УНИВЕРЗИТЕТ У БЕОГРАДУ
ИНСТИТУТ ЗА ФИЗИКУ | БЕОГРАД
ИНСТИТУТ ОД НАЦИОНАЛНОГ ЗНАЧАЈА ЗА РЕПУБЛИКУ СРБИЈУ

Effect of higher orders in opacity on high-$p_\perp$ observables

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Република Србија
МИНИСТАРСТВО НАУКЕ, ТЕХНОЛОШКОГ РАЗВОЈА И ИНОВАЦИЈА

High-$p_\perp$ probes

- Rare high-$p_\perp$ particles are one of the main probes for inferring features of the Quark-Gluon Plasma, created at the RHIC and the LHC.
- Their interactions with QGP are dominated by radiative energy loss
- Realistic high-$p_\perp$ radiative energy loss model is crucial for understanding QGP properties
- Test of model validity: comparison of numerical predictions with experimental data
- Numerical framework to generate predictions for high-$p_\perp$ observables (e.g. R_{AA} and v_2) is also needed

Motivation

- pQCD high-$p_\perp$ radiative energy loss – medium assumptions:
  - Optically thin → One scattering center (SHSA) GLV NPB 594, 371; DGLV NPA 733, 265; HT NPA 696, 788; Prog. PPNP 66,41
  - Optically thick → Infinite number of scattering centers (MSSA, Big Bang) BDMPS-Z NPB 484, 265; NPB 531, 403; JETP Lett. 63, 952; JETP Lett. 65, 615; ASW PRD 69, 114003; AMY JHEP 12, 009
- Realistically, short finite-size droplets of QGP are created at RHIC and LHC (several *fms*, *λ≈1 fm)*

Relaxing these approximations to the case of a finite number of scattering centers is required!

- Current theoretical attempts to address this highly nontrivial problem need to be more conclusive or complete (C. Andres et al., JHEP 2007, 114; JHEP 2103, 102; Y. Mehtar-Tani et al, JHEP 1907, 057; JHEP 2006, 187; PRD 98, 094010; PLB 795, 502)
- Lacking phenomenological studies (not tested against experimental data)

These corrections need to be implemented in both analytical calculations (i.e., radiative energy loss model) and numerical framework!

The dynamical energy loss formalism

✓ **Includes:**

- QCD medium of finite size and finite temperature
- The medium consists of dynamical (i.e., moving) partons
- Based on finite T field theory and generalized HTL approach

M. Djordjevic, PRC 74, 064907; PRC 80, 064909, M. Djordjevic, U. Heinz, PRL 101, 022302

- Applicable to both light and heavy flavor M. Djordjevic and M. Gyulassy, NPA 733, 265
- Finite magnetic mass effects M. Djordjevic and M. Djordjevic, PLB 709, 229
- Running coupling M. Djordjevic and M. Djordjevic, PLB 734, 286

⇩

Developed up to the 1st order in opacity

⇩

For improved QGP application: going beyond this approximation is required

Higher orders in opacity: analytical calculations

In static QGP (D(GLV)):

$$x\frac{dN^{(n)}}{dx\,d^2\mathbf{k}} = \int_0^L dz_1 \cdots \int_{z_{n-1}}^L dz_n \int \prod_{i=1}^n \left(d^2\mathbf{q}_i \frac{v^2(\mathbf{q}_i) - \delta^2(\mathbf{q}_i)}{\lambda(z)} \right) \times \frac{C_R \alpha_s(Q_k^2)}{\pi^2} \left(-2\,\mathbf{C}_{(1\cdots n)} \cdot \mathbf{B}_n \left[\cos \sum_{k=2}^n \omega_{(k\cdots n)} \Delta z_k - \cos \sum_{k=1}^n \omega_{(k\cdots n)} \Delta z_k \right] \right)$$

S. Wicks, arXiv:0804.4704, M. Djordjevic and M. Gyulassy, NPA 733, 265

$$\mathbf{C}_{(i_1 i_2 \ldots i_m)} = \frac{(\mathbf{k} - \mathbf{q}_{i_1} - \mathbf{q}_{i_2} - \ldots - \mathbf{q}_{i_m})}{\chi^2 + (\mathbf{k} - \mathbf{q}_{i_1} - \mathbf{q}_{i_2} - \ldots - \mathbf{q}_{i_m})^2}$$

$$\mathbf{B}_i = \mathbf{H} - \mathbf{C}_i$$

$$\mathbf{H} = \frac{\mathbf{k}}{\chi^2 + \mathbf{k}^2}$$

Inverse of formation time:

$$\omega_{(m\ldots n)} = \frac{\chi^2 + (\mathbf{k} - \mathbf{q}_m - \ldots \mathbf{q}_n)^2}{2xE}$$

$$\chi^2 \equiv M^2 x^2 + m_g^2$$

$$\alpha_s(Q^2) = \frac{4\pi}{(11 - 2/3n_f)\ln(Q^2/\Lambda_{QCD})}$$

$$Q_k^2 = \frac{\mathbf{k}^2 + M^2 x^2 + m_g^2}{x}$$

In dynamical QGP:

M. Djordjevic, PRC 80, 064909; M. Djordjevic and U. Heinz, PRL 101, 022302; M. Djordjevic, PLB 709, 229

Effective potential $\left[\frac{\mu_E^2}{\pi(\mathbf{q}^2+\mu_E^2)^2}\right]_{\text{stat}} \rightarrow \left[\frac{\mu_E^2 - \mu_M^2}{\pi(\mathbf{q}^2+\mu_E^2)(\mathbf{q}^2+\mu_M^2)}\right]_{\text{dyn}}$

Mean free path $\frac{1}{\lambda_{\text{stat}}} = c(n_f)\frac{1}{\lambda_{\text{dyn}}} = 6\frac{1.202}{\pi^2}\frac{1+n_f/4}{1+n_f/6}3\alpha_S T$

Obtained explicit analytical expressions up to 4th order in opacity within Dynamical energy loss formalism for the first time.

S. Stojku, BI, I. Salom, M. Djordjevic, arXiv:2303.14527

Analytical calculations: examples

Highly oscillatory, and difficult to converge!

1st order in opacity:

$$\left(\frac{dN_g^{(1)}}{dx}\right) = \frac{2C_R}{\pi x}\int\frac{d^2\mathbf{k}}{\pi}\int\frac{d^2\mathbf{q}_1}{\pi}\,\alpha_s(Q_k^2)\frac{L}{\lambda_{dyn}}\frac{\mu_E^2-\mu_M^2}{(\mathbf{q}_1^2+\mu_E^2)(\mathbf{q}_1^2+\mu_M^2)}\frac{\chi^2(\mathbf{q}_1\cdot(\mathbf{q}_1-\mathbf{k}))+(\mathbf{q}_1\cdot\mathbf{k})(\mathbf{k}-\mathbf{q}_1)^2}{(\chi^2+\mathbf{k}^2)(\chi^2+(\mathbf{k}-\mathbf{q}_1)^2)^2}\left(1-\frac{\sin(L\omega_{(1)})}{L\omega_{(1)}}\right)$$

2nd order in opacity:

$$\left(\frac{dN_g^{(2)}}{dx}\right)_1 = \frac{2C_R}{\pi x}\int\frac{d^2\mathbf{k}}{\pi}\iint\frac{d^2\mathbf{q}_1}{\pi}\frac{d^2\mathbf{q}_2}{\pi}\,\alpha_s(Q_k^2)\frac{1}{\lambda_{dyn}^2}\frac{\mu_E^2-\mu_M^2}{(\mathbf{q}_1^2+\mu_E^2)(\mathbf{q}_1^2+\mu_M^2)}\frac{\mu_E^2-\mu_M^2}{(\mathbf{q}_2^2+\mu_E^2)(\mathbf{q}_2^2+\mu_M^2)}$$
$$\frac{\chi^2(\mathbf{q}_2\cdot(\mathbf{q}_1+\mathbf{q}_2-\mathbf{k}))+(\mathbf{q}_2\cdot\mathbf{k})(\mathbf{k}-\mathbf{q}_2)^2+(\mathbf{k}\cdot\mathbf{q}_1)(\mathbf{q}_2\cdot(\mathbf{q}_2-2\mathbf{k}))+\mathbf{k}^2(\mathbf{q}_2\cdot\mathbf{q}_1)}{(\chi^2+\mathbf{k}^2)(\chi^2+(\mathbf{k}-\mathbf{q}_2)^2)(\chi^2+(\mathbf{k}-\mathbf{q}_1-\mathbf{q}_2)^2)}$$
$$\frac{1}{\omega_{(2)}}\left(\frac{\omega_{(2)}\cos(L(\omega_{(2)}+\omega_{(12)}))}{(\omega_{(2)}+\omega_{(12)})\,\omega_{(12)}}+L\sin(L\omega_{(2)})-\frac{(\omega_{(2)}-\omega_{(12)})\cos(L\omega_{(2)})}{\omega_{(2)}\omega_{(12)}}-\frac{\omega_{(12)}}{\omega_{(2)}(\omega_{(2)}+\omega_{(12)})}\right)$$

3rd order in opacity:

$$\left(\frac{dN_g^{(3)}}{dx}\right)_1 = \frac{2C_R}{\pi x}\int\frac{d^2\mathbf{k}}{\pi}\iiint\frac{d^2\mathbf{q}_1}{\pi}\frac{d^2\mathbf{q}_2}{\pi}\frac{d^2\mathbf{q}_3}{\pi}$$
$$\alpha_s(Q_k^2)\frac{1}{\lambda_{dyn}^3}\frac{\mu_E^2-\mu_M^2}{(\mathbf{q}_1^2+\mu_E^2)(\mathbf{q}_1^2+\mu_M^2)}\frac{\mu_E^2-\mu_M^2}{(\mathbf{q}_2^2+\mu_E^2)(\mathbf{q}_2^2+\mu_M^2)}\frac{\mu_E^2-\mu_M^2}{(\mathbf{q}_3^2+\mu_E^2)(\mathbf{q}_3^2+\mu_M^2)}$$
$$\frac{\chi^2(\mathbf{q}_3\cdot(\mathbf{q}_1+\mathbf{q}_2+\mathbf{q}_3-\mathbf{k}))+(\mathbf{q}_3\cdot\mathbf{k})(\mathbf{k}-\mathbf{q}_3)^2+(\mathbf{k}\cdot(\mathbf{q}_1+\mathbf{q}_2))(\mathbf{q}_3\cdot(\mathbf{q}_3-2\mathbf{k}))+\mathbf{k}^2(\mathbf{q}_3\cdot(\mathbf{q}_1+\mathbf{q}_2))}{(\chi^2+\mathbf{k}^2)(\chi^2+(\mathbf{k}-\mathbf{q}_3)^2)(\chi^2+(\mathbf{k}-\mathbf{q}_1-\mathbf{q}_2-\mathbf{q}_3)^2)}$$
$$\left(\frac{\omega_{(3)}\omega_{(123)}+2\omega_{(23)}\omega_{(123)}-\omega_{(23)}^2-\omega_{(3)}\omega_{(23)}}{\omega_{(23)}^2(\omega_{(3)}+\omega_{(23)})^2\omega_{(123)}}\sin\left(L(\omega_{(3)}+\omega_{(23)})\right)-\frac{\omega_{(123)}\sin\left(L\omega_{(3)}\right)}{\omega_{(3)}\omega_{(23)}^2\left(\omega_{(23)}+\omega_{(123)}\right)}\right.$$
$$\left.+\frac{\sin\left(L\left(\omega_{(3)}+\omega_{(23)}+\omega_{(123)}\right)\right)}{\omega_{(123)}\left(\omega_{(23)}+\omega_{(123)}\right)\left(\omega_{(3)}+\omega_{(23)}+\omega_{(123)}\right)}-\frac{L\cos\left(L\left(\omega_{(3)}+\omega_{(23)}\right)\right)}{\omega_{(23)}\left(\omega_{(3)}+\omega_{(23)}\right)}\right)$$

4th order in opacity:

$$\left(\frac{dN_g^{(4)}}{dx}\right)_1 = \frac{2C_R}{\pi x}\int\frac{d^2\mathbf{k}}{\pi}\iiiint\frac{d^2\mathbf{q}_1}{\pi}\frac{d^2\mathbf{q}_2}{\pi}\frac{d^2\mathbf{q}_3}{\pi}\frac{d^2\mathbf{q}_4}{\pi}$$
$$\alpha_s(Q_k^2)\frac{1}{\lambda_{dyn}^4}\frac{\mu_E^2-\mu_M^2}{(\mathbf{q}_1^2+\mu_E^2)(\mathbf{q}_1^2+\mu_M^2)}\frac{\mu_E^2-\mu_M^2}{(\mathbf{q}_2^2+\mu_E^2)(\mathbf{q}_2^2+\mu_M^2)}\frac{\mu_E^2-\mu_M^2}{(\mathbf{q}_3^2+\mu_E^2)(\mathbf{q}_3^2+\mu_M^2)}\frac{\mu_E^2-\mu_M^2}{(\mathbf{q}_4^2+\mu_E^2)(\mathbf{q}_4^2+\mu_M^2)}$$
$$\frac{\chi^2(\mathbf{q}_4\cdot(\mathbf{q}_1+\mathbf{q}_2+\mathbf{q}_3+\mathbf{q}_4-\mathbf{k}))+(\mathbf{q}_4\cdot\mathbf{k})(\mathbf{k}-\mathbf{q}_4)^2+(\mathbf{k}\cdot(\mathbf{q}_1+\mathbf{q}_2+\mathbf{q}_3))(\mathbf{q}_4\cdot(\mathbf{q}_4-2\mathbf{k}))+\mathbf{k}^2(\mathbf{q}_4\cdot(\mathbf{q}_1+\mathbf{q}_2+\mathbf{q}_3))}{(\chi^2+\mathbf{k}^2)(\chi^2+(\mathbf{k}-\mathbf{q}_4)^2)(\chi^2+(\mathbf{k}-\mathbf{q}_1-\mathbf{q}_2-\mathbf{q}_3-\mathbf{q}_4)^2)}$$
$$\left(-\frac{L\sin\left(L\left(\omega_{(4)}+\omega_{(34)}+\omega_{(234)}\right)\right)}{\omega_{(234)}\left(\omega_{(34)}+\omega_{(234)}\right)\left(\omega_{(4)}+\omega_{(34)}+\omega_{(234)}\right)}\right.$$
$$-\frac{\cos\left(L\left(\omega_{(4)}+\omega_{(34)}+\omega_{(234)}+\omega_{(1234)}\right)\right)}{\omega_{(1234)}\left(\omega_{(234)}+\omega_{(1234)}\right)\left(\omega_{(34)}+\omega_{(234)}+\omega_{(1234)}\right)\left(\omega_{(4)}+\omega_{(34)}+\omega_{(234)}+\omega_{(1234)}\right)}$$
$$+\frac{F_{41}}{\omega_{(234)}^2\left(\omega_{(34)}+\omega_{(234)}\right)^2\left(\omega_{(4)}+\omega_{(34)}+\omega_{(234)}\right)^2\omega_{(1234)}}\cos\left(L(\omega_{(4)}+\omega_{(34)}+\omega_{(234)})\right)$$
$$+\frac{\omega_{(1234)}\cos\left(L\left(\omega_{(4)}+\omega_{(34)}\right)\right)}{\omega_{(34)}\left(\omega_{(4)}+\omega_{(34)}\right)\omega_{(234)}^2\left(\omega_{(234)}+\omega_{(1234)}\right)}-\frac{\omega_{(1234)}\cos\left(L\omega_{(4)}\right)}{\omega_{(4)}\omega_{(34)}\left(\omega_{(34)}+\omega_{(234)}\right)^2\left(\omega_{(34)}+\omega_{(234)}+\omega_{(1234)}\right)}$$
$$\left.+\frac{\omega_{(1234)}}{\omega_{(4)}\left(\omega_{(4)}+\omega_{(34)}\right)\left(\omega_{(4)}+\omega_{(34)}+\omega_{(234)}\right)^2\left(\omega_{(4)}+\omega_{(34)}+\omega_{(234)}+\omega_{(1234)}\right)}\right),$$

$$F_{41}=(\omega_{(34)}+\omega_{(234)})\left[(\omega_{(4)}+\omega_{(34)})(\omega_{(234)}-\omega_{(1234)})+\omega_{(234)}^2-3\omega_{(234)}\omega_{(1234)}\right]-\omega_{(4)}\omega_{(234)}\omega_{(1234)}$$

S. Stojku, BI, I. Salom, M. Djordjevic, arXiv:2303.14527

Higher orders in opacity: Numerical calculations

S. Stojku, BI, I. Salom, M. Djordjevic, arXiv:2303.14527

Representative contribution:

$$\left(\frac{dN_g^{(3)}}{dx}\right)_1 = \frac{2C_R}{\pi x}\int\frac{d^2\mathbf{k}}{\pi}\iiint\frac{d^2\mathbf{q}_1}{\pi}\frac{d^2\mathbf{q}_2}{\pi}\frac{d^2\mathbf{q}_3}{\pi}$$
$$\alpha_s(Q_k^2)\frac{1}{\lambda_{dyn}^3}\frac{\mu_E^2-\mu_M^2}{(\mathbf{q}_1^2+\mu_E^2)(\mathbf{q}_1^2+\mu_M^2)}\frac{\mu_E^2-\mu_M^2}{(\mathbf{q}_2^2+\mu_E^2)(\mathbf{q}_2^2+\mu_M^2)}\frac{\mu_E^2-\mu_M^2}{(\mathbf{q}_3^2+\mu_E^2)(\mathbf{q}_3^2+\mu_M^2)}$$
$$\frac{\chi^2(\mathbf{q}_3\cdot(\mathbf{q}_1+\mathbf{q}_2+\mathbf{q}_3-\mathbf{k}))+(\mathbf{q}_3\cdot\mathbf{k})(\mathbf{k}-\mathbf{q}_3)^2+(\mathbf{k}\cdot(\mathbf{q}_1+\mathbf{q}_2))(\mathbf{q}_3\cdot(\mathbf{q}_3-2\mathbf{k}))+\mathbf{k}^2(\mathbf{q}_3\cdot(\mathbf{q}_1+\mathbf{q}_2))}{(\chi^2+\mathbf{k}^2)(\chi^2+(\mathbf{k}-\mathbf{q}_3)^2)(\chi^2+(\mathbf{k}-\mathbf{q}_1-\mathbf{q}_2-\mathbf{q}_3)^2)}$$
$$\left(\frac{\omega_{(3)}\omega_{(123)}+2\omega_{(23)}\omega_{(123)}-\omega_{(23)}^2-\omega_{(3)}\omega_{(23)}}{\omega_{(23)}^2(\omega_{(3)}+\omega_{(23)})^2\omega_{(123)}}\sin\left(L(\omega_{(3)}+\omega_{(23)})\right)-\frac{\omega_{(123)}\sin\left(L\omega_{(3)}\right)}{\omega_{(3)}\omega_{(23)}^2\left(\omega_{(23)}+\omega_{(123)}\right)}\right.$$
$$\left.+\frac{\sin\left(L\left(\omega_{(3)}+\omega_{(23)}+\omega_{(123)}\right)\right)}{\omega_{(123)}\left(\omega_{(23)}+\omega_{(123)}\right)\left(\omega_{(3)}+\omega_{(23)}+\omega_{(123)}\right)}-\frac{L\cos\left(L\left(\omega_{(3)}+\omega_{(23)}\right)\right)}{\omega_{(23)}\left(\omega_{(3)}+\omega_{(23)}\right)}\right)$$

⇨ Highly oscillatory, and difficult to converge!

⇩ Up to 3rd order: required 70 000 CPUh to converge.

⇩

Numerical integration w.r.t. **k** and **q** → $\frac{dN^g}{dx}$ up to 3rd order in opacity

⇩

Implemented into generalized DREENA-C (Dynamical Radiative and Elastic ENergy loss Approach, C-constant T medium) framework to generate predictions for high-$p_\perp$ R_{AA} and v_2.

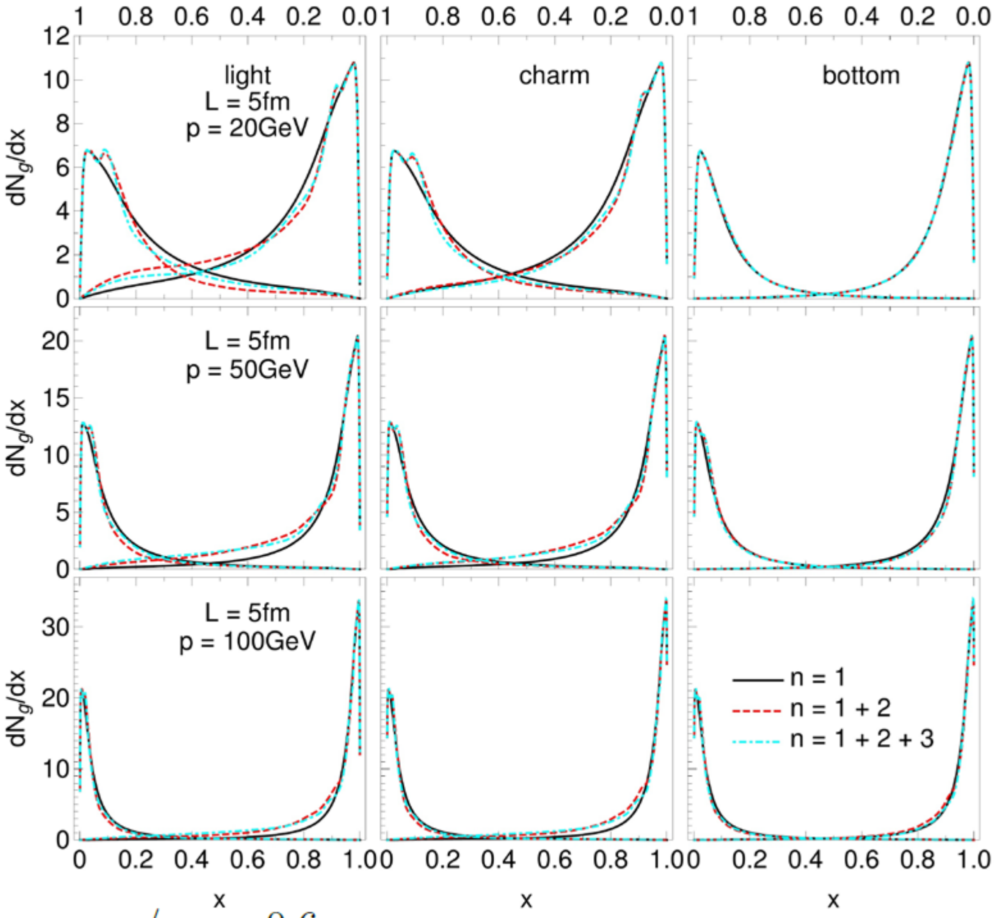
DREENA talk by Dusan Zigic
Tue 11:25
https://GitHub.com/DusanZigic/DREENA-A

Effect of higher orders in opacity on $\frac{dN^g}{dx}$

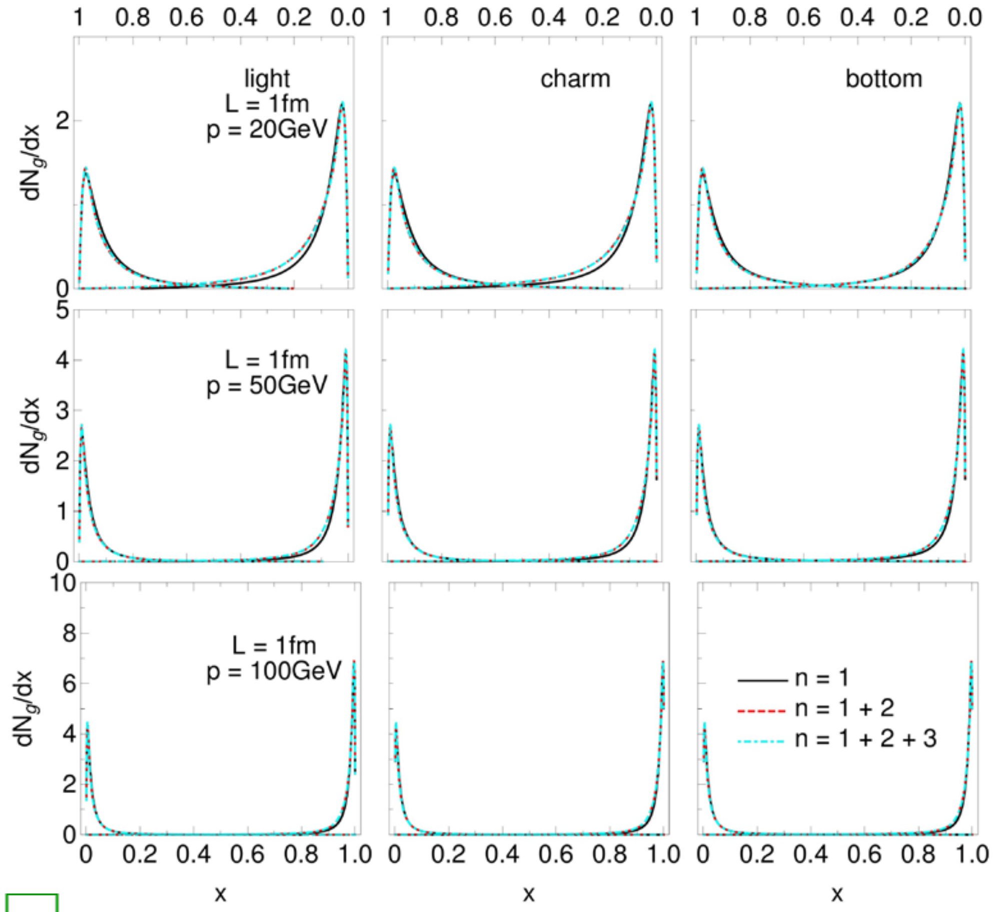
S. Stojku, Bl. I. Salom, M. Djordjevic, arXiv:2303.14527

x - fraction of initial parton's energy carried away by radiated gluon

$\mu_M/\mu_E = 0.4$



$\mu_M/\mu_E = 0.6$



Bottom quark practically unaffected.

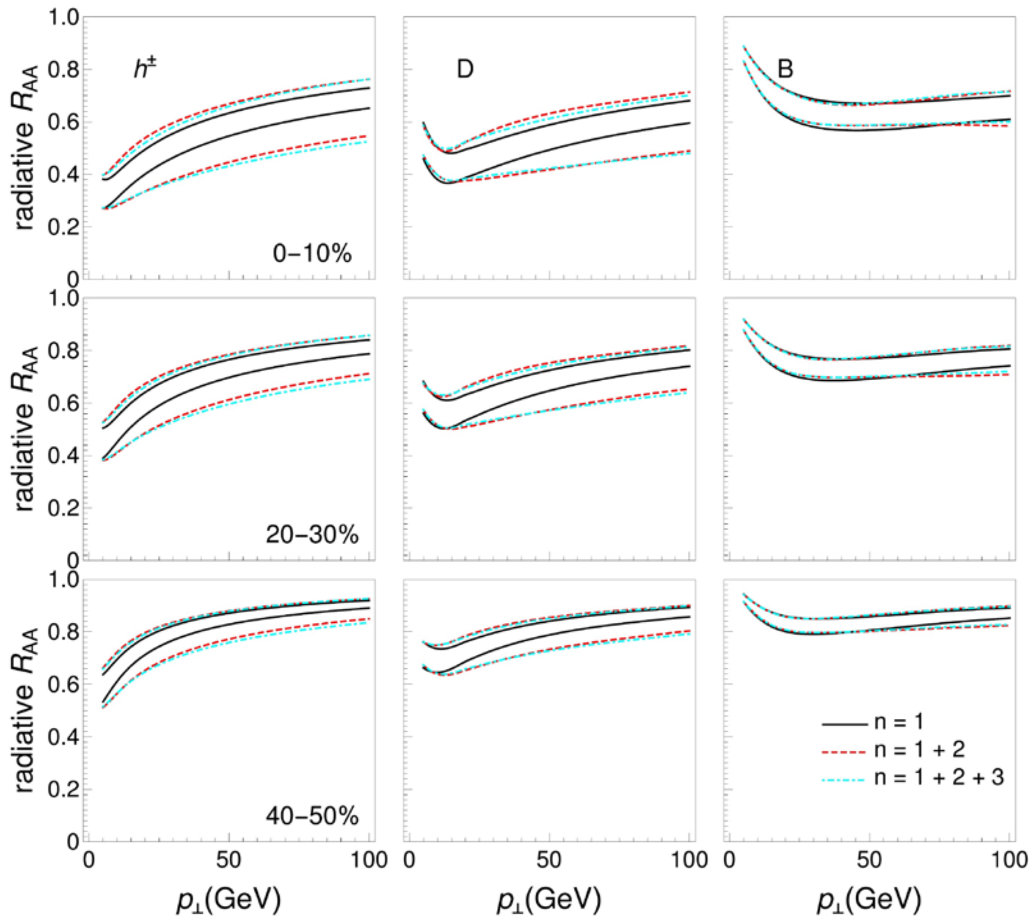
Light and charm quarks moderately affected.

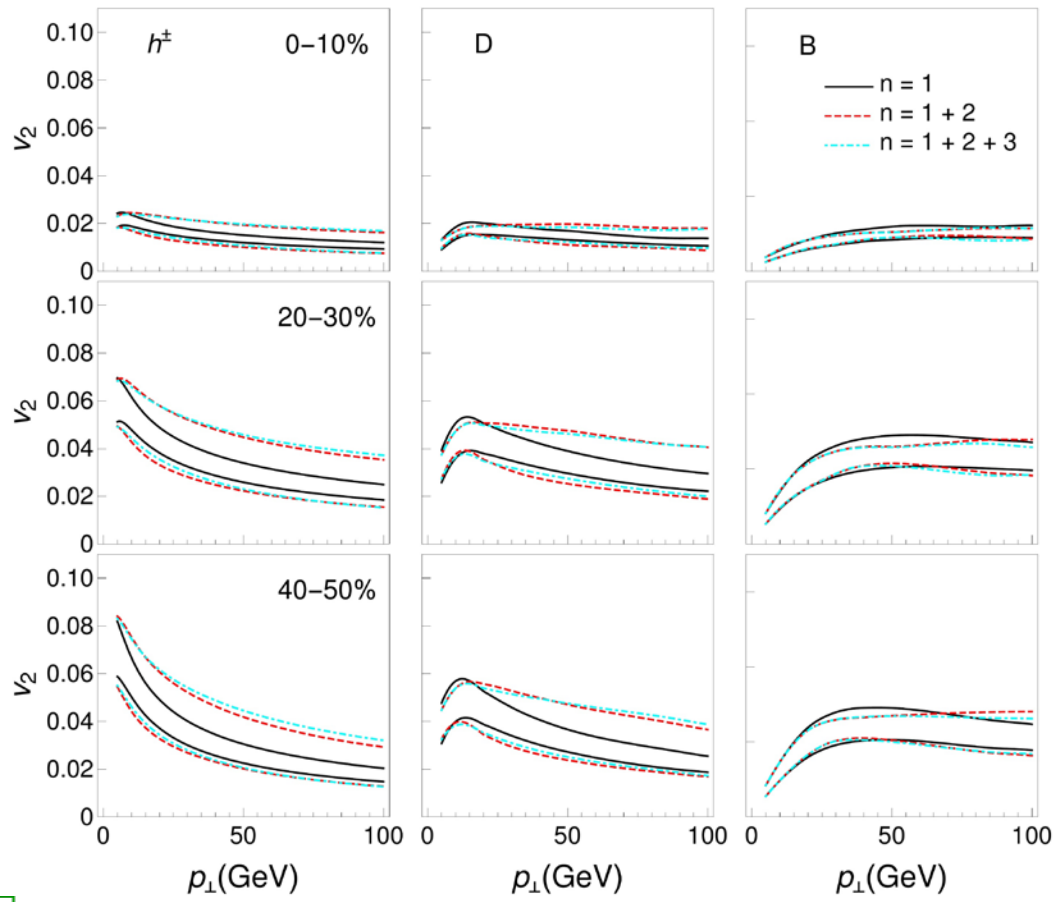
What is the effect on observables?

Diminishes with the increase of high-$p_\perp$ partons energy and mass, as well as with the decrease of QCD medium size.

PRD 81 091501; PRD 69, 014506

Effect of higher orders in opacity on radiative R_{AA} and v_2





3rd overlaps with 2nd order.

2nd order sufficient!

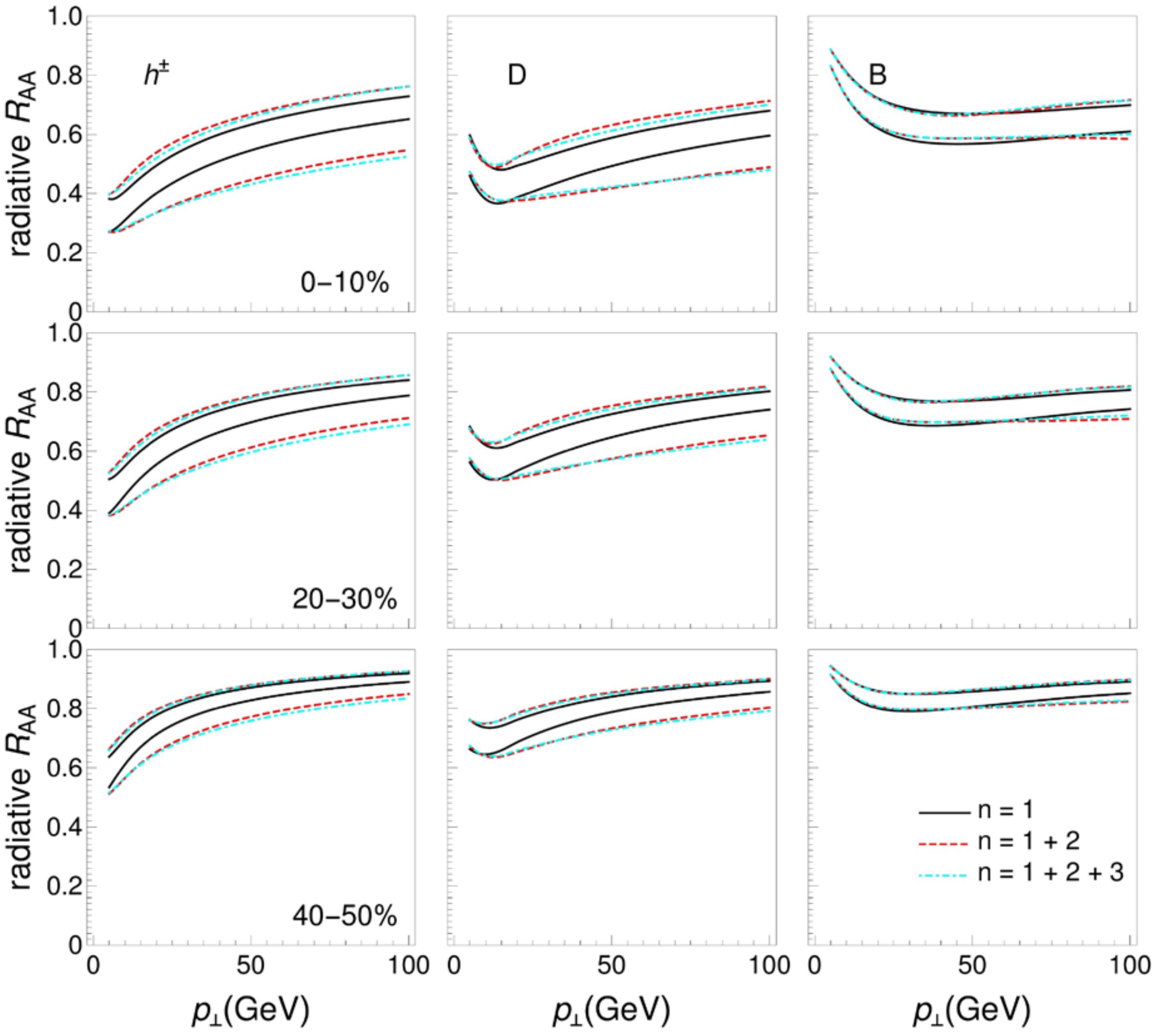
M. Djordjevic and M. Gyulassy, NPA 733, 265; C. Andres et al., JHEP 07, 114

Qualitatively and quantitatively similar effect on R_{AA} and v_2: Decreases for more peripheral collisions. Effect on B meson insignificant (short formation time; approaches incoherent limit), whereas increases with decreasing mass.

Effect of higher orders in opacity on radiative R_{AA} (I)

(dependence on magnetic mass)

S. Stojku, BI, I. Salom, M. Djordjevic, arXiv:2303.14527



Surprisingly, the effect is opposite in sign for limiting values of $\frac{\mu_M}{\mu_E}$

For $\frac{\mu_M}{\mu_E} = 0.6$: negligible and decreases energy loss (increases R_{AA})

For $\frac{\mu_M}{\mu_E} = 0.4$: noticeable and increases energy loss (decreases R_{AA})

What is the origin of such behavior?

PRD 81, 091501; PRD 69, 014506

Effective potential in dynamical QCD medium: theoretical analysis

S. Stojku, BI, I. Salom, M. Djordjevic, arXiv:2303.14527

M. Djordjevic, PLB 709, 229

$$v(\mathbf{q}) = v_L(\mathbf{q}) - v_T(\mathbf{q})$$

$$v_L(\mathbf{q}) = \frac{1}{\pi}\left(\frac{1}{(\mathbf{q}^2+\mu_{pl}^2)} - \frac{1}{(\mathbf{q}^2+\mu_E^2)}\right), \quad v_T(\mathbf{q}) = \frac{1}{\pi}\left(\frac{1}{(\mathbf{q}^2+\mu_{pl}^2)} - \frac{1}{(\mathbf{q}^2+\mu_M^2)}\right)$$

$$\mu_{pl} = \mu_E/\sqrt{3}$$

$\mu_{pl} < \mu_E$

⇓

Longitudinal (electric) contribution to effective potential is always positive.

⇓

Radiative energy loss > 0 (R_{AA} < 1).

⇓

Transverse (magnetic) contribution to effective potential changes sign.

⇓

$\mu_M/\mu_E = 0.6$

If $\mu_M > \mu_{pl}$ decreases radiative energy loss (increases R_{AA})

If $\mu_M < \mu_{pl}$ increases radiative energy loss (decreases R_{AA})

$\mu_M/\mu_E = 0.4$

⇒

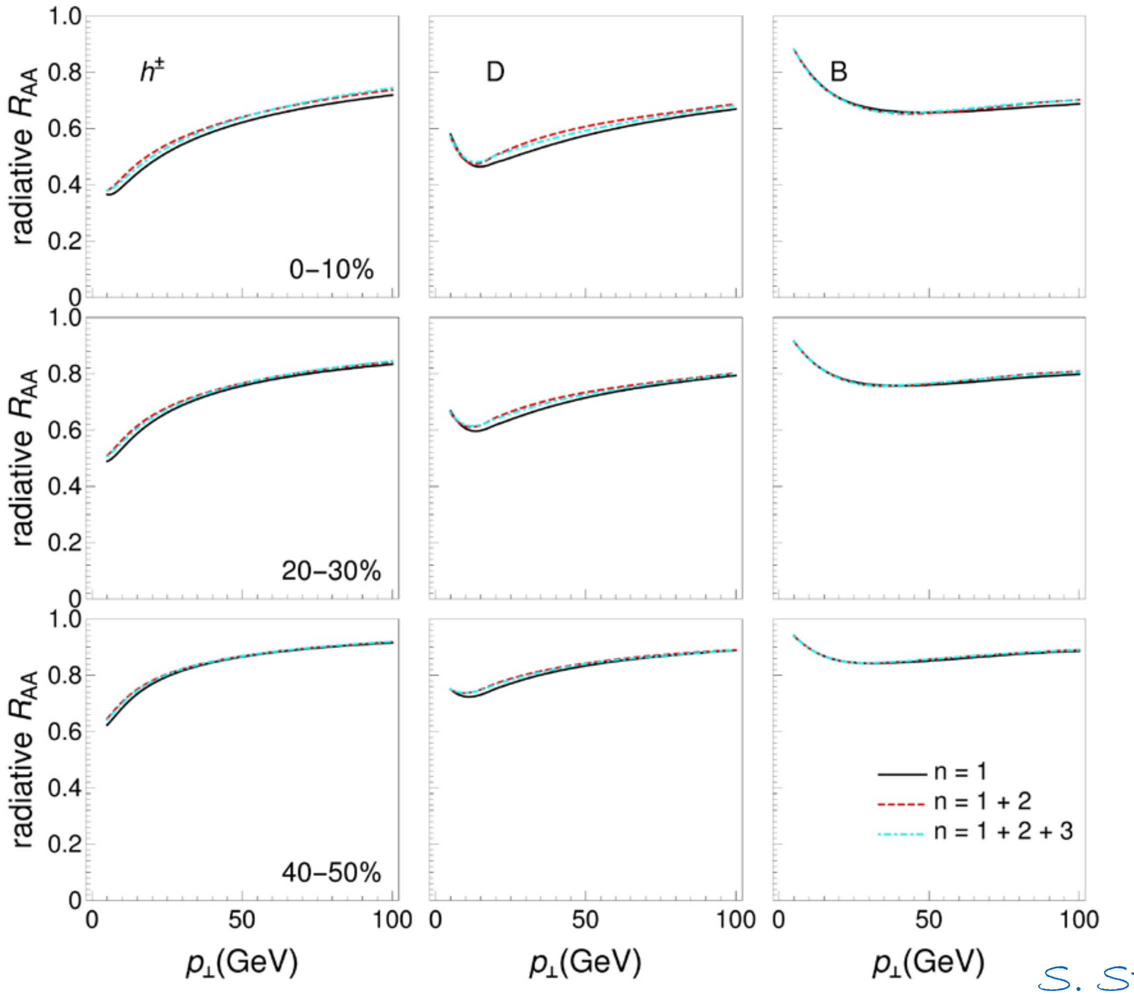
Consistent with previous slide and provides possible explanation of R_{AA} behavior.

Effect of higher orders in opacity on electric contribution to R_{AA} in dynamical QCD medium:

Electric effective potential is well-defined within dynamical QCD medium, as μ_E is set from lattice QCD, consistently with perturbative calculations.

A. Peshier, arXiv:0601119

$$v(\mathbf{q}) = v_L(\mathbf{q})$$



Unexpectedly, electric contribution to radiative energy loss in a dynamical QCD medium is insignificantly affected.

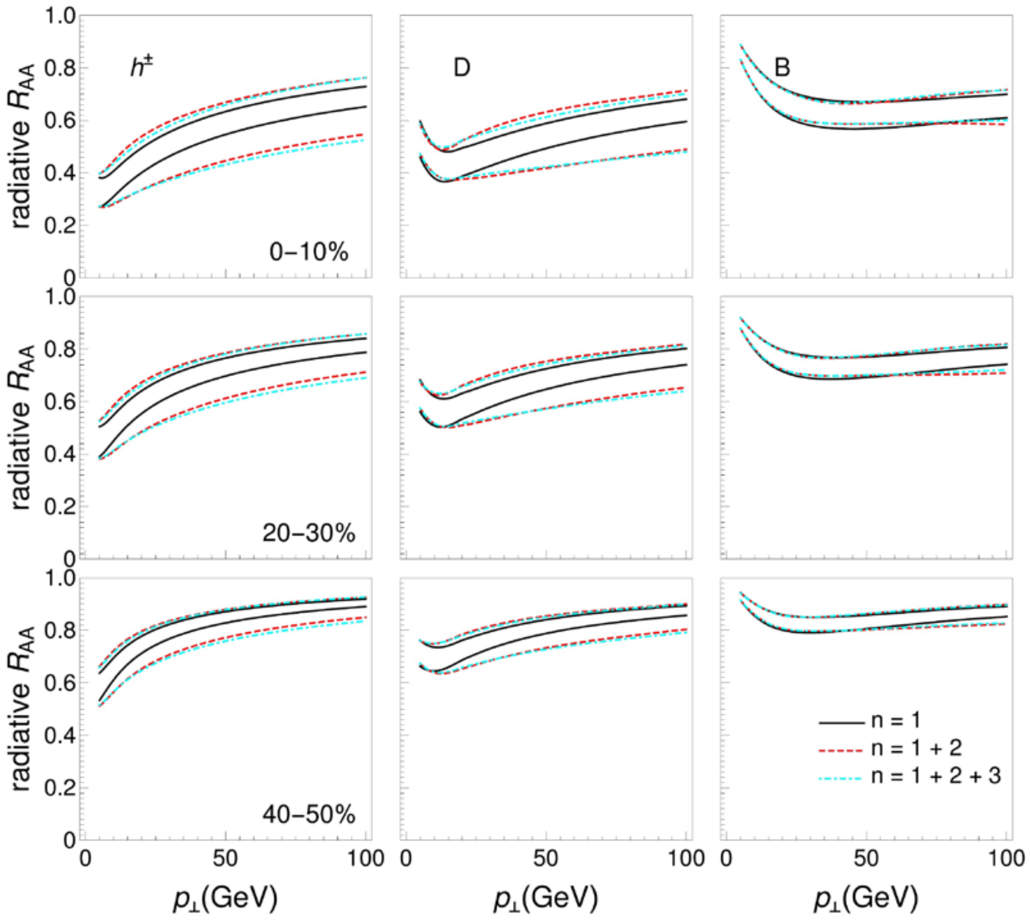
Higher orders in opacity dominantly influence magnetic contribution to radiative energy loss.

Sign and magnitude of the effect is controlled by μ_M value.

S. Stojku, BI, I. Salom, M. Djordjevic, arXiv:2303.14527

Effect of higher orders in opacity on radiative RAA (II)

$\mu_{pl}/\mu_E = 1/\sqrt{3}$

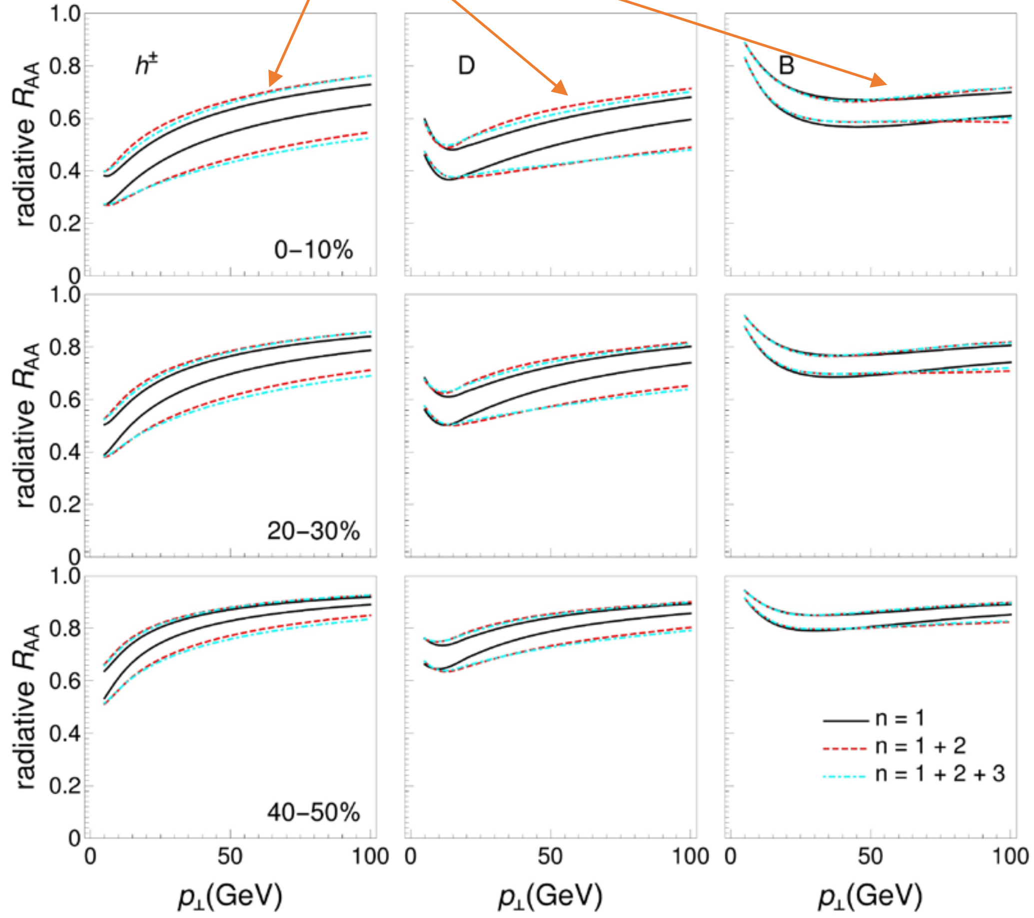


For $\frac{\mu_M}{\mu_E} = 0.4$ (μ_M substantialy smaller than μ_{pl}):
Effect significant and decreases R_{AA}
For $\frac{\mu_M}{\mu_E} = 0.6$ (μ_M close to, but a bit larger than μ_{pl}):
Effect negligible and slightly increases R_{AA}

In agreement with our analytical arguments.

Effect of higher orders in opacity on radiative RAA (II)

The upper set of curves: $\mu_M/\mu_E = 0.6$



$\mu_{pl}/\mu_E = 1/\sqrt{3}$

For $\frac{\mu_M}{\mu_E} = 0.4$ (μ_M substantialy smaller than μ_{pl}):
Effect significant and decreases R_{AA}
For $\frac{\mu_M}{\mu_E} = 0.6$ (μ_M close to, but a bit larger than μ_{pl}):
Effect negligible and slightly increases R_{AA}

In agreement with our analytical arguments.

2+1 flavor lQCD:

$0.58 < \mu_M/\mu_E < 0.64$

S. Borsanyi et al., JHEP 1504, 138

The effect of higher orders in opacity on R_{AA} is negligible ($\lesssim 5\%$) in a dynamical medium!

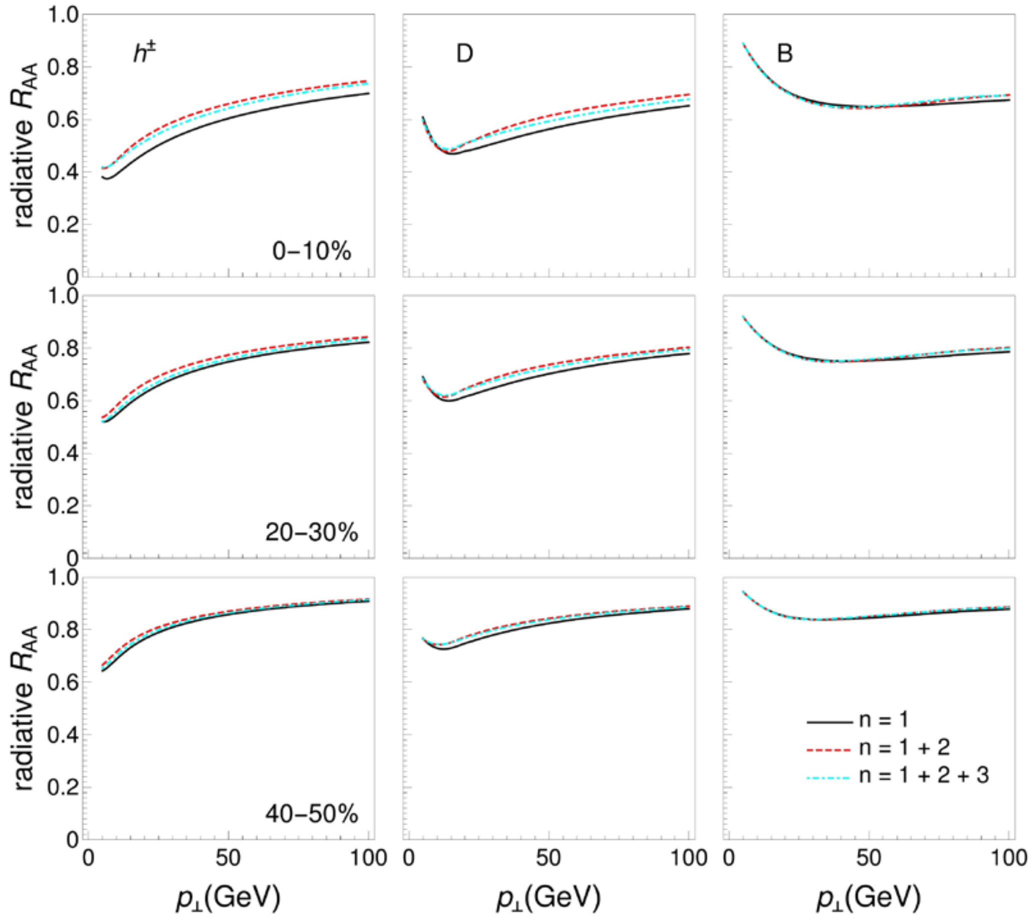
Effect of higher orders in opacity on radiative R_{AA} in *static* QCD medium (DGLV)

Static QCD medium approximation → only electric contribution

S. Stojku, BI, I. Salom, M. Djordjevic, arXiv:2303.14527

Static effective potential: $\frac{\mu_E^2}{\pi(\mathbf{q}^2+\mu_E^2)^2}$ Static mfp: $\lambda_{\text{stat}}^{-1} = 6\frac{1.202}{\pi^2}\frac{1+n_f/4}{1+n_f/6}\lambda_{\text{dyn}}^{-1}$

$\lambda_{dyn}^{-1} = 3\alpha_s(Q_v^2)T$



The effect is slightly larger than in the dynamical case, but yet small (≲6%).

For optically thin *static* medium models the effect of higher order in opacity on high-$p_\perp$ observables is still insignificant.

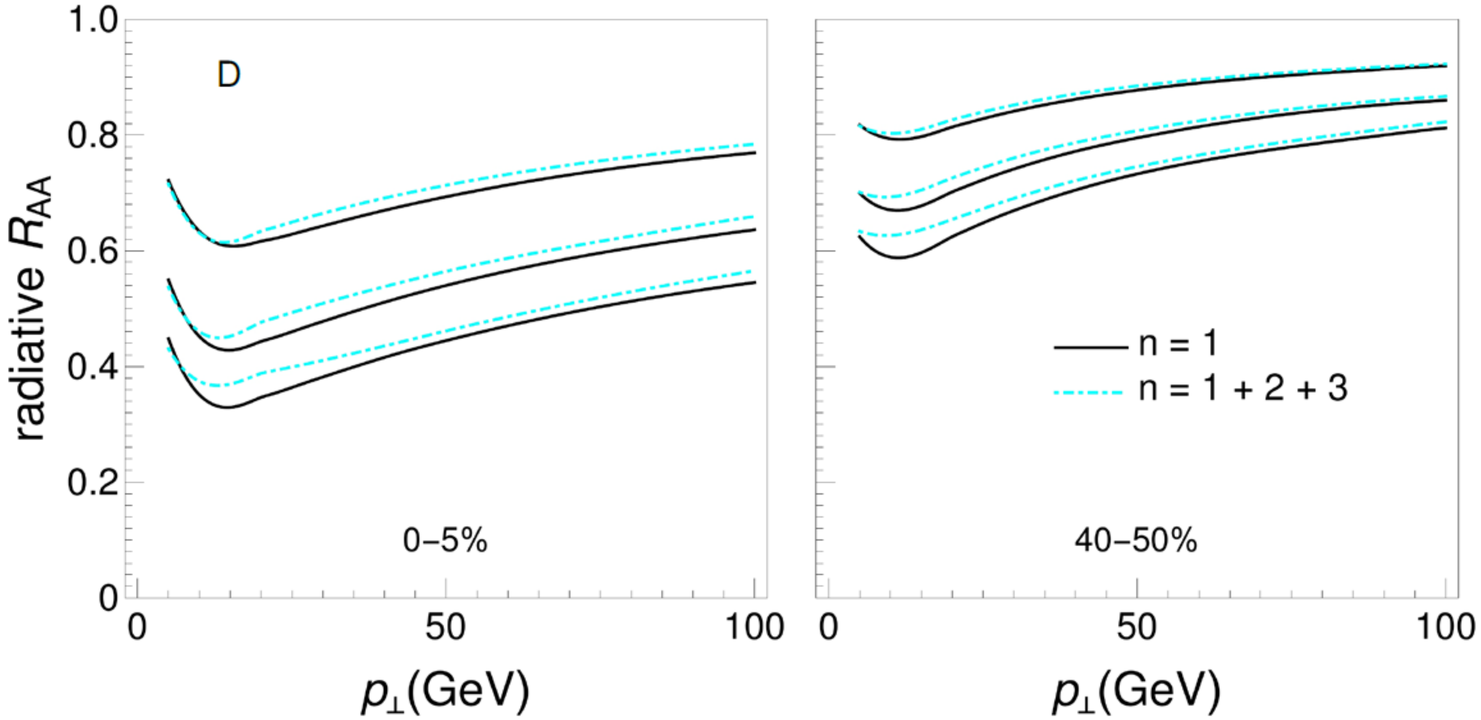
1st order in opacity is an adequate approximation for optically thin *static* medium.

Effect of higher orders in opacity in evolving medium: a preview

Quite a demanding task, out of the scope of this talk

We mimic medium evolution by assuming different medium T

[S. Stojku, BI, I. Salom, M. Djordjevic, arXiv:2303.14527]



Uppermost curves: T=200 MeV
Middle curves: T=400 MeV
Lowest curves: T=600 MeV

$\mu_M/\mu_E = 0.6$

The effect is nearly the same and small regardless of the medium T.

We expect the conclusions to remain the same for evolving medium.

The higher orders in opacity should be negligible in evolving medium.

Conclusions

- ✓We included a finite number of scattering centers in our dynamical energy loss (and DGLV) formalism (analytical expressions up to 4th order in opacity), and DREENA-C numerical framework
- ✓Bottom probes practically unaffected (short gluon formation time, i.e., incoherent limit)
- ✓Charm and light probes: 2nd order in opacity sufficient
- ✓In static QCD medium (DGLV): 1st order in opacity approximation adequate
- ✓In dynamic QCD medium: Surprisingly, impact on electric contribution to radiative energy loss negligible
- ✓Magnetic contribution to radiative energy loss sensitive to higher orders in opacity. The sign and magnitude of the effect depend on the magnetic mass value
- ✓For most recent magnetic screening estimates higher orders can be safely disregarded.
- ✓1st order in opacity approximation is adequate for a finite size medium created at the RHIC and LHC
- ✓The same conclusions should remain valid when evolving medium is introduced.

Thank you for your attention!

BACK UP

Analytical calculations: examples

$$\left(\frac{dN_g}{dx}\right) = \left(\frac{dN_g^{(1)}}{dx}\right) + \left(\frac{dN_g^{(2)}}{dx}\right)_1 - \left(\frac{dN_g^{(2)}}{dx}\right)_2$$
$$+ \left(\frac{dN_g^{(3)}}{dx}\right)_1 - \left(\frac{dN_g^{(3)}}{dx}\right)_2 - \left(\frac{dN_g^{(3)}}{dx}\right)_3 + \left(\frac{dN_g^{(3)}}{dx}\right)_4$$
$$+ \left(\frac{dN_g^{(4)}}{dx}\right)_1 - \left(\frac{dN_g^{(4)}}{dx}\right)_2 - \left(\frac{dN_g^{(4)}}{dx}\right)_3 + \left(\frac{dN_g^{(4)}}{dx}\right)_4$$
$$- \left(\frac{dN_g^{(4)}}{dx}\right)_5 + \left(\frac{dN_g^{(4)}}{dx}\right)_6 + \left(\frac{dN_g^{(4)}}{dx}\right)_7 - \left(\frac{dN_g^{(4)}}{dx}\right)_8$$

1st order in opacity:

$$\left(\frac{dN_g^{(1)}}{dx}\right) = \frac{2C_R}{\pi x}\int\frac{d^2\mathbf{k}}{\pi}\int\frac{d^2\mathbf{q}_1}{\pi}\,\alpha_s(Q_k^2)\frac{L}{\lambda_{dyn}}\frac{\mu_E^2-\mu_M^2}{(\mathbf{q}_1^2+\mu_E^2)(\mathbf{q}_1^2+\mu_M^2)}$$
$$\frac{\chi^2(\mathbf{q}_1\cdot(\mathbf{q}_1-\mathbf{k}))+(\mathbf{q}_1\cdot\mathbf{k})(\mathbf{k}-\mathbf{q}_1)^2}{(\chi^2+\mathbf{k}^2)(\chi^2+(\mathbf{k}-\mathbf{q}_1)^2)^2}\left(1-\frac{\sin(L\omega_{(1)})}{L\omega_{(1)}}\right)$$

2nd order in opacity:

$$\left(\frac{dN_g^{(2)}}{dx}\right)_1 = \frac{2C_R}{\pi x}\int\frac{d^2\mathbf{k}}{\pi}\iint\frac{d^2\mathbf{q}_1}{\pi}\frac{d^2\mathbf{q}_2}{\pi}\,\alpha_s(Q_k^2)\frac{1}{\lambda_{dyn}^2}\frac{\mu_E^2-\mu_M^2}{(\mathbf{q}_1^2+\mu_E^2)(\mathbf{q}_1^2+\mu_M^2)}\frac{\mu_E^2-\mu_M^2}{(\mathbf{q}_2^2+\mu_E^2)(\mathbf{q}_2^2+\mu_M^2)}$$
$$\frac{\chi^2(\mathbf{q}_2\cdot(\mathbf{q}_1+\mathbf{q}_2-\mathbf{k}))+(\mathbf{q}_2\cdot\mathbf{k})(\mathbf{k}-\mathbf{q}_2)^2+(\mathbf{k}\cdot\mathbf{q}_1)(\mathbf{q}_2\cdot(\mathbf{q}_2-2\mathbf{k}))+\mathbf{k}^2(\mathbf{q}_2\cdot\mathbf{q}_1)}{(\chi^2+\mathbf{k}^2)(\chi^2+(\mathbf{k}-\mathbf{q}_2)^2)(\chi^2+(\mathbf{k}-\mathbf{q}_1-\mathbf{q}_2)^2)}$$
$$\frac{1}{\omega_{(2)}}\left(\frac{\omega_{(2)}\cos(L(\omega_{(2)}+\omega_{(12)}))}{(\omega_{(2)}+\omega_{(12)})\,\omega_{(12)}}+L\sin(L\omega_{(2)})-\frac{(\omega_{(2)}-\omega_{(12)})\cos(L\omega_{(2)})}{\omega_{(2)}\omega_{(12)}}-\frac{\omega_{(12)}}{\omega_{(2)}(\omega_{(2)}+\omega_{(12)})}\right)$$

$$\left(\frac{dN_g^{(2)}}{dx}\right)_2 = \frac{2C_R}{\pi x}\int\frac{d^2\mathbf{k}}{\pi}\int\frac{d^2\mathbf{q}_2}{\pi}\,\alpha_s(Q_k^2)\frac{1}{\lambda_{dyn}^2}\frac{\mu_E^2-\mu_M^2}{(\mathbf{q}_2^2+\mu_E^2)(\mathbf{q}_2^2+\mu_M^2)}$$
$$\frac{\chi^2(\mathbf{q}_2\cdot(\mathbf{q}_2-\mathbf{k}))+(\mathbf{q}_2\cdot\mathbf{k})(\mathbf{k}-\mathbf{q}_2)^2}{(\chi^2+\mathbf{k}^2)(\chi^2+(\mathbf{k}-\mathbf{q}_2)^2)^2}\frac{\sin(L\omega_{(2)})\,(L\omega_{(2)}-\sin(L\omega_{(2)}))}{\omega_{(2)}^2}$$

3rd order in opacity:

$$\left(\frac{dN_g^{(3)}}{dx}\right)_1 = \frac{2C_R}{\pi x}\int\frac{d^2\mathbf{k}}{\pi}\iiint\frac{d^2\mathbf{q}_1}{\pi}\frac{d^2\mathbf{q}_2}{\pi}\frac{d^2\mathbf{q}_3}{\pi}$$
$$\alpha_s(Q_k^2)\frac{1}{\lambda_{dyn}^3}\frac{\mu_E^2-\mu_M^2}{(\mathbf{q}_1^2+\mu_E^2)(\mathbf{q}_1^2+\mu_M^2)}\frac{\mu_E^2-\mu_M^2}{(\mathbf{q}_2^2+\mu_E^2)(\mathbf{q}_2^2+\mu_M^2)}\frac{\mu_E^2-\mu_M^2}{(\mathbf{q}_3^2+\mu_E^2)(\mathbf{q}_3^2+\mu_M^2)}$$
$$\frac{\chi^2(\mathbf{q}_3\cdot(\mathbf{q}_1+\mathbf{q}_2+\mathbf{q}_3-\mathbf{k}))+(\mathbf{q}_3\cdot\mathbf{k})(\mathbf{k}-\mathbf{q}_3)^2+(\mathbf{k}\cdot(\mathbf{q}_1+\mathbf{q}_2))(\mathbf{q}_3\cdot(\mathbf{q}_3-2\mathbf{k}))+\mathbf{k}^2(\mathbf{q}_3\cdot(\mathbf{q}_1+\mathbf{q}_2))}{(\chi^2+\mathbf{k}^2)(\chi^2+(\mathbf{k}-\mathbf{q}_3)^2)(\chi^2+(\mathbf{k}-\mathbf{q}_1-\mathbf{q}_2-\mathbf{q}_3)^2)}$$
$$\left(\frac{\omega_{(3)}\omega_{(123)}+2\omega_{(23)}\omega_{(123)}-\omega_{(23)}^2-\omega_{(3)}\omega_{(23)}}{\omega_{(23)}^2(\omega_{(3)}+\omega_{(23)})^2\omega_{(123)}}\sin\left(L(\omega_{(3)}+\omega_{(23)})\right)-\frac{\omega_{(123)}\sin\left(L\omega_{(3)}\right)}{\omega_{(3)}\omega_{(23)}^2\left(\omega_{(23)}+\omega_{(123)}\right)}\right.$$
$$\left.+\frac{\sin\left(L\left(\omega_{(3)}+\omega_{(23)}+\omega_{(123)}\right)\right)}{\omega_{(123)}\left(\omega_{(23)}+\omega_{(123)}\right)\left(\omega_{(3)}+\omega_{(23)}+\omega_{(123)}\right)}-\frac{L\cos\left(L\left(\omega_{(3)}+\omega_{(23)}\right)\right)}{\omega_{(23)}\left(\omega_{(3)}+\omega_{(23)}\right)}\right),$$

$$\left(\frac{dN_g^{(3)}}{dx}\right)_2 = \frac{C_R}{\pi x}\int\frac{d^2\mathbf{k}}{\pi}\iint\frac{d^2\mathbf{q}_1}{\pi}\frac{d^2\mathbf{q}_3}{\pi}\alpha_s(Q_k^2)\frac{1}{\lambda_{dyn}^3}\frac{\mu_E^2-\mu_M^2}{(\mathbf{q}_1^2+\mu_E^2)(\mathbf{q}_1^2+\mu_M^2)}\frac{\mu_E^2-\mu_M^2}{(\mathbf{q}_3^2+\mu_E^2)(\mathbf{q}_3^2+\mu_M^2)}$$
$$\frac{\chi^2(\mathbf{q}_3\cdot(\mathbf{q}_1+\mathbf{q}_3-\mathbf{k}))+(\mathbf{q}_3\cdot\mathbf{k})(\mathbf{k}-\mathbf{q}_3)^2+(\mathbf{k}\cdot\mathbf{q}_1)(\mathbf{q}_3\cdot(\mathbf{q}_3-2\mathbf{k}))+\mathbf{k}^2(\mathbf{q}_3\cdot\mathbf{q}_1)}{(\chi^2+\mathbf{k}^2)(\chi^2+(\mathbf{k}-\mathbf{q}_3)^2)(\chi^2+(\mathbf{k}-\mathbf{q}_1-\mathbf{q}_3)^2)}$$
$$\left(\frac{\left(\frac{3\omega_{(13)}}{2}-\omega_{(3)}\right)\sin(2L\omega_{(3)})}{\omega_{(3)}^3\omega_{(13)}}-\frac{2\omega_{(13)}\sin(L\omega_{(3)})}{\omega_{(3)}^3(\omega_{(3)}+\omega_{(13)})}+\frac{\sin\left(2L\left(\omega_{(3)}+\frac{\omega_{(13)}}{2}\right)\right)}{\left(\omega_{(3)}+\frac{\omega_{(13)}}{2}\right)\omega_{(13)}\left(\omega_{(3)}+\omega_{(13)}\right)}-\frac{L\cos\left(2L\omega_{(3)}\right)}{\omega_{(3)}^2}\right)$$

Analytical calculations: examples

3rd order in opacity:

$$\left(\frac{dN_g^{(3)}}{dx}\right)_3 = \frac{2C_R}{\pi x}\int\frac{d^2\mathbf{k}}{\pi}\iint\frac{d^2\mathbf{q}_2}{\pi}\frac{d^2\mathbf{q}_3}{\pi}\alpha_s(Q_k^2)\frac{1}{\lambda_{dyn}^3}\frac{\mu_E^2-\mu_M^2}{(\mathbf{q}_2^2+\mu_E^2)(\mathbf{q}_2^2+\mu_M^2)}\frac{\mu_E^2-\mu_M^2}{(\mathbf{q}_3^2+\mu_E^2)(\mathbf{q}_3^2+\mu_M^2)}$$
$$\frac{\chi^2(\mathbf{q}_3\cdot(\mathbf{q}_2+\mathbf{q}_3-\mathbf{k}))+(\mathbf{q}_3\cdot\mathbf{k})(\mathbf{k}-\mathbf{q}_3)^2+(\mathbf{k}\cdot\mathbf{q}_2)(\mathbf{q}_3\cdot(\mathbf{q}_3-2\mathbf{k}))+\mathbf{k}^2(\mathbf{q}_3\cdot\mathbf{q}_2)}{(\chi^2+\mathbf{k}^2)(\chi^2+(\mathbf{k}-\mathbf{q}_3)^2)(\chi^2+(\mathbf{k}-\mathbf{q}_2-\mathbf{q}_3)^2)}$$
$$\left(\frac{\sin\left(2L\left(\frac{\omega_{(3)}}{2}+\omega_{(23)}\right)\right)}{4\omega_{(23)}^2\left(\frac{\omega_{(3)}}{2}+\omega_{(23)}\right)}-\frac{\sin\left(L\omega_{(3)}\right)}{2\omega_{(23)}^2\omega_{(3)}}+\frac{\frac{\sin\left(L\left(\omega_{(3)}+\omega_{(23)}\right)\right)}{\omega_{(3)}+\omega_{(23)}}-L\cos\left(L\left(\omega_{(3)}+\omega_{(23)}\right)\right)}{\omega_{(23)}\left(\omega_{(3)}+\omega_{(23)}\right)}\right),$$

$$\left(\frac{dN_g^{(3)}}{dx}\right)_4 = \frac{C_R}{\pi x}\int\frac{d^2\mathbf{k}}{\pi}\int\frac{d^2\mathbf{q}_3}{\pi}\alpha_s(Q_k^2)\frac{1}{\lambda_{dyn}^3}\frac{\mu_E^2-\mu_M^2}{(\mathbf{q}_3^2+\mu_E^2)(\mathbf{q}_3^2+\mu_M^2)}\frac{\chi^2(\mathbf{q}_3\cdot(\mathbf{q}_3-\mathbf{k}))+(\mathbf{q}_3\cdot\mathbf{k})(\mathbf{k}-\mathbf{q}_3)^2}{(\chi^2+\mathbf{k}^2)(\chi^2+(\mathbf{k}-\mathbf{q}_3)^2)^2}$$
$$\frac{1}{\omega_{(3)}^2}\left(-\frac{\sin\left(L\omega_{(3)}\right)}{\omega_{(3)}}+\frac{\sin\left(2L\omega_{(3)}\right)}{2\omega_{(3)}}+\frac{\sin\left(3L\omega_{(3)}\right)}{3\omega_{(3)}}-L\cos\left(2L\omega_{(3)}\right)\right),$$

4th order in opacity:

$$\left(\frac{dN_g^{(4)}}{dx}\right)_1 = \frac{2C_R}{\pi x}\int\frac{d^2\mathbf{k}}{\pi}\iiiint\frac{d^2\mathbf{q}_1}{\pi}\frac{d^2\mathbf{q}_2}{\pi}\frac{d^2\mathbf{q}_3}{\pi}\frac{d^2\mathbf{q}_4}{\pi}$$
$$\alpha_s(Q_k^2)\frac{1}{\lambda_{dyn}^4}\frac{\mu_E^2-\mu_M^2}{(\mathbf{q}_1^2+\mu_E^2)(\mathbf{q}_1^2+\mu_M^2)}\frac{\mu_E^2-\mu_M^2}{(\mathbf{q}_2^2+\mu_E^2)(\mathbf{q}_2^2+\mu_M^2)}\frac{\mu_E^2-\mu_M^2}{(\mathbf{q}_3^2+\mu_E^2)(\mathbf{q}_3^2+\mu_M^2)}\frac{\mu_E^2-\mu_M^2}{(\mathbf{q}_4^2+\mu_E^2)(\mathbf{q}_4^2+\mu_M^2)}$$
$$\frac{\chi^2(\mathbf{q}_4\cdot(\mathbf{q}_1+\mathbf{q}_2+\mathbf{q}_3+\mathbf{q}_4-\mathbf{k}))+(\mathbf{q}_4\cdot\mathbf{k})(\mathbf{k}-\mathbf{q}_4)^2+(\mathbf{k}\cdot(\mathbf{q}_1+\mathbf{q}_2+\mathbf{q}_3))(\mathbf{q}_4\cdot(\mathbf{q}_4-2\mathbf{k}))+\mathbf{k}^2(\mathbf{q}_4\cdot(\mathbf{q}_1+\mathbf{q}_2+\mathbf{q}_3))}{(\chi^2+\mathbf{k}^2)(\chi^2+(\mathbf{k}-\mathbf{q}_4)^2)(\chi^2+(\mathbf{k}-\mathbf{q}_1-\mathbf{q}_2-\mathbf{q}_3-\mathbf{q}_4)^2)}$$
$$\left(-\frac{L\sin\left(L\left(\omega_{(4)}+\omega_{(34)}+\omega_{(234)}\right)\right)}{\omega_{(234)}\left(\omega_{(34)}+\omega_{(234)}\right)\left(\omega_{(4)}+\omega_{(34)}+\omega_{(234)}\right)}\right.$$
$$-\frac{\cos\left(L\left(\omega_{(4)}+\omega_{(34)}+\omega_{(234)}+\omega_{(1234)}\right)\right)}{\omega_{(1234)}\left(\omega_{(234)}+\omega_{(1234)}\right)\left(\omega_{(34)}+\omega_{(234)}+\omega_{(1234)}\right)\left(\omega_{(4)}+\omega_{(34)}+\omega_{(234)}+\omega_{(1234)}\right)}$$
$$+\frac{F_{41}}{\omega_{(234)}^2\left(\omega_{(34)}+\omega_{(234)}\right)^2\left(\omega_{(4)}+\omega_{(34)}+\omega_{(234)}\right)^2\omega_{(1234)}}\cos\left(L(\omega_{(4)}+\omega_{(34)}+\omega_{(234)})\right)$$
$$+\frac{\omega_{(1234)}\cos\left(L\left(\omega_{(4)}+\omega_{(34)}\right)\right)}{\omega_{(34)}\left(\omega_{(4)}+\omega_{(34)}\right)\omega_{(234)}^2\left(\omega_{(234)}+\omega_{(1234)}\right)}-\frac{\omega_{(1234)}\cos\left(L\omega_{(4)}\right)}{\omega_{(4)}\omega_{(34)}\left(\omega_{(34)}+\omega_{(234)}\right)^2\left(\omega_{(34)}+\omega_{(234)}+\omega_{(1234)}\right)}$$
$$\left.+\frac{\omega_{(1234)}}{\omega_{(4)}\left(\omega_{(4)}+\omega_{(34)}\right)\left(\omega_{(4)}+\omega_{(34)}+\omega_{(234)}\right)^2\left(\omega_{(4)}+\omega_{(34)}+\omega_{(234)}+\omega_{(1234)}\right)}\right),$$

4th order in opacity:

$$\left(\frac{dN_g^{(4)}}{dx}\right)_2 = \frac{C_R}{\pi x}\int\frac{d^2\mathbf{k}}{\pi}\iiint\frac{d^2\mathbf{q}_1}{\pi}\frac{d^2\mathbf{q}_2}{\pi}\frac{d^2\mathbf{q}_4}{\pi}$$
$$\alpha_s(Q_k^2)\frac{1}{\lambda_{dyn}^4}\frac{\mu_E^2-\mu_M^2}{(\mathbf{q}_1^2+\mu_E^2)(\mathbf{q}_1^2+\mu_M^2)}\frac{\mu_E^2-\mu_M^2}{(\mathbf{q}_2^2+\mu_E^2)(\mathbf{q}_2^2+\mu_M^2)}\frac{\mu_E^2-\mu_M^2}{(\mathbf{q}_4^2+\mu_E^2)(\mathbf{q}_4^2+\mu_M^2)}$$
$$\frac{\chi^2(\mathbf{q}_4\cdot(\mathbf{q}_1+\mathbf{q}_2+\mathbf{q}_4-\mathbf{k}))+(\mathbf{q}_4\cdot\mathbf{k})(\mathbf{k}-\mathbf{q}_4)^2+(\mathbf{k}\cdot(\mathbf{q}_1+\mathbf{q}_2))(\mathbf{q}_4\cdot(\mathbf{q}_4-2\mathbf{k}))+\mathbf{k}^2(\mathbf{q}_4\cdot(\mathbf{q}_1+\mathbf{q}_2))}{(\chi^2+\mathbf{k}^2)(\chi^2+(\mathbf{k}-\mathbf{q}_4)^2)(\chi^2+(\mathbf{k}-\mathbf{q}_1-\mathbf{q}_2-\mathbf{q}_4)^2)}$$
$$\left(\frac{2\left(\omega_{(24)}\left(2\omega_{(4)}^2+3\omega_{(24)}\omega_{(4)}+\omega_{(24)}^2\right)-\left(2\omega_{(4)}^2+6\omega_{(24)}\omega_{(4)}+3\omega_{(24)}^2\right)\omega_{(124)}\right)\cos\left(L\left(2\omega_{(4)}+\omega_{(24)}\right)\right)}{\omega_{(24)}^2\left(\omega_{(4)}+\omega_{(24)}\right)^2\left(2\omega_{(4)}+\omega_{(24)}\right)^2\omega_{(124)}}\right.$$
$$-\frac{2\cos\left(L\left(2\omega_{(4)}+\omega_{(24)}+\omega_{(124)}\right)\right)}{\omega_{(124)}\left(\omega_{(24)}+\omega_{(124)}\right)\left(\omega_{(4)}+\omega_{(24)}+\omega_{(124)}\right)\left(2\omega_{(4)}+\omega_{(24)}+\omega_{(124)}\right)}+\frac{\omega_{(124)}\cos\left(2L\omega_{(4)}\right)}{\omega_{(4)}^2\omega_{(24)}^2\left(\omega_{(24)}+\omega_{(124)}\right)}$$
$$-\frac{2L\sin\left(L\left(2\omega_{(4)}+\omega_{(24)}\right)\right)}{\omega_{(24)}\left(\omega_{(4)}+\omega_{(24)}\right)\left(2\omega_{(4)}+\omega_{(24)}\right)}-\frac{2\omega_{(124)}\cos\left(L\omega_{(4)}\right)}{\omega_{(4)}^2\left(\omega_{(4)}+\omega_{(24)}\right)^2\left(\omega_{(4)}+\omega_{(24)}+\omega_{(124)}\right)}$$
$$\left.+\frac{\omega_{(124)}}{\omega_{(4)}^2\left(2\omega_{(4)}+\omega_{(24)}\right)^2\left(2\omega_{(4)}+\omega_{(24)}+\omega_{(124)}\right)}\right),$$

$$\left(\frac{dN_g^{(4)}}{dx}\right)_3 = \frac{C_R}{2\pi x}\int\frac{d^2\mathbf{k}}{\pi}\iiint\frac{d^2\mathbf{q}_1}{\pi}\frac{d^2\mathbf{q}_3}{\pi}\frac{d^2\mathbf{q}_4}{\pi}$$
$$\alpha_s(Q_k^2)\frac{1}{\lambda_{dyn}^4}\frac{\mu_E^2-\mu_M^2}{(\mathbf{q}_1^2+\mu_E^2)(\mathbf{q}_1^2+\mu_M^2)}\frac{\mu_E^2-\mu_M^2}{(\mathbf{q}_3^2+\mu_E^2)(\mathbf{q}_3^2+\mu_M^2)}\frac{\mu_E^2-\mu_M^2}{(\mathbf{q}_4^2+\mu_E^2)(\mathbf{q}_4^2+\mu_M^2)}$$
$$\frac{\chi^2(\mathbf{q}_4\cdot(\mathbf{q}_1+\mathbf{q}_3+\mathbf{q}_4-\mathbf{k}))+(\mathbf{q}_4\cdot\mathbf{k})(\mathbf{k}-\mathbf{q}_4)^2+(\mathbf{k}\cdot(\mathbf{q}_1+\mathbf{q}_3))(\mathbf{q}_4\cdot(\mathbf{q}_4-2\mathbf{k}))+\mathbf{k}^2(\mathbf{q}_4\cdot(\mathbf{q}_1+\mathbf{q}_3))}{(\chi^2+\mathbf{k}^2)(\chi^2+(\mathbf{k}-\mathbf{q}_4)^2)(\chi^2+(\mathbf{k}-\mathbf{q}_1-\mathbf{q}_3-\mathbf{q}_4)^2)}$$
$$\left(-\frac{2L\sin\left(L\left(\omega_{(4)}+2\omega_{(34)}\right)\right)}{\omega_{(34)}^2\left(\omega_{(4)}+2\omega_{(34)}\right)}-\frac{4\cos\left(L\left(\omega_{(4)}+2\omega_{(34)}+\omega_{(134)}\right)\right)}{\omega_{(134)}\left(\omega_{(34)}+\omega_{(134)}\right)\left(2\omega_{(34)}+\omega_{(134)}\right)\left(\omega_{(4)}+2\omega_{(34)}+\omega_{(134)}\right)}\right.$$
$$+\frac{\left(2\omega_{(34)}\left(\omega_{(4)}+2\omega_{(34)}\right)-\left(3\omega_{(4)}+8\omega_{(34)}\right)\omega_{(134)}\right)\cos\left(L\left(\omega_{(4)}+2\omega_{(34)}\right)\right)}{\omega_{(34)}^3\left(\omega_{(4)}+2\omega_{(34)}\right)^2\omega_{(134)}}-\frac{\omega_{(134)}\cos\left(L\omega_{(4)}\right)}{\omega_{(4)}\omega_{(34)}^3\left(2\omega_{(34)}+\omega_{(134)}\right)}$$
$$\left.+\frac{4\omega_{(134)}}{\left(\omega_{(4)}+\omega_{(34)}\right)}\left(\frac{\cos\left(L\left(\omega_{(4)}+\omega_{(34)}\right)\right)}{\omega_{(34)}^3\left(\omega_{(34)}+\omega_{(134)}\right)}+\frac{1}{\omega_{(4)}\left(\omega_{(4)}+2\omega_{(34)}\right)^2\left(\omega_{(4)}+2\omega_{(34)}+\omega_{(134)}\right)}\right)\right),$$

Analytical calculations: examples

4th order in opacity:

$$\left(\frac{dN_g^{(4)}}{dx}\right)_4 = \frac{C_R}{3\pi x}\int\frac{d^2\mathbf{k}}{\pi}\iint\frac{d^2\mathbf{q}_1}{\pi}\frac{d^2\mathbf{q}_4}{\pi}\alpha_s(Q_k^2)\frac{1}{\lambda_{dyn}^4}\frac{\mu_E^2-\mu_M^2}{(\mathbf{q}_1^2+\mu_E^2)(\mathbf{q}_1^2+\mu_M^2)}\frac{\mu_E^2-\mu_M^2}{(\mathbf{q}_4^2+\mu_E^2)(\mathbf{q}_4^2+\mu_M^2)}$$
$$\frac{\chi^2(\mathbf{q}_4\cdot(\mathbf{q}_1+\mathbf{q}_4-\mathbf{k}))+(\mathbf{q}_4\cdot\mathbf{k})(\mathbf{k}-\mathbf{q}_4)^2+(\mathbf{k}\cdot\mathbf{q}_1)(\mathbf{q}_4\cdot(\mathbf{q}_4-2\mathbf{k}))+\mathbf{k}^2(\mathbf{q}_4\cdot\mathbf{q}_1)}{(\chi^2+\mathbf{k}^2)(\chi^2+(\mathbf{k}-\mathbf{q}_4)^2)(\chi^2+(\mathbf{k}-\mathbf{q}_1-\mathbf{q}_4)^2)}$$
$$\frac{1}{\omega_{(4)}^3}\left(-\frac{6\omega_{(4)}^3\cos\left(L\left(3\omega_{(4)}+\omega_{(14)}\right)\right)}{\omega_{(14)}\left(\omega_{(4)}+\omega_{(14)}\right)\left(2\omega_{(4)}+\omega_{(14)}\right)\left(3\omega_{(4)}+\omega_{(14)}\right)}+\left(\frac{1}{\omega_{(14)}}-\frac{11}{6\omega_{(4)}}\right)\cos(3L\omega_{(4)})\right.$$
$$\left.-L\sin\left(3L\omega_{(4)}\right)-\frac{3\omega_{(14)}\cos\left(L\omega_{(4)}\right)}{4\omega_{(4)}^2+2\omega_{(14)}\omega_{(4)}}+\frac{3\omega_{(14)}\cos\left(2L\omega_{(4)}\right)}{\left(\omega_{(4)}+\omega_{(14)}\right)\omega_{(4)}}+\frac{\omega_{(14)}}{9\omega_{(4)}^2+3\omega_{(14)}\omega_{(4)}}\right),$$

$$\left(\frac{dN_g^{(4)}}{dx}\right)_5 = \frac{C_R}{\pi x}\int\frac{d^2\mathbf{k}}{\pi}\iiint\frac{d^2\mathbf{q}_2}{\pi}\frac{d^2\mathbf{q}_3}{\pi}\frac{d^2\mathbf{q}_4}{\pi}$$
$$\alpha_s(Q_k^2)\frac{1}{\lambda_{dyn}^4}\frac{\mu_E^2-\mu_M^2}{(\mathbf{q}_2^2+\mu_E^2)(\mathbf{q}_2^2+\mu_M^2)}\frac{\mu_E^2-\mu_M^2}{(\mathbf{q}_3^2+\mu_E^2)(\mathbf{q}_3^2+\mu_M^2)}\frac{\mu_E^2-\mu_M^2}{(\mathbf{q}_4^2+\mu_E^2)(\mathbf{q}_4^2+\mu_M^2)}$$
$$\frac{\chi^2(\mathbf{q}_4\cdot(\mathbf{q}_2+\mathbf{q}_3+\mathbf{q}_4-\mathbf{k}))+(\mathbf{q}_4\cdot\mathbf{k})(\mathbf{k}-\mathbf{q}_4)^2+(\mathbf{k}\cdot(\mathbf{q}_2+\mathbf{q}_3))(\mathbf{q}_4\cdot(\mathbf{q}_4-2\mathbf{k}))+\mathbf{k}^2(\mathbf{q}_4\cdot(\mathbf{q}_2+\mathbf{q}_3))}{(\chi^2+\mathbf{k}^2)(\chi^2+(\mathbf{k}-\mathbf{q}_4)^2)(\chi^2+(\mathbf{k}-\mathbf{q}_2-\mathbf{q}_3-\mathbf{q}_4)^2)}$$
$$\frac{1}{\omega_{(234)}^2}\left(\frac{2\omega_{(234)}^3}{\omega_{(4)}\left(\omega_{(4)}+\omega_{(34)}\right)\left(\omega_{(4)}+\omega_{(34)}+\omega_{(234)}\right)^2\left(\omega_{(4)}+\omega_{(34)}+2\omega_{(234)}\right)}+\frac{\cos\left(L\left(\omega_{(4)}+\omega_{(34)}\right)\right)}{\omega_{(34)}\left(\omega_{(4)}+\omega_{(34)}\right)}\right.$$
$$-\frac{2\omega_{(234)}^3\cos\left(L\omega_{(4)}\right)}{\omega_{(4)}\omega_{(34)}\left(\omega_{(34)}+\omega_{(234)}\right)^2\left(\omega_{(34)}+2\omega_{(234)}\right)}-\frac{2L\omega_{(234)}\sin(L(\omega_{(4)}+\omega_{(34)}+\omega_{(234)}))}{\left(\omega_{(34)}+\omega_{(234)}\right)\left(\omega_{(4)}+\omega_{(34)}+\omega_{(234)}\right)}$$
$$\left.-\frac{\cos\left(L\left(\omega_{(4)}+\omega_{(34)}+2\omega_{(234)}\right)\right)}{\left(\omega_{(34)}+2\omega_{(234)}\right)\left(\omega_{(4)}+\omega_{(34)}+2\omega_{(234)}\right)}-\frac{2\omega_{(234)}(\omega_{(4)}+2\omega_{(34)}+2\omega_{(234)})\cos(L(\omega_{(4)}+\omega_{(34)}+\omega_{(234)}))}{\left(\omega_{(34)}+\omega_{(234)}\right)^2\left(\omega_{(4)}+\omega_{(34)}+\omega_{(234)}\right)^2}\right)$$

$$\left(\frac{dN_g^{(4)}}{dx}\right)_6 = \frac{C_R}{\pi x}\int\frac{d^2\mathbf{k}}{\pi}\iint\frac{d^2\mathbf{q}_2}{\pi}\frac{d^2\mathbf{q}_4}{\pi}\alpha_s(Q_k^2)\frac{1}{\lambda_{dyn}^4}\frac{\mu_E^2-\mu_M^2}{(\mathbf{q}_2^2+\mu_E^2)(\mathbf{q}_2^2+\mu_M^2)}\frac{\mu_E^2-\mu_M^2}{(\mathbf{q}_4^2+\mu_E^2)(\mathbf{q}_4^2+\mu_M^2)}$$
$$\frac{\chi^2(\mathbf{q}_4\cdot(\mathbf{q}_2+\mathbf{q}_4-\mathbf{k}))+(\mathbf{q}_4\cdot\mathbf{k})(\mathbf{k}-\mathbf{q}_4)^2+(\mathbf{k}\cdot\mathbf{q}_2)(\mathbf{q}_4\cdot(\mathbf{q}_4-2\mathbf{k}))+\mathbf{k}^2(\mathbf{q}_4\cdot\mathbf{q}_2)}{(\chi^2+\mathbf{k}^2)(\chi^2+(\mathbf{k}-\mathbf{q}_4)^2)(\chi^2+(\mathbf{k}-\mathbf{q}_2-\mathbf{q}_4)^2)}$$
$$\left(\frac{\omega_{(24)}}{8\omega_{(4)}^2\left(\omega_{(4)}+\frac{\omega_{(24)}}{2}\right)^2\left(\omega_{(4)}+\omega_{(24)}\right)}-\frac{L\sin\left(L\left(2\omega_{(4)}+\omega_{(24)}\right)\right)+\frac{\left(\frac{3\omega_{(4)}}{2}+\omega_{(24)}\right)\cos\left(2L\left(\omega_{(4)}+\frac{\omega_{(24)}}{2}\right)\right)}{\left(\omega_{(4)}+\frac{\omega_{(24)}}{2}\right)\left(\omega_{(4)}+\omega_{(24)}\right)}}{\left(\omega_{(4)}+\frac{\omega_{(24)}}{2}\right)\left(\omega_{(4)}+\omega_{(24)}\right)\omega_{(24)}}\right.$$
$$\left.+\frac{\left(\frac{\cos\left(2L\omega_{(4)}\right)}{2\omega_{(4)}}-\frac{\omega_{(4)}\cos\left(2L\left(\omega_{(4)}+\omega_{(24)}\right)\right)}{4\left(\frac{\omega_{(4)}}{2}+\omega_{(24)}\right)\left(\omega_{(4)}+\omega_{(24)}\right)}\right)}{\omega_{(4)}\omega_{(24)}^2}-\frac{\omega_{(24)}\cos\left(L\omega_{(4)}\right)}{\omega_{(4)}^2\left(\frac{\omega_{(4)}}{2}+\omega_{(24)}\right)\left(\omega_{(4)}+\omega_{(24)}\right)^2}\right),$$

$$\left(\frac{dN_g^{(4)}}{dx}\right)_7 = \frac{C_R}{2\pi x}\int\frac{d^2\mathbf{k}}{\pi}\iint\frac{d^2\mathbf{q}_3}{\pi}\frac{d^2\mathbf{q}_4}{\pi}\alpha_s(Q_k^2)\frac{1}{\lambda_{dyn}^4}\frac{\mu_E^2-\mu_M^2}{(\mathbf{q}_3^2+\mu_E^2)(\mathbf{q}_3^2+\mu_M^2)}\frac{\mu_E^2-\mu_M^2}{(\mathbf{q}_4^2+\mu_E^2)(\mathbf{q}_4^2+\mu_M^2)}$$
$$\frac{\chi^2(\mathbf{q}_4\cdot(\mathbf{q}_3+\mathbf{q}_4-\mathbf{k}))+(\mathbf{q}_4\cdot\mathbf{k})(\mathbf{k}-\mathbf{q}_4)^2+(\mathbf{k}\cdot\mathbf{q}_3)(\mathbf{q}_4\cdot(\mathbf{q}_4-2\mathbf{k}))+\mathbf{k}^2(\mathbf{q}_4\cdot\mathbf{q}_3)}{(\chi^2+\mathbf{k}^2)(\chi^2+(\mathbf{k}-\mathbf{q}_4)^2)(\chi^2+(\mathbf{k}-\mathbf{q}_3-\mathbf{q}_4)^2)}$$
$$\frac{1}{\omega_{(34)}^2\left(\frac{\omega_{(4)}}{2}+\omega_{(34)}\right)}\left(\frac{2\omega_{(34)}^3}{\omega_{(4)}\left(\omega_{(4)}+\omega_{(34)}\right)\left(\omega_{(4)}+2\omega_{(34)}\right)\left(\omega_{(4)}+3\omega_{(34)}\right)}-\frac{\left(\frac{\omega_{(4)}}{2}+2\omega_{(34)}\right)\cos\left(L\left(\omega_{(4)}+2\omega_{(34)}\right)\right)}{\left(\omega_{(4)}+2\omega_{(34)}\right)\omega_{(34)}}\right.$$
$$\left.-L\sin\left(L\left(\omega_{(4)}+2\omega_{(34)}\right)\right)-\frac{\left(\frac{\omega_{(4)}}{2}+\omega_{(34)}\right)\left(\frac{\cos\left(L\omega_{(4)}\right)}{\omega_{(4)}}-\frac{6\cos\left(L\left(\omega_{(4)}+\omega_{(34)}\right)\right)}{\omega_{(4)}+\omega_{(34)}}+\frac{2\cos\left(L\left(\omega_{(4)}+3\omega_{(34)}\right)\right)}{\omega_{(4)}+3\omega_{(34)}}\right)}{3\omega_{(34)}}\right),$$

$$\left(\frac{dN_g^{(4)}}{dx}\right)_8 = \frac{C_R}{3\pi x}\int\frac{d^2\mathbf{k}}{\pi}\int\frac{d^2\mathbf{q}_4}{\pi}\alpha_s(Q_k^2)\frac{1}{\lambda_{dyn}^4}\frac{\mu_E^2-\mu_M^2}{(\mathbf{q}_4^2+\mu_E^2)(\mathbf{q}_4^2+\mu_M^2)}\frac{\chi^2\mathbf{q}_4\cdot(\mathbf{q}_4-\mathbf{k})+(\mathbf{q}_4\cdot\mathbf{k})(\mathbf{k}-\mathbf{q}_4)^2}{(\chi^2+\mathbf{k}^2)(\chi^2+(\mathbf{k}-\mathbf{q}_4)^2)^2}$$
$$\frac{1}{\omega_{(4)}^3}\left(\frac{1}{12\omega_{(4)}}-L\sin(3L\omega_{(4)})-\frac{\cos(L\omega_{(4)})}{2\omega_{(4)}}+\frac{3\cos(2L\omega_{(4)})}{2\omega_{(4)}}-\frac{5\cos(3L\omega_{(4)})}{6\omega_{(4)}}-\frac{\cos(4L\omega_{(4)})}{4\omega_{(4)}}\right)$$

Numerical framework: DREENA-C

D. Zigic, I. Salom, J. Auvinen, M. Djordjevic and M. Djordjevic, JPG 46, no.8, 085101

- Light and heavy flavor production

(Z.B. Kang, I. Vitev, H. Xing, PLB 718, 482; R. Sharma, I. Vitev and B.W. Zhang, PRC 80, 054902)

- Dynamical energy loss

(M. Djordjevic, PRC 80, 064909; M. Djordjevic and U. Heinz, PRL 101, 022302)

- Multi-gluon fluctuations

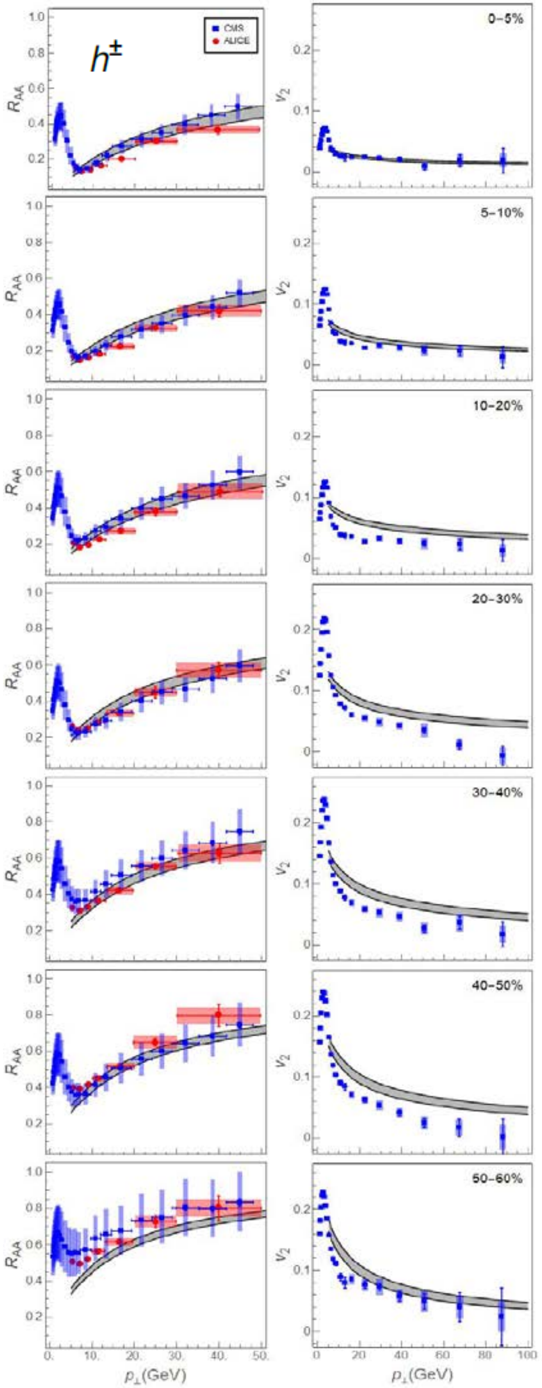
(M. Gyulassy, P. Levai, I. Vitev, PLB 538, 282)

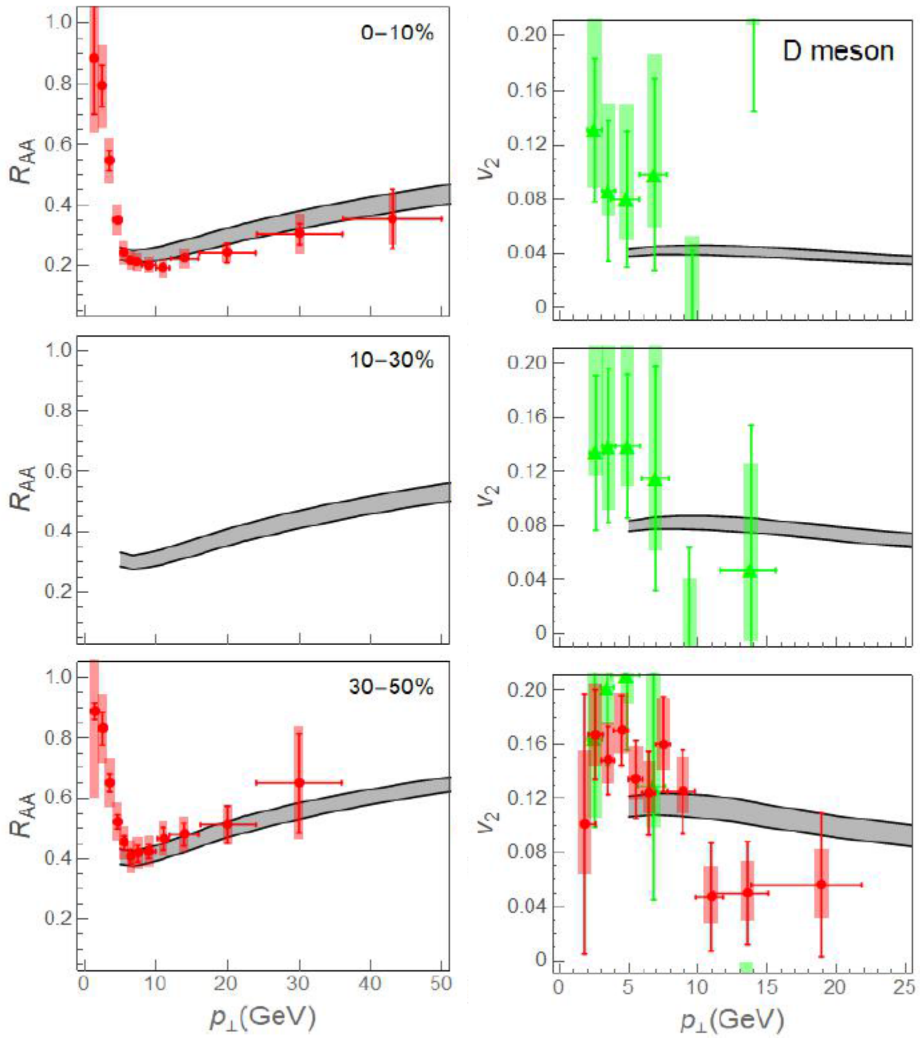
- Path-length fluctuations, hard sphere restriction $r < R_A$ introduced in WS nuclear density distribution

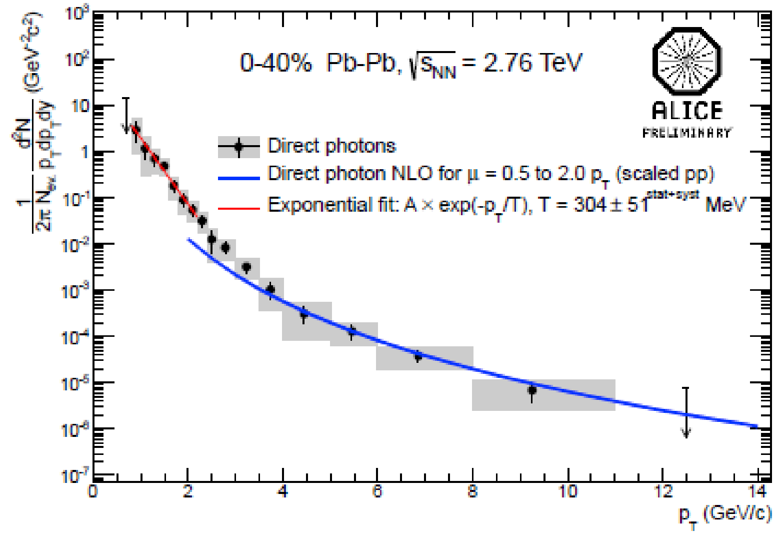
(A. Dainese, EPJ C33, 495; D. Zigic, I. Salom, J. Auvinen, M. Djordjevic and M. Djordjevic, JPG 46, 085101)

- Fragmentation for light and heavy flavor

(D. de Florian, R. Sassot, M. Stratmann, PRD 75, 114010; M. Cacciari, P. Nason, JHEP 0309 , 006, E. Braaten, K.-M. Cheung, S. Fleming and T. C. Yuan, PRD 51, 4819; V. G. Kartvelishvili, A.K. Likhoded, V.A. Petrov, PLB 78, 615)


h±
CMS
ALICE
R_AA
v_2
0–5%
5–10%
10–20%
20–30%
30–40%
40–50%
50–60%
p⊥(GeV)


D meson
R_AA
v_2
0–10%
10–30%
30–50%
p⊥(GeV)



ALICE: NPA 904-905 573c

(T_{eff}) of 304 MeV for 0-40% centrality 2.76 TeV Pb+Pb

$$T^3 \sim \frac{\frac{dN_g}{dy}}{V} \rightarrow T = c\left(\frac{\frac{dN_g}{dy}}{N_{part}}\right)^{1/3}$$

For each centrality region.

$$V \sim N_{part}$$

$$\frac{\frac{dN_g}{dy}}{N_{part}} \sim \frac{\frac{dN_{ch}}{dy}}{N_{part}/2}$$

measured

M. Gyulassy, P. Levai and I. Vitev, NPB 594 371
M. Djordjevic, M. Djordjevic and B. Blagojevic, PLB 737, 298

$$\alpha_s(Q^2) = \frac{4\pi}{(11 - 2/3 n_f)\ln(Q^2/\Lambda_{QCD})}$$

$$\frac{\mu_E^2}{\Lambda_{QCD}^2} \ln\left(\frac{\mu_E^2}{\Lambda_{QCD}^2}\right) = \frac{1 + n_f/6}{11 - 2/3\, n_f} \left(\frac{4\pi T}{\Lambda_{QCD}}\right)^2$$

$$Q_k^2 = \frac{\mathbf{k}^2 + M^2 x^2 + m_g^2}{x}$$

$$Q_v^2 = ET$$