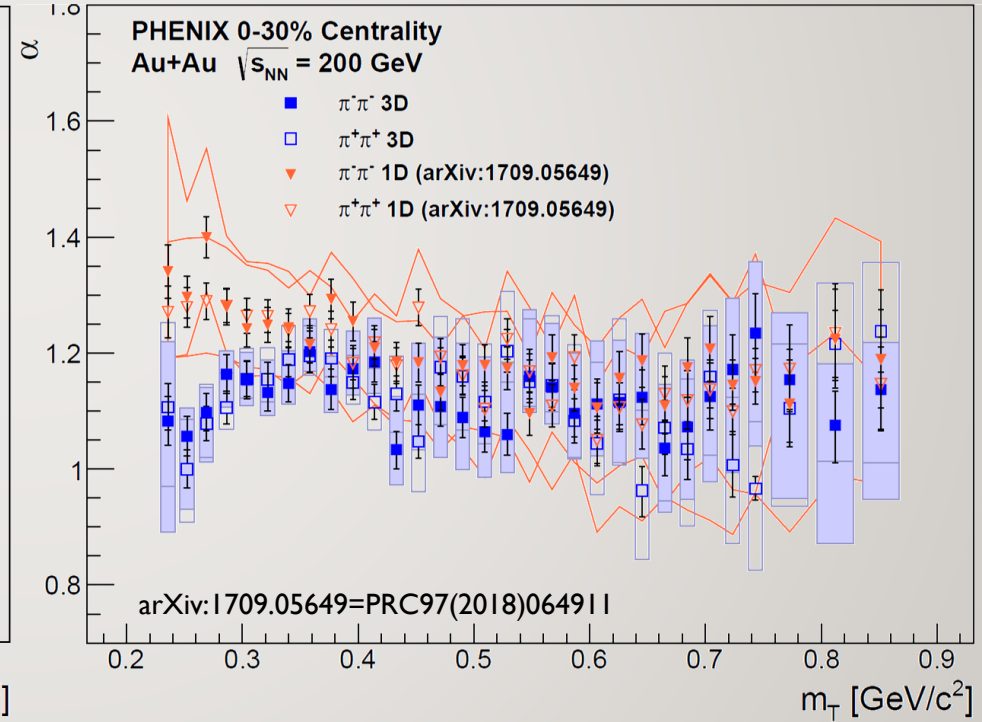
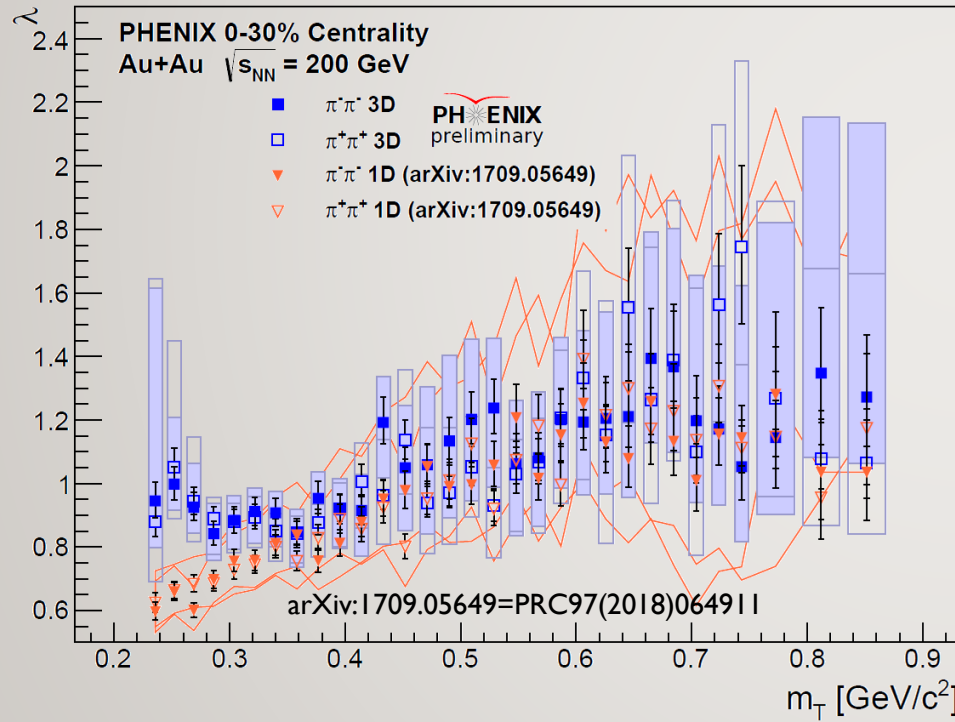
- Femtoscopy done in 3D: Bertsch-Pratt pair frame (out/side/long coordinates)
- Physical parameters: $R_{out/side/long}$, λ , α measured versus pair m_T
- Fit in this case: modified log-likelihood (small statistics in peak range)





3D VERSUS 1D: STRENGTH λ AND SHAPE α

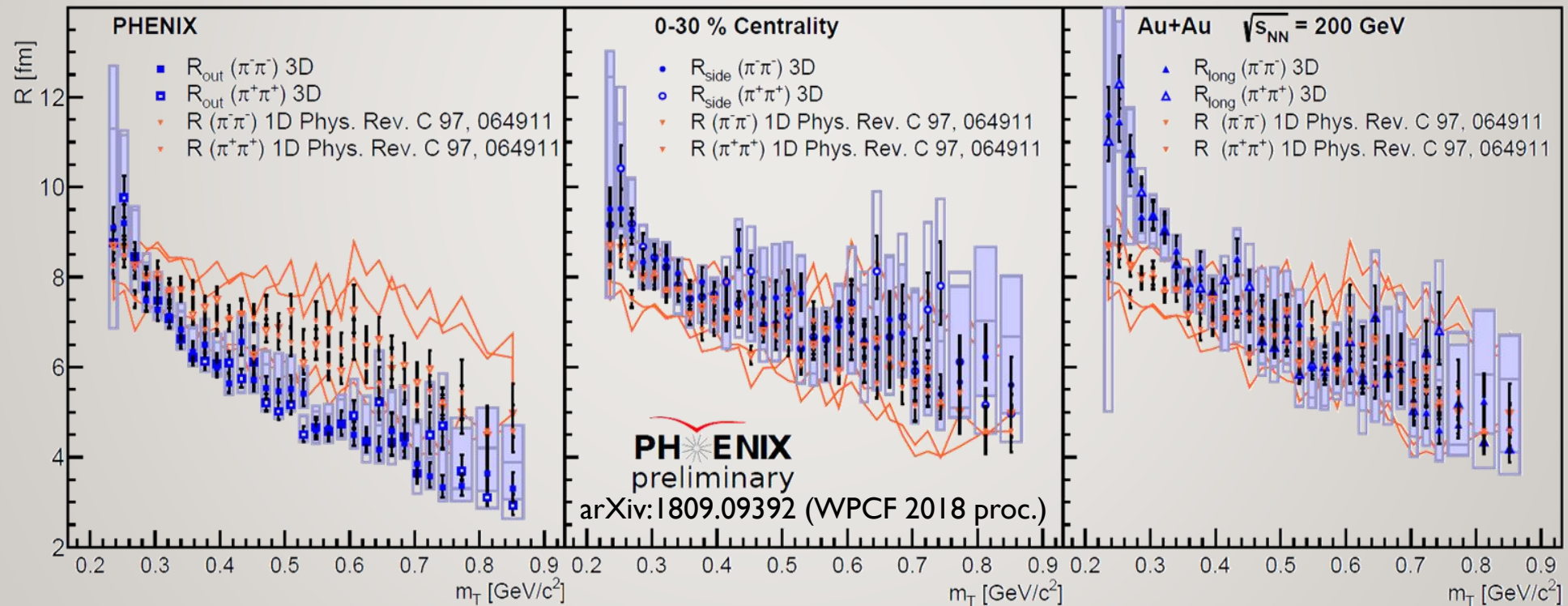
- Compatible with 1D (Q_{LCMS}) measurement of PRC97(2018)064911
- Small discrepancy at small m_T : due to large R_{long} at small m_T ?





LÉVY SCALES IN 3D

- Compatibility with 1D Lévy analysis
- Similar decreasing trend as Gaussian HBT radii, but hydro prediction based on Gaussian source
- Asymmetric source for small m_T , validity of Coulomb-approximation?



LEVY HBT

SHAPE

SCALE+STRENGTH

OTHER EXP



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OPEN QUESTIONS

- Collision energy and centrality dependence of Lévy parameters?
 - Non-monotonicity in $\alpha(\sqrt{s_{NN}})$ or $\alpha(\text{centrality})$?
 - Hole in $\lambda(m_T)$ at low $\sqrt{s_{NN}}$? Really due to η' ?
- Reason for the appearance of Lévy distributions for pions?
 - What is the Lévy exponent for kaons?
 - Kaons have smaller total cross-section thus larger mean free path, heavier tail?
 - Does m_T scaling hold for Lévy scale R ?
- Correlation strength versus core-halo picture: are there other effects?
 - Three-particle correlations may show if coherence or other effects play a role
 - Other effects may also play a role (finite meson sizes, random field phase shift, etc)

LEVY HBT

SHAPE

SCALE+STRENGTH

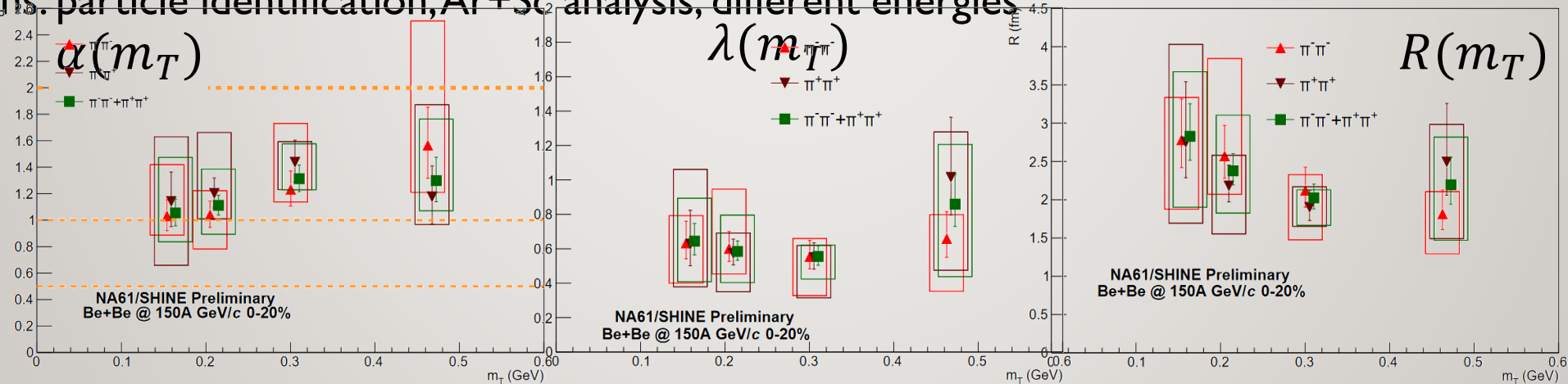
OTHER EXP



RESULTS AT NA61/SHINE

- Be+Be collisions at 150 AGeV beam momentum (17.3 AGeV in c.m.s.)
- Lévy fits describe correlation functions
 - Shape parameter α : far from Gaussian and CEP conjecture
 - Strength parameter λ : nearly constant as previous SPS results, unlike RHIC
 - Spatial scale R : weakly decreasing trend \rightarrow hydro

- Plans: particle identification, Ar+Sc analysis, different energies



B. Porfy [NA61] WPCFI9 & PoS CORFU2018 (2019) 184

LEVY HBT

SHAPE

SCALE+STRENGTH

OTHER EXP

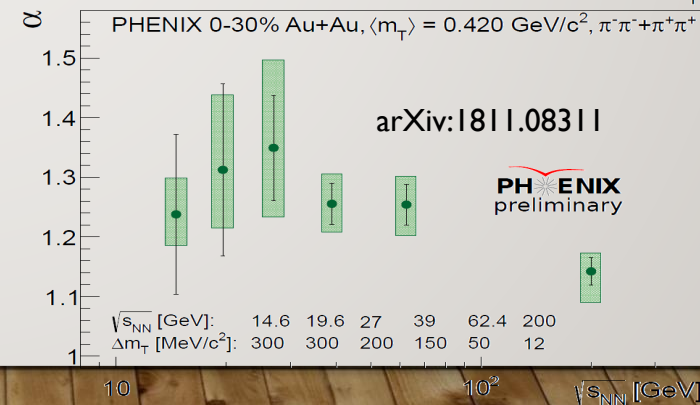
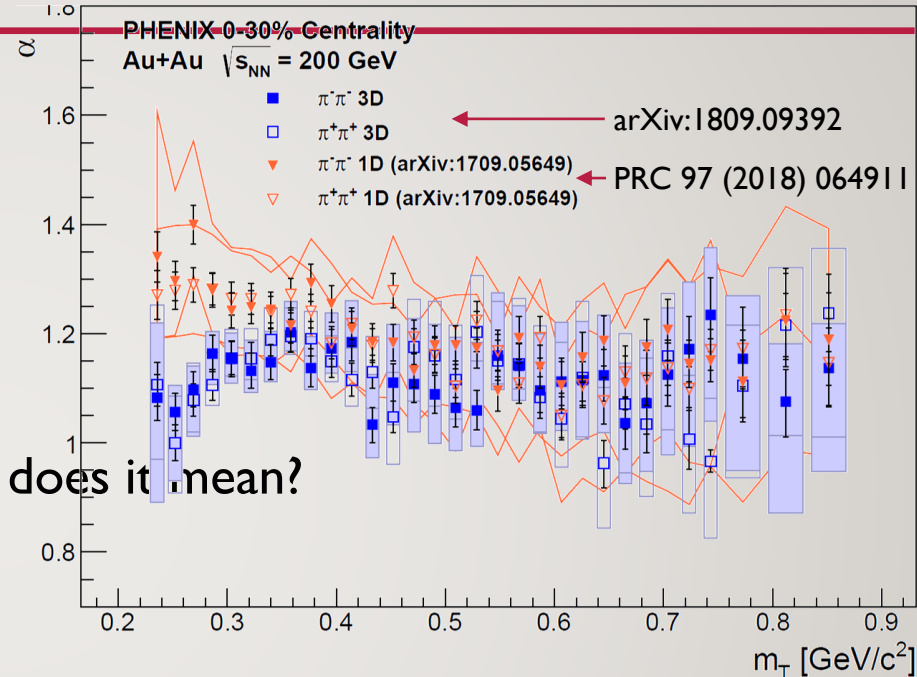
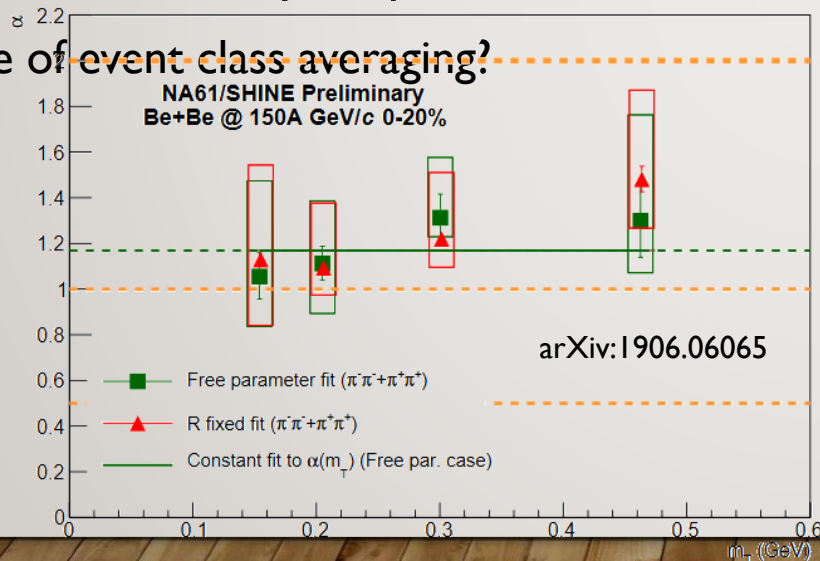


LÉVY HBT MEASUREMENTS

- Many experimental results
 - PHENIX Au+Au: $\alpha \approx 1 - 1.5$
 - STAR Au+Au: ongoing
 - NA61 Be+Be: $\alpha \approx 1 - 1.5$
 - CMS Pb+Pb: $\alpha = 1$ fixed

• Where does this Lévy shape come from? What does it mean?

- Role of event class averaging?





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THE EPOS MODEL

- **E**nergy conserving quantum-mechanical multiple scattering approach, based on **P**artons ladders, **O**ff-shell remnants, and **S**plitting of parton ladders
 - K.Werner et al., PRC82 (2010) 044904, PRC89 (2014) 064903, ...
- Based on Monte-Carlo simulation
- Theoretical framework: parton-based Gribov-Regge theory (PBGRT)
- Three main parts of the model:
 - Core-Corona division (based on dE/dx of string segments)
 - Hydrodynamical evolution (vHLLE 3D+1 viscous hydro)
 - Hadronic cascades (UrQMD afterburner)
- Effects/components to be turned on or off (on top of Core):
 - Corona
 - Rescattering
 - Decays

LEVY HBT

EPOS ANALYSIS

SUMMARY



TWO-PARTICLE SPATIAL CORRELATIONS

- Object to be investigated: two-particle source

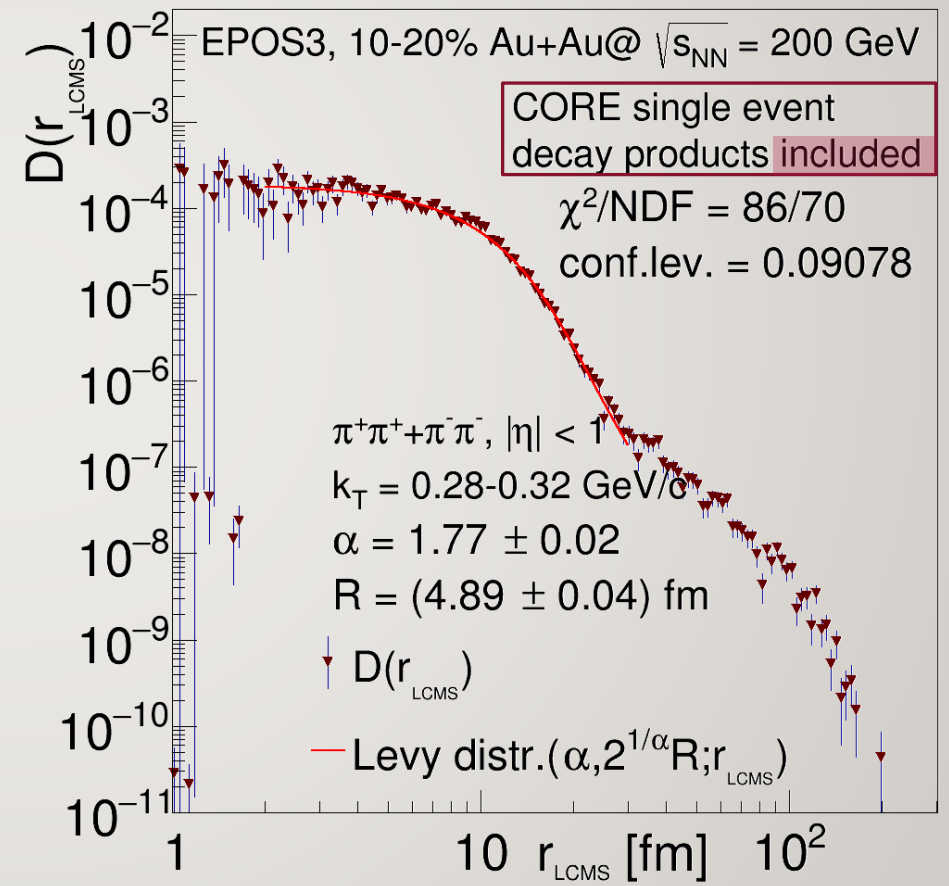
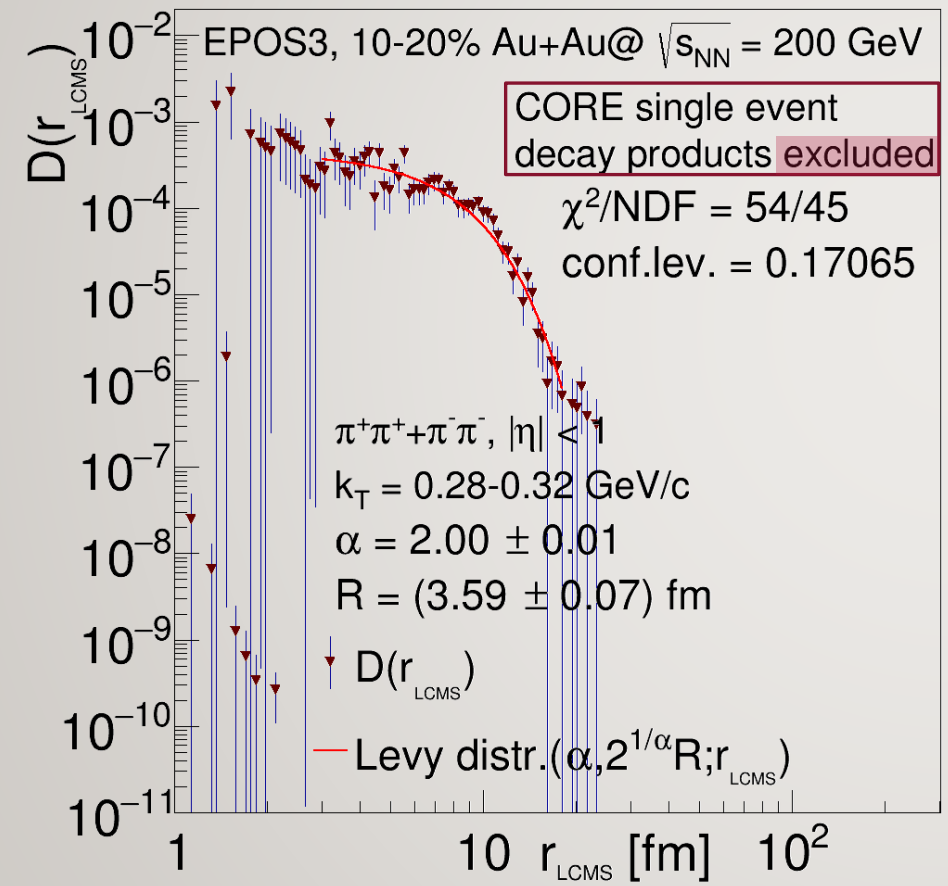
$$D(r, K) = \int d^4\rho S\left(\rho + \frac{r}{2}, K\right) S\left(\rho - \frac{r}{2}, K\right)$$

- Experimental results measure power-law tails, Lévy shapes
 - Measure momentum-space correlations, reconstruct $D(r)$ or fit its parameters
- Why do these Lévy shapes appear?
 - What physics does contribute to it? Rescattering, decays?
 - What role does event averaging have in it?
Cimerman, Plumberg, Tomasik, Phys.Part.Nucl. 51 (2020) 282, PoS ICHEP2020 538
 - What do specific α values mean?
- Event generator models (like EPOS) – direct access to pair-source!
 - Phenomenological investigations of $D(r)$ possible
 - Effects can be turned off or on, investigated separately



EXAMPLE SINGLE EVENT, CORE ONLY

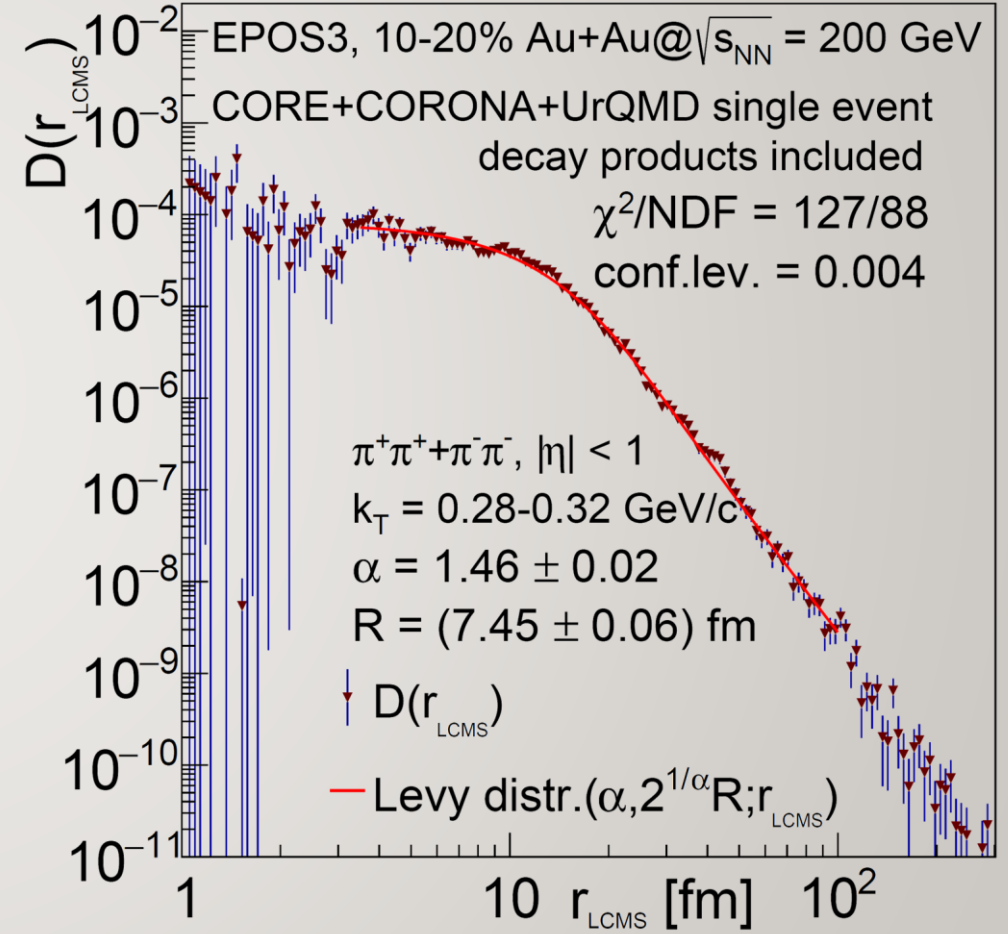
- Gaussian shape without decays, additional structure with decays





EXAMPLE EVENT, CORE+CORONA+URQMD

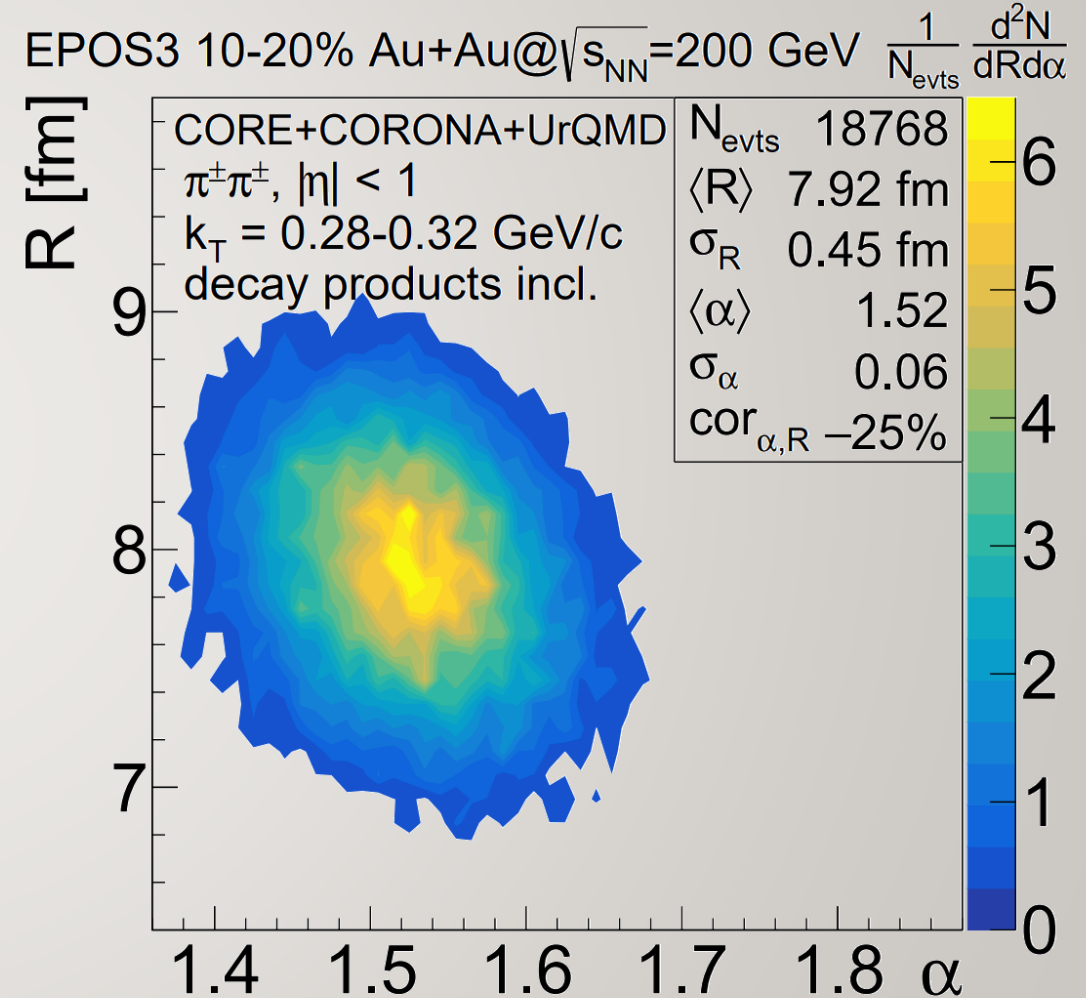
- Investigating $D(r)$ event-by-event
- Lévy-fits provide good description (2-100 fm range)
- Repeat such fits for thousands of events
- Extract α, R distribution





DISTRIBUTION OF α , R PARAMETERS

- Normal distribution of α , R for given centrality & k_T
- Extract mean and std.dev,
- Investigate centrality & k_T dependence
- k_T dependence investigated around the peak of the pair- k_T distr. to have adequate stat.

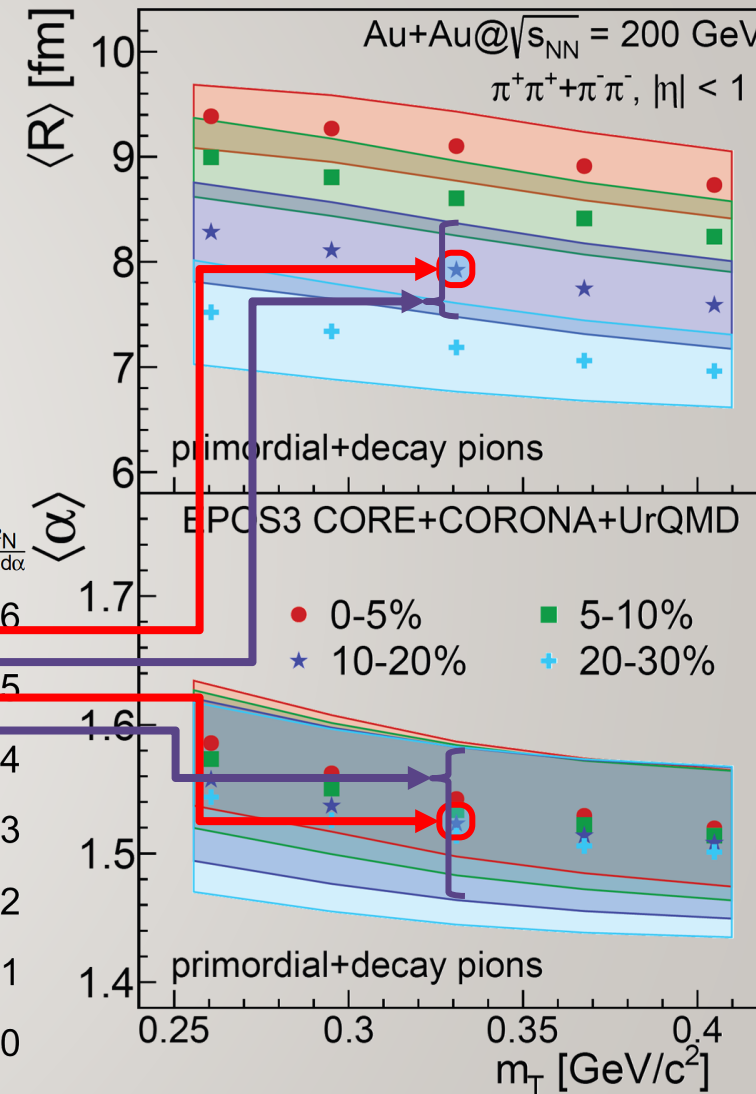
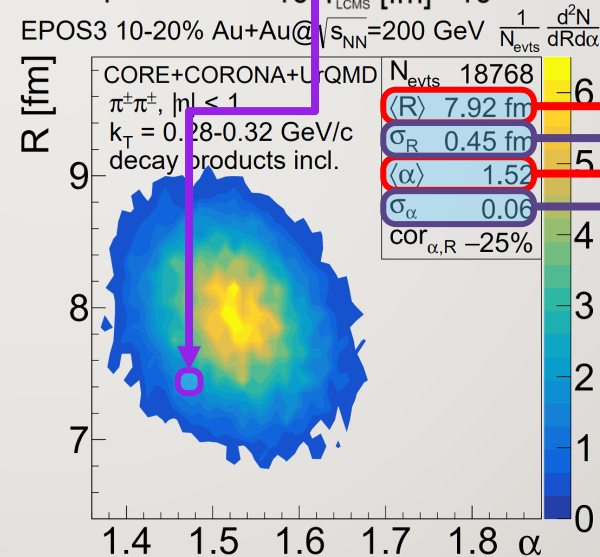
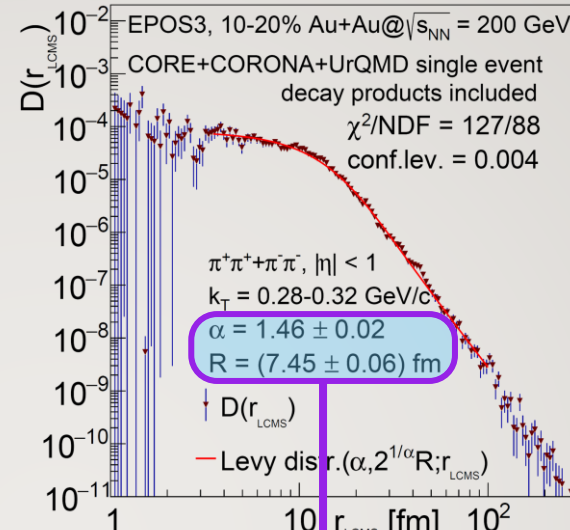




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EPOS SUMMARY

- $D(r)$ calculated in EPOS evt-by-evt
- Lévy fits done evt-by-evt
- Non-Gaussianity in single events
- Extracting mean, & std.dev. of R, α
- m_T & centrality dependence



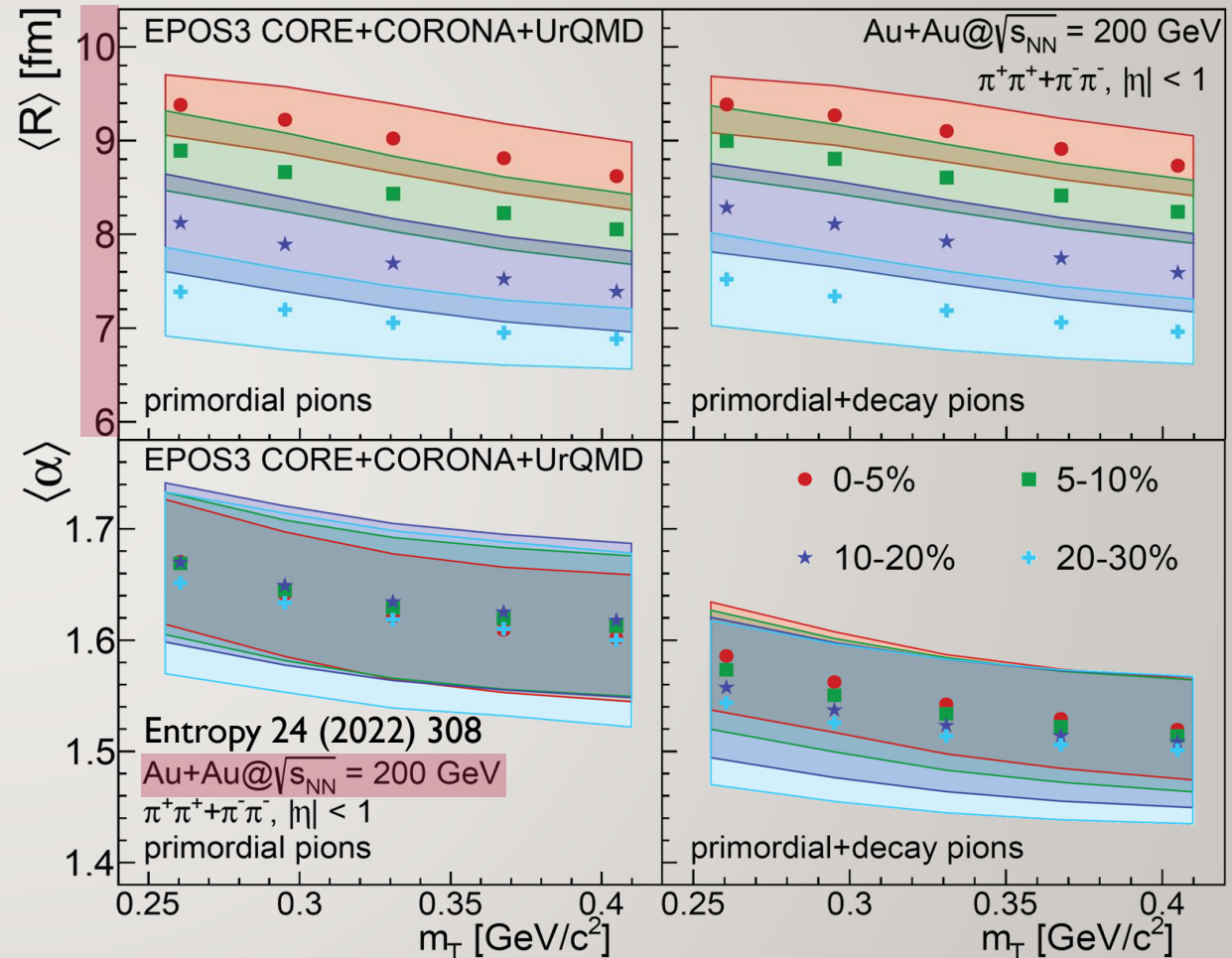
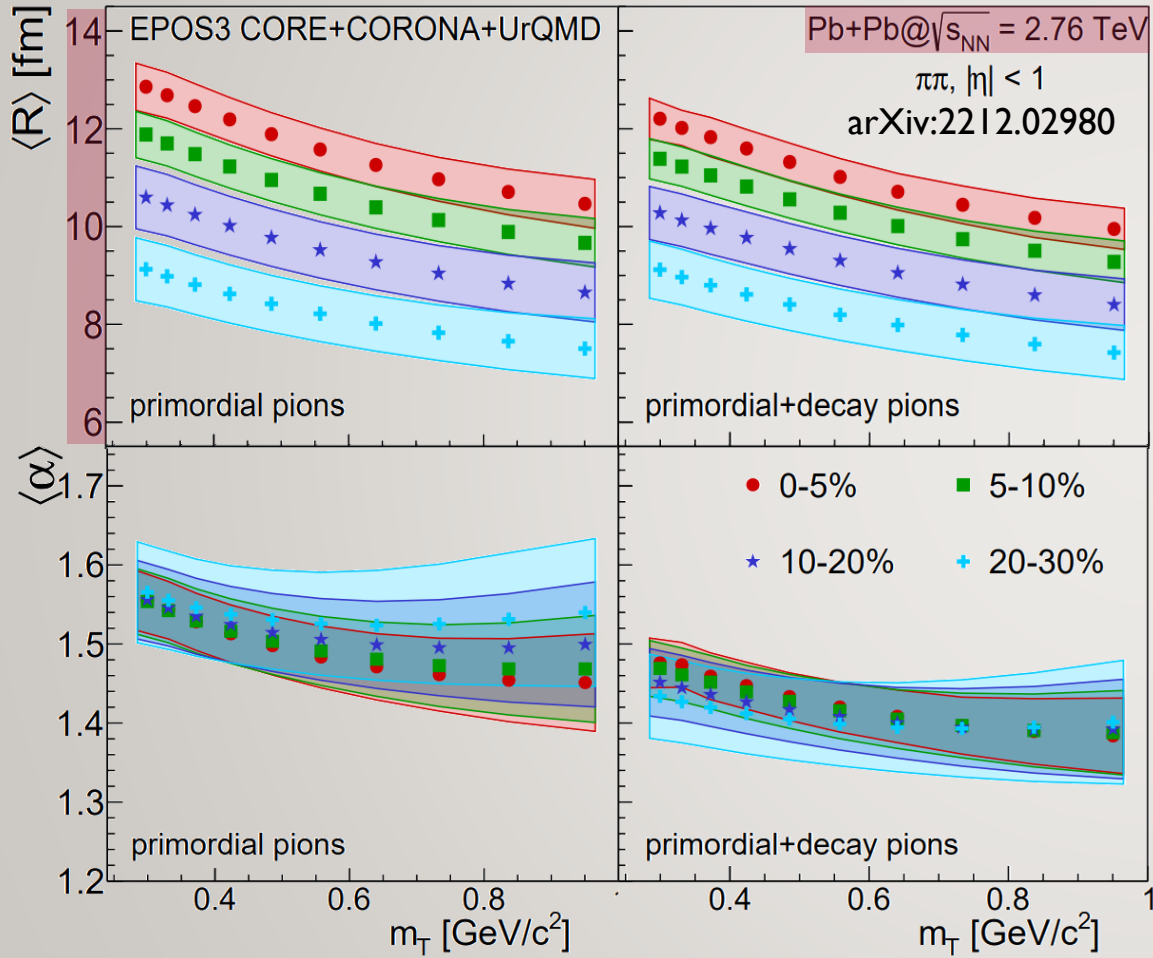


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EPOS @ LHC

VS

RHIC





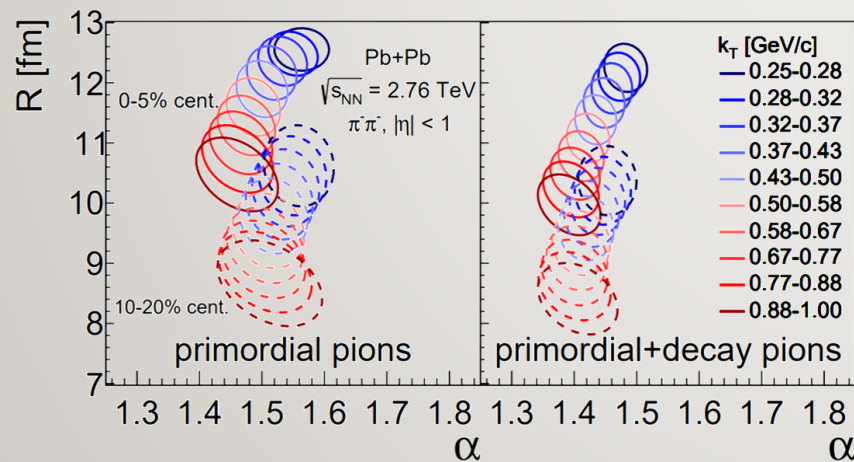
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EPOS IN 2.76 TEV PBPB, EVENT-BY-EVENT

- Pion and kaon pair distributions calculated in individual EPOS events

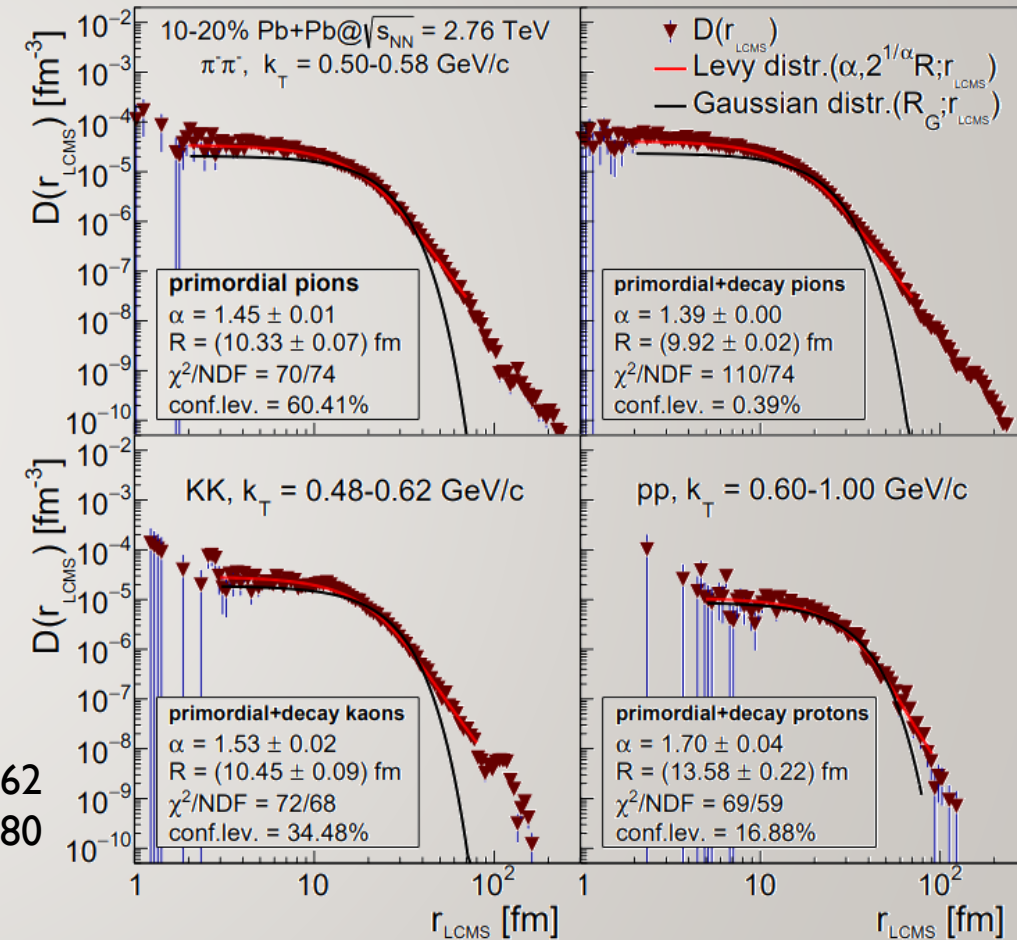
$$D(r_{LCMS}) = \int d\Omega dt D(t, r_x, r_y, r_z)$$

- Lévy source parameters determined for 800k events separately
 - Fit limits: from 2-5 fm to 70-100 fm
 - Criterion for acceptance: confidence level > 0.1%
 - Strongly non-Gaussian shapes observed
- Separately for various centrality and k_T classes, primordial or decays



arXiv:2201.07962
arXiv:2212.02980

EPOS3 CORE+CORONA+UrQMD, single event



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A CROSS-CHECK: THREE-PION LÉVY HBT

- Recall: two particle correlation strength $\lambda = f_C^2$ where $f_C = N_{\text{core}}/N_{\text{total}}$
- Generalization for higher order correlations: $\lambda_2 = f_C^2, \lambda_3 = 2f_C^3 + 3f_C^2$
- If there is partial coherence (p_C):

$$\lambda_2 = f_C^2 [(1 - p_C)^2 + 2p_C(1 - p_C)]$$

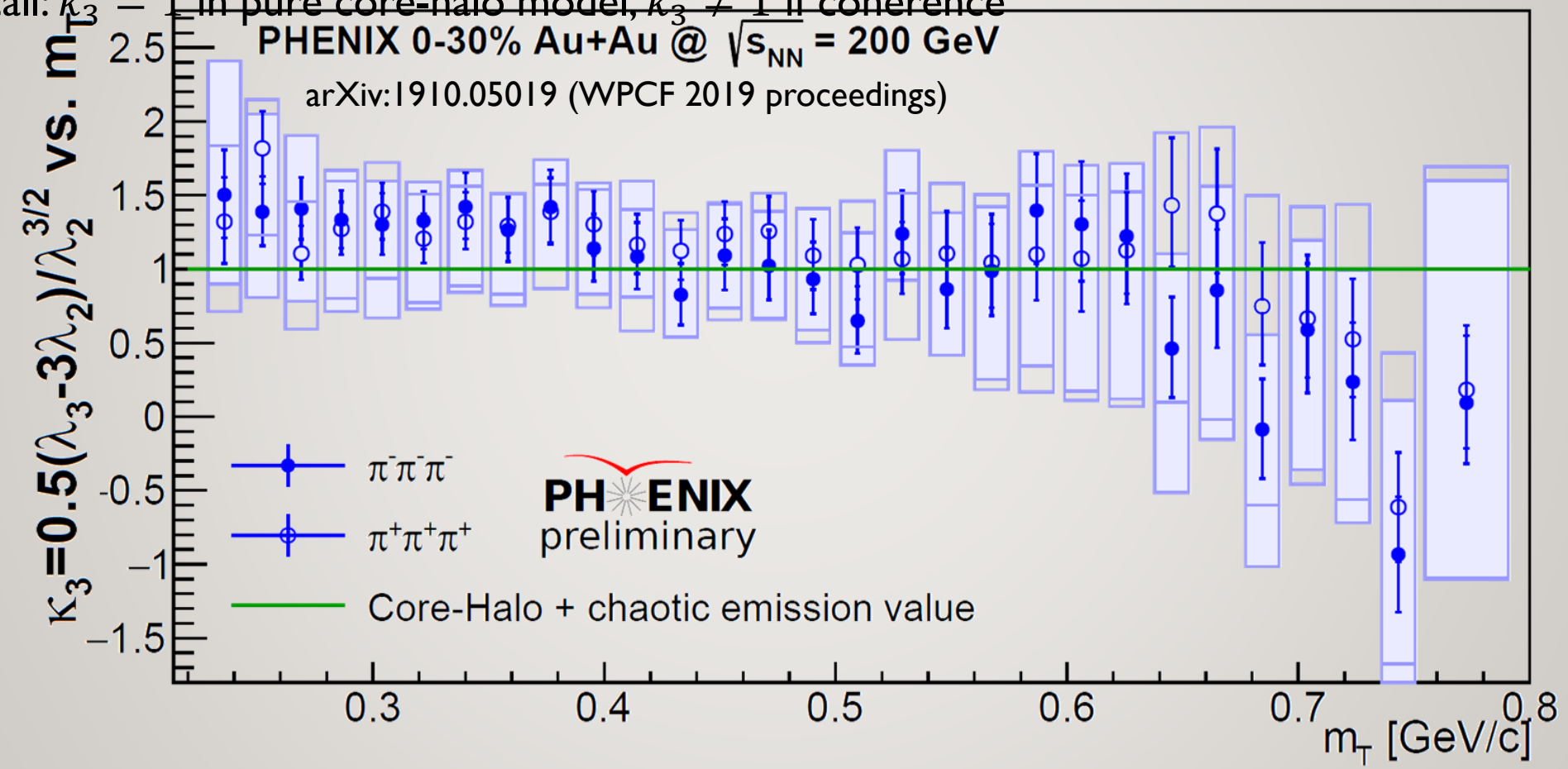
$$\lambda_3 = 2f_C^3 [(1 - p_C)^3 + 3p_C(1 - p_C)^2] + 3f_C^2 [(1 - p_C)^2 + 2p_C(1 - p_C)]$$

- Introduce core-halo independent parameter $\kappa_3 = \frac{\lambda_3 - 3\lambda_2}{2\sqrt{\lambda_2}^3}$
 - does not depend on f_C
 - $\kappa_3 = 1$ if no coherence
- Finite meson sizes?
 - Gavrilik, SIGMA 2 (2006) 074 [hep-ph/0512357]
- Phase shift (a la Aharonov-Bohm) in hadron gas?
 - Random fields create random phase shift, on average distorts Bose-Einstein correlations
Csanád et al., Gribov-90 (2021) 261-273 [arXiv:2007.07167]



TEST OF CORE-HALO MODEL / COHERENCE

- Recall: $\kappa_3 = 1$ in pure core-halo model, $\kappa_3 \neq 1$ if coherence





SHAPE ANALYSIS AT STAR

- Gaussian fit: unacceptable description
- Levy fit somewhat better, but still additional effects present
- Low Q behavior not captured by any of the two

