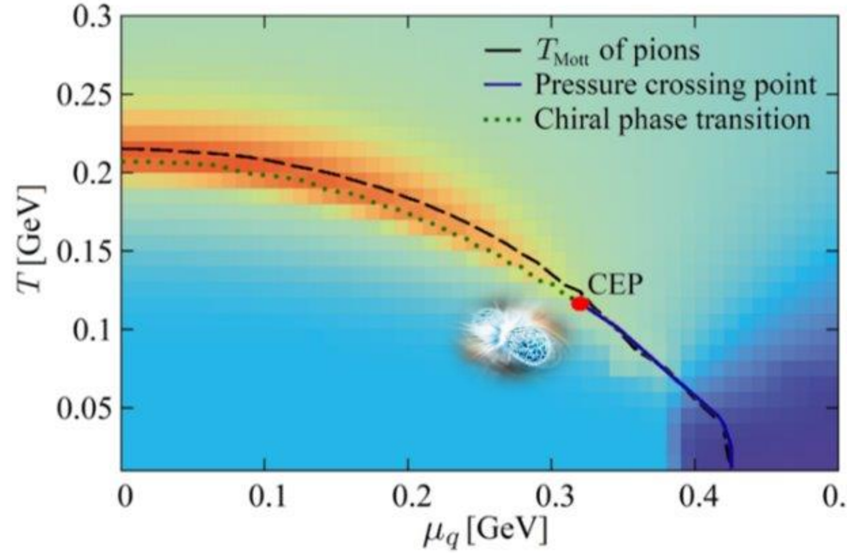
Transport coefficients and comparison of the QGP evolution in transport and hydro approach

Olga Soloveva

P. Moreau, L. Oliva, V. Voronyuk,
V. Kireyeu, T. Song,
E. Bratkovskaya

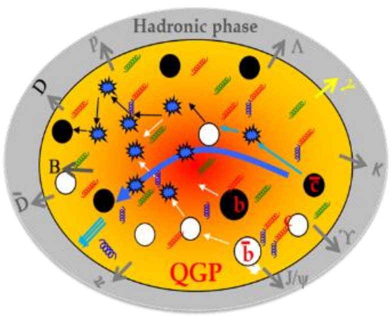


Exploring Quark-Gluon Plasma through soft and hard probes

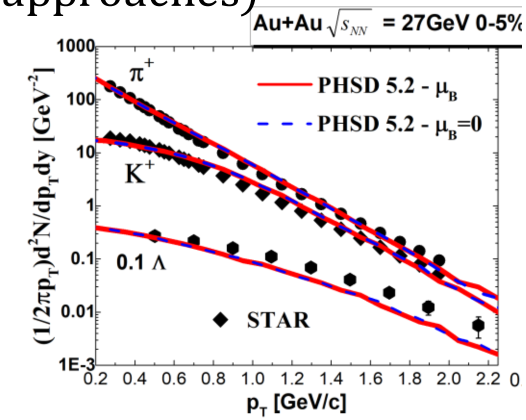
30 May, 2023

Properties of QGP: transport coefficients

! One has to specify transport and microscopic properties as well as EoS for theoretical simulations of HICs (hydro / transport approaches)



Evolution of QCD medium



Hybrid simulations:

vHLLE/Music+UrQMD/SMASH

Iu.A. Karpenko, P. Huovinen, H. Petersen and M. Bleicher PRC 91 (2015), 064901.

CORE-CORONA – EPOS (K. Werner talk), DCCI(Y. Kanakuba)

MUFFIN – next talk

Transport simulations with QGP phase:

Catania transport – QuasiParticle Model

F. Scardina, S. K. Das, V. Minissale, S. Plumari, and V. Greco, PRC 96, 044905 (2017).

AMPT – PNJL EoS (Mean field potentials)

K.J. Sun, C. M. Ko, and Z.-W. Lin, PRC 103(2021)

PHSD – off-shell transport approach derived from Kadanoff-Baym many-body theory (Quantumm Boltzmann) with hadronic and QGP phase – 2PI Dynamical QuasiParticle Model

W. Cassing, E. Bratkovskaya, PRC 78 (2008) 034919

P. Moreau, O. S , L. Oliva, T. Song, W. Cassing, E. Bratkovskaya, PRC 100 (2019) , 014911;

O. S, P. Moreau, L. Oliva, V. Voronyuk, V. Kireyeu, T. Song, E. Bratkovskaya, Particles 3 (2020)

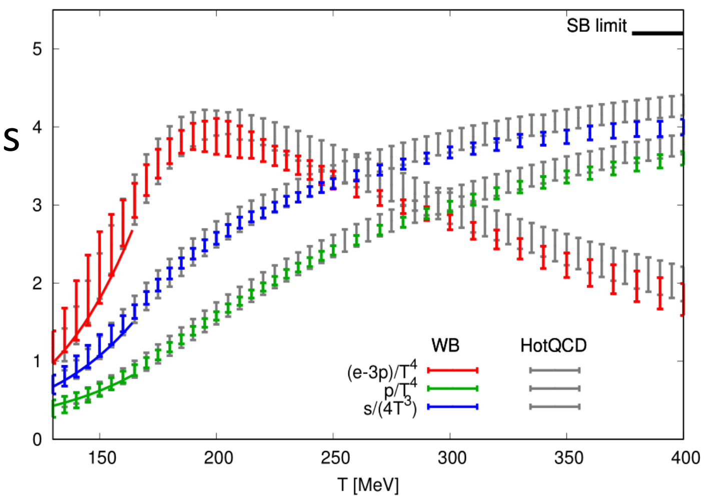
Properties of QGP: Terra incognita

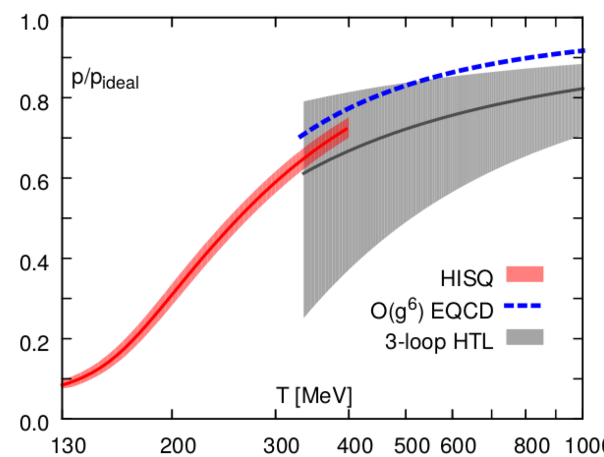
On practice: effective models
for QGP

EoS(ε,n)
$\boldsymbol{\sigma(\sqrt{s}, m_q, m_q, T, \mu_B)}$
$\mathbf{m}(\boldsymbol{T}, \boldsymbol{\mu_B})$

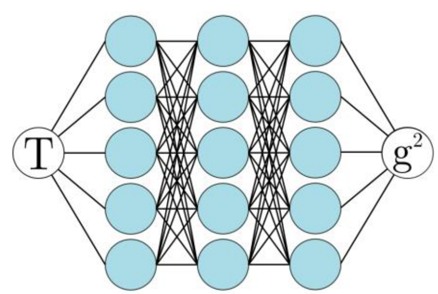
! QPM enables to estimate simultaneously of the EoS and transport coefficients also including jet and charm coefficients (talk by I Grishmanovskii)

Phenomenological models are based on lattice QCD thermodynamics





ML facilitated QGP description – by minimizing the loss function (can be chosen in various forms)
Output: spectral function, coupling constant – which later can be used for the extraction of transport coefficients



O.S., A. Palermo, E. Bratkovskaya in preparation

Properties of QGP: transport coefficients

Hydrodynamics

$$\begin{cases} \partial_\mu T^{\mu\nu} = 0 \quad T^{\mu\nu} = -P g^{\mu\nu} + w u^\mu u^\nu + \Delta T^{\mu\nu} \\ \partial_\mu J^\mu_B = 0 \quad J^\mu_B = n_B u^\mu + \Delta J^\mu_B \end{cases}$$

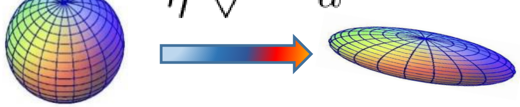
$$\Delta T^{\mu\nu} \rightarrow \boxed{\eta}\left(D^\mu u^\nu + D^\nu u^\mu + \tfrac{2}{3}\Delta^{\mu\nu}\partial_\rho u^\rho\right) - \boxed{\zeta}\Delta^{\mu\nu}\partial_\rho u^\rho$$

$$\Delta J^\mu_B = \boxed{\kappa_B} D^\mu\left(\frac{\mu_B}{T}\right)$$

Transport coefficients:

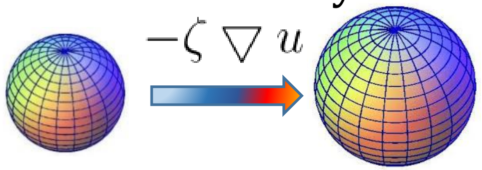
Shear viscosity

$\eta \nabla^{<\mu} u^{\nu>}$



Bulk viscosity

$-\zeta \nabla u$

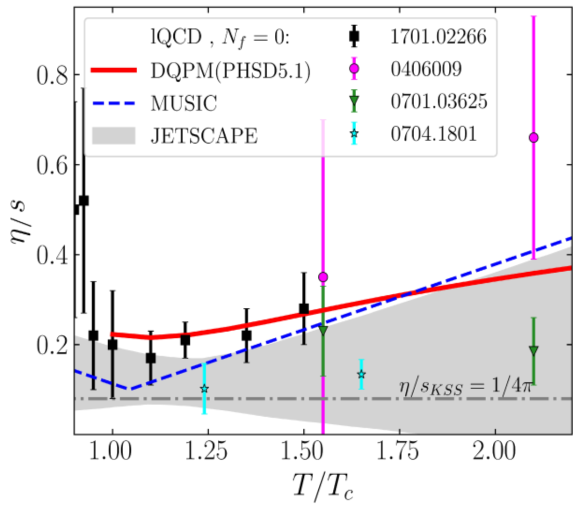


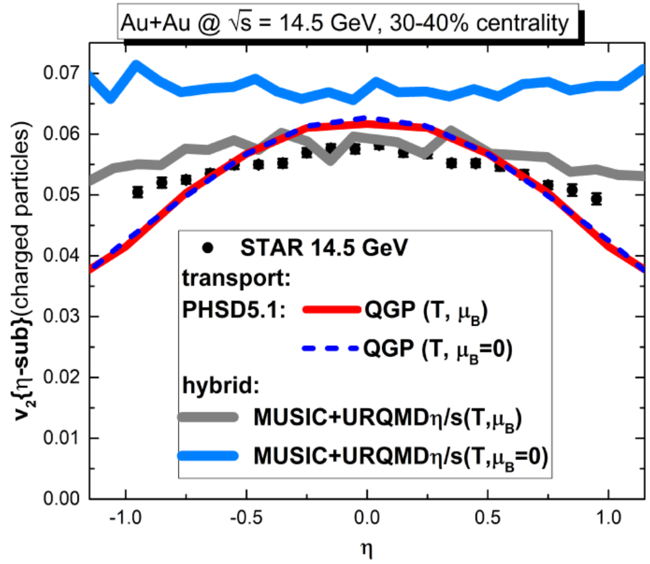
(B, Q, S) diffusion coefficients

$\kappa_q \nabla \frac{\mu_q}{T}$

Evolution of QGP: **!** same EoS but different transport coefficients

Transport coefficients can serve a bridge for comparison transport and hydro

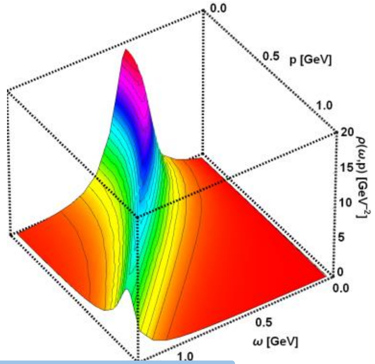




MUSIC: C. Shen, S.Alzhrani, PRC 102 (2020) 1, 014909

Dynamical Quasi-Particle Model

The QGP phase is described in terms of strongly-interacting quasiparticles - quarks and gluons with Lorentzian spectral functions:



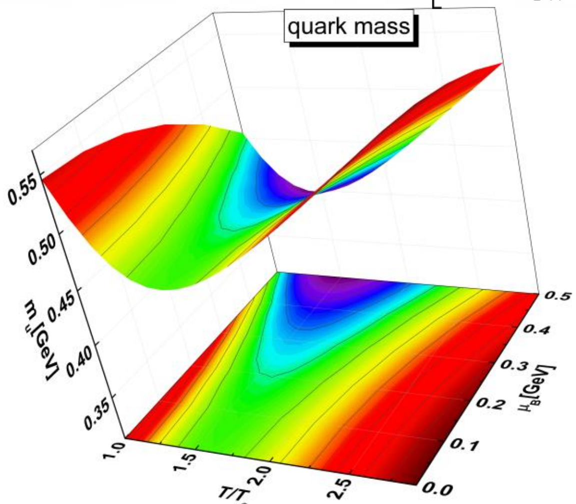
$$\rho_j(\omega,\mathbf{p}) = \frac{\gamma_j}{\tilde{E}_j}\left(\frac{1}{(\omega-\tilde{E}_j)^2+\gamma_j^2} - \frac{1}{(\omega+\tilde{E}_j)^2+\gamma_j^2}\right)$$

$$\equiv \frac{4\omega\gamma_j}{\left(\omega^2-\mathbf{p}^2-M_j^2\right)^2+4\gamma_j^2\omega^2}$$

resummed propagators: $\Delta_i(\omega,\mathbf{p}) = \frac{1}{\omega^2-\mathbf{p}^2-\Pi_i}$ & self-energies: $\Pi_i = m_i^2 - 2i\gamma_i\omega$

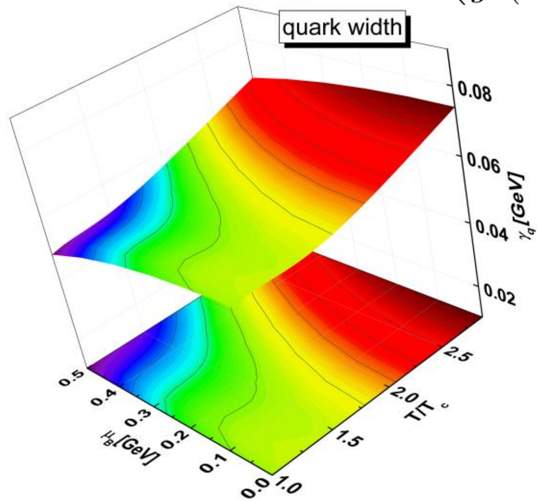
Re $\mathbf{\Pi_i}$: thermal mass (M_g, M_q)

$$m^2_{q(\bar{q})}(T,\mu_B) = C_q\frac{g^2(T,\mu_B)}{4}T^2\left[1+\left(\frac{\mu_B}{3\pi T}\right)^2\right]$$



Im $\mathbf{\Pi_i}$: interaction width (γ_g, γ_q)

$$\gamma_j(T,\mu_B) = \frac{1}{3}C_j\frac{g^2(T,\mu_B)T}{8\pi}\ln\left(\frac{2c_m}{g^2(T,\mu_B)}+1\right)$$



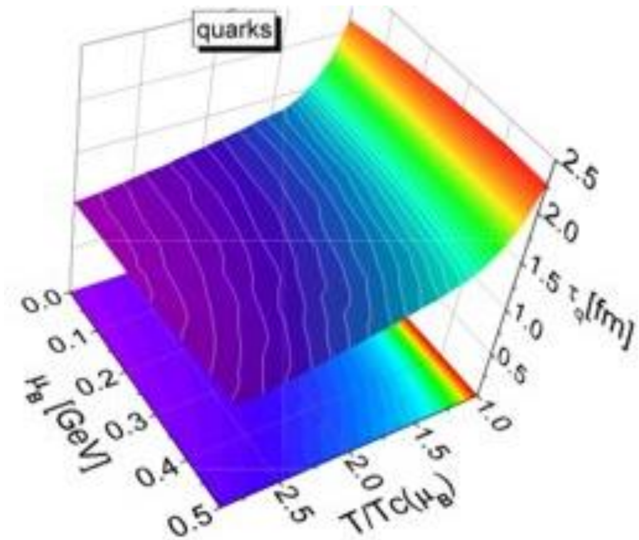
Peshier, Cassing, PRL 94 (2005) 172301; Cassing, NPA 791 (2007) 365; H. Berrehrah, E. Bratkovskaya, T. Steinert, W. Cassing, Int. J. Mod. Phys. E 25 (2016), 164200; P. Moreau, O. Soloveva, L. Oliva, T. Song, W. Cassing, E. Bratkovskaya, PRC 100 (2019), 014911;

Transport coefficients at finite μ_B

$$\eta^{\rm RTA}(T,\mu_B) = \frac{1}{15T}\sum_{i=q,\bar{q},g}\int \frac{d^3p}{(2\pi)^3}\frac{\mathbf{p}^4}{E_i^2}\tau_i(\mathbf{p},T,\mu_B)\, d_i(1\pm f_i)f_i$$

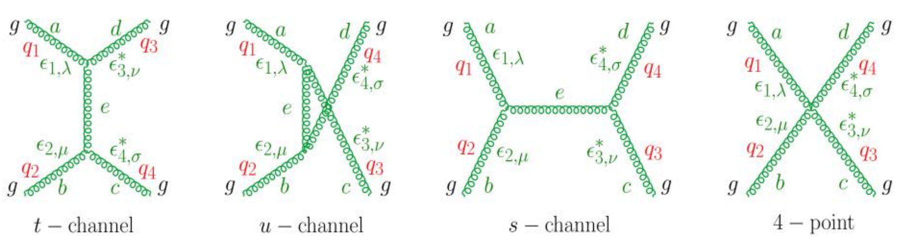
$$\tau_i(\mathbf{p},T,\mu_B) = \frac{1}{\Gamma_i(\mathbf{p},T,\mu_B)}$$

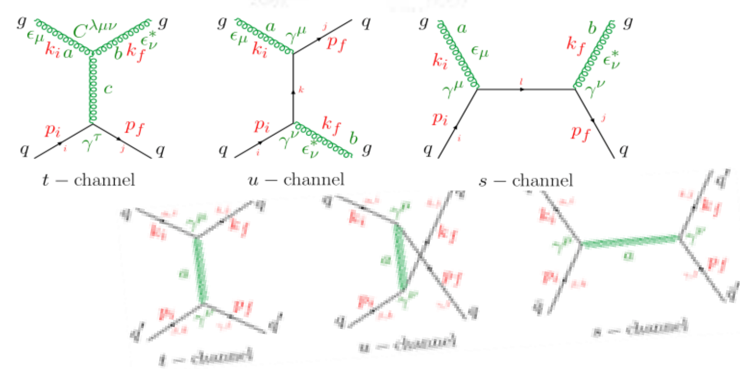
$$\langle \mathcal{O}\rangle^{\rm on} = \frac{1}{2E_i}\sum_{j=q,\bar{q},g} d_j f_j \int \frac{d^3p_j}{(2\pi)^3 2E_j} \times \int \frac{d^3p_1}{(2\pi)^3 2E_1}\int \frac{d^3p_2}{(2\pi)^3 2E_2} \times (1\pm f_1)(1\pm f_2)\mathcal{O}|\overline{\mathcal{M}}|^2(2\pi)^4\delta^{(4)}(p_i+p_j-p_1-p_2)$$



- $\mathcal{O}=1$ – to the scattering rate Γ,
- $\mathcal{O}=(p_T^2-p_T'^2)$ – jet transport coefficient $\hat{q}$,
- $\mathcal{O}=(E-E')$ – energy loss $\Delta E = dE/dx$,
- $\mathcal{O}=(p_L-p_L')$ – drag coefficient $\mathcal{A}$,

2 ↔2 scatterings



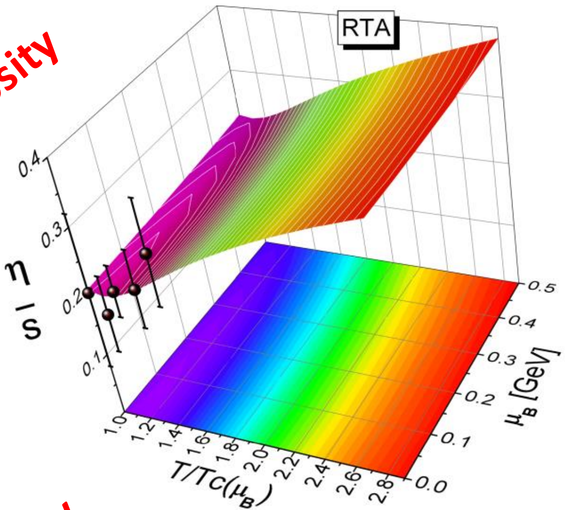


O. S., P. Moreau and E. Bratkovskaya, PRC 101 (2020), 045203

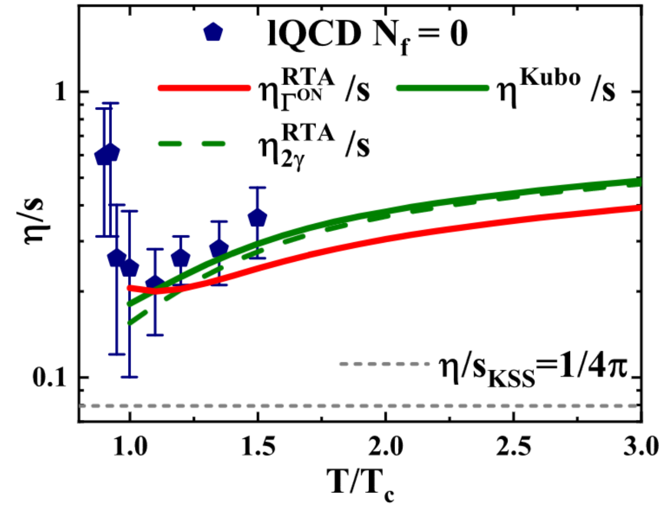
- Good agreement with lQCD predictions and Bayesian estimates
- Light increase with μ_B in the crossover region for viscosities and electric conductivity

Transport coefficients at finite μ_B

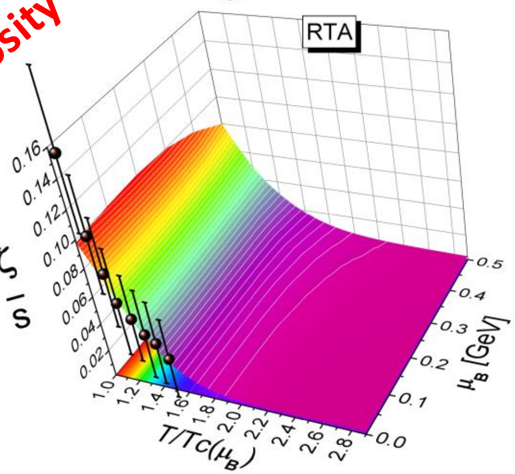
Specific shear viscosity

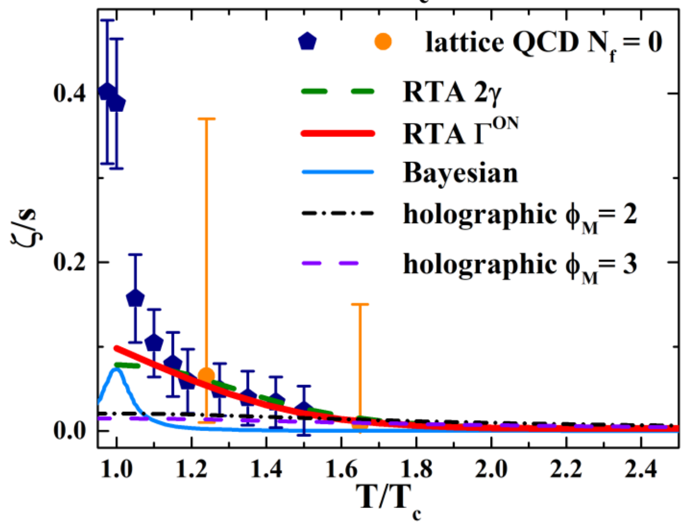


μ_B



Specific bulk viscosity





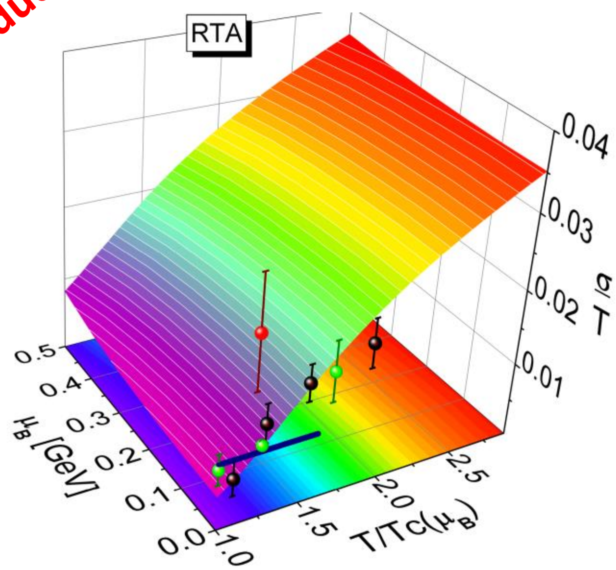
O. S., P. Moreau and E. Bratkovskaya, PRC 101 (2020), 045203

- Good agreement with lQCD predictions and Bayesian estimates
- Light increase with μ_B in the crossover region for viscosities and electric conductivity

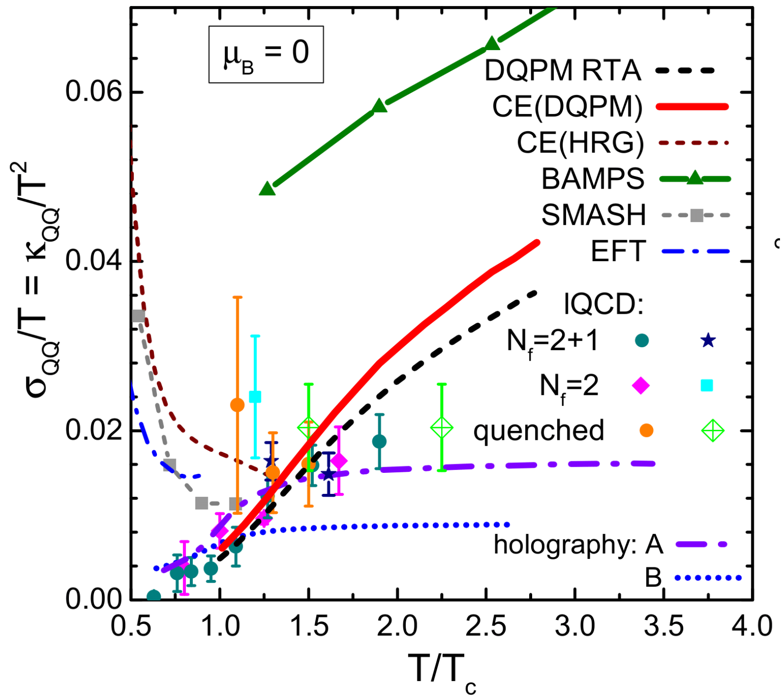
Transport coefficients at finite μ_B

$$\sigma qq'(T,\mu_q) = \frac{1}{3T}\sum_{i=q,\bar{q}} q_i \int \frac{d^3p}{(2\pi)^3}\frac{\mathbf{p}^2}{E_i^2}\tau_i(\mathbf{p},T,\mu_q) \times \left(\frac{E_i n_{q'}}{\epsilon+p} - q'_i\right) d_i(1\pm f_i) f_i$$

Electric conductivity



μ_B

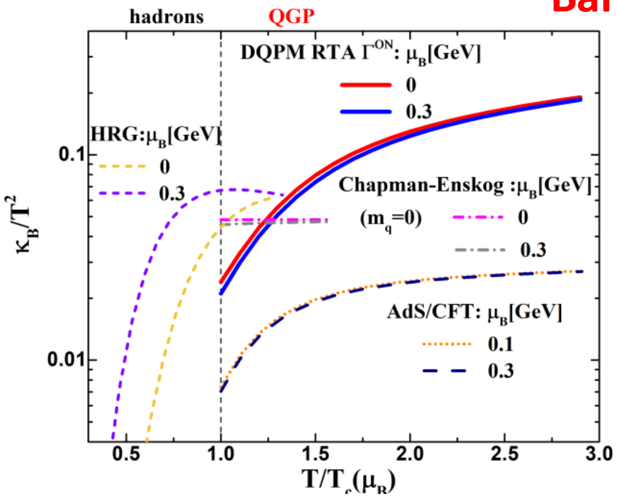


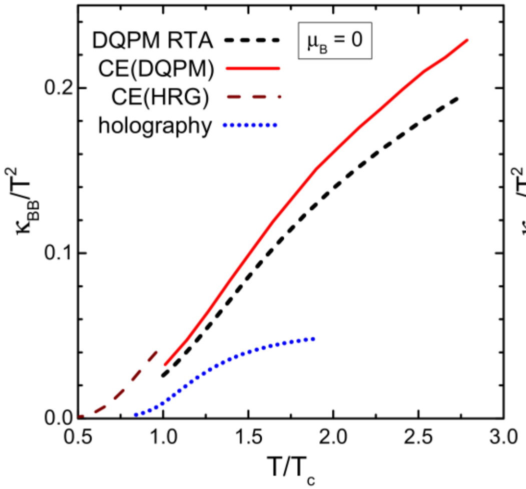
- Light increase with μ_B in the crossover region for shear and bulk viscosities and electric conductivity
- Electric conductivity consist mostly of quark sector!

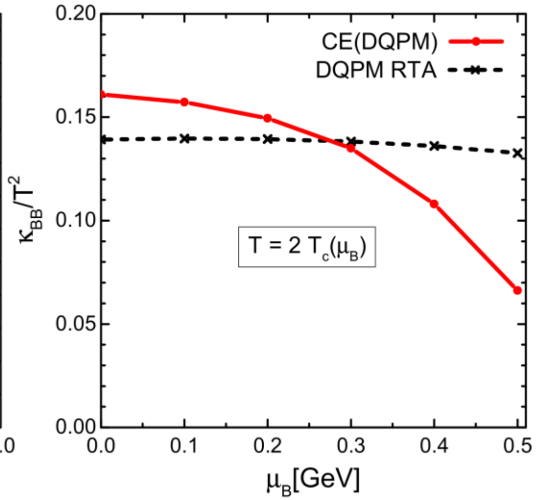
O. S., P. Moreau and E. Bratkovskaya, PRC 101 (2020), 045203

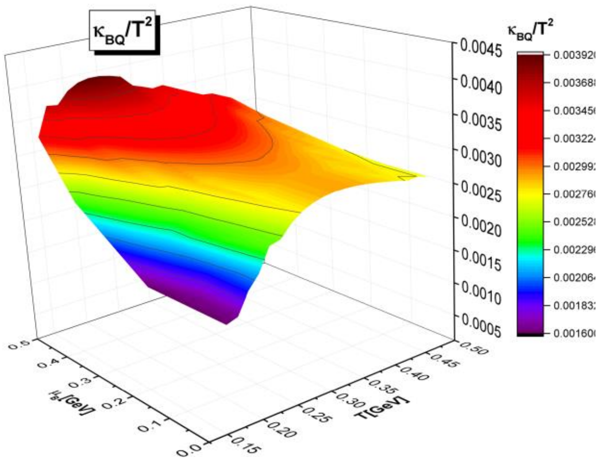
Transport coefficients at finite μ_B

Baryon diffusion coefficient









+ Full diffusion coefficient matrix has been evaluated $\sigma_q = \kappa_q/T$

$$\begin{pmatrix} j_B^\mu \\ j_Q^\mu \\ j_S^\mu \end{pmatrix} = \begin{pmatrix} \kappa_{BB} & \kappa_{BQ} & \kappa_{BS} \\ \kappa_{QB} & \kappa_{QQ} & \kappa_{QS} \\ \kappa_{SB} & \kappa_{SQ} & \kappa_{SS} \end{pmatrix} \cdot \begin{pmatrix} \nabla^\mu \alpha_B \\ \nabla^\mu \alpha_Q \\ \nabla^\mu \alpha_S \end{pmatrix}$$

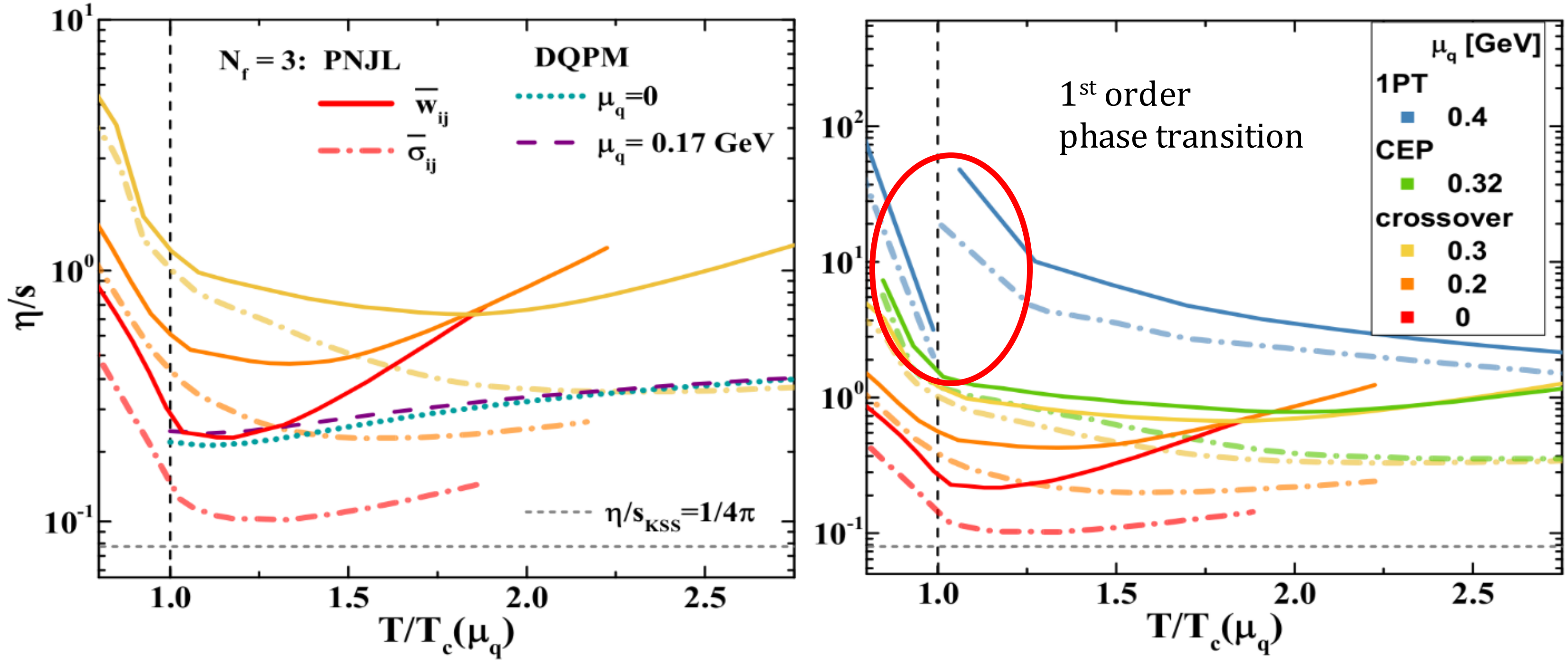
J. A. Fotakis, O. S., C. Greiner, O. Kaczmarek and E. Bratkovskaya PRD 104 (2021) , 034014

- Light increase with μ_B in the crossover region for shear and bulk viscosities and electric conductivity
- Baryon diffusion coefficients decrease with μ_B

O. S., P. Moreau and E. Bratkovskaya, PRC 101 (2020), 045203

Specific shear viscosity at high μ_B

PNJL vs DQPM



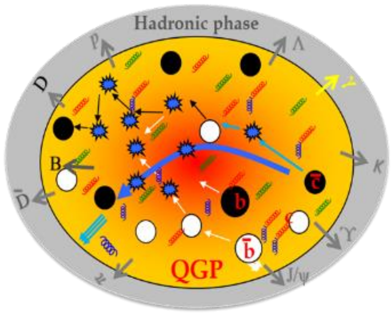
O. S., D. Fuseau, J. Aichelin and E. Bratkovskaya, PRC 103 (2021) no.5, 054901

- CEP: $(\mathrm{T}, \mu_B) = (110{,}960)$ MeV , $\mu_B/\mathrm{T} = 8.73$
- 1st order PT at high μ_B
- same symmetries for the quarks as QCD

Chiral masses (M_l, M_s)

Drastic change of T–dependence for all transport coefficients after 1st order phase transition

Modelling HICs: PHSD



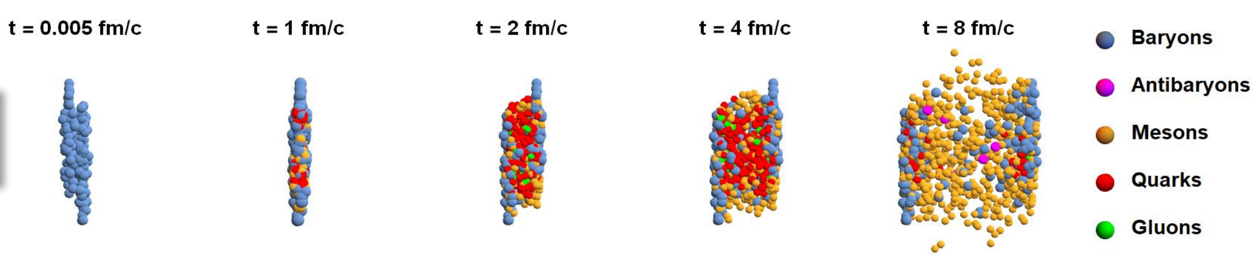
QGP out-of equilibrium ←→ HIC

Parton-Hadron-String-Dynamics (PHSD)

Non-equilibrium microscopic transport approach for the description of strongly-interacting hadronic and partonic matter created in heavy-ion collisions

Dynamics: based on the solution of generalized off-shell transport equations derived from Kadanoff-Baym many-body theory

200 GeV
b=2fm



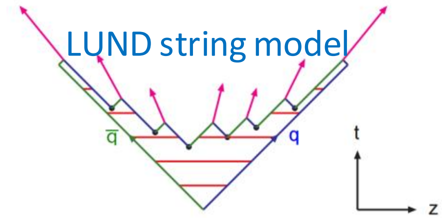
W. Cassing, E. Bratkovskaya, PRC 78 (2008) 034919; NPA831 (2009) 215; W. Cassing, EPJ ST 168 (2009) 3; …….;
P. Moreau, O. Soloveva , L. Oliva, T. Song, W. Cassing, E. Bratkovskaya, PRC 100 (2019) , 014911;
O. Soloveva, P. Moreau, L. Oliva, V. Voronyuk, V. Kireyeu, T. Song, E. Bratkovskaya, Particles 3 (2020), 178-192;….

Stages of collisions in PHSD

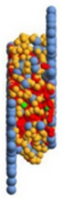
Initial A+A collision

String formation in primary NN collisions

decays to pre-hadrons (baryons and mesons)



Partonic phase



- Formation of a QGP state if $\varepsilon > \varepsilon_{critical}$:
  Dissolution of pre-hadrons to partons DQPM

$$\epsilon_c(WB) = 0.4\ GeV/fm^3$$

massive quarks/gluons and mean-field energy

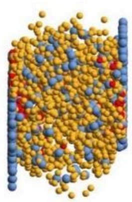
(quasi-)elastic collisions :

$$q+q \to q+q \qquad g+q \to g+q$$
$$q+\bar{q} \to q+\bar{q} \qquad g+\bar{q} \to g+\bar{q}$$
$$\bar{q}+\bar{q} \to \bar{q}+\bar{q} \qquad g+g \to g+g$$

inelastic collisions:

$$q+\bar{q} \to g$$
$$g \to q+\bar{q}$$

Hadronization

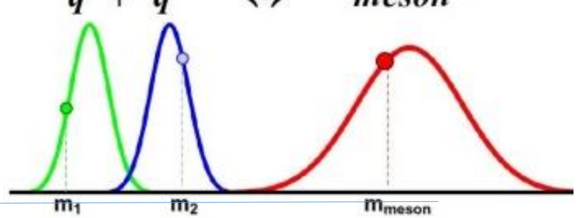


Hadronization to colorless off-shell mesons and baryons

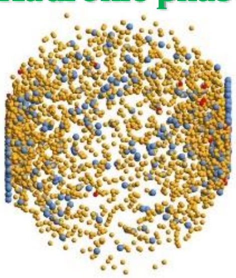
$$g \to q+\bar{q}, \qquad q+\bar{q} \leftrightarrow meson\ ('\ string\ ')$$

$$q+q+q \leftrightarrow baryon\ ('\ string\ ')$$

Strict 4-momentum and quantum number conservation



Hadronic phase



Hadron-string interactions – off-shell HSD

W. Cassing, E. Bratkovskaya, PRC 78 (2008) 034919; NPA831 (2009) 215;
W. Cassing, EPJ ST 168 (2009) 3

PHSD5: extraction of T and μ_B

For each space-time cell of the PHSD:

$$T^{\mu\nu}(x) = \sum_i \int_0^\infty \frac{d^3p_i}{(2\pi)^3} f_i(E_i) \frac{p_i^\mu p_i^\nu}{E_i}$$ → **Diagonalize in LRF** → ε^{PHSD}

1. Calculate the local energy density ε^{PHSD} and baryon density n_B^{PHSD}

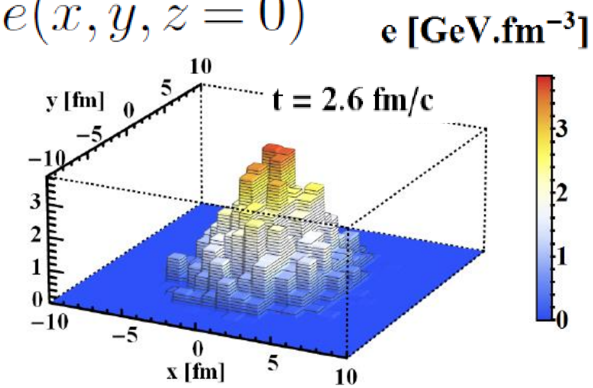
2. use lQCD relations (up to 6th order):

$$\frac{n_B}{T^3} \approx \chi_2^B(T)\left(\frac{\mu_B}{T}\right) + \ldots$$

$$\Delta\epsilon/T^4 \approx \frac{1}{2}\left(T\frac{\partial\chi_2^B(T)}{\partial T} + 3\chi_2^B(T)\right)\left(\frac{\mu_B}{T}\right)^2 + \ldots$$

Use baryon number susceptibilities χ_n from lQCD
Obtain T, μ_B by solving sistem of coupled eqs using ε, n_B

$e(x, y, z = 0)$



Input:

ε^{PHSD} and n_B^{PHSD}

Output:

T, μ_B

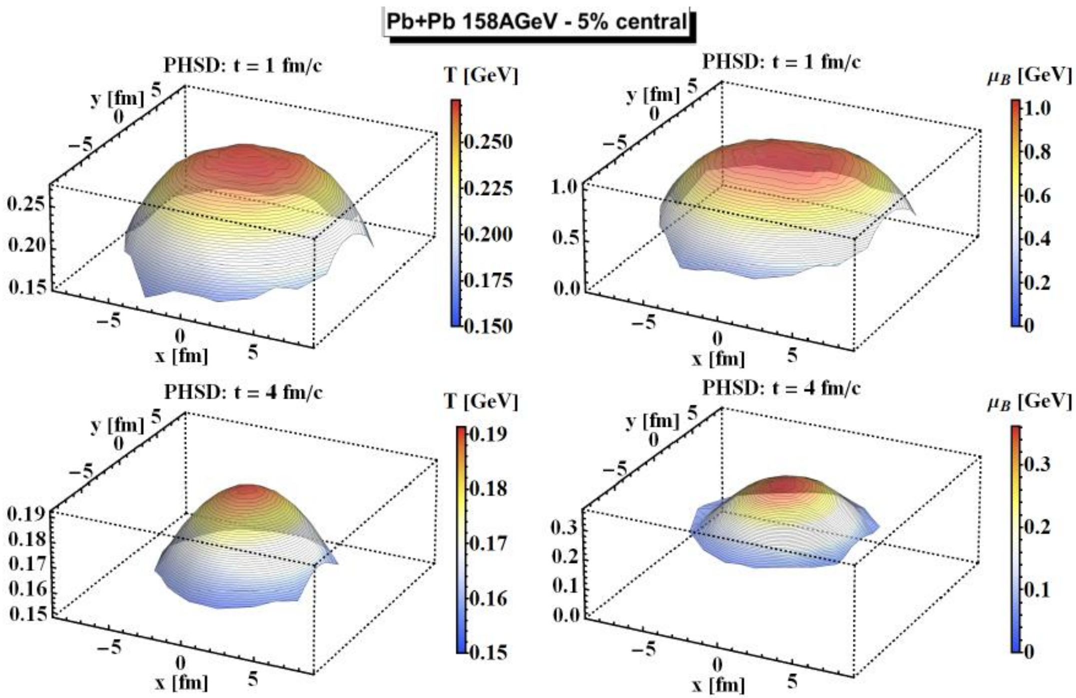
for details see P. Moreau, O. S., L. Oliva, T. Song, W. Cassing, E. Bratkovskaya
arXiv:1903.10157, PRC 100 (2019) no. 1, 014911

PHSD: QGP evolution in HICs

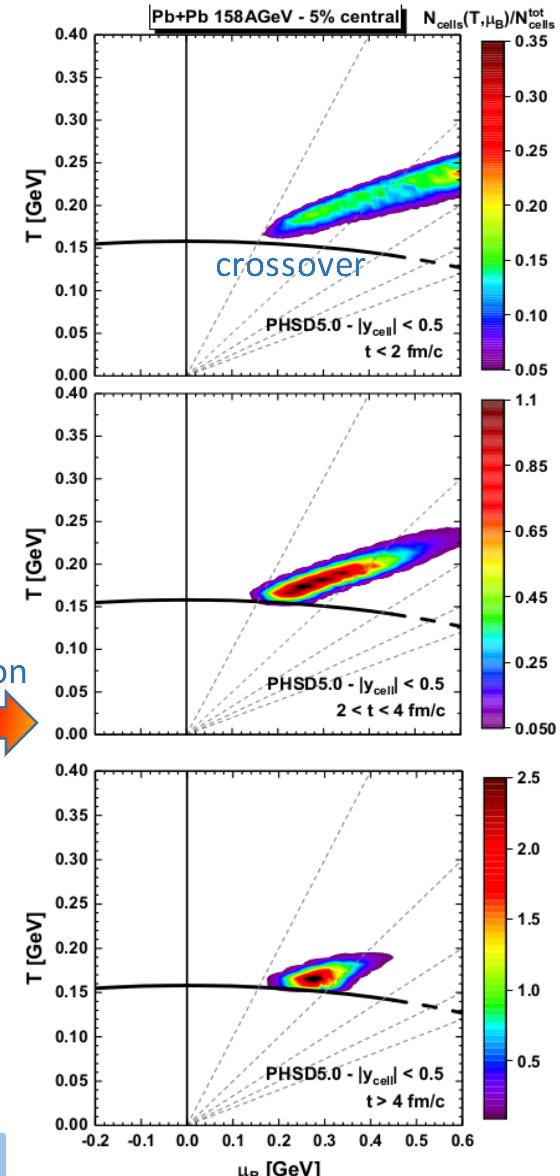
Input:
ε^{PHSD} and n_B^{PHSD}

Output:
T, μ_B

The T -profile in (x;y) & μ_B profile in (x;y)
at midrapidity ($|y_{cell}|$< 1) at fixed times(1 and 4 fm/c)



time evolution

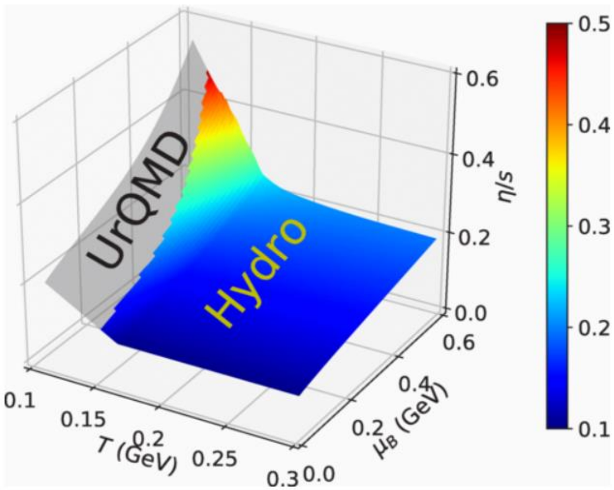


Path through the phase diagram is not trivial and localized

Evolution of QGP: still big difference

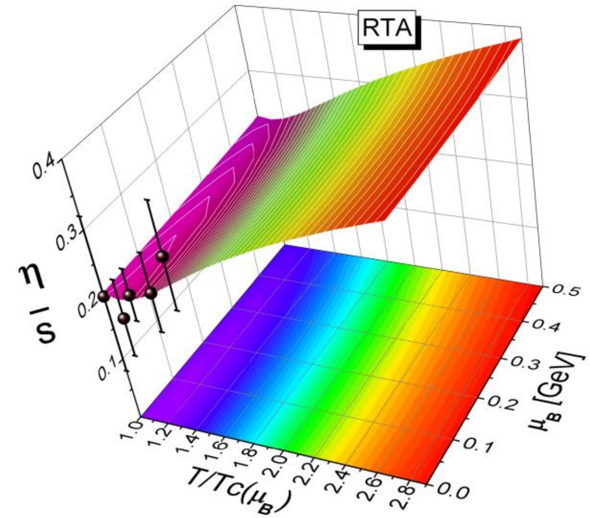
MUSIC: $\tau_0 = 2.2 fm/c$

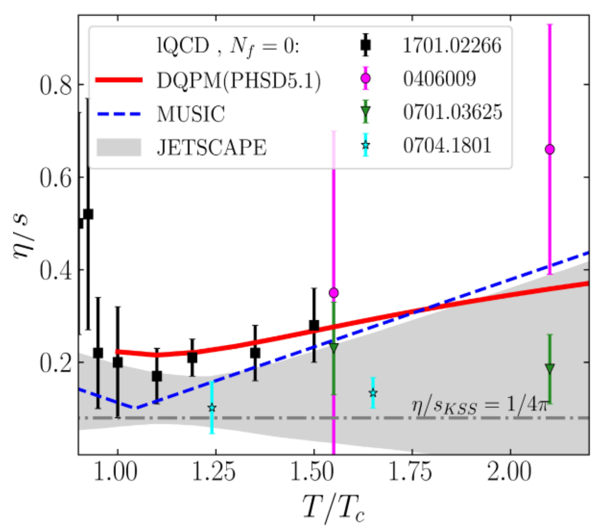
$\epsilon_{SW} = 0.26\ GeV/fm^3$

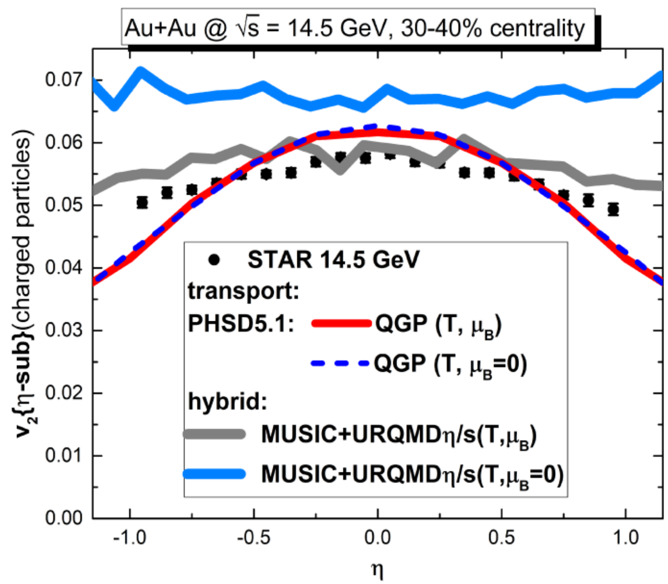


PHSD: QGP phase

$\epsilon_c(WB) = 0.4\ GeV/fm^3$







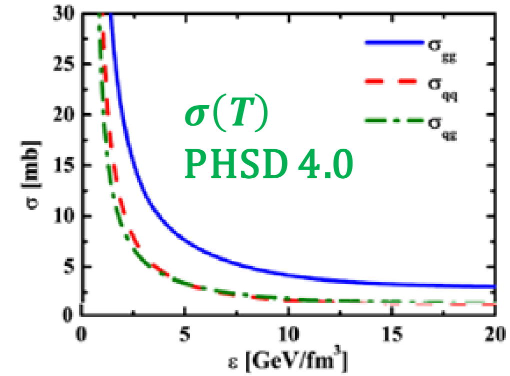
MUSIC: C. Shen, S.Alzhrani, PRC 102 (2020) 1, 014909

PHSD

- **PHSD 4.0 : only isotropic $\sigma(T)$ and $\rho(T)$**

parton cross sections

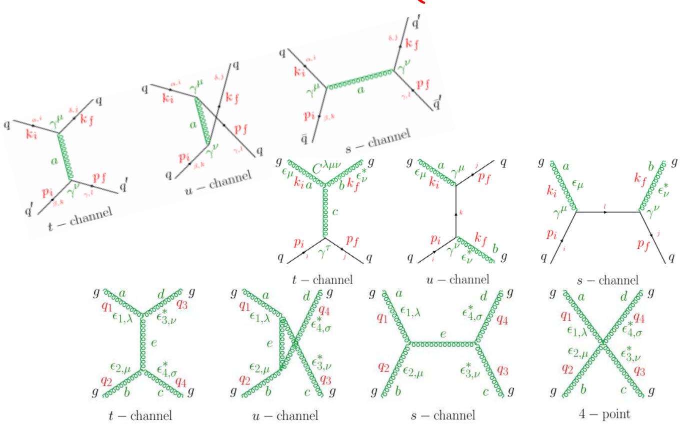
parton spectral function (masses and widths)

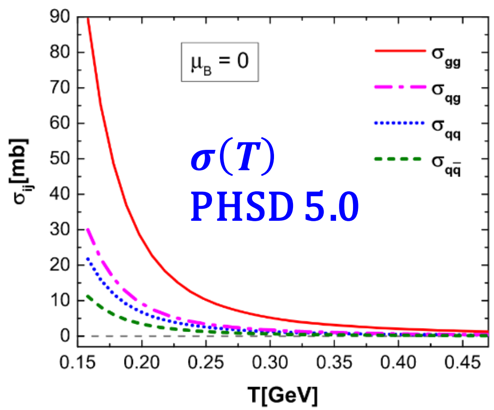


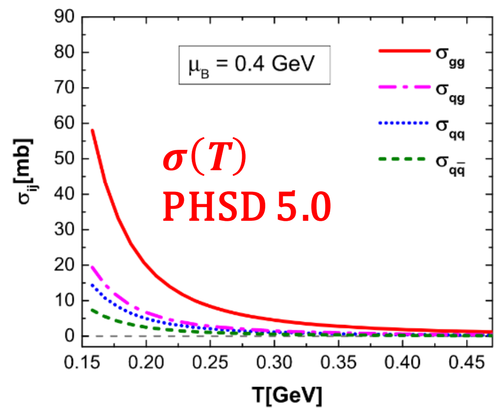
new PHSD 5 : angular dependence of $d\sigma/d\cos\theta$

- **PHSD 5.0 : with $\sigma(\sqrt{s}, m_1, m_2, T, \mu_B = 0)$ and $\rho(T, \mu_B = 0)$**

- **PHSD 5.0 : with $\sigma(\sqrt{s}, m_1, m_2, T, \mu_B)$ and $\rho(T, \mu_B)$**







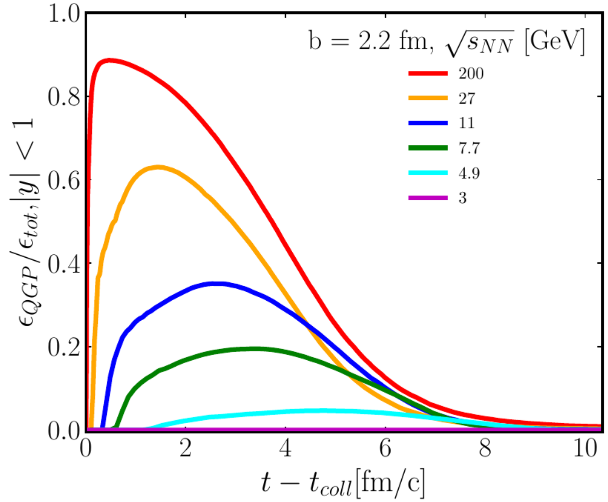
P. Moreau, O. Soloveva , L. Oliva, T. Song, W. Cassing, E. Bratkovskaya, PRC 100 (2019) , 014911;
O. Soloveva, P. Moreau, L. Oliva, V. Voronyuk, V. Kireyeu, T. Song, E. Bratkovskaya, Particles 3 (2020), 178-192

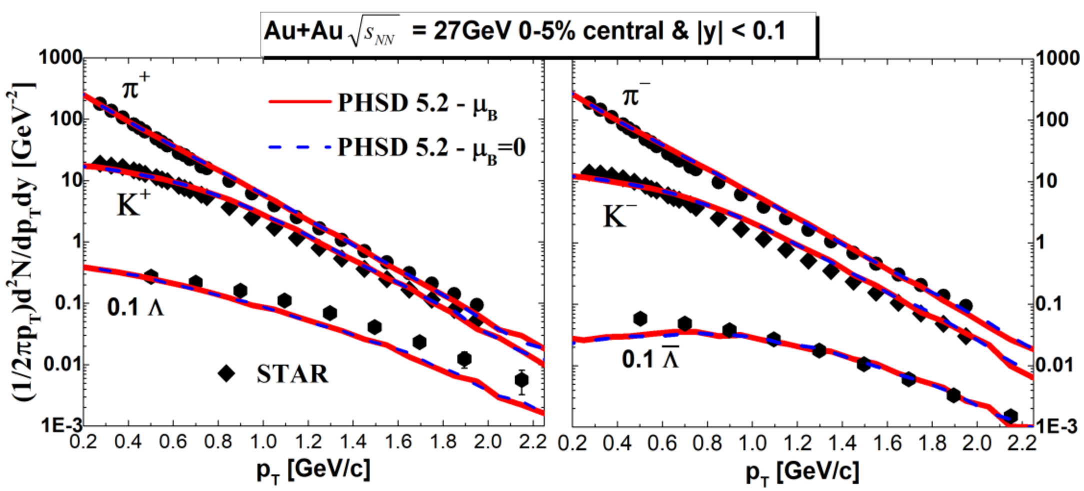
Results for ($\sqrt{s_{NN}}$ = 200 GeV - 7 GeV)

- No visible effects on p_T-spectra, dN/dy of μ_B-dependence
- Small effect of the angular dependence of **dσ/dcosθ**

! QGP fraction is small at low $\sqrt{s_{NN}}$

at high $\sqrt{s_{NN}}$ - low μ_B





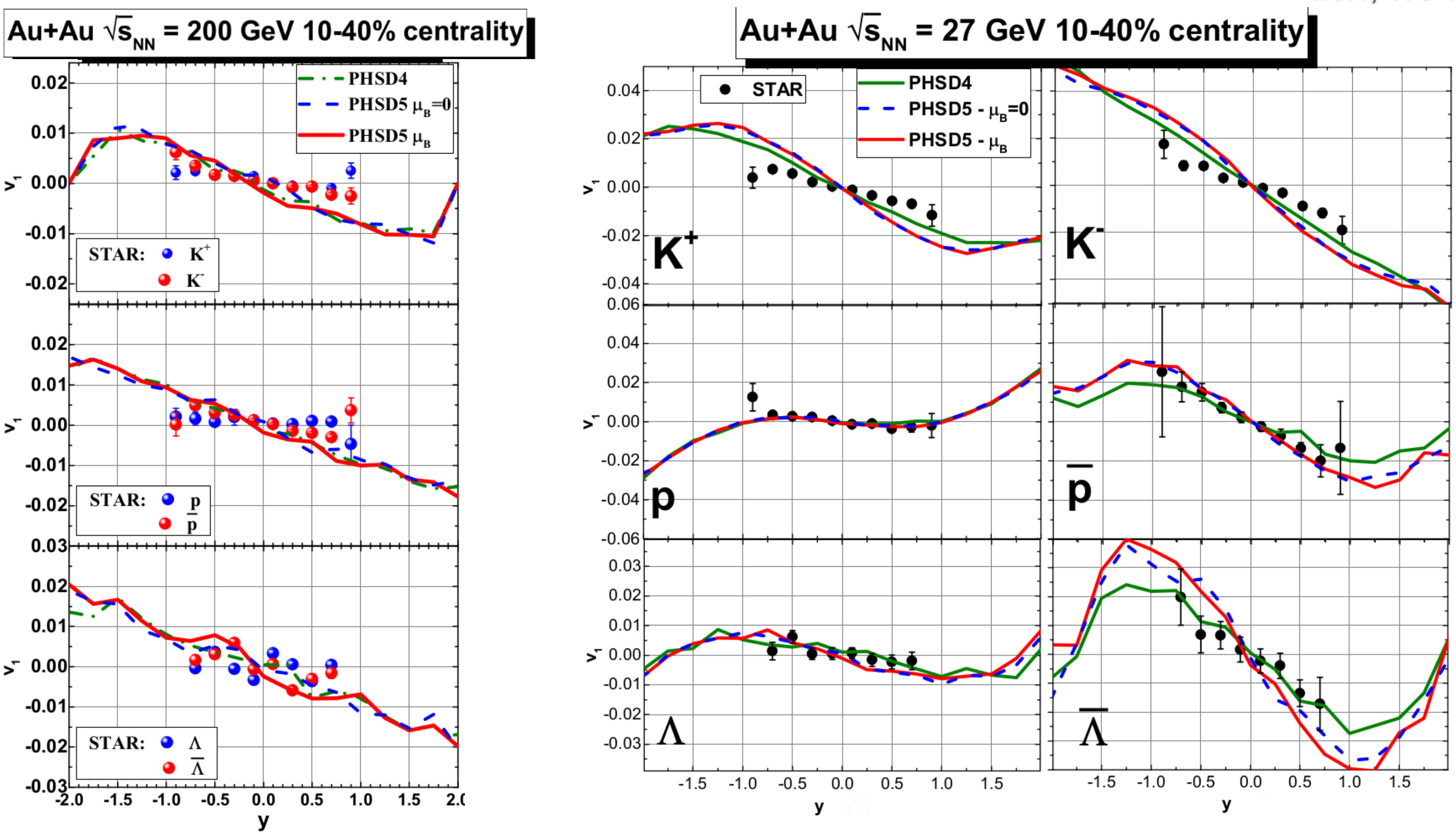
P. Moreau, O. Soloveva , L. Oliva, T. Song, W. Cassing, E. Bratkovskaya, PRC 100 (2019) , 014911;
O. Soloveva, P. Moreau, L. Oliva, V. Voronyuk, V. Kireyeu, T. Song, E. Bratkovskaya, Particles 3 (2020), 178-192

Directed flow ($\sqrt{s_{NN}} = 200\ GeV\ vs\ 27\ GeV$)

$$v_1 = \left\langle \frac{p_x}{p_T} \right\rangle$$

- Influence of the QGP dynamics on final particles observables
- Weak μ_B –dependence – small fraction of QGP or low μ_B

Particles 3 (2020) no.1, 178-192

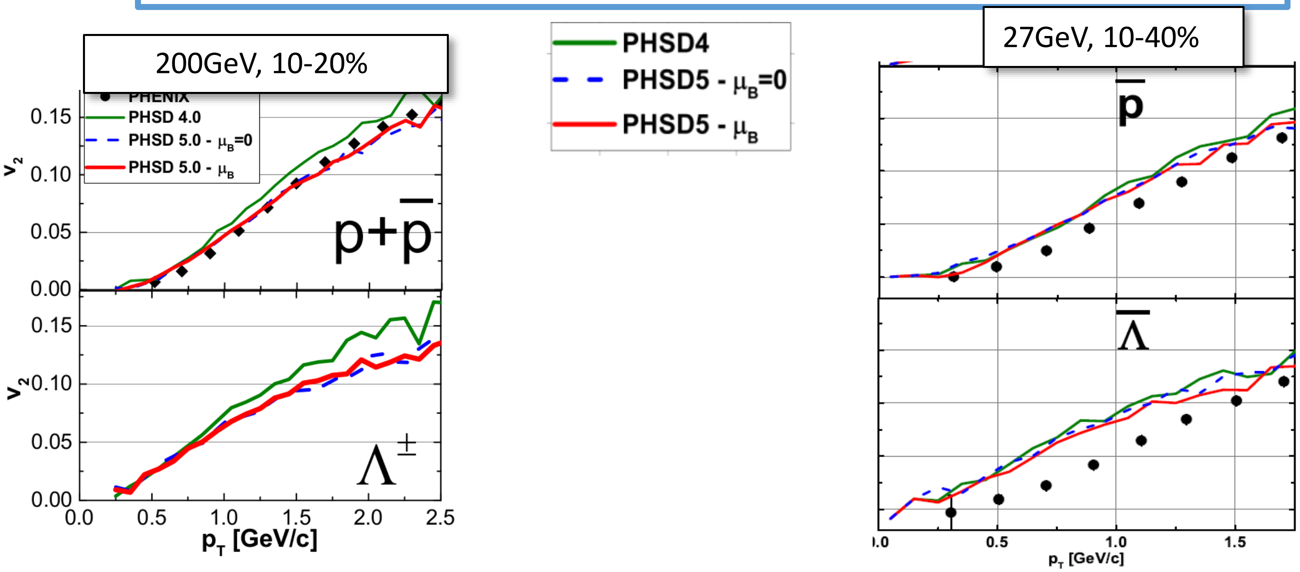


Elliptic flow ($\sqrt{s_{NN}} = 200\ GeV\ vs\ 27 GeV$)

$$v_2 = \left\langle \frac{p_x^2 - p_y^2}{p_T^2} \right\rangle$$

- Weak μ_B -dependence
- Small effect of the angular dependence of **dσ/dcosθ**
- Strong flavor dependence

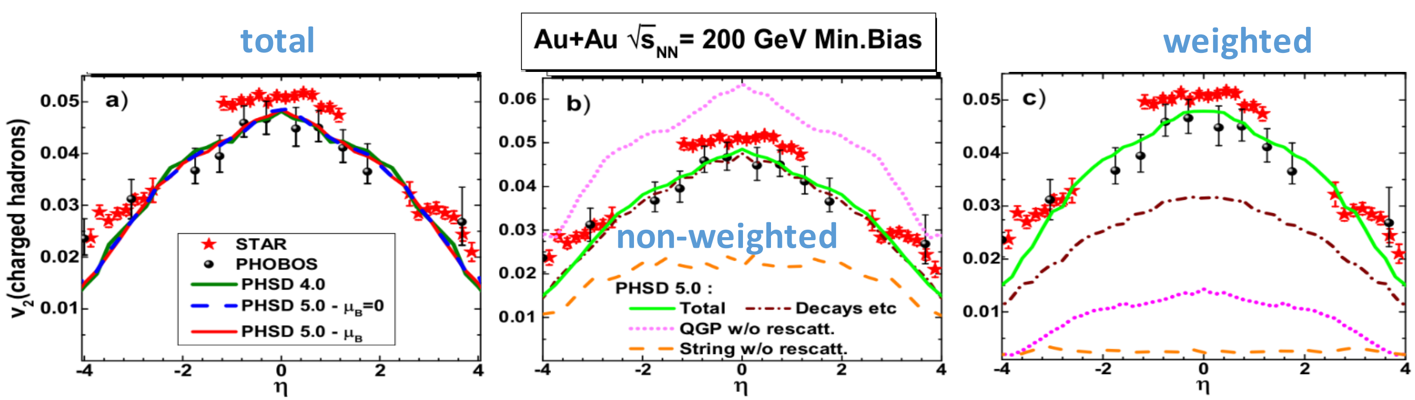
Particles 3 (2020) no.1, 178-192



Channel decomposition :

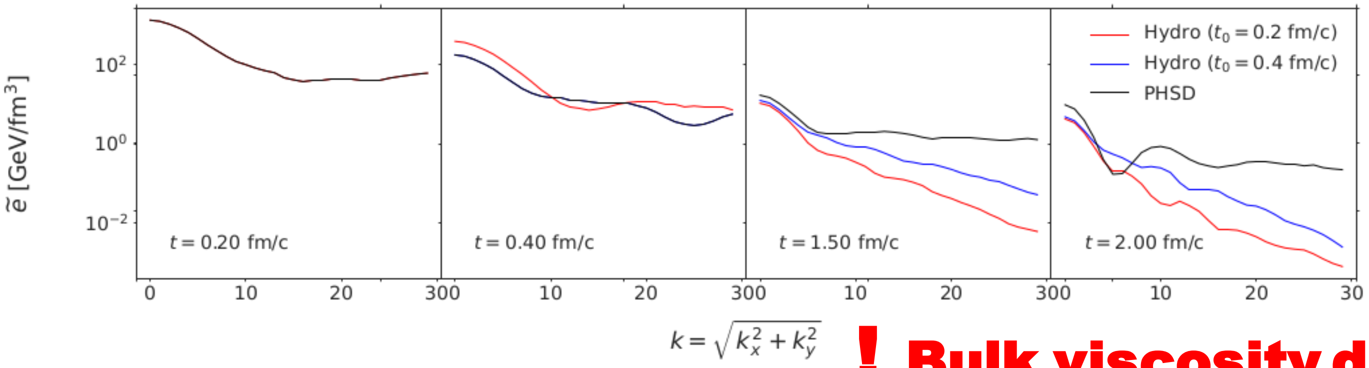
total

weighted



Evolution of QGP: comparison pA

$$\tilde{e}(k_x,k_y) = \frac{1}{m}\frac{1}{n}\sum_{x=0}^{m-1}\sum_{y=0}^{n-1} e(x,y)e^{2\pi i\left(\frac{xk_x}{L_x m}+\frac{yk_y}{L_y n}\right)}$$

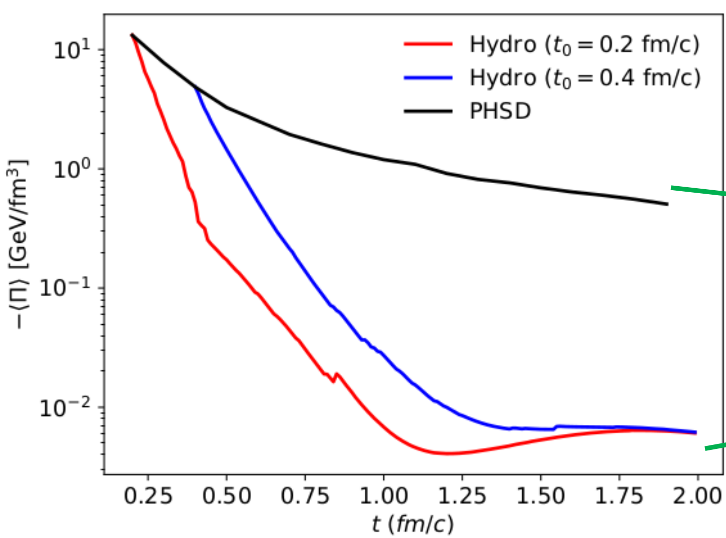


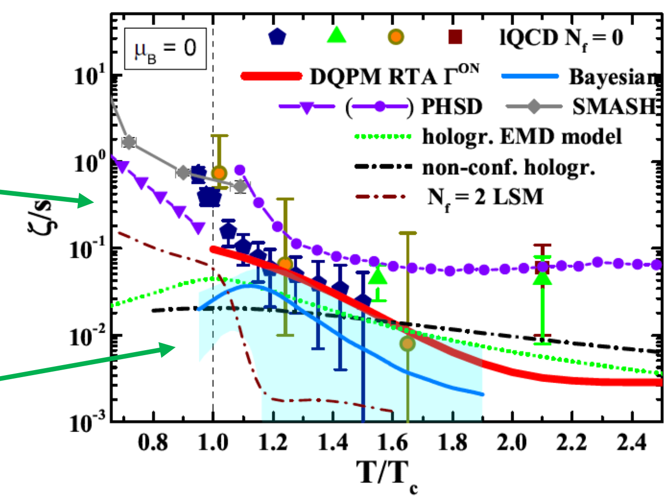
PHSD vs VISHNew:
Lucia Oliva, Wenkai Fan, Pierre Moreau, Steffen A. Bass, Elena Bratkovskaya
PRC 106(4), 044910 (2022)

! Bulk viscosity drives **hydro evolution of Π -> can effect mean <pT>**

W.Zhao, S.Ryu, C.Shen, B.Schenke,
PRC 107 (2023) no.1, 014904

$$\langle \Pi \rangle = \frac{\int d^2\mathbf{x}_T\, \Pi\, e(x,y)}{\int d^2\mathbf{x}_T\, e(x,y)}$$





Summary

Transport properties of the strongly-interacting QGP matter at finite **T** and μ_B have been investigated.
Influence of an order of a phase transition on thermodynamic and transport properties has been studied.

- Sizable bulk viscosity should be considered for hydro simulation in order to reproduce better experimental observables at RHIC and LHC

Evolution of the QGP matter created in HICs and the sensitivity of the bulk and flow observables on the QGP interactions and transport properties have been explored by the simulations within the PHSD transport approach

- High-μ_B regions are probed at low $\sqrt{s_{NN}}$ or high rapidity regions
  Moreover, QGP fraction is small at low $\sqrt{s_{NN}}$: small effect seen in observables
- μ_B-dependence of QGP interactions is more pronounced in observables for strange hadrons and antiprotons

Thank you for your attention!

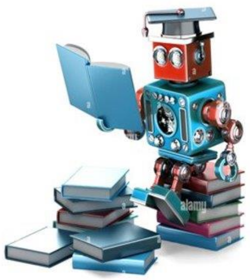
Bonus: ML for HEP

Theory:

- Classify phases of matter - phase transitions in QCD
- Properties of dense matter :
- Exploring QCD matter in extreme conditions with Machine Learning (recent review: https://arxiv.org/abs/2303.15136)

Jet flavour identification:

› https://arxiv.org/abs/1407.5675 - CNN, Josh Cogan et al;
› https://arxiv.org/abs/1603.09349 - DNN for jets, Pierre Baldi et al;
› https://arxiv.org/abs/1701.05927 - GAN for jets, Luke de Oliveira et al;
› https://arxiv.org/abs/1702.00748 - RNN for jets, Gilles Louppe et al;



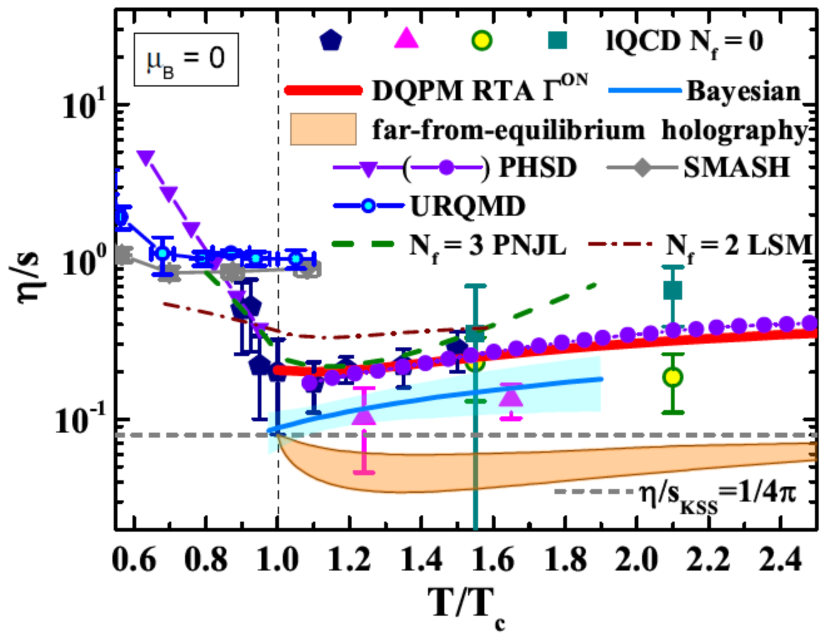
And much more in

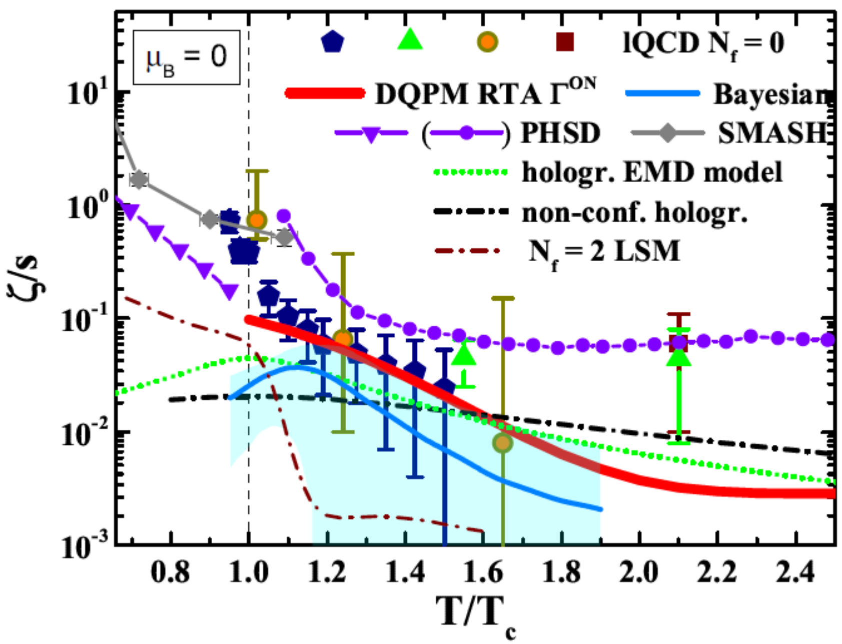
Living Review of ML for Particle Physics ->

https://github.com/iml-wg/HEPML-LivingReview

Uncertainties in viscosities of QGP

Model predictions: from first principles to effective models – quest for consistency





! Effective models of QGP using the same EoS predict completely different transport coefficients

Evolution of QGP: MUSIC setup

MUSIC:

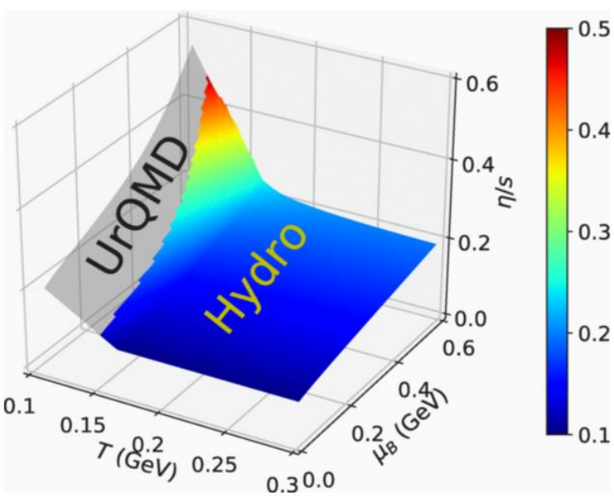
$$M(x,y) = \int \tau_0 d\eta_s e(x,y,\eta_s)\cosh(\eta_s - y_{\rm CM}) \quad (12)$$

$$0 = \int \tau_0 d\eta_s e(x,y,\eta_s)\sinh(\eta_s - y_{\rm CM}). \quad (13)$$

We choose a symmetric rapidity profile parameterization w.r.t $y_{\rm CM}$ for the local energy density [11],

$$e(x,y,\eta_s;y_{\rm CM}) = \mathcal{N}_e(x,y) \times \exp\left[-\frac{(|\eta_s - y_{\rm CM}| - \eta_0)^2}{2\sigma_\eta^2}\theta(|\eta_s - y_{\rm CM}| - \eta_0)\right]. (14)$$

$$\epsilon_{SW} = 0.26\ GeV/fm^3$$



$$f^A_{n_B}(\eta_s) = \mathcal{N}_{n_B}\left\{\theta(\eta_s - \eta_{B,0})\exp\left[-\frac{(\eta_s - \eta_{B,0})^2}{2\sigma^2_{B,\rm out}}\right] + \theta(\eta_{B,0} - \eta_s)\exp\left[-\frac{(\eta_s - \eta_{B,0})^2}{2\sigma^2_{B,\rm in}}\right]\right\} \quad (17)$$

and

$$f^B_{n_B}(\eta_s) = \mathcal{N}_{n_B}\left\{\theta(\eta_s + \eta_{B,0})\exp\left[-\frac{(\eta_s + \eta_{B,0})^2}{2\sigma^2_{B,\rm in}}\right] + \theta(-\eta_{B,0} - \eta_s)\exp\left[-\frac{(\eta_s + \eta_{B,0})^2}{2\sigma^2_{B,\rm out}}\right]\right\}. \quad (18)$$

$\sqrt{s_{\rm NN}}$ (GeV)	τ_0 (fm/c)	η_0	σ_η	$\eta_{B,0}$	$\sigma_{B,\rm in}$	$\sigma_{B,\rm out}$
AuAu & dAu @ 200	1.0	2.5	0.6	3.5	2.0	0.1
AuAu & dAu @ 62.4	1.0	2.25	0.3	2.7	1.9	0.2
AuAu & dAu @ 39	1.3	1.9	0.3	2.2	1.6	0.2
AuAu@27	1.4	1.6	0.3	1.8	1.5	0.2
AuAu & dAu @ 19.6	1.8	1.3	0.3	1.5	1.2	0.2
AuAu@14.5	2.2	1.15	0.3	1.4	1.15	0.2
AuAu@7.7	3.6	0.9	0.2	1.05	1.0	0.1
PbPb@17.3	1.8	1.25	0.3	1.6	1.2	0.2
PbPb@8.77	3.5	0.95	0.2	1.2	1.0	0.1

C. Shen, S.Alzhrani, PRC 102 (2020) 1, 014909

DQPM: EoS

Entropy and baryon density in the quasiparticle limit (G. Baym 1998, Blaizot et al. 2001):

$$s^{dqp} = - \int \frac{d\omega}{2\pi} \frac{d^3p}{(2\pi)^3} \left[d_g \frac{\partial n_B}{\partial T} \left(\mathrm{Im}(\ln -\Delta^{-1}) + \mathrm{Im}\,\Pi\, \mathrm{Re}\,\Delta \right) + \sum_{q=u,d,s} d_q \frac{\partial n_F(\omega-\mu_q)}{\partial T} \left(\mathrm{Im}(\ln -S_q^{-1}) + \mathrm{Im}\,\Sigma_q\, \mathrm{Re}\,S_q \right) + \sum_{\bar{q}=\bar{u},\bar{d},\bar{s}} d_{\bar{q}} \frac{\partial n_F(\omega+\mu_q)}{\partial T} \left(\mathrm{Im}(\ln -S_{\bar{q}}^{-1}) + \mathrm{Im}\,\Sigma_{\bar{q}}\, \mathrm{Re}\,S_{\bar{q}} \right) \right]$$

$$n^{dqp} = - \int \frac{d\omega}{2\pi} \frac{d^3p}{(2\pi)^3} \left[\sum_{q=u,d,s} d_q \frac{\partial n_F(\omega-\mu_q)}{\partial \mu_q} \left(\mathrm{Im}(\ln -S_q^{-1}) + \mathrm{Im}\,\Sigma_q\, \mathrm{Re}\,S_q \right) + \sum_{\bar{q}=\bar{u},\bar{d},\bar{s}} d_{\bar{q}} \frac{\partial n_F(\omega+\mu_q)}{\partial \mu_q} \left(\mathrm{Im}(\ln -S_{\bar{q}}^{-1}) + \mathrm{Im}\,\Sigma_{\bar{q}}\, \mathrm{Re}\,S_{\bar{q}} \right) \right]$$

- Input: entropy density as a f(T , $\boldsymbol{\mu_B = 0}$)

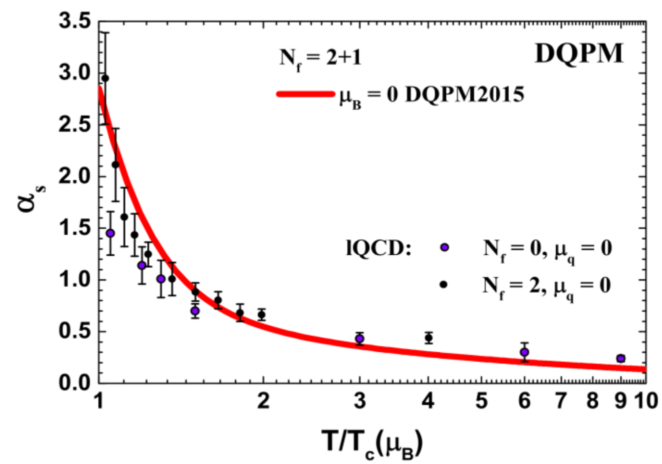
$$g^2(s/s_{SB}) = d\left((s/s_{SB})^e - 1\right)^f$$

$$s^{DQPM}(\Pi, \Delta, S_q, \Sigma) = s^{lattice}$$

fix the parameters

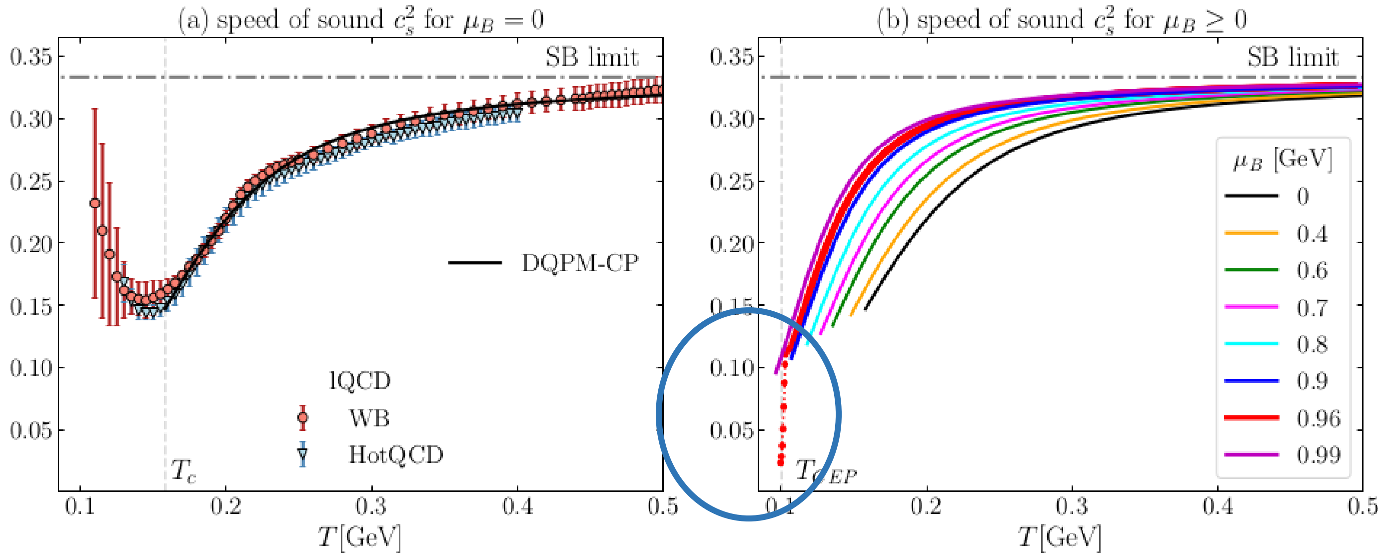
- Scaling hypothesis for the crossover region at finite $\boldsymbol{\mu_B}$

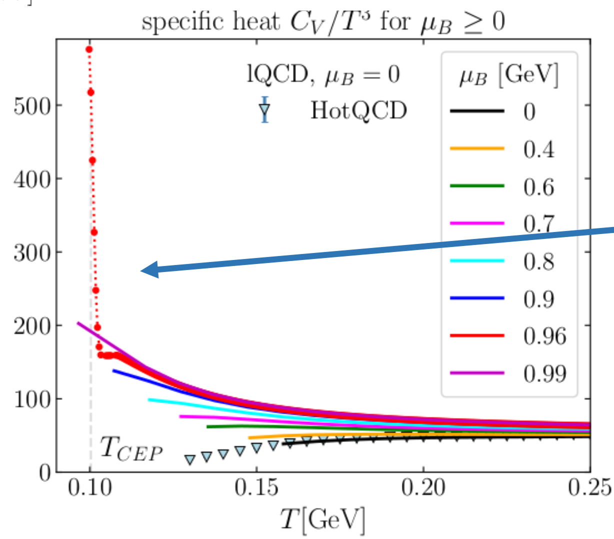
$$g^2(T/T_c, \mu_B) = g^2\left(\frac{T^*}{T_c(\mu_B)}, \mu_B = 0\right) \quad \text{with} \quad T^* = \sqrt{T^2 + \mu_q^2/\pi^2}$$



Speed of sound

- EoS : for μ_B/T <2 agreement with lQCD for μ_B/T >6 agreement with pQCD



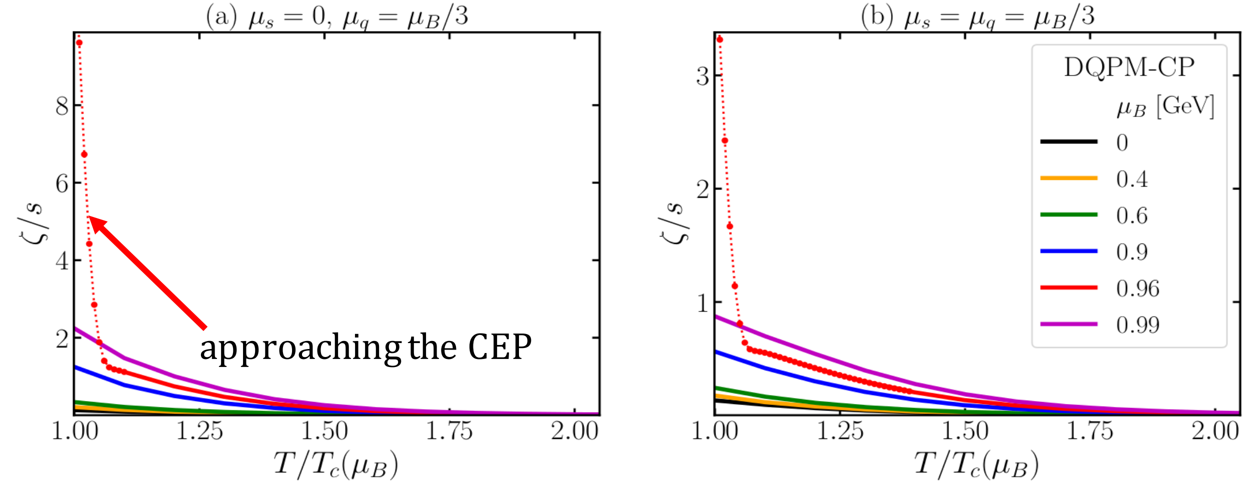


Near CEP critical scaling can be seen:

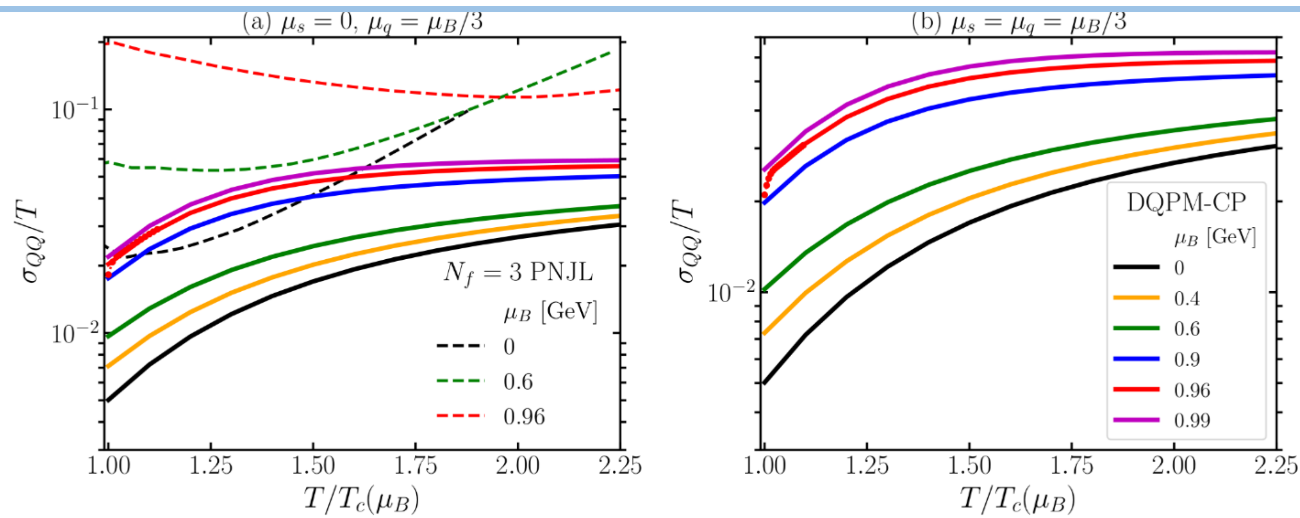
$$\ln(C_V) = -\alpha \cdot \ln(T - T_{CEP}) + const$$

O. S., J. Aichelin and E. Bratkovskaya, PRD 105 (2022) 054011

Shear and bulk viscosities near the CEP



- Sudden rise of specific bulk viscosity approaching the CEP



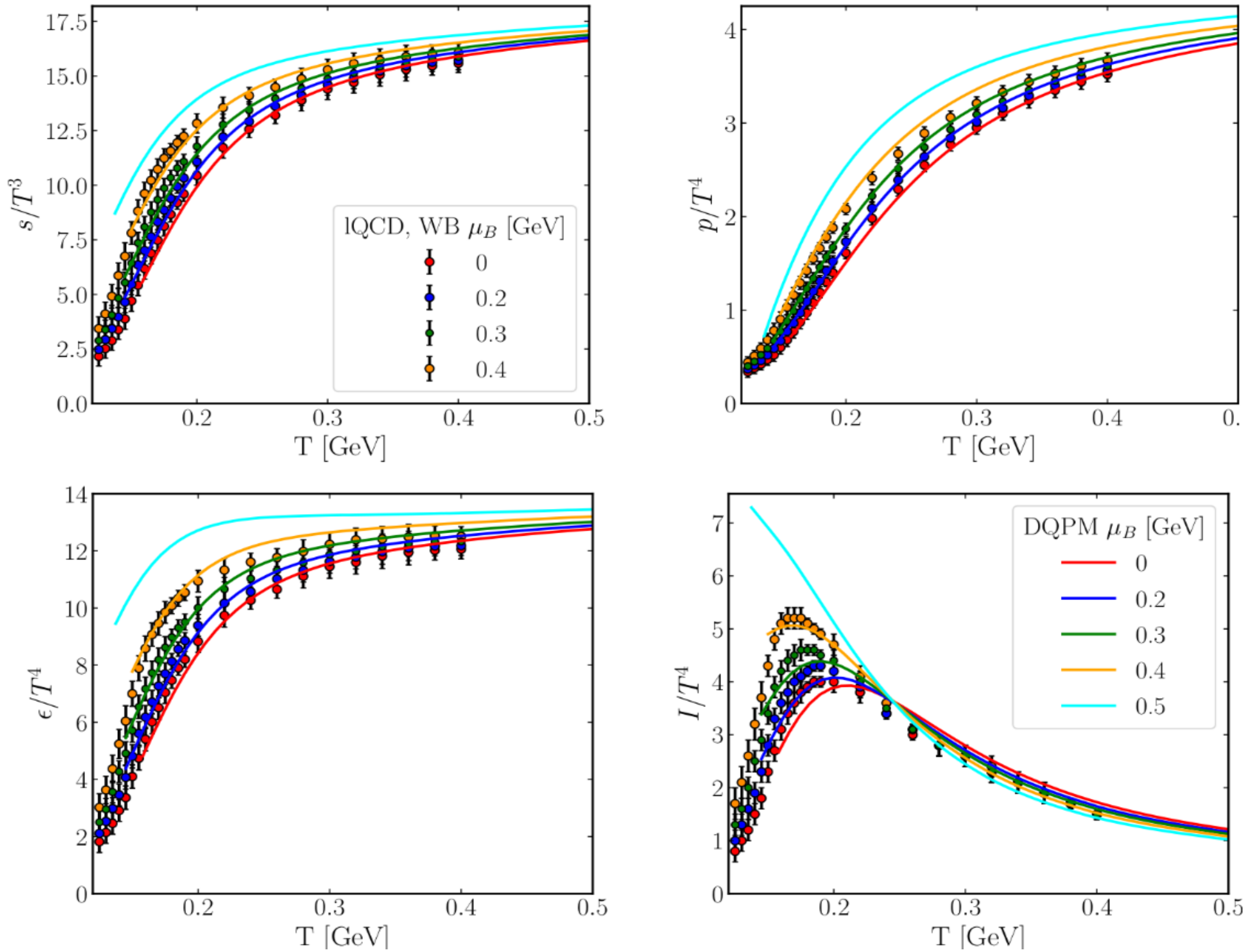
- B,Q,S diffusion coefficients have pronounced $\boldsymbol{\mu_B}, \boldsymbol{\mu_S}$-dependence
- Only small increase approaching the CEP

O. S., J. Aichelin and E. Bratkovskaya, PRD 105 (2022) 054011

DQPM: EoS – crossover region

Input:
lattice EoS
$\mu_B = 0$
(red dots)

Output:
DQPM EoS
$\mu_B \geq 0$
(lines)



How to evaluate transport coefficient?

- **Kubo formalism: transport coefficients are expressed through correlation functions of stress-energy tensor**

used in lattice QCD, transport approaches(hadrons), effective models

$$\eta = \frac{1}{20}\lim_{\omega\to 0}\frac{1}{\omega}\int d^4x\, e^{i\omega t}\left\langle\left[\mathcal{S}^{ij}(t,\mathbf{x}),\, \mathcal{S}^{ij}(0,\mathbf{0})\right]\right\rangle\theta(t)$$

$$\zeta = \frac{1}{2}\lim_{\omega\to 0}\frac{1}{\omega}\int d^4x\, e^{i\omega t}\langle[\mathcal{P}(t,\mathbf{x}),\, \mathcal{P}(0,\mathbf{0})]\rangle\theta(t)$$

$$\mathcal{S}^{ij} = T^{ij} - \delta^{ij}\mathcal{P}$$

$$\mathcal{P} = -\tfrac{1}{3}T^{i}{}_{i}$$

R. Lang and W. Weise, EPJ. A 50, 63 (2014) (NJL model)
A. Harutyunyan et al,PRD 95, 114021, (2017)

Kinetic theory:

- **Relaxation time approximation(RTA): consider relaxation time**

$$\frac{df_a^{\rm eq}}{dt} = \mathcal{C}_a = -\frac{f_a^{\rm eq}\phi_a}{\tau_a}$$

P. Chakraborty and J. I. Kapusta, PRC 83,014906 (2011)
G.S. Rocha, M. N. Ferreira, G. S. Denicol and J. Noronha, PRD 106 (2022) no.3, 036022

- **Chapman-Enskog: expand the distribution in terms of the Knudsen number**

J. A. Fotakis et al, PRD 101 (2020) 7, 076007 (HRG)

And more!

Holographic models: AdS/CFT correspondence

D. T. Son and A. O. Starinets, JHEP 0603, 052 (2006)
M. Attems et al , JHEP 10 (2016), 155.
J. Grefa, M. Hippert, J. Noronha, J. Noronha-Hostler, I. Portillo, C. Ratti and R. Rougemont, PRD 106 (2022) no.3, 034024 <- near CEP and across the first-order line

Transport coefficients at finite μ_B

Jet transport coefficient – significant E-dependence

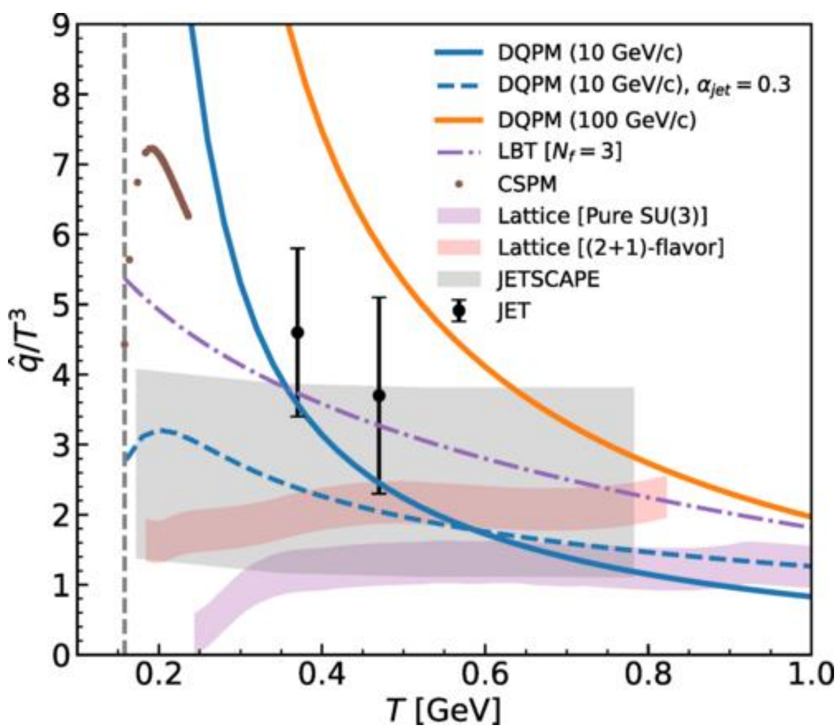
2→2 and 2→3 partonic scatterings

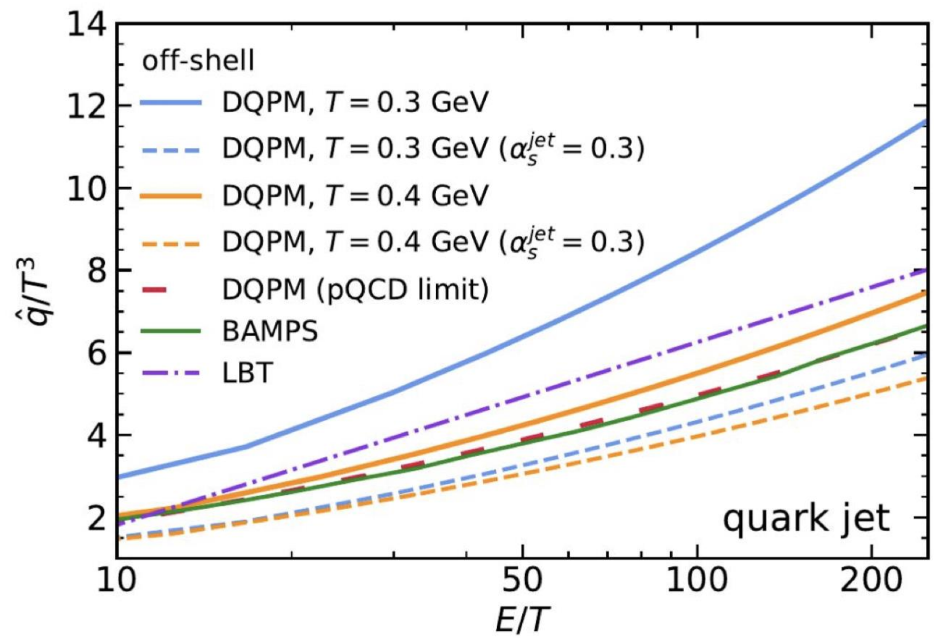
$$\langle \mathcal{O} \rangle^{\rm on} = \frac{1}{2E_i} \sum_{j=q,\bar{q},g} d_j f_j \int \frac{d^3p_j}{(2\pi)^3 2E_j} \times \int \frac{d^3p_1}{(2\pi)^3 2E_1} \int \frac{d^3p_2}{(2\pi)^3 2E_2} \times (1 \pm f_1)(1 \pm f_2) \mathcal{O} |\overline{\mathcal{M}}|^2 (2\pi)^4 \delta^{(4)}(p_i + p_j - p_1 - p_2)$$

$$\mathcal{O} = |\vec{p_T} - \vec{p_T}'|^2 \rightarrow \langle O \rangle = \hat{q}$$

$$\mathcal{O} = (E - E') \rightarrow \langle O \rangle = dE/dx$$

Most energy loss implementations don't take this dependence into account

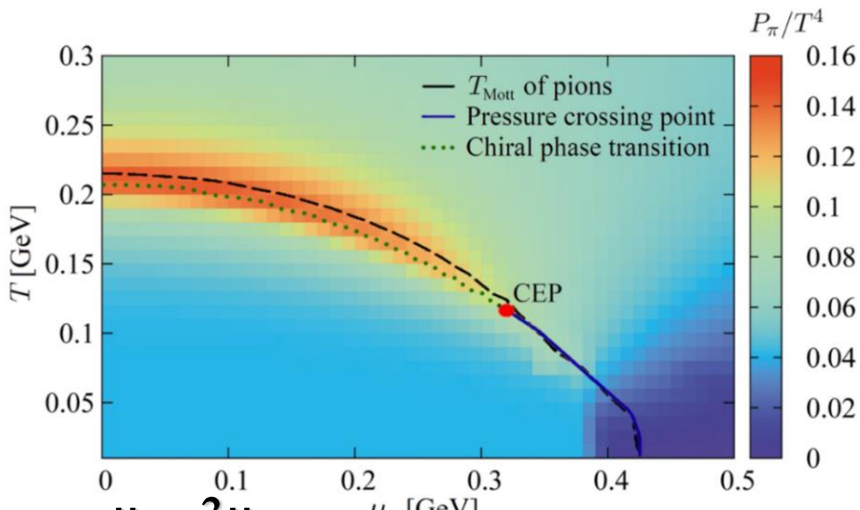




Grishmanovskii, T. Song, O.S., C. Greiner and E. Bratkovskaya, PRC 106 (2022) no.1, 014903

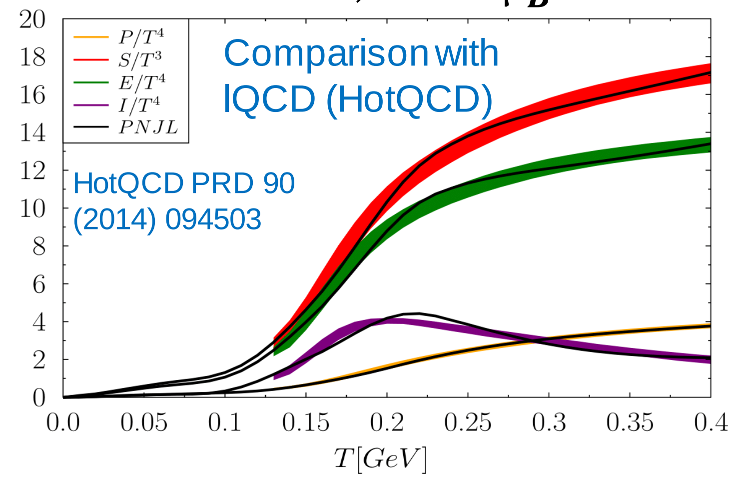
QGP in the Polyakov extended NJL model

- PNJL allows for prediction of macroscopic properties of QGP at finite T and μ_B
- & QGP transport coefficients for $0 \leq \mu_B \leq 1.2$ GeV



$\mu_B = 3\mu_q$

➢ Parameters fixed, EoS at $\mu_B = 0$:



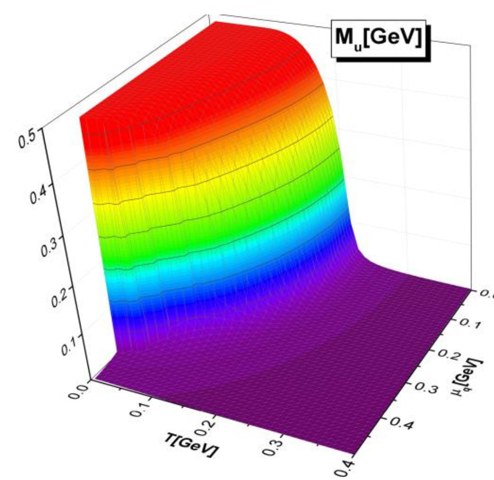
➢ CEP: $(T, \mu_B) = (110,960)$ MeV, $\mu_B/T = 8.73$

➢ 1st order PT at high μ_B

➢ same symmetries for the quarks as QCD

Chiral masses (M_l, M_s)

$$m_i = m_{0i} - 4G\langle\langle\bar{\psi}_i\psi_i\rangle\rangle + 2K\langle\langle\bar{\psi}_j\psi_j\rangle\rangle\langle\langle\bar{\psi}_k\psi_k\rangle\rangle$$



Improved thermodynamics by NNLO in Ω and Polyakov loop

J. M. Torres-Rincon, J. Aichelin PRC 96 (2017) 4 045205

D. Fuseau, T. Steinernert, J. Aichelin PRC 101 (2020) 6 065203

PNJL relaxation times

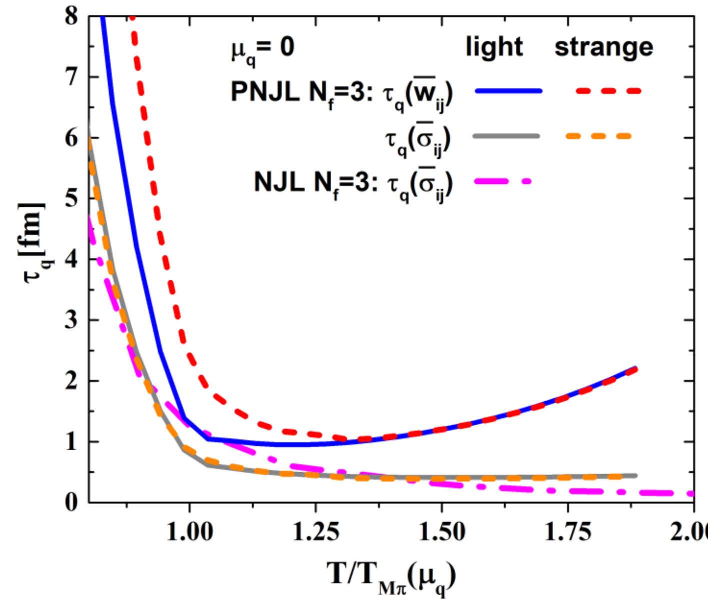
$$\tau_i(\mathbf{p},T,\mu_B) = \frac{1}{\Gamma_i(\mathbf{p},T,\mu_B)}$$

$$\Gamma_i^{\text{on}}(\mathbf{p}_i,T,\mu_q) = \frac{1}{2E_i}\sum_{j=q,\bar{q},g}\int\frac{d^3p_j}{(2\pi)^3 2E_j}\, d_j\, f_j(E_j,T,\mu_q)$$

$$\int\frac{d^3p_3}{(2\pi)^3 2E_3}\int\frac{d^3p_4}{(2\pi)^3 2E_4}(1\pm f_3)(1\pm f_4)$$

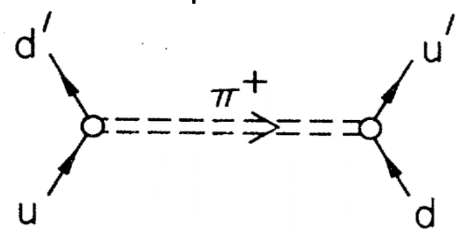
$$|\bar{\mathcal{M}}|^2(p_i,p_j,p_3,p_4)\,(2\pi)^4\delta^{(4)}\,(p_i+p_j-p_3-p_4)$$

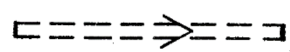
Relaxation times(PNJL vs NJL)



qq - interactions:

4 point interaction -> meson exchange(π,Ϭ,η,η',K,.. for s,t,u channels)

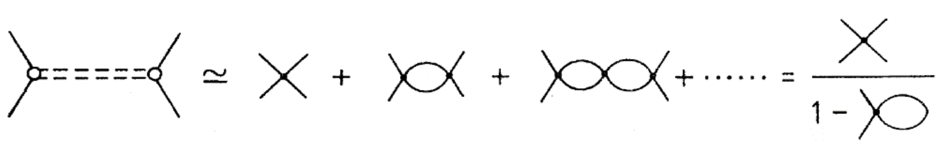




$$=(i\gamma_5)\tau^{(-)}\frac{-ig^2_{\pi qq}}{k^2-m^2_\pi}(i\gamma_5)\tau^{(+)}$$

meson propagator $\mathscr{D} = \dfrac{2ig_m}{1-2g_m\Pi^{\pm}_{ff'}(k_0,\vec{k})}$

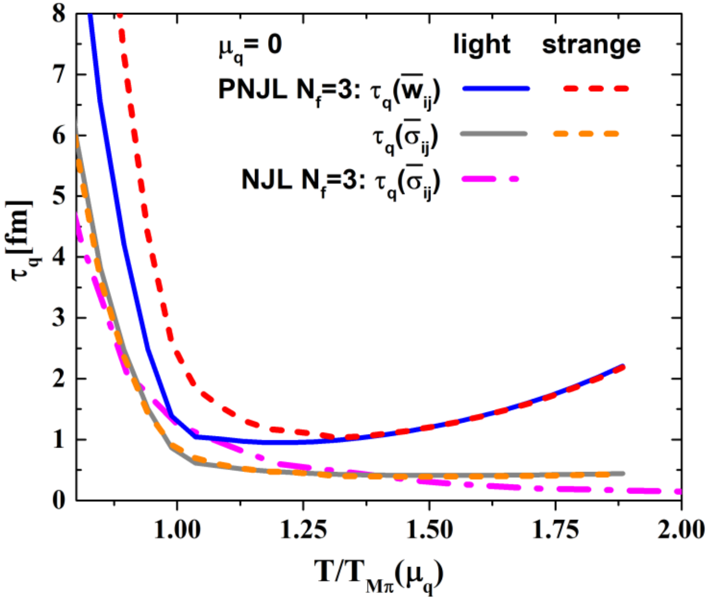
Effective interaction in RPA



PNJL relaxation times

$$\tau_i(\mathbf{p},T,\mu_B) = \frac{1}{\Gamma_i(\mathbf{p},T,\mu_B)}$$

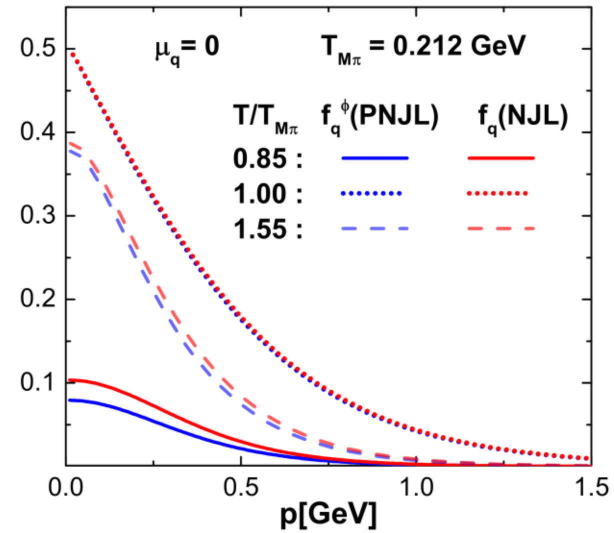
$$\Gamma_i^{on}(\mathbf{p}_i,T,\mu_q) = \frac{1}{2E_i}\sum_{j=q,\bar{q},g}\int\frac{d^3p_j}{(2\pi)^3 2E_j}\, d_j\, f_j(E_j,T,\mu_q)$$
$$\int\frac{d^3p_3}{(2\pi)^3 2E_3}\int\frac{d^3p_4}{(2\pi)^3 2E_4}(1\pm f_3)(1\pm f_4)$$
$$|\bar{\mathcal{M}}|^2(p_i,p_j,p_3,p_4)\ (2\pi)^4\delta^{(4)}\ (p_i+p_j-p_3-p_4)$$



Modified distribution functions:

Polyakov loop contributions

$$f_q \to f_q^{\Phi}(\mathbf{p},T,\mu) = \frac{(\bar{\Phi}+2\Phi e^{-(E_\mathbf{p}-\mu)/T})e^{-(E_\mathbf{p}-\mu)/T}+e^{-3(E_\mathbf{p}-\mu)/T}}{1+3(\bar{\Phi}+\Phi e^{-(E_\mathbf{p}-\mu)/T})e^{-(E_\mathbf{p}-\mu)/T}+e^{-3(E_\mathbf{p}-\mu)/T}},$$



QGP in the Polyakov extended NJL model

- PNJL model based on effective Lagrangian with the same symmetries for the quark dof as QCD

$$\mathscr{L}_{PNJL} = \sum_i \bar{\psi}_i (iD - m_{0i} + \mu_i \gamma_0)\psi_i$$
$$+ G \sum_a \sum_{ijkl} \left[(\bar{\psi}_i \; i\gamma_5 \tau^a_{ij} \psi_j) \; (\bar{\psi}_k \; i\gamma_5 \tau^a_{kl} \psi_l) + (\bar{\psi}_i \tau^a_{ij} \psi_j) \; (\bar{\psi}_k \tau^a_{kl} \psi_l) \right]$$
$$- K \det_{ij} \left[\bar{\psi}_i \; (-\gamma_5) \psi_j \right] - K \det_{ij} \left[\bar{\psi}_i \; (+\gamma_5) \psi_j \right]$$
$$- \mathcal{U}(T; \Phi, \bar{\Phi})$$

Polyakov-loop effective potential fitted to the YM

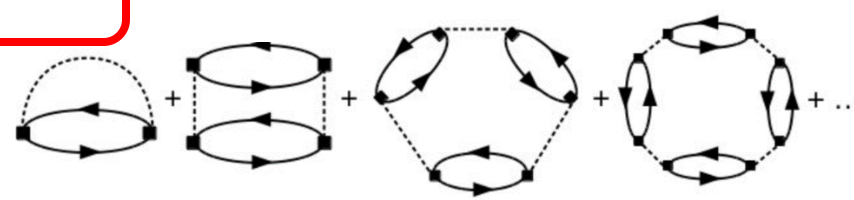
5 parameters fixed by vacuum values K,π masses, η-η'mass splitting , π decay constant, Chiral condensate

Improvements:

➢ Next to leading order in Nc(O(1/Nc)^0) of the grand-canonical potential : presence of the mesons below Tc

$$\Omega_{\rm PNJL}(T, \mu_i) = \Omega_q^{(-1)}(T, \mu_i) + \sum_{M \in J^\pi = \{0^+, 0^-\}} \Omega_M^{(0)}(T, \mu_M(\mu_i)) + \mathcal{U}_{glue}(T) \; ,$$

J. M. Torres-Rincon, J. Aichelin PRC 96 (2017) 4 045205
D. Fuseau, T. Steinernert, J. Aichelin PRC 101 (2020) 6 065203



Modification of the gluon potential due to the presence of the quark

Specific shear viscosity to conductivity