Determining the onset of color coherence with energy correlators

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CPHT, École polytechnique

Exploring the QGP through soft and hard probes,
Belgrade, May 29 - 31 2023

CA, Dominguez, Elayavalli, Holguin, Marquet, Moult, arXiv: 2209.11236
CA, Dominguez, Holguin, Marquet, Moult, arXiv: 2303.03413

CPHT Centre de Physique Théorique

MARIE CURIE ACTIONS

l'X ÉCOLE POLYTECHNIQUE

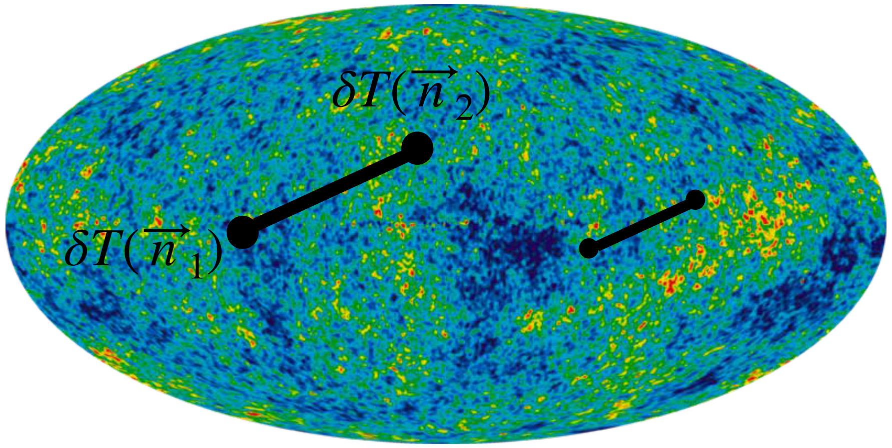
Correlation functions

- What are they?

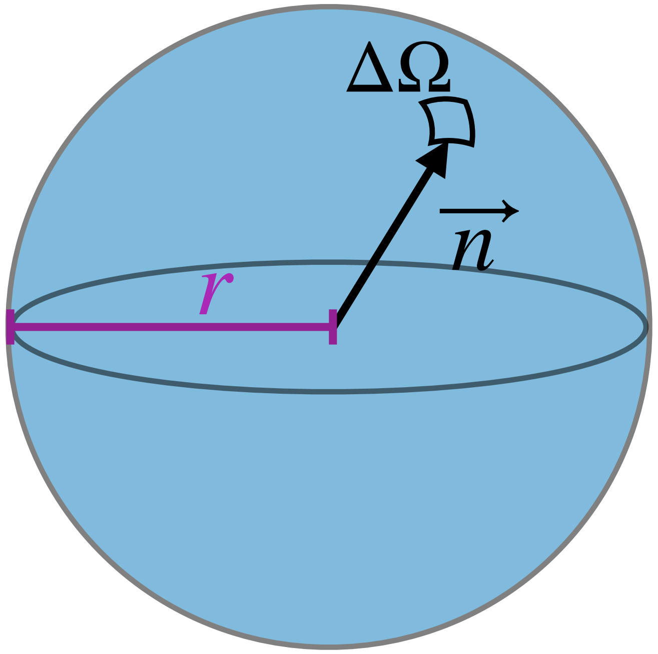
$$\mathrm{Corr}_2(X, Y) = \langle XY \rangle - \langle X \rangle \langle Y \rangle$$

$$\mathrm{Corr}_3(X, Y, Z) = \langle XYZ \rangle - \langle X \rangle \langle YZ \rangle - \langle Y \rangle \langle XZ \rangle - \langle Z \rangle \langle XY \rangle + 2\langle X \rangle \langle Y \rangle \langle Z \rangle$$

- In physics: usually $\langle X_i \rangle = 0 \Rightarrow \langle X_1, X_2, \ldots X_n \rangle$ is the *n*-point correlator



Energy correlators



- Correlators $\langle \varepsilon(\vec{n}_1)\varepsilon(\vec{n}_2)\cdots\varepsilon(\vec{n}_k)\rangle$ of the energy flux:

$$\varepsilon(\vec{n}) = \lim_{r\to\infty}\int \mathrm{d}t\, r^2 n^i\, T_{0i}(t, r\vec{n})$$

They naturally remove the soft physics with NO grooming!

- 1-point correlator: $\langle \varepsilon(\vec{n})\rangle \propto \sum_i E_i$ Total energy flux through an area element

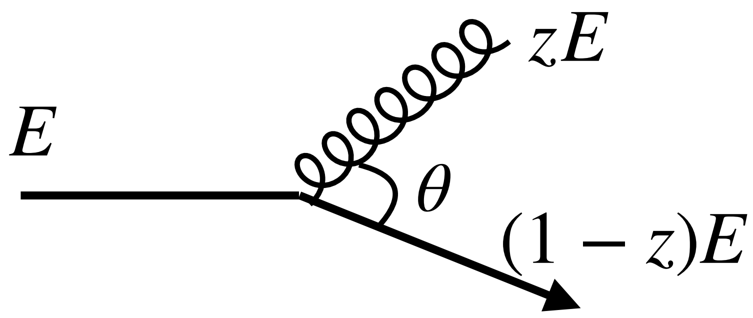
- 2-point correlator:

Inclusive cross section to produce two particles *i* and *j*

$$\frac{\langle \varepsilon^n(\vec{n_1})\varepsilon^n(\vec{n_2})\rangle}{Q^{2n}} = \frac{1}{\sigma}\sum_{ij}\frac{\mathrm{d}\sigma_{ij}}{\mathrm{d}\vec{n}_i\mathrm{d}\vec{n}_j}\frac{E_i^n E_j^n}{Q^{2n}}\delta^{(2)}(\vec{n}_i - \vec{n}_1)\delta^{(2)}(\vec{n}_j - \vec{n}_2)$$

Hard scale of the process

2-point correlator



- As function of the relative angle only:

$$\frac{d\Sigma^{(n)}}{d\theta} = \int d\vec{n}_{1,2} \frac{\langle \epsilon^n(\vec{n}_1)\epsilon^n(\vec{n}_2)\rangle}{Q^{2n}} \delta^{(2)}(\vec{n}_1 \cdot \vec{n}_2 - \cos\theta)$$

  - Infrared and collinear safe for $n = 1$
  - For divergences $1 < n \leq 2$ can be absorbed into track or fragmentation functions

- 2-point correlator for a quark jet: $Q = E$

$$\frac{d\Sigma^{(n)}}{d\theta} = \frac{1}{\sigma_{qg}} \int dz \frac{d\sigma_{qg}}{dz d\theta} z^n (1-z)^n + \mathcal{O}\left(\frac{\mu_s}{E}\right)$$

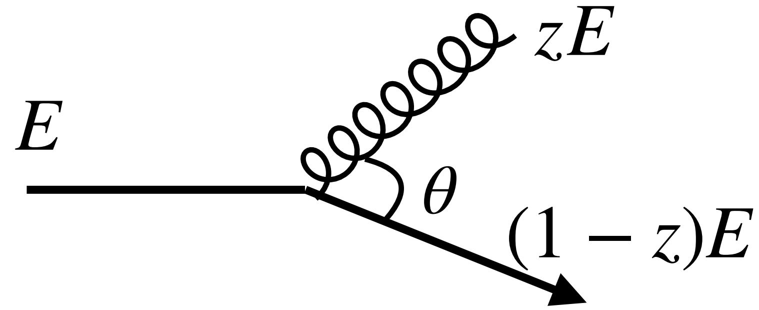
μ_s a softer scale over which the cross section is inclusive

Inclusive cross section

  - Additional energy loss ($E_q + E_g \neq E$) is subleading
  - qq and gg contributions are higher order

EEC in vacuum

(In the perturbative regime)



- EEC for a **massless** quark jet in vacuum at LO:

$$\frac{d\sigma_{qg}^{\rm vac}}{dz d\theta} = \frac{\alpha_s C_F \sigma}{\pi} \frac{1+(1-z)^2}{z\,\theta} + \mathcal{O}(\alpha_s^2, \theta) \quad \Rightarrow \quad \frac{d\Sigma^{(1)}}{d\theta} \sim \frac{1}{\theta}$$

- EEC for a massless quark jet in vacuum at NLO + NLL resummation:

$$\frac{d\Sigma^{(1)}}{d\theta} \sim \frac{1}{\theta^{1-\gamma(3)}}$$

Power-law behavior

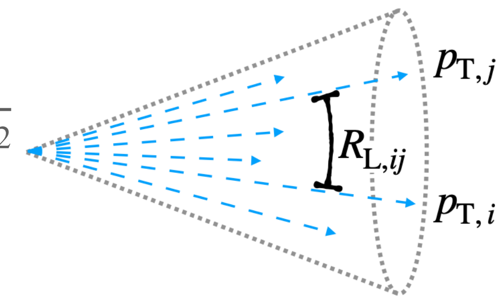
$\gamma(3)$ is the twist-2 spin-3 QCD anomalous dimension

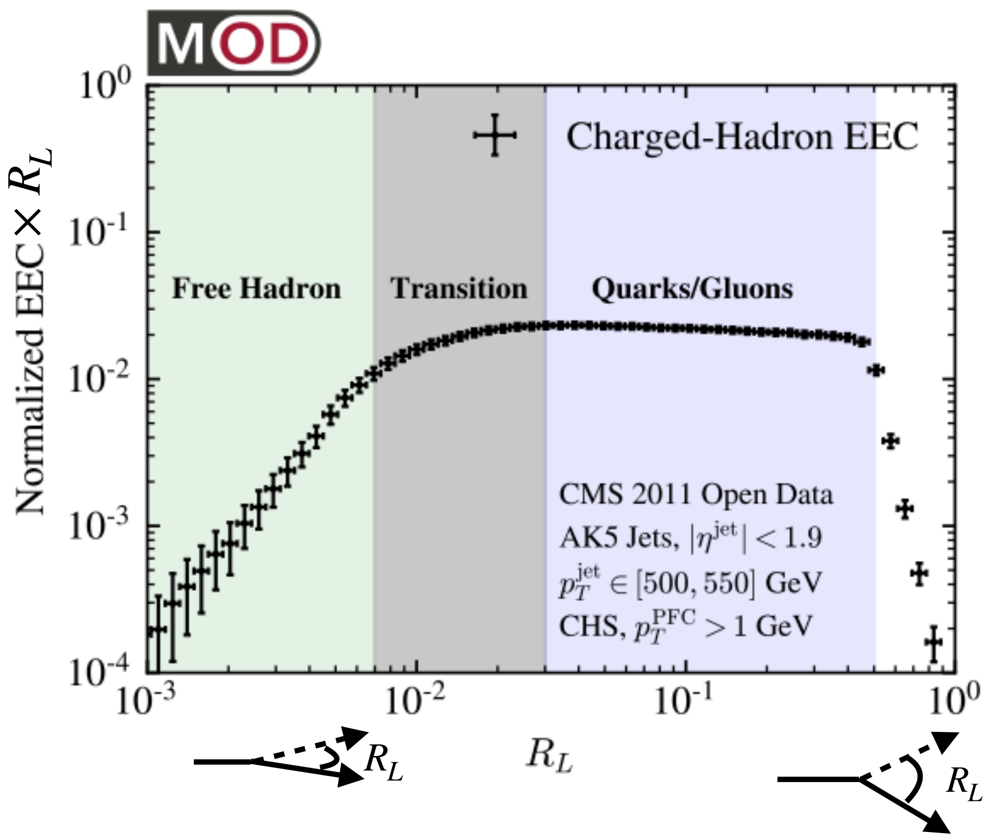
Hoffman, Maldacena, 0803.1467
Chen, Moult, Sandor, Zhu, 2202.04085

- Higher-orders, soft physics, quark/gluon ratios can change the overall normalization but not the power-law behavior

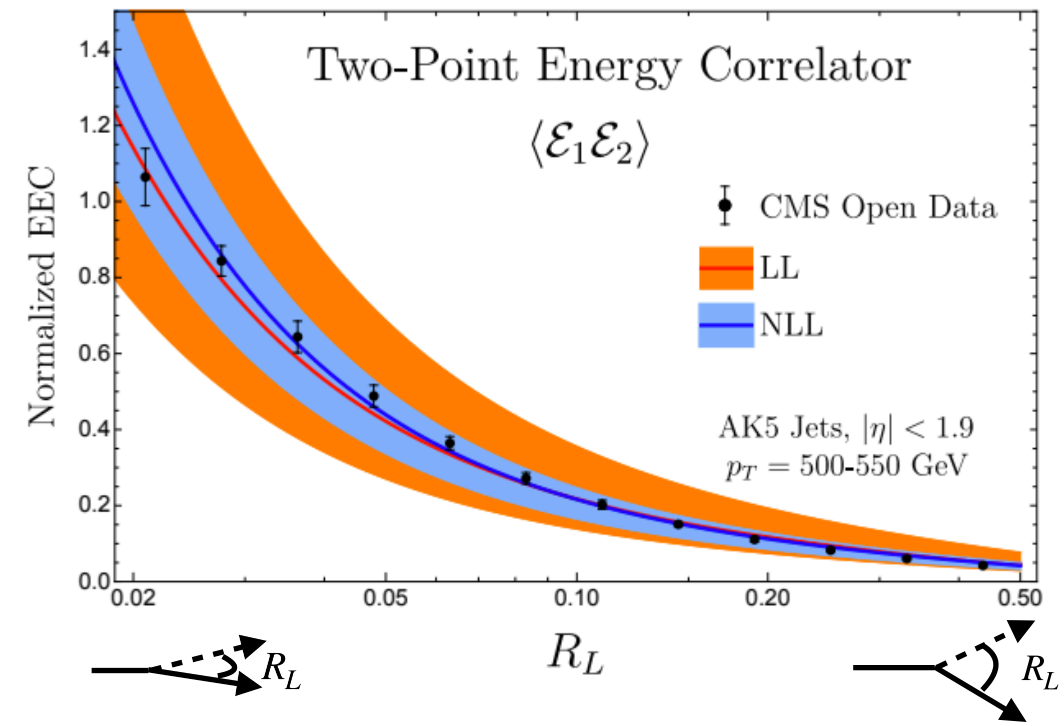
EEC in vacuum

$R_L = \sqrt{\Delta\phi^2 + \Delta\eta^2}$





Komiske, Moult, Thaler, Zhu 2201.07800



Lee, Meçaj, Moult 2205.03414

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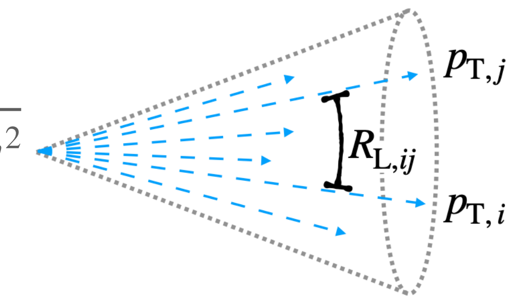

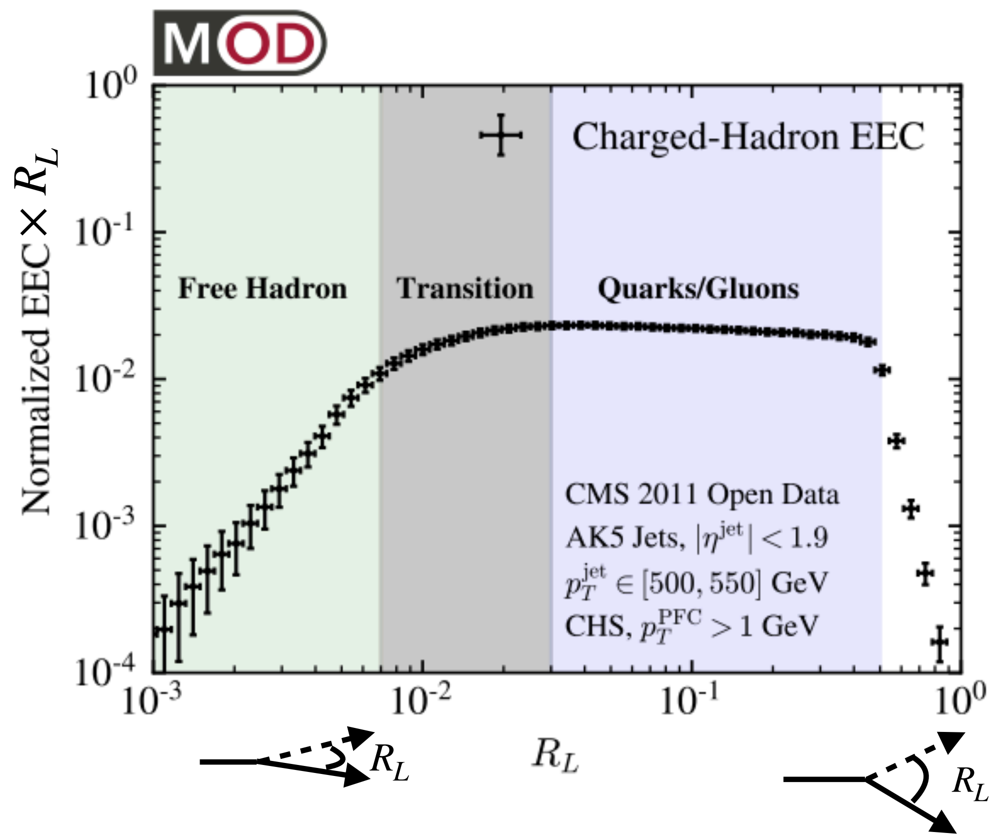

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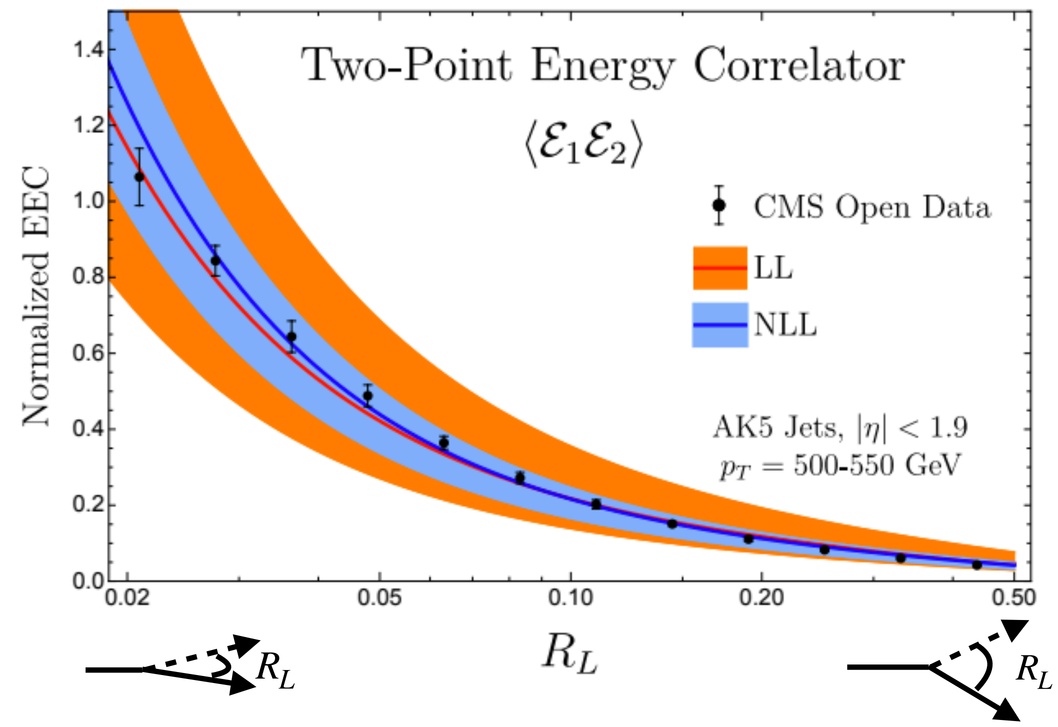

Lee, Meçaj, Moult 2205.03414

- Clear separation between perturbative and non-perturbative regimes
- p-p baseline under control
- Reduced sensitivity to soft physics

EEC in vacuum

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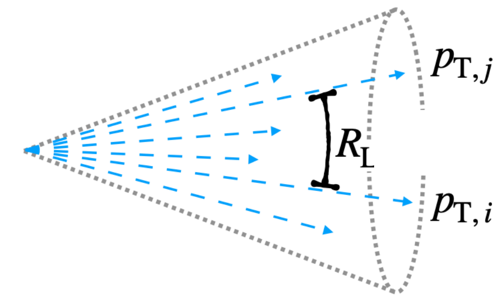
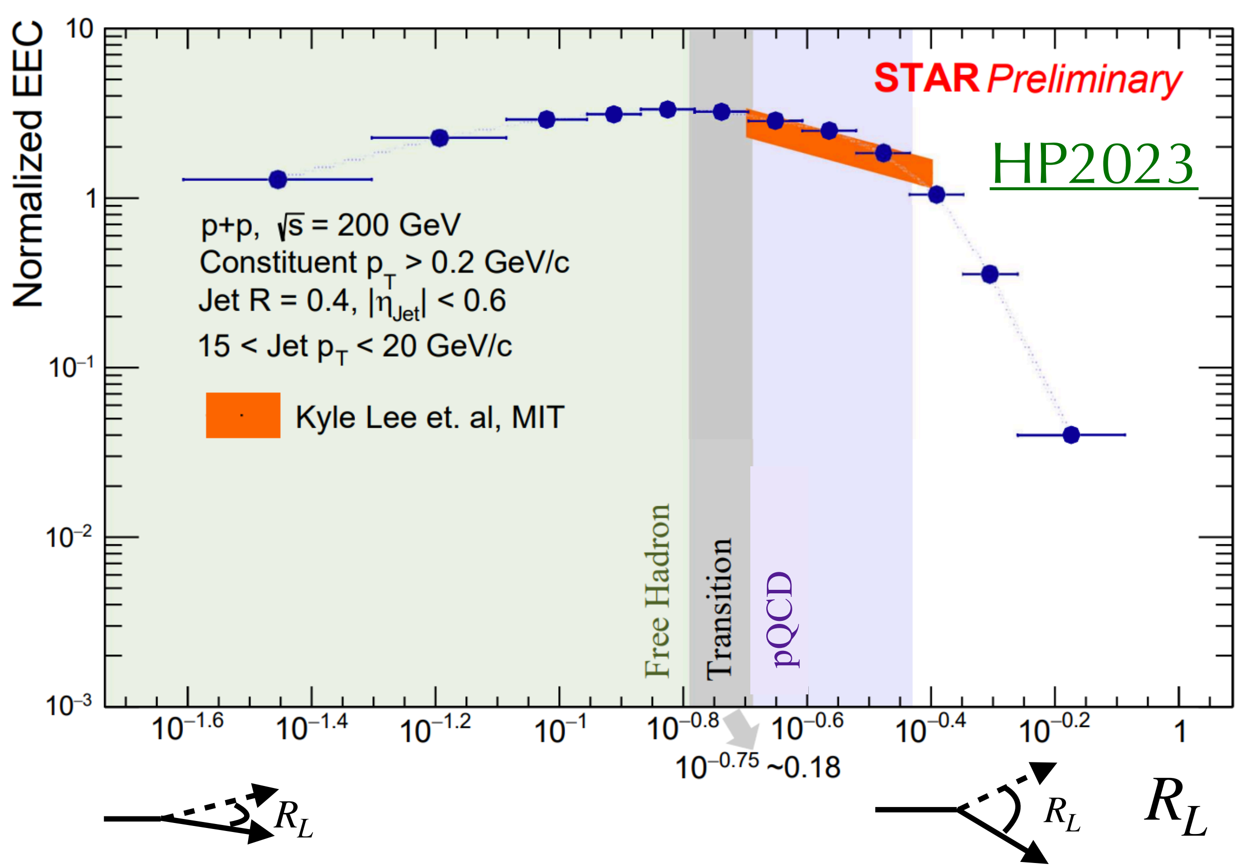
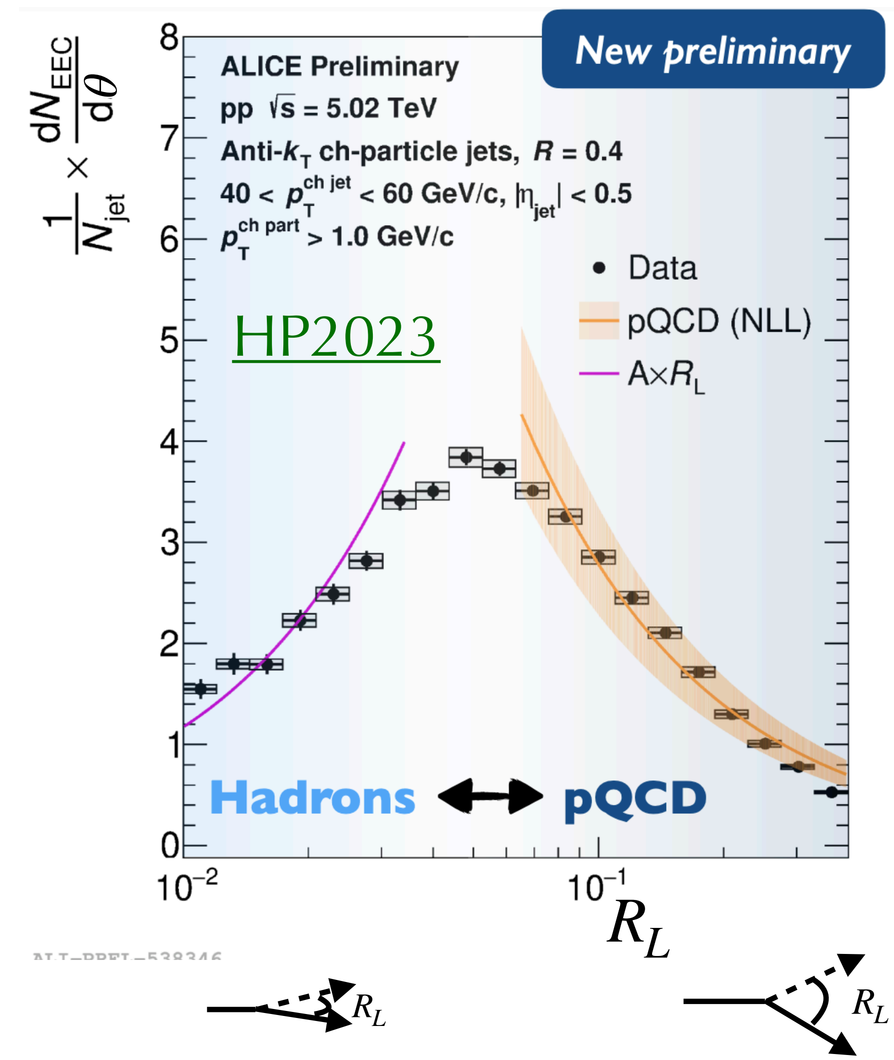


- **First measurements** of the EEC in **p-p collisions** announced in HP2023





- **Good agreement** with **pQCD** predictions

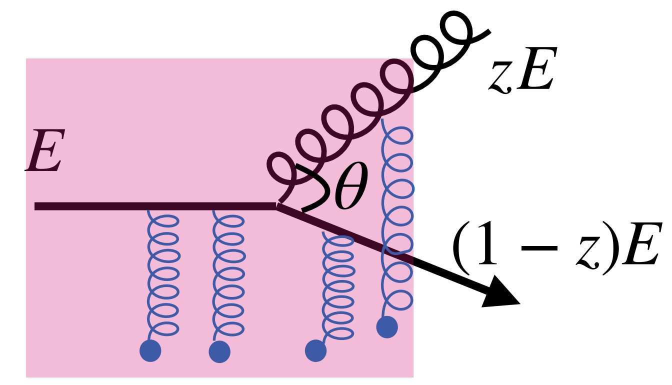
Energy correlators in HICs

- **Background** is expected **to be less of an issue**
  - **Energy weighting** removes most of the soft physics, specially if one increases the power in the energy weighting
  - **Uncorrelated background** does not **affect** the shape of the correlations, **only** the **normalization**

- Observables are **not event-by-event**
  - Fluctuations are less important
  - Require large statistics
  - Cannot be used to tag events

EEC in HICs

- EEC for a massless quark **heavy-ion** jet:



$$\frac{d\Sigma^{(n)}}{d\theta} = \frac{1}{\sigma_{qg}} \int dz \, \frac{d\sigma_{qg}}{dz d\theta} z^n (1-z)^n + \mathcal{O}\left(\frac{\mu_s}{E}\right)$$

- We can always define F_{med} such as

$$\frac{d\sigma_{qg}}{d\theta dz} = \left(1 + F_{\text{med}}(z,\theta)\right) \frac{d\sigma_{qg}^{\text{vac}}}{d\theta dz} \qquad F_{\text{med}}(z,\theta) \xrightarrow{\theta<\theta_L} 0$$

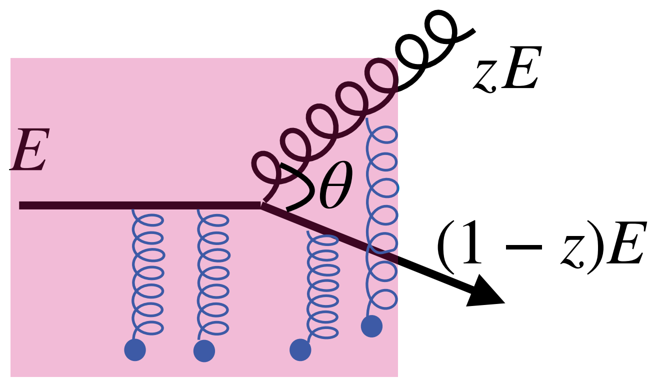
- We do not expect medium modification at small angles, thus vacuum collinear resummation should still be valid

$$\frac{d\Sigma^{(n)}}{d\theta} = \left(\frac{1}{\sigma_{qg}} \int dz \, \left(g^{(n)}(\theta,\alpha_s) + F_{\text{med}}(z,\theta)\right) \frac{d\sigma^{\text{vac}}}{d\theta dz} z^n (1-z)^n\right)\left(1 + \mathcal{O}\left(\frac{\bar{\mu}_s}{Q}\right)\right) + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{\theta Q}\right)$$

$$g^{(1)}(\theta,\alpha) = \theta^{\gamma(3)} + \mathcal{O}(\theta) \quad \Longrightarrow \quad \frac{d\Sigma^{(1)\,\text{vac}}}{d\theta} \sim \frac{1}{\theta^{1-\gamma(3)}}$$

Evaluation of the in-medium splitting

$$\frac{d\sigma_{qg}}{d\theta dz} = \left(1 + F_{\rm med}(z,\theta)\right) \frac{d\sigma^{\rm vac}_{qg}}{d\theta dz}$$



- Well understood in the soft limit $z \to 0$ (with zE finite) or when all transverse momenta are integrated over, thus losing the angle dependence

Mehtar-Tani, Barata, Soto-Ontoso, Tywoniuk, 1903.00506, 2004.02323, 2106.07402

CA, Apolinario, Martinez, Dominguez, 2002.01517, 2011.06522

Stojku, Ilic, Salom, Djordjevic, 2303.14527

- For the energy correlator calculation is is crucial to **keep z finite and also the angle dependence**

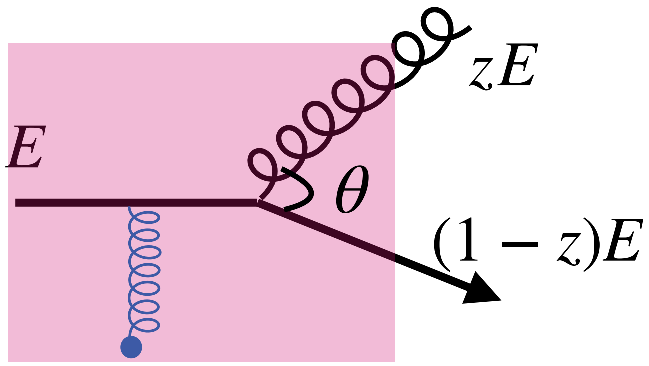
- **Complete (multiple scatterings)** medium-induced emission spectrum **keeping z** and **θ not yet available**

Recent results for the $\gamma \to q\bar{q}$ case (computationally costly) Isaksen, Tywoniuk, 2303.12119

Beyond the soft limit

- Two available approaches:
  - **Opacity expansion:**
    - $N = 1$ result

      Ovanesyan, Vitev 1103.1074, 1109.5619



    - Highly complicated recursive relations to go to all orders

      Sievert, Vitev, 1807.03799 Sievert, Vitev, Yoon, 1903.06170

  - ***Tilted* Wilson lines (multiple scatterings resummed):**

    Dominguez, Milhano, Salgado, Tywoniuk, Vila, 1907.03653

    - Assumes *semi-hard* splittings (z not too small)
    - All partons propagate along straight line trajectories

      Isaksen, Tywoniuk 2107.02542

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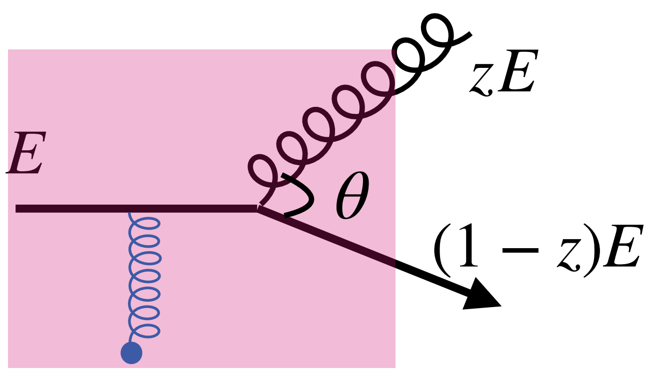

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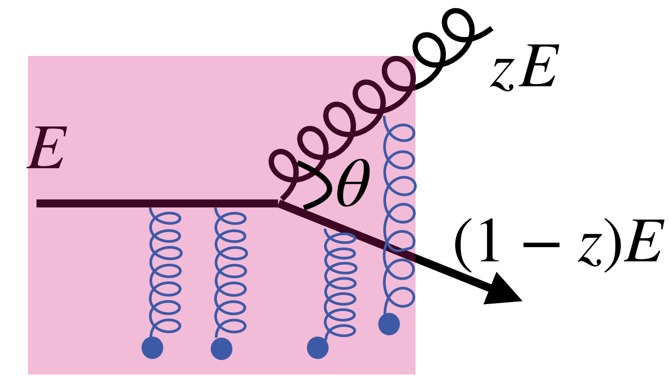
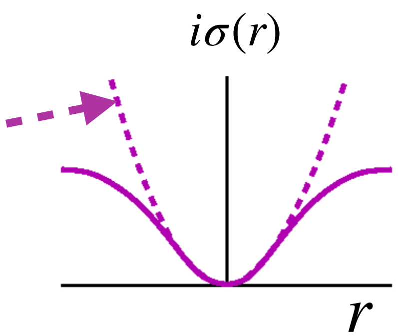
Our model



- Medium is assumed to be **static and uniform**, with length L
- **Harmonic oscillator** (HO) approximation employed $n\sigma(r) \approx \frac{1}{2}\hat{q}r^2$



- The strength of the interactions is encoded in the **jet quenching paramete**r $\hat{q}$, which measures the average transverse momentum transferred per unit length
- Emissions with a long formation time are not sensitive to the medium and therefore are emitted as in vacuum
- Multiple medium scatterings destroy the color coherence between the daughter partons

Time and angular scales (HO)

- For a static medium of length L within the HO one can read off the relevant scales directly from the formulas:

  2 competing angular scales: θ_L and θ_c

  - (Vacuum) formation time:

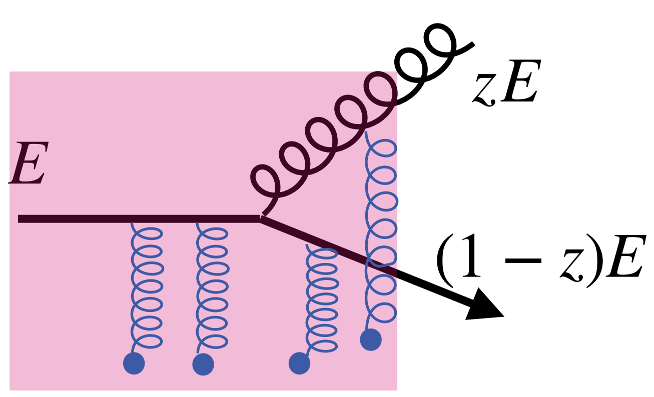
$$t_f = \frac{2}{z(1-z)E\theta^2} \xrightarrow{t_f \leq L} \theta_L \sim (EL)^{-1/2}$$

Below θ_L all emissions have a formation time larger than L

  - Decoherence time:

$$S_{12}(\tau) = e^{-\frac{1}{12}\hat{q}(1+z^2)\theta^2\tau^3} \qquad t_d \sim (\hat{q}\theta^2)^{-1/3} \xrightarrow{t_d \leq L} \theta_c \sim (\hat{q}L^3)^{-1/2}$$

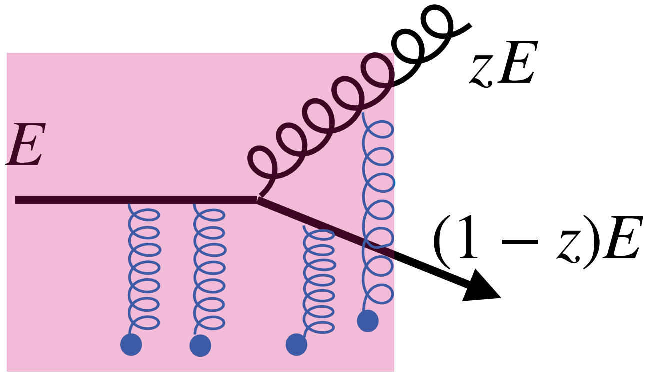
Below θ_c splittings do not color decohere and the medium does not resolve them



If $\theta_L > \theta_c$: θ_c becomes irrelevant

Time and angular scales (HO)

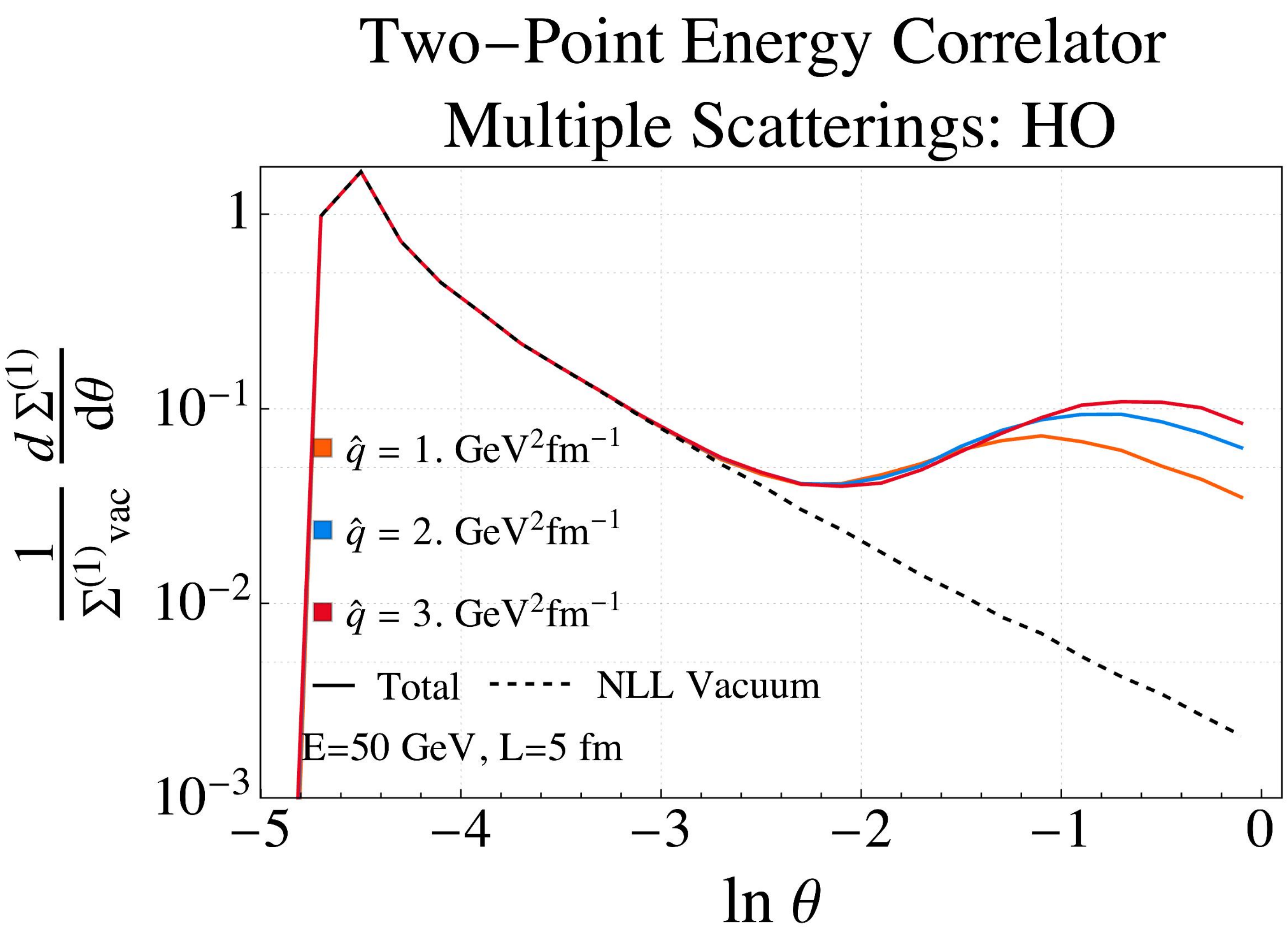
Can be extended to include a more **realistic interactions or expanding media**, but then we would not know the scales directly from the equations



If $\theta_L > \theta_c$: θ_c becomes irrelevant

Results HO

$\theta_L \gg \theta_c \left(E \ll \hat{q}L^2\right)$



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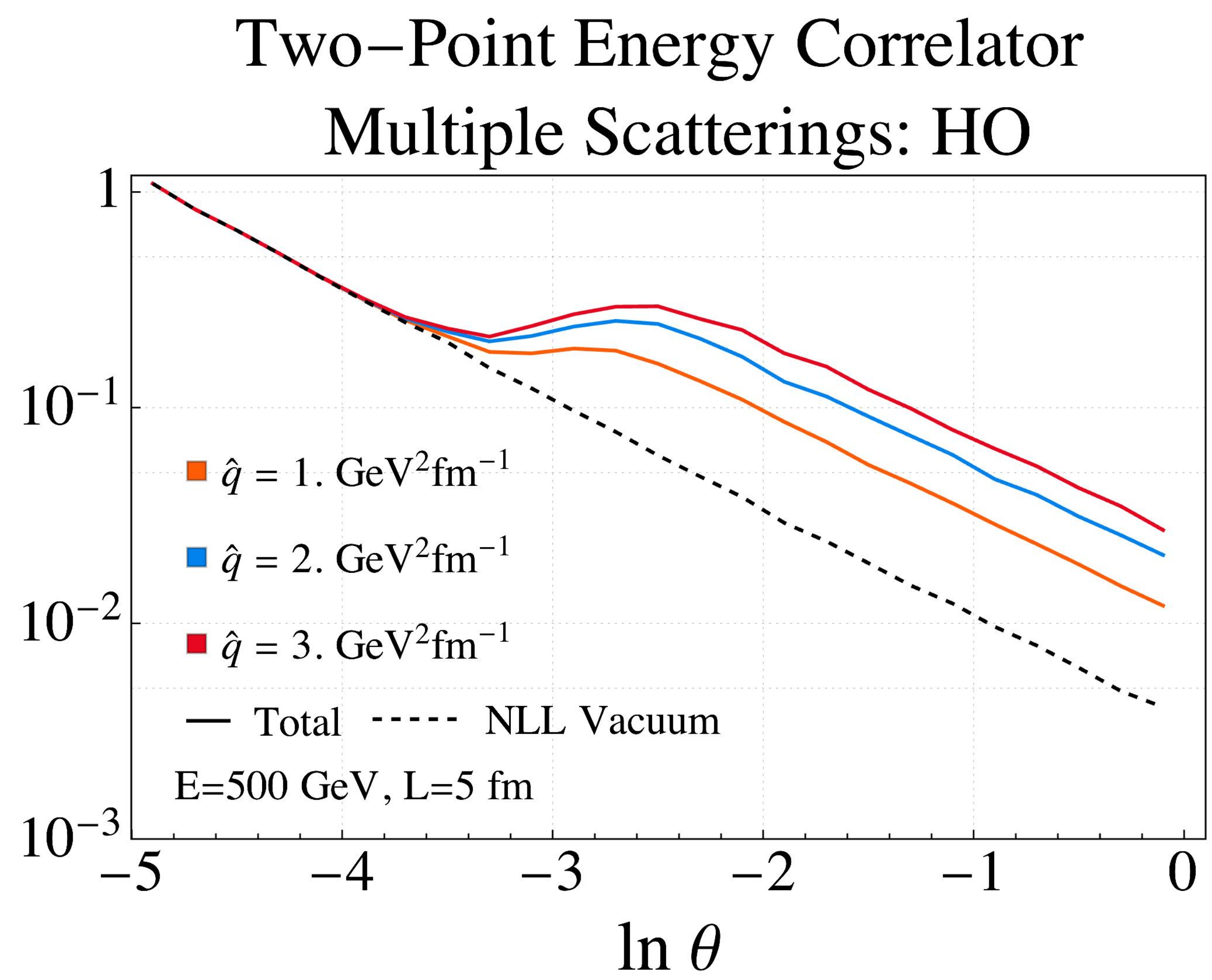

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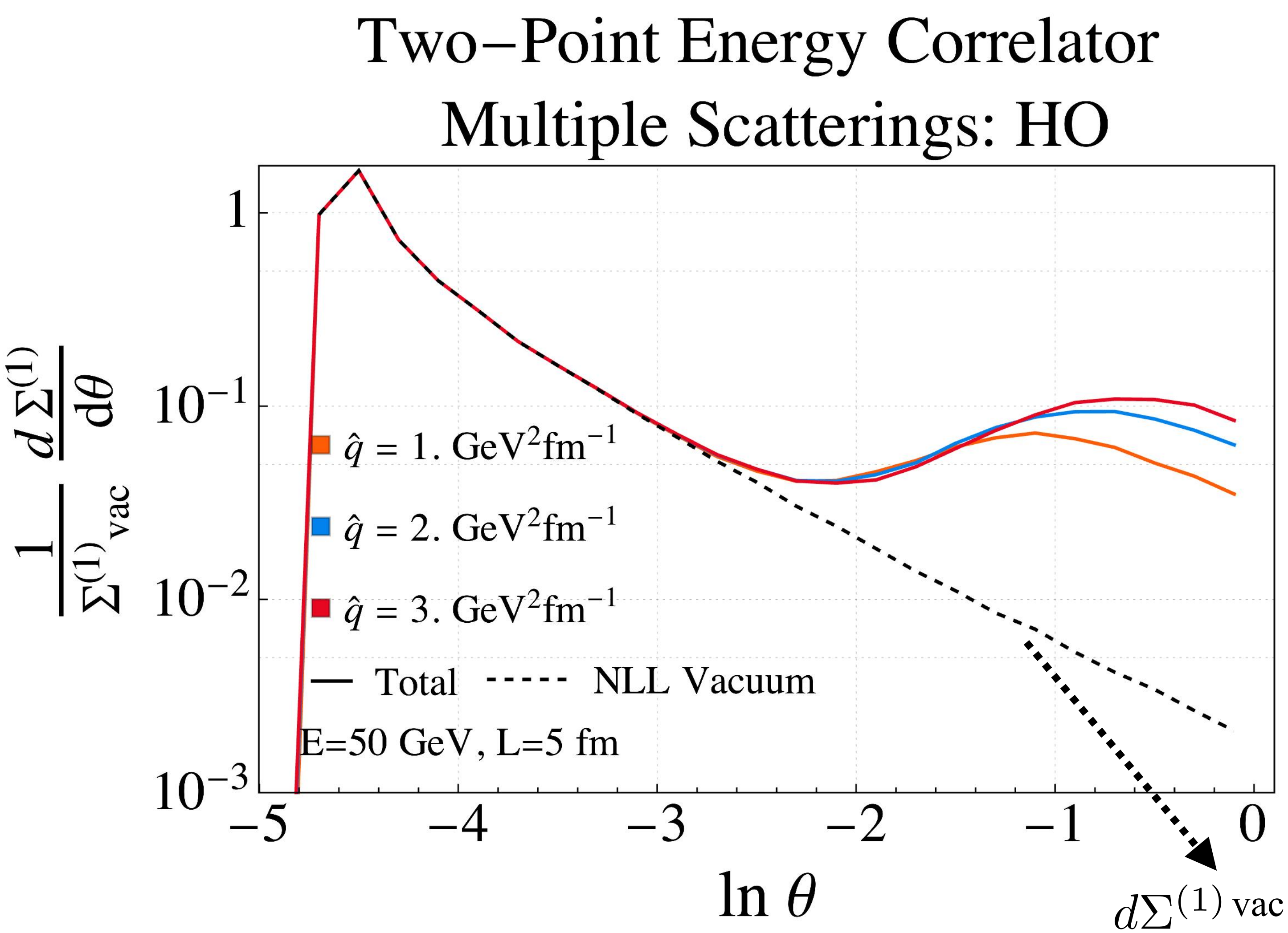

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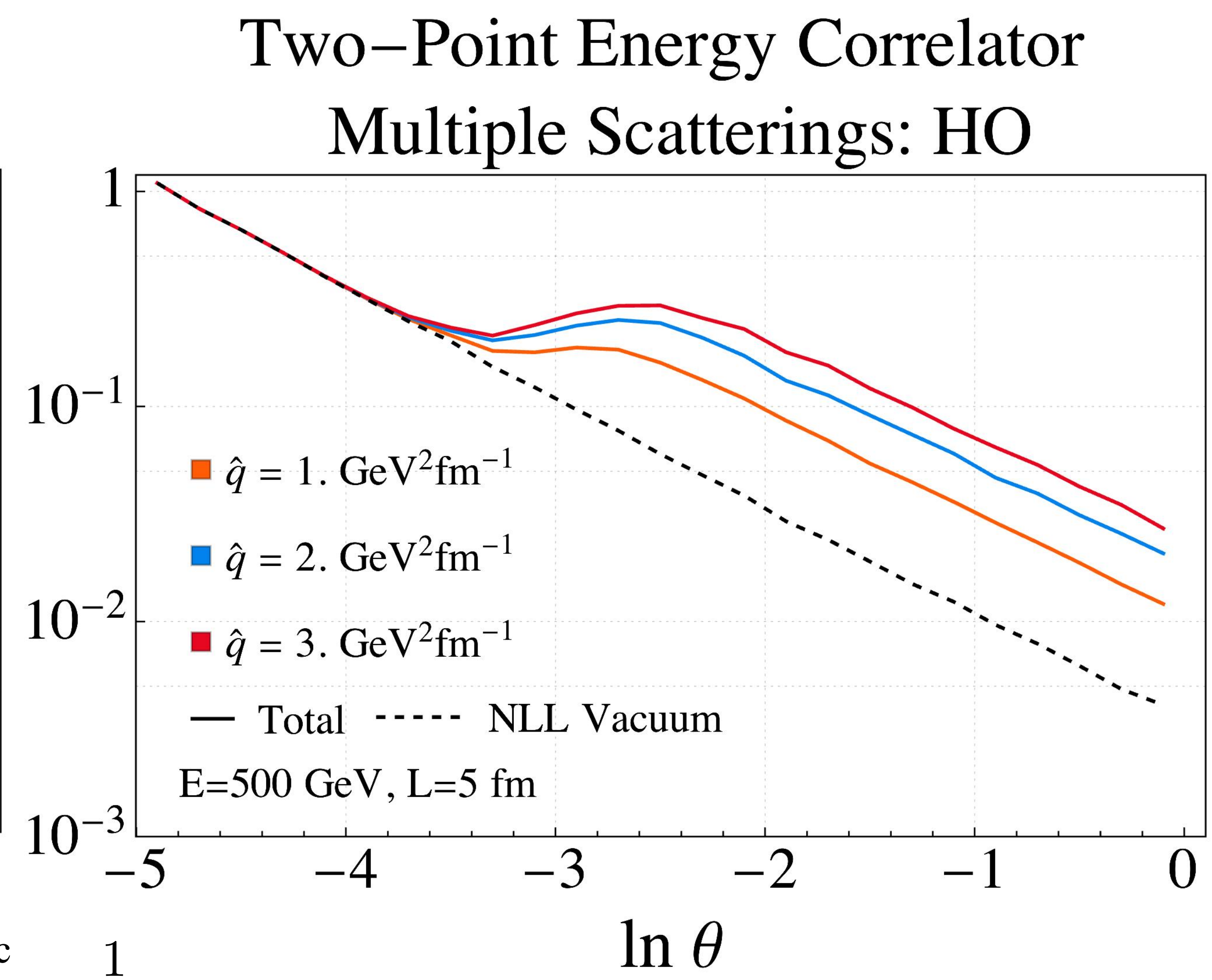
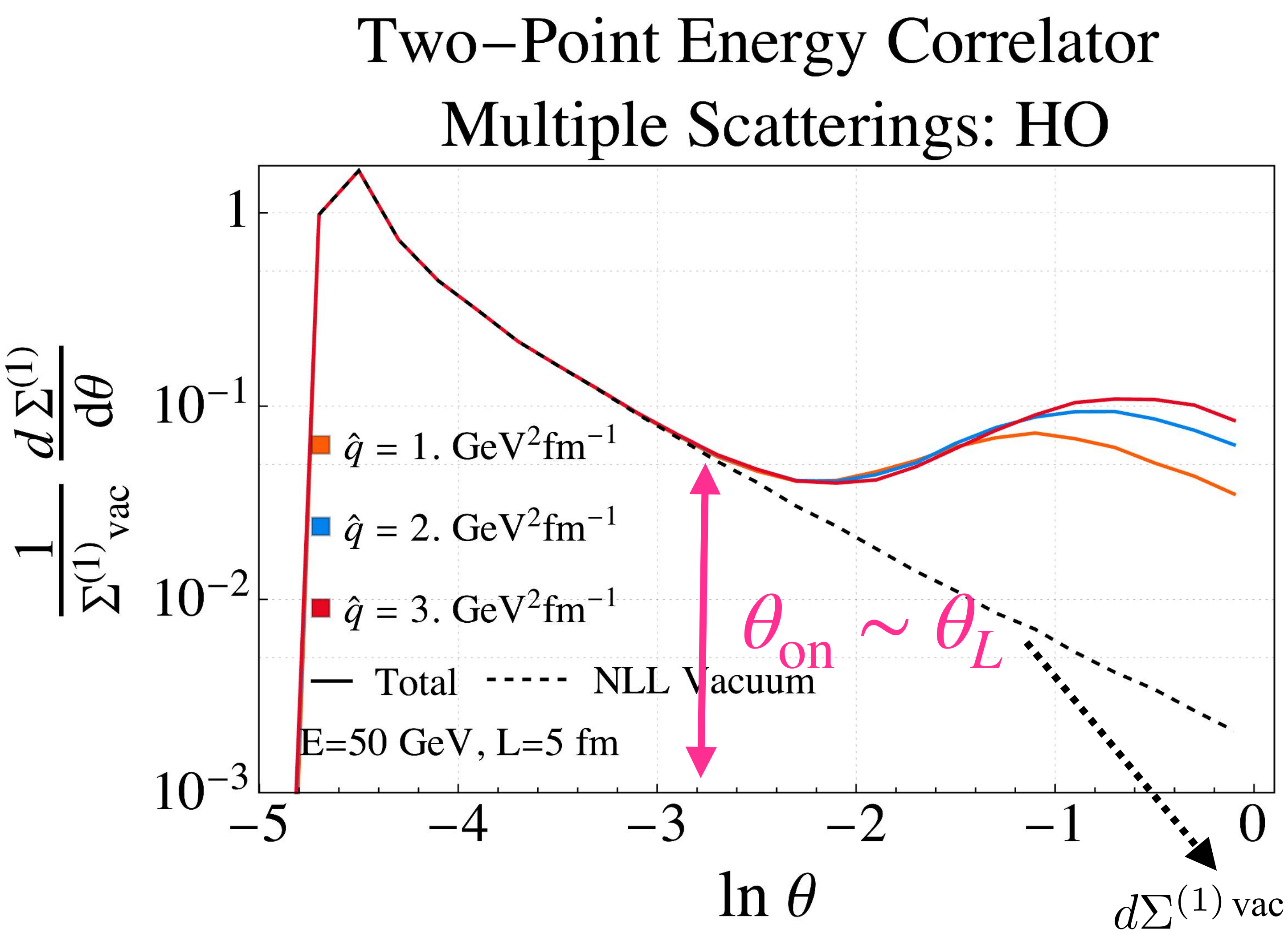


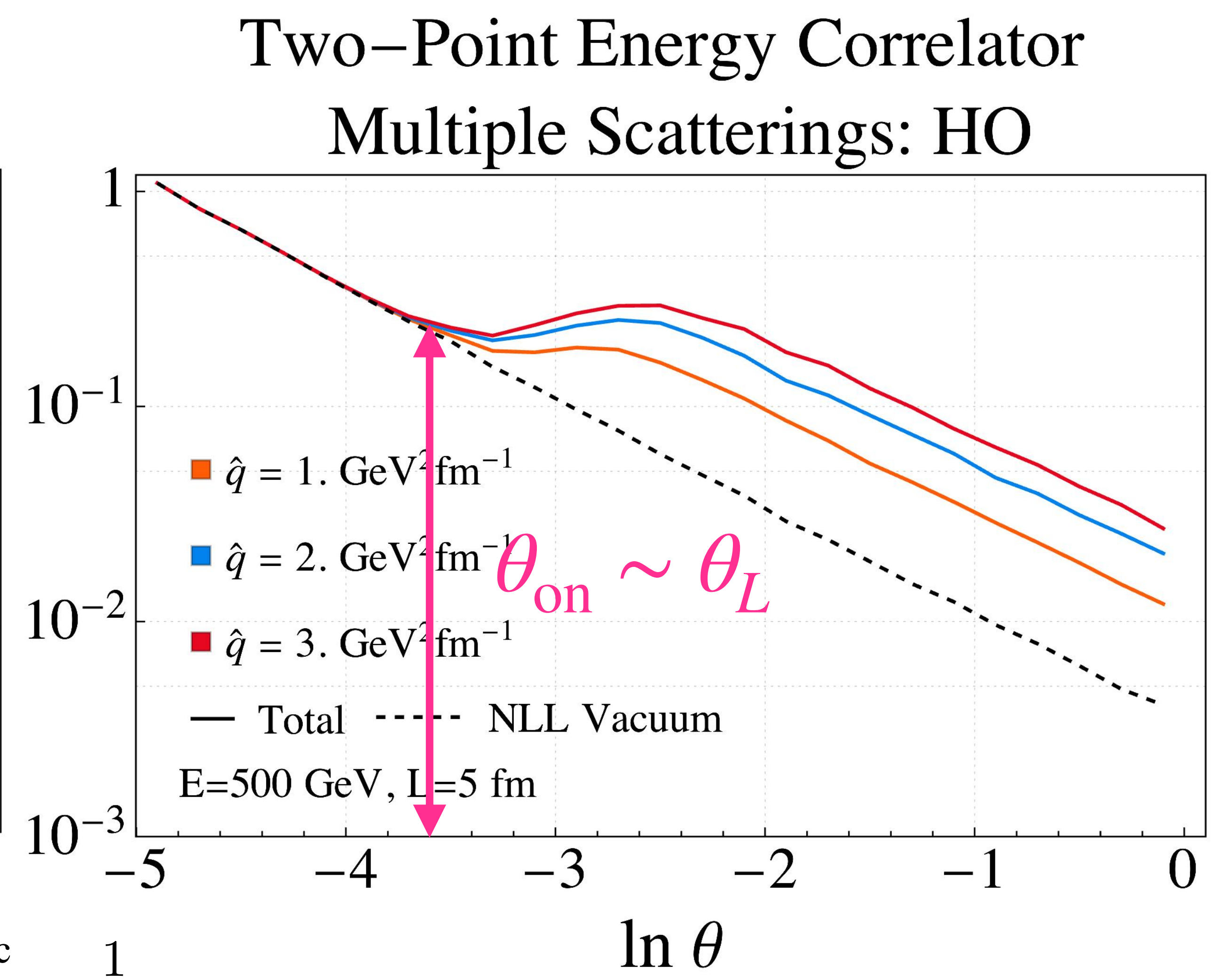
$$\frac{d\Sigma^{(1)\,\rm vac}}{d\theta} \sim \frac{1}{\theta^{1-\gamma(3)}}$$

- **No** medium-induced enhancement at **small angles**

Results HO

$\theta_L \gg \theta_c \left(E \ll \hat{q}L^2\right)$ $\theta_L \ll \theta_c \left(E \gg \hat{q}L^2\right)$



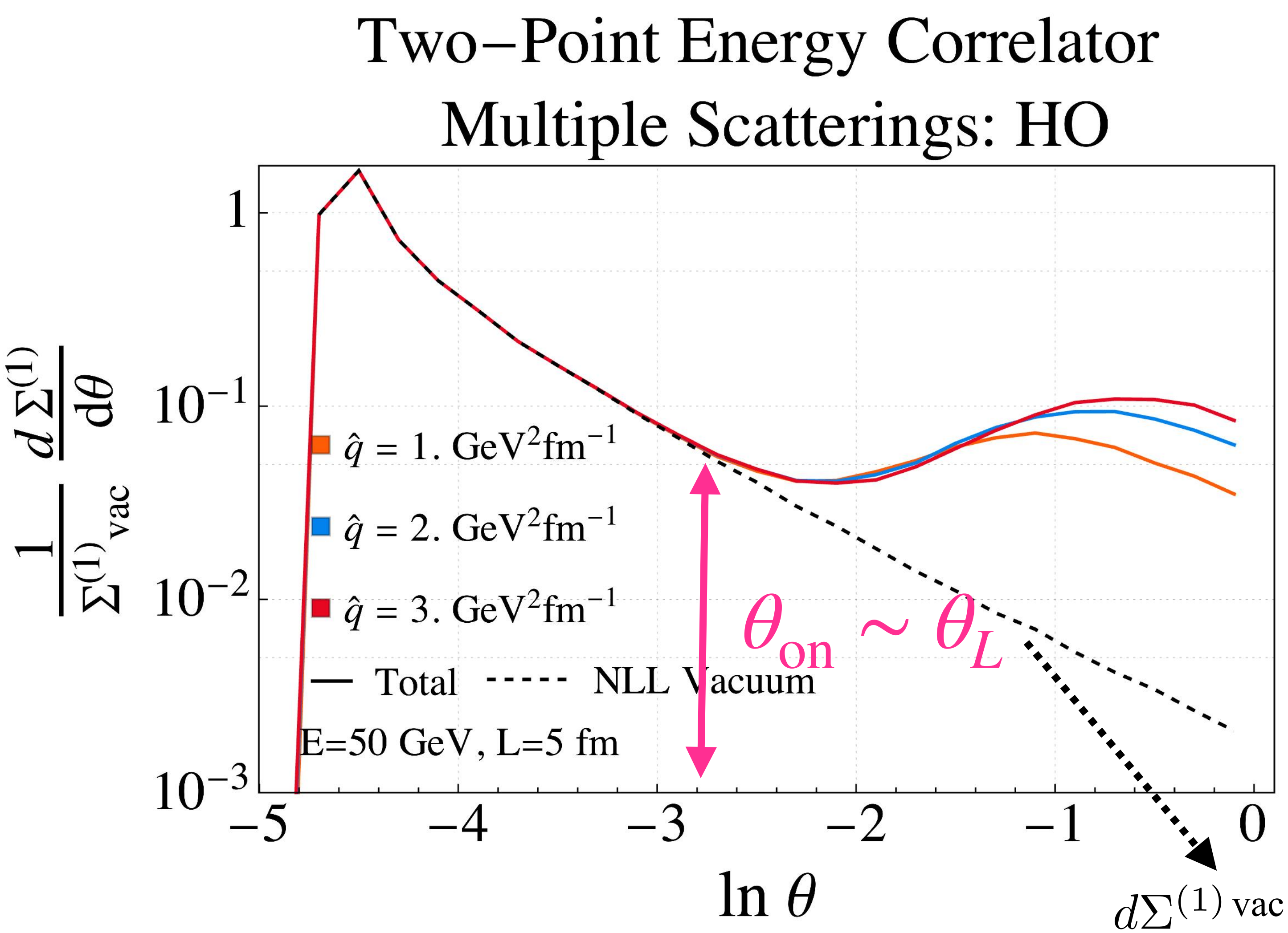


$$\frac{d\Sigma^{(1)\,\text{vac}}}{d\theta} \sim \frac{1}{\theta^{1-\gamma(3)}}$$

- **No** medium-induced enhancement at **small angles**
- Onset angle seems to be independent of $\hat{q}$

Results HO

$\theta_L \gg \theta_c (E \ll \hat{q}L^2)$



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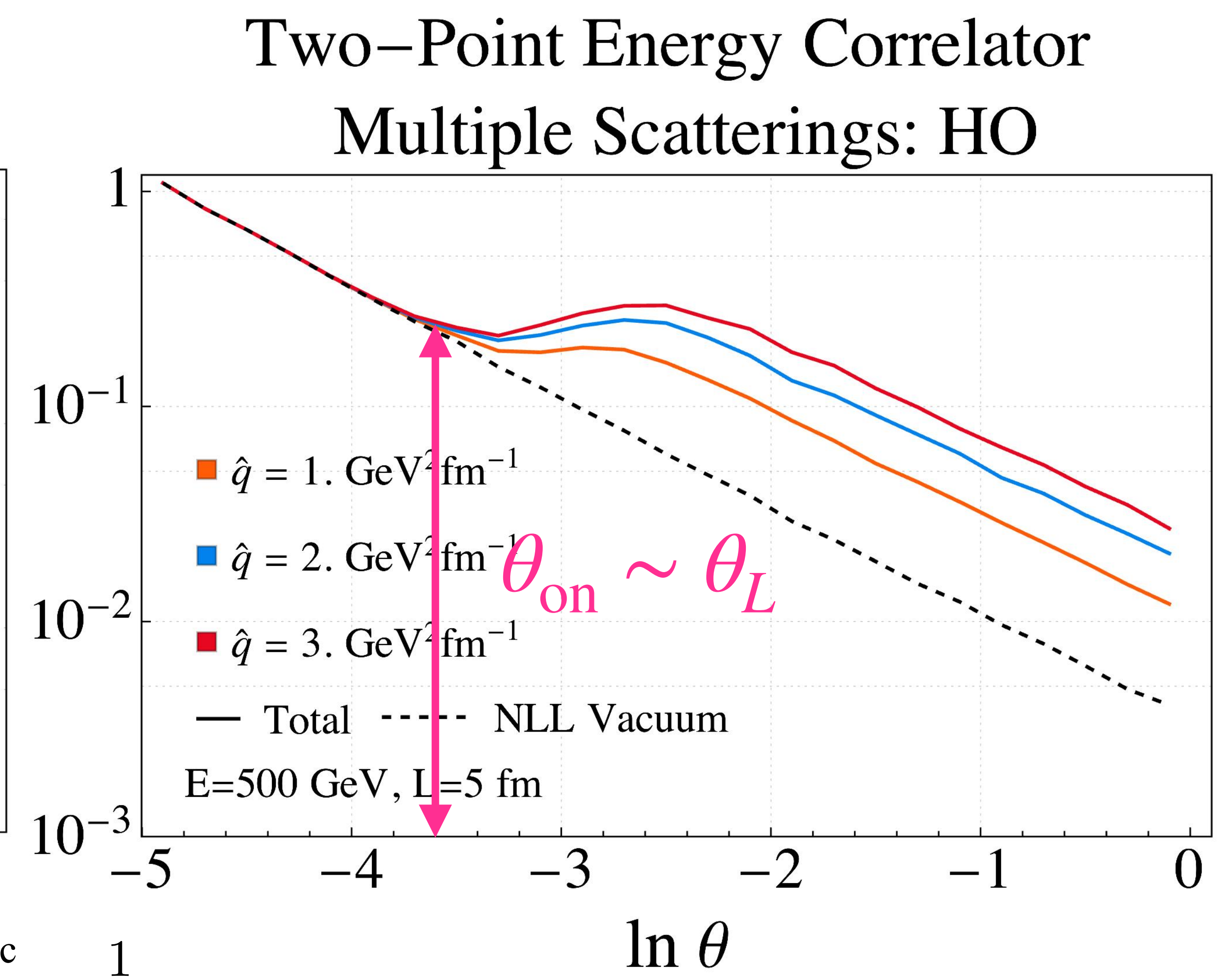
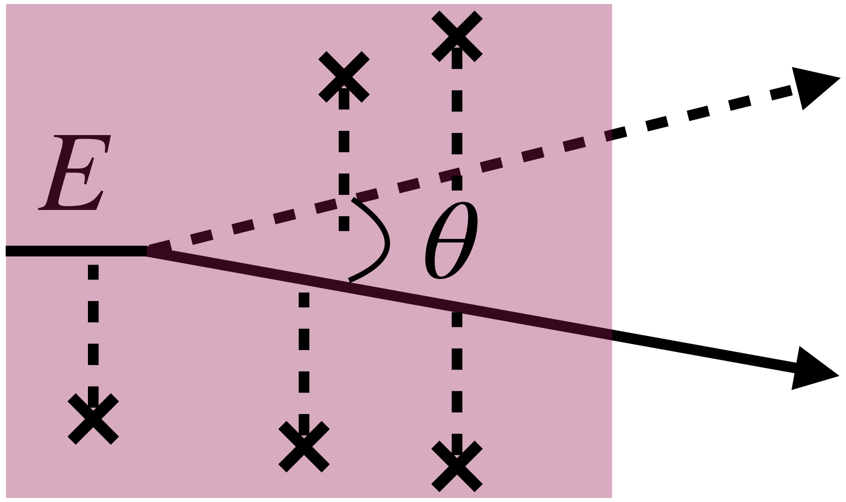


- **No** medium-induced enhancement at **small angles**
- Onset angle seems to be independent of $\hat{q}$
- Varying $\hat{q}$ has different effects in the two regimes

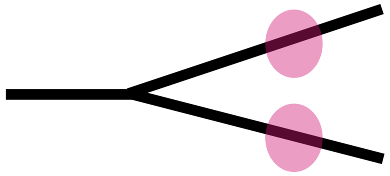
Interpretation

$\theta_L \gg \theta_c \quad (E \ll \hat{q}L^2)$

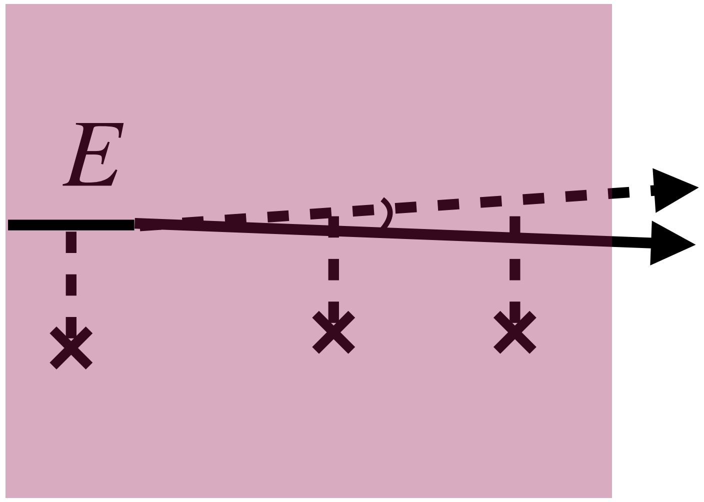


For $\theta \gg \theta_L \Rightarrow \theta \gg \theta_c$

The medium resolves the emission

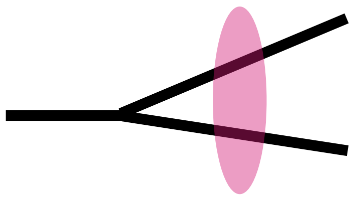


$\theta_L \ll \theta_c \quad (E \gg \hat{q}L^2)$

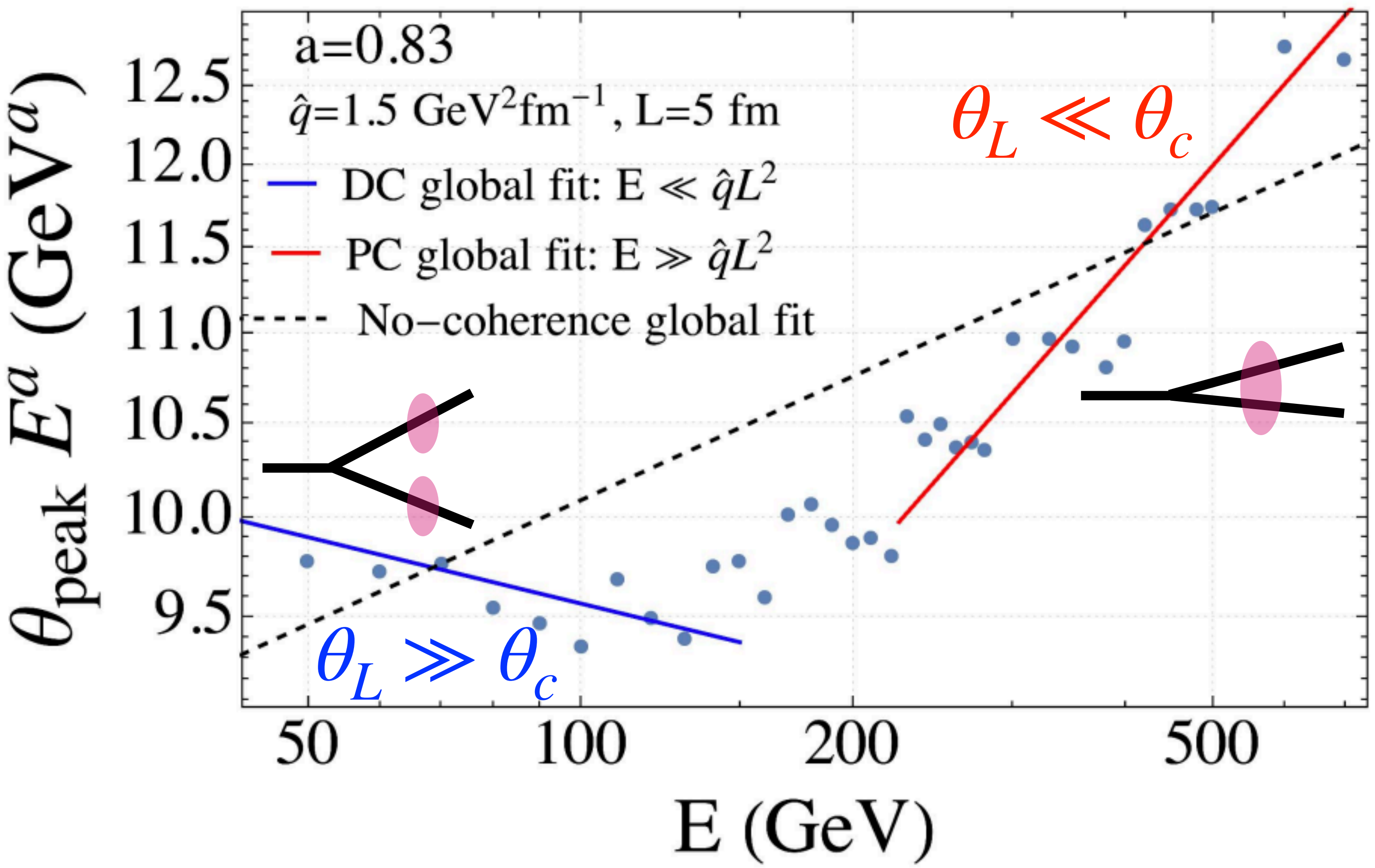


For $\theta_c \gg \theta \gg \theta_L$:

The medium does NOT resolve the emission



Coherence transition

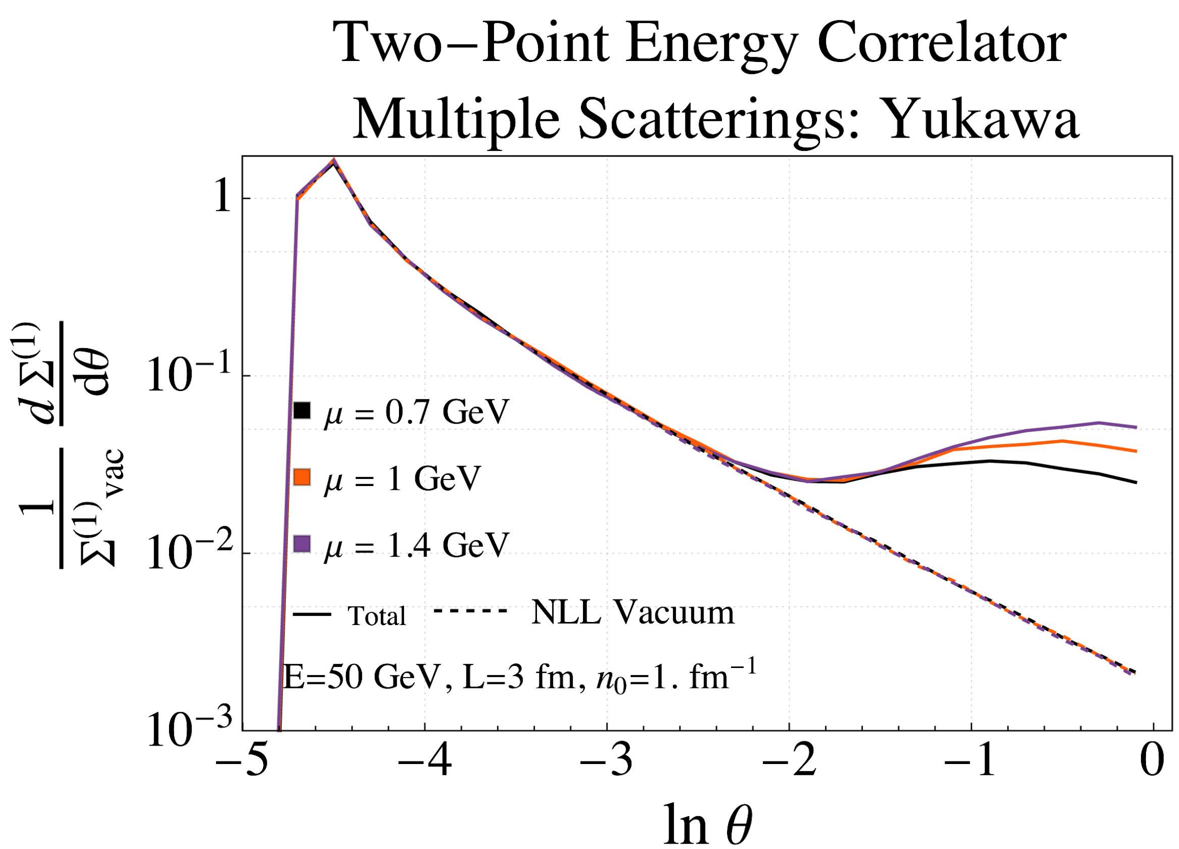


- Extracted the peak angle $\theta_{\rm peak}$ for 332 sets of parameters with $E \in [50{,}700]$ GeV, $L \in [0.2{,}10]$ fm, $\hat{q} \in [1{,}3]\,\mathrm{GeV}^2/\mathrm{fm}$

- Performed separate fits in the two different regions for the scaling behavior of the peak angle with respect to the 3 parameters

Results with a Yukawa interaction

$\theta_L \gg \theta_c$



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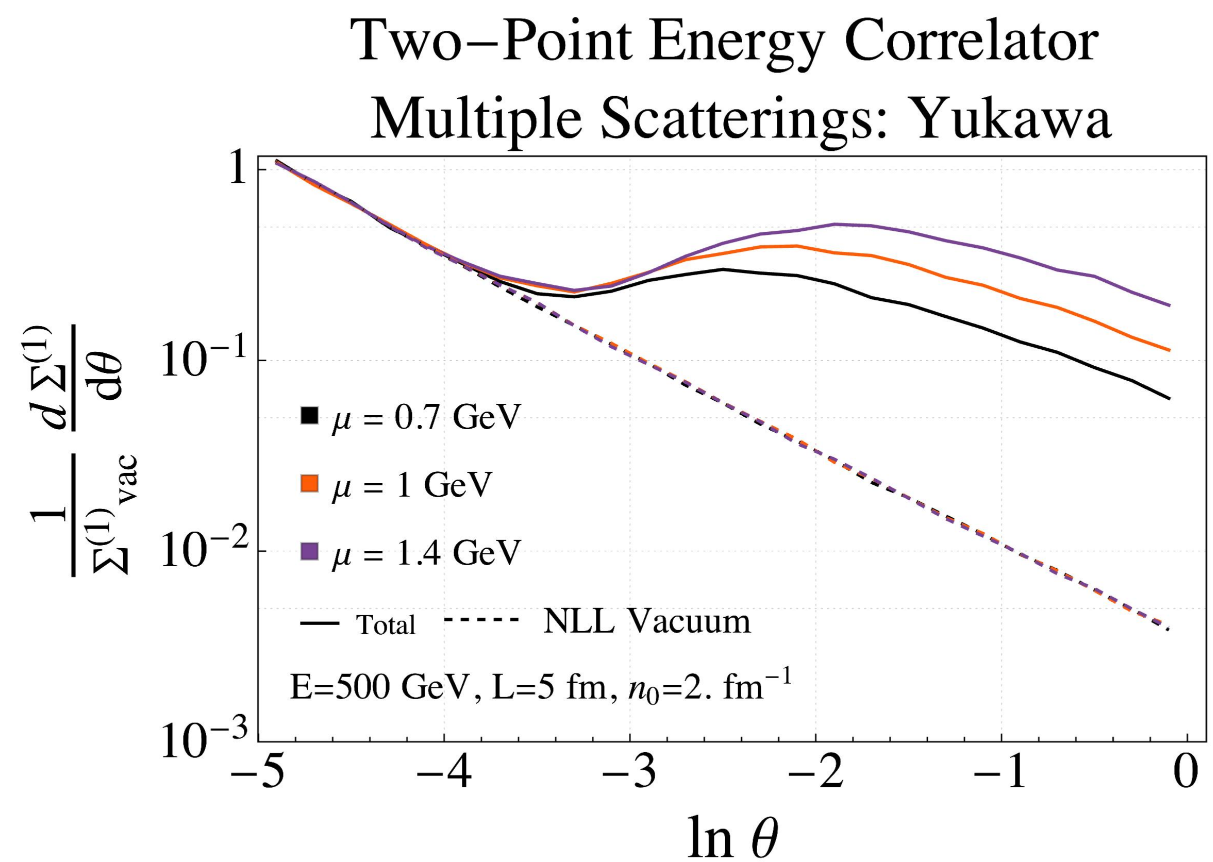

$$V_{\rm yuk}(\boldsymbol{q}) = \frac{8\pi\mu^2}{\left(\boldsymbol{q}^2+\mu^2\right)^2}$$

$$\sigma(\boldsymbol{q}) \equiv -V(\boldsymbol{q}) + (2\pi)^2\delta^2(\boldsymbol{q})\int_l V(\boldsymbol{l})$$

Results with a Yukawa interaction

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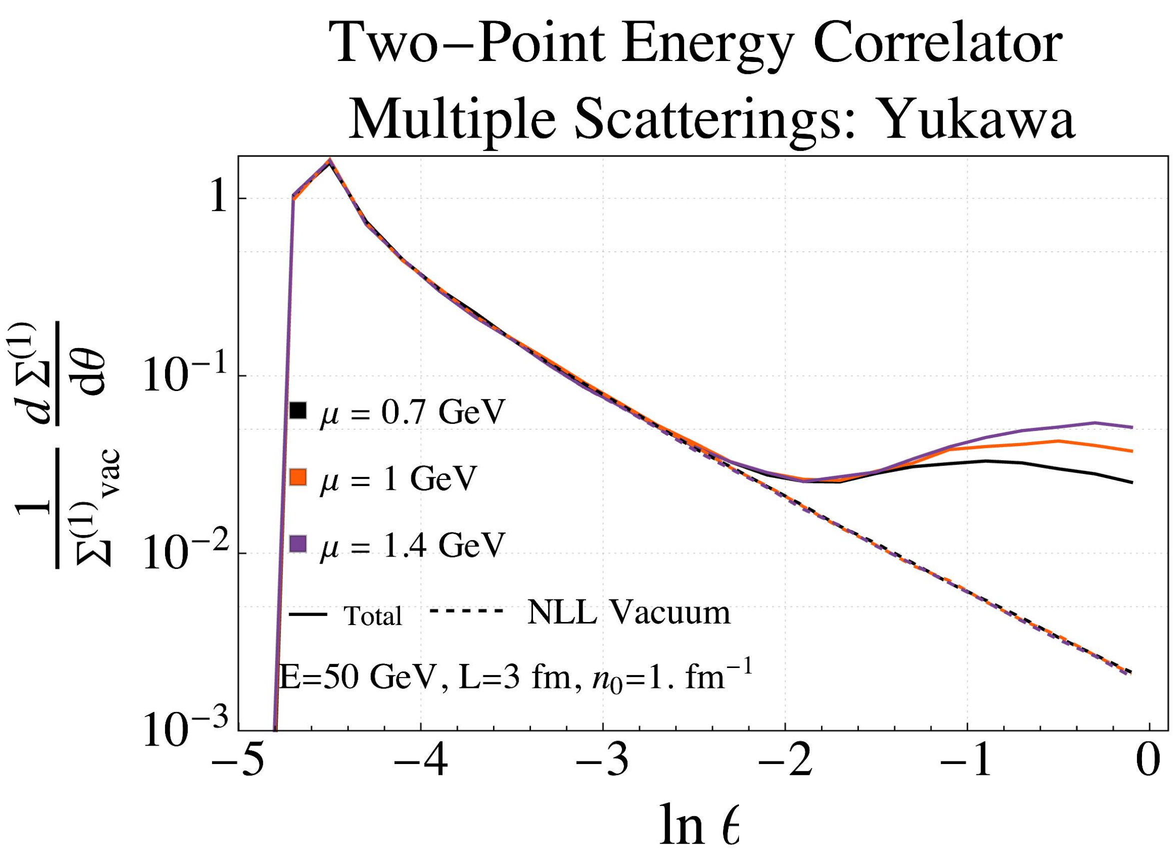

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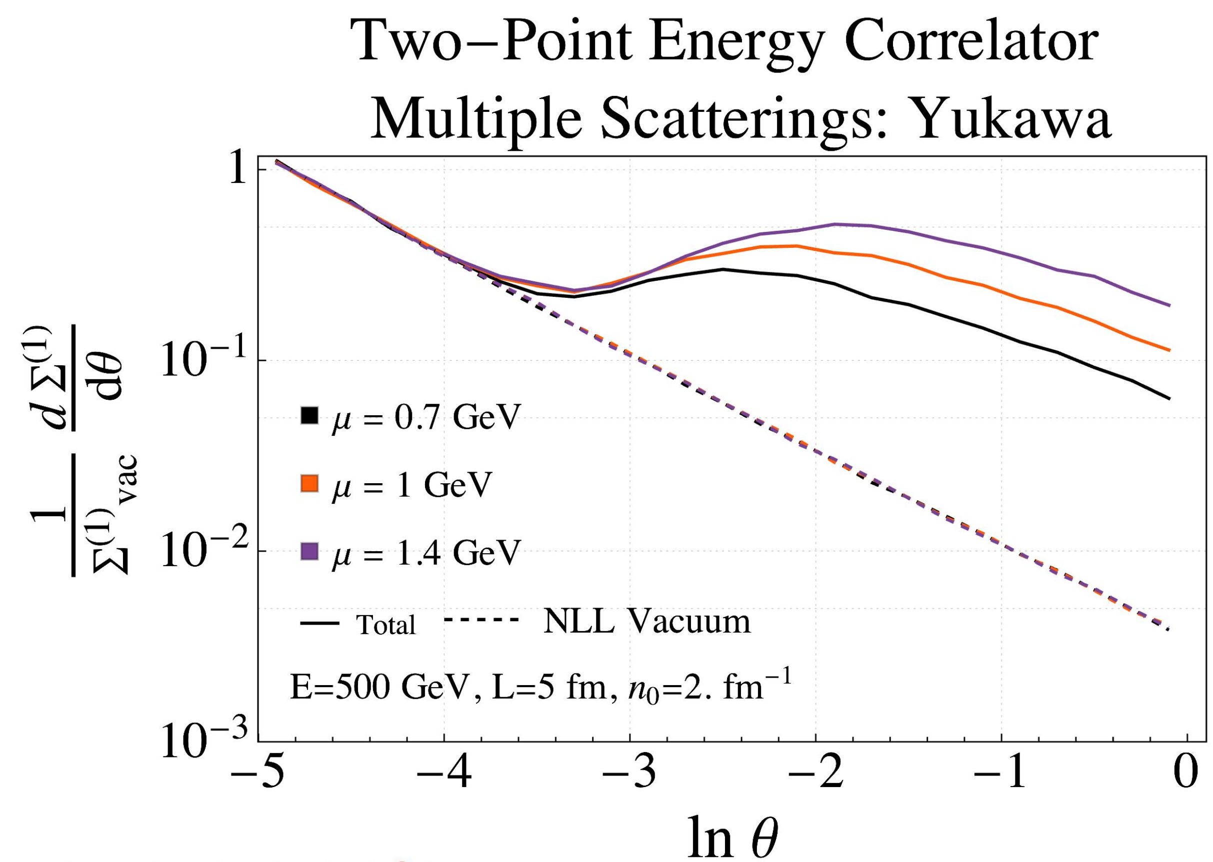

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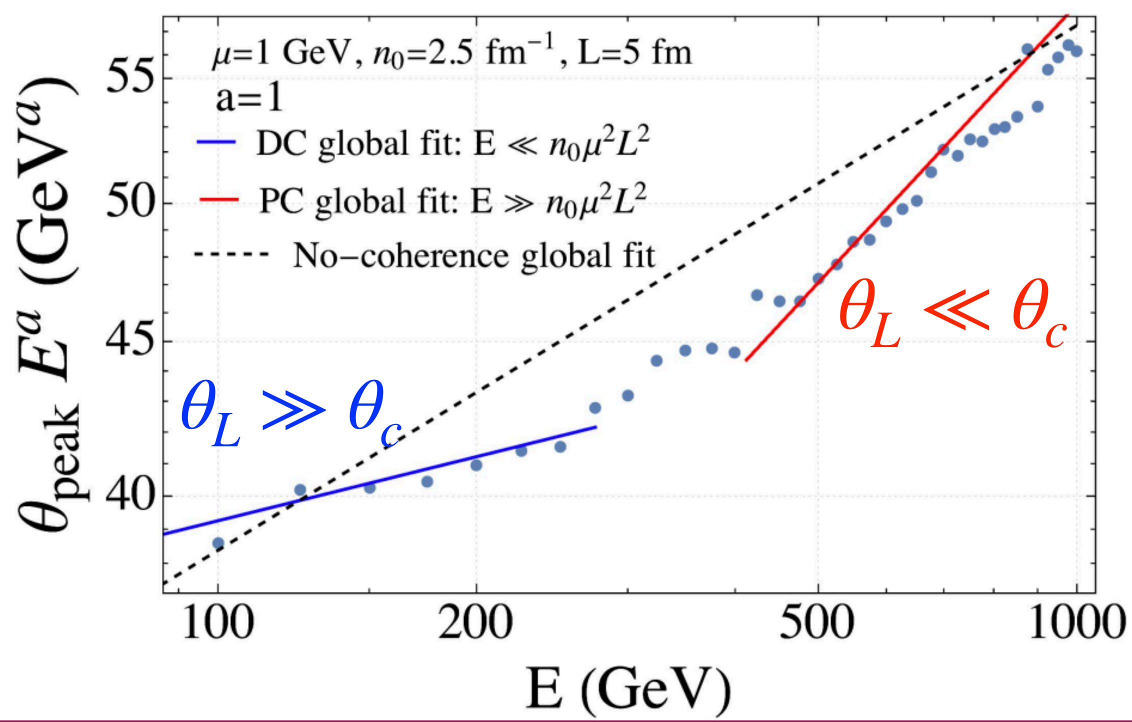
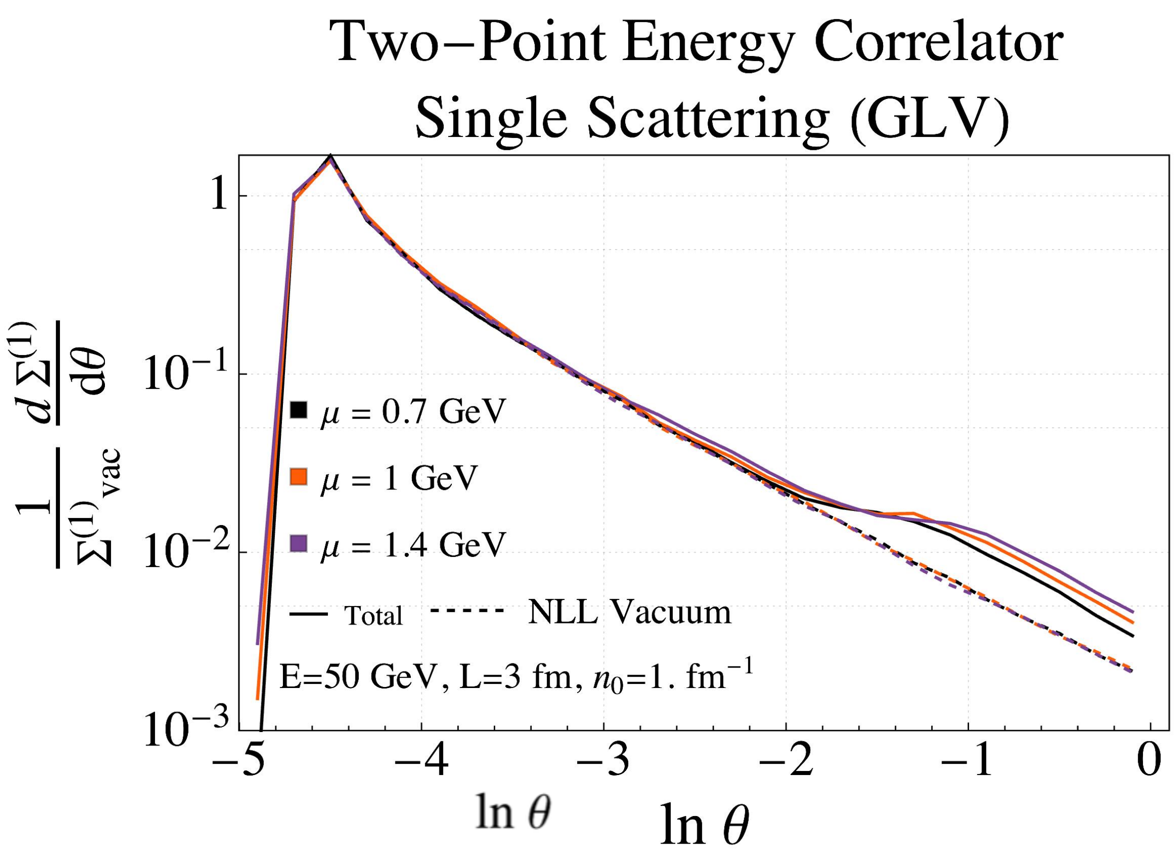


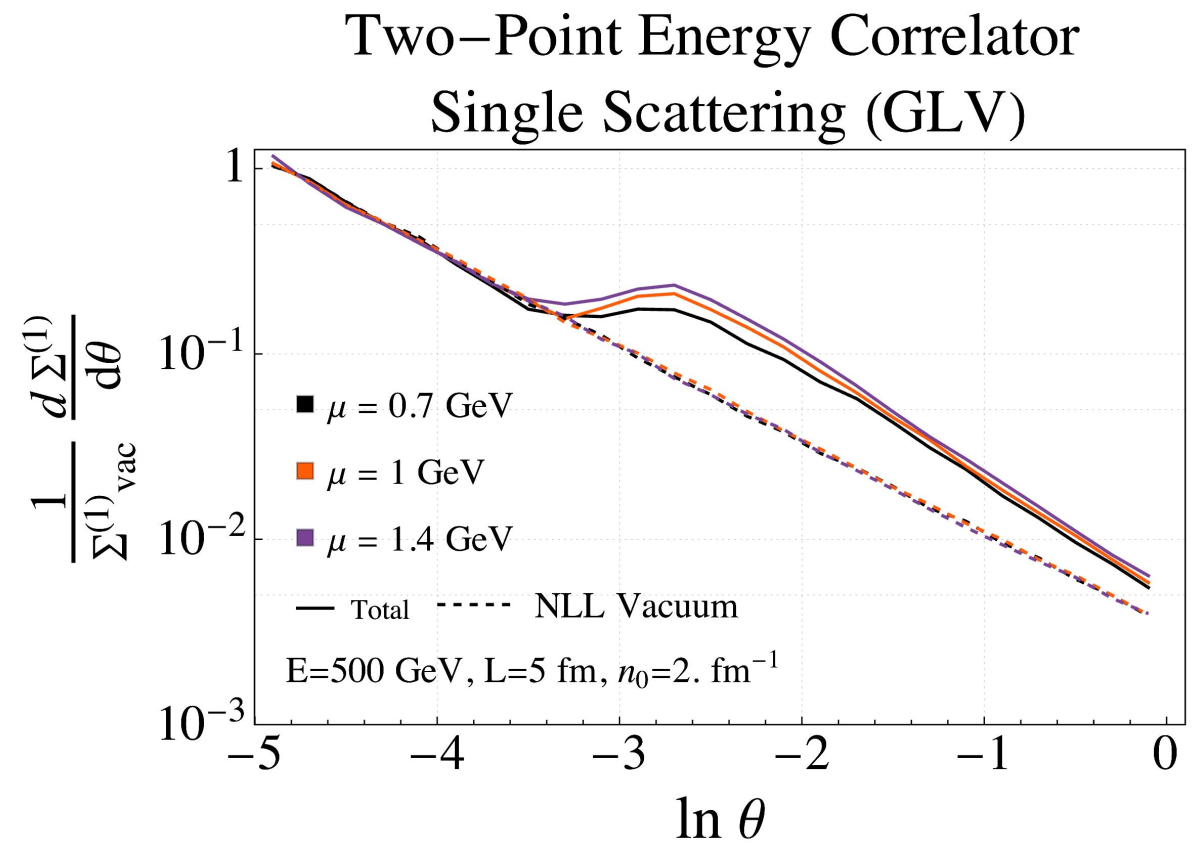
Onset of color **coherence is NOT a feature of the HO** approximation

Results GLV

$\theta_L \gg \theta_c$

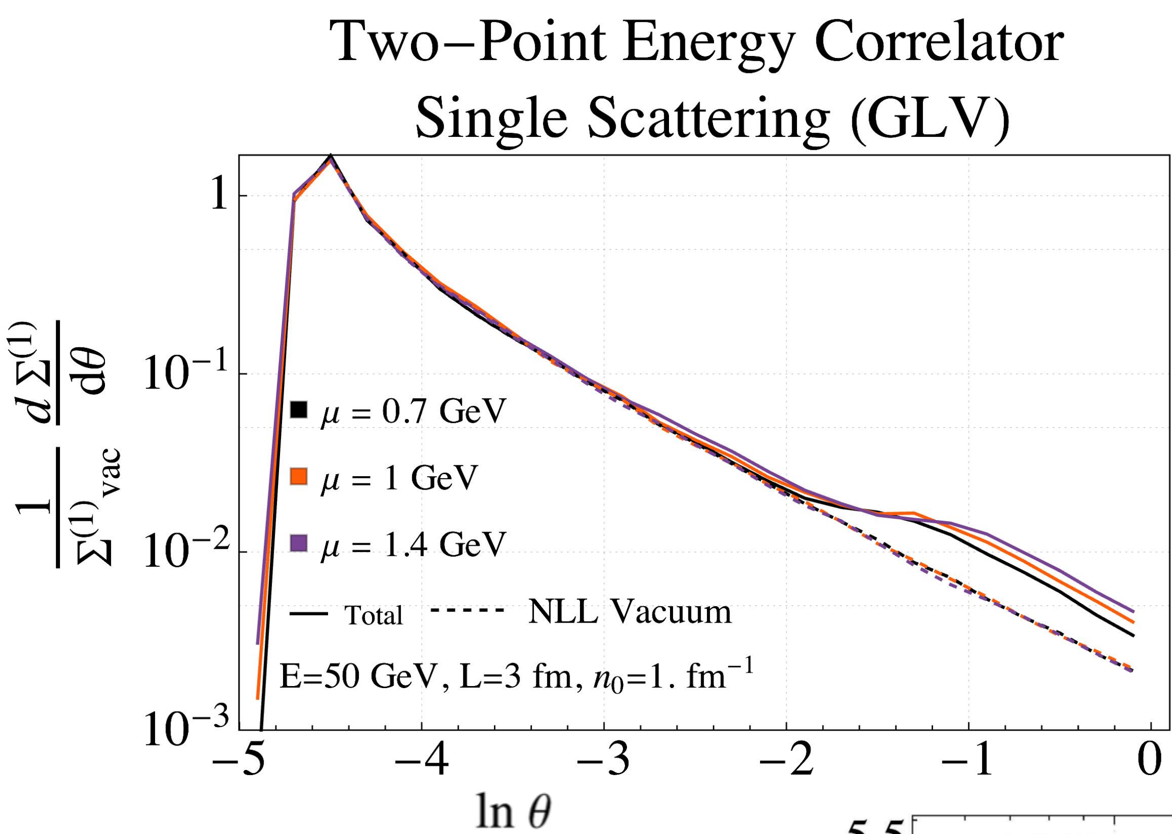


$\theta_L \ll \theta_c$

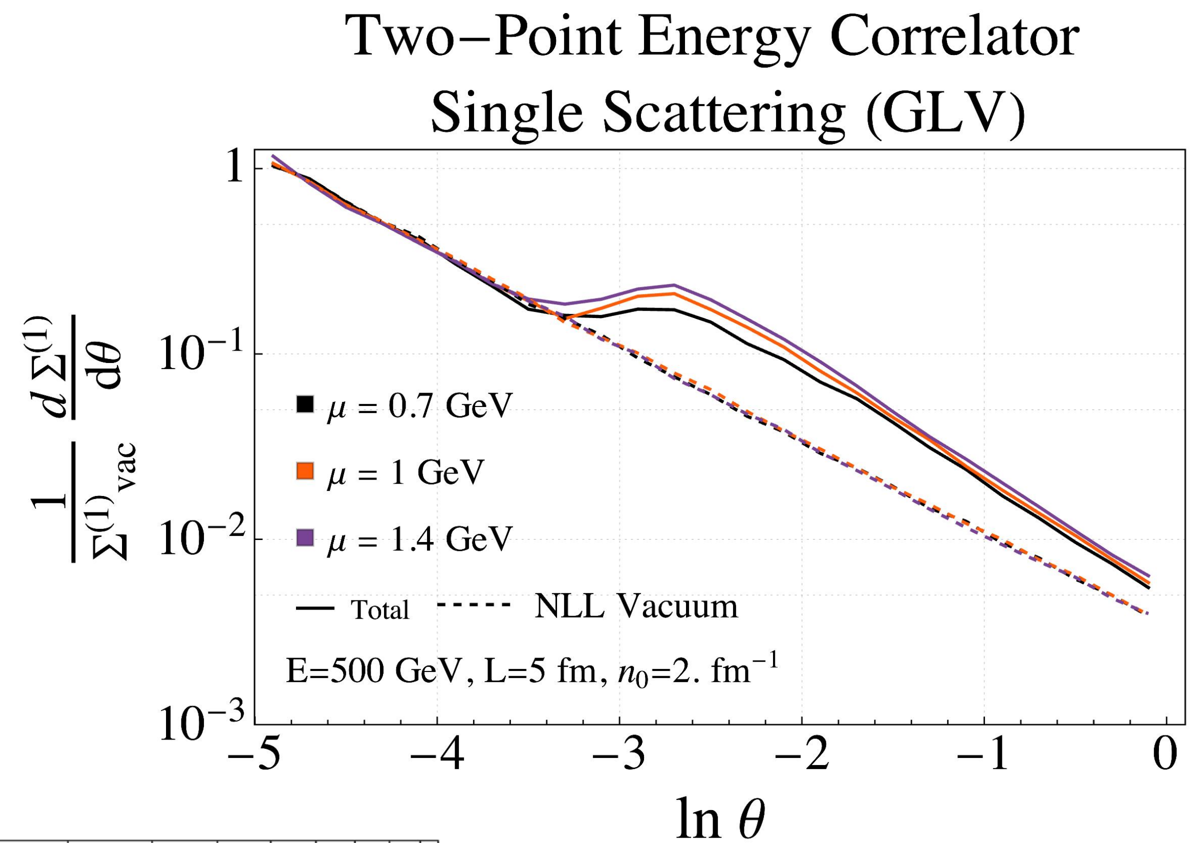


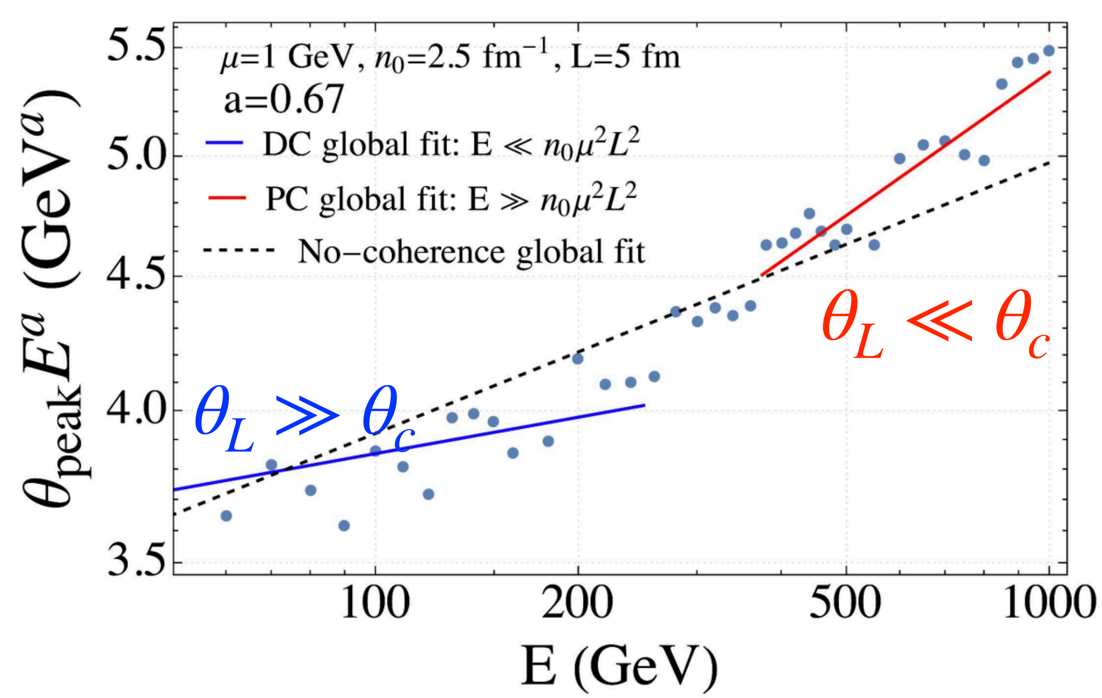
Results GLV

$\theta_L \gg \theta_c$



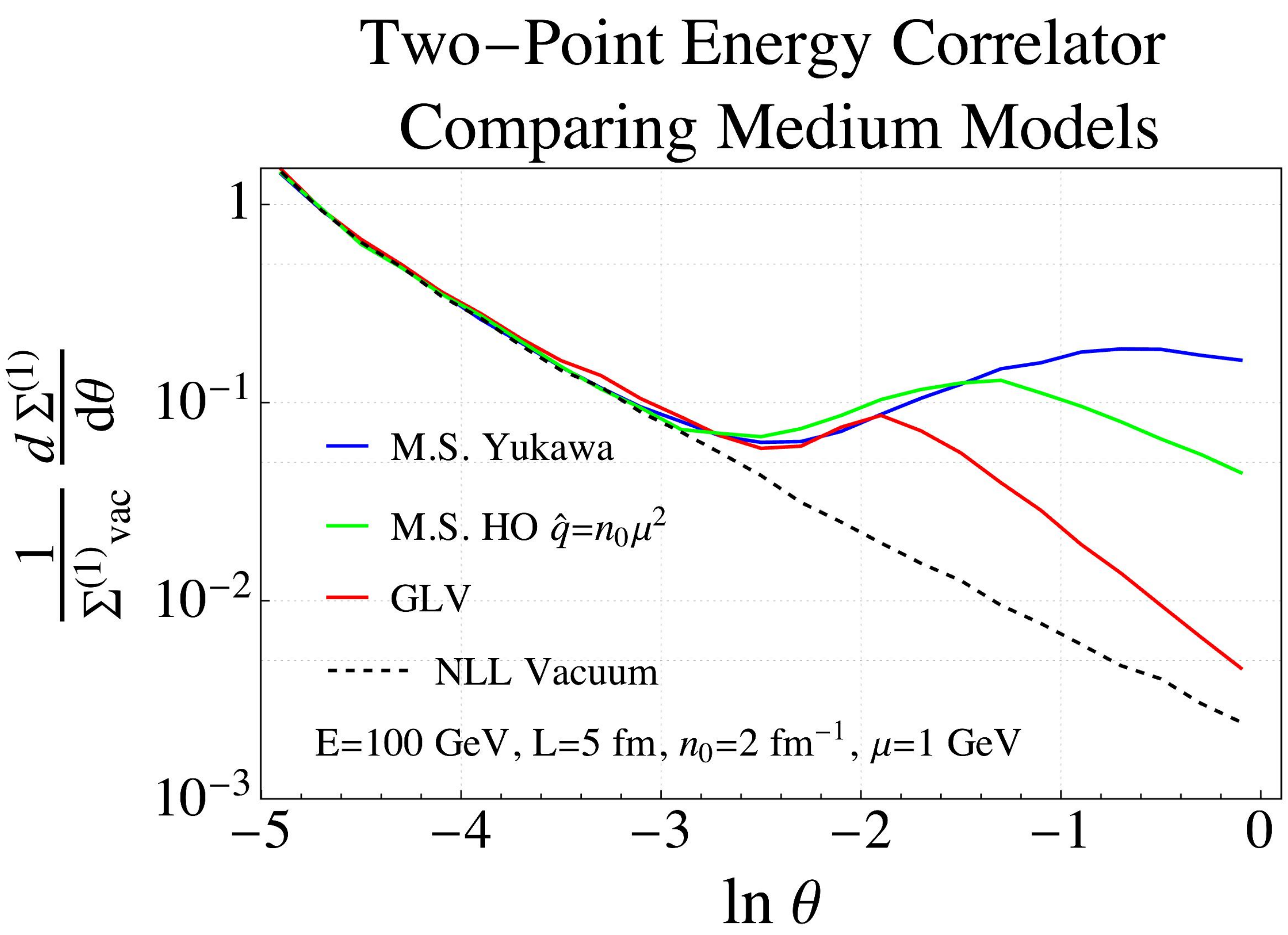
$\theta_L \ll \theta_c$

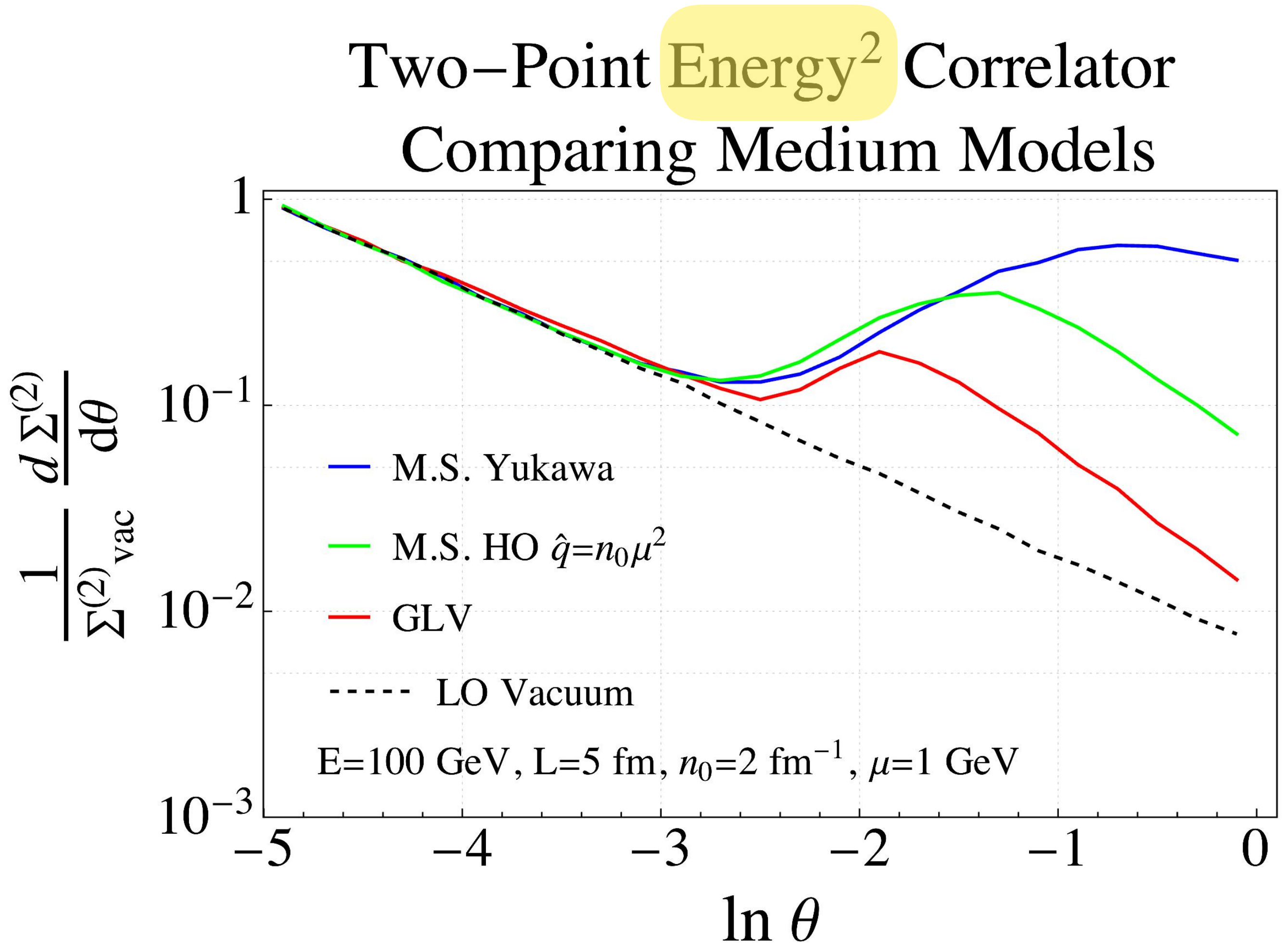




Coherence transition not as clearly observed **as** in the **multiple scattering** case

Higher point correlators





Conclusions

- Energy Correlators provide a powerful tool to understand jets in HICs
  - Broadly insensitive to soft physics: hadronization, and background are usually subleading
  - Can be computed perturbatively
  - Experimentally accessible
- Characteristic features of the calculation of the in-medium splittings are clearly imprinted in these observables
- 2-point correlator provides an angular variable that can be used to probe color coherence
  - Main features seem to be model independent, though transitions between regions are less sharp for the GLV case

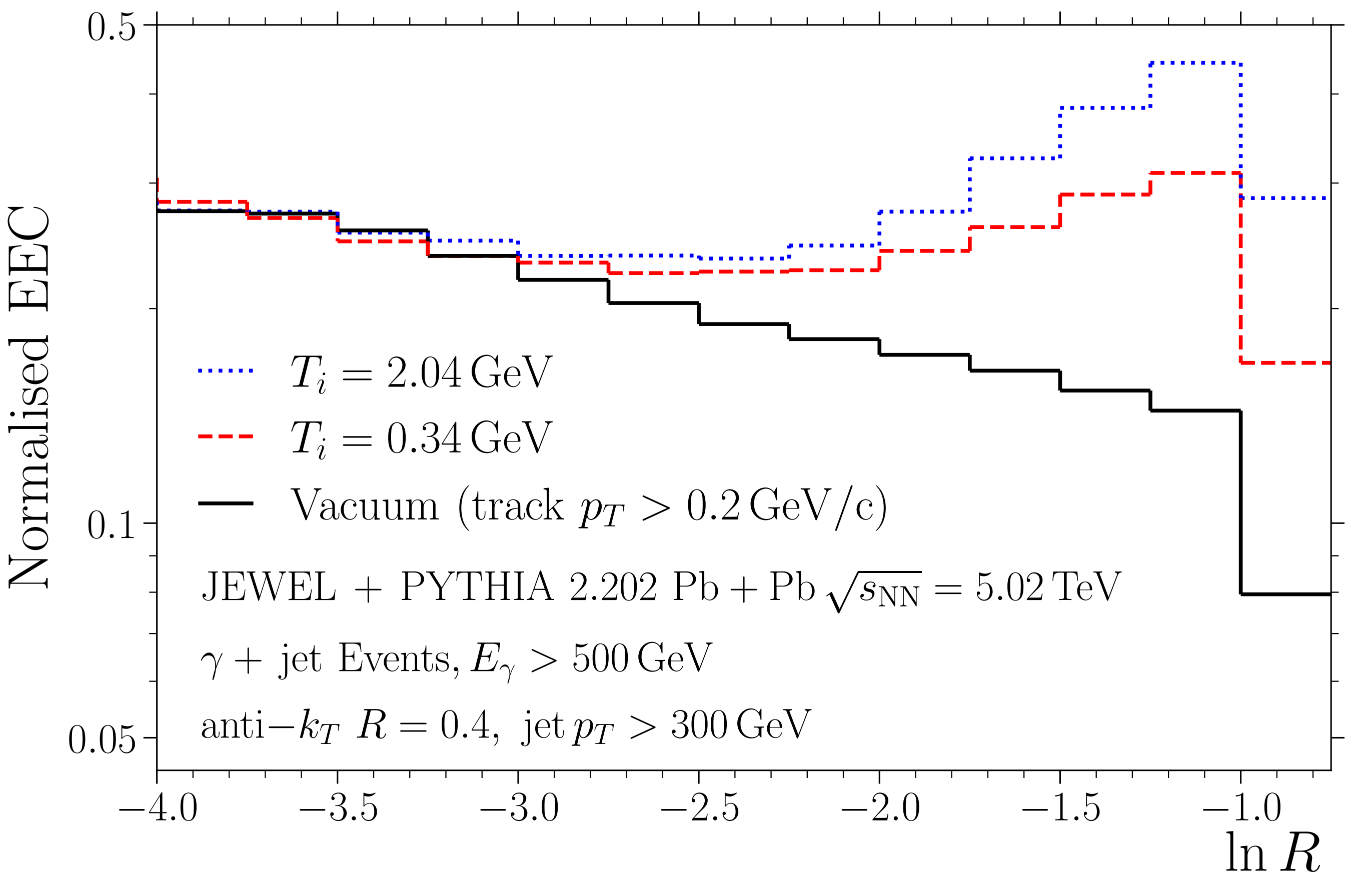
Outlook

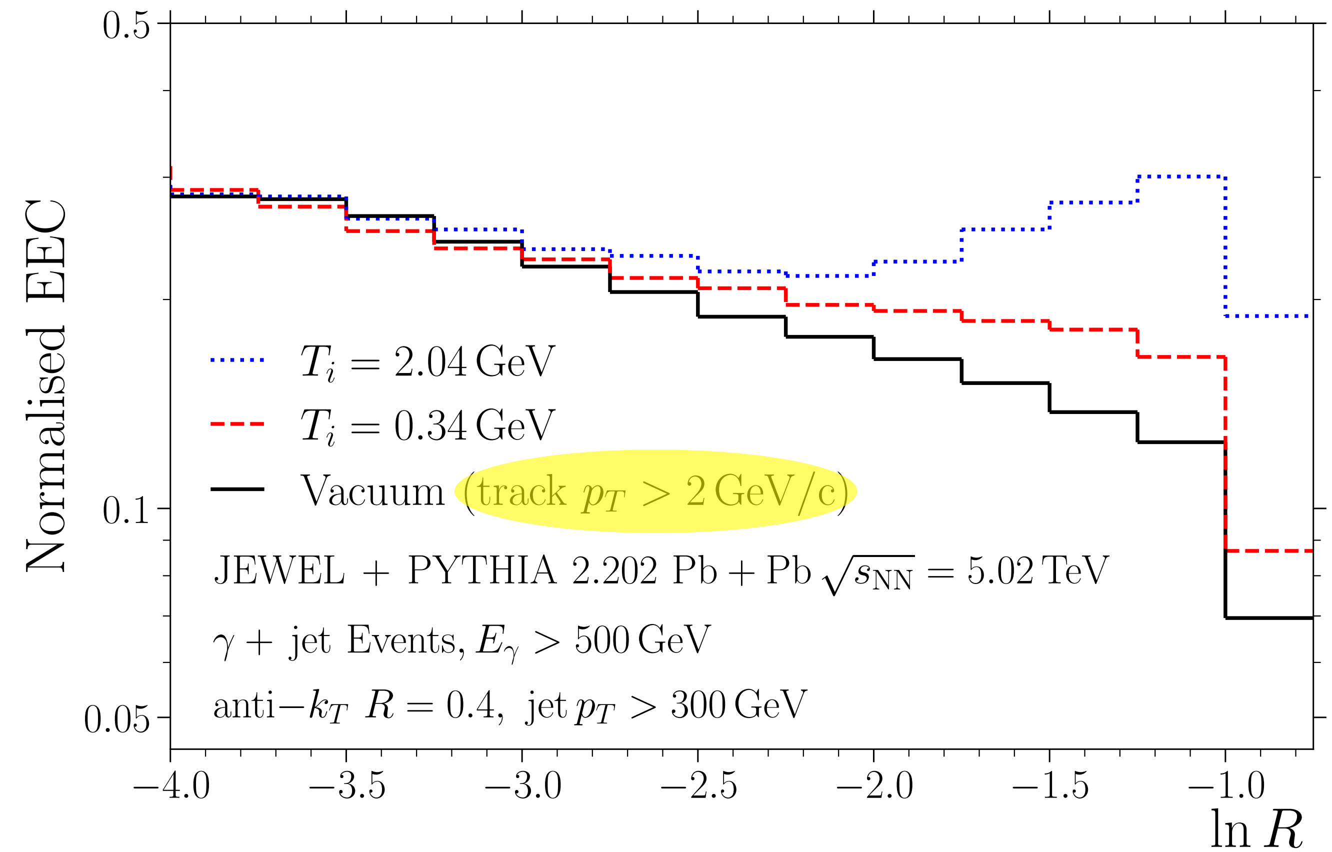
- Lots of new exciting developments!
- **Expanding** media
  - Using energy correlators to find the relevant angular scales
- **Heavy quarks**
  - Can be used to measure the dead-cone
- **Monte Carlo studies**
  - Test resilience to background
  - Test the effects of having the full parton shower

Thank you!

Results from JEWEL

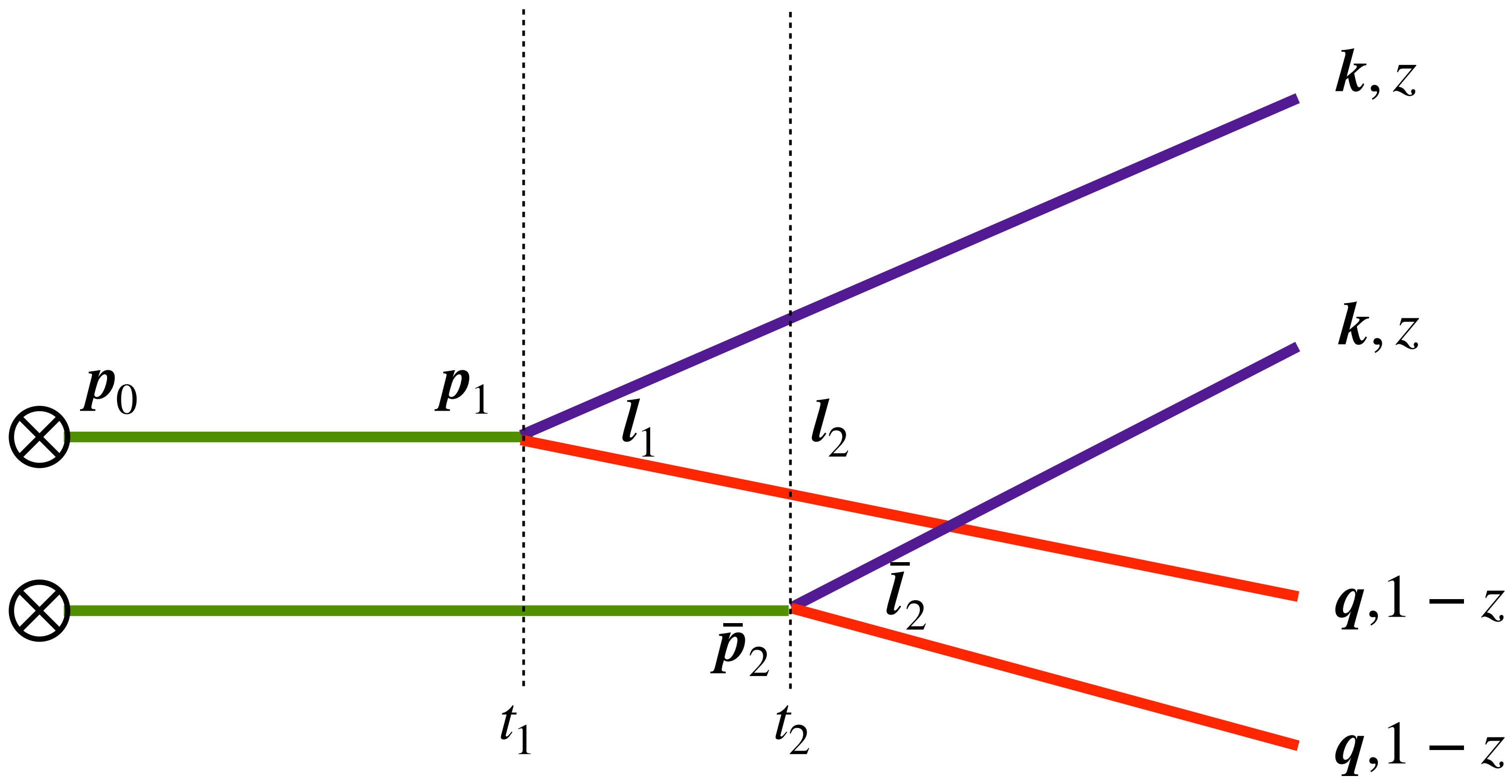
- An analysis on JEWEL is on the way





Features in the curves seem resilient against a hadron cut $p_T \gtrsim 2\,\text{GeV}$

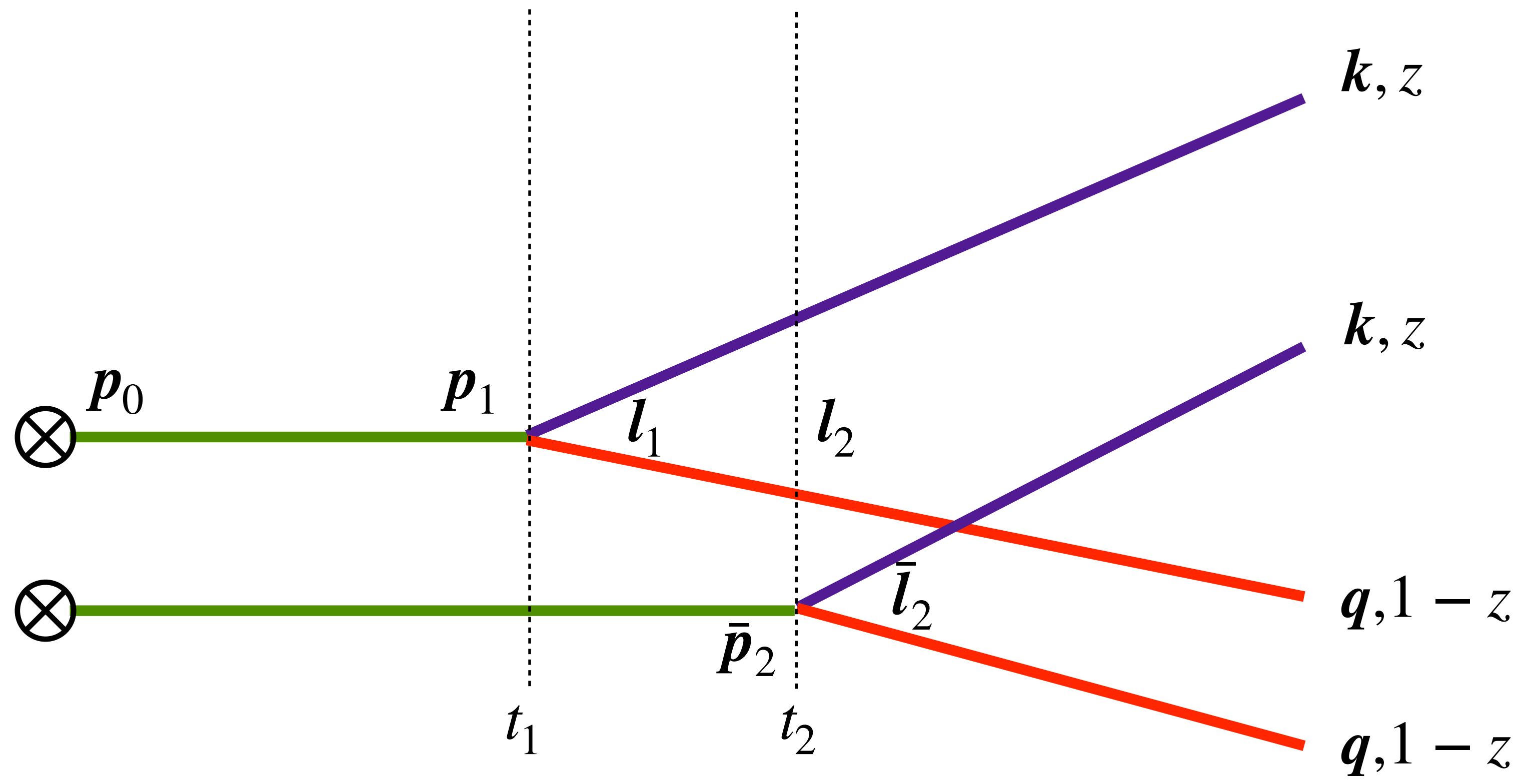
Double differential cross section



$$\frac{d\sigma}{d\Omega_k d\Omega_q} = \frac{g^2}{z(1-z)E^2} P_{a\to bc}(z)\, 2\mathrm{Re} \int_{\boldsymbol{p}_0\boldsymbol{p}_1\bar{\boldsymbol{p}}_2\boldsymbol{l}_1\boldsymbol{l}_2\bar{\boldsymbol{l}}_2} \int_{t_0}^{\infty} dt_1 \int_{t_1}^{\infty} dt_2 (\boldsymbol{l}_1 \cdot \bar{\boldsymbol{l}}_2)$$
$$\times\, \mathcal{S}^{(4)}((1-z)\boldsymbol{k} - z\boldsymbol{q}, L; \boldsymbol{l}_2, \bar{\boldsymbol{l}}_2, t_2; \boldsymbol{k}+\boldsymbol{q}-\bar{\boldsymbol{p}}_2, z)$$
$$\times\, \mathcal{K}^{(3)}(\boldsymbol{l}_2, t_2; \boldsymbol{l}_1, t_1; \bar{\boldsymbol{p}}_2 - \boldsymbol{p}_1, z)\, \mathcal{P}_{R_a}(\boldsymbol{p}_1 - \boldsymbol{p}_0; t_1, t_0) \frac{d\sigma_{hard}}{d\Omega_{p_0}}$$

Blaizot, Iancu, FD, Mehtar-Tani 1209.4585
Apolinario, Armesto, Milhano, Salgado 1407.0599

Double differential cross section



$$\frac{d\sigma}{d\Omega_k d\Omega_q} = \frac{g^2}{z(1-z)E^2} P_{a\to bc}(z)\, 2\mathrm{Re} \int_{\boldsymbol{p}_0 \boldsymbol{p}_1 \bar{\boldsymbol{p}}_2 \boldsymbol{l}_1 \boldsymbol{l}_2 \bar{\boldsymbol{l}}_2} \int_{t_0}^{\infty} dt_1 \int_{t_1}^{\infty} dt_2 (\boldsymbol{l}_1 \cdot \bar{\boldsymbol{l}}_2)$$

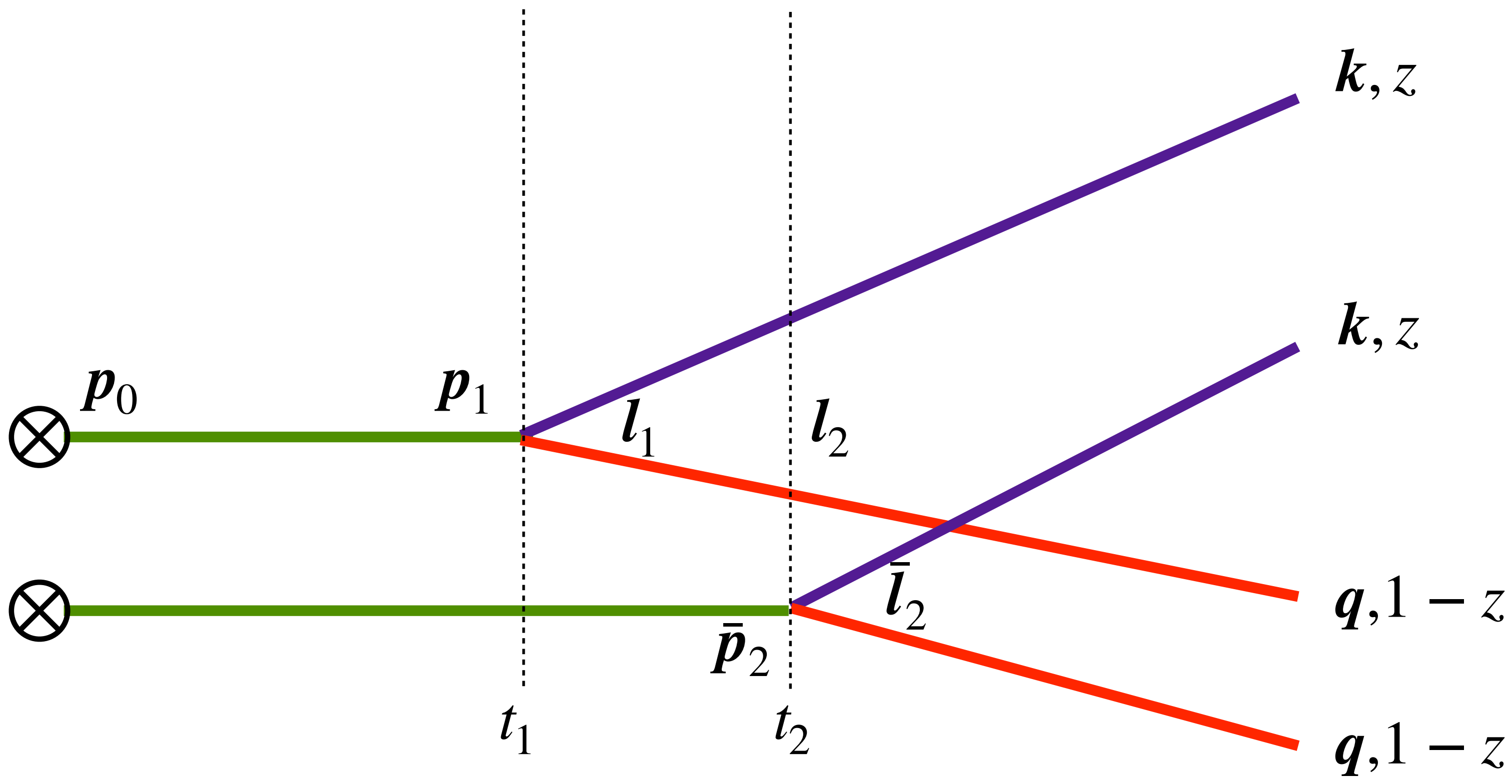
$$\times\, \mathcal{S}^{(4)}((1-z)\boldsymbol{k} - z\boldsymbol{q}, L; \boldsymbol{l}_2, \bar{\boldsymbol{l}}_2, t_2; \boldsymbol{k} + \boldsymbol{q} - \bar{\boldsymbol{p}}_2, z)$$

$$\times\, \mathcal{K}^{(3)}(\boldsymbol{l}_2, t_2; \boldsymbol{l}_1, t_1; \bar{\boldsymbol{p}}_2 - \boldsymbol{p}_1, z)\, \mathcal{P}_{R_a}(\boldsymbol{p}_1 - \boldsymbol{p}_0; t_1, t_0) \frac{d\sigma_{hard}}{d\Omega_{p_0}}$$

$$\left\langle \mathcal{G}\mathcal{G}^{\dagger} \right\rangle$$

Blaizot, Iancu, FD, Mehtar-Tani 1209.4585
Apolinario, Armesto, Milhano, Salgado 1407.0599

Double differential cross section



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$$\times\, \mathcal{S}^{(4)}((1-z)\boldsymbol{k} - z\boldsymbol{q}, L; \boldsymbol{l}_2, \bar{\boldsymbol{l}}_2, t_2; \boldsymbol{k}+\boldsymbol{q}-\bar{\boldsymbol{p}}_2, z)$$

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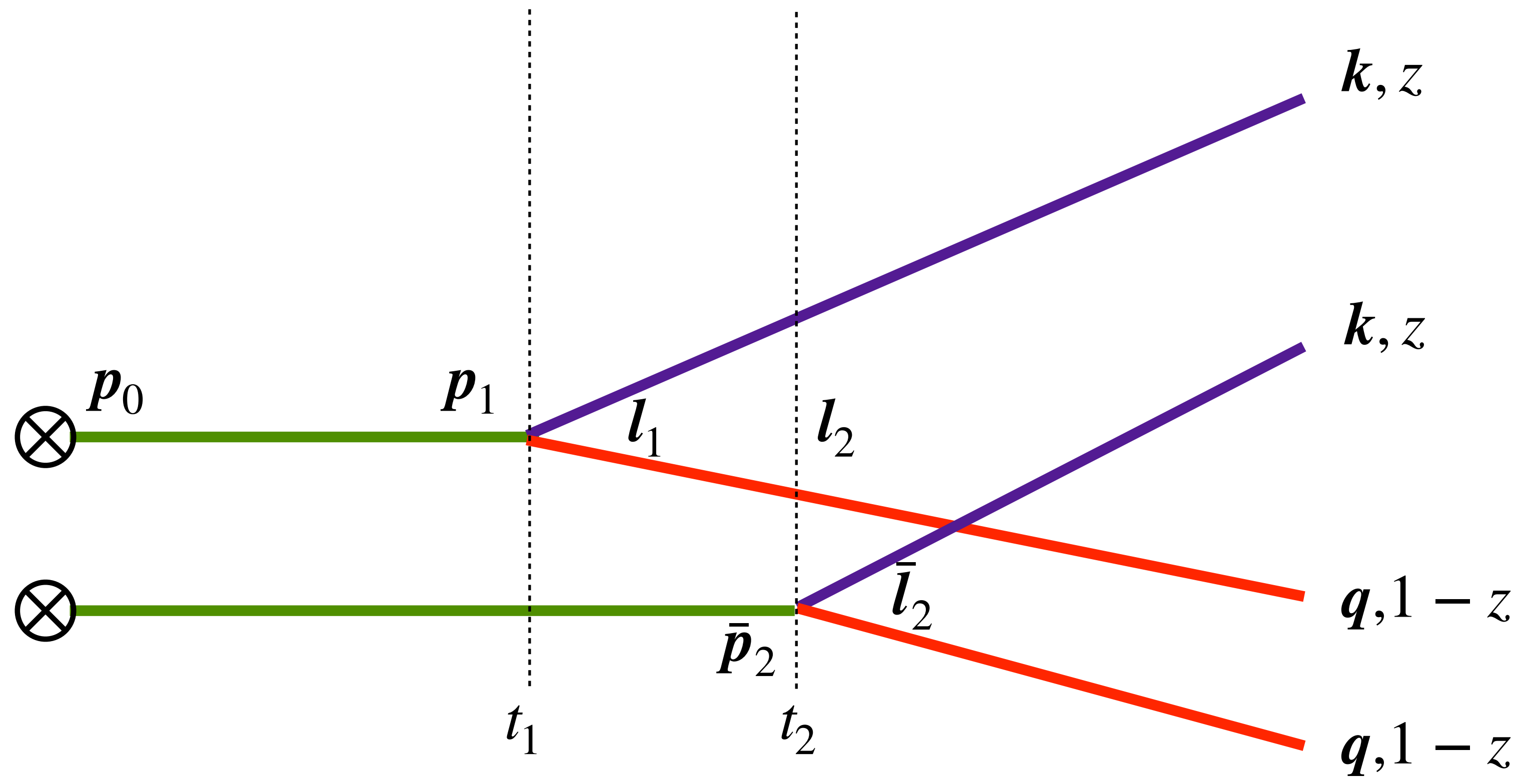
$\left\langle \mathcal{G}\mathcal{G}\mathcal{G}^\dagger \right\rangle$ $\left\langle \mathcal{G}\mathcal{G}^\dagger \right\rangle$

Blaizot, Iancu, FD, Mehtar-Tani 1209.4585
Apolinario, Armesto, Milhano, Salgado 1407.0599

Double differential cross section



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$\langle \mathcal{G}\mathcal{G}\mathcal{G}^\dagger\mathcal{G}^\dagger \rangle$ (for $\mathcal{S}^{(4)}$)

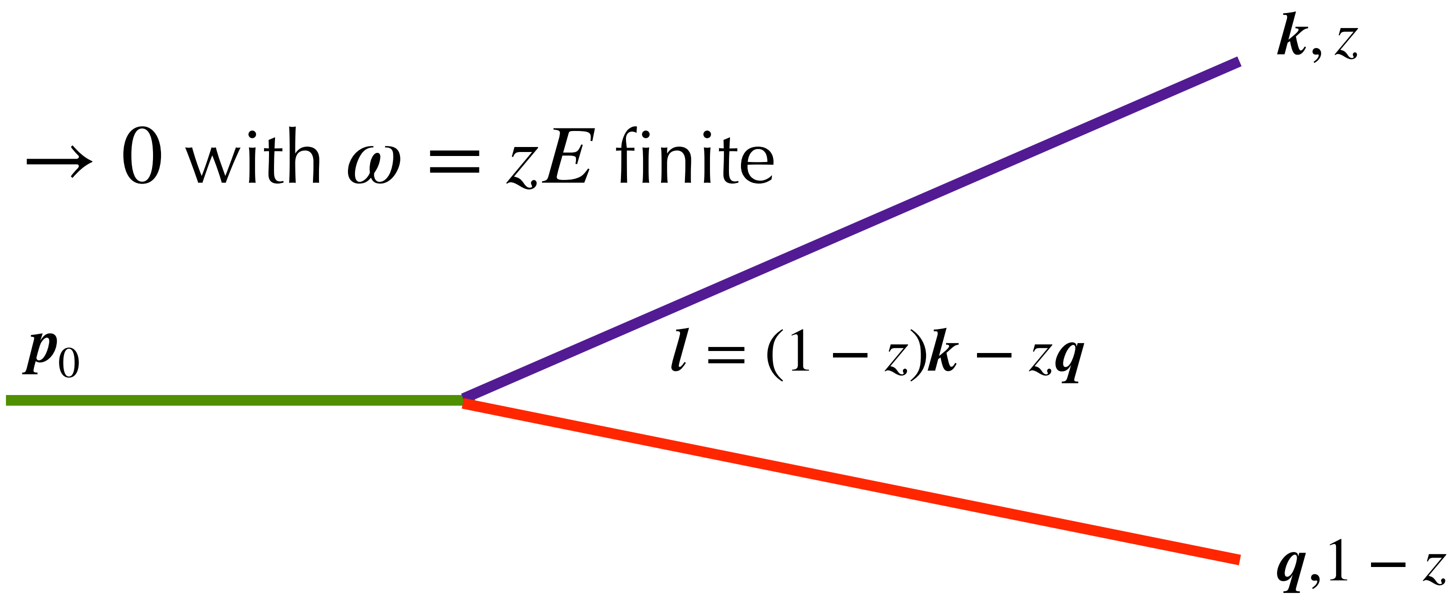
$\langle \mathcal{G}\mathcal{G}\mathcal{G}^\dagger \rangle$ (for $\mathcal{K}^{(3)}$)

$\langle \mathcal{G}\mathcal{G}^\dagger \rangle$ (for $\mathcal{P}_{R_a}$)

Blaizot, Iancu, FD, Mehtar-Tani 1209.4585
Apolinario, Armesto, Milhano, Salgado 1407.0599

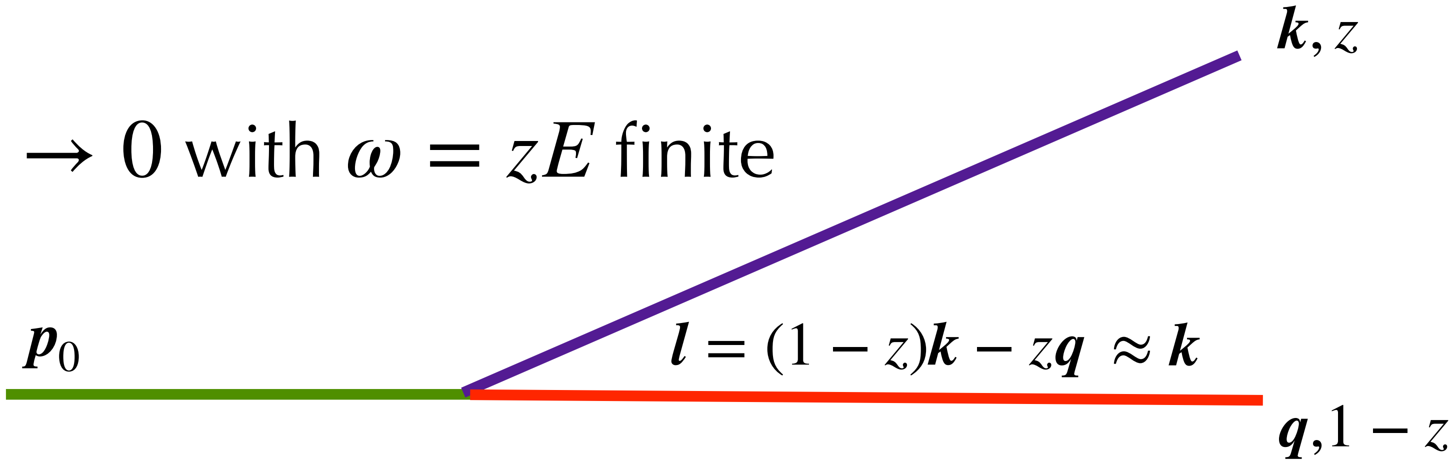
Soft limit

$z \to 0$ with $\omega = zE$ finite



Soft limit

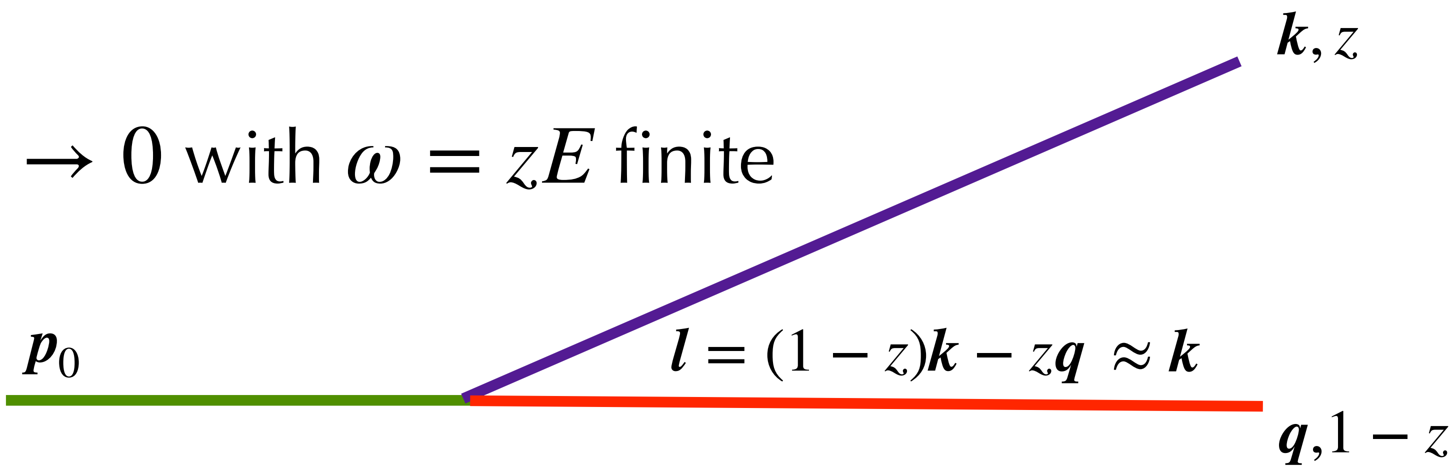
$z \to 0$ with $\omega = zE$ finite



Angle of emission depends only on transverse momentum of the soft particle

Soft limit

$z \to 0$ with $\omega = zE$ finite



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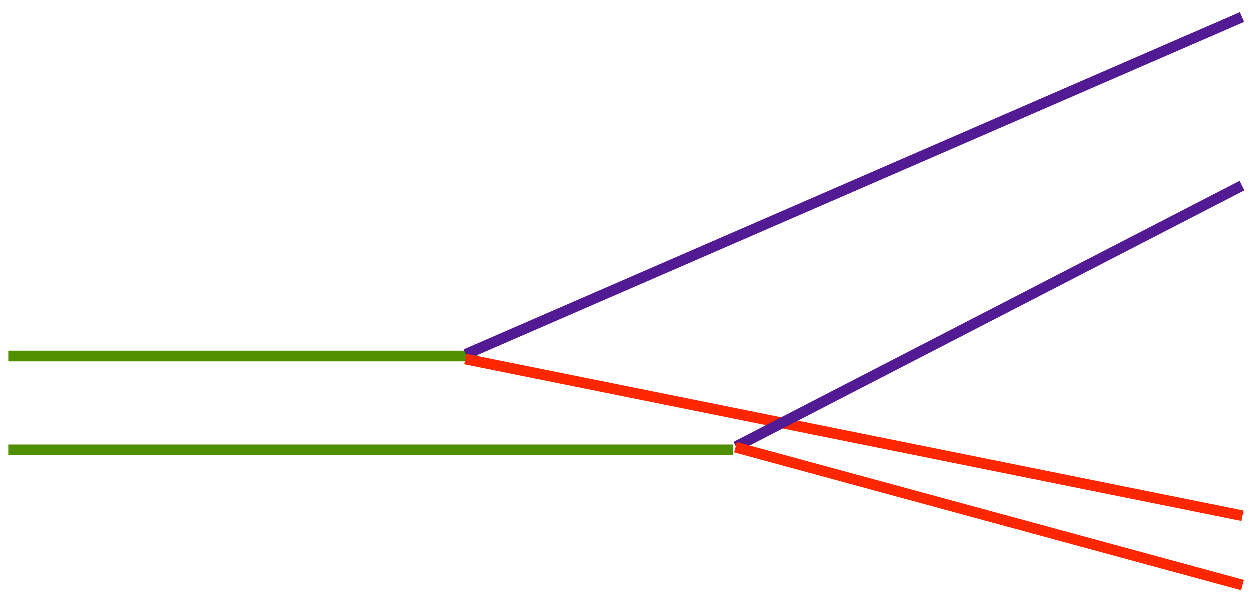

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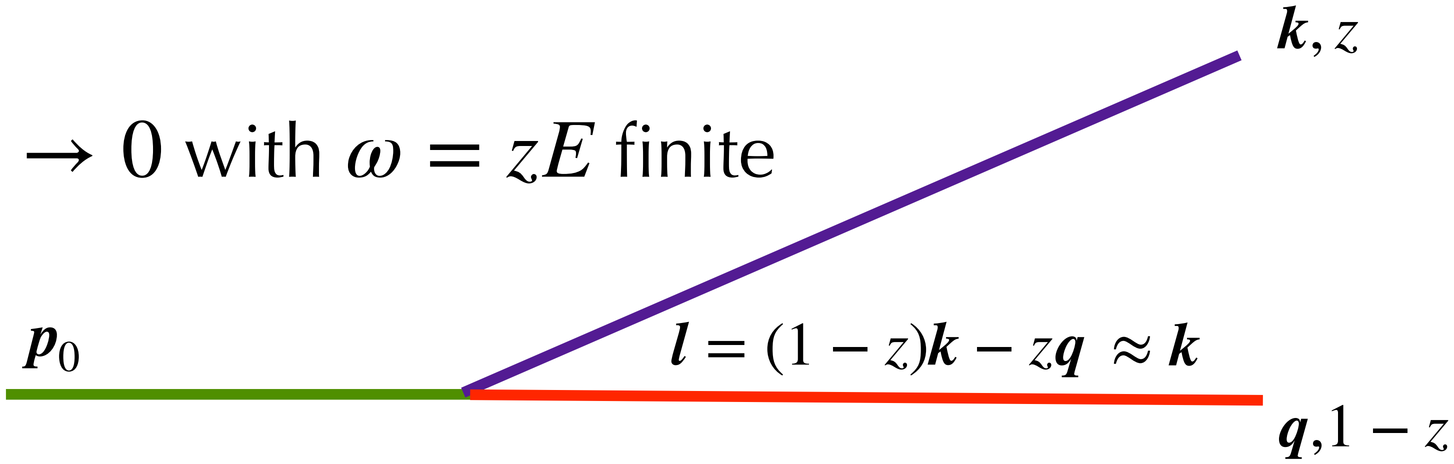

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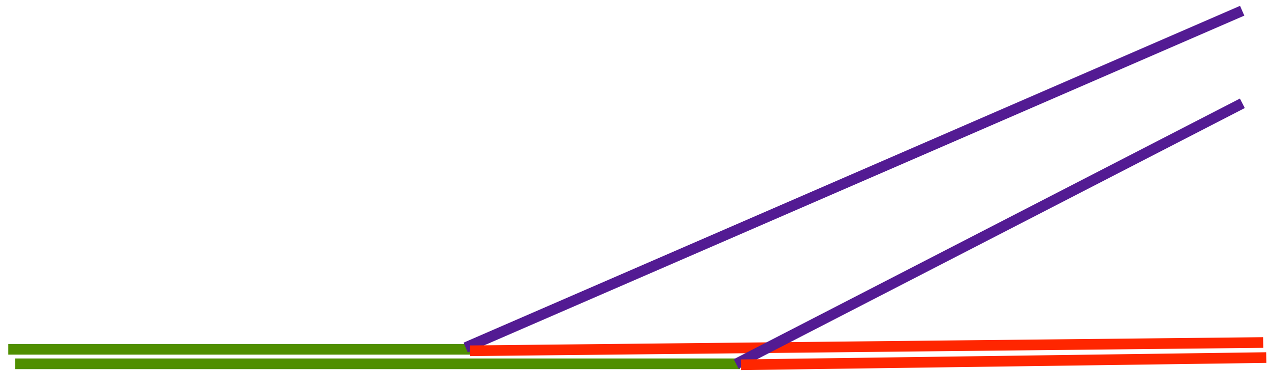

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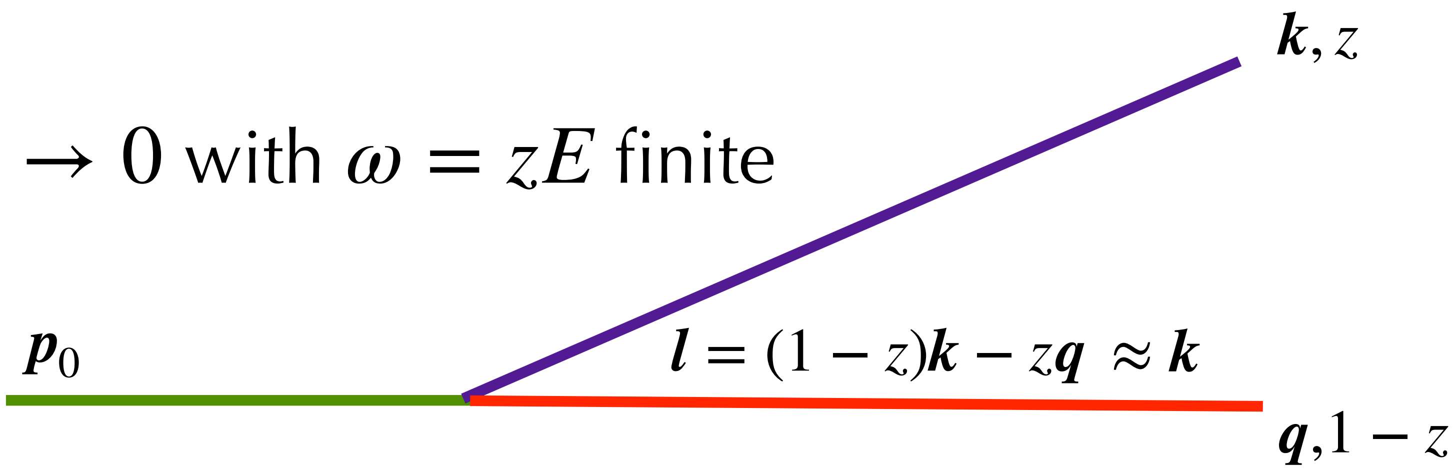

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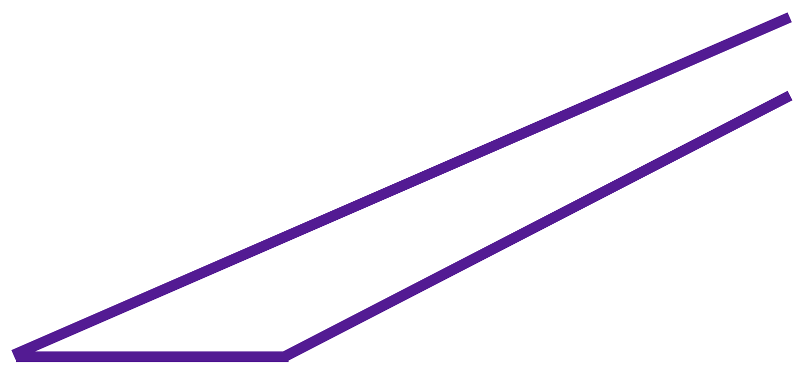
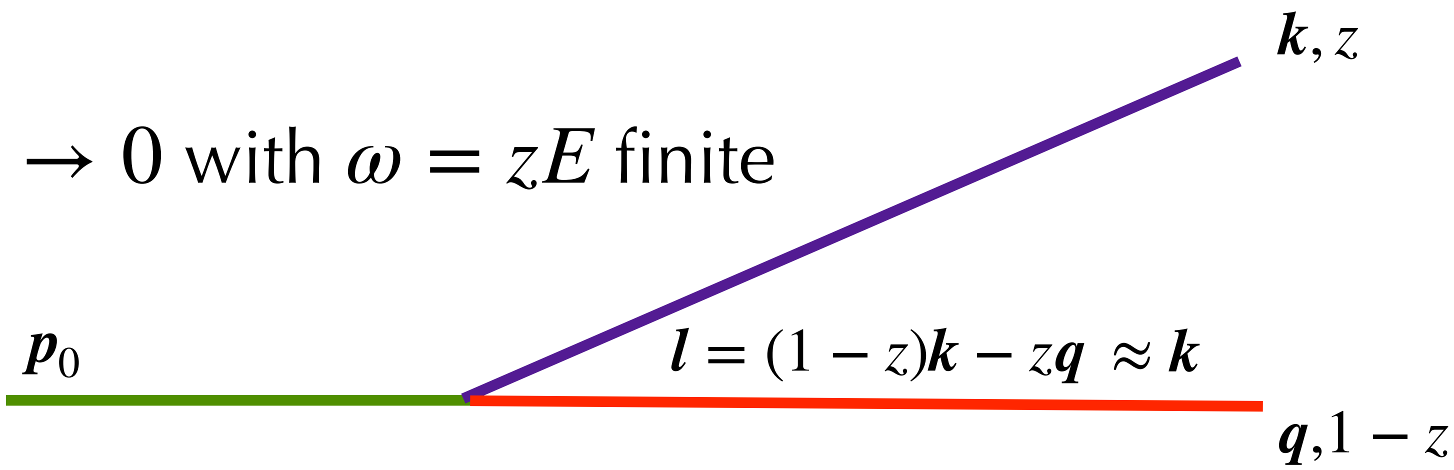


Initial and final broadening of the hard particle cancels out

Soft limit

$z \to 0$ with $\omega = zE$ finite



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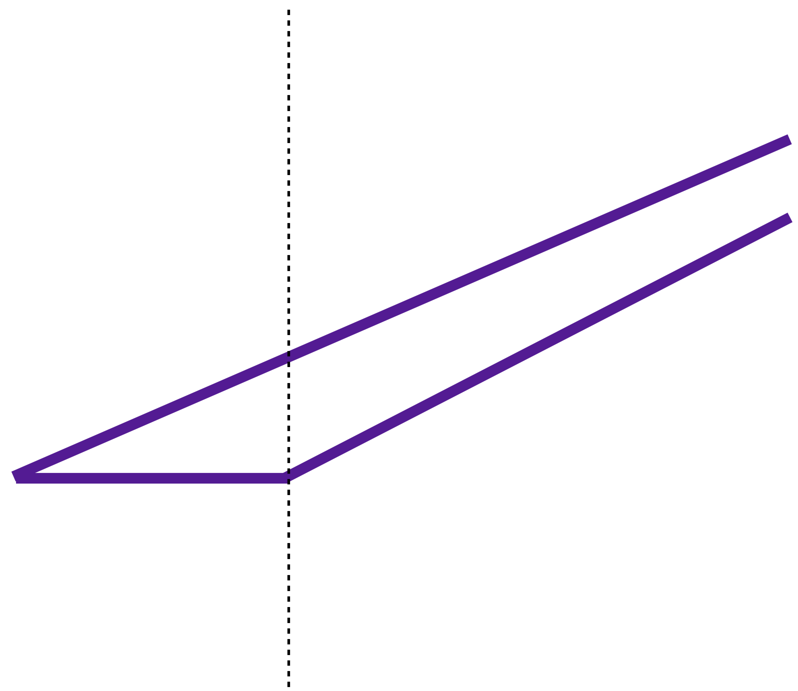

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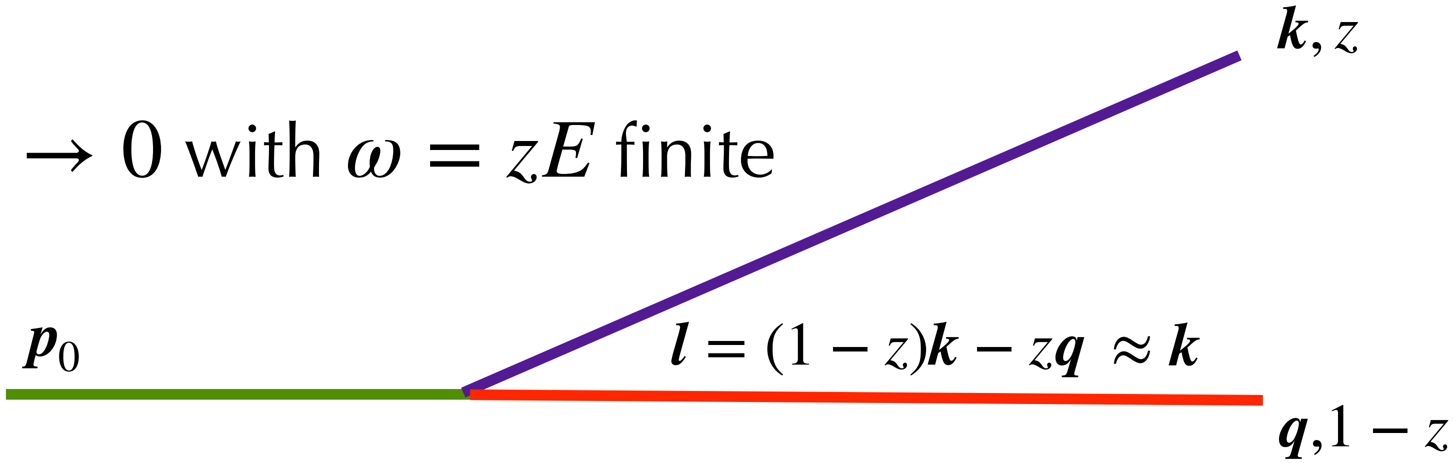

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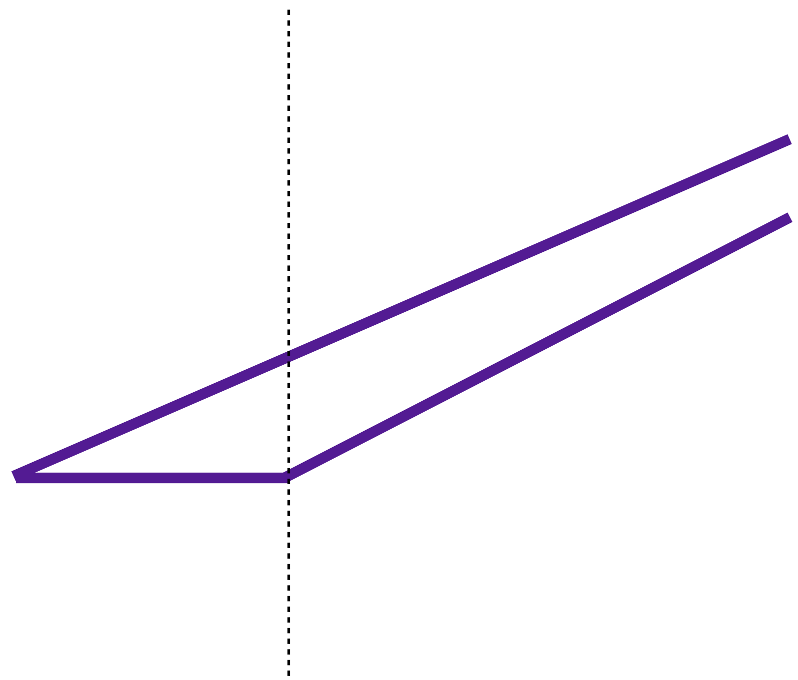
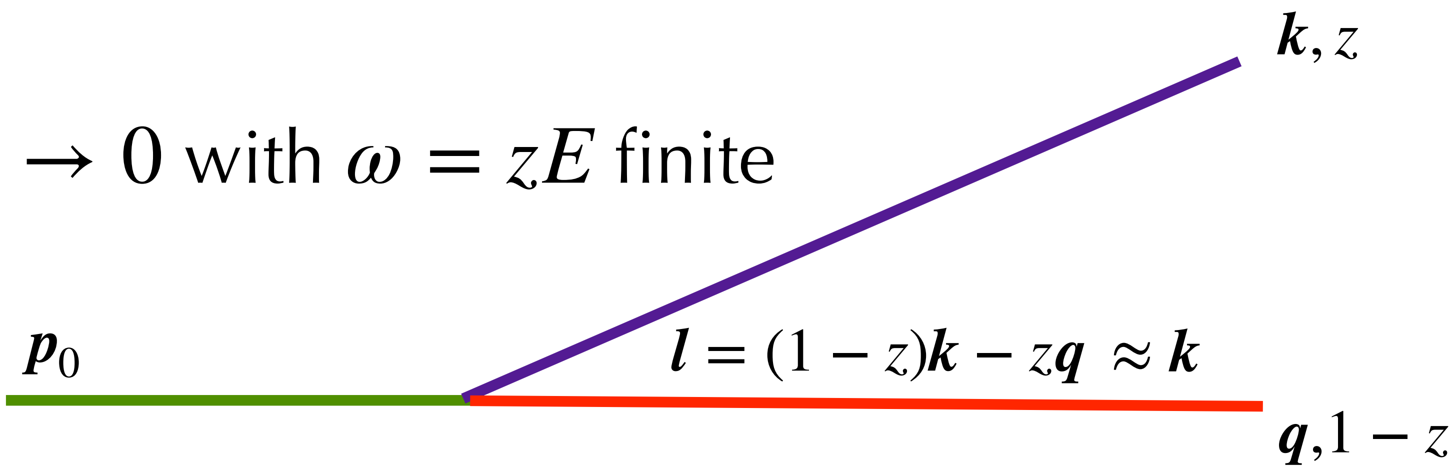


Initial and final broadening of the hard particle cancels out

$$\omega \frac{dI}{d\omega d^2\boldsymbol{k}} = \frac{2\alpha_s C_R}{(2\pi)^2\omega^2} \mathrm{Re} \int_0^\infty dt' \int_0^{t'} dt \int_{\boldsymbol{pq}} \boldsymbol{p}\cdot\boldsymbol{q}\, \widetilde{\mathcal{K}}(t', \boldsymbol{q}; t, \boldsymbol{p}) \mathcal{P}(\infty, \boldsymbol{k}; t', \boldsymbol{q})$$

Soft limit

$z \to 0$ with $\omega = zE$ finite



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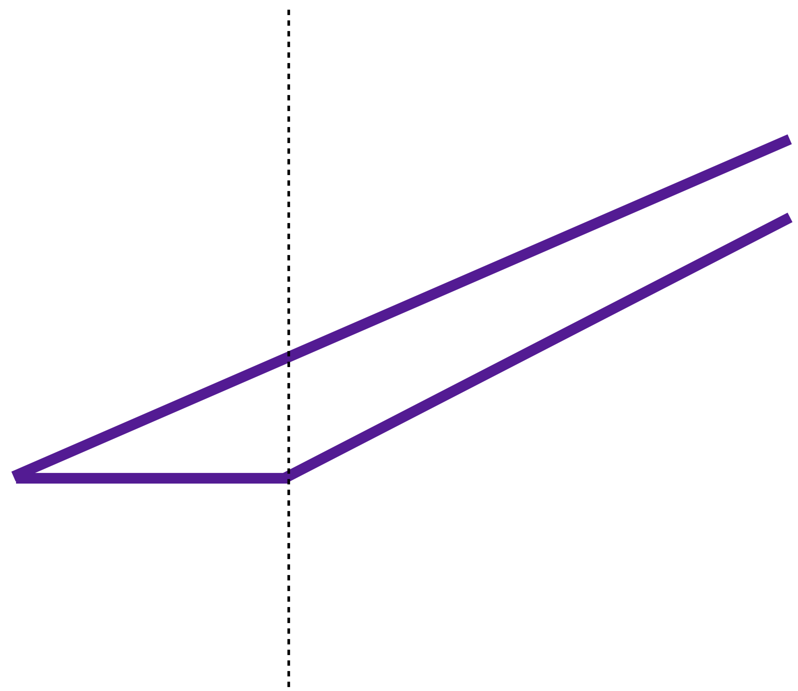

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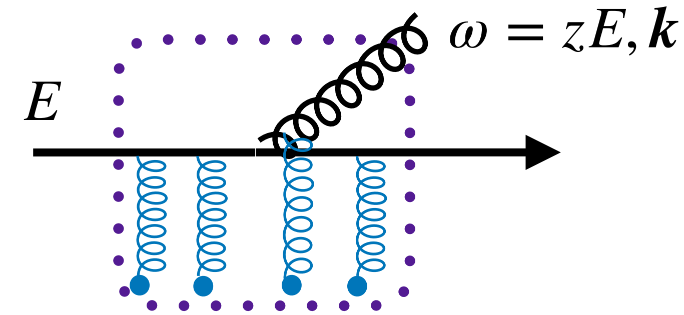
Recently evaluated numerically with multiple scatterings and realistic interactions

Andres, Apolinario, FD 2002.01517
Andres, FD, Gonzalez Martinez 2011.06522

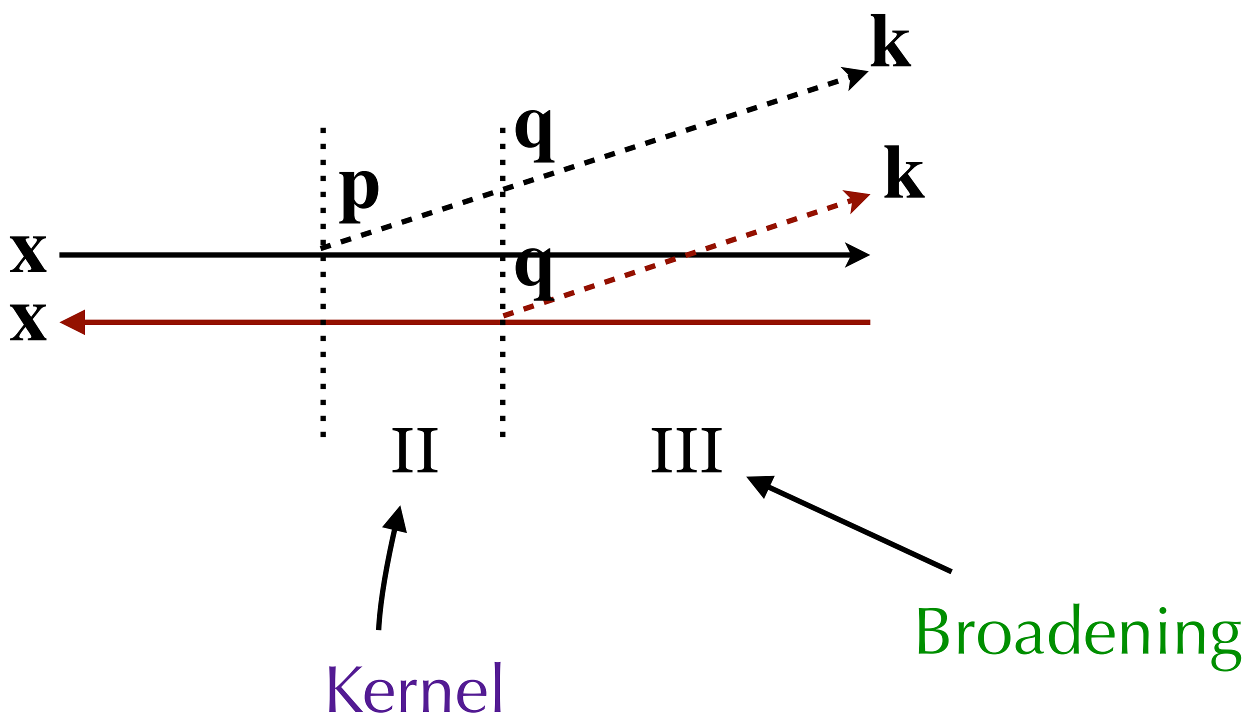
Medium-induced gluon spectrum

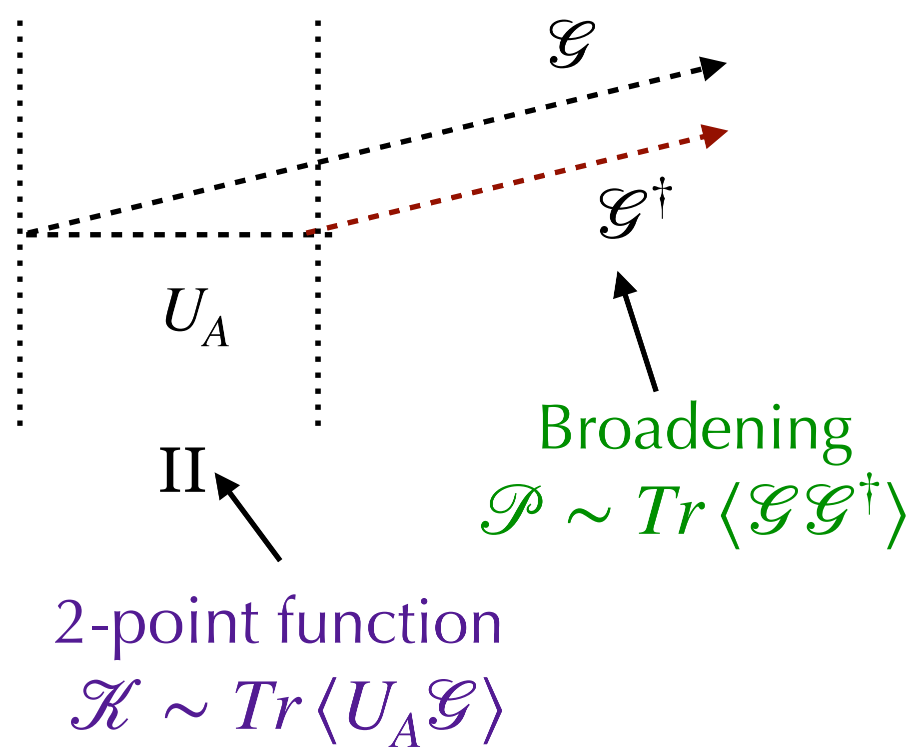


- For a soft emitted gluon ($z \ll 1$)

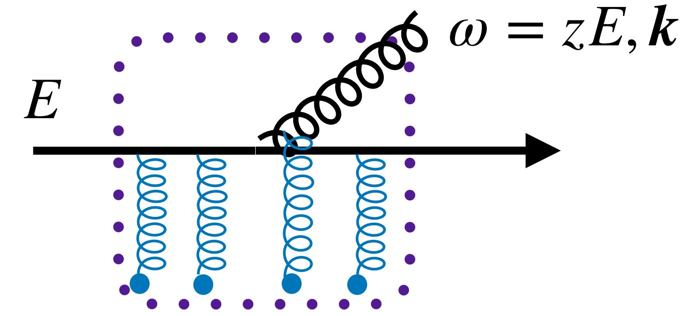
$$\omega \frac{dI}{d\omega d^2\boldsymbol{k}} = \frac{2\alpha_s C_R}{(2\pi)^2\omega^2} \text{Re} \int_0^\infty dt' \int_0^{t'} dt \int_{\boldsymbol{pq}} \boldsymbol{p}\cdot\boldsymbol{q}\, \widetilde{\mathcal{K}}(t', \boldsymbol{q}; t, \boldsymbol{p}) \mathcal{P}(\infty, \boldsymbol{k}; t', \boldsymbol{q})$$

BDMPS-Z





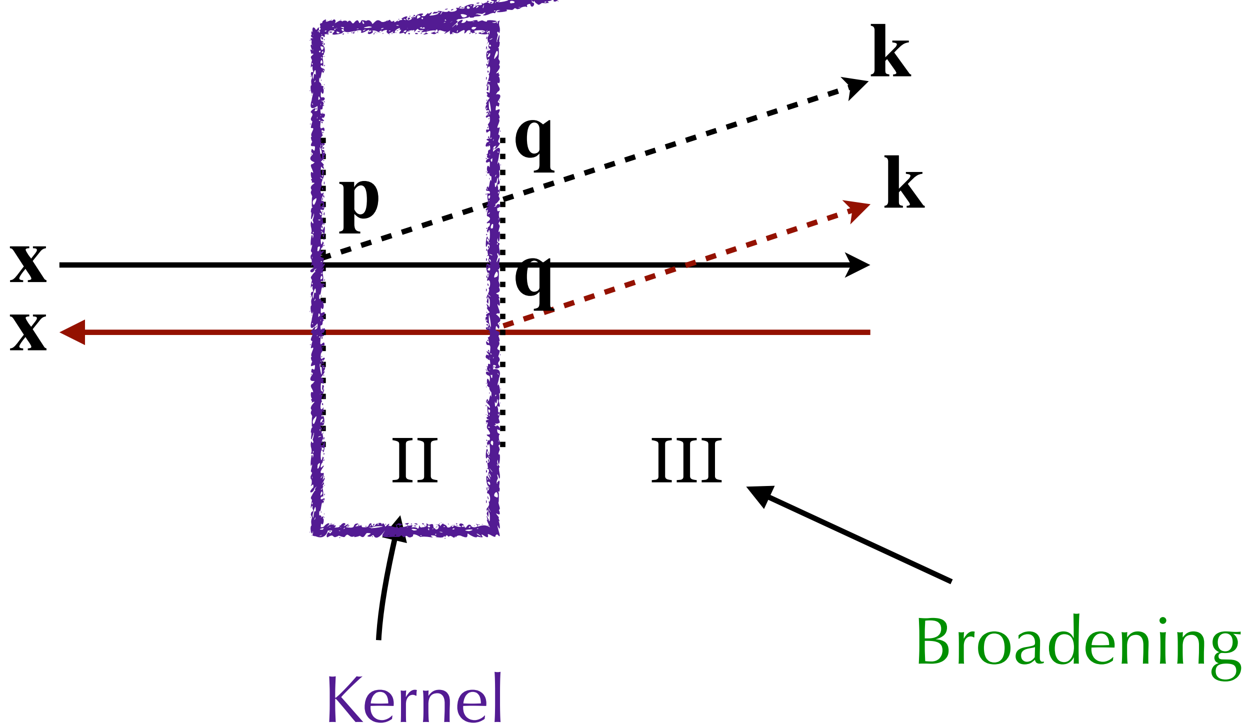
Medium-induced gluon spectrum

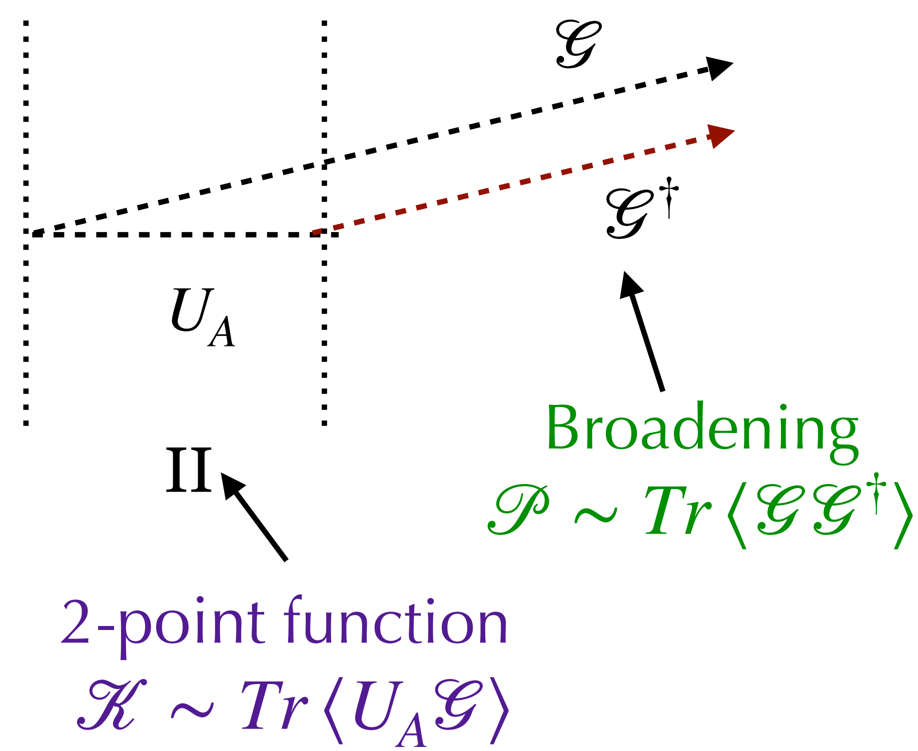


- For a soft emitted gluon ($z \ll 1$)

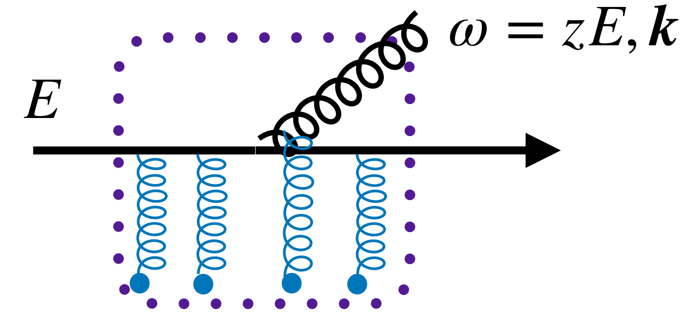
$$\omega \frac{dI}{d\omega\, d^2\boldsymbol{k}} = \frac{2\alpha_s C_R}{(2\pi)^2\omega^2}\,\mathrm{Re}\int_0^\infty dt' \int_0^{t'} dt \int_{\boldsymbol{pq}} \boldsymbol{p}\cdot\boldsymbol{q}\,\widetilde{\mathcal{K}}(t',\boldsymbol{q};t,\boldsymbol{p})\,\mathcal{P}(\infty,\boldsymbol{k};t',\boldsymbol{q})$$

BDMPS-Z





Medium-induced gluon spectrum



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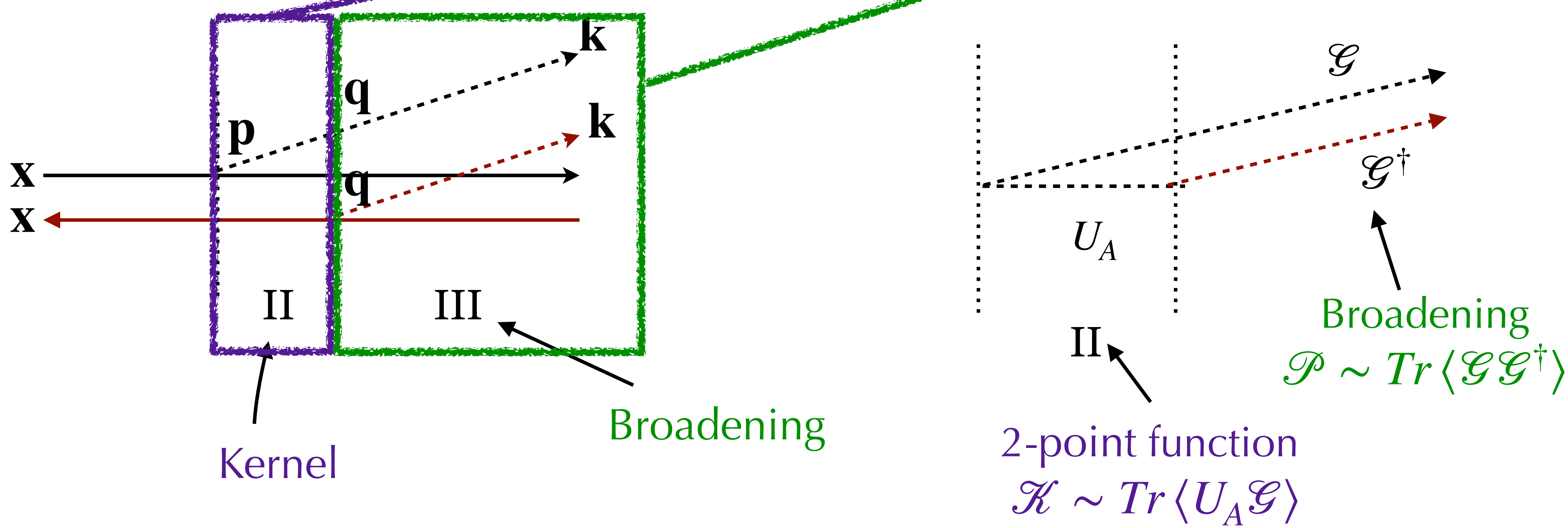
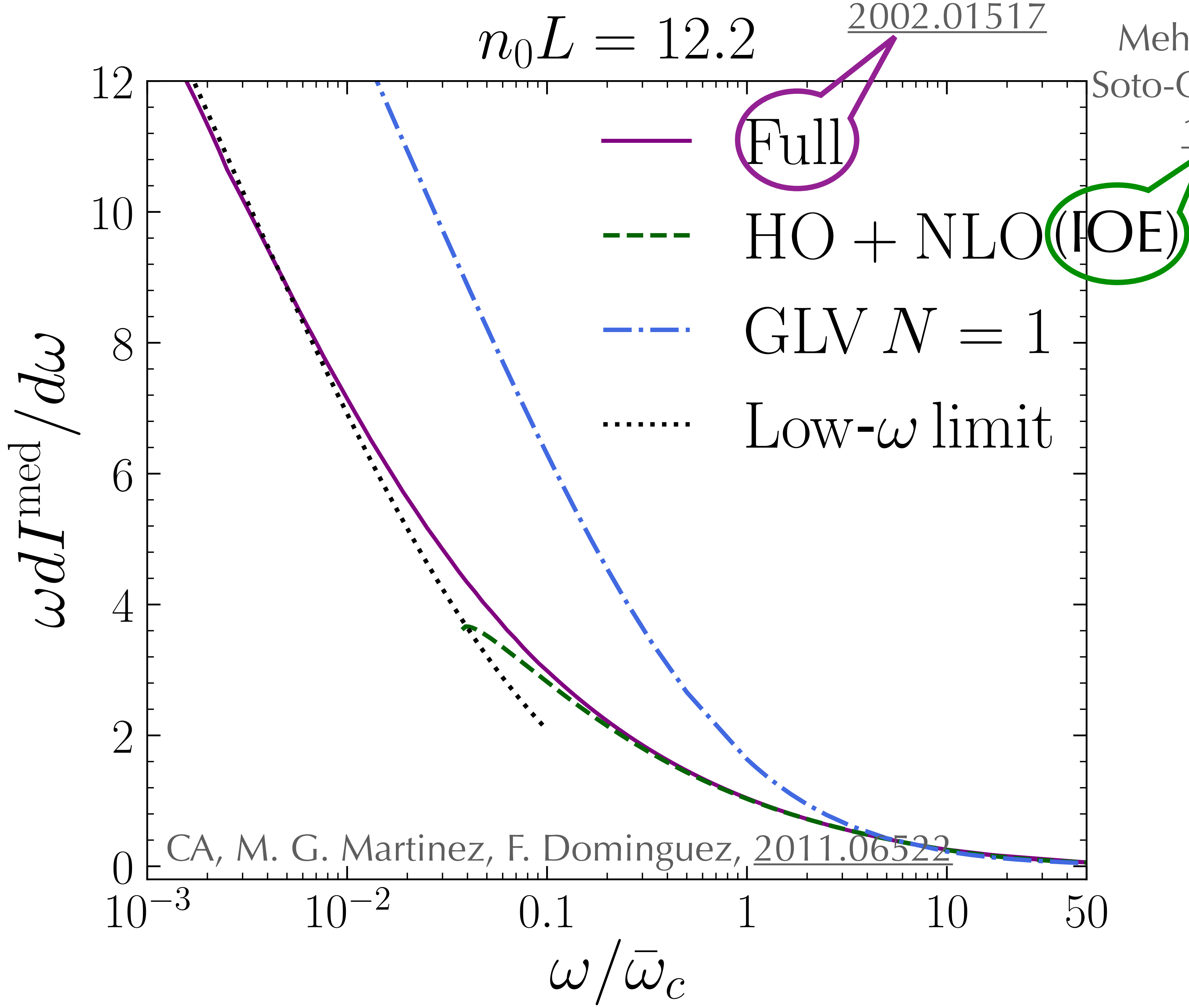
BDMPS-Z



Medium-induced radiation

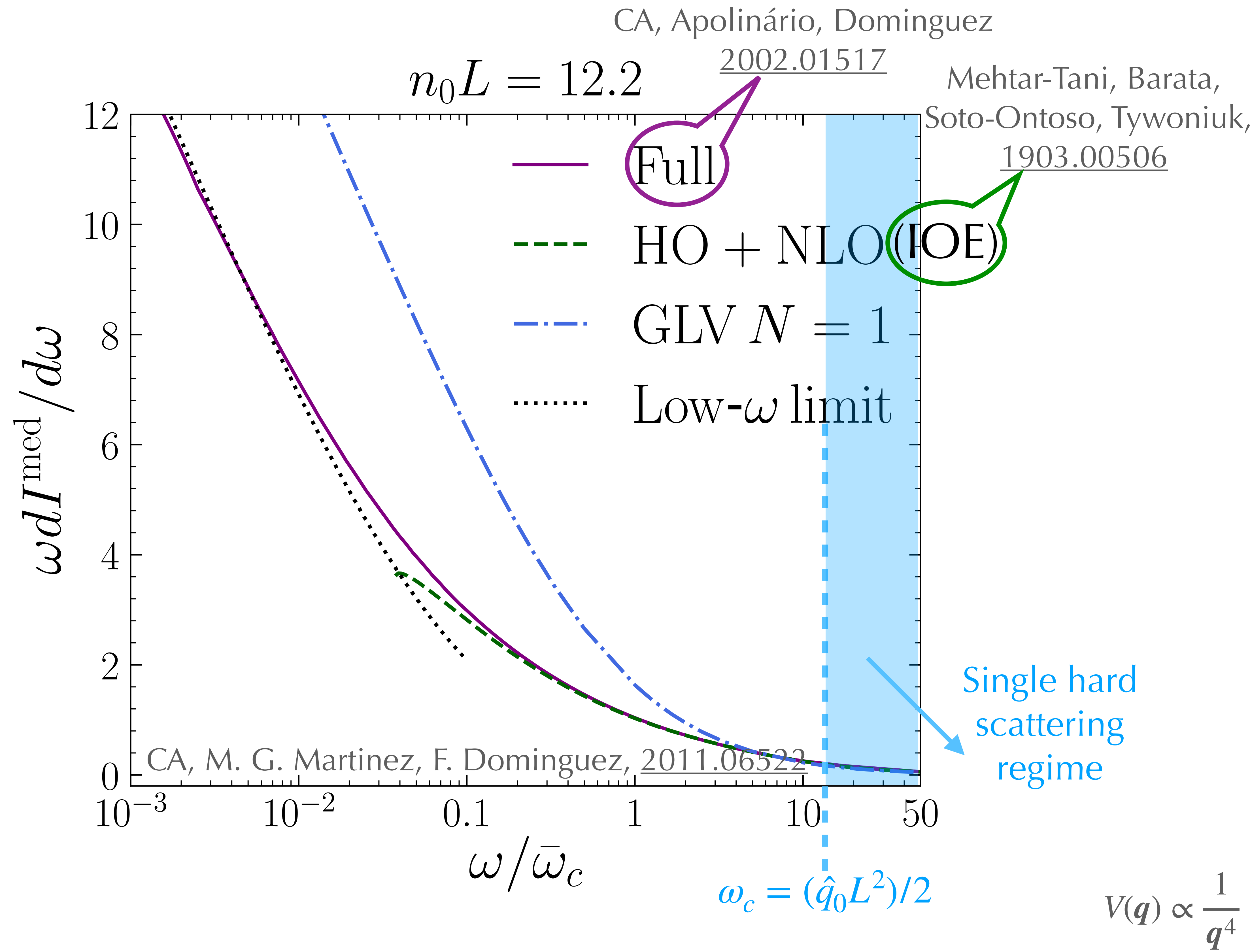
CA, Apolinário, Dominguez
2002.01517

Mehtar-Tani, Barata,
Soto-Ontoso, Tywoniuk,
1903.00506

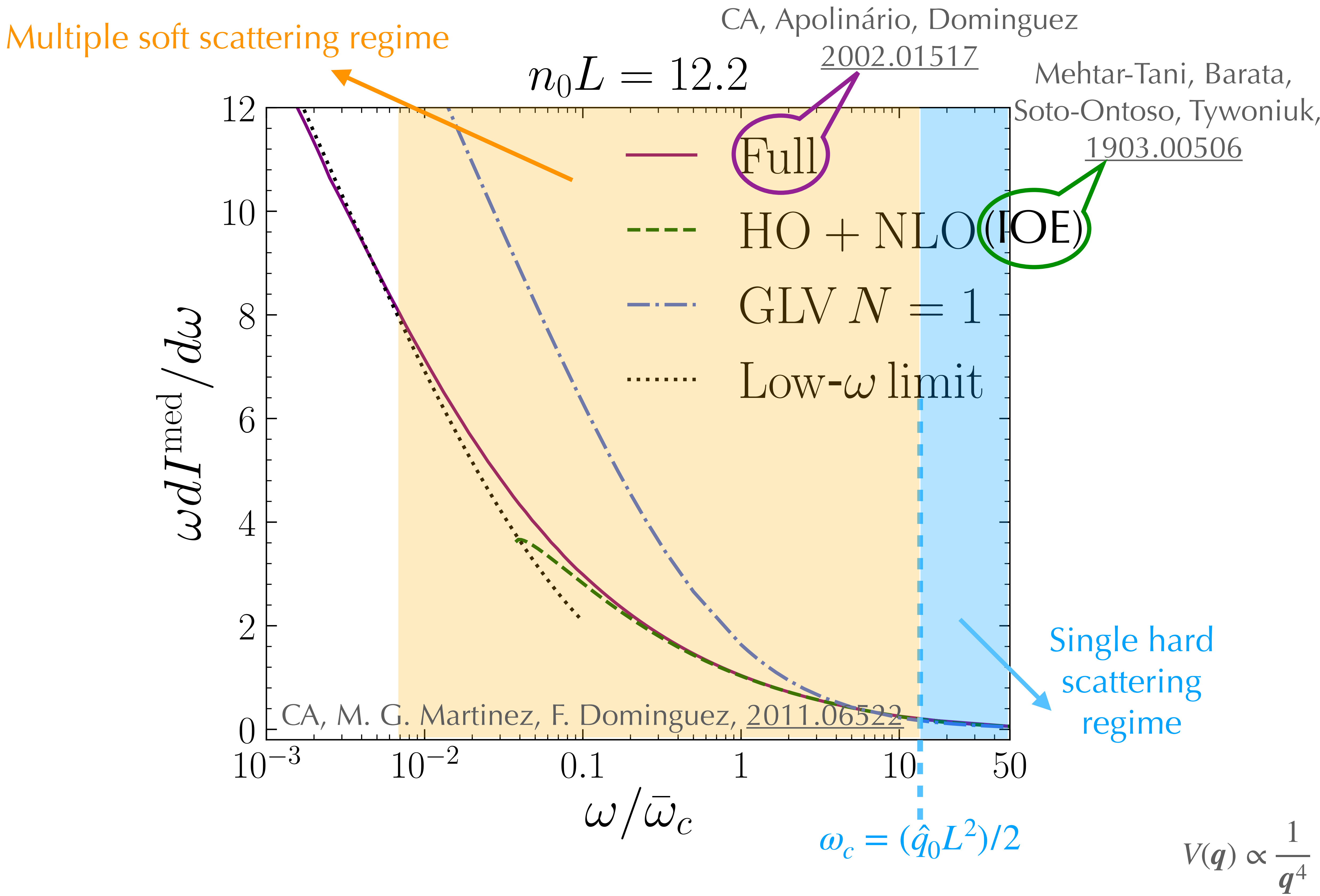


$V(\boldsymbol{q}) \propto \frac{1}{\boldsymbol{q}^4}$

Medium-induced radiation



Medium-induced radiation



Medium-induced radiation

Multiple soft scattering regime

CA, Apolinário, Dominguez
2002.01517

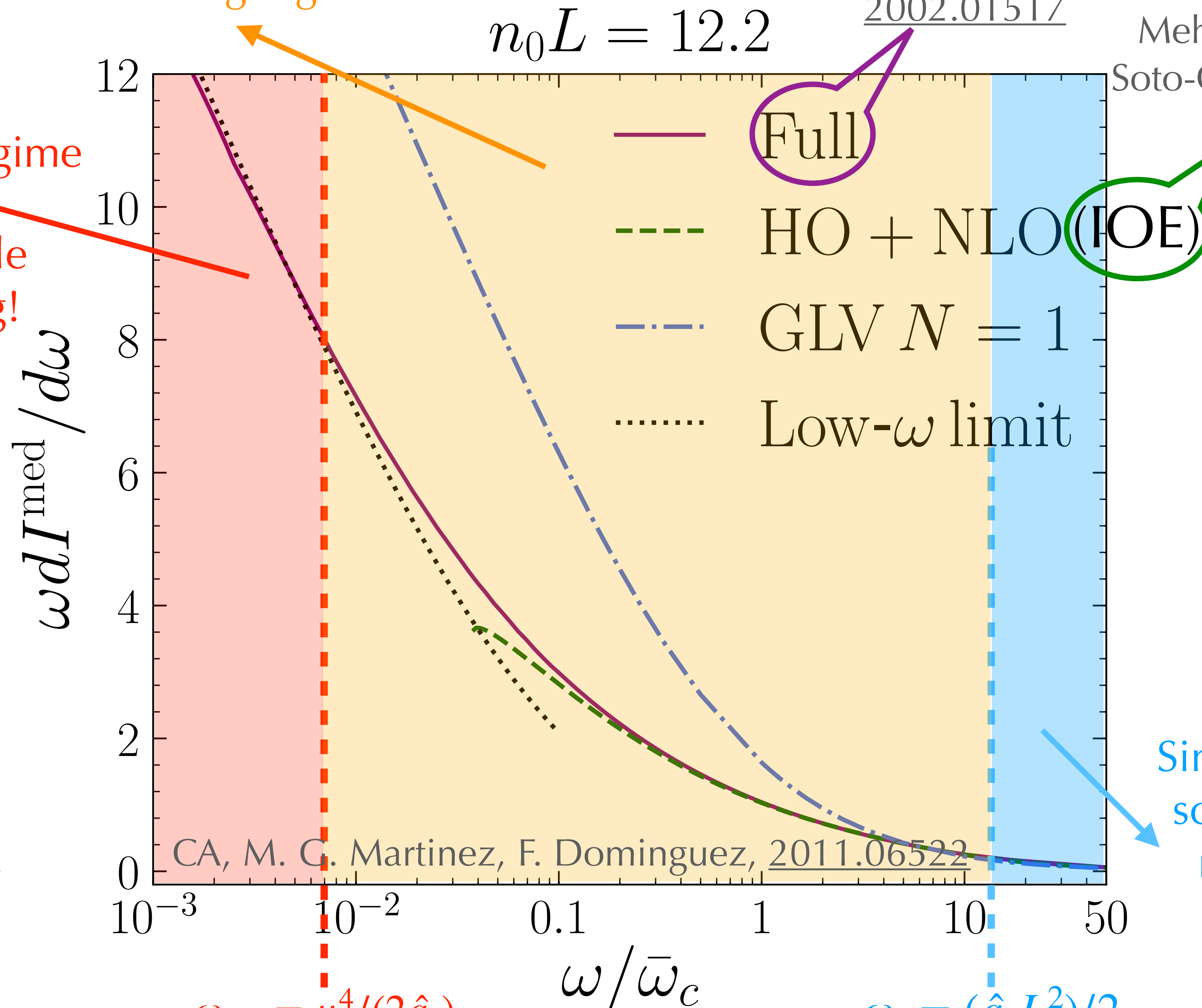
Mehtar-Tani, Barata,
Soto-Ontoso, Tywoniuk,
1903.00506

Bethe-Heitler regime

Not just a single
hard scattering!

Generalized
to the ROE

Isaksen, Takacs
and Tywoniuk
arXiv:2206.0281



Single hard
scattering
regime

$\omega_{\rm BH} = \mu^4/(2\hat{q}_0)$

$\omega_c = (\hat{q}_0 L^2)/2$

$V(\boldsymbol{q}) \propto \frac{1}{\boldsymbol{q}^4}$

Semi-hard approximation

Dominguez, Milhano, Salgado, Tywoniuk, Vila 1907.03653
Isaksen, Tywoniuk 2107.02542

- Use high-energy limit of propagators: vacuum propagator times a Wilson line in the classical trajectory

$$\mathcal{G}_R(t_2,\boldsymbol{p}_2;t_1,\boldsymbol{p}_1;\omega) \to (2\pi)^2\delta^{(2)}(\boldsymbol{p}_2-\boldsymbol{p}_1)\, e^{-i\frac{p_2^2}{2\omega}(t_2-t_1)} V_R(t_2,t_1;[\boldsymbol{n}t])$$

- Calculate averages of Wilson lines in the large-N_c limit (calculations also available for finite N_c). All averages can be expressed in terms of fundamental dipoles and quadrupoles

$$\frac{1}{N_c}\left\langle \mathrm{Tr}\, V_1 V_2^\dagger \right\rangle = S_{12} \qquad \frac{1}{N_c}\left\langle \mathrm{Tr}\, V_1 V_2^\dagger V_{\bar{2}} V_{\bar{1}}^\dagger \right\rangle = Q$$