Naučnom veću Instituta za fiziku

Predmet: Molba za pokretanje postupka za za izbor u zvanje naučni savetnik

Molim Naučno veće Instituta za fiziku u Beogradu da u skladu Pravilnikom o postupku i načinu vrednovanja i kvantitativnom iskazivanju naučno-istraživačkih rezultata istraživača pokrene postupak za moj izbor u zvanje naučni savetnik.
U prilogu dostavljam:

1. Mišljenje rukovodica projekta sa predlogom članova komisije za izbor u zvanje
2. Stručnu biografiju
3. Pregled naučne aktivnosti
4. Elemente za kvalitativnu analizu naučnog doprinosa
5. Elemente za kvantitativnu analizu naučnog doprinosa
6. Spisak objavljenih radova i njihove kopije
7. Potrvde o citiranosti radova (arhive INSPIRE i Google Scholar)
8. Fotokopiju o rešenja o izboru u prethodno zvanje
9. Dodatke

Beograd, 19.04.2018.

dr Branislav Cvetković
viši naučni saradnik Instituta za fiziku

Бранислав Цветковић

Naučnom veću Instituta za fiziku
Univerziteta u Beogradu

Mišljenje rukovodioca projekta o izboru dr Branislava Cvetkovića u zvanje naučni savetnik

Dr Branislav Cvetković je zaposlen na Institutu za fiziku u Beogradu u Grupi za gravitaciju, čestice i polja. Angažovan je na projektu osnovnih istraživanja ON 171031 "Fizičke implikacije modifikovanog prostor-vremena" Ministarstva prosvete, nauke i tehnološkog razvoja Srbije, i radi na temama vezanim za gradijentne teorije gravitacije. Dr Cvetković je jedan od vodećih istraživača na našem projektu, a u periodu od prethodnih pet godina ostvario je veoma značajne rezultate, objavio veliki broj radova u vrhunskim časopisima a doktorant koga vodi je pred odbranom doktorske teze (izveštaj o tezi je na uvidu javnosti na Fizičkom fakultetu). S obzirom da dr Branislav Cvetković ispunjava sve uslove predvidjene Pravilnikom o postupku, načinu vrednovanja i kvantitativnom iskazivanju naučnoistraživačkih rezultata istraživača MPNTR za naučnog savetnika, predlažem Naučnom veću Instituta za fiziku da pokrene postupak za njegov izbor u ovo zvanje.

Za izbor dr Branislava Cvetkovića u zvanje naučni savetnik predlažem sledeću komisiju:

Prof. dr Voja Radovanović, redovni profesor, Fizički fakultet

Dr Milutin Blagojević, naučni savetnik u penziji, Institut za fiziku

Dr Branislav Sazdović, naučni savetnik, Institut za fiziku

Dr Milovan Vasilić, naučni savetnik, Institut za fiziku.

Beograd, 17. april 2018.

Rukovodilac projekta ON171031

Maja Burić

prof. dr Maja Burić

Biografski podaci

Branislav Cvetković je rodjen 14.01.1979. u Beogradu, gde je završio osnovnu školu. Srednju školu, Matematičku gimnaziju, završio je 1998. kao djak generacije. Učestovao je na Medjunarodnim olimpijadama iz matematike (Argentina 1997) i fizike (Island 1998). Iste 1998. godine upisao je Fizički fakultet Univerziteta u Beogradu, smer Teorijska i eksperimentalna fizika, koji je završio 2002. godine sa prosečnom ocenom 10.

Postdiplomske studije na Fiziǩom fakultetu, smer Teorijska fizika elementarnih čestica i gravitacije, upisao je 2002. godine. Magistrirao je 17.03.2005. sa temom ,,Kanonska struktura trodimenzione gravitacije sa torzijom". Mentor magistarske teze je bio Milutin Blagojević.

Od 01.11.2003. radi na Institutu za fiziku kao saradnik projekata ,,Gradijente teorije gravitacije: dinamika i simetrija", ,,Alternativne teorije gravitacije" (od 01.01.2006. do 31.12.2010.) i ,,Fizičke implikacija modifikovanog prostor-vremena" (od 01.01.2011.), koji su finansirani od strane Ministarstva prosvete, nauke i tehnološkog razvoja vlade Republike Srbije.

Doktorsku disertaciju pod naslovom ,,Asimptotska struktura trodimenzione gravitacije sa torzijom" odbranio je 06.03.2008. na Fizičkom fakultetu Univerziteta u Beogradu. Mentor disertacije bio je dr Milutin Blagojević. U oktobru 2008. izabran je u zvanje naučni saradnik, a u septembru 2013. u zvanje viši naučni saradnik.

Od 01.06.2010. do 01.12.2010. boravio je na postdoktorskom usavršavanju, kao stipendista Ministarstva nauke, na Institutu za teorijsku fiziku Tehničkog univerziteta u Beču u grupi dr Danijela Grumilera. Tokom boravka u Beču bavio se holografskom strukturom trodimenzione Čern-Sajmonsove gravitacije.

U tri navrata tokom 2009, 2012. i 2015. godine boravio je u poseti Katoličkom univerzitetu u Valparaisu u okviru saradnje sa dr Oliverom Mišković i dr Rodrigom Oleom, poslednji put kao gostujući profesor.

Aktivan je kao referi za časopise Physical Review Letters, Physical Review D, Classical and Quantum Gravity, Journal of physics A: Mathematical and Theoretical, International Journal of Modern Physics D, European Journal of Physics. Čileanska nacionalna fondacija za nauku FONDECYT angažovala ga je od 2010. kao referija za ocenu projekata.

Do sada je objavio dvadeset devet radova u vodećim medjunarodnim časopisima kategorije M21, jedan rad u medjunarodnom časopisu M22, jedan rad u medjunarodnom časopisu kategorije M23, koji su prema podacima baze INSPIRE citirani 433 puta, od toga 292 bez autocitata sa h-faktorom 12. Prema bazi Google Scholar radovi dr Cvetkovica citirani su 491 put sa h-faktorom 12.

Pod rukovodstvom dr Cvetkovića je u završnoj fazi izrada jedne doktorske disertacije na Fizičkom fakultetu Univerziteta u Beogradu, a bio je i mentor jednog diplomskog master rada na istom fakultetu, kao i komentor prilikom izrade jednog diplomskog rada na Katoličkom Univerzitetu u Valparaisu.

Od školske 2013/2014. godine nastavnik je na doktorskim studijama na Fizičkom fakultetu Univerziteta u Beogradu za užu naučnu oblast Kvantna polja, čestice i gravitacija na predmetu Teorija gravitacije 2.

Od školske 2004/2005. radi kao spoljni saradnik – profesor fizike, u Matematičkoj gimnaziji, gde je tokom školske 2006/2007. obavljao i funkciju pomoćnika direktora, a u periodu od 2013. do 2017. bio je član Školskog odbora. Aktivno je učestvovao u obeležavanju Svetske godine fizike 2005. U periodu od 2003. do 2005. učestvovao je u radu Komisije za takmičenje iz fizike, dok je 2008. bio zamenik lidera na Medjunarodnoj olimpijadi iz fizike održanoj u Vijetnamu.

Od 2014. godine je zamenik predsednika Upravnog odbora Instituta za fiziku.

Autor je više zbirki zadataka za učenike osnovnih i srednjih škola u izdanju Zavoda za udžbenike iz Beograda i Istočnog Sarajeva.

Oženjen je i ima jednu ćerku.

Pregled naučne aktivnosti

Naučno-istraživački rad dr Branislava Cvetkovića odvija se u oblasti teorijske fizike gravitacije, preciznije lokalne Poenkareove teorije. Kandidat je objavio ukupno 31 rad u medjunardonim časopisima sa recenzijom, ot toga 29 kategorije M21 i po 1 rad kategorije M22 i M23.

Za vreme postdiplomskih studija (2002-2005) na Fizičkom fakultetu u Beogradu kandidat se bavio Hamiltonovom analizom Milke-Beklerovog (MB) modela. Magistrirao je na temi ,,Kanonska struktura trodimenzione gravitacije sa torzijom", koja je uradjena pod mentorstvom dr Milutina Blagojevića.

Tokom izrade doktorata (2005-2008) kandidat je nastavio da se bavi MB modelom, gde su u radu:

- M. Blagojević and B. Cvetković, Black hole entropy in 3D gravity with torsion, Class. Quantum Grav. **23** (2006) 4781,

dobijeni veoma značajni rezultati vezani za termodinamičke osobine crnih rupa – pokazano je da torzija utiče na vrednost entropije crne rupe i da je dobijeni rezultat u skladu sa prvim zakonom termodinamike. Doktorsku disertaciju pod naslovom ,,Asimptotska struktura trodimenzione gravitacije sa torzijom" odbranio je 06.03.2008. na Fizičkom fakultetu Univerziteta u Beogradu. Mentor disertacije bio je dr Milutin Blagojević.

Nakon doktorata istraživanje kandidata takodje je vezanom za modele 3D gravitacije. Pronadjena su i ispitane su osobine rešenja sa električnim i magnetnim poljem za 3D gravitaciju kuplovanu sa Maksvelovom i nelinearnom elektrodinamikom. Pokazano je da geometrijske osobine ovih rešenje zavise od vrednosti centalnih naboja u MB modelu. Kandidat je 2009. postigao značajne rezultate u ispitivanju kanonske strukture topološki masivne gravitacije (TMG). U radu

- M. Blagojević and B. Cvetković, Canonical structure of topologically massive gravity with a cosmological constant, JHEP**05**(2009)073,

razrešena je kontroverza, prisutna u tadašnjoj literauri oko broja propagirajućih stepeni slobode u ovoj teoriji i utvrdjeno je da za TMG on iznosi jedan, a ne tri. Ispitana je kanonska struktura Bergšof-Hom-Taundzendove (BHT) gravitacije, dok je u radu:

- M. Blagojević and B. Cvetković, Extra gauge symmetries in BHT gravity, JHEP**03**(2011)139.

pokazano da je fenomen parcijalne bezmasenosti, tj. pojave da za specifičan izbor parametara broj propagirajućih stepeni slobode smanjuje za 1, vezan isključivo za linearnu aproksimaciju, a ne za nelinearnu teoriju koja u posmatranoj tački ne poseduje dodatnu lokalnu simetriju.

Tokom 2010. kandidat je boravio na postdoktorskom usavršavanju, kao stipendista Ministarstva nauke, na Institutu za teorijsku fiziku Tehničkog univerziteta u Beču u grupi dr Danijela Grumilera. Tokom boravka u Beču bavio sam se holografskom strukturom trodimenzione Čern-Sajmonsove gravitacije, koja je veoma detaljno ispitana u radu

- H. Afshar, B.Cvetković, S. Ertl, D. Grumiller and N. Johansson, Conformal Chern-Simons holography, Phys. Rev. D **85**, 064033 (2012).

Nakon prethodnog izbora u zvanje u periodu (2013-2018) istraživanje kandidata se odvijalo u okviru nekoliko tema.

Trodimenziona teorija gravitacije sa propagirajućom torzijom. Tokom proučavanja MB modela od početka devedesetih godina prošlog veka postignuti su mnogobrojni značajni rezultati. Medjutim, MB model je topološki, tj. ne poseduje propagirajuće stepene slobode. Zbog toga model sa propagirajućom torzijom predstavlja ,,realističniju" teoriju gravitacije.

Kandidat je rad na opštoj teoriji gravitacije sa propagijaćom torzijom koja ne narušava parnost započeo 2012. godine, kada je ispitan čestični spektar teorije oko prostora M_3, kao i formulacije prvog reda koja je veoma pogodna za konstrukciju generatora lokalne simetrije i ispitivanje kanonske strukture u AdS sektoru.

U saradnji sa kolegama iz Čilea ispitani su osnovni aspekti AdS/CFT korespondencije za 3D gravitaciju sa torzijom. Izabran je konziztentan holografski anzac, formulisan je poboljšani pristup Neter-Vordovim identitetima za teoriju na granici. I za MB model i za model sa propagirajućom torzjom dobijene su konačne struje spina i energije impulsa i izračunate anomalije.

Zatim je pažnja posvećena Hamiltovoj strukturi skalarnog sektora teorije. Stabilnost Hamiltonove strukture u odnosu na linearizaciju je iskorišćena za identifikaciju dinamički prihvatljivog skupa parametara u dejstvu.

Konstruisana su i talasna rešenja 3D gravitacije sa propagirajućom torzijom, najpre uopšteni pp-talasi, a zatim i generalisani Sikloševi talasi. Pokazano je da Oliva-Tempo-Troncozo crna rupa, konformno ravno rešenje BHT gravitacije, predstavlja rešenje Lokalne Poeankareove teorije u 3D za specifičan izbor parametara. Korišćenjem pouzdanog kanonskog pristupa izračunati su održani naboji za ovo rešenje, koji zadovoljavaju prvi zakon termodinamike, čime je pokazano da je Abot-Dezer-Tekinov pristup neadekvatan za izračunavanje naboja ovog rešenja. Konstruisana je Vaidija ekstenzija ovog rešenja, čija posebna podklasa poseduje asimptotsku konformnu simetriju.

Od prethodnog izbora u zvanje iz ove tematike objavljeni su sledeći radovi:

- M. Blagojević, B. Cvetković , O. Misković and R. Olea, Holography in 3D AdS gravity with torsion, JHEP**1305**(2013)103.

- M. Blagojević, B. Cvetković, M. Vasilić, Exotic black holes with torsion, Phys.Rev. D **88**, 101501 (2013).

- M. Blagojević and B. Cvetković, Three-dimensional gravity with propagating torsion: Hamiltonian structure of the scalar sector, Phys.Rev. D**88**, 104032 (2013).

- M. Blagojević and B. Cvetković, Gravitational waves with torsion in 3D, Phys. Rev. D **90**, 044006 (2014).

- M. Blagojević and B. Cvetković, Siklos waves with torsion in 3D, JHEP**11**(2014)141.

- M. Blagojević and B. Cvetković, Vaidya-like exact solutions with torsion, JHEP**05**(2015)101.

- M. Blagojević and B. Cvetković, Conformally flat black holes in Poincaré gauge theory, Phys. Rev D **93**, 044018 (2016).

Talasna rešenja u Lokalnoj Poenkareovoj teoriji. Druga istraživačka tema kojom se kandidat bavio je nalaženje i ispitivanje osobina gravitacionih talasa, koji su posebno dobili na aktuelnosti nakon nedavne eksperimentalne potvrde njihovog postojanja. Kandidat se bavio talasnim rešenjima sa torzijom u 4D lokalnoj Poenkareovoj teoriji.

Iako je u literaturi bila poznata talasna rešenja sa torzijom, nedostajao je sistematski pristup konstruisanju anzaca na nivou osnovnih dinamičkih varijabli lokalne Poenkareove teorije, tetrada i Lorencove koneksije. Pokazano je da se upravo polazeći od anzaca u kome je koneksija generalisana u odnosu na Rimanovu teoriju, ali ipak zadržava njene glavne osobine mogu konstruisati uopšteni Siklоševi talasi sa torzijom, kao i uopšteni pp-talasi sa torzijom. Ispitane su i osobine posebnih rešenja kao što su uopšteno Kaigorovljevo rešenje, eksponencijalno i homogeno rešenje, i pokazano je da u slučaju jedne familije uopštenih pp-talasa torzija dinamički odredjuje oblik metrike.

U poslednjem objavljenom radu iz ove oblasti u dejstvo su uključeni i članovi kvadratični po torziji i krivini koji narušavaju parnost. Ovaj rad predstavlja uvod u izučavanje opšte lokalne Poenkareove teorije koja ima veoma zanimljivu kanonsku strukturu i čestični spektar oko ravnog prostora, što je potrvrdjeno u radu koji je poslat u štampu.

Iz ove tematike objavljeni su sledeći radovi:

- M. Blagojević and B. Cvetković, Siklos waves in Poincaré gauge theory, Phys. Rev. D **92**, 024047 (2015).
- M. Blagojević and B. Cvetković, Generalized *pp* waves in Poincar gauge theory, Phys. Rev D **95**, 104018 (2017).
- M. Blagojević, B. Cvetković and Y. N. Obukhov, Generalized plane waves in Poincaré gauge theory of gravity, Phys. Rev. D **96**, 064031 (2017).

Lavlokova teorija gravitacije sa torzijom. Lavlokova teorija gravitacije predstavlja minimalističko uopstenje OTR i jedna je od alternativnih teorija gravitacije koja je predmet aktivnih istraživanja još od ranih sedamdesetih godina. Iako je u literaturi povremeno posvećivana pažnja ispitivanju Lavlokove gravitacije sa torzijom ta oblast je još uvek nedovoljno istražena, jer je nalaženje rešenja sa torzijom tehnički veoma komplikovano, budući da se ispostavlja da su jednačine kretanja neretko ,,preodredjene".

Doprinos kandidata u ovoj oblasti ogleda se u konstrukciji novih rešenja sa torzijom: BTZ crnog prstena i sferno-simetrične crne rupe sa torzijom. Identifikovani su sektori teorije u kojima postoje ova rešenja, koja su nadjena u slučaju petodimenzione teorije i ispitane su njihove geometrijske karakteristike i termodinamičke osobine.

Poseban sektor Lavlokove gravitacije predstavlja Lavlokova Čern-Sajmonsova gravitacija koja je posebno pogodna za proučavanje sa stanovišta AdS/CFT korespondencije, budući da Feferman-Grahamov razvoj osnovnih dinamičkih varijablo sadrži konačan broj članova. Nadjene su asimptotske simetrije u AdS sektoru i pokazano je da se one sastoje od lokalnih translacija, lokalnih Lorencovih rotacija, dilatacija i ne-Abelovih lokalnih tranformacija. Izračunate su 1-tačkaste funkcije: struje energije-impulsa i spina u dualnoj koformnoj teoriji polja i zapisani su odgovarajući Vordovi identiteti. Pokazano je da holografska teorija poseduje Vajlovu anomaliju, kao i da je ne-Abelova lokalna simetrija narušena na kvantnom nivou.

Objavljeni radovi iz ove oblasti su

- B. Cvetković and D. Simić, 5D Lovelock gravity: New exact solutions with torsion, Phys. Rev. D **94**, 084037 (2016).
- B. Cvektović, O. Miskovic and B. Cvetković, Holography in Lovelock Chern-Simons AdS gravity, Phys. Rev. D **96**, 044027 (2017).
- B. Cvetković and D. Simić, A black hole with torsion in 5D Lovelock gravity, Class. Quantum Grav. **35** (2018) 055005 (13pp).

Elementi za kvalitativnu ocenu rada kandidata

1 Kvalitet naučnih rezulata

1.1 Naučni nivo i značaj rezultata, uticaj naučnih radova

Dr Branislav Cvetković je tokom naučne karijere objavio ukupno 31 rad u medjunarodnim časopisima sa recenzijom, od čega 29 kategorije M21, 1 kategorije M22 i 1 kategorije M23. Ukupan impakt faktor radova je 139.23. Od odluke Naučnog veća o predlogu za sticanje zvanja viši naučni saradnik dr Cvetkovic je objavio 13 radova kategorije M21. Ukupan impakt faktor ovih radova je 63.19. Kvalitet naučnih radova dr Cvetkovića se može proceniti, izmedju ostalog, prema kvalitetu časopisa u kojima su objavljeni: dr Cvetković je do sada objavio 8 radova u časopisu Journal of High Energy Physics (IF=6.22), jednom od najuglednjih časopisa iz oblasti fizike visokih energija, kao i 14 radova u časopisu Physical Review D (IF=4.57) koji je jedan od najznačajnijih časopisa za fiziku gravitacije, čestica i polja. Dva rada doktora Cvetkovića u časopisu Phys. Rev. D objavljeni su kao "rapid communication".

Najznačajniji radovi dr Cvetkovića u poslednjih nekoliko godina su

[1] H. Afshar, B.Cvetković, S. Ertl, D. Grumiller and N. Johansson, Conformal Chern-Simons holography, Phys. Rev. D **85**, 064033 (2012), IF=4.691, citiran 49 puta

[2] M. Blagojević, B. Cvetković , O. Misković and R. Olea, Holography in 3D AdS gravity with torsion, JHEP**1305**(2013)103, IF=6.220, citiran 12 puta

[3] M. Blagojević and B. Cvetković, Siklos waves with torsion in 3D, JHEP**11**(2014)141, IF=6.111, citiran 7 puta

[4] M. Blagojević and B. Cvetković, Vaidya-like exact solutions with torsion, JHEP**05**(2015)101, IF=6.023, citiran 6 puta

[5] M. Blagojević, B. Cvetković and Y. N. Obukhov, Generalized plane waves in Poincaré gauge theory of gravity, Phys. Rev. D **96**, 064031 (2017), IF=4.557, citiran 1 put

U radu [1] je veoma detaljno ispitana je holografska struktura trodimenzione Čern-Sajmonsove gravitacije za skup različitih asimptotskih uslova. U radu [2] u saradnji sa kolegama iz Čilea ispitani su osnovni aspekti AdS/CFT korespondencije za 3D gravitaciju sa torzijom. Izabran je konzistentan holografski anzac, formulisan je poboljšani pristup Neter-Vordovim identitetima za teoriju na granici o dobijene su konačne struje spina i energije impulsa i izračunate anomalije. Talasna rešenja 3D gravitacije sa propagirajućom torzijom, generalisani Siklośevi talasi konstruisani su u radu [3]. U radu [4] je pokazano je da Oliva-Tempo-Troncozo crna rupa, konformno ravno rešenje BHT gravitacije, predstavlja rešenje

Lokalne Poeankareove teorije u 3D za specifičan izbor parametara. Konstruisana je Vaidija ekstenzija ovog rešenja, čija posebna podklasa poseduje asimptotsku konformnu simetriju. U radu [5] pokazano je da se polazeći od anzaca u kome je koneksija generalisana u odnosu na Rimanovu teoriju, ali ipak zadržava njene glavne osobine mogu konstruisati pp-talasi sa torzijom. U dejstvo su uključeni i članovi kvadratični po torziji i krivini koji narušavaju parnost. Ovaj rad predstavlja uvod u izučavanje opšte lokalne Poenkareove teorije koja ima veoma zanimljivu kanonsku strukturu i čestični spektar oko ravnog prostora.

1.2 Pozitivna citiranost radova kandidata

Prema podacima baze inSPIRE na dan 18.04.2018. radovi doktora Cvetkovića citirani su ukupno 435 puta, 292 puta bez autocitata, sa h-faktorom 12. Prema podacima baze Google Scholar radovi su citirani ukupno 494 puta (videti prilog o citiranosti). Najveći broj citata imaju rasovi objavjeni u časopisima Journal of high energy physics i Physical Review D. Rad M. Blagojević and B. Cvetković, Canonical structure of topologically massive gravity with a cosmological constant, JHEP**05**(2009)073, citiran je ukupno 55 puta bez autocitata.

1.3 Parametri kvaliteta časopisa

Dr Branislav Cvetković je tokom karijere objavio ukupno 31 rad u časopisima sa ISI liste od toga 29 kategorije M21, 1 kategorije M22 i 1 kategorije M23. Ukupan impalt faktor radova je ukupan impakt faktor radova je 139.23. Od odluke Naučnog veća o predlogu za sticanje zvanja viši naučni saradnik dr Cvetkovic je objavio 13 radova kategorije M21. Ukupan impakt faktor ovih radova je 63.19.

Zbirno prikazano dr Cvetković je objavio:

- 8 radova u Journal of High Energy Physics, (srednji IF=5.931)
- 16 radova u Physical Review D (srednji IF=4.728)
- 5 radova u Classical and Quantum Gravity (srednji IF=2.981)
- 1 rad u Modern Physics Letters A (srednji IF=1.418)

Nakon odluke Naučnog veća o predlogu za sticanje zvanja viši naučni saradnik dr Cvetkovic je objavio:

- 3 rada u Journal of High Energy Physics (srednji IF=6.118)
- 9 radova u Physical Review D (srednji IF=4.635)
- 1 rad u Classical and Quantum Gravity (srednji IF=3.119)

1.4 Stepen samostalnosti i stepen učešća u realizaciji radova u naučnim centrima u zemlji i inostranstvu

Od izbora u prethodno zvanje dr Cvetković je pokrenuo pravce istraživanja koji se nisu ranije izučavali u Srbiji. Primena aspekata AdS/CFT korenspondencije na teorije gravitacije sa torzijom, izučava se u bliskoj saradnji sa kolegama iz Čilea (Katolički univerzitet u Valparaisu i Univerzitet Andreas Beljo u Santjagu). Dr Cvetković je deo veliki doprinos u formulisanju holografskog anzaca u okviru lokalne Poenkareove teorije, uopštavanju Neter-Vordovih identiteta na teorije sa torzijom i razumevanje Rimanovog limita za zakone održanja.

Drugi pravac istraživanja odnosi se na nalaženje talasnih rešenja u okviru teorija gravitacije sa torzijom. Doprinos dr Cvetkovića ogledao se u nalaženju sistematskog pristupa za nalaženje odgovarajućeg anzaca za Lorencovu koneksiju, koji dovodi do identifikacije sektora teorije u kome egzistiraju talasna rešenja, esplicitnoj konstrukciji i analizi osobina dobijenih partikularnih rezšenja.

Treći pravac istraživanja je Lavlokova gravitacija sa torzijom, koja predstavlja i temu doktorske disertacije Dejana Simića.

2 Angažovanost u razvoju uslova za naučni rad, obrazovanju i formiranju nauňih kadrova

Pod mentorstvom dr Branislava Cvetkovića radi se jedna doktorska disertacija na Fizičkom fakultetu Univerziteta u Beogradu. Doktorska teza Dejan Simića pod naslovom ,,Lavlokova gravitacija sa torzijom: egzaktna rešenja, kanonska i holografska struktura" sa pratećim izveštajem za pregled i ocenu disertacije koji je sačinila komisija u sastavu prof. dr Maja Burić, prof. dr Voja Radovanović i dr Branislav Cvetković se nalazi se na uvidu javnosti na Fizičkom fakultetu u Beogradu. Očekuje se da će teza biti odbranjena početkom leta ove godine.

Pod mentorstvom dr Branislava Cvetkovića na Fizičkom fakultetu Univerziteta u Beogradu odbranjen je i jedan master rad studentkinje Marije Tomašević pod naslovom ,,Kretanje četica u polju OTT crne rupe". Dr Branislav Cvetković bio je i komentor diplomskog rada Constance Belen Calender Olivares pod naslovom "Chemistry of three-dimensional black holes in AdS space" koji je odbranjen na Katoličkom Univerzitetu u Valparaisu. Trenutno je u toku izdrada još jednog master rada na Fizikom fakultetu.

Dr Branislav Cvetković je angažovan na doktorskim studijama na Fizčkom fakultetu u uokviru uže naučne oblasti Kvantna polja, čestice i gravitacija kao nastavnik na predmetu Teorija gravitacije 2.

Od 2004. radi kao spoljni saradnik – profesor fizike u Matematičkoj gimnaziji. Njegovi učenici postižu zapažene rezultate na državnim i medjunarodnim takmičenjima iz fizike.

3 Normiranje broja koautorskih radova, patenata i tehničkih rešenja

Radovi dr Cvetkocića su teorijski i najveći broj ima samo dva autora. Medju radovima dr Cvetkovića objavljenim u periodu nakon odluke Naučnog veća o predlogu za sticanje zvanja viši nauňi saradnik jedan rad ima četiri, jedan rad tri dok svi ostali radovi imaju samo dva autora. Ukupan broj normiranih M bodova je 106.5, odnosno 102.5 nakon normiranja.

4 Rukovodjenje projektima, potprojektima i projektnim zadacima

Dr Branislav Cvetković rukovodi potprojektom ,,Gradijente teorije gravitacije" u okviru projekta ON171031 ,,Fizičke implikacije modifikovanog prostor-vremena" koji je finansiran od strane Ministarstva prosvete, nauke i thenološkog razvoja Republike Srbije. U periodu od 2008. do 2010. rukovodio je potprojektom ,,Torzija i nemetričnost u gravitaciji i teoriji struna/brana" u okviru projekta 141036 ,,Alternativne teorije gravitacije", koji je bio finansiran od strane Ministarstva nauke vlade Republike Srbije.

Od 2015. Branislav Cvetkovć je zamenik člana uprave (MC substitute) COST akcije "Quantum Structure of Spacetime" kojim rukovodi prof. Ričard Sabo.

Kada je 2016. godine raspisan konkurs za nove projekte Ministarstva prosvete nauke i thenološkog razvoja dr Branislav Cvetković je bio prijavljen kao rukovodilac projekta ,,Kvantno prostorvreme".

5 Aktivnost u naučno stručnim društvima

Dr Branislav Cvetković je recenzent za vodeće medjunarodne časopise iz fizike kao što su: Physical Review Letters, Physical Review D, Classical and Quantum Gravity, Journal of physics A: Mathematical and Theoretical, International Journal of Modern Physics D, European Journal of Physics. Čileanska nacionalna fondacija za nauku FONDECYT angažovala ga je od 2010. kao referija za ocenu projekata.

Dr Branislav Cvetković je od 2012. do 2014. bio član Odeljenja za nauku i visoko obrazovanje Društva fizičara Srbije (NIVO DFS). Od 2003. do 2004. bio je član Komisije za takmičenja učenika srednih škola. Bio je zamenik lidera na Medjunarodnoj olimpijadi iz fizike održanoj u Vijetnamu 2008. Aktivno je učestvovao u obeležavanju Svetske godine fizike 2005, kao koordinator takmičenja ,,Otkrivamo talente za fiziku".

Dr Branislav Cvetković je bio član Organizacionog komiteta više medjunarodnih konferencija, kao što su 2018 Workshop on Gravity, Holography, Strings and Noncommutative Geometry (Beograd 2018), Gravity: new ideas for unsovled problems (Divčibare 2011), Gravity: new ideas for unsovled problems II (Divčibare 2013), 5th MATHEMATICAL PHYSICS MEETING: Summer School and Conference on Modern Mathematical Physics (Beograd 2010).

6 Uticajnost naučnih rezultata

Uticajnost naučnih rezultata kandidata ogleda se u broju citata koji su navedeni u tački 1 ovog priloga, kao i priloga o citiranosti. Značaj rezultata kandidata je takodje opisan u tački 1.

7 Konkretan doprinos kandidata u realizaciji radova u centrima u zemlji i inostranstvu

Dr Cvetković je pokrenuo pravce istraživanja koji se nisu ranije izučavali u Srbiji. Primena aspekata AdS/CFT korenspondencije na teorije gravitacije sa torzijom, izučava se u bliskoj saradnji sa kolegama iz Čilea (Katolički univerzitet u Valparaisu i Univerzitet Andreas Beljo u Santjagu). Dr Cvetković je deo veliki doprinos u formulisanju holografskog anzaca u okviru lokalne Poenkareove teorije, uopštavanju Neter-Vordovih identiteta na teorije sa torzijom i razumevanje Rimanovog limita za zakone održanja.

Drugi pravac istraživanja odnosi se na nalaženje talasnih rešenja u okviru teorija gravitacije sa torzijom. Doprinos dr Cvetkovića ogledao se u nalaženju sistematskog pristupa za nalaženje odgovarajućeg anzaca za Lorencovu koneksiju, koji dovodi do identifikacije sektora teorije u kome egzistiraju talasna rešenja, esplicitnoj konstrukciji i analizi osobina dobijenih partikularnih rezšenja.

Treći pravac istraživanja je Lavlokova gravitacija sa torzijom, koja predstavlja i temu doktorske disertacije Dejana Simića.

Dr Cvetkovi je učestvovao i svim segmentima izrade svih radova od prethodnog izbora u zvanje od definisanja teme, analitičkog računa, provere rezultata korišćenjem softerskih paketa Mathematica i Reduce pa do procesa objavljivanja kroz komunikaciju sa recenzentima i editorima časopisa budući da je gotovo kod svih radova on "corresponding autor"

8 Uvodna predavanja na konferencijama i druga predavanja

Nakon prethodnog izbora u zvanje dr Cvetković je održao sledeća predavanja po pozivu:

1. *Generalized plane waves in Poincaré gauge theory of gravity*, 9th Mathematical Physics Meeting: School and Conference on Modern Mathematical Physics (MPHYS9), 18-23 septembar 2018, Beograd.

2. *Siklos waves in gravity with torsion*, Universtity Andreas Bello, Santiago, Chile 27.10.2015.

3. *General Relativity - Introduction, Overview and Perspectives*, GR100 Centennial of General Relativity, Beograd 23.06.2015.

4. *3D gravity with propagating torsion*, Savremena matematička fizika i njene primene, Banja Luka 13.09.2014.

5. *Holografija u 3D gravitaciji sa torzijom*, Gravity: new ideas for unsolved problems II, Divčibare 19-22. septembar 2013.

Do prethodnog izbora u zvanje dr Cvetković je odrzao sledeća predavanja po pozivu:

1. *Extra gauge symmetries in BHT gravity*, Gravity: new ideas for unsolved problems, Divčibare 12-14. septembar 2011.

2. *Conserved charges in 3D gravity*, Technical University Vienna, 08.06.2010.

3. *Canonical structure of topologically massive gravity with a cosmological constant*, Universidad Catholica de Valparaiso, Chile, 15.05.2009.

Elementi za kvantitativnu ocenu rada kandidata

Dr Branislav Cvetković je tokom naučne karijere objavio ukupno 31 rad u medjunarodnim časopisima sa recenzijom, od čega 29 kategorije M21, 1 kategorije M22 i 1 kategorije M23. Ukupan impakt faktor radova je 139.23. Od odluke Naučnog veća o predlogu za sticanje zvanja viši naučni saradnik dr Cvetkovic je objavio 13 radova kategorije M21. Ukupan impakt faktor ovih radova je 63.19.

Prema podacima baze inSPIRE na dan 18.04.2018. radovi doktora Cvetkovića citirani su ukupno 435 puta, 292 puta bez autocitata, sa h-faktorom 12. Prema podacima baze Google Scholar radovi su citirani ukupno 494 puta (videti prilog o citiranosti). Najveći broj citata imaju rasovi objavjeni u časopisima Journal of high energy physics i Physical Review D. Rad M. Blagojević and B. Cvetković, Canonical structure of topologically massive gravity with a cosmological constant, JHEP**05**(2009)073, citiran je ukupno 55 puta bez autocitata.

Ostvareni rezultati u periodu nakon odluke Naučnog veća o predlogu za sticanje zvanja viši naučni saradnik sumirani su u tabeli 1:

Tabela 1

Kategorija	M bodova po radu	Broj radova	Ukupno M bodova	Normiranih M bodova
M21	8	13	104	100
M33	1	2	2	2
M34	0.5	1	0.5	0.5

Poredjenje sa minimalnim kvantitatvnim uslovima za izbor u zvanje naučni savetnik dato je u tabeli 2:

Tabela 2

Minimalan broj M bodova		Ostvareni rezultati	Ostvareni normirani rezultati
Ukupno	70	106.5	102.5
M10+M20+M31+M32+M33+M41+M42+M90	50	106	102
M11+M12+M21+M22+M23	35	104	100

Spisak radova dr Branislava Cvetkovića

Radovi u vrhunskim medjunarodnim časopisima M21

♣ **Do prethodnog izbora u zvanje**:

- M. Blagojević and B. Cvetković, Black hole entropy in 3D gravity with torsion, Class. Quantum Grav. **23** (2006) 4781.
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- M. Blagojević, B. Cvetković and Y. N. Obukhov, Generalized plane waves in Poincaré gauge theory of gravity, Phys. Rev. D **96**, 064031 (2017).
- B. Cvetković and D. Simić, A black hole with torsion in 5D Lovelock gravity, Class. Quantum Grav. **35** (2018) 055005 (13pp).

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♣ **Do prethodnog izbora u zvanje**:

- B. Cvetković and M. Blagojević , Stability of 3D black hole with torsion, Mod. Phys. Lett. A, Vol. **22**, No. 40 (2007) 3047-3055.

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♣ **Do prethodnog izbora u zvanje**:

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- M. Blagojević and B. Cvetković, Generalized plane waves in Poincaré gauge theory of gravity, *Book of abstracts – 9th MATHEMATICAL PHYSICS MEETING: School and Conference on Modern Mathematical Physics*, 22.

M63 - 1 rad

♣ **Do prethodnog izbora u zvanje**:

- B. Cvetković and M. Blagojević, Supersymmetric 3D black hole with torsion, J. Res. Phys, Vol.**31**, No 2 (2007) 102-105.

Objavljeni radovi

Published for SISSA by Springer
Received: *March 1, 2015*
Accepted: *April 16, 2015*
Published: *May 20, 2015*

JHEP05(2015)101

Vaidya-like exact solutions with torsion

M. Blagojević and B. Cvetković

Institute of Physics, University of Belgrade,
P.O. Box 57, Belgrade, 11001 Serbia

E-mail: mb@ipb.ac.rs, cbranislav@ipb.ac.rs

Abstract: Starting from the Oliva-Tempo-Troncoso black hole, a solution of the Bergshoeff-Hohm-Townsend massive gravity, a class of the Vaidya-like exact vacuum solutions with torsion is constructed in the three-dimensional Poincaré gauge theory. A particular subclass of these solutions is shown to possess the asymptotic conformal symmetry. The related canonical energy contains a contribution stemming from torsion.

Keywords: Classical Theories of Gravity, Space-Time Symmetries

ArXiv ePrint: 1502.07105

doi:10.1007/JHEP05(2015)101

Contents

1 Introduction

Poincaré gauge theory (PGT) is a modern field-theoretic approach to gravity, proposed in the early 1960s by Kibble and Sciama [1, 2]. Compared to Einstein's general relativity (GR), PGT is based on using both the torsion and the curvature to describe the underlying Riemann-Cartan (RC) geometry of spacetime [3–6]. Investigations of PGT in three-dimensional (3D) spacetime are expected to improve our understanding of both the geometric and dynamical role of torsion in a realistic, four-dimensional gravitational theory. Systematic studies of 3D PGT started with the Mielke-Baekler model [7], introduced in the 1990s as a PGT extension of GR. However, this model is, just like GR, a topological theory without propagating degrees of freedom. In PGT, such an unrealistic dynamical feature can be quite naturally improved by going over to Lagrangians that are quadratic in the field strengths [8, 9], as in the standard gauge theories.

Relying on our experience with GR, we know that exact solutions of a gravitational theory are essential for its physical interpretation. In the context of 3D PGT, exact solutions were first studied in the Mielke-Baekler model; for a review, see chapter 17 in ref. [6]. Recently, our research interest moved toward exact solutions in a more dynamical framework of the quadratic PGT. The first step in this direction was made by constructing the Bañados-Teitelboim-Zanelli (BTZ) black hole with torsion [9]. Then, we showed that gravitational waves can be naturally incorporated into the PGT dynamical framework [10, 11]. The purpose of the present work is to examine a PGT generalization of the Oliva-Tempo-Troncoso (OTT) black hole [12], see also [13], as well as its Vaidya-like extension [14].

The OTT black hole is an exact solution of the Bergshoeff-Hohm-Townsend (BHT) massive gravity [15], a Riemannian model defined by adding a specific combination of curvature-squared terms to the Hilbert-Einstein action. Generically, the BHT gravity with a cosmological constant admits two distinct maximally symmetric vacua. However, when the coupling constants satisfy a specific critical condition, these two vacua coincide. It is exactly in this case that the OTT black hole is a vacuum solution of the BHT gravity.[1] Going a step further, Maeda [14] formulated a Vaidya-like extension of the OTT black hole, assuming the presence of a null dust fluid as a *matter field*. In this paper, we construct a Vaidya-OTT spacetime with torsion as an exact *vacuum solution* of PGT.

The paper is organized as follows. In section 2, we describe the static OTT black hole as a Riemannian solution of PGT in vacuum. In particular, the canonical expression for the gravitational energy is shown to be directly compatible with the first law of black hole thermodynamics. In section 3, we introduce a Vaidya extension of the OTT metric in the manner of Maeda [14]; the resulting Riemannian geometry is not compatible with the PGT dynamics in vacuum. Then, in section 4, we construct a Vaidya-OTT geometry with torsion as an exact vacuum solution of PGT. In section 5, we apply canonical methods to show that a specific subclass of these solutions is characterized by the asymptotic conformal symmetry. The canonical Vaidya-OTT energy is found; apart from the OTT term, it contains a contribution stemming from torsion. The associated surface term of the canonical generator for time translations is a generalization of the more standard expression [16], used in ref. [17] to calculate energies for a number of exact solutions in 3D gravity. Finally, section 6 is devoted to concluding remarks, while appendices contain some technical details.

Working in PGT, we use the following conventions: the Latin indices $(i, j, k, \dots)$ refer to the local Lorentz frame, the Greek indices $(\mu, \nu, \rho, \dots)$ refer to the coordinate frame, b^i is the triad field (coframe 1-form), $\omega^{ij} = -\omega^{ji}$ is a connection 1-form, the respective field strengths are the torsion $T^i = db^i + \omega^i{}_m \wedge b^m$ and the curvature $R^{ij} = d\omega^{ij} + \omega^i{}_k \wedge \omega^{kj}$ (2-forms); the Lie dual of an antisymmetric form X^{ij} is $X_i := -\varepsilon_{ijk} X^{jk}/2$, the Hodge dual of a form α is ${}^\star\alpha$, and the exterior product of forms is implicit.

2 OTT black hole in PGT

We begin our considerations by showing that the static OTT black hole, a vacuum solution of the BHT gravity with a unique AdS ground state [12], is also a *Riemannian solution* of PGT, in spite of the fact that PGT represents quite a different dynamical framework [9].

[1] For the canonical aspects of the full nonlinear theory in the critical regime, see refs. [18, 19].

JHEP05(2015)101

2.1 Geometric aspects

The metric of the static OTT spacetime is given by

$$ds^2 = N^2 dt^2 - \frac{dr^2}{N^2} - r^2 d\varphi^2 \,, \qquad N^2 := -\mu + Br + \frac{r^2}{\ell^2} \,, \tag{2.1}$$

where μ and B are integration constants. The Killing horizons are determined by the condition $N^2 = 0$:

$$r_\pm = \frac{\ell^2}{2}\left(-B \pm \sqrt{B^2 + 4\mu/\ell^2}\right) .$$

When at least r_+ is real and positive, and $\ell^2 > 0$, the OTT metric defines a static and spherically symmetric AdS black hole; for $B = 0$, it reduces to the BTZ black hole.

Given the metric (2.1), one can choose the associated triad field in the form

$$b^0 = N dt \,, \qquad b^1 = \frac{dr}{N} \,, \qquad b^2 = r d\varphi \,, \tag{2.2}$$

so that $ds^2 = \eta_{ij} b^i \otimes b^j$, where $\eta = \text{diag}(+1, -1, -1)$. Then, treating the OTT black hole as a Riemannian object, we use the Christoffel connection,

$$\omega^{01} = -N' b^0 \,, \qquad \omega^{02} = 0 \,, \qquad \omega^{12} = \frac{N}{r} b^2 \,, \tag{2.3}$$

where $N' := \partial N / \partial r$, to calculate the curvature 2-form:

$$\begin{aligned}
R^{01} &= (N'N)' b^0 b^1 = \frac{1}{\ell^2} b^0 b^1 \,, \\
R^{02} &= \frac{1}{r} N' N b^0 b^2 = \left(\frac{B}{2r} + \frac{1}{\ell^2}\right) b^0 b^2 \,, \\
R^{12} &= \frac{1}{r} N' N b^1 b^2 = \left(\frac{B}{2r} + \frac{1}{\ell^2}\right) b^1 b^2 \,.
\end{aligned} \tag{2.4a}$$

For $B \neq 0$, the scalar curvature has a singularity at $r = 0$:

$$R = \frac{6}{\ell^2} + \frac{2B}{r} \,. \tag{2.4b}$$

Nonvanishing irreducible components of the curvature are:

$$^{(6)}R^{ij} = \frac{1}{6} R b^i b^j \,, \qquad ^{(4)}R^{ij} = R^{ij} - {}^{(6)}R^{ij} \,.$$

In this geometry, the Cotton 2-form C^i, defined by

$$C^i := \nabla L^i \,, \qquad L^i = (Ric)^i - \frac{1}{4} R b^i \,, \tag{2.5}$$

is vanishing, so that the OTT spacetime is conformally flat. This is not a surprise since the OTT metric is also a solution of the conformal gravity [13].

Now, we shall show that the OTT black hole is a Riemannian solution of PGT in vacuum.

JHEP05(2015)101

2.2 Riemannian sector of PGT

Lagrangian dynamics of PGT if expressed in terms of its basic field variables, the triad field b^i and the RC connection ω^{ij}, and the related field strengths, the torsion T^i and the curvature R^{ij}. The general parity-preserving gravitational Lagrangian of PGT is quadratic in the field strengths, see appendix A. In the Riemannian sector of PGT, torsion vanishes and L_G is expressed only in terms of the curvature. For ${}^{(5)}R_{ij} = 0$, we have

$$L_G = -{}^\star(a_0 R + 2\Lambda_0) + \frac{1}{2} R^{ij\star}\left(b_4{}^{(4)}R_{ij} + b_6{}^{(6)}R_{ij}\right), \tag{2.6}$$

and the general vacuum PGT field equations (A.2) reduce to a simpler form:

$$\begin{aligned} &(1\text{ST}) \qquad E_i = 0\,, \\ &(2\text{ND}) \qquad \nabla H_{ij} = 0\,, \end{aligned} \tag{2.7}$$

where E_i and H_{ij} are given in (A.5). The field equations produce the following result:

$$\begin{aligned} &(2\text{ND}) \quad \Rightarrow \quad b_4 + 2b_6 = 0\,, \\ &(1\text{ST}) \quad \Rightarrow \quad b_4 - 2a_0\ell^2 = 0\,, \qquad a_0 + 2\ell^2\Lambda_0 = 0\,. \end{aligned} \tag{2.8}$$

Thus, the OTT black hole is an exact vacuum solution in the Riemannian sector of PGT, provided the four Lagrangian parameters $(a_0, b_4, b_6, \Lambda_0)$ satisfy the above three conditions.

2.3 Gravitational energy and entropy

Asymptotically, for large r, the OTT geometry takes the AdS form. Based on the canonical approach described in appendix B and section 5, one finds that the only nontrivial conserved charge of this geometry is the gravitational energy,

$$E = \frac{1}{4G}\left(\mu + \frac{1}{4}B^2\ell^2\right), \tag{2.9}$$

whereas the angular momentum M vanishes. The result is obtained from the canonical generator of time translations, the surface term of which contains a new contribution with respect to the more standard situation, see refs. [16, 17] and subsection 5.2.

Remarkably, the canonical expression for E is directly compatible with the first law of black hole thermodynamics. Indeed, using the OTT central charges (subsection 5.3)

$$c^\pm = 24\pi \cdot 2a_0\ell = \frac{3\ell}{G}, \tag{2.10}$$

the Cardy formula produces the following expression for the entropy:

$$S = 4\pi\ell\sqrt{E/4G}\,. \tag{2.11}$$

Then, by introducing the Hawking temperature,

$$T = \frac{1}{4\pi}\,\partial_r N^2\big|_{r=r_+} = \frac{1}{2\pi\ell}\sqrt{4GE}\,, \tag{2.12}$$

one can directly verify the first law of the black hole thermodynamics:

$$\delta E = T\delta S\,. \tag{2.13}$$

Since the entropy vanishes for $E = 0$, the state with $E = 0$ can be naturally regarded as the ground state of the OTT family of black holes [20].

The canonical energy (2.9) coincides with the shifted OTT energy $\Delta M = M - M_0$, introduced by Giribet et al. [20], where $M = \mu/4G$ is interpreted as the conserved charge and $M_0 = -B^2/16G$. The quantity ΔM is defined to respect Cardy's formula for the entropy, and it has the role of thermodynamic energy in the first law. In the canonical approach, the conserved charge E is the same object as the thermodynamic energy.

3 Vaidya extension of the OTT metric

To obtain a Vaidya extension of the OTT metric, we first make a coordinate transformation from the Schwarzschild-like time coordinate t to a new coordinate u, such that

$$dt = du + dr/N^2\,. \tag{3.1}$$

The physical meaning of u is obtained by noting that $u =$ const. corresponds to a radially outgoing null ray, $dr/dt = N^2$, see ref. [21]. Then, following Maeda [14], we introduce a Vaidya extension of the OTT black hole by making B a function of u, $B = B(u)$, but leaving μ as a constant. The Vaidya-OTT metric defines a time dependent spherically symmetric geometry:

$$ds^2 = N^2 du^2 + 2dudr - r^2 d\varphi^2\,. \tag{3.2}$$

In the new coordinates $x^\mu = (u, r, \varphi)$, it is convenient to choose the triad field as

$$b^+ := du\,, \qquad b^- := Hdu + dr\,, \qquad b^2 := rd\varphi\,, \tag{3.3}$$

where $H = N^2/2$, so that the line element becomes $ds^2 = \eta_{ij} b^i b^j$, with

$$\eta_{ij} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$

The dual frame h_i, defined by $h_i \rfloor b^j = \delta_i^j$, is given by

$$h_+ = \partial_u - H\partial_r\,, \qquad h_- = \partial_r\,, \qquad h_2 = \frac{1}{r}\partial_\varphi\,.$$

For vanishing torsion, one can use the Riemannian connection

$$\omega^{+-} = -H'b^+\,, \qquad \omega^{+2} = -\frac{1}{r}b^2\,, \qquad \omega^{-2} = \frac{1}{r}Hb^2\,, \tag{3.4}$$

to calculate the related curvature 2-form R^{ij}. Then, following the procedure described in the previous section, one finds that the PGT field equations (2.7) imply:

$$\begin{aligned} &(\text{2ND}) \qquad \Rightarrow \qquad b_4 + 2b_6 = 0\,, \\ &(\text{1ST}) \qquad \Rightarrow \qquad b_4 - 2a_0\ell^2 = 0\,, \qquad a_0 + 2\ell^2\Lambda = 0\,, \qquad \underline{\dot{B} = 0}\,, \end{aligned} \tag{3.5}$$

where $\dot{B} := \partial_u B$. Thus, the Vaidya-OTT metric with $\dot{B} \neq 0$ is *not a Riemannian solution of PGT in vacuum.*

In order to overcome a similar barrier in the BHT gravity, Maeda [14] introduced the Vaidya-OTT solution in the presence of *matter*, represented by a null dust fluid. The energy density of this fluid is expressed directly in terms of the metric function $B(u)$, which remains dynamically undetermined. Based on our experience with exact wave solutions in PGT [10, 11], we expect that the presence of torsion could lead to a consistent description of the Vaidya-OTT dynamics *in vacuum*. Further exposition confirms this expectation.

4 Vaidya-OTT solution with torsion

4.1 Geometry of the ansatz

Following the logic of our approach to exact wave solutions in PGT [10, 11], we propose to look for a Vaidya-OTT solution with torsion using the following two assumptions:

(i) The new triad field retains the form (3.3);

(ii) The RC connection is obtained from the Riemannian expression (3.4) by the rule $H \to H + K$, where $K = K(u)$:

$$\omega^{+-} = -H' b^+ , \qquad \omega^{+2} = -\frac{1}{r} b^2 , \qquad \omega^{-2} = \frac{1}{r}(H + K) b^2 . \tag{4.1}$$

The new function K is expected to *compensate* the presence of the problematic $\dot{B}$ term in the Riemannian field equations (3.5). Geometrically, K defines the torsion of spacetime. Indeed, using $T^i := \nabla b^i$ one obtains:

$$T^+, T^- = 0 , \qquad T^2 = \frac{1}{r} K b^+ b^2 . \tag{4.2}$$

The nonvanishing irreducible components of the torsion are ${}^{(1)}T^i$ and ${}^{(2)}T^i$.

To complete the geometric description of our ansatz, we use the connection (4.1) to calculate the RC curvature 2-form:

$$\begin{aligned}
R^{+-} &= H'' b^+ b^- = \frac{1}{\ell^2} b^+ b^- , \\
R^{+2} &= \frac{1}{r} H' b^+ b^2 = \left(\frac{1}{\ell^2} + \frac{B}{2r} \right) b^+ b^2 , \\
R^{-2} &= \frac{1}{r} H' b^- b^2 + \frac{1}{r} \left(\dot{H} + \dot{K} + H' K \right) b^+ b^2 .
\end{aligned} \tag{4.3}$$

For $B \neq 0$, the scalar curvature is singular at $r = 0$:

$$R = \frac{6}{\ell^2} + \frac{2B}{r} .$$

The nonvanishing irreducible components of the curvature are ${}^{(6)}R^{ij}$ and ${}^{(4)}R^{ij} = R^{ij} - {}^{(6)}R^{ij}$.

With the adopted geometric structure of our ansatz, the general PGT Lagrangian (A.1) becomes effectively of the form

$$L_G = -{}^\star(a_0 R + 2\Lambda_0) + T^{i\star}(a_1 {}^{(1)}T_i + a_2 {}^{(2)}T_i) + \frac{1}{2} R^{ij\,\star}(b_4 {}^{(4)}R_{ij} + b_6 {}^{(6)}R_{ij}) . \tag{4.4}$$

JHEP05(2015)101

4.2 Solutions

With a given geometry of our ansatz, we now wish to find the metric function H and the torsion function K as solutions of the vacuum PGT field equations (A.2). To ensure a smooth limit to the standard OTT black hole for $B \to$ const., we impose the conditions (2.8) on the Lagrangian parameters. Then, the field equations (A.2) take the form

$$\begin{aligned} &(\text{2ND}) \qquad 2\dot{K} + BK = 0\,, \qquad a_1, a_2 = 0\,, \\ &(\text{1ST}) \qquad \dot{B}\ell^2 + 2K = 0\,. \end{aligned} \tag{4.5}$$

The conditions $a_1, a_2 = 0$ effectively eliminate the T^2 terms from the Lagrangian. Moreover, the second term in R^{-2} vanishes on-shell. Such a reduction of R^{ij} to its OTT form (with $\dot{H}, K = 0$) is a manifestation of the compensating role of the torsion function K.

By combining the above two equations, one obtains

$$2K - \frac{1}{4}B^2\ell^2 = -K_0\ell^2\,, \qquad \dot{B} + \frac{1}{4}B^2 = K_0\,, \tag{4.6}$$

where K_0 is an integration constant, the first integral of the field equations (4.5). Introducing a new constant E by $K_0\ell^2 = 4GE - \mu$, the first equation takes the form

$$4GE = \mu + \frac{1}{4}B^2\ell^2 - 2K\,, \tag{4.7}$$

where E is recognized as a RC generalization of the gravitational energy (2.9). The conservation law of E is defined with respect to the evolution along u, $dE/du = 0$. However, $dt = du + dr/N^2$ implies $t = u + \mathcal{O}_1$, so that asymptotically, one expects E to be conserved also with respect to the Schwarzschild-like time t. In the next section, this argument is confirmed by canonical methods.

Depending on the value of K_0, there exist three branches of solutions.

1. $K_0 = C_1^2$. Apart from the trivial case $B = 2C_1$, $K = 0$, one finds:

$$B = 2C_1 \tanh\frac{C_1}{2}(u + C_2)\,, \qquad K = -\frac{C_1^2\ell^2}{2\cosh^2\frac{C_1}{2}(u + C_2)}\,. \tag{4.8}$$

2. $K_0 = -C_1^2$. By replacing $C_1 \to iC_1$ in the solution (4.8), one obtains:

$$B = -2C_1 \tan\frac{C_1}{2}(u + C_2)\,, \qquad K = \frac{C_1^2\ell^2}{2\cos^2\frac{C_1}{2}(u + C_2)}\,. \tag{4.9}$$

3. $K_0 = 0$.

$$B = \frac{4}{u + C_2}\,, \qquad K = \frac{2\ell^2}{(u + C_2)^2}\,. \tag{4.10}$$

The solutions in branches 2 and 3 are singular at finite values of u, whereas the solutions in branch 1 are perfectly regular, and physically most appealing.

In figure 1, we illustrate a typical form of the solutions from branch 1. Since $B(u)$ and $K(u)$, as well as their derivatives, are bounded functions, the field strengths (4.2) and (4.3) approach asymptotically to a Riemannian AdS spacetime. This motivates us to examine the corresponding asymptotic structure in more details.

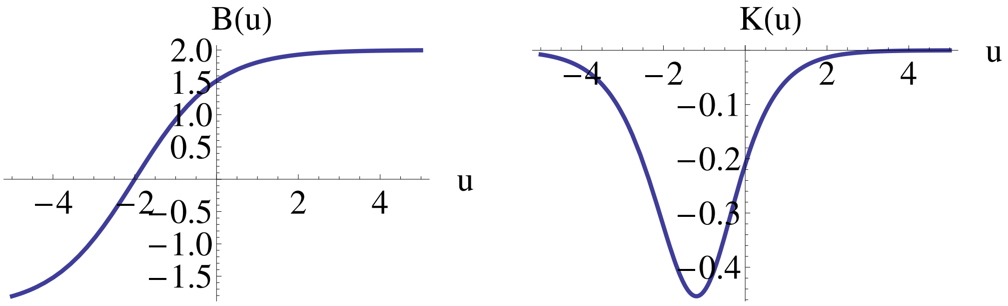

Figure 1. Branch 1 solutions for $B(u)$ and $K(u)$, with C_1, $\ell = 1$, $C_2 = 2$.

5 Asymptotic symmetry

In this section, we use the canonical approach to analyze the asymptotic symmetry associated to the Vaidya-OTT solution with torsion in branch 1.

5.1 AdS asymptotic conditions

Transition from the OTT to the Vaidya-OTT triad is realized not only by making B a function of u, but also by going over to a new triad basis, as can be seen by comparing eqs. (2.2) and (3.3). The new basis allowed us to introduce the RC geometry by the simple rules formulated in subsection 4.1. Then, requiring the *invariance under the AdS group* $\mathrm{SO}(2,2)$, see [22], one arrives at the following set of the Vaidya-OTT asymptotic states:

$$b^i{}_\mu = \bar{b}^i{}_\mu + B^i{}_\mu , \qquad \bar{b}^i{}_\mu = \begin{pmatrix} 1 & 0 & 0 \\ \frac{r^2}{2\ell^2} & 1 & 0 \\ 0 & 0 & r \end{pmatrix}, \qquad B^i{}_\mu := \begin{pmatrix} \mathcal{O}_1 & \mathcal{O}_3 & \mathcal{O}_1 \\ \mathcal{O}_{-1} & \mathcal{O}_1 & \mathcal{O}_{-1} \\ \mathcal{O}_0 & \mathcal{O}_2 & \mathcal{O}_0 \end{pmatrix}, \tag{5.1a}$$

and

$$\omega^i{}_\mu = \bar{\omega}^i{}_\mu + \Omega^i{}_\mu , \qquad \bar{\omega}^i{}_\mu = -\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & \frac{r^2}{2\ell^2} \\ \frac{r}{\ell^2} & 0 & 0 \end{pmatrix}, \qquad \Omega^i{}_\mu := \begin{pmatrix} \mathcal{O}_1 & \mathcal{O}_3 & \mathcal{O}_1 \\ \mathcal{O}_{-1} & \mathcal{O}_1 & \mathcal{O}_{-1} \\ \mathcal{O}_0 & \mathcal{O}_2 & \mathcal{O}_0 \end{pmatrix}, \tag{5.1b}$$

where ω^i is the Lie dual of ω^{ij}, and $\bar{b}^i{}_\mu$ and $\bar{\omega}^i{}_\mu$ refer to the background configuration with $\mu, B = 0$, representing the massless BTZ black hole. These states are invariant under the set of restricted local Poincaré transformations, defined by the parameters

$$\begin{aligned} &\xi^u = \ell U + \mathcal{O}_2 , \qquad \xi^r = -r\ell\partial_u U + \mathcal{O}_0 , \\ &\xi^\varphi = \Phi - \frac{\ell}{r}\partial_\varphi U + \mathcal{O}_2 , \end{aligned} \tag{5.2a}$$

$$\begin{aligned} &\theta^+ = -\frac{\ell}{r}\partial_\varphi U + \mathcal{O}_2 , \qquad \theta^- = \frac{r}{2\ell}\partial_\varphi U + \mathcal{O}_0 , \\ &\theta^2 = \ell\partial_u U + \mathcal{O}_1 . \end{aligned} \tag{5.2b}$$

JHEP05(2015)101

Here, the functions $U = U(u, \varphi)$ and $\Phi = \Phi(u, \varphi)$ are such that the combinations $U^\pm = U \pm \Phi$ satisfy the conditions $\partial_\pm U^\mp = 0$, where $x^\pm = u/\ell \pm \varphi$. Since $u = t + \mathcal{O}_1$ for large r, these conditions define the asymptotic conformal group in 2D.

In spite of certain technical differences between the asymptotic requirements (5.1) and (B.1), the corresponding commutator algebras have the same form. Using the composition law of the restricted Poincaré parameters to leading order, the commutator algebra associated to (5.1) is found to have the form of two independent Virasoro algebras,

$$i[\ell_m^\pm, \ell_n^\pm] = (m - n)\ell_{m+n}^\pm \,, \tag{5.3}$$

where $\ell_n^\pm = \delta_0(U^\pm = e^{\pm inx^+})$. The respective central charges $c^\pm$ will be determined by the canonical methods.

To complete the analysis of the asymptotic conditions, we presented in appendix C an additional set of asymptotic requirements, motivated by the form of torsion in (4.2).

5.2 Canonical generators

In order to examine the canonical structure of the quadratic PGT, we use the *first-order formulation* [23], as it leads to a particularly simple construction of the canonical generator, the form of which can be found in eq. (5.7) of ref. [9]. In this formulation, one introduces two new variables, τ_i and ρ_{mn}, such that their on-shell values are $\tau_i = H_i$ and $\rho_{mn} = H_{mn}$. Since the canonical generator G acts on basic dynamical variables via the Poisson bracket operation, it is required to be a differentiable phase-space functional. For a given set of asymptotic conditions, this property is ensured by adding a suitable surface Γ term to G, such that $\tilde{G} = G + \Gamma$ is *both differentiable and finite* phase-space functional [24, 25]. To examine the differentiability of G, we start from the form of its variation:

$$\begin{aligned}
\delta G &= -\int_\Sigma d^2x(\delta G_1 + \delta G_2)\,, \\
\delta G_1 &= \varepsilon^{t\alpha\beta}\xi^\mu \left(b^i{}_\mu \partial_\alpha \delta\tau_{i\beta} + \omega^i{}_\mu \partial_\alpha \delta\rho_{i\beta} + \tau^i{}_\mu \partial_\alpha \delta b_{i\beta} + \rho^i{}_\mu \partial_\alpha \delta\omega_{i\beta}\right) + R\,, \\
\delta G_2 &= \varepsilon^{t\alpha\beta}\theta^i \partial_\alpha \delta\rho_{i\beta} + R\,.
\end{aligned} \tag{5.4}$$

Here, the coherently oriented volume 2-form on the spatial section Σ of spacetime is normalized to $d^2x = dr d\varphi$, the variation is performed in the set of asymptotic states, R stands for regular terms, and ρ^i is the Lie dual of $\rho_{mn} = H_{mn}$, the on-shell value of which reads

$$H_{ij} = -2a_0\varepsilon_{ijk}b^k - 4a_0\ell^2\varepsilon_{ijk}\hat{L}^k\,, \tag{5.5}$$

and $\hat{L}^k$ is the "symmetrized" Schouten 1-form, $\hat{L}_k = L_{(km)}b^m$, see (2.5).

In what follows, we restrict our considerations by two specific assumptions that characterize both the OTT black hole and the Vaidya-OTT solution with torsion:

(1) The torsion squared-terms in L_G effectively vanish, that is $\tau_i = 0$;

(2) ${}^{(5)}R^{ij} = 0$.

The asymptotic conditions (5.1) imply $\delta G_2 = R$, so that the surface term in the improved generator $\tilde{G} = G + \Gamma$ is determined by the variational equations

$$\delta\Gamma = \int_0^{2\pi} d\varphi(\xi^t \delta\mathcal{E} + \xi^\varphi \delta\mathcal{M})\,, \tag{5.6a}$$

$$\delta\mathcal{E} := \frac{1}{2}\left(\omega^{ij}{}_t \delta H_{ij\varphi} + \delta\omega_{ij\varphi} H^{ij}{}_t\right)\,, \tag{5.6b}$$

$$\delta\mathcal{M} := \frac{1}{2}\left(\omega^{ij}{}_\varphi \delta H_{ij\varphi} + \delta\omega_{ij\varphi} H^{ij}{}_\varphi\right)\,, \tag{5.6c}$$

where we used $u = t + \mathcal{O}_1$, and the boundary $\partial\Sigma$ is parametrized by the coordinate φ.

Finding a solution for $\mathcal{E}$ from the variational equation (5.6b) demands rather involved considerations, based on the asymptotic conditions (5.1) and (C.1). As shown in appendix C, the surface term for time translations can be written in the form

$$\Gamma[\xi^t] = \int_0^{2\pi} d\varphi\, \xi^t \mathcal{E}\,, \tag{5.7a}$$

$$\mathcal{E} = \frac{1}{2}\big(\omega^{ij}{}_t \Delta H_{ij\varphi} + \Delta\omega^{ij}{}_\varphi \bar{H}_{ijt}\big) - \frac{1}{4}\big(\Delta\omega^{ij}{}_t \Delta H_{ij\varphi} - \Delta\omega^{ij}{}_\varphi \Delta H_{ijt}\big)\,, \tag{5.7b}$$

where $\Delta X := X - \bar{X}$ is the difference between any form X and its boundary value $\bar{X}$. On the other hand, equation (5.6c) leads to a simple surface term for spatial rotations:

$$\Gamma[\xi^\varphi] = \int_0^{2\pi} d\varphi\, \xi^\varphi \mathcal{M}\,, \qquad \mathcal{M} = \frac{1}{2}\omega^{ij}{}_\varphi H_{ij\varphi}\,. \tag{5.7c}$$

Both $\Gamma[\xi^t]$ and $\Gamma[\xi^\varphi]$ are finite phase-space functionals (see appendix C).

The boundary terms for $\xi^t = 1$ and $\xi^\varphi = 1$,

$$E = \int_0^{2\pi} d\varphi\, \mathcal{E}\,, \qquad M = \int_0^{2\pi} d\varphi\, \mathcal{M}\,, \tag{5.8}$$

represent the energy and angular momentum of the system, respectively. Calculated on the Vaidya-OTT configuration, these expressions take the values

$$E = \frac{1}{4G}\left(\mu + \frac{1}{4}B^2\ell^2 - 2K\right)\,, \qquad M = 0\,. \tag{5.9}$$

The form of E confirms the result (4.7) obtained from the Lagrangian field equations. In the canonical formalism, the conservation laws for E and M follow from the Poisson bracket algebra of the asymptotic symmetry [13].

The expression for energy defined by equation (5.7b) consists of two pieces. As shown in ref. [17], the first piece is sufficient to correctly describe the energy content of a number of solutions in 3D gravity with/without torsion and topologically massive gravity. However, when applied to the (Vaidya-)OTT solution, this piece is not sufficient; in particular, it produces the incorrect coefficient $1/2$ for the B^2 term in (5.9). The second piece in (5.7b) is closely related to the presence of the Br term in the OTT metric. Thus, our result (5.7b) represents a generalization of the energy formula used in [17] to the (Vaidya-)OTT case.

5.3 Canonical algebra of asymptotic symmetries

The asymptotic symmetry is described by the Poisson bracket algebra of the improved generators. Rather then performing a direct calculation, the form of this algebra can be found by a more instructive method. To show how it works, we introduce the notation $\tilde{G}' = \tilde{G}[U', \Phi']$, and similarly for $\tilde{G}''$ and $\tilde{G}'''$. Then, according to the main theorem of ref. [25], one can conclude that the Poisson bracket algebra has the form

$$\{\tilde{G}'', \tilde{G}'\} = \tilde{G}''' + C''' , \tag{5.10}$$

where the parameters of $\tilde{G}'''$ are defined by the composition law of the asymptotic Poincaré transformations, and C''' is the central charge term. In order to calculate C''', one should note that the algebra (5.10) implies $\delta_0'\Gamma'' \approx \Gamma''' + C'''$, where $\delta_0'\Gamma''$ is determined by the relations (5.6), and C''' is identified as the field independent piece on the right-hand side. Then, going over to the Fourier modes $L_n^{\pm}$ of $\tilde{G}$, the algebra (5.10) takes the form of two independent Virasoro algebras,

$$i\{L_m^{\pm}, L_n^{\pm}\} = (m-n)L_{n+m}^{\pm} + \frac{c^{\pm}}{12} n^3 \delta_{m,-n} , \tag{5.11}$$

where the classical central charges are equal to each other, $c^{\pm} = c$, with

$$c = \frac{3\ell}{G} . \tag{5.12}$$

Thus, the value of c is found to be twice the GR value $c_0 = 3\ell/2G$.

6 Concluding remarks

In this paper, we constructed a Vaidya-like extension of the OTT black hole as an exact solution of the quadratic PGT in vacuum. The construction is realized in two steps.

First, we showed that the OTT black hole is a Riemannian vacuum solution of PGT, provided the coupling constants satisfy certain requirements. The black hole energy is calculated from the canonical generator for time translations, the surface term of which is a suitable generalization of the more standard expression that can be found in ref. [16], see also ref. [17]. The canonical energy E is compatible with the first law of black hole thermodynamics, in agreement with the equality of E to the shifted OTT energy [20].

Then, following Maeda [14], we introduced a Vaidya-like extension of the OTT black hole; however, this extension is not a Riemannian solution of PGT in vacuum. To overcome this difficulty, we introduced a suitable ansatz for the connection possessing a nontrivial torsion content, making thereby the resulting Vaidya-OTT geometry an exact vacuum solution of PGT. As far as the asymptotic structure of the Vaidya-OTT solution is concerned, one should note that: (a) the surface term of the canonical generator for time translations has the same structure as in the OTT case, (b) the canonical energy differs from the OTT black hole energy by a contribution stemming from the torsion, and (c) central charges of the asymptotic algebra are the same as in the OTT black hole case.

Since the OTT solution is known to exists also for positive or vanishing $1/\ell^2$ [12], most of the present results could be straightforwardly extended to these sectors.

Acknowledgments

This work was supported by the Serbian Science Foundation under Grant No. 171031. The results are checked using the Excalc package of the computer algebra system Reduce.

A PGT field equations

In this appendix, we give a brief account of the PGT field equations, based on ref. [9]. The parity-invariant gravitational Lagrangian $L_G = L_G(b^i, T^j, R^{mn})$ (3-form) is at most quadratic in the torsion T^i and the curvature R^{ij}:

$$\begin{aligned} L_G &= -{}^\star(a_0 R + 2\Lambda_0) + T^{i\star}(a_1{}^{(1)}T_i + a_2{}^{(2)}T_i + a_3{}^{(3)}T_i) \\ &\quad + \frac{1}{2}R^{ij}\left(b_4{}^{(4)}R_{ij} + b_5{}^{(5)}R_{ij} + b_6{}^{(6)}R_{ij}\right), \end{aligned} \tag{A.1}$$

where ${}^{(n)}T^i$ and ${}^{(n)}R^{ij}$ are irreducible components of the respective field strengths, and a_0 is normalized by $a_0 = /16\pi G$. By varying L_G with respect to b^i and ω^{ij}, one obtains the vacuum field equations that can be written in a compact form as

$$\begin{aligned} &(\text{1ST}) \qquad \nabla H_i + E_i = 0\,, \\ &(\text{2ND}) \qquad \nabla H_{ij} + E_{ij} = 0\,. \end{aligned} \tag{A.2}$$

Here, $H_i := \partial L_G/\partial T^i$ and $H_{ij} := \partial L_G/\partial R^{ij}$ are the covariant momenta:

$$\begin{aligned} H_i &= 2^\star(a_1{}^{(1)}T_i + a_2{}^{(2)}T_i + a_3{}^{(3)}T_i)\,, \\ H_{ij} &= -2a_0\varepsilon_{ijm}b^m + H'_{ij}\,, \\ H'_{ij} &:= 2^\star\left(b_4{}^{(4)}R_{ij} + b_5{}^{(5)}R_{ij} + b_6{}^{(6)}R_{ij}\right), \end{aligned} \tag{A.3}$$

and $E_i := \partial L_G/\partial b^i$ and $E_{ij} = \partial L_G/\partial \omega^{ij}$ are the energy-momentum and spin currents:

$$\begin{aligned} E_i &= h_i \rfloor L_G - (h_i \rfloor T^m)H_m - \frac{1}{2}(h_i \rfloor R^{mn})H_{mn}\,, \\ E_{ij} &= -(b_i H_j - b_j H_i)\,. \end{aligned} \tag{A.4}$$

In the Riemannian sector ($T^i = 0$) with ${}^{(5)}R_{ij} = 0$, H_i and E_{ij} vanish, and the simplified field equations take the form displayed in (2.7), with

$$\begin{aligned} H_{ij} &= -2a_0\varepsilon_{ijm}b^m + \frac{b_6 + 2b_4}{3}\,R\varepsilon_{ijk}b^k - 2b_4\varepsilon_{ij}{}^m(Ric)_{mk}b^k\,, \\ E_i &= \mathcal{L}_G{}^\star b_i - R^{mn}{}_{ik}b^k H_{mn}\,. \end{aligned} \tag{A.5}$$

Here, we used $L_G = \mathcal{L}_G\,\hat\epsilon$, and $\hat\epsilon$ is the volume 3-form.

B Asymptotic conditions for the OTT black hole

The action of the AdS Killing vectors on the OTT black hole configuration, described in section 2, leads to the associated asymptotic conditions that are *relaxed* with respect to the Brown-Henneaux ones:

$$b^i{}_\mu = \bar{b}^i{}_\mu + B^i{}_\mu\,, \qquad B^i{}_\mu := \begin{pmatrix} \mathcal{O}_0 & \mathcal{O}_3 & \mathcal{O}_0 \\ \mathcal{O}_1 & \mathcal{O}_2 & \mathcal{O}_1 \\ \mathcal{O}_0 & \mathcal{O}_3 & \mathcal{O}_0 \end{pmatrix}\,, \tag{B.1a}$$

and

$$\omega^i{}_\mu = \bar{\omega}^i{}_\mu + \Omega^i{}_\mu\,, \qquad \Omega^i{}_\mu := \begin{pmatrix} \mathcal{O}_0 & \mathcal{O}_3 & \mathcal{O}_0 \\ \mathcal{O}_1 & \mathcal{O}_2 & \mathcal{O}_1 \\ \mathcal{O}_0 & \mathcal{O}_3 & \mathcal{O}_0 \end{pmatrix}\,. \tag{B.1b}$$

Here, ω^i is the Lie dual of ω^{ij}, and $\bar{b}^i{}_\mu$ and $\bar{\omega}^i{}_\mu$ refer to the AdS background (with $B = \mu = 0$). These conditions are invariant under the asymptotic Poincaré transformations, defined by the set of restricted local parameters (ξ^μ, θ^i) that can be found in ref. [22]. The conditions (B.1) are a PGT generalization of those discussed in [12].

Following the procedure described in section 5, one can find the conserved charges of the OTT black hole, the energy E and the angular momentum M. Moreover, the canonical algebra of the asymptotic symmetry is represented by two independent Virasoro algebras with equal central charges $c^\mp = c$. The values of E, M and c are given in subsection 2.3.

C Refined asymptotic conditions

Equation (4.2) implies that the Vaidya-OTT solution has only one nonvanishing component of torsion: $T^2{}_{u\varphi} = K$. Clearly, this property is not valid on the whole set of asymptotic states. In order to ensure finiteness of the improved canonical generators, we find it necessary to make further restrictions of the asymptotic conditions (5.1) by demanding the highest order terms in $T^i{}_{\mu\nu}$ to vanish:

$$\begin{aligned}
T^+{}_{u\varphi}:&\quad r\left(\Omega^+{}_u + \frac{1}{\ell^2}B^+{}_\varphi\right) + (\Omega^2{}_\varphi + B^2{}_u) = \mathcal{O}_1\,,\\
T^+{}_{ur}:&\quad \frac{r}{\ell^2}B^+{}_r + \Omega^2{}_r + \frac{1}{r}B^+{}_u = \mathcal{O}_3\,,\\
T^+{}_{r\varphi}:&\quad r\Omega^+{}_r + B^2{}_r + \frac{1}{r}B^+{}_\varphi = \mathcal{O}_3\,,
\end{aligned} \tag{C.1a}$$

$$\begin{aligned}
T^-{}_{u\varphi}:&\quad r\left(\Omega^-{}_u + \frac{1}{\ell^2}B^-{}_\varphi\right) + \frac{r^2}{2\ell^2}(\Omega^2{}_\varphi + B^2{}_u) = \mathcal{O}_0\,,\\
T^-{}_{ur}:&\quad \frac{r}{\ell^2}B^-{}_r + \frac{r^2}{2\ell^2}\Omega^2{}_r - \Omega^2{}_u - \frac{1}{r}B^-{}_u = \mathcal{O}_1\,,\\
T^-{}_{r\varphi}:&\quad \Omega^2{}_\varphi + \frac{r^2}{2\ell^2}B^2{}_r + r\Omega^-{}_r + \frac{1}{r}B^-{}_\varphi = \mathcal{O}_1\,,\\
T^2{}_{u\varphi}:&\quad \frac{r^2}{2\ell^2}(\Omega^+{}_\varphi + B^+{}_u) - (B^-{}_u + \Omega^-{}_\varphi) = \mathcal{O}_0\,,
\end{aligned} \tag{C.1b}$$

$$T^2{}_{ur}: \qquad \frac{r^2}{2\ell^2}\Omega^+{}_r - (\Omega^-{}_r + \Omega^+{}_u) = \mathcal{O}_2\,,$$

$$T^2{}_{r\varphi}: \qquad \frac{r^2}{2\ell^2}B^+{}_r + \Omega^+{}_\varphi - B^-{}_r = \mathcal{O}_2\,. \tag{C.1c}$$

Now, we use the asymptotic conditions (5.1) and (C.1) to derive the surface terms (5.7) and prove their finiteness. First, we show that $\mathcal{E}$ satisfies the variational equation (5.6b):

$$\begin{aligned}\delta\mathcal{E} &= \frac{1}{2}\left(\omega^{ij}{}_t\delta H_{ij\varphi} + \delta\omega_{ij\varphi}H^{ij}{}_t\right)\\ &\quad+\frac{1}{4}\left(\delta\omega^{ij}{}_t\Delta H_{ij\varphi} - \Delta\omega^{ij}{}_t\delta H_{ij\varphi} - \delta\omega^{ij}{}_\varphi\Delta H_{ijt} + \Delta\omega^{ij}{}_\varphi\Delta H_{ijt}\right)\\ &= \frac{1}{2}\left(\omega^{ij}{}_t\delta H_{ij\varphi} + \delta\omega_{ij\varphi}H^{ij}{}_t\right) + \mathcal{O}_1\,.\end{aligned} \tag{C.2}$$

Next, we prove that the surface term for time translations is finite:

$$\begin{aligned}\mathcal{E} &= 2a_0\left(\frac{r^2}{2\ell^2}\Omega^+{}_\varphi + \Omega^-{}_\varphi - r\Omega^2{}_u\right) + \mathcal{O}_0\\ &= -a_0\frac{r^3}{\ell^2}\left(\frac{1}{r}B^+{}_u + \frac{r}{\ell^2}B^+{}_r + \Omega^2{}_r\right) + \mathcal{O}_0 = \mathcal{O}_0\,.\end{aligned} \tag{C.3}$$

Finally, we derive the finiteness of the surface term for spatial rotations:

$$\begin{aligned}\mathcal{M} &= -a_0\varepsilon^{imn}\omega_{mn\varphi}\left(b_{i\varphi} + 2\ell^2 L_{(ij)}b^j{}_\varphi\right)\\ &= 2a_0\left(rB^2{}_u - B^-{}_\varphi - \frac{1}{2}\frac{r^2}{\ell^2}B^+{}_\varphi\right) - 4a_0\ell^2\left(r(Ric)_{(+2)} + \frac{r^2}{2\ell^2}(Ric)_{(-2)}\right) + \mathcal{O}_0\\ &= \mathcal{O}_0\,.\end{aligned} \tag{C.4}$$

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JHEP05(2015)101

Published for SISSA by Springer
Received: *October 8, 2014*
Accepted: *November 10, 2014*
Published: *November 25, 2014*

JHEP11(2014)141

Siklos waves with torsion in 3D

M. Blagojević and B. Cvetković

Institute of Physics, University of Belgrade,
Pregrevica 118, 11080 Belgrade-Zemun, Serbia

E-mail: mb@ipb.ac.rs, cbranislav@ipb.ac.rs

Abstract: Starting from the Siklos waves in general relativity with a cosmological constant, interpreted as gravitational waves on the anti-de Sitter background, a new class of exact torsion waves is constructed in the framework of three-dimensional gravity with propagating torsion. In the asymptotic limit, the geometry of torsion waves takes the anti-de Sitter form. In the sector with massless torsion modes, we found a set of asymptotic conditions that leads to the conformal asymptotic symmetry.

Keywords: Classical Theories of Gravity, Field Theories in Lower Dimensions, Conformal and W Symmetry

ArXiv ePrint: 1410.0800

doi:10.1007/JHEP11(2014)141

Contents

JHEP11(2014)141

1 Introduction

Exact gravitational waves have been an important subject of investigation in general relativity (GR) from the early 1920s; for a review, see [1–4]. Most of the activity on the subject has been focused on asymptotically flat models, the solutions of GR without a cosmological constant. From 1980s, exact gravitational waves have been studied also in GR with a cosmological constant (GR_Λ) [5–7], see also [8]; for higher-dimensional extensions, see [9–13]. In particular, exact gravitational waves with an AdS asymptotic behavior attracted a lot of interest in regard to the AdS/CFT correspondence [14, 15]. Moreover, some of these solutions "may serve as exact models of the propagation of primordial gravitational waves and may be relevant for the (hypothetical) cosmological wave background" [16].

To properly understand dynamical complexities of gravity, one often relies on technically simplified three-dimensional (3D) models (for a review and an extensive list of references, see [17, 18]). In 3D, both GR and GR_Λ are topological theories without propagating degrees of freedom, in which nontrivial wave solutions can exist only in the presence of *matter sources* [19–21]. To avoid such a degenerate situation, one is naturally motivated to study alternative gravitational models possessing true dynamical degrees of freedom. The well-known models of this type, formulated in the context of *Riemannian geometry* of spacetime, are Topological massive gravity and New massive gravity [22–24]. Their dynamical properties allow for the existence of gravitational waves *in vacuum*; see, for instance, Ayón-Beato et al. [25].

In the early 1960s, a new approach to gravitational dynamics was proposed, based on a modern, gauge-field-theoretic approach, known as the *Poincaré gauge theory* (PGT) (see [26, 27], for a textbook exposition of PGT, [28] for an up-to-date status of PGT, including its 3D version, and [29]) with an underlying *Riemann-Cartan (RC) geometry* of spacetime, characterized by both the *curvature* and the *torsion*. In a topological version of the three-dimensional PGT, gravitational waves with torsion were constructed in the presence of matter sources by Obukhov [30]. However, genuine gravitational waves are those that can propagate in spacetime regions without matter. Further investigations of the PGT, with a Lagrangian that is at most quadratic in the field strengths (quadratic PGT), revealed a rich dynamical structure, expressed, in particular, by the existence of propagating torsion modes [31]. In a recent paper [32],[1] we used quadratic PGT to construct exact torsion waves in vacuum as a generalization of the plane-fronted waves from GR.

In the present paper, we continue the investigation of genuine gravitational waves with torsion in 3D, by focusing on the anti-de Sitter (AdS) background. We found a new class of exact torsion waves in vacuum, representing a PGT extension of the Siklos waves in GR_Λ [33], see also [8, 16]. In the linear approximation, this class is associated to spin-2 torsion excitations around the AdS background. In the sector of massless torsion modes, we found a set of asymptotic conditions that leads to a conformal asymptotic symmetry, characterized by two independent Virasoro algebras with central charges. On the other hand, massive torsion waves show kind of an oscillatory behavior in the asymptotic region.

The paper is organized as follows. In section 2, we give an overview of the Siklos waves in the three-dimensional GR_Λ. In section 3, we construct a new wave solution in PGT, taking the metric to be of the Siklos form, whereas the torsion piece of the connection is assumed to possess only the tensorial irreducible component. The solutions of the field equations are found and classified according to the values of the mass parameter μ^2, associated to the spin-2 torsion modes. For $\mu^2 \geq 0$ (no tachyons), the asymptotic limit of the Siklos waves with torsion is shown to be represented by Riemannian AdS spacetimes. In section 4, we study the form of the AdS asymptotic conditions for $\mu^2 \geq 0$. It turns out that a well-defined asymptotic structure exists only in the massless sector. The corresponding central charges of the asymptotic symmetry are found in section 5, and section 6 is devoted to concluding remarks. Finally, two appendices contain some technical details.

Here are our conventions: the Latin indices $(i, j, k, \dots)$ refer to the local Lorentz (co)frame and run over $(+, -, 2)$, b^i is the triad field (coframe 1-form), h_i is the dual basis (frame), totally antisymmetric tensor ε^{ijk} is normalized to $\varepsilon^{+-2} = 1$; the Greek indices $(\mu, \nu, \rho, \dots)$ refer to the coordinate frame; the Lie dual of an antisymmetric form X^{jk} is $X_i := -\varepsilon_{ijk}X^{jk}/2$, the Hodge dual of a form α is ${}^\star\alpha$, and the exterior product of forms is implicit.

[1]Here, the reader can find references of earlier studies of exact gravitational waves with torsion in 4D.

2 Siklos waves

In 1980s, Siklos [33] found a special class of exact gravitational waves propagating on the AdS background, the physical interpretation of which was investigated in detail by Podolský [16]. In the Poincaré coordinates $x^\mu = (u, v, y)$, the Siklos metric in 3D has the form

$$ds^2 = \frac{\ell^2}{y^2}\left[2du(Hdu + dv) - dy^2\right], \tag{2.1}$$

with $H = H(u, y)$, which is equivalent to a subclass of the Kundt metric [8, 16]. The wave fronts are labeled by $u =$ const., v is an affine parameter along the corresponding rays generated by the Killing vector field ∂_v that is null but not covariantly constant, and for $H = 0$ the metric reduces to the AdS background (see appendix A). We choose the triad field b^i (1-form) to be

$$b^+ := \frac{\ell}{y}du\,, \quad b^- := \frac{\ell}{y}(Hdu + dv)\,, \quad b^2 = \frac{\ell}{y}dy\,, \tag{2.2}$$

so that the line element is given by $ds^2 = \eta_{ij}b^i b^j$, with the half-null Lorentz metric

$$\eta_{ij} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$

The dual frame basis h_i, defined by $h_i \rfloor b^j = \delta^j_i$, is given by

$$h_+ = \frac{y}{\ell}(\partial_u - H\partial_v)\,, \qquad h_- = \frac{y}{\ell}\partial_v\,, \qquad h_2 = \frac{y}{\ell}\partial_y\,.$$

The related Riemannian connection ω^{ij} (1-form) can be written in a compact form as

$$\omega^{ij} = \bar\omega^{ij} - \frac{1}{\ell}\varepsilon^{ij}{}_m k^m \left(yH'\right) k_n b^n\,. \tag{2.3a}$$

Here, prime denotes a derivative with respect to y, the first term $\bar\omega^{ij}$ describes the background AdS geometry,

$$\bar\omega^{+-} = 0\,, \qquad \bar\omega^{+2} = \frac{1}{\ell}b^+\,, \qquad \bar\omega^{-2} = \frac{1}{\ell}b^-\,, \tag{2.3b}$$

and the second one is the radiation piece, characterized by the null vector $k^m := (0, 1, 0)$, with $k_m = (1, 0, 0)$.

Next, we calculate the Riemannian curvature,

$$R^{ij} = \frac{1}{\ell^2}b^i b^j - \frac{1}{\ell^2}\varepsilon^{ij}{}_m k^m \left(y^2H'' - yH'\right) k^{n\star}b_n\,, \tag{2.4a}$$

whereupon the Ricci curvature $(Ric)^i = -h_j \rfloor R^{ij}$ and the scalar curvature $R = h_i \rfloor (Ric)^i$ are found to be

$$\begin{aligned}(Ric)^i &= \frac{2}{\ell^2}b^i + \frac{1}{\ell^2}k^i\left(y^2H'' - yH'\right)k_n b^n\,,\\ R &= \frac{6}{\ell^2}\,.\end{aligned} \tag{2.4b}$$

JHEP11(2014)141

When the Siklos metric satisfies the vacuum field equation of GR_Λ with $\Lambda \sim -1/\ell^2$, the metric function H takes a simple form:

$$y^2 H'' - yH' = 0 \quad \Rightarrow \quad H = D_1(u) + D_2(u) y^2 \,. \tag{2.5}$$

However, this solution is *trivial*. Indeed, since the radiation piece of the curvature vanishes on shell, we have $R^{ij} = b^i b^j/\ell^2$, and the geometry of spacetime is fixed, it has the AdS form. Nontrivial AdS waves can exist in GR_Λ only in the presence of matter [19–21], but to have *vacuum* AdS waves, one has to change the gravitational dynamics. As we shall see, transition to quadratic PGT allows the existence of genuine AdS waves with torsion.

3 Siklos waves with torsion

Basic gravitational variables of PGT are the triad field b^i and the Lorentz connection ω^{ij} (1-forms), and the related field strengths are the torsion $T^i = db^i + \omega^i{}_m b^m$ and the curvature $R^{ij} = d\omega^{ij} + \omega^i{}_m \omega^{mj}$ (2-forms). Relying on PGT, we now introduce a geometric extension of the Siklos waves (2.1) to genuine Siklos waves with torsion.

3.1 Ansatz

In order to *preserve the radiation nature* of the Siklos metric, we assume that the form of the triad field in PGT remains the same as in eq. (2.2). Essentially the same idea can be applied also to the connection [32]: starting from the Riemannian connection (2.3), we assume that the new, RC connection is given by

$$\omega^{ij} = \bar{\omega}^{ij} - \frac{1}{\ell}\varepsilon^{ij}{}_m k^m (yG) k_n b^n \,, \tag{3.1a}$$

where

$$G := H' + K \,, \qquad K = K(u,y) \,. \tag{3.1b}$$

Geometrically, the new function K in the connection is related to the torsion:

$$T^i := \nabla b^i = -\frac{yK}{\ell} k^i k^{n\star} b_n \,. \tag{3.2}$$

For $K = 0$, the torsion vanishes, and the connection becomes equivalent to $\bar{\omega}^{ij}$. The only nonvanishing irreducible component of T^i is its tensorial piece ${}^{(1)}T^i$ [32], so that

$${}^{(1)}T^i = T^i \,.$$

Using the above ansatz for the connection, one can calculate the RC curvatures:

$$\begin{aligned} R^{ij} &= \frac{1}{\ell^2} b^i b^j - \frac{1}{\ell^2}\varepsilon^{ij}{}_m k^m \left(y^2 G' - yH'\right) k^{n\star} b_n \,, \\ (Ric)^i &= \frac{2}{\ell^2} b^i + \frac{1}{\ell^2} k^i \left(y^2 G' - yH'\right) k_n b^n \,, \\ R &= \frac{6}{\ell^2} \,. \end{aligned} \tag{3.3}$$

The quadratic curvature invariant takes the form

$$R^{ij\star}R_{ij} = \frac{6}{\ell^4}{}^\star 1\,.$$

The only nonvanishing irreducible components of R^{ij} are:

$${}^{(6)}R^{ij} = \frac{1}{6}Rb^ib^j\,, \qquad {}^{(4)}R^{ij} = R^{ij} - {}^{(6)}R^{ij}\,.$$

For more details on the irreducible decomposition of the field strengths, see ref. [32].

In what follows, the specific forms of both the metric function H and the torsion function K will be determined by the PGT field equations.

3.2 Lagrangian dynamics of PGT

The PGT dynamics is described by a Lagrangian 3-form $L_G = L_G(b^i, T^i, R^{ij})$, which is assumed to be at most quadratic in the field strengths (quadratic PGT) and parity invariant. In conformity with our ansatz, the Lagrangian is chosen to have the form

$$\begin{aligned} L_G = &-a_0\varepsilon_{ijk}b^iR^{jk} - \frac{1}{3}\Lambda_0\varepsilon_{ijk}b^ib^jb^k \\ &+T^{i\,\star}\left(a_1{}^{(1)}T_i\right) + \frac{1}{2}R^{ij\,\star}\left(b_4{}^{(4)}R^{ij} + b_6{}^{(6)}R^{ij}\right)\,. \end{aligned} \tag{3.4}$$

Indeed, the only nonvanishing irreducible components of the field strengths appearing in L_G are ${}^{(1)}T^i$, ${}^{(4)}R^{ij}$ and ${}^{(6)}R^{ij}$, and a_1, b_4, b_6 are the corresponding coupling constants. Then, the PGT field equations in vacuum are found to be (see section III.A of ref. [32]):

$$\begin{aligned} (1ST):\quad & \left(a_0\ell^2 - b_4 - b_6\right)\left(yH'' - H'\right) + \left(a_0\ell^2 - a_1\ell^2 - b_4 - b_6\right)yK' = 0\,, \\ & 2a_0\ell^2 + b_6 + 2\ell^4\Lambda_0 = 0\,, \\ (2ND):\quad & b_4\left[y^2\left(H''' + K''\right) + yK'\right] - \left(a_0\ell^2 - a_1\ell^2 - b_6\right)K = 0\,. \end{aligned} \tag{3.5}$$

These equations are checked using the Excalc package of the computer algebra system Reduce. Using the expression for $(1ST)'$, one finds that $(2ND)$ can be rewritten as

$$y^2K'' + yK' + \ell^2\mu^2K = 0\,, \qquad \mu^2 = \frac{(a_1 - a_0 - b_6\lambda)(a_0 + b_4\lambda + b_6\lambda)}{b_4a_1}\,,$$

where $\lambda := -1/\ell^2$. Finally, after introducing the notation

$$\hat{y} = \frac{y}{\ell}\,, \qquad m^2 = \ell^2\mu^2\,,$$

the two field equations take a more compact form:

$$\begin{aligned} (1ST) \quad & \hat{y}H'' - H' = \ell C\hat{y}K'\,, \qquad C := \frac{a_1}{a_0 + b_4\lambda + b_6\lambda} - 1\,, \\ (2ND) \quad & \hat{y}^2K'' + \hat{y}K' + m^2K = 0\,, \end{aligned} \tag{3.6}$$

where prime now denotes differentiation with respect to $\hat{y}$. As one can see, it is the presence of torsion ($K \neq 0$) that makes the metric of the AdS wave nontrivial ($\hat{y}H'' - H' \neq 0$). Equations (3.6) define a new class of Siklos waves — the Siklos waves with torsion.

JHEP11(2014)141

3.3 Solutions

The coefficient m^2 in $(2ND)$ is the (dimensionless) mass parameter associated to the spin-2 excitation of the torsion field around the AdS background, see [31, 32]. The absence of tachyons requires $m^2 \geq 0$. In this subsection, we construct the exact Sikos waves with torsion, and classify them according to the values of m^2.

(1) $m^2 > 0$. The Euler (or Euler-Fuchs, Euler-Cauchy) differential equation $(2ND)$ is solved by the ansatz $K = \hat{y}^\alpha$, which yields $\alpha^2 + m^2 = 0$. For $m^2 > 0$, we have $\alpha = \pm im$, so that $K = \hat{y}^{\pm im} = e^{\pm im \ln \hat{y}}$, or equivalently,

$$K = A(u)\cos(m \ln \hat{y}) + B(u)\sin(m \ln \hat{y})\,. \tag{3.7a}$$

By substituting this result into $(1ST)$, one finds the related solution for H:

$$H = D_1 + D_2\hat{y}^2 + \frac{\ell C m}{1+m^2}\hat{y}\left[A(u)\sin(m \ln \hat{y}) - B(u)\cos(m \ln \hat{y})\right]\,. \tag{3.7b}$$

The first two terms, which represent a solution of the homogeneous equation $\hat{y}H'' - H' = 0$, can be geometrically disregarded, as they do not influence the values of the field strengths.

In the asymptotic limit $\hat{y} \to 0$, the torsion and the radiation piece of the curvature, ${}^{(4)}R_{ij}$, vanish, as follows from the relations

$$\begin{aligned} \lim_{\hat{y}\to 0} \hat{y}K &= 0\,, \\ \lim_{\hat{y}\to 0}\Big[\hat{y}^2(H''+K') - \hat{y}H'\Big] = \lim_{\hat{y}\to 0}\Big[\hat{y}^2K' + \ell C\hat{y}^2K'\Big] &= 0\,. \end{aligned} \tag{3.8}$$

Thus, the asymptotic geometry of our solution is given by the Riemannian AdS spacetime.

(2) $m^2 = 0$. In order to have a smooth Minkowskian limit for $\ell^2 \to \infty$, the condition $m^2 = 0$ is realized by demanding [31]

$$a_1 - a_0 + b_6/\ell^2 = 0\,. \tag{3.9}$$

As a consequence, the solution for the massless torsion wave is given by

$$\begin{aligned} K &= C_1 + C_2 \ln \hat{y}\,, \\ H &= D_1 + D_2\hat{y}^2 - \ell C C_2 \hat{y}\,. \end{aligned} \tag{3.10}$$

As before, one can choose $D_1 = D_2 = 0$ without loss of generality, so that the asymptotic limit of the solution is again given by the Riemannian AdS spacetime.

(3) $m^2 < 0$. Although the spin-2 torsion modes are now tachyons, we present the related exact wave solution, for the sake of completeness:

$$\begin{aligned} K &= A\hat{y}^m + B\hat{y}^{-m}\,, \\ H &= \frac{\ell C m}{m^2-1}\left(A\hat{y}^{1+m} - B\hat{y}^{1-m}\right)\,. \end{aligned} \tag{3.11}$$

The asymptotic behavior depends on the value of m.

JHEP11(2014)141

4 Asymptotic conditions

In our study of the asymptotic conditions, we assume that the topology of the spacetime manifold M is $R\times\Sigma$, where R is interpreted as time, and Σ is a spatial section of spacetime, whose boundary $\partial\Sigma$ is topologically a circle. The asymptotic analysis is simplified by introducing a new set of local coordinates (t,φ), given by $u=(t+\ell\varphi)/\sqrt{2}$, $v=(t-\ell\varphi)/\sqrt{2}$, such that the boundary $\partial\Sigma$ at $y=0$ is parametrized by the angular coordinate φ.

As we have seen in the previous section, in the asymptotic limit $y\to 0$, the geometry of our torsion wave is described by the Riemannian AdS spacetime. This property motivates us to examine asymptotic conditions based on the following requirements:

(a) asymptotic configurations include the torsion wave geometry;

(b) they are invariant under the action of the AdS group SO(2, 2);

(c) asymptotic symmetries have well defined canonical generators.

Specific aspects of these criteria depend on the value of the mass parameter μ^2.

4.1 Massive torsion waves

For $\mu^2>0$, the characteristic functions H and K can be represented in the form

$$H=yW_0\,,\qquad K=W_0\,, \tag{4.1a}$$

where W_0 is a *generic* wave "oscillatory" function,

$$W_0:=C_1(u)\cos(m\ln y/\ell)+C_2(u)\sin(m\ln y/\ell)\,. \tag{4.1b}$$

In spite of this oscillatory behavior, both the torsion and the wave piece of the curvature tend to zero when $y\to 0$.

In the matrix notation, the components of the Siklos metric (2.1) read

$$g_{\mu\nu}=\frac{\ell^2}{y^2}\begin{pmatrix}2H & 1 & 0\\ 1 & 0 & 0\\ 0 & 0 & -1\end{pmatrix}.$$

Asymptotically, for $y\to 0$, we have $g_{uu}\sim W_0/y$, so that, to leading order in $1/y$, $g_{\mu\nu}$ reduces to the AdS metric $\bar g_{\mu\nu}$. In the asymptotic analysis, we use $\mathcal{O}(y^nW_0)$ to denote a term that is *at most* proportional to y^nW_0 when $y\to 0$. Thus, the Siklos metric is of the type

$$g_{\mu\nu}=\bar g_{\mu\nu}+G_{\mu\nu}\,,\qquad G_{\mu\nu}:=\begin{pmatrix}\mathcal{O}(W_0/y) & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}.$$

Looking at the action of the AdS Killing vectors (appendix A) on $g_{\mu\nu}$, one finds that the general requirements (a) and (b) are fulfilled by the following asymptotic configurations:

$$g_{\mu\nu}=\bar g_{\mu\nu}+G_{\mu\nu}\,,\qquad G_{\mu\nu}:=\begin{pmatrix}\mathcal{O}_{-1} & \mathcal{O}_0 & \mathcal{O}_0\\ \mathcal{O}_0 & \mathcal{O}_0 & \mathcal{O}_0\\ \mathcal{O}_0 & \mathcal{O}_0 & \mathcal{O}_0\end{pmatrix}, \tag{4.2}$$

where $\mathcal{O}_n := \mathcal{O}(y^n W_0)$. The asymptotic form (4.2), but with $\mathcal{O}_n = \mathcal{O}(y^n)$, was studied earlier by Afshar et al. [34, 35], in the context of Conformal Chern-Simons gravity.

The asymptotic conditions (4.2) are preserved by the local translations of the form

$$\begin{aligned}
\xi^u &= \varepsilon^u(u) + \frac{y^2}{4}\partial_v^2\varepsilon^v(v) + \mathcal{O}_3\,,\\
\xi^v &= \varepsilon^v(v) + \frac{y^2}{4}\partial_u^2\varepsilon^u(u) + \mathcal{O}_3\,,\\
\xi^2 &= \frac{y}{2}(\partial_u\varepsilon^u + \partial_v\varepsilon^v) + \mathcal{O}_3\,.
\end{aligned} \tag{4.3}$$

These parameters are essentially of the Brown-Henneaux type [35, 36].

In the next step, one could try to extend these considerations to the variables b^i and ω^{ij}. However, a problem arises when we return to our general requirement (c). Namely, although the field strengths T^i and R^{ij} have an AdS asymptotic limit, the asymptotic behavior of b^i and ω^{ij} is determined by the function W_0, which oscillates when $y \to 0$. Thus, the basic dynamical variables have *no asymptotic limit*, and one is not able to define surface terms of the canonical generators. Thus, one cannot formulate a boundary theory, and in particular, the AdS/CFT correspondence is not well defined.

4.2 Massless torsion waves

In the sector with massless torsion modes, the form of our wave solution is displayed in eq. (3.10). As we noted before, the geometrically irrelevant term $D_1 + D_2 y^2$ in H can be removed by choosing $D_1 = D_2 = 0$, whereupon the characteristic functions H and K are of the generic form

$$H = C_0(y/\ell)\,, \qquad K = C_1 + C_2\ln(y/\ell)\,. \tag{4.4}$$

The asymptotic geometry of the solution is described by the AdS spacetime. In this section, we discuss the asymptotic structure of the massless torsion wave (4.4).

Quite generally, the wave triad (2.2) can be written in the form $b^i{}_\mu = \bar{b}^i{}_\mu + B^i{}_\mu$, where $\bar{b}^i$ is the AdS triad, and the only nonvanishing component of $B^i{}_\mu$ is $B^-{}_u = \ell H/y = C_0$. Then, in accordance with the general requirements (a) and (b), we choose the following asymptotic form of the triad field:

$$b^i{}_\mu = \bar{b}^i{}_\mu + B^i{}_\mu\,, \qquad B^i{}_\mu := \begin{pmatrix} \mathcal{O}_1 & \mathcal{O}_1 & \mathcal{O}_1\\ \mathcal{O}_0 & \mathcal{O}_1 & \mathcal{O}_1\\ \mathcal{O}_1 & \mathcal{O}_1 & \mathcal{O}_1 \end{pmatrix}\,, \tag{4.5}$$

where $\mathcal{O}_n := \mathcal{O}(y^n)$. These conditions impose the following restriction on the local Poincaré parameters $(\xi^\rho, \varepsilon^{ij})$:

$$\delta_0 b^i{}_\mu := \varepsilon^{ijk}\theta_j b_{k\mu} - (\partial_\mu\xi^\rho)b^i{}_\rho - \xi^\rho\partial_\rho b^i{}_\mu = B^i{}_\mu\,,$$

where θ^i is the Lie dual of ε_{mn}. As a consequence, the asymptotic parameters of local translations take the form displayed in eq. (4.3), whereas the asymptotic parameters of

Lorentz rotations are found to be

$$\begin{aligned}\theta^+ &= \frac{y}{2}\partial_v^2\varepsilon^v + \mathcal{O}_2\,,\\ \theta^- &= -\frac{y}{2}\partial_u^2\varepsilon^u + \mathcal{O}_2\,,\\ \theta^2 &= \frac{1}{2}(\partial_v\varepsilon^v - \partial_u\varepsilon^u) + \mathcal{O}_2\,.\end{aligned} \tag{4.6}$$

Next, we wish to examine whether the asymptotic behavior of the RC connection (3.1) can be made compatible with the already found form of the asymptotic Poincaré parameters. First, we introduce the Lie-dual connection ω^i:

$$\omega^+ = \frac{1}{\ell}b^+\,, \qquad \omega^- = -\frac{1}{\ell}b^- + \frac{y}{\ell}Gb^+\,, \qquad \omega^2 = 0\,. \tag{4.7}$$

The form of K implies that the asymptotic conditions on the connection should contain log terms. By combining the expression (4.7) for ${\omega^i}_\mu$ with the asymptotic formulas for $b^\pm$ and $G = H' + K$, we find it suitable to assume

$${\omega^i}_\mu = {\bar\omega^i}_\mu + {\Omega^i}_\mu\,, \qquad {\Omega^i}_\mu := \frac{1}{\ell}\begin{pmatrix} \mathcal{O}_1 & \mathcal{O}_1 & \mathcal{O}_1 \\ \mathcal{O}(\ln y/\ell) & \mathcal{O}_1 & \mathcal{O}(y\ln y/\ell) \\ \mathcal{O}(y\ln y/\ell) & \mathcal{O}_1 & \mathcal{O}_1 \end{pmatrix}. \tag{4.8}$$

As it turns out, the asymptotic invariance of ${\omega^i}_\mu$,

$$\delta_0{\omega^i}_\mu := -\partial_\mu\theta^i - \varepsilon^{ijk}\omega_{j\mu}\theta_k - \partial_\mu\xi^\rho{\omega^i}_\rho - \xi^\rho\partial_\rho{\omega^i}_\mu = {\Omega^i}_\mu\,,$$

does not impose any new restriction of the asymptotic Poincaré parameters (4.3) and (4.6).

In order to clarify the interpretation of our asymptotic conditions, we wish to find the commutator algebra of the asymptotic Poincaré transformations. To do that, we note that the composition law of the asymptotic transformations, to lowest order in y, reads

$$(\varepsilon^u)''' = (\varepsilon^u)'\partial_u(\varepsilon^u)'' - (\varepsilon^u)''\partial_u(\varepsilon^u)'\,, \tag{4.9}$$

and similarly for ε^v. Then, introducing the notation

$$\ell_n^+ := -\frac{1}{\sqrt{2}}\delta_0\left(\varepsilon^u = \ell e^{inu\sqrt{2}/\ell}, \varepsilon^v = 0\right), \qquad \ell_n^- := -\frac{1}{\sqrt{2}}\delta_0\left(\varepsilon^u = 0, \varepsilon^v = \ell e^{inv\sqrt{2}/\ell}\right),$$

the commutator algebra of the asymptotic symmetry takes the form of two independent Virasoro algebras:

$$i\left[\ell_m^\pm, \ell_n^\pm\right] = (m-n)\ell_{m+n}^\pm\,. \tag{4.10}$$

The related central charges are discussed in the next section.

5 Canonical form of the asymptotic symmetry

In this section, we use the canonical approach to analyze the asymptotic symmetry in the massless sector, including the values of the central charges.

JHEP11(2014)141

To simplify the analysis, we follow Nester [37] in applying the *first-order formulation* to the quadratic PGT. In this formalism, the Lagrangian (3.4) is written in the form

$$L_G = T^i\tau_i + \frac{1}{2}R^{ij}\rho_{ij} - V\left(b^i, \tau_i, \rho_{ij}\right) - \frac{1}{3}\Lambda\varepsilon_{ijk}b^ib^jb^k\,. \tag{5.1}$$

Here, τ_i and ρ_{ij} are new, *independent* variables, and V is a function quadratic in τ_i and ρ_{ij}, chosen so that, on shell, we have $\tau_i = H_i$ and $\rho_{ij} = H_{ij}$, where $H_i = \partial L_G/\partial T^i$ and $H_{ij} = \partial L_G/\partial R^{ij}$ are the covariant field momenta associated to the original Lagrangian (3.4). Explicit form of V is described in ref. [31], and it ensures the first-order formulation (5.1) to be *equivalent* to (3.4). Thus, the variation of L_G with respect to τ_i and ρ_{ij} yields

$$\begin{aligned} \tau_i &= 2a_1{}^\star T_i\,, \\ \rho_{ij} &= -2\left(a_0 - \frac{1}{6}b_6R\right)\varepsilon_{ijk}b^k + 2b_4{}^{\star(4)}R_{ij}\,, \end{aligned} \tag{5.2}$$

in accordance with the forms of H_i and H_{ij} defined by the Lagrangian (3.4).

Asymptotic symmetries are best described in the canonical formalism. In the first order formulation of PGT, the canonical gauge generator is a functional $G[\varphi,\pi]$ on the phase space, the form of which is defined in eqs. (5.7) of ref. [31]. The canonical generator acts on the phase-space variables (φ,π) via the Poisson (or Dirac) bracket operation, defined in terms of the functional derivatives. A functional $F[\varphi,\pi] = \int d^2x f(\varphi,\partial_\alpha\varphi,\pi,\partial_\alpha\pi)$ is differentiable (or regular) if its variation has the form $\delta F = \int d^2x\,[A(x)\delta\varphi + B(x)\delta\pi]$. In order to ensure this property for our generator G, we have to improve its form by adding an appropriate surface term Γ [38]. The improved canonical generator $\tilde G := G + \Gamma$ has been calculated in appendix B; it is both finite and differentiable (well-defined).

The Poisson bracket (PB) algebra of the improved generators could be found by a direct calculation, but we rather rely on another, more instructive method. Introducing a convenient notation, $\tilde G' = \tilde G[\varepsilon^{u\prime}, \varepsilon^{v\prime}]$ and similarly for $\tilde G''$ and $\tilde G'''$, we use the main theorem of ref. [39], which states that the PB of two well-defined generators must also be a well-defined generator, to conclude that the PB algebra has the form

$$\{\tilde G'', \tilde G'\} = \tilde G''' + C'''\,. \tag{5.3a}$$

Here, the parameters of $\tilde G'''$ are defined by the composition law (4.9), and C''' is the central charge of the algebra. A simple reformulation of this formula, given by

$$\{\tilde G'', \tilde G'\} = \delta_0'\tilde G'' \approx \delta_0'\Gamma''\,, \tag{5.3b}$$

represents a powerful tool for calculating the central charge. Indeed, the previous two equations imply

$$\delta_0'\Gamma'' \approx \Gamma''' + C'''\,. \tag{5.3c}$$

Now, since C''' does not depend on the basic dynamical variables and Γ''' vanishes on the AdS background (see appendix B), the evaluation of $\delta_0'\Gamma''$ on the AdS background yields the final expression for C''':

$$\overline{\delta_0'\Gamma''} = C'''\,. \tag{5.4}$$

JHEP11(2014)141

An explicit calculation based on the results of appendix B yields

$$\frac{\sqrt{2}}{\ell}C''' = -\left(a_0 - \frac{b_6}{\ell^2}\right)\int_0^{2\pi} d\varphi \left(\varepsilon^{u\prime\prime}\partial_u^3\varepsilon^{u\prime} + \varepsilon^{v\prime\prime}\partial_v^3\varepsilon^{v\prime}\right) . \tag{5.5}$$

This result, combined with eq. (5.3a), completes the derivation of the canonical PB algebra.

A more familiar form of this algebra is obtained by introducing the Fourier modes of the improved generator:

$$L_n^+ := -\frac{1}{\sqrt{2}}\tilde{G}\left(\varepsilon^u = \ell e^{inu\sqrt{2}/l}, \varepsilon^v = 0\right) , \qquad L_n^- := -\frac{1}{\sqrt{2}}\tilde{G}\left(\varepsilon^v = \ell e^{inv\sqrt{2}/l}, \varepsilon^u = 0\right) .$$

Then, the canonical algebra (5.3a) takes the form of two independent Virasoro algebras with central charges,

$$i\left\{L_m^\pm, L_n^\pm\right\} = (m-n)L_{m+n}^\pm + \frac{c^\pm}{12}m^3\delta_{m+n} , \tag{5.6}$$

where the central charges are equal to each other:

$$c^\pm = \left(1 - \frac{b_6}{a_0\ell^2}\right)c_0 . \tag{5.7}$$

Note that the coupling constant b_6 modifies the GR_Λ central charge $c_0 := 3\ell/2G$, and for $b_6 < a_0\ell^2$, the central charge $c^\pm$ is positive.

6 Concluding remarks

In this paper, we found a new class of exact vacuum solutions of the three-dimensional PGT, the class of Siklos waves with torsion. Asymptotic geometry of these solutions is described by the Riemannian AdS spacetime. In the sector of massless torsion modes, we found a set of asymptotic conditions for which the asymptotic symmetry is described by two independent Virasoro algebras with equal central charges $c^\pm$, the values of which differ from the GR_Λ result.

Further studies of the massless sector might help us to clarify the role of torsion in the AdS/CFT correspondence.

Acknowledgments

This work was supported by the Serbian Science Foundation under Grant No. 171031.

A AdS and Siklos spacetimes in 3D

In this appendix, we review basic aspects of the three-dimensional AdS and Siklos spacetimes; see for instance [4, 40, 41] and [7, 8, 16], respectively.

The AdS space in 3D, with topology $S^1 \times R^2$, can be defined in terms of the hypersurface

$$H_3 : \qquad \bar{u}^2 - \bar{x}^2 - \bar{y}^2 + \bar{v}^2 = \ell^2 ,$$

embedded in a 4-dimensional Minkowski space with metric $\eta_{ab} = (1, -1, -1, 1)$. The metric on H_3 has the form

$$ds^2 = d\bar{u}^2 - d\bar{x}^2 - d\bar{y}^2 + d\bar{v}^2\,, \tag{A.1}$$

its isometry group is $\mathrm{SO}(2,2)$, and the scalar curvature is $R = 6/\ell^2$.

The space H_3 can be covered by the global coordinates (t, ρ, φ),

$$\begin{aligned} \bar{u} &= \ell \cosh\rho \cos t\,, & \bar{x} &= \ell \sinh\rho \cos\varphi\,, \\ \bar{v} &= \ell \cosh\rho \sin t\,, & \bar{y} &= \ell \sinh\rho \sin\varphi\,, \end{aligned}$$

with $t \in [-\pi, \pi], \rho \in [0, \infty)$, for which the metric takes the form

$$ds^2 = \ell^2 \left[dt^2 \cosh^2\rho - \left(d\rho^2 + \sinh^2\rho d\varphi^2\right)\right]. \tag{A.2}$$

However, since t is an angle, there are closed timelike curves in H_3. The problem can be cured by replacing the S^1 time $t \in [-\pi, \pi]$ by a new, R^1 time $t \in (-\infty, +\infty)$, changing thereby the topology from $S^1 \times R^2$ to R^3. The space obtained in this way is known as the *universal covering* of the AdS space. According to the commonly accepted terminology, it is this space that is called the AdS space; we denote it by AdS_3. A simple form of the AdS_3 metric is obtained in the Schwarzschild-like coordinates $r = \ell \sinh\rho, \ell t \to t$.

Let us now parametrize AdS_3 by introducing the Poincaré coordinates:

$$\tau = \frac{-\bar{v}}{\bar{u} + \bar{x}}\,, \qquad x = \frac{\bar{y}}{\bar{u} + \bar{x}}\,, \qquad y = \frac{\ell}{\bar{u} + \bar{x}}\,.$$

They do not cover the whole space, but only one of the regions where $\bar{u} + \bar{x}$ has a definite sign. In these regions, the metric has the form

$$ds^2 = \frac{\ell^2}{y^2}\left(2du dv - dy^2\right), \tag{A.3}$$

where $u = (\tau + x)/\sqrt{2}$, $v = (\tau - x)/\sqrt{2}$, and the boundary is located at $y = 0$.

The Killing vectors $\xi = \xi^\mu \partial_\mu$ for the metric (A.3) are defined by the conditions

$$\delta_0 g_{\mu\nu} := -\partial_\mu \xi^\rho g_{\rho\nu} - \partial_\nu \xi^\rho g_{\rho\mu} - \xi^\rho \partial_\rho g_{\mu\nu} = 0\,.$$

They produce a set of requirements on ξ^μ, the solutions of which define a basis of six independent AdS Killing vectors $\xi_{(m)}$:

$$\begin{aligned} \xi_{(1)} &= (\ell, 0, 0)\,, & \xi_{(4)} &= (0, 2v, y)\,, \\ \xi_{(2)} &= (0, \ell, 0)\,, & \xi_{(5)} &= \left(\frac{u^2}{\ell}, \frac{y^2}{2\ell}, \frac{uy}{\ell}\right), \\ \xi_{(3)} &= (u, -v, 0)\,, & \xi_{(6)} &= \left(\frac{y^2}{2\ell}, \frac{v^2}{\ell}, \frac{vy}{\ell}\right). \end{aligned} \tag{A.4}$$

Turning now to the class of Siklos spacetimes (2.1), we note that it is equivalent to a subclass of Kundt spacetimes, defined by the metric

$$ds^2 = 2\left(\frac{q}{p}\right)^2 dU\left(\bar{H}dU + dV\right) - \frac{1}{p^2}dY^2\,, \tag{A.5}$$

where $\bar{H} = \bar{H}(U, Y)$, and

$$p := 1 + \frac{\lambda}{4} Y^2 \,, \qquad q := \left(1 + \sqrt{-\frac{\lambda}{4}}\, Y\right)^2 \,,$$

with $\lambda := -1/\ell^2$. Indeed, by introducing the new coordinates

$$Y = -2\ell \frac{y + 1/2}{y - 1/2} \,, \qquad U = 2\ell u \,, \qquad V = 2\ell v \,,$$

one ends up with the Siklos metric (2.1), where the new function $H = H(u, y)$ is defined by $H(u, y) := \bar{H}(U, Y)|_{U=U(u), Y=Y(y)}$.

For general H, the only Killing vector of the Siklos metric is $\xi_{(2)} = \ell \partial_v$, but for some specific forms of H there can be more Killing vectors; for instance, $\xi_{(1)} = \ell \partial_u$ when H is independent of u, or the maximal number of six Killing vectors (A.4) when $H = 0$.

B Improving the canonical generator

In this appendix, we construct the improved gauge generator for the massless sector of our solution.

Gauge symmetries of the first-order Lagrangian (5.1) are described by the canonical gauge generator G, the form of which can be found in eqs. (5.7) of ref. [31]. To examine the differentiability of G, we start from the form of its variation:

$$\begin{aligned}
\delta G &= -\int_\Sigma d^2x (\delta G_1 + \delta G_2) \,, \\
\delta G_1 &= -\varepsilon^{t\alpha\beta} \xi^\mu \left(b^i{}_\mu \partial_\alpha \delta\tau_{i\beta} + \omega^i{}_\mu \partial_\alpha \delta\rho_{i\beta} + \tau^i{}_\mu \partial_\alpha \delta b_{i\beta} + \rho^i{}_\mu \partial_\alpha \delta\omega_{i\beta} \right) + R \,, \\
\delta G_2 &= -\varepsilon^{t\alpha\beta} \theta^i \partial_\alpha \delta\rho_{i\beta} + R \,.
\end{aligned} \tag{B.1}$$

Here, the variation is performed in the set of asymptotic states, R stands for regular terms and ρ^i is the Lie dual of ρ_{mn}:

$$\rho_i = 2\left(a_0 - \frac{b_6}{6} R\right) b_i + 2b_4 \left((Ric)_{(ik)} - \frac{1}{3} R\eta_{ik}\right) b^k \,.$$

Moreover, the coherently oriented volume 2-form on Σ, expressed in the new coordinates (t, φ, y), is normalized to $d^2x = dy d\varphi$. Together with $\varepsilon^{y\varphi} := \varepsilon^{ty\varphi} = 1$, this is in accordance with the conventions used in ref. [31].

As one can see, G is not differentiable, but the problem can be corrected by going over to the improved canonical generator $\tilde{G} := G + \Gamma$, where the surface term Γ is constructed so that $\delta\tilde{G} = R$. In the process, transition to surface integrals is performed with the help of the Stokes formula:

$$\int_\Sigma d^2x\, \partial_\alpha v^\alpha = \int_{\partial\Sigma} df_\alpha v^\alpha = \int_0^{2\pi} d\varphi v^y \,, \qquad df_\alpha = \varepsilon_{\alpha\beta} dx^\beta \,.$$

JHEP11(2014)141

Thus, using (B.1) and the asymptotic conditions (4.5) and (4.8), the surface term Γ in the improved generator $\tilde{G} \equiv G + \Gamma$ is found to have the following form:

$$\begin{aligned}
\Gamma &= \Gamma_u + \Gamma_v \,, \\
\frac{\sqrt{2}}{\ell}\Gamma_u &= -2\left(a_0 - \frac{b_6}{\ell^2}\right)\int_0^{2\pi} d\varphi \epsilon^u \frac{1}{y}\left(B^-{}_u - B^-{}_v\right) + 2\ell a_1 \int_0^{2\pi} d\varphi \epsilon^u \partial_y \left(B^-{}_u - B^-{}_v\right) \\
&\quad + \frac{2b_4}{\ell}\int_0^{2\pi} d\varphi \epsilon^u \left(\partial_y \Omega^-{}_u - \partial_u \Omega^-{}_y + \frac{1}{y}\frac{B^-{}_u}{\ell}\right) , \qquad \text{(B.2a)} \\
\frac{\sqrt{2}}{\ell}\Gamma_v &= 2\left(a_0 - \frac{b_6}{\ell^2}\right)\int_0^{2\pi} d\varphi \epsilon^v \frac{\ell}{y}\left(\Omega^+{}_u - \Omega^+{}_v + \frac{1}{\ell}B^+{}_u - \frac{1}{\ell}B^+{}_v\right) . \qquad \text{(B.2b)}
\end{aligned}$$

The result for Γ_u is simplified with the help of the condition $a_0 - b_6/\ell^2 - a_1 = 0$, which is used in eq. (3.9) to define the massless sector of the torsion wave. The factors $\sqrt{2}/\ell$ appear as an effect of the change of coordinates $(t, \varphi) \to (u, v)$ in the components of B^i and Ω^i.

The above construction shows that $\tilde{G}$ is differentiable provided it is finite, and the finiteness of $\tilde{G}$ follows from the finiteness of $\Gamma \equiv \Gamma_u + \Gamma_v$. The term Γ_v is seen to be finite directly from the adopted asymptotic conditions, whereas the finiteness of Γ_u depends on the validity of an additional relation:

$$-\left(a_0 - \frac{b_6}{\ell^2} - \frac{b_4}{\ell^2}\right) B^-{}_u + \frac{b_4}{\ell} y \partial_y \Omega^-{}_u = \mathcal{O}_1 \,. \qquad \text{(B.3)}$$

To clarify this situation, we note that the original set of the asymptotic conditions, given in eqs. (4.5) and (4.8), can be extended using the following general principle: the expressions that vanish on-shell should have an arbitrarily fast asymptotic decrease, as no solution of the field equations is thereby lost. This principle allow us to derive the needed relation (B.3) as the ($\mu = v$, $i = +$) component of the field equation

$$\varepsilon^{\mu\nu\rho}\left(\nabla_\mu \rho_{i\nu} + \varepsilon_{ijk} b^j{}_\nu \tau^k{}_\rho\right) = 0 \,. \qquad \text{(B.4)}$$

The surface terms (B.2) are used in section 5 to calculate the canonical algebra of the improved gauge generators. Note, in particular, that Γ vanishes on the AdS background.

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Published for SISSA by Springer
Received: *January 11, 2013*
Accepted: *May 2, 2013*
Published: *May 21, 2013*

Holography in 3D AdS gravity with torsion

Milutin Blagojević,[a] Branislav Cvetković,[a] Olivera Miskovic[b] and Rodrigo Olea[c]

[a] *Institute of Physics, University of Belgrade, P.O. Box 57, 11001 Belgrade, Serbia*
[b] *Instituto de Física, Pontificia Universidad Católica de Valparaíso, Casilla 4059, Valparaíso, Chile*
[c] *Universidad Andres Bello, Departamento de Ciencias Físicas, República 220, Santiago, Chile*

E-mail: mb@ipb.ac.rs, cbranislav@ipb.ac.rs, olivera.miskovic@ucv.cl, rodrigo.olea@unab.cl

Abstract: Basic aspects of the AdS/CFT correspondence are studied in the framework of 3-dimensional gravity with torsion. After choosing a consistent holographic ansatz, we formulate an improved approach to the Noether-Ward identities for the boundary theory. The method is applied first to the topological Mielke-Baekler model, and then to the more interesting (parity-preserving) 3-dimensional gravity with propagating torsion. In both cases, we find the finite holographic energy-momentum and spin currents and obtain the associated (anomalous) Noether-Ward identities.

Keywords: AdS-CFT Correspondence, Classical Theories of Gravity, Space-Time Symmetries, Anomalies in Field and String Theories

ArXiv ePrint: 1301.1237

doi:10.1007/JHEP05(2013)103

JHEP05(2013)103

Contents

1 Introduction

According to the idea of AdS/CFT correspondence [1], to any asymptotically anti-de Sitter (AdS) gravitational theory on a $(d+1)$-dimensional spacetime M, there corresponds a d-dimensional conformal field theory (CFT) on the boundary ∂M. This duality is of the weak/strong coupling type: the weak coupling regime of the gravitational theory is related to the strong coupling regime of the boundary CFT, and vice versa.

JHEP05(2013)103

Following a widely spread belief that general relativity (GR) is the most reliable approach for studying the gravitational phenomena, the analysis of the AdS/CFT correspondence has been carried out mostly in the realm of Riemannian geometry, leading to a number of highly interesting results [2, 3]. However, one should note that, for nearly five decades, there exists a modern gauge-theoretic conception of gravity, characterized by a Riemann-Cartan geometry of spacetime. In this approach, known as Poincaré gauge theory (PGT) [4–6], both the torsion and the curvature carry the gravitational dynamics. In spite of its well-founded dynamical structure, the use of this framework for studying the AdS/CFT correspondence is still in a rather rudimentary phase. In this regard, we wish to mention the work of Bañados et al. [7], who studied the holographic currents in the 5-dimensional (5D) Chern-Simons gravity with torsion, and the paper of Klemm and Tagliabue [8], investigating the holographic structure of the Mielke-Baekler (MB) model of 3D gravity with torsion [9]. In 4D, Petkou [10] examined holographic aspects of Einstein-Cartan theory amended by topological torsional invariants.

In order to properly understand dynamical features of gravity with torsion, one is naturally led to consider technically simplified models with the same conceptual features. An important and useful model of this type is the MB model of topological 3D gravity with torsion [9], introduced in the early 1990s. Further investigations along these lines led to a number of remarkable results; for more details, see [11, 12] and references therein. Of particular interest for our present work is the existence of a holographic structure, as discussed in [8]. However, in the MB model (like in GR with a cosmological constant) there are no propagating degrees of freedom. In order to overcome this unrealistic feature of the gravitational dynamics, a systematic study of 3D gravity with *propagating* torsion has been recently initiated in [12], see also [13]. The present work is aimed at investigating holographic aspects of 3D gravity with (propagating) torsion, in order to reexamine the compatibility of the concept of torsion with the basic aspects of the AdS/CFT correspondence, and moreover, to understand the dynamical role of the new CFT sources associated with torsion.

The paper is organized as follows. In section 2, we discuss general holographic features of 3D gravity with torsion, *with or without* the propagating torsion modes. After choosing a suitable ansatz for the gravitational variables, we derive the related consistency conditions, which tell us that the maximal boundary symmetry consists of the local Poincaré transformations and dilatations. In section 3, we propose an improved treatment of the corresponding Noether-Ward identities for the boundary theory. In section 4, we use this approach to reexamine the holographic structure of the *topological* 3D gravity with torsion; our results confirm the analysis of Klemm and Tagliabue [8], based on a different technique. Then, in section 5, we turn to the main subject of the present work — the study of holography in 3D gravity with *propagating* torsion. We find that the maximal boundary symmetry is reduced by the existence of the conformal anomaly. The improved formalism ensures that these results do not depend on the value of torsion on the boundary.

Our conventions are given by the following rules. In 3D spacetime M, the Latin indices $(i, j, k, \dots)$ refer to the local Lorentz frame, the Greek indices $(\mu, \nu, \rho, \dots)$ refer to the coordinate frame, the metric components in the local Lorentz frame are $\eta_{ij} = (+1, -1, -1)$, totally antisymmetric tensor ε^{ijk} is normalized by $\varepsilon^{012} = +1$, and symmetric and antisymmetric pieces of a tensor X_{ij} are $X_{(ij)} = \frac{1}{2}(X_{ij} + X_{ji})$ and $X_{[ij]} = \frac{1}{2}(X_{ij} - X_{ji})$, respectively. Next, the $(1+2)$ decomposition of spacetime is described in terms of the suitable coordinates $x^\mu = (\rho, x^\alpha)$, where ρ is a radial coordinate and x^α are local coordinates on the boundary ∂M; in the local Lorentz frame, this decomposition is expressed by $i = (1, a)$. Then, on 2D boundary ∂M (which is orthogonal to the radial direction), we have $\eta_{ab} = (+1, -1)$ and $\varepsilon^{ab} := \varepsilon^{a1b}$, with $\varepsilon^{02} = +1$. Finally, we use the Stokes theorem in the form $\int \partial_\lambda V^\lambda d^3x = \int V^\rho d^2x$, where $V^\lambda = (V^\rho, V^\alpha)$ is a vector density, and d^3x and d^2x are coherently oriented volume forms on M and ∂M, respectively.

2 Holographic ansatz

In this section, we introduce a general setting for 3D gravity with torsion and discuss a suitable holographic ansatz for the basic dynamical variables.

Three-dimensional gravity with torsion can be naturally described in the framework of PGT [11, 12], where basic gravitational variables are the triad field $\hat{e}^i$ and the Lorentz connection $\hat{\omega}^{ij} = -\hat{\omega}^{ji}$ (1-forms), the corresponding field strengths are $\hat{T}^i = d\hat{e}^i + \hat{\omega}^i{}_j \wedge \hat{e}^j$ and $\hat{R}^{ij} = d\hat{\omega}^{ij} + \hat{\omega}^i{}_k \wedge \hat{\omega}^{kj}$ (2-forms), and the covariant derivative $\hat{\nabla} = d + \frac{1}{2}\hat{\omega}^{ij}\Sigma_{ij}$ (1-form) acts on local Lorentz spinors/tensors in accordance with their spinorial structure, encoded in the form of the spin matrix Σ_{ij}. The antisymmetry of $\hat{\omega}^{ij}$ ensures that the underlying geometric structure of spacetime is given by the Riemann-Cartan (RC) geometry, in which $\hat{e}^i$ is an orthonormal frame, $\hat{g} = \eta_{ij}\hat{e}^i \otimes \hat{e}^j$ is the metric of spacetime, $\hat{\omega}^{ij}$ is the metric-compatible connection, $\hat{\nabla}\hat{g} = 0$, and $\hat{T}^i$ and $\hat{R}^{ij}$ are the torsion and the RC curvature of spacetime, respectively. In our convention, hatted variables are 3D objects. Clearly, general features of PGT make it dynamically quite different from Riemannian theories, such as, for instance, topologically massive gravity [14, 15] or the Bergshoeff-Hohm-Townsend gravity [16].

In 3D, to any antisymmetric form $\hat{X}^{ij}$ there corresponds its Lie dual form $\hat{X}_k$, defined by $\hat{X}^{ij} = -\varepsilon^{ijk}\hat{X}_k$. Replacing $\hat{\omega}^{ij}, \hat{R}^{ij}$ with their Lie duals $\hat{\omega}^i, \hat{R}^i$, we have:

$$\hat{T}^i = d\hat{e}^i + \varepsilon^i{}_{jk}\hat{\omega}^j \wedge \hat{e}^k\,, \qquad \hat{R}^i = d\hat{\omega}^i + \frac{1}{2}\varepsilon^i{}_{jk}\hat{\omega}^j \wedge \hat{\omega}^k \tag{2.1}$$

In local coordinates x^μ, we can write $\hat{e}^i = \hat{e}^i{}_\mu dx^\mu$, $\hat{\omega}^i = \hat{\omega}^i{}_\mu dx^\mu$, and the action of local Poincaré transformations on the basic dynamical variables reads:

$$\begin{aligned}
\delta_0\hat{e}^i{}_\mu &= -\varepsilon^{ijk}\hat{e}_{j\mu}\hat{\theta}_k - (\partial_\mu\hat{\xi}^\lambda)\hat{e}^i{}_\lambda - \hat{\xi}^\lambda\partial_\lambda\hat{e}^i{}_\mu\,,\\
\delta_0\hat{\omega}^i{}_\mu &= -\hat{\nabla}_\mu\hat{\theta}^i - (\partial_\mu\hat{\xi}^\lambda)\hat{\omega}^i{}_\lambda - \hat{\xi}^\lambda\partial_\lambda\hat{\omega}^i{}_\mu\,.
\end{aligned} \tag{2.2}$$

Here, δ_0 is the form variation of a field, the parameters $\hat{\theta}^i$ and $\hat{\xi}^\mu$ describe local Lorentz transformations and local translations, respectively, and $\hat{\nabla}_\mu\hat{\theta}^i = \partial_\mu\hat{\theta}^i + \varepsilon^i{}_{jk}\hat{\omega}^j{}_\mu\hat{\theta}^k$.

Specific features of the RC geometry in 2D are described in appendix A.

2.1 Restricting the local Poincaré symmetry

In order to study the holographic structure of 3D gravity with torsion, we assume that spacetime M is a 3D manifold with a boundary ∂M at spatial infinity; more precisely, M is asymptotically diffeomorphic to $R \times \partial M$. The gravitational content of M implies that its geometric structure is of the RC type, whereas its dynamics is determined by choosing an action integral, which produces the field equations. Given the field equations, the asymptotic behavior of M is controlled by the asymptotic conditions. In the asymptotic region, M can be suitably parametrized by the local coordinates $x^\mu = (\rho, x^\alpha)$, where ρ is a radial coordinate, such that $\rho = 0$ on ∂M. The asymptotic conditions are formulated as certain conditions on the gravitational variables $\hat{e}^i$ and $\hat{\omega}^i$ near the boundary at $\rho = 0$.

The (asymptotic) radial foliation of M is an analog of the temporal foliation in the standard canonical formalism, with time line replaced by the radial line; early ideas on dynamical evolutions along spatial directions can be found in [17]. In this framework, Poincaré gauge invariance implies that $\hat{e}^i{}_\rho$ and $\hat{\omega}^i{}_\rho$ are unphysical variables, so that their values can be fixed by suitable gauge conditions. Although gauge conditions have no influence on the physical content in the bulk, the boundary dynamics is very sensitive to their form. Based on the experience with the Mielke-Baekler (MB) topological model of 3D gravity with torsion [8, 11], we impose the following six *gauge conditions*:

$$\hat{e}^i{}_\rho = (\hat{e}^1{}_\rho, \hat{e}^a{}_\rho) = \left(\frac{\ell}{\rho}, 0\right), \tag{2.3a}$$

$$\hat{\omega}^i{}_\rho = (\hat{\omega}^1{}_\rho, \hat{\omega}^a{}_\rho) = \left(\frac{p\ell}{2\rho}, 0\right), \tag{2.3b}$$

which break the Lorentz and the translational gauge invariance; ℓ is the AdS radius. As we shall see below, the parameter p controls the strength of both the torsion and the curvature on M. Next, we impose an *extra condition*:

$$\hat{e}^1{}_\alpha = 0\,, \tag{2.4}$$

which is equivalent to $\hat{e}_i{}^\rho = 0$ and is known as the “radial gauge” (an analog of the standard “time gauge”). Geometrically, it ensures that the radial direction coincides with the normal to ∂M, which greatly simplifies the calculations. In particular, the matrix representation of $\hat{e}^i{}_\mu$ becomes block diagonal. Finally, combining (2.4) with a suitable ansatz for $\hat{e}^a{}_\alpha$ and $\hat{\omega}^i{}_\alpha$, we can write:

$$\hat{e}^i{}_\alpha = (\hat{e}^1{}_\alpha, \hat{e}^a{}_\alpha) = \left(0, \frac{1}{\rho} e^a{}_\alpha\right), \tag{2.5a}$$

$$\hat{\omega}^i{}_\alpha = (\hat{\omega}^1{}_\alpha, \hat{\omega}^a{}_\alpha) = \left(\omega_\alpha, \frac{1}{\rho} k^a{}_\alpha\right), \tag{2.5b}$$

where $e^a{}_\alpha(\rho, x), \omega_\alpha(\rho, x)$ and $k^a{}_\alpha(\rho, x)$ are assumed to be finite and differentiable functions of ρ at $\rho = 0$, such that, near the boundary, they have the form

$$\begin{aligned} e^a{}_\alpha(\rho, x) &= \bar{e}^a{}_\alpha(x) + \mathcal{O}(\rho)\,, \\ \omega_\alpha(\rho, x) &= \bar{\omega}_\alpha(x) + \mathcal{O}(\rho)\,, \end{aligned} \tag{2.6}$$

and similarly for $\kappa^a{}_\alpha(\rho, x)$. Here, $\mathcal{O}(\rho)$ tends to zero when $\rho \to 0$, a bar over $e^a{}_\alpha$ denotes the value of $e^a{}_\alpha$ at the boundary $\rho = 0$, and similarly for $\bar{\omega}_\alpha$. Note, in particular, that the conditions (2.6) allow the presence of $\rho^n \ln\rho$ terms for $n > 0$, but not for the leading term $n = 0$. The inverse of $\hat{e}^i{}_\mu$ has the form

$$\begin{aligned} \hat{e}_i{}^\rho &= (\hat{e}_1{}^\rho, \hat{e}_a{}^\rho) = \left(\frac{\rho}{\ell}, 0\right) \,, \\ \hat{e}_i{}^\alpha &= (\hat{e}_1{}^\alpha, \hat{e}_a{}^\alpha) = (0, \rho e_a{}^\alpha) \,. \end{aligned} \tag{2.7}$$

The geometric interpretation of $e^a{}_\alpha, \omega_\alpha$ and $k^a{}_\alpha$ will be given in the next subsection.

Based on these conditions, we will investigate the holographic structure of 3D gravity with torsion, assuming *the absence of matter*. In particular, we shall study two complementary dynamical situations, described by

(a) MB model of topological 3D gravity with torsion, and
(b) general (parity-preserving) 3D gravity with propagating torsion.

For later convenience, we note that the metric defined by (2.3) and (2.5),

$$ds^2 = \hat{g}_{\mu\nu} dx^\mu dx^\nu = -\frac{\ell^2 d\rho^2}{\rho^2} + \frac{1}{\rho^2} g_{\alpha\beta} dx^\alpha dx^\beta \,,$$

where $g_{\alpha\beta} := e^a{}_\alpha e^b{}_\beta \eta_{ab}$ is regular at $\rho = 0$ and takes the usual Fefferman-Graham form [18]. For $\rho = 0$, the full metric has a pole of order two, which is typical for asymptotically AdS spaces, and directly related to the pole of order one in the triad field (2.5a).

In the rest of the paper, we use the units in which the AdS radius is $\ell = 1$.

Comment on (2.6). Any assumption on the asymptotic form of dynamical variables restricts the set of possible solutions of the field equations. In general, depending on the model-dependent dynamical features, expansions of the fields in (2.6) could contain logarithmic terms or power series of different order. However, having in mind that the holographic structure of the general 3D gravity model (b) has not been studied before, our intention is not to make the most general holographic analysis, which would be technically extremely complex, but to identify its basic holographic features. Furthermore, since both models (a) and (b) possess asymptotically AdS black hole solution [12], it is quite natural to expect that those features can be successfully revealed by focusing on the AdS asymptotic sector of the Brown-Henneaux type [11, 20].

To be more specific, let us mention that certain holographic aspects of the MB model in the Chern-Simons formulation have been studied earlier by Klemm and Tagliabue [8]. Their results strongly suggest that, in the MB model, our assumption (2.6) should be restricted to the following form:

$$\begin{aligned} \hat{e}^a{}_\alpha(\rho, x) &= \bar{e}^a{}_\alpha + \rho^2 s^a{}_\alpha + \mathcal{O}_4 \,, \\ \hat{\omega}_\alpha(\rho, x) &= \bar{\omega}_\alpha + \mathcal{O}_2 \,, \end{aligned} \tag{2.8}$$

where $\mathcal{O}_n$ is a term that tends to zero as ρ^n or faster, when $\rho \to 0$. Moreover, we expect the same sector to be of prime interest for the holographic structure of the general 3D gravity model (b). As we shall see, the results obtained in sections 4 and 5 justify our expectations. In this section, however, we continue using only (2.6).

2.2 Residual gauge symmetries

A field theory is defined by both the field equations and the asymptotic (boundary) conditions. The concept of asymptotic symmetry is of fundamental importance for understanding basic aspects of the boundary dynamics. Since the conditions (2.3), (2.5) and (2.6) control the form of dynamical variables in the asymptotic region near $\rho = 0$, they have a decisive influence on the asymptotic symmetry. The asymptotic symmetry is defined by a subset of gauge transformations that leaves the asymptotic conditions invariant. Thus, the parameters of the asymptotic (or residual) gauge transformations are defined by the consistency requirements

$$
\begin{aligned}
-\varepsilon^{ijk}\hat{e}_{j\mu}\hat{\theta}_k - (\partial_\mu\hat{\xi}^\lambda)\hat{e}^i{}_\lambda - \hat{\xi}^\lambda\partial_\lambda\hat{e}^i{}_\mu &= 0\,,\\
-\nabla_\mu\hat{\theta}^i - (\partial_\mu\hat{\xi}^\lambda)\hat{\omega}^i{}_\lambda - \hat{\xi}^\lambda\partial_\lambda\hat{\omega}^i{}_\mu &= 0\,,
\end{aligned}
$$

where $\hat{e}^i{}_\mu$ and $\hat{\omega}^i{}_\mu$ are taken to satisfy (2.3) and (2.5).

Starting with these conditions, we first find the restrictions stemming from the invariance of $\hat{e}^1{}_\rho, \hat{e}^a{}_\rho, \hat{e}^1{}_\alpha$, and $\hat{\omega}^1{}_\rho$, respectively:

$$
\begin{aligned}
\hat{\xi}^\rho &= \rho f(x)\,,\\
\partial_\rho\hat{\xi}^\alpha &= \rho g^{\alpha\beta}\partial_\beta f\,,\\
\hat{\theta}^a &= \rho\varepsilon^{ab}e_b{}^\alpha\partial_\alpha f\,,\\
\partial_\rho\hat{\theta}^1 &= -\rho\omega^\alpha\partial_\alpha f\,.
\end{aligned}
\tag{2.9a}
$$

There relations give rise to the following radial radial expansion of the local parameters:

$$
\begin{aligned}
\hat{\xi}^\rho &= \rho f(x)\,,\\
\hat{\xi}^\alpha &= \xi^\alpha(x) + \frac{1}{2}\rho^2\bar{g}^{\alpha\beta}\partial_\beta f + \rho^2\mathcal{O}(\rho)\,,\\
\hat{\theta}^a &= \rho\varepsilon^{ab}\bar{e}_b{}^\alpha\partial_\alpha f + \rho\mathcal{O}(\rho)\,,\\
\hat{\theta}^1 &= \theta(x) - \frac{\rho^2}{2}\bar{\omega}^\alpha\partial_\alpha f + \rho\mathcal{O}(\rho^2)\,.
\end{aligned}
\tag{2.9b}
$$

Thus, the residual symmetry is expressed in terms of the *four boundary parameters*: $\xi^\alpha(x), \theta(x)$ and $f(x)$.

In the next step, we find the restrictions produced by the invariance of $\hat{\omega}^a{}_\rho$ and $\hat{\omega}^a{}_\alpha$, respectively:

$$
\begin{aligned}
&\left[\left(\varepsilon^{ab} - \frac{p}{2}\eta^{ab} + k^{ab}\right)e_b{}^\beta + \rho\varepsilon^{ab}(\partial_\rho e_b{}^\beta)\right]\partial_\beta f = 0\,,\\
&\delta_0 k^a{}_\alpha = [-\varepsilon^{ab}k_{b\alpha}\theta - (\partial_\alpha\xi^\beta)k^a{}_\beta - \xi^\beta\partial_\beta k^a{}_\alpha] + fk^a{}_\alpha + \mathcal{O}(\rho)\,.
\end{aligned}
\tag{2.10}
$$

Assuming that $f(x)$ is an arbitrary function on ∂M, we have $\partial_\beta f \neq 0$, and the first relation defines $k^a{}_\alpha$ in terms of the $e^a{}_\alpha$:

$$
k^{ab} = \frac{p}{2}\eta^{ab} - \varepsilon^{ab} - \rho\varepsilon^{ac}e_c{}^\beta\partial_\rho e^b{}_\beta\,,
\tag{2.11}
$$

where $k^{ab} = k^a{}_\alpha e_b{}^\alpha$. The second relation in (2.10) defines the transformation law for $k^a{}_\alpha$; it shows that, at the boundary, $k^a{}_\alpha$ is a tensorial object with respect to local Poincaré transformations combined with dilatations. As shown in appendix A, $K_{ab} = \varepsilon_{cb} k^c{}_a$ is the extrinsic curvature of ∂M.

Finally, we wish to examine the implications of the invariance conditions for $\hat{e}^a{}_\alpha$ and $\hat{\omega}^1{}_\alpha$. Using (2.5), these condition yield, in the lowest order of the radial expansion, the following transformation rules for the boundary fields:

$$\begin{aligned} \delta_0 \bar{e}^a{}_\alpha &= \delta_P \bar{e}^a{}_\alpha + f \bar{e}^a{}_\alpha \,, \\ \delta_0 \bar{\omega}_\alpha &= \delta_P \bar{\omega}_\alpha + \varepsilon_{ab} \bar{e}^a{}_\alpha \bar{e}^{b\beta} \partial_\beta f \,, \end{aligned} \tag{2.12}$$

where $\delta_P \bar{e}^a{}_\alpha$ and $\delta_P \bar{\omega}_\alpha$ are the local Poincaré transformations in 2D:

$$\begin{aligned} \delta_P \bar{e}^a{}_\alpha &= -\varepsilon^a{}_c \theta \bar{e}^c{}_\alpha - (\partial_\alpha \xi^\beta) \bar{e}^a{}_\beta - \xi \cdot \partial \bar{e}^a{}_\alpha \,, \\ \delta_P \bar{\omega}_\alpha &= -\partial_\alpha \theta - (\partial_\alpha \xi^\beta) \bar{\omega}_\beta - \xi \cdot \partial \bar{\omega}_\alpha \,, \end{aligned} \tag{2.13}$$

and f defines local dilatations. Thus, we conclude the following:

- The residual symmetry transformations (2.12) belong to the Weyl group of local Poincaré transformations plus dilatations, whereas $\bar{e}^a{}_\alpha$ and $\bar{\omega}_\alpha$ are recognized as the vielbein and the spin connection of the boundary RC geometry.

The transformation rule for $\bar{e}^a{}_\alpha$ can be used to calculate how the residual symmetries act on the boundary metric $\bar{g}_{\alpha\beta} = \eta_{ab} \bar{e}^a{}_\alpha \bar{e}^b{}_\beta$. Restricting our attention to dilatations ($f \neq 0$), we obtain $\delta_f \bar{g}_{\alpha\beta} = 2f\, \bar{g}_{\alpha\beta}$. For more details, see appendix B.

The results obtained in this subsection are based only on the adopted holographic conditions (2.3), (2.5) and (2.6). We consider them as being kinematical, in the sense that they are not influenced by the dynamical arguments encoded in (2.8). Another useful set of kinematical relations is found by calculating the expressions for the torsion and the curvature tensors, based on (2.3), (2.5) and (2.6). As shown in appendix C.1, the result is of the form

$$\hat{T}_{ijk} = p\varepsilon_{ijk} + \mathcal{O}(\rho) \,, \qquad \hat{R}_{ijk} = q\varepsilon_{ijk} + \mathcal{O}(\rho) \,, \tag{2.14a}$$

where

$$q := \frac{p^2}{4} - 1 \,. \tag{2.14b}$$

Thus, to lowest order in ρ, the parameter p defines both the torsion and the curvature of spacetime.

In sections 4 and 5, we shall combine these results with (2.8) to study the specific dynamical models.

3 Noether-Ward identities

It is clear from the previous discussion that the residual gauge symmetries (2.9) are also kinematical. They are *maximal* gauge symmetries that we can expect to find on the boundary. Indeed, after choosing an action integral, the corresponding field equations may impose additional restrictions on these symmetries. In this section, we shall study the gravitational Noether identities (also called generalized conservation laws) induced by the maximal gauge symmetries (2.9), and interpret them as the corresponding Ward identities of the boundary CFT.

To make these ideas more precise, consider a 3D gravitational system without matter in an asymptotically AdS spacetime, with solutions characterized by independent boundary values of $e^a{}_\alpha$ and ω_α. The quasilocal energy-momentum and spin currents of the system are calculated by varying the action with respect to the boundary values of $e^a{}_\alpha$ and ω_α. The variation produces a bulk term, which is proportional to the field equations, and a boundary term. The on-shell value of the gravitational action, suitably *renormalized*, is given as a *finite* 2D functional $I_{\rm ren}[e,\omega]$ on ∂M. Next, consider a set of quantum fields ϕ on ∂M, coupled to the external gravitational fields (sources) $e^a{}_\alpha$ and ω_α, and described by an action integral $I[\phi;e,\omega]$. The corresponding effective action $W[e,\omega]$ is defined by the functional average over ϕ:

$$e^{iW[e,\omega]} = \int_{\partial M} D\phi e^{iI[\phi;e,\omega]}\,. \tag{3.1a}$$

In the semiclassical approximation, the AdS/CFT correspondence can be expressed by identifying the effective action with $I_{\rm ren}[e,\omega]$:

$$W[e,\omega] = I_{\rm ren}[e,\omega]\,. \tag{3.1b}$$

Using this identification, we can calculate the gravitational *Noether identities* for $I_{\rm ren}[e,\omega]$ and identify them as the *Ward identities* for the 1-point functions derived from $W[e,\omega]$, provided the functional measure is invariant under the residual gauge symmetries.

We consider gravity theories whose Lagrangians are at most quadratic in the first derivatives of the spin connection and the vielbein. The corresponding field equations are obtained integrating by parts, such that the surface term,

$$\delta I_{\rm on-shell} = \int d^2x\,\left(P_i^\nu\,\delta\hat{e}^i{}_\nu + Q_i^\nu\,\delta\hat{\omega}^i{}_\nu\right)\,, \tag{3.2}$$

does not contain derivatives of the variations of the fields.

The gauge choice (2.3)–(2.5), when used in the above formula, produces a surface term expressed in terms of the boundary quantities

$$\delta I_{\rm on-shell} = \int d^2x\,\left(p_i^\alpha\delta e^a{}_\alpha + q^\alpha\delta\omega{}_\alpha + \tilde{q}_a^\alpha\delta k^a{}_\alpha\right)\,. \tag{3.3}$$

It is clear that the PGT formulation of gravity also allows to impose boundary conditions different than keeping the vielbein and spin connection fixed at the conformal boundary. However, a theory with boundary conditions other than a Dirichlet one does not lead itself to a holographic description in the usual AdS/CFT framework.

In fact, in the metric formalism, the last term in (3.3) is related to the variation of the extrinsic curvature that is usually traded off for the variation of metric by a Gibbons-Hawking-type term. When a Gibbons-Hawking-type term cannot be constructed for a given theory, the only way out is to consider that the extrinsic curvature and the metric are related asymptotically.

The fact that the leading-order in the expansion of the extrinsic curvature is the same as the leading-order of the boundary metric for Riemannian AdS spacetimes suggests that

JHEP05(2013)103

there is an asymptotic relation between the extrinsic curvature and the vielbein in theories with torsion; such a relation in PGT is given by (2.11). Note that, as showed in appendix A, only the symmetric part of the extrinsic curvature is Riemannian, and the antisymmetric one explicitly depends on torsion. Once appropriate counterterms are added, the variation of the renormalized PGT action can be written as

$$\delta\ I_{\text{ren}} = -\int_{\partial M} d^2x \left(\tau^\alpha{}_a \delta e^a{}_\alpha + \sigma^\alpha \delta\omega_\alpha\right)\ , \tag{3.4}$$

whereby the standard duality between gravity and a boundary CFT is recovered.

The form of the expected Noether identities is based on the residual symmetry transformations (2.12) and (2.13). Quite generally, the invariance of the renormalized action under these transformations can be written in the form

$$\delta\ I_{\text{ren}} = -\int_{\partial M} d^2x \left(\tau^\alpha{}_a \delta_0 e^a{}_\alpha + \sigma^\alpha \delta_0\omega_\alpha\right) = 0\,, \tag{3.5a}$$

where

$$\tau^\alpha{}_a := -\frac{\delta I_{\text{ren}}}{\delta e^a{}_\alpha}\,, \qquad \sigma^\alpha := -\frac{\delta I_{\text{ren}}}{\delta\omega_\alpha}\,, \tag{3.5b}$$

are the energy-momentum and spin currents (tensor densities) of our dynamical system.

Restricting our attention first to the local translations (with parameters ξ^α) and then to the local Lorentz transformations (with parameter θ), we arrive at the corresponding Noether identities:

$$e^a{}_\beta \nabla_\alpha \tau^\alpha{}_a = \tau^\alpha{}_a T^a{}_{\beta\alpha} + \sigma^\alpha F_{\beta\alpha} - \omega_\beta(\nabla_\alpha\sigma^\alpha + \varepsilon^{ab}\tau_{ab})\,, \tag{3.6a}$$

$$\nabla_\beta \sigma^\beta = -\varepsilon^{ab}\tau_{ab}\,, \tag{3.6b}$$

which are also known as the generalized conservation laws of $\tau^\alpha{}_a$ and σ^β. Note that if the second Noether identity (3.6b) is fulfilled, the last term in (3.6a) can be omitted. Similarly, the invariance of $I_{\text{ren}}[e^a{}_\alpha, \omega_\alpha]$ under dilatations leads to

$$\tau - \nabla_\beta\big(\varepsilon_{ab}\sigma^a e^{b\beta}\big) = 0\,, \tag{3.6c}$$

where $\tau := \tau^a{}_a$ is the trace of the energy momentum tensor.

Although the gravitational dynamics in the bulk is described by a RC geometry, with $\hat\omega^i{}_\mu$ and $\hat e^i{}_\mu$ as independent fields, it may happen that some solutions on the boundary are Riemannian, that is, characterized by a vanishing torsion, $T_{abc} = 0$. For such solutions, the boundary connection ω_α is no longer independent of the vielbein $e^a{}_\alpha$. Nevertheless, as we are going to show, the Noether-Ward identities are still of the form (3.6), but now, ω_α takes on the Riemannian value $\tilde\omega_\alpha$. In a way, this might have been expected, since the transformation properties of $\tilde\omega_\alpha$ are the same as those of ω_α, and these properties play a crucial role in defining the boundary symmetry.

When the boundary torsion vanishes, the connection takes the Riemannian form (A.3). However, we find it more convenient to use an equivalent but more compact expression:

$$\tilde\omega_\alpha = -\varepsilon_{ab}\varepsilon^{\gamma\delta}\varepsilon_{\alpha\beta}e^{a\beta}\partial_\gamma e^b{}_\delta\,. \tag{3.7}$$

Now, starting from the Riemannian renormalized action $\tilde{I}_{\rm ren} = I_{\rm ren}[e^a{}_\alpha, \tilde{\omega}_\alpha]$, we find that the related spin current $\Sigma^\alpha := -\delta\tilde{I}_{\rm ren}/\delta\omega_\alpha$ vanishes, whereas the energy-momentum current $\Theta^\alpha{}_a := -\delta\tilde{I}_{\rm ren}/\delta e^a{}_\alpha$ has an additional contribution stemming from the last term in (3.5a):

$$\Theta^\alpha{}_a = \tilde{\tau}^\alpha{}_a - \tilde{\nabla}_\beta \left(\varepsilon^{\alpha\beta} e^{-1} \tilde{\sigma}_a\right) . \tag{3.8}$$

Here, $\tilde{X}$ denotes the Riemannian limit of a RC object X; in particular, $\tilde{\nabla}_\alpha f_a = \partial_\alpha f_a - \varepsilon_{ac}\tilde{\omega}_\alpha f^c$. Then, the Noether identities for the action $\tilde{I}_{\rm ren}$ are found to be:

$$\begin{aligned} e^a{}_\beta \tilde{\nabla}_\alpha \Theta^\alpha{}_a + \tilde{\omega}_\beta \varepsilon^{ab}\Theta_{ab} &= 0\,, && \text{(3.9a)} \\ \varepsilon^{ab}\Theta_{ab} &= 0\,, && \text{(3.9b)} \\ \Theta &= 0\,. && \text{(3.9c)} \end{aligned}$$

Since $\Theta^\alpha{}_a$ is a tensor density, the first relation, which is a condition for diffeomorphism invariance, is seen to coincide with the condition (4.10) in Klemm et al. [8]. When the Lorentz invariance is satisfied, (3.9a) reduces to the usual form $D_\alpha(e^{-1}\Theta^\alpha{}_\beta) = 0$, where D_α is the Riemannian covariant derivative. The remaining two relations are the standard Riemannian conditions for the Lorentz and Weyl invariance, respectively. Using $T_{abc} = 0$, as well as the identity $\varepsilon^{\alpha\beta}\tilde{\nabla}_\alpha\tilde{\nabla}_\beta f_a = -\frac{1}{2}\varepsilon^{\alpha\beta}\tilde{F}_{\alpha\beta}\varepsilon_{ab}f^b$, one can transform (3.9) into

$$\begin{aligned} e^a{}_\beta \tilde{\nabla}_\alpha \tilde{\tau}^\alpha{}_a - \tilde{\sigma}^\alpha F_{\beta\alpha} + \tilde{\omega}_\beta(\tilde{\nabla}_\beta\tilde{\sigma}^\beta + \varepsilon^{ab}\tilde{\tau}_{ab}) &= 0\,, && \text{(3.10a)} \\ \varepsilon^{ab}\tilde{\tau}_{ab} + \tilde{\nabla}_\beta\tilde{\sigma}^\beta &= 0\,, && \text{(3.10b)} \\ \tilde{\tau} - \tilde{\nabla}_\beta(\varepsilon_{ab}\tilde{\sigma}^a e^{b\beta}) &= 0\,. && \text{(3.10c)} \end{aligned}$$

Hence, the Riemannian identities (3.9) coincide with those obtained from (3.6) in the limit $T_{abc} \to 0$, as expected. This proves the following theorem:

– In the context of PGT, the form (3.6) of Noether identities can be used for both Riemann-Cartan and Riemannian boundary geometries.

According to the AdS/CFT correspondence, relations (3.6) are interpreted as the maximal set of Ward identities that can be found in the boundary CFT. If the field equations happen to be incompatible with the above symmetries, some of the Ward identities may be violated, leading to the appearance of *quantum anomalies*.

4 Holography in topological 3D gravity with torsion

In this section, we analyze the validity of the Noether-Ward identities (3.6), in the MB model of topological 3D gravity with torsion [9, 11], described by the action

$$I_{\rm MB} = \int \left(2a\hat{e}^i\hat{R}_i - \frac{1}{3}\Lambda_0\varepsilon_{ijk}\hat{e}^i\hat{e}^j\hat{e}^k + \alpha_3 L_{\rm CS}(\hat{\omega}) + \alpha_4\hat{e}^i\hat{T}_i\right) , \tag{4.1}$$

where $L_{\rm CS}(\hat{\omega}) = \hat{\omega}_i d\hat{\omega}^i + \frac{1}{3}\varepsilon_{ijk}\hat{\omega}^i\hat{\omega}^j\hat{\omega}^k$ is the Chern-Simons Lagrangian for the Lorentz connection, $a = 1/16\pi G$ is the gravitational constant, Λ_0 is a (bare) cosmological constant,

α_3, α_4 are dimensionful coupling constants, the wedge product signs $\wedge$ are omitted for simplicity, and the matter contribution is absent.

The vacuum field equations read

$$\hat{T}_{ijk} = p\varepsilon_{ijk} \,, \qquad \hat{R}_{ijk} = q\varepsilon_{ijk} \,, \tag{4.2a}$$

where the parameters p and q are defined in terms of the coupling constants $a, \Lambda, \alpha_3, \alpha_4$. The spacetime described by these equations is maximally symmetric, at least locally. Moreover, in the AdS sector, the effective cosmological constant is negative,

$$\Lambda_{\text{eff}} := q - \frac{p^2}{4} = -1 \,. \tag{4.2b}$$

By comparing these equations with (2.14), it follows that the parameter p from our ansatz should be identified with the parameter p in the MB model.

Our analysis is based on using the AdS asymptotic conditions (2.8). For an interesting asymptotic correspondence between the MB model and topologically massive gravity, see [21].

4.1 Analysis of the field equations

The subset of the field equations (4.2a) that describes the *radial evolution* of the system is given by $(ijk) = (11c), (a1c)$. The first pair of equations takes the form

$$\hat{T}_{11c} = 0 \,, \qquad \hat{R}_{11c} = 0 \,. \tag{4.3a}$$

Using the expressions for $\hat{T}_{ijk}$ and $\hat{R}_{ijk}$ calculated in appendix C, one finds that the first equation is identically satisfied, whereas the second one implies that ω_α is the Lorentz connection at the boundary,

$$\omega_\alpha = \omega_\alpha(x) \,. \tag{4.3b}$$

The second pair of equations reads:

$$\hat{T}_{a1c} = -p\varepsilon_{ac} \,, \qquad \hat{R}_{a1c} = -q\varepsilon_{ac} \,. \tag{4.4a}$$

After introducing the radial expansion (2.6), the first equation in (4.4a) yields that s_{ab} is symmetric,

$$\varepsilon^{ab} s_{ab} = 0 \,. \tag{4.4b}$$

This result simplifies the second equation in (4.4a); relying on $(\text{C.5})_3$, the piece of the zeroth order in ρ implies that the effective cosmological constant Λ_{eff} is negative, see (4.2b), whereas the piece of order ρ^2 leads to a finite radial expansion of $e_{c\beta}$:

$$e_{c\beta} = \bar{e}_{c\beta} + \rho^2 \bar{s}_{c\beta} \,. \tag{4.4c}$$

Such an expansion that terminates at ρ^2 is a generalization of the result known for GR in 3D; in higher dimensions, the result holds when the Weyl tensor vanishes [22]. As a simple consequence, the radial expansion of k^{ab} is also finite:

$$k^{ab} = \frac{p}{2}\eta^{ab} - \varepsilon^{ab} + 2\rho^2 \varepsilon^{ac} s^b{}_c \,.$$

JHEP05(2013)103

Using the above results, the nontrivial content of the remaining $(1bc)$ and (abc) field equations is expressed in terms of the following *radial contraints*:

$$T_{abc} = 0\,, \tag{4.5a}$$
$$R - 4s^c{}_c = \mathcal{O}_2\,, \tag{4.5b}$$
$$\nabla_\alpha s_{b\beta} - \nabla_\beta s_{b\alpha} = 0\,. \tag{4.5c}$$

In particular, we see that the boundary torsion vanishes.

4.2 Counterterm and boundary currents

Now, we introduce the boundary currents and verify their Noether-Ward identities.

The variation of the MB action, calculated on shell, reduces to a surface integral:

$$\delta I_{\text{MB}} = \int_{\partial M} d^2x \varepsilon^{\alpha\beta} \left(2a\hat{e}^i{}_\alpha \delta\hat{\omega}_{i\beta} + \alpha_3 \hat{\omega}^i{}_\alpha \delta\hat{\omega}_{i\beta} + \alpha_4 \hat{e}^i{}_\alpha \delta\hat{e}_{i\beta}\right)\,. \tag{4.6}$$

Each of these three terms can be written in more details as:

$$\begin{aligned}
2a\varepsilon^{\alpha\beta}\hat{e}^i{}_\alpha\delta\hat{\omega}_{i\beta} &= \frac{2a}{\rho^2}\varepsilon^{\alpha\beta}\left[\frac{p}{2}e^b{}_\alpha\delta e_{b\beta} - \varepsilon_{ab}e^a{}_\alpha\delta e^b{}_\beta - 2\rho^2\varepsilon_{ab}s^a{}_\alpha\delta e^b{}_\beta\right] + \delta\Delta_1\,,\\
\alpha_3\varepsilon^{\alpha\beta}\hat{\omega}^i{}_\alpha\delta\hat{\omega}_{i\beta} &= \frac{\alpha_3}{\rho^2}\varepsilon^{\alpha\beta}\left[qe^b{}_\alpha\delta e_{b\beta} - 2p\rho^2\varepsilon^{ab}s_{a\alpha}\delta e_{b\beta} + 4\rho^2 s^b{}_\alpha\delta e_{b\beta}\right]\\
&\quad -\alpha_3\varepsilon^{\alpha\beta}\omega_\alpha\delta\omega_\beta\,,\\
\alpha_4\varepsilon^{\alpha\beta}\hat{e}^i{}_\alpha\delta\hat{e}_{i\beta} &= \frac{\alpha_4}{\rho^2}\varepsilon^{\alpha\beta}e^b{}_\alpha\delta e_{b\beta}\,,
\end{aligned}$$

where $\delta\Delta_1$ is a total variation with

$$\Delta_1 := 4a\varepsilon^{\alpha\beta}\varepsilon_{ab}s^a{}_\alpha e^b{}_\beta = -4a\bar{e}s^c{}_c\,,$$

and $e := \det(e^a{}_\alpha)$. Then, the identity $ap + \alpha_3 q + \alpha_4 = 0$, see ref. [11], implies that the sum of the first three terms in the above expressions vanishes, whereupon the only *divergent* term in δI_{MB} is also a total variation, $\delta\Delta_2$, with

$$\Delta_2 := -\frac{a}{\rho^2}\varepsilon^{\alpha\beta}\varepsilon_{ab}e^a{}_\alpha e^b{}_\beta = \frac{2a}{\rho^2}\bar{e}(1 + \rho^2 s^c{}_c)\,.$$

Since the boundary integral of $\Delta_1 + \Delta_2$ appears in δI_{MB} as a total variation, it can be subtracted from I_{MB} to obtain an *improved* variational principle. The integral

$$I_{\text{ct}} := \int_{\partial M} d^2x(\Delta_1 + \Delta_2) = 2a\int_{\partial M} d^2x\bar{e}\left(\frac{1}{\rho^2} - s^c{}_c\right) \tag{4.7a}$$

is usually called the *counterterm*. Before discussing its role in the new variational principle, let us rewrite I_{ct} in an equivalent form as

$$I_{\text{ct}} := a\int_{\partial M} d^2x\tilde{e}K\,, \tag{4.7b}$$

where K is the trace of the extrinsic curvature (A.5), and $\tilde{e}^a{}_\alpha = \bar{e}^a{}_\alpha/\rho$ is the induced vielbein at the boundary. The expression for I_{ct} is just one-half of the Gibbons-Hawking term

($I_{\rm GH}$), the result that naturally appears in the Chern-Simons formulation of GR in 3D, as discussed by Bañados and Méndez [23], and by Mišković and Olea [24] (for an interesting approach to counterterms in higher dimensional gravity, see [25]). On the other hand, using the field equation (4.5b), we can express the finite piece of the counterterm, $s^c{}_c$, in terms of the scalar curvature R, but since R is a topological invariant, its contribution to $I_{\rm ct}$ can be disregarded. Thus, effectively, the counterterm can be written as a covariant object, determined by a local function of $\tilde{e}^a{}_\alpha$:

$$I_{\rm ct} = 2a \int d^2x \tilde{e} = I_{\rm GH} - 2a \int d^2x \tilde{e}\,, \tag{4.7c}$$

where the last term is the usual *local counterterm* of Balasubramanian and Kraus [26], obtained in the context of 3D GR. It is interesting to note that the nonlinear Chern-Simons term in the MB action does not contribute to the counterterm, in agreement with the analysis of [7].

Since the total variation $\delta I_{\rm ct}$ is a divergent piece of $\delta I_{\rm MB}$, we are quite naturally led to introduce the *renormalized* (or, more precisely, the improved) MB action:

$$I^{\rm ren}_{\rm MB} := I_{\rm MB} - I_{\rm ct}\,, \tag{4.8}$$

such that it has well-defined functional derivatives and produces finite boundary currents, on-shell.

Note that, although the counterterm (4.7a) ensures that the variation of $I^{\rm ren}_{\rm MB}$ is finite and differentiable, one can verify that the value of the renormalized action $I^{\rm ren}_{\rm MB}$ is logarithmically divergent. Similarly as in GR, the logarithmic term is proportional to the Euler topological invariant eR, which is why it does not influence the variation of $I^{\rm ren}_{\rm MB}$. The logarithmic terms, even though topological in three dimensions, are important to be included, because the renormalized gravitational action is identified with the free energy in the dual boundary field theory.

Finally, by noting that

$$\delta I^{\rm ren}_{\rm MB} = \int_{\partial M} d^2x \varepsilon^{\alpha\beta}\left[-4\left(a+\frac{\alpha_3 p}{2}\right)\varepsilon_{ab}s^a{}_\alpha \delta e^b{}_\beta + 4\alpha_3 s_{b\alpha}\delta e^b{}_\beta - \alpha_3\omega_\alpha\delta\omega_\beta\right]\,, \tag{4.9}$$

we can use (3.5b) to obtain the energy-momentum and spin currents on the boundary:

$$\begin{aligned} \tau^\beta{}_b &= 4\left(a+\frac{\alpha_3 p}{2}\right)\varepsilon^{\alpha\beta}\varepsilon_{ab}s^a{}_\alpha - 4\alpha_3\varepsilon^{\alpha\beta}s_{b\alpha}\,, \\ \sigma^\beta &= -\alpha_3\varepsilon^{\beta\alpha}\omega_\alpha\,. \end{aligned} \tag{4.10}$$

4.3 Boundary symmetries and anomalies

Now, we wish to check the expected Noether-Ward identities (3.6).

Using the radial constraints (4.5), we find the following on-shell relations:

$$\nabla_\beta \tau^\beta{}_b = 0\,, \qquad \nabla_\beta\sigma^\beta = -\frac{1}{2}\varepsilon^{bc}\tau_{bc}\,. \tag{4.11}$$

Comparing with (3.6b), we see that the Lorentz invariance of the effective 2D theory is violated, and the *Lorentz anomaly* reads:

$$A_{\rm L} := \nabla_\beta \sigma^\beta + \varepsilon^{bc}\tau_{bc} = \frac{1}{2}\varepsilon^{bc}\tau_{bc} = -\frac{1}{2}\alpha_3 \bar{e} R\,. \tag{4.12}$$

The coefficient α_3, multiplying the topological (Euler) density $\bar{e}R$, is proportional to the difference of the classical central charges $c^\mp$ of the Mielke-Baekler model [11]:

$$c^\mp = 24\pi\left[a\ell + \alpha_3\left(\frac{p\ell}{2} \mp 1\right)\right].$$

Next, $(4.11)_1$ implies that the translation invariance condition (3.6b) is reduced to the form $0 = \sigma^\beta F_{\alpha\beta} + \omega_\alpha \nabla_\beta \sigma^\beta$. Using the relations

$$\nabla_\beta \sigma^\beta = \frac{1}{2}\alpha_3 e R\,, \qquad \sigma^\beta F_{\alpha\beta} = -\frac{1}{2}\alpha_3 \omega_\alpha e R\,,$$

we conclude that local translations are a correct boundary symmetry. Hence, there is *no translational* anomaly:

$$A_{\rm T} := e^a{}_\beta \nabla_\alpha \tau^\alpha{}_a - \tau^\alpha{}_a T^a{}_{\beta\alpha} - \sigma^\beta F_{\alpha\beta} + \omega_\alpha(\nabla_\beta\sigma^\beta + \varepsilon^{ab}\tau_{ab}) = 0\,. \tag{4.13}$$

Finally, in order to verify the Noether identity for dilatations (3.6c), we use (4.4b) and (4.5b) to obtain

$$\begin{aligned} \tau^c{}_c &= -4\bar{e}\left(a + \frac{\alpha_3 p}{2}\right) s^c{}_c = -\bar{e}\left(a + \frac{\alpha_3 p}{2}\right) R\,, \\ \nabla_\beta\left(\varepsilon_{ab}\sigma^a e^{b\beta}\right) &= -\alpha_3 \partial_\beta(\bar{e}\omega^\beta)\,. \end{aligned} \tag{4.14}$$

Thus, the dilatational Noether identity is violated, and the violation is measured by a quantity which is usually called the *conformal* (or Weyl) *anomaly*:

$$A_C := \tau^c{}_c - \nabla_\beta\left(\varepsilon_{ab}\sigma^a e^{b\beta}\right) = -\left(a + \frac{\alpha_3 p}{2}\right)\bar{e}R + \alpha_3\partial_\beta(\bar{e}\omega^\beta)\,. \tag{4.15}$$

Here, the coefficient of $\bar{e}R$ is proportional to the sum of the central charges.

In treating the boundary symmetries of the MB model, Klemm et al. [8] followed a different approach, based on using the Riemannian connection in the renormalized action. Nevertheless, our results for anomalies coincide with theirs, in agreement with the theorem proved in section 3. The full strength of this theorem will be seen in the more interesting case of 3D gravity with propagating torsion, where the complicated field equations may lead to either vanishing or nonvanishing boundary torsion. However, we will be able to derive the Noether-Ward identities without recourse to the value of the boundary torsion.

5 Holography in 3D gravity with propagating torsion

In this section, we analyze the holographic structure of 3D gravity with propagating torsion, assuming parity invariance [12], and using the AdS asymptotic conditions (2.8).

JHEP05(2013)103

5.1 Lagrangian and the field equations

Assuming the absence of matter, dynamical content of 3D gravity with propagating torsion is defined by the action integral

$$I = \int d^3x\, \hat{e}\mathcal{L}_G\,, \tag{5.1}$$

where $\hat{e} = \det(\hat{e}^i{}_\mu)$, and the gravitational Lagrangian $\mathcal{L}_G$ is at most *quadratic* in the torsion and the curvature. Assuming parity invariance, the general form of $\mathcal{L}_G$ is given by [12]

$$\mathcal{L}_G = -a\hat{R} - 2\Lambda_0 + \mathcal{L}_{T^2} + \mathcal{L}_{R^2}\,. \tag{5.2a}$$

The quadratic terms can be conveniently be written in the form

$$\begin{aligned}
\mathcal{L}_{T^2} &= \frac{1}{4}\hat{T}^{ijk}\mathcal{H}_{ijk}\,, &\qquad \mathcal{H}_{ijk} &:= a_1\,{}^{(1)}\hat{T}_{ijk} + a_2\,{}^{(2)}\hat{T}_{ijk} + a_3\,{}^{(3)}\hat{T}_{ijk}\,,\\
\mathcal{L}_{R^2} &= \frac{1}{8}\hat{R}^{ijkl}\mathcal{H}_{ijkl}\,, &\qquad \mathcal{H}_{ijkl} &:= b_4\,{}^{(1)}\hat{R}_{ijkl} + b_5\,{}^{(2)}\hat{R}_{ijkl} + b_6\,{}^{(3)}\hat{R}_{ijkl}\,,
\end{aligned} \tag{5.2b}$$

where we introduced the covariant field momenta $\mathcal{H}_{ijk}$ and $\mathcal{H}_{ijkl}$, which are *linear* in the irreducible components of the torsion, ${}^{(n)}\hat{T}_{ijk}$, and the curvature, ${}^{(n)}\hat{R}_{ijkl}$. An equivalent form of these two terms, which is more convenient for practical calculations, is given by:

$$\begin{aligned}
\mathcal{H}_{ijk} &= 4(\alpha_1\hat{T}_{ijk} + \alpha_2\hat{T}_{[kj]i} + \alpha_3\hat{T}_{ijk})\,,\\
\mathcal{L}_{R^2} &= \hat{R}^{ij}\mathcal{H}_{ij}\,, \qquad \mathcal{H}_{ij} = \beta_1\hat{R}_{ij} + \beta_2\hat{R}_{ji} + \beta_3\eta_{ij}\hat{R}\,.
\end{aligned} \tag{5.2c}$$

The expression for $\mathcal{L}_{R^2}$ is obtained using the fact that the Weyl tensor identically vanishes in 3D, and the new coupling constants (α_k, β_k) are expressed in terms of the (a_k, b_k) as [12]

$$\begin{aligned}
\alpha_1 &= \frac{1}{6}(2a_1 + a_3)\,, &\qquad \alpha_2 &= \frac{1}{3}(a_1 - a_3)\,, &\qquad \alpha_3 &= \frac{1}{2}(a_2 - a_1)\,,\\
\beta_1 &= \frac{1}{2}(b_4 + b_5)\,, &\qquad \beta_2 &= \frac{1}{2}(b_4 - b_5)\,, &\qquad \beta_3 &= \frac{1}{12}(b_6 - 4b_4)\,.
\end{aligned}$$

The variation of the action (5.1) with respect to $\hat{e}^i{}_\mu$ and $\hat{\omega}^{ij}{}_\mu$ $(= -\varepsilon^{ij}\hat{\omega}_\mu)$ produces two gravitational field equations, displayed in equations (2.13) of ref. [12]. Without matter contribution, these equations, transformed to the local Lorentz basis, take the form:

$$\begin{aligned}
&\nabla^m\mathcal{H}_{imj} + \frac{1}{2}\mathcal{H}_i{}^{mn}(-T_{jmn} + 2\eta_{jm}V_n) - t_{ij} = 0\,,\\
&t_{ij} := \eta_{ij}\mathcal{L}_G - T^{mn}{}_i\mathcal{H}_{mnj} + 2a\hat{R}_{ji} - 2(\hat{R}^n{}_i\mathcal{H}_{nj} - \hat{R}_j{}^{nm}{}_i\mathcal{H}_{nm})\,,
\end{aligned} \tag{5.3a}$$

where t_{ij} is the energy-momentum tensor of gravity, and

$$2aT_{kij} + 2T^m{}_{ij}(\mathcal{H}_{mk} - \eta_{mk}\mathcal{H}) + 4\nabla_{[i}(\mathcal{H}_{j]k} - \eta_{j]k}\mathcal{H}) + \varepsilon_{ijn}\varepsilon^{mr}{}_k\mathcal{H}_{mr}{}^n = 0\,, \tag{5.3b}$$

with $\mathcal{H} = \mathcal{H}^k{}_k$.

In the near-boundary expansion, the leading order of the field equations (5.3), corresponding to $\rho = 0$, reduces to the following relations involving the coupling constants:

$$p(a + qb_6 + 2a_3) = 0\,, \tag{5.4a}$$

JHEP05(2013)103

$$aq - \Lambda_0 + \frac{1}{2}p^2 a_3 - \frac{1}{2}q^2 b_6 = 0\,. \tag{5.4b}$$

As shown in [12], these relations ensure that the AdS configuration, as well as the black hole with torsion, are solutions of the present theory. However, quadratic equations (5.4) allow to have two different solution for the effective cosmological constant $\Lambda_{\text{eff}} = p - q^2/4$, and consequently, two different AdS vacua. For a particular choice of parameters ($p = 0, a - b_6 q = 0$), the two vacua coincide [12]. For an analysis of this situation in the Bergshoeff-Hohm-Townsend gravity, see refs. [27, 28].

5.2 Equations of motion

In this section, we discuss the consistency of the near-boundary analysis of the field equations (5.3), given in appendix D, with the holographic description of the asymptotic theory.

The leading order of the field equations is given by eqs. (5.4). These equations constrain the coupling constants and, therefore, restrict the form of the allowed gravity actions.

Equations linear in ρ are given by the algebraic system (D.1), (D.2), (D.6) and (D.8) for the vector $\bar{V}_a = \bar{T}^b{}_{ba}$, which defines the complete torsion tensor in 2D (appendix A). These equations allow not only Riemann-Cartan but also Riemann boundary geometries. However, thanks to the theorem proved in section 3, we can study the Noether-Ward identities in these two cases quite generally, without making an explicit distinction between them.

The order ρ^2 of the field equations is given in (D.3)–(D.5) and (D.7). These are algebraic equations in the tensor s_{ab}, which is related to the extrinsic curvature K_{ab} (appendix D). More precisely, these equations determine the antisymmetric part $\varepsilon^{ab}s_{ab}$ and the trace $s^c{}_c$ as local expressions of the boundary curvature and torsion. In particular, for the vanishing torsion we have $\varepsilon^{ab}s_{ab} = 0$ and $s^c{}_c = \frac{1}{4}R$, as in the MB model.

Here, in contrast to the MB model, the radial expansion goes beyond ρ^2, but the cubic and higher order terms do not affect our results in the $\rho \to 0$ limit.

Let us emphasize that, in our near-boundary analysis, we were not able to determine the symmetric traceless part s'_{ab} of s_{ab}. We can understand this situation by noting that s'_{ab} is a *nonlocal* function that requires a global solution. Such nonlocal terms are parts of the (nonlocal) 1-point functions of the boundary CFT. On the other hand, physical objects, such as the conformal anomaly, are always local. This is a general feature of the boundary currents in an effective theory.

In the next section, we calculate the boundary currents of the effective CFT.

5.3 Boundary currents

In the absence of matter, the variation of the (gravitatonal) action, evaluated *on-shell*, takes the form

$$\delta I_{\text{on-shell}} = \int d^3x\, \partial_\mu \left\{ 2\varepsilon^{\mu\nu\lambda}\hat{e}^k{}_\lambda \left[\delta\hat{e}^i{}_\nu \varepsilon^{jm}{}_k \mathcal{H}_{ijm} + \delta\hat{\omega}^i{}_\nu \left(a\,\eta_{ik} + \mathcal{H}_{ki} - \eta_{ki}\mathcal{H} \right) \right] \right\}. \tag{5.5}$$

After expressing $\delta I_{\text{on-shell}}$ as a boundary integral, we will use the field equations to find the renormalized 2D action. Then, in accordance with (3.5b), we will identify the energy-momentum and the spin boundary currents as the objects (1-point functions) coupled to the sources $\bar{e}^a{}_\alpha$ and $\bar{\omega}_\alpha$ in the boundary CFT. To do that, we write the action corresponding to the Lagrangian (5.2a) as

$$I = I_{EC} + I_{\Lambda_0} + I_{T^2} + I_{R^2}\,. \tag{5.6}$$

The variation of the term I_{EC}, linear in the scalar curvature, is known from the MB model:

$$\begin{aligned}\delta I_{EC} &= \frac{ap}{\rho^2}\int_{\partial\mathcal{M}} d^2x\varepsilon^{\alpha\beta}e^a{}_\alpha\delta e_{a\beta} - 4a\int_{\partial\mathcal{M}} d^2x\varepsilon^{\alpha\beta}\varepsilon^{ab}s_{\alpha a}\delta e_{b\beta}\\ &\quad+\delta\int_{\partial M} d^2x(\Delta_1+\Delta_2)\,,\end{aligned} \tag{5.7a}$$

where the total variation contains two pieces, one finite and the other divergent:

$$\begin{aligned}\Delta_1 &:= 4a\varepsilon^{\alpha\beta}\varepsilon_{ab}s^a{}_\alpha e^b{}_\beta = -4aes^c{}_c\,,\\ \Delta_2 &:= -\frac{a}{\rho^2}\varepsilon^{\alpha\beta}\varepsilon_{ab}e^a{}_\alpha e^b{}_\beta = \frac{2a}{\rho^2}\bar{e}(1+\rho^2 s^c{}_c)\,.\end{aligned}$$

The variation of the cosmological term does not contribute to the boundary integrals. Next, we vary the term quadratic in torsion:

$$\delta I_{T^2} = \frac{2a_3 p}{\rho^2}\int_{\partial\mathcal{M}} d^2x\varepsilon^{\alpha\beta}e^a{}_\alpha\delta e_{a\beta} + \frac{2a_3}{\rho^2}\int_{\partial\mathcal{M}} d^2x(\hat{\mathcal{A}}-p)\varepsilon^{\alpha\beta}e^a{}_\alpha\delta e_{a\beta}\,. \tag{5.8}$$

Note that the second piece, containing the axial torsion, is a finite 2D integral.

Finally, the variation of the term quadratic in curvature yields:

$$\begin{aligned}\delta I_{R^2} &= 2\int_{\mathcal{M}} d^3x\varepsilon^{\mu\nu\sigma}\left(\mathcal{H}_{\sigma i} - b_{i\sigma}\mathcal{H}\right)\partial_\mu\delta\hat{\omega}^i{}_\nu\\ &= 2\int_{\partial\mathcal{M}} d^2x\varepsilon^{\alpha\beta}\left[\mathcal{H}_{\alpha 1}\delta\omega_\beta + \left(\mathcal{H}_{ca} - \eta_{ac}\mathcal{H}\right)\frac{1}{\rho^2}e^c{}_\alpha\delta k^a{}_\beta\right]\,,\end{aligned} \tag{5.9}$$

where

$$\mathcal{H}_{ca} - \eta_{ca}\mathcal{H} = \eta_{ca}b_6 q + \eta_{ca}\rho^2\left[b_6 p(\varepsilon\cdot s) - \frac{b_6 - b_4}{6}(R - 4s^\gamma{}_\gamma)\right] + 2\varepsilon_{ca}\rho^2 b_5(\varepsilon\cdot s)\,,$$

and $\varepsilon\cdot s := \varepsilon^{fg}s_{fg}$. The first piece of δI_{R^2} has the form

$$A := 2\beta_2\int_{\partial\mathcal{M}} d^2x\varepsilon^{\alpha\beta}\left(\frac{p}{2}\varepsilon_{ac} - \eta_{ac}\right)V^c\,\bar{e}^a{}_\alpha\delta\omega_\beta\,. \tag{5.10a}$$

The second piece can be conveniently written as the sum of two terms, $B + C$, where:

$$\begin{aligned}B &:= \frac{b_6 qp}{\rho^2}\int_{\partial\mathcal{M}} d^2x\varepsilon^{\alpha\beta}e_{a\alpha}\delta e^a{}_\beta - 4b_6 q\int_{\partial\mathcal{M}} d^2x\varepsilon^{\alpha\beta}\delta e_{a\alpha}\varepsilon^{af}s_{\beta f}\\ &\quad+\delta\int_{\partial M} d^2x\Delta_3\,,\end{aligned} \tag{5.10b}$$

$$\Delta_3 := 4b_6 q\varepsilon^{\alpha\beta}\varepsilon^{ab}e_{a\alpha}s_{\beta b} - \frac{b_6 q}{\rho^2}\varepsilon^{\alpha\beta}\varepsilon^{ab}e_{a\alpha}e_{b\beta} = \frac{qb_6}{\rho^2}\bar{e}K\,.$$

JHEP05(2013)103

and

$$C := 2\int_{\partial\mathcal{M}} d^2x\varepsilon^{\alpha\beta}\left[b_6p(\varepsilon\cdot s) - \frac{b_6-b_4}{6}(R-4s^\gamma{}_\gamma)\right]e_{a\alpha}\left(\frac{p}{2}\eta^{ab}-\varepsilon^{ab}\right)\delta e_{b\beta}$$
$$-4b_5\int_{\partial\mathcal{M}} d^2x\varepsilon^{\alpha\beta}(\varepsilon\cdot s)e^c{}_\beta\left(\frac{p}{2}\eta_{ca}-\varepsilon_{ca}\right)\delta e^a{}_\alpha\,. \tag{5.10c}$$

Now, the first terms in $\delta I_{\rm EC}$, $\delta I_{\rm T^2}$ and A are divergent, but their sum vanishes as a consequence of (5.4a). The sum $I_{\rm ct} := \int d^2x(\Delta_1+\Delta_2+\Delta_3)$, which appears in δI as a total variation and is also divergent, is recognized as the *counterterm*; when subtracted from I, it defines the renormalized action $I_{\rm ren} = I - I_{\rm ct}$, see the next subsection for more details. The variation of $I_{\rm ren}$ is finite:

$$\begin{aligned}\delta I_{\rm ren} = &-4a\int_{\partial\mathcal{M}} d^2x\varepsilon^{\alpha\beta}\varepsilon^{ac}s_{\beta c}\delta e_{a\alpha}\,,\\ &+4a_3\int_{\partial\mathcal{M}} d^2x(\varepsilon\cdot s)\varepsilon^{\alpha\beta}e^a{}_\alpha\delta e_{a\beta}\,,\\ &+A-4b_6q\int d^2x\varepsilon^{\alpha\beta}\varepsilon^{af}s_{\beta f}\delta e_{a\alpha}+C\,.\end{aligned} \tag{5.11}$$

From this result, one can identify the spin and the energy-momentum boundary currents, or equivalently, the 1-point functions of an effective 2D quantum theory, as:

$$\sigma^\beta = (b_4-b_5)\varepsilon^{\beta\alpha}\left(\frac{p}{2}\varepsilon_{ac}-\eta_{ac}\right)V^c\,\bar e^a{}_\alpha\,, \tag{5.12a}$$
$$\begin{aligned}\tau^\alpha{}_a = &\,4(a+b_6q)\varepsilon^{\alpha\beta}\varepsilon_{ac}s_\beta{}^c + 4a_3(\varepsilon\cdot s)\varepsilon^{\alpha\beta}e_{a\beta}\\ &-\varepsilon^{\alpha\beta}\frac{b_6-b_4}{3}(R-4s^\gamma{}_\gamma)e^b{}_\beta\left(\frac{p}{2}\eta_{ba}-\varepsilon_{ba}\right)\\ &+2\varepsilon^{\alpha\beta}\left[\left(b_6\frac{p^2}{2}-2b_5\right)\eta_{ba}+p(b_5-b_6)\varepsilon_{ba}\right]e^b{}_\beta(\varepsilon\cdot s)\,.\end{aligned} \tag{5.12b}$$

5.4 Renormalized action

Before we continue to examine the Noether-Ward identities of the boundary currents (5.12), let us stress that the variation of the full action I contains the total variation of the divergent term $I_{\rm ct}$, which can be compactly expressed as

$$I_{ct} = \frac{(a+qb_6)}{\rho^2}\int_{\partial M} d^2x\bar eK = (a+qb_6)\int_{\partial M} d^2x\tilde eK\,. \tag{5.13a}$$

Note that the factor $(a+qb_6)$ is proportional to the central charge of the theory [12]. Subtracting this counterterm from the original action I yields the renormalized action,

$$I_{\rm ren} = I - I_{ct} = I-(a+qb_6)\int_{\partial M} d^2x\tilde eK\,, \tag{5.13b}$$

the variation of which produces the finite boundary currents (5.12).

One should observe that here, like in the MB model or GR, the counterterm is of the Gibbons-Hawking type, but with a modified factor which involves the $\hat R^2$ coupling constant b_6. All the other quadratic terms in the action give finite contributions that need not be regularized. Similarly as in the previous section, we can decompose $I_{\rm ct}$ into the Balasubramanian-Kraus type local counterterm and the finite term proportional to $\int d^2x\bar{e}s^c{}_c$, which becomes, on shell, a local function of the boundary curvature and torsion.

We would like to emphasize that, in even boundary dimensions, there is a logarithmic term in the field expansions related to the variation of the conformal anomaly, i.e., to its functional derivative with respect to the corresponding source. In Einstein-Hilbert gravity, however, its coefficient is obtained as a variation with respect to the boundary metric of the conformal anomaly which is topological invariant in two dimensions, such that it can be dropped out in the holographic renormalization procedure [19]. For the present holographic analysis with torsion, the field equations can be also solved consistently without adding such type of terms. This seems to be a reflection of the fact that the coefficients of the log terms in both the vielbein and the spin connection are related to the variation of the Weyl anomaly that turns out to be, as we show below, a topological invariant, even when the torsional degrees of freedom are taken into account.

Similar type of a logarithmic term also appears in the action evaluated on-shell. Namely, the counterterm (5.13a) ensures a differentiable and finite *variation* of the action $I_{\rm ren}$, but the action itself contains a log term whose coefficient is related to topological invariants. As mentioned in section4, inclusion of these terms is important in the full renormalized action that is identified with the free energy of the dual CFT.

These invariants are the same as those appearing in the conformal anomaly, the form of which will be obtained in the next subsection.

5.5 Boundary symmetries and anomalies

To simplify the derivation of the boundary symmetries and make it more direct, we rewrite the spin and the energy-momentum current in a more compact way. First, using the expression (5.9), we write the spin current in the form

$$\sigma^\beta = -2\varepsilon^{\alpha\beta}\mathcal{H}_{\alpha 1} = 2\varepsilon^{\beta\alpha}\left(\hat{e}^a{}_\alpha\mathcal{H}_{a1}\right)|_{\rho=0}\,. \tag{5.14}$$

In what follows, we shall omit the sign $|_{\rho=0}$ for simplicity. After isolating the counterterm, the energy-momentum tensor becomes

$$\begin{aligned}\tau^\beta{}_b = {}& 4(a+qb_6)\varepsilon^{\alpha\beta}\varepsilon_{cb}s_\alpha{}^c - \frac{2a_3}{\rho^2}\varepsilon^{\alpha\beta}e_{b\alpha}\left(\hat{\mathcal{A}}-p\right)\\ &-\frac{2}{\rho^2}\varepsilon^{\alpha\beta}e^c{}_\alpha\left(\mathcal{H}_{cg}-\eta_{cg}\mathcal{H}-\eta_{cg}b_6q\right)\left(\frac{p}{2}\delta^g_b-\varepsilon^g{}_b\right)\,.\end{aligned} \tag{5.15a}$$

Then, using (5.4a), we obtain an equivalent form of $\tau^\beta{}_b$:

$$\begin{aligned}\tau^\beta{}_b = {}& -\frac{2}{\rho^2}(a+qb_6)\varepsilon^{\alpha\beta}(k_{b\alpha}-\varepsilon_{ab}e^a{}_\alpha) - \frac{2a_3}{\rho^2}\varepsilon^{\alpha\beta}e_{b\alpha}\hat{\mathcal{A}}\\ &-\frac{2}{\rho^2}\varepsilon^{\alpha\beta}e^c{}_\alpha\left(\mathcal{H}_{cg}-\eta_{cg}\mathcal{H}-\eta_{cg}b_6q\right)\left(\frac{p}{2}\delta^g_b-\varepsilon^g{}_b\right)\,.\end{aligned} \tag{5.15b}$$

Note that the trace of $\tau^\alpha{}_a$ is given by

$$\tau = \bar{e}\left[-4(a+b_6q)s^a{}_a - \frac{2}{\rho^2}\left(\mathcal{H}^a{}_a - 2\mathcal{H} - 2b_6q + \frac{p}{2}\varepsilon^{ab}\mathcal{H}_{ab}\right)\right]\,. \tag{5.16}$$

Lorentz invariance. To verify the conservation law of the spin current (5.14), we start from the relations:

$$\begin{aligned}
\nabla_\alpha\sigma^\alpha &= \hat\nabla_\alpha\sigma^\alpha = \frac{\bar e}{\rho^2}\varepsilon^{bc}\left(\hat T^a{}_{bc}\mathcal{H}_{a1} + 2\hat\nabla_b\mathcal{H}_{c1}\right) ,\\
\varepsilon^{ab}\tau_{ab} &= -\frac{2\bar e}{\rho^2}(a+b_6q+2a_3)(\hat{\mathcal{A}}-p) - \frac{2\bar e}{\rho^2}\left(\varepsilon^{bc}\mathcal{H}_{bc} + \frac{p}{2}\mathcal{H}^c{}_c - p\mathcal{H} - pb_6q\right)\\
&\equiv \frac{\bar e}{\rho^2}(a+2a_3-\mathcal{H}^c{}_c)\varepsilon^{ab}\hat T_{1ab} = -\frac{2\bar e}{\rho^2}(a+2a_3-\mathcal{H}^c{}_c)\hat{\mathcal{A}} .
\end{aligned}$$

Then, using the field equation $(1ab)$ in the form

$$-2\big(a-\mathcal{H}^c{}_c+2a_3\big)\hat{\mathcal{A}} + \varepsilon^{bc}(\hat T^a{}_{bc}\mathcal{H}_{a1} + 2\hat\nabla_b\mathcal{H}_{c1}) = 0 ,$$

the Lorentz invariance condition is found to be satisfied on shell:

$$A_L \equiv \nabla_\alpha\sigma^\alpha + \varepsilon^{ab}\tau_{ab}{=}0 . \tag{5.17}$$

Thus, our parity-invariant model (5.2) is Lorentz-invariant, in contrast to the situation in the MB model, where the Chern-Simons term violates this invariance, see (4.12).

Translation invariance. Let us now examine the invariance under local translations. First, we note that the validity of the Lorentz invariance condition (3.6b) implies that the last term on the right-hand-side of (3.6a) vanishes. Next, we calculate the divergence of the energy-momentum tensor:

$$\begin{aligned}
\nabla_\beta\tau^\beta{}_a &= \frac{2\bar e}{\rho^2}\left[(a+b_6q)\left(\frac{1}{\rho}\hat R_{1a} - \left(\frac{p}{2}\varepsilon_{ab}-\eta_{ab}\right)V^b\right) + 2\nabla^b\mathcal{H}_{a1b} - a_3(p-\hat{\mathcal{A}})\varepsilon_{ab}V^b\right]\\
&\quad + \frac{\bar e}{\rho^3}\left(\varepsilon_a{}^b + \frac{p}{2}\delta^a_b\right)\Big[2\varepsilon^{cd}\hat\nabla_c(\mathcal{H}_{db}-\eta_{db}\mathcal{H}) + 2\hat{\mathcal{A}}\mathcal{H}_{1b} + 2\hat{\mathcal{A}}\mathcal{H}_{b1} - 2k_b{}^d\mathcal{H}_{d1}\\
&\quad + \hat T^f{}_{cd}\varepsilon^{cd}\left(\mathcal{H}_{fb} - \eta_{fb}\mathcal{H} - b_6q\eta_{fb}\right)\Big] .
\end{aligned}$$

Making use of (5.4a) and the (abc) field equation (appendix D), the above result is simplified:

$$\begin{aligned}
\nabla_\beta\tau^\beta{}_a = \frac{2\bar e}{\rho^2}\Big[&(a+b_6q)\frac{1}{\rho}\hat R_{1a} + 2\nabla^b\mathcal{H}_{a1b} - \alpha_3\left(\frac{p}{2}\varepsilon_{ab}+\eta_{ab}\right)V^b\\
&+a_3(\hat{\mathcal{A}}+p)\varepsilon_{ab}v^b + \frac{1}{\rho}\left(\varepsilon_{ab}+\frac{p}{2}\eta_{ab}\right)(\hat{\mathcal{A}}\mathcal{H}^b{}_1 - k^{db}\mathcal{H}_{d1})\Big] .
\end{aligned} \tag{5.18}$$

Then, using the relations

$$\begin{aligned}
\sigma^\beta F_{\alpha\beta} &= \bar e R\mathcal{H}_{\alpha1} ,\\
\tau^{cb}T_{bac} &= 4\bar e(a+b_6q)s_{ab}V^b + \frac{2\bar e}{\rho^2}a_3(p-\hat{\mathcal{A}})\varepsilon_{ab}V^b\\
&\quad -\frac{2\bar e}{\rho^3}(\mathcal{H}_{ac}-\eta_{ac}\mathcal{H}-\eta_{ac}b_6q)\hat R_1{}^c ,
\end{aligned} \tag{5.19}$$

we finally obtain:

$$A_T = \nabla_\beta\tau^b{}_a - \sigma^\beta F_{a\beta} - \tau^{bc}T_{cab}$$

$$= \frac{\bar{e}}{\rho^2}\left[-4\rho^2(a+b_6q+\alpha_3)s_{ab}V^b - \rho R\mathcal{H}_{a1} + \frac{1}{\rho}(2\hat{R}_{11}+\hat{R})\mathcal{H}_{a1}\right.$$
$$\left. + \frac{2}{\rho}\left(\varepsilon_{ab}+\frac{p}{2}\eta_{ab}\right)(\hat{\mathcal{A}}\mathcal{H}^b{}_1 - k^{db}\mathcal{H}_{d1})\right]$$
$$= -4\bar{e}s_a{}^b\left[\left(a+b_6q+\alpha_3-\frac{b_4-b_5}{2}\left(1+\frac{p^2}{4}\right)\right)V_b + p\frac{b_4-b_5}{2}\varepsilon_{bc}V^c\right]=0\,, \quad (5.20)$$

where, in the last line, we again used the (abc) field equation.

This proves the translation invariance on the boundary.

Conformal anomaly. Let us now examine the dilatation invariance by calculating the expression $A_C = \tau - \nabla_\beta(\varepsilon_{ab}\sigma^a e^{b\beta})$. We start with

$$A_C = \bar{e}\left[-4(a+b_6q)s^c{}_c + 4p(b_5-b_6)(\varepsilon\cdot s) + \frac{2}{3}(b_6-b_4)(R-4s^c{}_c)\right]$$
$$+(b_4-b_5)\nabla_\beta\left[\bar{e}\left(\frac{p}{2}\varepsilon_{ab}-\eta_{ab}\right)e^{a\beta}V^b\right]\,.$$

Then, the identity

$$\nabla_\beta\left[\bar{e}\left(\frac{p}{2}\varepsilon_{ab}-\eta_{ab}\right)e^{a\beta}V^b\right] = \bar{e}\left[\left(\frac{p}{2}\varepsilon_{ab}-\eta_{ab}\right)\nabla^a V^b + V^a V_a\right]\,,$$

and the 2nd order piece of equation $(1ab)$, lead to:

$$A_C = \bar{e}\Big[-(a+b_6q)R + 4b_6pq(\varepsilon\cdot s) + \left(a+q\frac{b_6+2b_4}{3}\right)(R-4s^c{}_c)$$
$$-(q+2)(b_4-b_5)(\nabla_a V^a - V_a V^a) + p(b_4-b_5)\varepsilon^{ab}\nabla_a V_b)\Big]\,.$$

Finally, by using equations $(1a)$ and (11), we obtain the conformal anomaly:

$$A_C = -(a+b_6q)\bar{e}R + [2\alpha_3 - (q+2)(b_4-b_5)]\,\bar{e}(\nabla_a V^a - V_a V^a)$$
$$+p(b_4-b_5)\bar{e}\varepsilon^{ab}\nabla_a V_b\,. \quad (5.21)$$

Since the conformal symmetry is broken, the boundary symmetry is reduced to the local Poincaré invariance.

The first term in A_C, proportional to $\bar{e}R = \partial_\alpha(2\varepsilon^{\alpha\beta}\omega_\beta)$, is a topological density (related to the topological invariant $\int d^2x\bar{e}\bar{R}$); the related factor $(a+b_6q)$ is proportional to the central charge of the theory [12]. Since the Weyl weights of $e^a{}_\alpha, T^a{}_{\beta\gamma}, V^a, \nabla_a V^a$ are $+1,+1,-1,-2$, respectively, the remaining two terms in A_C are seen to be invariant under local dilatations. For details of the classification of conformal anomalies, see [29].

A closer inspection of the Weyl invariants leads to the identities:

$$W_1 := \bar{e}(\nabla_a V^a - V_a V^a) = \partial_\alpha(\varepsilon^{\alpha\beta}e_{a\beta}\varepsilon^{ab}V_b)\,,$$
$$W_2 := \varepsilon^{ab}\nabla_a V_b = \partial_\alpha(\varepsilon^{\alpha\beta}e^a{}_\beta V_a)\,. \quad (5.22)$$

In particular, the first identity can be written in the language of differential forms as

$$N' \equiv T^a\,{}^*T_a - e^a\nabla\,{}^*T_a = d(e^a\,{}^*T_a)\,, \quad (5.23)$$

where we used $V_a = \varepsilon_{ab}{}^*T^b$. The 2-form N', which represents W_1, has an interesting resemblance with the Nieh-Yan 4-form [30, 31]. Similarly, a Nie-Yan-like representation for W_2 is obtained by the replacement ${}^*T_a \to T_a$ in (5.23). The integrals of W_1 and W_2 over the boundary are topological invariants, the nature of which will be studied elsewhere.

A theory with parameters for which the conformal anomaly vanishes is known as the critical gravity. For such a critical choice of parameters, the bulk theory may acquire logarithmic modes, which leads to a logarithmic CFT at the boundary. For general properties of gravities at the critical point, see e.g. [32].

6 Concluding remarks

In this paper, we presented an analysis of the AdS/CFT correspondence in the realm of 3D gravity with torsion, with an underlying RC geometry of spacetime.

Starting with a suitable holographic ansatz and its consistency condition, we found that the expected boundary symmetry is described by local Poincaré transformations plus dilatations. Based on an improved form of the Noether-Ward identities, we first analyzed the holographic features of the MB model, where we confirmed the results of Klemm and Tagliabue [8], derived by a different technique. Then, turning our attention to the more interesting case of 3D gravity with propagating torsion, we obtained the holographic conformal anomaly, with contributions stemming from both the curvature and the torsion invariants. As a consequence, the boundary symmetry is reduced to the local Poincaré invariance. The improved treatment of the Noether-Ward identities, being independent of the value of torsion on the boundary, significantly simplifies the calculations.

An interesting problem for further study is to clarify how torsion affects the structure of the dual CFT. A simple approach would be to study the specific PGT sectors containing only one of the six propagating torsion modes, with $J^P = 0^{\pm}, 1, 2$ [12].

Acknowledgments

One of us (O.M.) would like to thank Max Bañados, Gastón Giribet, Julio Oliva, Jorge Zanelli and all the other participants of the *Workshop on String Theory, Gravity and Fields*, held in Buenos Aires in October 2012, for their useful comments in the final stage of the preparation of this manuscript. This work was supported by the Serbian Science Foundation under Grant No. 171031 and the Chilean FONDECYT Grants No. 1090357 and No. 1110102. O.M. thanks DII-PUCV for support through the project No. 123.711/2011. The work of R.O. is financed in part by the UNAB grant DI-117-12/R.

A On the RC geometry in 2D

In 2D, the Lorentz connection, which is Abelian, has only one independent "internal" component, $\omega^{ab}{}_\alpha = -\varepsilon^{ab}\omega_\alpha$, and the local Poincaré transformations of $e^a{}_\alpha$ and ω_α have the form (2.12). The corresponding field strengths, the curvature and the torsion, are given by

$$R^{ab}{}_{\alpha\beta} = -\varepsilon^{ab} F_{\alpha\beta}\,, \qquad\qquad F_{\alpha\beta} := \partial_\alpha\omega_\beta - \partial_\beta\omega_\alpha\,,$$

$$T^a{}_{\alpha\beta} = \nabla_\alpha e^a{}_\beta - \nabla_\beta e^a{}_\alpha\,, \qquad \nabla_\alpha e^a{}_\beta := \partial_\alpha e^a{}_\beta - \varepsilon^a{}_c \omega_\alpha e^c{}_\beta\,. \tag{A.1}$$

The Ricci tensor and the scalar curvature read:

$$R^a{}_c = -\varepsilon^{ab} F_{cb}\,, \qquad R = -\varepsilon^{ab} F_{ab}\,. \tag{A.2a}$$

As a consequence:

$$R_{ab} = \frac{1}{2}\eta_{ab} R\,, \qquad F_{ab} = \frac{1}{2}\varepsilon_{ab} R\,, \tag{A.2b}$$

and the Ricci tensor is always symmetric. The torsion tensor, with only two independent components, is completely determined by its vector piece $V_a = T^b{}_{ba}$ as

$$T^a{}_{bc} = \delta^a_b V_c - \delta^a_c V_b\,.$$

When the torsion vanishes, the connection becomes Riemannian:

$$\tilde{\omega}_\alpha = \frac{1}{2}\varepsilon^{ab}(c_{abc} - c_{cab} + c_{bca}) e^c{}_\alpha\,, \qquad c^a{}_{\alpha\beta} := \partial_\alpha e^a{}_\beta - \partial_\beta e^a{}_\alpha\,, \tag{A.3}$$

see also (3.7).

In the Gauss-normal radial foliation, the unit normal to the boundary ∂M has the form

$$n_i = (n_1, n_a) = \frac{\hat{e}_i{}^\rho}{\sqrt{-\hat{g}^{\rho\rho}}} = (1, 0, 0)\,,$$

with $n^2 = -1$. The extrinsic curvature (the second fundamental form) of ∂M is defined by $K_{ij} = \hat{\nabla}_i n_j$. The only nonvanishing components of K_{ij} are

$$K_{ab} := \hat{\nabla}_a n_b = -\varepsilon_{bc}\hat{\omega}^c{}_a = \varepsilon_{cb} k^c{}_a = \frac{p}{2}\varepsilon_{ab} + \eta_{ab} - 2\rho^2 s_{ab}\,, \tag{A.4}$$

where we used $k^c{}_a := k^c{}_\alpha e_a{}^\alpha$. In particular:

$$\begin{aligned} K_{(ab)} &= \eta_{ab} - 2\rho^2 s_{(ab)}\,, \qquad K^b{}_b = 2 - 2\rho^2 s^c{}_c\,, \\ \varepsilon^{ab} K_{ab} &= -p - 2\rho^2 \varepsilon^{ab} s_{ab} \equiv -\hat{\mathcal{A}}\,. \end{aligned} \tag{A.5}$$

The last equation gives an interesting geometric interpretation of the axial torsion $\hat{\mathcal{A}}$. For $\hat{\mathcal{A}} = 0$, K_{ab} reduces to the standard Riemannian form.

B Residual symmetries to second order

At the end of section 2, we showed that the residual symmetry group with the parameter $f(x)$, defined by (2.9), acts as local dilatation on the leading order of the metric, $\bar{g}_{\alpha\beta}$. From (2.9), we can also find the transformation rule for the second order of the vielbein, $s^a{}_\alpha$, and extend the result of section 2 to the second order of the metric, $g_{(2)\alpha\beta}$.

Indeed, using the definitions $\bar{g}_{\alpha\beta} = \eta_{ab}\bar{e}^a{}_\alpha \bar{e}^b{}_\beta$ and $g_{(2)\alpha\beta} = s_{\alpha\beta} + s_{\beta\alpha}$, and restricting our attention to dilatations ($f \neq 0$), we obtain:

$$\begin{aligned} \delta_f \bar{g}_{\alpha\beta} &= 2f\, \bar{g}_{\alpha\beta}\,, \\ \delta_f g_{(2)\alpha\beta} &= 2f\, g_{(2)\alpha\beta} - 2\bar{e}_{a(\alpha}\bar{\nabla}_{\beta)} f^a + 2f^\gamma \bar{T}_{(\alpha\beta)\gamma}\,, \end{aligned} \tag{B.1}$$

where $f_\alpha := \frac{1}{2}\partial_\alpha f$. In the limit when torsion vanishes, this result reduces to the Penrose-Brown-Henneaux transformation [33, 34], which was derived in Riemannian GR and used to study universal properties of trace anomalies.

C Field strengths and covariant momenta

C.1 Torsion and curvature

The results of this subsection are obtained using the expression (2.11) for k^{ab}.

In the local Lorentz basis, the torsion components are:

$$\begin{aligned}
\hat{T}^1{}_{1c} &= 0\,,\\
\hat{T}^1{}_{bc} &= \varepsilon_{ce}k^e{}_b - \varepsilon_{be}k^e{}_c = -\varepsilon_{bc}k^e{}_e\,,\\
\hat{T}^a{}_{1c} &= -\varepsilon^{ae}\left[\varepsilon_{ec} + \left(\frac{p}{2}\eta_{ec} + k_{ec}\right)\right] + \rho e_c{}^\gamma\partial_\rho e^a{}_\gamma\,,\\
\hat{T}^a{}_{bc} &= \rho T^a{}_{bc}\,,
\end{aligned}\tag{C.1}$$

and the components of curvature read:

$$\begin{aligned}
\hat{R}_{11c} &= -\rho^2 e_c{}^\gamma\partial_\rho\omega_\gamma\,,\\
\hat{R}_{1bc} &= -\rho^2 F_{bc} + \varepsilon^{ed}k_{eb}k_{dc}\,,\\
\hat{R}_{a1c} &= -\left(k_{ac} + \frac{p}{2}\varepsilon_a{}^b k_{bc}\right) + \rho e_c{}^\gamma\partial_\rho k_{a\gamma}\,,\\
\hat{R}_{abc} &= \rho e_b{}^\beta e_c{}^\gamma(\nabla_\beta k_{a\gamma} - \nabla_\gamma k_{a\beta})\,.
\end{aligned}\tag{C.2}$$

The Ricci tensor and the scalar curvature are calculated from the relations:

$$\begin{aligned}
\hat{R}_{ik} &= -\varepsilon^{mn}{}_i\hat{R}_{mnk}\,,\\
\hat{R} &= -\varepsilon^{mnk}\hat{R}_{mnk} = \hat{R}^1{}_1 + \hat{R}^a{}_a\,.
\end{aligned}\tag{C.3}$$

Reduction. Equation (2.11), in which k_{ab} is expressed in terms of $e^a{}_\alpha$, simplifies the expressions (C.1) for the torsion:

$$\begin{aligned}
\hat{T}_{1bc} &= \varepsilon_{bc}\hat{\mathcal{A}}\,,\\
\hat{T}_{a1c} &= -\varepsilon_{ac}\hat{\mathcal{A}}\,,\\
\hat{T}_{abc} &= \rho T_{abc}\,,
\end{aligned}\tag{C.4}$$

where $\hat{\mathcal{A}}$ is the axial torsion:

$$\hat{\mathcal{A}} := \frac{1}{6}\varepsilon^{ijk}\hat{T}_{ijk} = p - \rho\varepsilon^{fg}e_f{}^\beta\partial_\rho e_{g\beta}\,.$$

Similarly, the curvature tensor reads:

$$\begin{aligned}
\hat{R}_{11c} &= -\rho^2 e_c{}^\beta\partial_\rho\omega_\beta\,,\\
\hat{R}_{1bc} &= \varepsilon_{bc}q - \rho^2 F_{bc} - \varepsilon_{bc}\frac{p}{2}(p-\hat{\mathcal{A}}) + \rho\varepsilon_{bc}e^{g\beta}\partial_\rho e_{g\beta} + Y_{bc}\,,\\
\hat{R}_{a1c} &= -\varepsilon_{ac}q + \left(\frac{p}{2}\varepsilon_{ac} - \eta_{ac}\right)(p-\hat{\mathcal{A}}) + \rho^3\varepsilon_{ab}e^{b\beta}\partial_\rho\left(\rho^{-1}\partial_\rho e_{c\beta}\right) + X_{ac}\,,\\
\hat{R}_{abc} &= \rho\left(\frac{p}{2}T_{abc} - \varepsilon_a{}^f T_{fbc}\right) + Z_{abc}\,,
\end{aligned}\tag{C.5}$$

where Y_{ac}, X_{ac} and Z_{abc} are given by

$$X_{ac} := \rho^2\varepsilon_a{}^f e_c{}^\beta\partial_\rho e_{b\gamma}\partial_\rho(e_f{}^\gamma e^b{}_\beta) = -\eta_{ac}\rho^3\partial_\rho\left[\rho^{-2}(p-\hat{\mathcal{A}})\right] - \rho(p-\hat{\mathcal{A}})e_c{}^\beta\partial_\rho e_{a\beta}\,,$$

$$
\begin{aligned}
Y_{bc} &= -\rho^2 \varepsilon_{fg} e^{f\beta} e^{g\gamma} (\partial_\rho e_{b\beta})(\partial_\rho e_{c\gamma}) \,, \\
Z_{abc} &= -\rho^2 \varepsilon_{af} e_b{}^\beta e_c{}^\gamma \big[\nabla_\beta(\partial_\rho e^{f\alpha} g_{\alpha\gamma}) - \nabla_\gamma(\partial_\rho e^{f\alpha} g_{\alpha\beta})\big] \,.
\end{aligned}
\tag{C.6}
$$

As a consequence, the Ricci tensor and the scalar curvature read:

$$
\begin{aligned}
\hat{R}_{11} &= \varepsilon^{bc} \hat{R}_{b1c} = 2q - p(p - \hat{\mathcal{A}}) - \rho^3 e^{c\beta} \partial_\rho \left(\rho^{-1} \partial_\rho e_{c\beta}\right) + (p - \hat{\mathcal{A}})^2 \,, \\
\hat{R}_{1c} &= -\varepsilon^{ab} \hat{R}_{abc} = \rho \left(\frac{p}{2} \varepsilon_{cb} V^b - V_c\right) - \varepsilon^{ab} Z_{abc} \,, \\
\hat{R}_{a1} &= \varepsilon_a{}^c \hat{R}_{11c} = -\rho^2 \varepsilon_a{}^c e_c{}^\beta \partial_\rho \omega_\beta \,, \\
\hat{R}_{ab} &= \varepsilon_a{}^c (\hat{R}_{c1b} - \hat{R}_{1cb}) = -2\eta_{ab} q + (p\eta_{ab} - \varepsilon_{ab})(p - \hat{\mathcal{A}}) \\
&\quad + \rho^3 e_a{}^\beta \partial_\rho \left(\rho^{-1} \partial_\rho e_{b\beta}\right) + \rho^2 R_{ab} - \rho \eta_{ab} e^{g\gamma} \partial_\rho e_{g\gamma} + \varepsilon_a{}^c (X_{cb} - Y_{cb}) \,, \\
\hat{R} &= -6q + 3p(p - \hat{\mathcal{A}}) + 2\rho^3 e^{c\beta} \partial_\rho (\rho^{-1} \partial_\rho e_{c\beta}) \\
&\quad + \rho^2 R - 2\rho e^{f\beta} \partial_\rho e_{f\beta} - \varepsilon^{ac} (2X_{ac} + Y_{ac}) \,,
\end{aligned}
$$

where

$$
\begin{aligned}
\varepsilon_a{}^c (X_{cb} - Y_{cb}) &= -\varepsilon_{ab} \rho^3 \partial_\rho \big[\rho^{-2}(p - \hat{\mathcal{A}})\big] - \rho(p - \hat{\mathcal{A}}) \varepsilon_a{}^c e_b{}^\beta \partial_\rho e_{c\beta} \\
&\quad + \rho^2 \varepsilon_a{}^c E^{\beta\gamma} (\partial_\rho e_{c\beta})(\partial_\rho e_{b\gamma}) \,, \\
-\varepsilon^{ac} (2X_{ac} + Y_{ac}) &= -2(p - \hat{\mathcal{A}})^2 - \rho^2 \varepsilon^{bc} E^{\beta\gamma} (\partial_\rho e_{b\beta})(\partial_\rho e_{c\gamma}) \,.
\end{aligned}
$$

C.2 Covariant momenta

Here, we rely on the conditions (2.8), which imply $X_{abc} = \mathcal{O}_4 = Y_{abc}$ and $Z_{abc} = \mathcal{O}_3$. The calculations in section 5 are greatly simplified if we first find the explicit form of the covariant momenta. In the torsion sector, we have:

$$
\begin{aligned}
\mathcal{H}_{11c} &= -2\alpha_3 \rho V_c \,, \\
\mathcal{H}_{1bc} &= -\mathcal{H}_{b1c} = 4(\alpha_1 - \alpha_2) \varepsilon_{bc} \hat{\mathcal{A}} \,, \\
\mathcal{H}_{abc} &= 2\rho \, (2\alpha_1 + \alpha_2 + \alpha_3) \, T_{abc} \,,
\end{aligned}
\tag{C.7}
$$

and in the curvature sector, we find:

$$
\begin{aligned}
\mathcal{H}_{11} &= 2(\beta_1 + \beta_2 + 3\beta_3) q - (\beta_1 + \beta_2 + 3\beta_3) p(p - \hat{\mathcal{A}}) \\
&\quad - \beta_3 \rho^2 \left(R - 4s^c{}_c\right) + \mathcal{O}_4 \,, \\
\mathcal{H}_{a1} &= \beta_2 \rho \left(\frac{p}{2} \varepsilon_{ac} - \eta_{ac}\right) V^c + \mathcal{O}_3 \,, \\
\mathcal{H}_{1a} &= \beta_1 \rho \left(\frac{p}{2} \varepsilon_{ac} - \eta_{ac}\right) V^c + \mathcal{O}_3 \,, \\
\mathcal{H}_{ab} &= -2(\beta_1 + \beta_2 + 3\beta_3) \eta_{ab} q + (\beta_1 + \beta_2 + 3\beta_3) \eta_{ab} p(p - \hat{\mathcal{A}}) \\
&\quad - (\beta_1 - \beta_2) \varepsilon_{ab} (p - \hat{\mathcal{A}}) + \frac{1}{2} \rho^2 (\beta_1 + \beta_2 + 2\beta_3) \eta_{ab} (R - 4\bar{e}^{f\beta} s_{f\beta}) + \mathcal{O}_4 \,.
\end{aligned}
\tag{C.8}
$$

D Radial expansion of the field equations

In this appendix, we display higher orders in ρ of the vacuum field equations (5.3), which are needed in our study of the Noether-Ward identities for 3D gravity with propagating torsion. To zeroth order in ρ, the content of these equations is displayed in (5.4). The parameter q is given in (2.14b) as $q = p^2/4 - 1$. In our notation, $\varepsilon \cdot s = \varepsilon^{ab} s_{ab}$ and $\mathcal{H} = \mathcal{H}^k{}_k$.

JHEP05(2013)103

(1). Let us start by considering the components $(ij) = (1a), (a1), (11)$ and (ab) of the first field equation (5.3a). The object t_{ij} is defined in the same equation. For each component (i,j), we display first a compact form, and then the fully expanded field equation.

$(1a)$:

$$\begin{aligned} \hat{\nabla}^m \mathcal{H}_{1am} + \frac{1}{2}\mathcal{H}_1{}^{mn}\hat{T}_{amn} - \mathcal{H}_{1a}{}^n V_n + t_{1a} &= 0\,, \\ 2\rho\left[(2\alpha_1+\alpha_2+\alpha_3)+\beta_1 q\right]\left(\frac{p}{2}\varepsilon_{ab}V^b - V_a\right) &= \mathcal{O}_3\,. \end{aligned} \tag{D.1}$$

$(a1)$:

$$\begin{aligned} \hat{\nabla}^m \mathcal{H}_{a1m} + \frac{1}{2}\mathcal{H}_a{}^{mn}\hat{T}_{1mn} - \mathcal{H}_{a1}{}^n V_n + t_{a1} &= 0\,, \\ -2\rho\left[a+\alpha_3+(b_6+\beta_2)q\right]V_a & \\ +p\rho\left[a-\alpha_3+8(\alpha_1-\alpha_2)+(b_6+\beta_2)q\right]\varepsilon_{ab}V^b &= \mathcal{O}_3\,. \end{aligned} \tag{D.2}$$

(11):

$$\begin{aligned} \hat{\nabla}^m \mathcal{H}_{11m} + \frac{1}{2}\mathcal{H}_1{}^{mn}\hat{T}_{1mn} - \mathcal{H}_{11}{}^n V_n + t_{11} &= 0\,, \\ -2\alpha_3\nabla_a V^a - \left[(2\alpha_1+\alpha_2-\alpha_3)+\beta_1 q\right]V_c V^c & \\ +\left[a-\left(2\beta_3-\frac{b_6}{2}\right)q\right](R-4s^\gamma{}_\gamma) & \\ -2p\left[a+4(\alpha_1-\alpha_2)-b_6 q\right](\varepsilon\cdot s) &= \mathcal{O}_2\,. \end{aligned} \tag{D.3}$$

(ab):

$$\begin{aligned} \hat{\nabla}^m \mathcal{H}_{abm} + \frac{1}{2}\mathcal{H}_a{}^{mn}\hat{T}_{bmn} - \mathcal{H}_{ab}{}^n V_n + t_{ab} &= 0\,, \\ 2(2\alpha_1+\alpha_2+\alpha_3)\nabla^c T_{abc} - \left[(2\alpha_1+\alpha_2+\alpha_3)+\beta_1 q\right]\eta_{ab}V_c V^c & \\ -\eta_{ab}\left(\beta_3-\frac{3b_6}{4}\right)q(R-4s^\gamma{}_\gamma) & \\ +2\eta_{ab}\left[a+4(\alpha_1-\alpha_2)-b_6 q\right]p(\varepsilon\cdot s) & \\ -4\varepsilon_{ab}\left[a+4(\alpha_1-\alpha_2)-(\beta_1-\beta_2-b_6)q\right](\varepsilon\cdot s) &= \mathcal{O}_2\,. \end{aligned} \tag{D.4}$$

(2). Now, we turn to the components $(kij) = (a1b), (11b), (1ab)$ and (cab) of the second field equation (5.3b).

$(a1b)$:

$$\begin{aligned} 2\hat{T}^c{}_{1b}\left(a\eta_{ca}+\mathcal{H}_{ca}-\eta_{ca}\mathcal{H}\right)+2\hat{\nabla}_1(\mathcal{H}_{ba}-\eta_{ba}\mathcal{H})-2\hat{\nabla}_b\mathcal{H}_{1a}-\varepsilon_{bc}\varepsilon^f{}_a(\mathcal{H}_{1f}{}^c-\mathcal{H}_{f1}{}^c) &= 0\,, \\ -2\beta_1\left(\frac{p}{2}\varepsilon_{af}-\eta_{af}\right)\nabla_b V^f + 2p\eta_{ab}\left[2b_6+(\beta_1-\beta_2)\right](\varepsilon\cdot s) & \\ -4\varepsilon_{ab}\left[a+4(\alpha_1-\alpha_2)+b_6(q+p^2/2)+(\beta_1-\beta_2)\right](\varepsilon\cdot s) & \\ -\left(\eta_{ab}-\frac{p}{2}\varepsilon_{ab}\right)\left(\frac{3b_6}{4}-\beta_3\right)(R-4s^\gamma{}_\gamma) &= \mathcal{O}_2\,. \end{aligned} \tag{D.5}$$

(11b):

$$2\hat{T}^c{}_{1b}\mathcal{H}_{c1} + 2\hat{\nabla}_1\mathcal{H}_{b1} - 2\hat{\nabla}_b(\mathcal{H}_{11} - \eta_{11}\mathcal{H}^k{}_k) + \varepsilon_{bc}\varepsilon^{fg}\mathcal{H}_{fg}{}^c = 0\,,$$
$$[-(2\alpha_1 + \alpha_2 + \alpha_3) - \beta_1 q]\,\rho V_b = \mathcal{O}_3\,. \qquad \text{(D.6)}$$

(1ab):

$$2\hat{T}^1{}_{ab}\,(a\eta_{11} + \mathcal{H}_{11} - \eta_{11}\mathcal{H}) + 2\hat{T}^c{}_{ab}\mathcal{H}_{c1} + 4\hat{\nabla}_{[a}\mathcal{H}_{b]1} + \varepsilon_{ab}\varepsilon^{fg}\mathcal{H}_{fg}{}^1 = 0\,,$$
$$4\left[a + 4(\alpha_1 - \alpha_2) + b_6(q + p^2/2) - 2(\beta_1 - \beta_2)\right](\varepsilon\cdot s)$$
$$+p\beta_2 V_c V^c - 2\beta_2\left(\frac{p}{2}\eta^{fg} - \varepsilon^{fg}\right)\nabla_f V_g - p\left(\frac{b_6}{4} + \beta_3\right)(R - 4s^\gamma{}_\gamma) = \mathcal{O}_2\,. \qquad \text{(D.7)}$$

(cab):

$$2\hat{T}^f{}_{ab}(a\eta_{fc} + \mathcal{H}_{fc} - \eta_{fc}\mathcal{H}) + 2\hat{T}^1{}_{ab}\mathcal{H}_{1c} + 4\hat{\nabla}_{[a}(\mathcal{H}_{b]c} - \eta_{b]c}\mathcal{H}) - \varepsilon_{ab}\varepsilon^f{}_c\mathcal{H}_{1f}{}^1 = 0\,,$$
$$\left[a + b_6 q - \beta_2\left(1 + \frac{p^2}{4}\right) + \alpha_3\right] T_{cab} - \beta_2 p\,\varepsilon_{ab}V_c = \mathcal{O}_2\,. \qquad \text{(D.8)}$$

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PHYSICAL REVIEW D **96,** 064031 (2017)

Generalized plane waves in Poincaré gauge theory of gravity

Milutin Blagojević* and Branislav Cvetković†
Institute of Physics, University of Belgrade, Pregrevica 118, 11080 Belgrade, Serbia

Yuri N. Obukhov‡
Theoretical Physics Laboratory, Nuclear Safety Institute,
Russian Academy of Sciences, B. Tulskaya 52, 115191 Moscow, Russia
(Received 11 July 2017; published 18 September 2017)

A family of exact vacuum solutions, representing generalized plane waves propagating on the (anti-)de Sitter background, is constructed in the framework of Poincaré gauge theory. The wave dynamics is defined by the general Lagrangian that includes all parity even and parity odd invariants up to the second order in the gauge field strength. The structure of the solution shows that the wave metric significantly depends on the spacetime torsion.

DOI: 10.1103/PhysRevD.96.064031

I. INTRODUCTION

The gauge principle, which was originally formulated by Weyl in the context of electrodynamics [1], now belongs to the key concepts which underlie the modern understanding of dynamical structure of fundamental physical interactions. Development of Weyl's idea, most notably in the works of Yang, Mills and Utiyama [2,3], resulted in the construction of the general gauge-theoretic framework for arbitrary non-Abelian groups of *internal* symmetries. Sciama and Kibble extended this formalism to the *spacetime* symmetries, and proposed a theory of gravity [4,5] based on the Poincaré group—a semidirect product of the group of spacetime translations times the Lorentz group. The importance of the Poincaré group in particle physics strongly supports the Poincaré gauge theory (PGT) as the most appropriate framework for description of the gravitational phenomena.

The "translational" gauge field potentials (corresponding to the subgroup of the spacetime translations) can be consistently identified with the spacetime coframe field, whereas the "rotational" gauge field potentials (corresponding to the local Lorentz subgroup) can be interpreted as the spacetime connection. This introduces the Riemann–Cartan geometry on the spacetime manifold, since one naturally recovers the torsion and the curvature as the Poincaré gauge field strengths [6–16] ("translational" and "rotational" one, respectively). The gravitational dynamics in PGT is determined by a Lagrangian that is assumed to be the function of the field strengths, the curvature and the torsion, and the dynamical setup is completed by including a suitable matter Lagrangian.

In the past, investigations of PGT were mostly focused on the class of models with quadratic parity symmetric Lagrangians of the Yang-Mills type, expecting that the results obtained for such a class should be sufficient to reveal essential dynamical features of the more complex general theory, for an overview see [17]. Recently, however, there has been a growing interest for the extended class of models with a general Lagrangian that includes both parity even and parity odd quadratic terms, see for instance [18–23]. An important difference between these two classes of PGT models is manifest in their particle spectra. Generically, the particle spectrum of the parity conserving PGT model consists of the massless graviton and eighteen massive torsion modes. The conditions for the absence of ghosts and tachyons impose serious restrictions on the propagation of these modes [24–29]. In contrast, a recent analysis of the general PGT [30] shows that the propagation of torsion modes is much less restricted. This is a new and physically interesting dynamical effect of the parity odd sector.

Based on the experience stemming from general relativity (GR), it is well known that exact solutions play an important role in understanding gravitational dynamics. An important class of these solutions consists of the gravitational waves [31–35], one of the best known families of exact solutions in GR. For many years, investigation of gravitational waves has been an interesting subject also in the framework of PGT [36–45], as well as in the metric-affine gravity theory which is obtained in the gauge-theoretic approach when the Poincaré group is extended to the general affine symmetry group [46–54]. Noticing that dynamical effects of the parity odd sector of PGT are not sufficiently well known, recently one of us [55] has studied exact plane wave solutions with torsion in vacuum, propagating on the flat background, for the case of the vanishing cosmological constant Λ. In another recent work [56] complementary results have been obtained, when the

*mb@ipb.ac.rs
†cbranislav@ipb.ac.rs
‡obukhov@ibrae.ac.ru

generalized pp waves with torsion were derived as exact vacuum solutions of the parity even PGT, but for the case of a nontrivial $\Lambda \neq 0$. In the present paper, we merge and extend these investigations by constructing the generalized plane waves with torsion as vacuum solutions of the general quadratic PGT with nonvanishing cosmological constant. The resulting structure offers a deeper insight into the dynamical role of the parity odd sector of PGT.

The paper is organized as follows. In the next Sec. II we present a condensed introduction to the Poincaré gauge gravity theory, giving the basic definitions and describing the main structures; more details can be found in [6–9]. In Sec. III we start with representing an (anti)-de Sitter spacetime as a gravitational wave and use the properties of the plane-fronted electromagnetic and gravitational waves discussed in [57] to formulate an ansatz for the gravitational wave in the Poincaré gauge gravity. The properties of the resulting curvature and torsion 2-forms are studied. In Sec. IV the set of differential equations for the wave variables is derived. It is worthwhile to note that the functions which describe the wave's profile satisfy a system of linear equations, even though the original field equations of the Poincaré gauge theory are highly nonlinear. Solutions of this system are constructed, and their properties are discussed. We demonstrate the consistency of the results obtained with the particle spectrum of the general Poincaré gauge gravity model. Finally, the conclusions are outlined in Sec. V.

Our basic notation and conventions are consistent with [7]. In particular, Greek indices $\alpha, \beta, \ldots = 0, \ldots, 3$, denote the anholonomic components (for example, of a coframe ϑ^α), while the Latin indices $i, j, \ldots = 0, \ldots, 3$, label the holonomic components (dx^i, e.g.). The anholonomic vector frame basis e_α is dual to the coframe basis in the sense that $e_\alpha \rfloor \vartheta^\beta = \delta^\beta_\alpha$, where $\rfloor$ denotes the interior product. The volume 4-form is denoted η, and the η-basis in the space of exterior forms is constructed with the help of the interior products as $\eta_{\alpha_1 \ldots \alpha_p} := e_{\alpha_p} \rfloor \ldots e_{\alpha_1} \rfloor \eta$, $p = 1, \ldots, 4$. They are related to the ϑ-basis via the Hodge dual operator *, for example, $\eta_{\alpha\beta} = {}^*(\vartheta_\alpha \wedge \vartheta_\beta)$. The Minkowski metric $g_{\alpha\beta} = \mathrm{diag}(+1, -1, -1, -1)$. All the objects related to the parity-odd sector (coupling constants, irreducible pieces of the curvature, gravitational wave potentials, etc) are marked by an overline, to distinguish them from the corresponding parity-even objects.

II. BASICS OF POINCARÉ GAUGE GRAVITY

The gravitational field is described by the coframe $\vartheta^\alpha = e_i^\alpha dx^a$ and connection $\Gamma_\alpha{}^\beta = \Gamma_{i\alpha}{}^\beta dx^i$ 1-forms. The translational and rotational field strengths read

$$T^\alpha = D\vartheta^\alpha = d\vartheta^\alpha + \Gamma_\beta{}^\alpha \wedge \vartheta^\beta, \tag{2.1}$$

$$R_\alpha{}^\beta = d\Gamma_\alpha{}^\beta + \Gamma_\gamma{}^\beta \wedge \Gamma_\alpha{}^\gamma. \tag{2.2}$$

As usual, the covariant differential is denoted D.

The gravitational Lagrangian 4-form is (in general) an arbitrary function of the geometrical variables:

$$V = V(\vartheta^\alpha, T^\alpha, R_\alpha{}^\beta). \tag{2.3}$$

Its variation with respect to the gravitational (translational and Lorentz) potentials yields the field equations

$$\mathcal{E}_\alpha := \frac{\delta V}{\delta \vartheta^\alpha} = -DH_\alpha + E_\alpha = 0, \tag{2.4}$$

$$\mathcal{C}^\alpha{}_\beta := \frac{\delta V}{\delta \Gamma_\alpha{}^\beta} = -DH^\alpha{}_\beta + E^\alpha{}_\beta = 0. \tag{2.5}$$

Here, the Poincaré *gauge field momenta* 2-forms are introduced by

$$H_\alpha := -\frac{\partial V}{\partial T^\alpha}, \qquad H^\alpha{}_\beta := -\frac{\partial V}{\partial R_\alpha{}^\beta}, \tag{2.6}$$

and the 3–forms of the *canonical* energy–momentum and spin for the gravitational gauge fields are constructed as

$$E_\alpha := \frac{\partial V}{\partial \vartheta^\alpha} = e_\alpha \rfloor V + (e_\alpha \rfloor T^\beta) \wedge H_\beta + (e_\alpha \rfloor R_\beta{}^\gamma) \wedge H^\beta{}_\gamma, \tag{2.7}$$

$$E^\alpha{}_\beta := \frac{\partial V}{\partial \Gamma_\alpha{}^\beta} = -\vartheta^{[\alpha} \wedge H_{\beta]}. \tag{2.8}$$

The field equations (2.4) and (2.5) are written here for the vacuum case. In the presence of matter, the right-hand sides of (2.4) and (2.5) contain the canonical energy-momentum and the canonical spin currents of the physical sources, respectively.

A. Quadratic Poincaré gravity models

The torsion 2-form can be decomposed into the 3 irreducible parts, whereas the curvature 2-form has 6 irreducible pieces. Their definition is presented in the Appendix.

The general quadratic model is described by the Lagrangian 4-form that contains all possible quadratic invariants of the torsion and the curvature:

$$\begin{aligned} V = \frac{1}{2\kappa c}\Big\{&(a_0 \eta_{\alpha\beta} + \bar{a}_0 \vartheta_\alpha \wedge \vartheta_\beta) \wedge R^{\alpha\beta} - 2\lambda_0 \eta \\ &- T^\alpha \wedge \sum_{I=1}^{3} [a_I {}^*({}^{(I)}T_\alpha) + \bar{a}_I {}^{(I)}T_\alpha]\Big\} \\ &- \frac{1}{2\rho} R^{\alpha\beta} \wedge \sum_{I=1}^{6} [b_I {}^*({}^{(I)}R_{\alpha\beta}) + \bar{b}_I {}^{(I)}R_{\alpha\beta}]. \end{aligned} \tag{2.9}$$

The Lagrangian has a clear structure: the first line is *linear* in the curvature, the second line collects *torsion quadratic* terms, whereas the third line contains the *curvature quadratic* invariants. Furthermore, each line is composed of the parity even pieces (first terms on each line), and the parity odd parts (last terms on each line). The dimensionless constant $\bar{a}_0 = \frac{1}{\xi}$ is inverse to the so-called Barbero-Immirzi parameter ξ, and the linear part of the Lagrangian —the first line in (2.9)—describes what is known in the literature as the Einstein-Cartan-Holst model. A special case $a_0 = 0$ and $\bar{a}_0 = 0$ describes the purely quadratic model without the Hilbert-Einstein linear term in the Lagrangian. In the Einstein-Cartan model, one puts $a_0 = 1$ and $\bar{a}_0 = 0$.

Besides that, the general PGT model contains a set of the coupling constants which determine the structure of quadratic part of the Lagrangian: ρ, a_1, a_2, a_3 and $\bar{a}_1, \bar{a}_2, \bar{a}_3$, $b_1, \ldots, b_6$ and $\bar{b}_1, \ldots, \bar{b}_6$. The overbar denotes the constants responsible for the parity odd interaction. We have the dimension $[\frac{1}{\rho}] = [\hbar]$, whereas a_I, $\bar{a}_I$, b_I and $\bar{b}_I$ are dimensionless. Moreover, not all of these constants are independent: we take $\bar{a}_2 = \bar{a}_3$, $\bar{b}_2 = \bar{b}_4$ and $\bar{b}_3 = \bar{b}_6$ because some of terms in (2.9) are the same in view of (A14)–(A16).

For the Lagrangian (2.9) from (2.6)–(2.8) we derive the gauge gravitational field momenta

$$H_\alpha = \frac{1}{\kappa c} h_\alpha, \tag{2.10}$$

$$H^\alpha{}_\beta = -\frac{1}{2\kappa c}(a_0 \eta^\alpha{}_\beta + \bar{a}_0 \vartheta^\alpha \wedge \vartheta_\beta) + \frac{1}{\rho} h^\alpha{}_\beta, \tag{2.11}$$

and the canonical energy-momentum and spin currents of the gravitational field

$$E_\alpha = \frac{1}{2\kappa c}(a_0 \eta_{\alpha\beta\gamma} \wedge R^{\beta\gamma} + 2\bar{a}_0 R_\alpha{}^\beta \wedge \vartheta_\beta - 2\lambda_0 \eta_\alpha + q^{(T)}_\alpha) + \frac{1}{\rho} q^{(R)}_\alpha, \tag{2.12}$$

$$E^\alpha{}_\beta = \frac{1}{2}(H^\alpha \wedge \vartheta_\beta - H_\beta \wedge \vartheta^\alpha). \tag{2.13}$$

For convenience, we introduced here the 2-forms which are linear functions of the torsion and the curvature, respectively, by

$$h_\alpha = \sum_{I=1}^{3} [a_I {}^*({}^{(I)}T_\alpha) + \bar{a}_I {}^{(I)}T_\alpha], \tag{2.14}$$

$$h^\alpha{}_\beta = \sum_{I=1}^{6} [b_I {}^*({}^{(I)}R^\alpha{}_\beta) + \bar{b}_I {}^{(I)}R^\alpha{}_\beta], \tag{2.15}$$

and the 3-forms which are quadratic in the torsion and in the curvature, respectively:

$$q^{(T)}_\alpha = \frac{1}{2}[(e_\alpha \rfloor T^\beta) \wedge h_\beta - T^\beta \wedge e_\alpha \rfloor h_\beta], \tag{2.16}$$

$$q^{(R)}_\alpha = \frac{1}{2}[(e_\alpha \rfloor R_\beta{}^\gamma) \wedge h^\beta{}_\gamma - R_\beta{}^\gamma \wedge e_\alpha \rfloor h^\beta{}_\gamma]. \tag{2.17}$$

By construction, (2.14) has the dimension of a length, $[h_\alpha] = [\ell]$, whereas the 2-form (2.15) is obviously dimensionless, $[h^\alpha{}_\beta] = 1$. Similarly, we find for (2.16) the dimension of length $[q^{(T)}_\alpha] = [\ell]$, and the dimension of the inverse length, $[q^{(R)}_\alpha] = [1/\ell]$ for (2.17).

The resulting *vacuum* Poincaré gravity field equations (2.4) and (2.5) then read:

$$\frac{a_0}{2}\eta_{\alpha\beta\gamma} \wedge R^{\beta\gamma} + \bar{a}_0 R_\alpha{}^\beta \wedge \vartheta_\beta - \lambda_0 \eta_\alpha + q^{(T)}_\alpha + \ell_\rho^2 q^{(R)}_\alpha - Dh_\alpha = 0, \tag{2.18}$$

$$a_0 \eta^\alpha{}_{\beta\gamma} \wedge T^\gamma + \bar{a}_0(T^\alpha \wedge \vartheta_\beta - T_\beta \wedge \vartheta^\alpha) + h^\alpha \wedge \vartheta_\beta - h_\beta \wedge \vartheta^\alpha - 2\ell_\rho^2 Dh^\alpha{}_\beta = 0. \tag{2.19}$$

The contribution of the curvature square terms in the Lagrangian (2.9) to the gravitational field dynamics in the Eqs. (2.18) and (2.19) is characterized by the parameter

$$\ell_\rho^2 = \frac{\kappa c}{\rho}. \tag{2.20}$$

Since $[\frac{1}{\rho}] = [\hbar]$, this new coupling parameter has the dimension of the area, $[\ell_\rho^2] = [\ell^2]$.

III. GRAVITATIONAL WAVES IN POINCARÉ GAUGE GRAVITY

Gravitational waves are of fundamental importance in physics, and recently the purely theoretical research in this area was finally supported by the first experimental evidence [58–60]. A general overview of the history of this fascinating subject can be found in [61–63].

A. (Anti)-de Sitter spacetime as a wave

Let us now discuss the four-dimensional manifold which can be viewed as an "(anti)-de Sitter spacetime in the wave disguise". As before [55], we use the same local coordinates which are divided into two groups: $x^i = (x^a, x^A)$, where $x^a = (x^0 = \sigma, x^1 = \rho)$ and $x^A = (x^2, x^3)$. Hereafter the indices from the beginning of the Latin alphabet label the coordinates σ and ρ parametrizing the wave rays, $a, b, c\ldots = 0,\ 1$, whereas the capital Latin indices, $A, B, C\ldots = 2, 3$, refer to coordinates x^A on the wave front.

The coframe 1-form is chosen as a direct generalization of the ansatz used in [55,57]:

$$\hat{\vartheta}^{\hat{0}} = \frac{q}{2p}[(\hat{U} + 1)d\sigma + d\rho], \tag{3.1}$$

$$\hat{\vartheta}^{\hat{1}} = \frac{q}{2p}[(\hat{U}-1)d\sigma + d\rho], \tag{3.2}$$

$$\hat{\vartheta}^{\hat{A}} = \frac{1}{p}dx^A, \qquad A = 2,3. \tag{3.3}$$

Here the three functions are given by the following expressions:

$$\hat{U} = -\frac{\lambda}{4}\rho^2, \tag{3.4}$$

$$p = 1 + \frac{\lambda}{4}\delta_{AB}x^Ax^B, \tag{3.5}$$

$$q = 1 - \frac{\lambda}{4}\delta_{AB}x^Ax^B. \tag{3.6}$$

The constant parameter λ is an arbitrary real number (which can be positive, negative or zero). As a result, the line element reads

$$ds^2 = \frac{1}{p^2}\{q^2(d\sigma d\rho + \hat{U}d\sigma^2) - \delta_{AB}dx^Adx^B\}. \tag{3.7}$$

The key object for the description of the wave configurations is the wave 1-form. On the basis of the earlier results [55], we introduce a wave 1-form k as

$$k \coloneqq d\sigma = \frac{p}{q}(\hat{\vartheta}^{\hat{0}} - \hat{\vartheta}^{\hat{1}}). \tag{3.8}$$

By construction, we have $k \wedge {}^*k = 0$. As before, the wave covector is $k_\alpha = e_\alpha \rfloor k$. Its (anholonomic) components are thus $k_\alpha = \frac{p}{q}(1,-1,0,0)$ and $k^\alpha = \frac{p}{q}(1,1,0,0)$. Hence, this is a null vector field, $k_\alpha k^\alpha = 0$.

The corresponding Riemannian connection $\hat{\Gamma}_\beta{}^\alpha$ is determined from

$$d\hat{\vartheta}^\alpha + \hat{\Gamma}_\beta{}^\alpha \wedge \hat{\vartheta}^\beta = 0, \tag{3.9}$$

and it reads explicitly (recall that $a,b,\ldots = 0,\ 1$ and $A,B,\ldots = 2,\ 3$)

$$\hat{\Gamma}_{\hat{0}}{}^{\hat{1}} = \hat{\Gamma}_{\hat{1}}{}^{\hat{0}} = -\frac{\lambda\rho}{2}k, \tag{3.10}$$

$$\hat{\Gamma}_B{}^a = \frac{p}{q}\hat{\vartheta}^a e_B \rfloor d\left(\frac{q}{p}\right), \tag{3.11}$$

$$\hat{\Gamma}_B{}^A = \frac{1}{p}(\hat{\vartheta}_B e^A \rfloor dp - \hat{\vartheta}^A e_B \rfloor dp). \tag{3.12}$$

Substituting (3.4)–(3.6), we straightforwardly find the curvature:

$$\hat{R}_\beta{}^\alpha = \lambda\hat{\vartheta}_\beta \wedge \hat{\vartheta}^\alpha. \tag{3.13}$$

Thus, the coframe and connection $(\hat{\vartheta}^\alpha, \hat{\Gamma}_\beta{}^\alpha)$, described by (3.1)–(3.3) and (3.10)–(3.12), represent the geometry of a torsionless (3.9) spacetime of constant curvature (3.13). Depending on the sign of λ, we have either a de Sitter or an anti-de Sitter space.

We mark the corresponding geometrical quantities by the hat over the symbols. This geometry will be used as a starting point for the construction of the plane wave solutions in the Poincaré gauge gravity with nontrivial cosmological constant.

It is worthwhile to note that the wave vector field k is a null geodesic in this geometry:

$$k \wedge {}^*k = 0, \qquad k \wedge {}^*\hat{D}k^\alpha = 0. \tag{3.14}$$

B. Generalized plane wave ansatz

We will construct new gravitational wave solutions in Poincaré gauge gravity theory by making use of the ansatz for the coframe and for the local Lorentz connection

$$\vartheta^\alpha = \hat{\vartheta}^\alpha + \frac{U}{2}\frac{q}{p}k^\alpha k, \tag{3.15}$$

$$\Gamma_\alpha{}^\beta = \hat{\Gamma}_\alpha{}^\beta + \frac{q}{p}(k_\alpha W^\beta - k^\beta W_\alpha)k. \tag{3.16}$$

Here the function $U = U(\sigma, x^A)$ determines the wave profile. The ansatz for the local Lorentz connection is postulated as a direct analogue of the construction used earlier in [55], and the vector variable $W^\alpha = W^\alpha(\sigma, x^A)$ satisfies the same orthogonality property, $k_\alpha W^\alpha = 0$, which is guaranteed by the choice

$$W^\alpha = \begin{cases} W^a = 0, & a = 0,1, \\ W^A = W^A(\sigma, x^B), & A = 2,3. \end{cases} \tag{3.17}$$

Consequently, the generalized ansatz for the Poincaré gauge potentials—coframe (3.15) and connection (3.16) —is described by the three variables $U = U(\sigma, x^B)$ and $W^A = W^A(\sigma, x^B)$. These should be determined from the gravitational field equations.

The ansatz (3.15) and (3.16) can be viewed as a non-Riemannian extension of the Kerr-Schild-Kundt construction developed recently [64–67] in general relativity and in modified gravity models. The original Kerr-Schild construction [34] in GR is underlain by the existence of preferred null directions. In our approach, the metric defined by the coframe (3.15) can be written in a typical Kerr-Schild form

$$g_{ij} = \hat{g}_{ij} + \frac{q}{p}Uk_ik_j, \tag{3.18}$$

where $\hat{g}_{ij}$ is the spacetime metric of the (anti)-de Sitter line element (3.7), and $k_i = \partial_i \rfloor k = \partial_i \rfloor d\sigma = (1,0,0,0)$ is the null vector with respect to both $\hat{g}_{ij}$ and g_{ij}. On the other hand, the orthogonality property of the vector W^α that defines the radiation piece of the connection (3.17), $k_\alpha W^\alpha = 0$, ensures typical radiation structure of the Riemann-Cartan field strengths, the torsion and the curvature.

The line element for this ansatz has the same form (3.7), with a replacement

$$\hat{U} \to \hat{U} + \frac{p}{q} U. \tag{3.19}$$

It is important to stress that the wave 1-form k is still defined by (3.8), which however can be recast into

$$k = d\sigma = \frac{p}{q}(\vartheta^{\hat{0}} - \vartheta^{\hat{1}}). \tag{3.20}$$

Consequently, the anholonomic components of the wave covector $k_\alpha = e_\alpha \rfloor k$ still have the values $k_\alpha = \frac{p}{q}(1, -1, 0, 0)$ and $k^\alpha = \frac{p}{q}(1, 1, 0, 0)$. As before, this is a null vector field, $k_\alpha k^\alpha = 0$.

One may wonder why does the factor $\frac{q}{p}$ appear in the ansatz (3.15) and (3.16). After all, it is always possible to absorb it by redefining U and W^A. However, it is convenient to keep this factor explicitly by noticing that the combination $\frac{q}{p} k^\alpha = (1, 1, 0, 0)$ has the constant values. It becomes clear then that the following differential relations are valid:

$$dk = 0, \qquad d\left(\frac{q}{p} k_\alpha\right) = 0. \tag{3.21}$$

Moreover, although Dk_α no longer vanishes, we find

$$k \wedge D\left(\frac{q}{p} k_\alpha\right) = k \wedge \hat{D}\left(\frac{q}{p} k_\alpha\right) = 0. \tag{3.22}$$

Taking this into account, we straightforwardly compute the torsion 2-form

$$T^\alpha = -k \wedge \frac{q}{p} k^\alpha \Theta, \tag{3.23}$$

where we introduced the 1-form

$$\Theta = \frac{1}{2} \underline{d} U + W_\alpha \vartheta^\alpha, \tag{3.24}$$

with the differential $\underline{d} := \vartheta^A e_A \rfloor d = dx^A \partial_A$ that acts in the transversal 2-space spanned by $x^A = (x^2, x^3)$.

The structure of the torsion is qualitatively the same as in the case of the vanishing parameter λ, see [55]. The structure of curvature is more nontrivial, though. A direct computation yields a 2-form

$$R_\alpha{}^\beta = \lambda \vartheta_\alpha \wedge \vartheta^\beta - k \wedge \frac{q}{p}(k_\alpha \Omega^\beta - k^\beta \Omega_\alpha), \tag{3.25}$$

where we introduced the vector-valued 1-form with the components

$$\Omega^\alpha = \begin{cases} \Omega^a = 0, & a = 0, 1, \\ \Omega^A = \hat{D} W^A + \frac{\lambda}{2} U \vartheta^A, & A = 2, 3. \end{cases} \tag{3.26}$$

The transversal covariant derivative is defined by

$$\hat{D} W^A = \underline{d} W^A + \hat{\Gamma}_B{}^A W^B. \tag{3.27}$$

Note that the Riemannian de Sitter connection (3.12) appears here (more exactly, the corresponding components of the Riemann-Cartan connection (3.16) coincide with the Riemannian components: $\Gamma_B{}^A = \hat{\Gamma}_B{}^A$).

Let us describe the geometry of the transversal 2-space spanned by $x^A = (x^2, x^3)$ explicitly. The volume 2-form reads $\underline{\eta} = \frac{1}{2} \eta_{AB} \vartheta^A \wedge \vartheta^B = \frac{1}{p^2} dx^2 \wedge dx^3$, where $\eta_{AB} = -\eta_{BA}$ is the 2-dimensional Levi-Civita tensor (with $\eta_{23} = 1$). Obviously this is a non-flat space. The corresponding Riemannian connection (3.12) yields a nontrivial curvature $\hat{R}_B{}^A = \lambda \vartheta_B \wedge \vartheta^A$ of a 2-dimensional de Sitter space. The volume 4-form of the spacetime manifold reads $\eta = \vartheta^{\hat{0}} \wedge \vartheta^{\hat{1}} \wedge \vartheta^{\hat{2}} \wedge \vartheta^{\hat{3}} = \frac{q^2}{2p^2} k \wedge d\rho \wedge \underline{\eta}$. For the wave 1-form we find the remarkable relation

$$^*k = -k \wedge \underline{\eta}. \tag{3.28}$$

We will denote the geometrical objects on the transversal 2-space by underlining them; for example, a 1-form $\underline{\phi} = \phi_A \vartheta^A$. The Hodge duality on this space is defined as usual via $\underline{^*}\vartheta_A = \underline{\eta}_A = e_A \rfloor \underline{\eta} = \eta_{AB} \vartheta^B$. With the help of (3.28), we can verify

$$^*(k \wedge \underline{\phi}) = k \wedge \underline{^*}\underline{\phi}. \tag{3.29}$$

The new 1-forms (3.24) and (3.26) have the obvious transversality properties:

$$k \wedge {}^*\Theta = 0, \qquad k \wedge {}^*\Omega^\alpha = 0, \qquad k_\alpha \Omega^\alpha = 0. \tag{3.30}$$

In accordance with (3.17) and (3.26), we have explicitly:

$$\Theta = \vartheta^A \left(\frac{1}{2} \hat{D}_A U - \delta_{AB} W^B\right), \tag{3.31}$$

$$\Omega^A = \vartheta^B \left(\hat{D}_B W^A + \frac{\lambda}{2} U \delta_B^A\right). \tag{3.32}$$

Here we denoted $\hat{D}_A = e_A \rfloor \hat{D}$. Applying the transversal differential to (3.24), and making use of (3.26), we find

$$\underline{d}\Theta = \Omega_\alpha \wedge \vartheta^\alpha. \tag{3.33}$$

In essence, this is equivalent to the Bianchi identity $DT^\alpha = R_\beta{}^\alpha \wedge \vartheta^\beta$ which is immediately checked by applying the covariant differential D to (3.23) and using (3.25). Note that it is crucial to use (3.21).

A further refinement of the generalized wave ansatz will be considered in Sec. IV C.

C. Irreducible decomposition of gravitational field strengths

Irreducible parts of the torsion and the curvature are as follows. The second (trace) and third (axial trace) irreducible part of the torsion are trivial, ${}^{(2)}T^\alpha = 0$ and ${}^{(3)}T^\alpha = 0$, and the first (pure tensor) piece is nontrivial:

$${}^{(1)}T^\alpha = T^\alpha = -k \wedge \frac{q}{p} k^\alpha \Theta. \tag{3.34}$$

At the same time, the curvature pieces ${}^{(3)}R^{\alpha\beta} = {}^{(5)}R^{\alpha\beta} = 0$, whereas

$${}^{(6)}R^{\alpha\beta} = \lambda \vartheta^\alpha \wedge \vartheta^\beta, \tag{3.35}$$

and for $I = 1, 2, 4$:

$${}^{(I)}R^{\alpha\beta} = 2k \wedge {}^{(I)}\Omega^{[\alpha} k^{\beta]} \frac{q}{p}. \tag{3.36}$$

Here ${}^{(1)}\Omega^\alpha + {}^{(2)}\Omega^\alpha + {}^{(4)}\Omega^\alpha = \Omega^\alpha$, and explicitly we have

$${}^{(1)}\Omega^\alpha = \frac{1}{2}(\Omega^\alpha - \vartheta^\alpha e_\beta \rfloor \Omega^\beta + \vartheta^\beta e^\alpha \rfloor \Omega_\beta), \tag{3.37}$$

$${}^{(2)}\Omega^\alpha = \frac{1}{2}(\Omega^\alpha - \vartheta^\beta e^\alpha \rfloor \Omega_\beta), \tag{3.38}$$

$${}^{(4)}\Omega^\alpha = \frac{1}{2} \vartheta^\alpha e_\beta \rfloor \Omega^\beta. \tag{3.39}$$

The transversal components of these objects are constructed in terms of the irreducible pieces of the 2×2 matrix $\hat{D}_B W^A$: symmetric traceless part, skew-symmetric part and the trace, respectively. Using (3.32), we derive ${}^{(I)}\Omega^A = {}^{(I)}\Omega^A{}_B \vartheta^B$, with

$${}^{(1)}\Omega^A{}_B = \frac{1}{2}(\hat{D}_B W^A + \hat{D}^A W_B - \delta^A_B \hat{D}_C W^C), \tag{3.40}$$

$${}^{(2)}\Omega^A{}_B = \frac{1}{2}(\hat{D}_B W^A - \hat{D}^A W_B), \tag{3.41}$$

$${}^{(4)}\Omega^A{}_B = \frac{1}{2}\delta^A_B(\hat{D}_C W^C + \lambda U). \tag{3.42}$$

One can demonstrate the following properties of these 1-forms:

$$\vartheta_\alpha \wedge {}^{(1)}\Omega^\alpha = 0, \qquad \vartheta_\alpha \wedge {}^{(2)}\Omega^\alpha = \vartheta_\alpha \wedge \Omega^\alpha, \tag{3.43}$$

$$\vartheta_\alpha \wedge {}^{(4)}\Omega^\alpha = 0, \qquad e_\alpha \rfloor {}^{(1)}\Omega^\alpha = -e_\alpha \rfloor \Omega^\alpha, \tag{3.44}$$

$$e_\alpha \rfloor {}^{(2)}\Omega^\alpha = 0, \qquad e_\alpha \rfloor {}^{(4)}\Omega^\alpha = 2e_\alpha \rfloor \Omega^\alpha, \tag{3.45}$$

$$k_\alpha {}^{(1)}\Omega^\alpha = -\frac{1}{2} k e_\alpha \rfloor \Omega^\alpha, \qquad k_\alpha {}^{(2)}\Omega^\alpha = 0, \tag{3.46}$$

$$k_\alpha {}^{(4)}\Omega^\alpha = \frac{1}{2} k e_\alpha \rfloor \Omega^\alpha, \qquad k \wedge {}^{*(2)}\Omega^\alpha = 0, \tag{3.47}$$

$$k \wedge {}^{*(1)}\Omega^\alpha = -k \wedge {}^{*(4)}\Omega^\alpha = -\frac{1}{2} k^\alpha \vartheta_\beta \wedge {}^*\Omega^\beta. \tag{3.48}$$

IV. FIELD EQUATIONS

Let us now turn to the quadratic Poincaré gauge model with the general Lagrangian (2.9), and allow for a nontrivial cosmological constant λ_0.

Substituting the torsion (3.34) and the curvature (3.35), (3.36), into (2.14) and (2.15), we find

$$h^\alpha = -k^\alpha Z \frac{q}{p}, \tag{4.1}$$

$$h^{\alpha\beta} = \lambda b_6 \eta^{\alpha\beta} + \lambda \bar{b}_6 \vartheta^\alpha \wedge \vartheta^\beta - 2k^{[\alpha} Z^{\beta]} \frac{q}{p}, \tag{4.2}$$

where we introduced the 2-forms

$$Z = a_1 {}^*(k \wedge \Theta) + \bar{a}_1 k \wedge \Theta, \tag{4.3}$$

$$Z^\alpha = \sum_{I=1,2,4} [b_I {}^*(k \wedge {}^{(I)}\Omega^\alpha) + \bar{b}_I k \wedge {}^{(I)}\Omega^\alpha]. \tag{4.4}$$

Making use of (3.30) and (3.43)–(3.48) we can show that

$$k \wedge h^\alpha = 0, \qquad k \wedge {}^*h^\alpha = 0, \qquad k_\alpha h^\alpha = 0. \tag{4.5}$$

As a result, substituting (4.2) into (2.16) and (2.17), we find $q^{(T)}_\alpha = 0$ and

$$q^{(R)}_\alpha = 2\lambda \frac{q}{p} k_\alpha \{-(b_4 + b_6) {}^*k e_\beta \rfloor \Omega^\beta + (\bar{b}_2 - \bar{b}_6) k \wedge \vartheta_\beta \wedge \Omega^\beta\}. \tag{4.6}$$

With an account of the properties (4.5), one can check that

$$Dh_\alpha = -\hat{D}\left(k_\alpha Z \frac{q}{p}\right), \tag{4.7}$$

$$Dh_{\alpha\beta} = -\hat{D}\left(2k_{[\alpha} Z_{\beta]} \frac{q}{p}\right) + \lambda b_6 \eta_{\alpha\beta\mu} \wedge T^\mu + \lambda \bar{b}_6 (T_\alpha \wedge \vartheta_\beta - T_\beta \wedge \vartheta_\alpha). \tag{4.8}$$

The transversal nature of Θ and Ω^A leads to a further simplification. In particular, using (3.29), we recast (4.3) and (4.4) into

$$Z = k \wedge \Xi, \qquad Z^A = k \wedge \Xi^A, \tag{4.9}$$

where we have introduced the 1-forms

$$\Xi = a_1 \,\underline{^*}\Theta + \bar{a}_1 \Theta, \tag{4.10}$$

$$\Xi^A = \sum_{I=1,2,4} [b_I \,\underline{^*}{}^{(I)}\Omega^A + \bar{b}_I {}^{(I)}\Omega^A]. \tag{4.11}$$

A. Wave equations

After all these preparations, we are in a position to write down the gravitational field equations for the quadratic Poincaré gauge model (2.9). Substituting the gravitational wave ansatz (3.15)–(3.16) into (2.18), we derive the first equation

$$(3a_0\lambda - \lambda_0)\eta_\alpha + \frac{q}{p} k_\alpha {}^*k(e_\beta \rfloor \Omega^\beta)[a_0 - 2\lambda\ell_\rho^2(b_4 + b_6)] + \frac{q}{p} k_\alpha k \wedge \{\vartheta_\beta \wedge \Omega^\beta[\bar{a}_0 + 2\lambda\ell_\rho^2(\bar{b}_2 - \bar{b}_6)] - \underline{d}\Xi\} = 0. \tag{4.12}$$

Contracting this with k^α, we find the value of the constant parameter in the wave ansatz:

$$\lambda = \frac{\lambda_0}{3a_0}, \tag{4.13}$$

and with an account of (3.28) and (4.10) we recast (4.12) into

$$[a_0 - 2\lambda\ell_\rho^2(b_4 + b_6)]\vartheta_A \wedge \underline{^*}\Omega^A + a_1 \underline{d}\,\underline{^*}\Theta - [\bar{a}_0 + \bar{a}_1 + 2\lambda\ell_\rho^2(\bar{b}_2 - \bar{b}_6)]\vartheta_A \wedge \Omega^A = 0. \tag{4.14}$$

The first two terms describe the parity-even model, whereas the last term accounts for the parity-odd sector.

Similarly, by gravitational wave ansatz (3.15)–(3.16) in (2.19), we obtain the second equation

$$k_a \frac{q}{p} k \wedge \{(a_0 + a_1 - 2\lambda\ell_\rho^2 b_6)\vartheta_B \wedge \underline{^*}\Theta + (\bar{a}_0 + \bar{a}_1 - 2\lambda\ell_\rho^2 \bar{b}_6)\vartheta_B \wedge \Theta - 2\ell_\rho^2 \hat{D}\Xi_B\} = 0. \tag{4.15}$$

Note here that the $[ab]$ and $[AB]$ components in (2.19) are satisfied identically, and only the mixed $[aB]$ components give rise to the result (4.15).

Equation (4.14) and the expression inside the curly bracket in (4.15) are both 2-forms on the 2-dimensional transversal space spanned by $x^A = (x^2, x^3)$, and thus (4.14) and (4.15) describe a system of three partial differential equations for the three variables $U = U(\sigma, x^B)$ and $W^A = W^A(\sigma, x^B)$. Substituting (3.31) and (3.32), we recast (4.14) and (4.15) into the final tensorial form

$$A_0(\hat{D}_A W^A + \lambda U) + a_1\left(\hat{D}_A W^A - \frac{1}{2}\hat{\Delta} U\right) - \bar{A}_0 \eta^{AB} \hat{D}_A \underline{W}_B = 0, \tag{4.16}$$

$$-A_1\left(\underline{W}_A - \frac{1}{2}\hat{D}_A U\right) + \bar{A}_1 \eta_{AB}\left(W^B - \frac{1}{2}\underline{\hat{D}}^B U\right) + \ell_\rho^2(\bar{b}_1 - \bar{b}_2)[\hat{D}_A(\eta^{BC}\hat{D}_B \underline{W}_C) + \eta_{AB}\underline{\hat{D}}^B(\hat{D}_C W^C + \lambda U)]$$
$$+\ell_\rho^2(b_1 + b_4)\left[-\hat{\Delta}\left(\underline{W}_A - \frac{1}{2}\hat{D}_A U\right) + \lambda\left(\underline{W}_A - \frac{1}{2}\hat{D}_A U\right) - \hat{D}_A(\hat{D}_B W^B + \lambda U) + \hat{D}_A\left(\hat{D}_B W^B - \frac{1}{2}\hat{\Delta} U\right)\right] = 0. \tag{4.17}$$

The 2-dimensional transversal space has the (anti)-de Sitter geometry and the corresponding covariant Laplacian reads

$$\hat{\Delta} = \delta^{AB}\hat{D}_A \hat{D}_B = p^2 \underline{\Delta}, \tag{4.18}$$

where $\underline{\Delta} = \delta^{AB}\partial_A \partial_B$ is the usual Laplace operator.

Note that $\bar{b}_4 = \bar{b}_2$. Here we denoted $\underline{W}_A = \delta_{AB} W^B$ and $\underline{\hat{D}}^A = \delta^{AB}\hat{D}_B$, and introduced the convenient abbreviations for the combinations of the coupling constants,

$$A_0 = a_0 - 2\lambda\ell_\rho^2(b_4 + b_6), \tag{4.19}$$

$$\bar{A}_0 = \bar{a}_0 + \bar{a}_1 + 2\lambda\ell_\rho^2(\bar{b}_2 - \bar{b}_6), \tag{4.20}$$

$$A_1 = a_0 + a_1 + 2\lambda\ell_\rho^2(b_1 - b_6), \tag{4.21}$$

$$\bar{A}_1 = \bar{a}_0 + \bar{a}_1 + 2\lambda\ell_\rho^2(\bar{b}_1 - \bar{b}_6). \tag{4.22}$$

The transversal covariant derivatives do not commute,

$$(\hat{D}_A \hat{D}_B - \hat{D}_B \hat{D}_A) W^C = \hat{R}_{ABD}{}^C W^D = 2\lambda \delta^C_{[A} \underline{W}_{B]}, \tag{4.23}$$

and we used this fact when deriving (4.16) and (4.17). Direct consequences of (4.23) are:

$$\eta^{BC}\hat{D}_B \hat{D}_C \underline{W}_A = \lambda \eta_{AB} W^B, \tag{4.24}$$

$$(\hat{\Delta}\hat{D}_A - \hat{D}_A\hat{\Delta})U = \lambda\hat{D}_A U. \tag{4.25}$$

It is worthwhile to notice that the derivatives of W^A appear in (4.16)–(4.17) only in combinations

$$\Omega := e_\alpha \rfloor \Omega^\alpha = \hat{D}_A W^A + \lambda U, \tag{4.26}$$

$$\Phi := \underline{{}^*}\underline{d}\,\underline{{}^*}\Theta = \hat{D}_A W^A - \frac{1}{2}\hat{\Delta}U, \tag{4.27}$$

$$\bar{\Phi} := \underline{{}^*}\underline{d}\Theta = -\eta^{AB}\hat{D}_A\underline{W}_B, \tag{4.28}$$

which have a clear geometrical meaning in terms of the curvature and the torsion.

The system (4.16)–(4.17) always admits a nontrivial solution for arbitrary quadratic Poincaré gauge model with any choices of the coupling constants. There are some interesting special cases.

B. Torsionless gravitational waves

The torsion (3.23) vanishes when $\Theta = 0$ which is realized, see (3.24) and (3.31), for

$$W^A = \frac{1}{2}\delta^{AB}\hat{D}_B U. \tag{4.29}$$

Substituting this into (4.16), we find

$$A_0\{\hat{\Delta}U + 2\lambda U\} = 0, \tag{4.30}$$

whereas (4.17) reduces to

$$\begin{aligned} &\ell_\rho^2(\bar{b}_1 - \bar{b}_2)\eta_{AB}\hat{\underline{D}}^B\{\hat{\Delta}U + 2\lambda U\} \\ &-\ell_\rho^2(b_1 + b_4)\hat{D}_A\{\hat{\Delta}U + 2\lambda U\} = 0. \end{aligned} \tag{4.31}$$

Accordingly, we conclude that the well-known torsionless wave solution of GR with the function U satisfying

$$p^2\underline{\Delta}U + 2\lambda U = 0 \tag{4.32}$$

is an exact solution of the generic quadratic Poincaré gauge gravity model. This is consistent with our earlier results on the torsion-free solutions in Poincaré gauge theory [16].

Moreover, the torsionless wave (4.29)–(4.30) represents a general solution for the purely torsion quadratic class of Poincaré models, since this is the only configuration admitted by the system (4.16)–(4.17) for $b_I = \bar{b}_I = 0$.

C. Torsion gravitational waves

The torsion-free ansatz (3.9) can be generalized to

$$W^A = \frac{1}{2}\delta^{AB}\hat{D}_B(U + V) + \frac{1}{2}\eta^{AB}\hat{D}_B\bar{V}, \tag{4.33}$$

with $V \neq 0$. The two scalar functions $V = V(\sigma, x^A)$ and $\bar{V} = \bar{V}(\sigma, x^A)$ define the non-Riemannian piece of the connection, stemming from torsion:

$$\begin{aligned} \Theta &= -\frac{1}{2}(\underline{d}V + \underline{{}^*}\underline{d}\,\bar{V}) \\ &= -\frac{1}{2}\vartheta^A(\hat{D}_A V - \eta_{AB}\hat{\underline{D}}^B\bar{V}). \end{aligned} \tag{4.34}$$

For the above choice, the metric and torsion contributions to the connection are described in a rather symmetric way, in terms of the three potentials $(U, V, \bar{V})$. In particular, we find for (4.26)–(4.28):

$$\Omega = \frac{1}{2}(\hat{\Delta}V + \hat{\Delta}U + 2\lambda U), \tag{4.35}$$

$$\Phi = \frac{1}{2}\hat{\Delta}V, \qquad \bar{\Phi} = \frac{1}{2}\hat{\Delta}\,\bar{V}. \tag{4.36}$$

Substituting (4.33) into (4.16) and (4.17), we derive

$$A_0\Omega + a_1\Phi + \bar{A}_0\bar{\Phi} = 0, \tag{4.37}$$

$$\begin{aligned} &\hat{D}_A\left\{-\frac{1}{2}A_1 V - \frac{1}{2}\bar{A}_1\bar{V} - \ell_\rho^2(b_1 + b_4)\Omega - \ell_\rho^2(\bar{b}_1 - \bar{b}_2)\bar{\Phi}\right\} \\ &+\eta_{AB}\hat{\underline{D}}^B\left\{-\frac{1}{2}A_1 V + \frac{1}{2}\bar{A}_1\bar{V} - \ell_\rho^2(b_1 + b_4)\bar{\Phi} + \ell_\rho^2(\bar{b}_1 - \bar{b}_2)\Omega\right\} = 0. \end{aligned} \tag{4.38}$$

One needs to pay attention to the noncommutativity of the covariant derivatives and use (4.23)–(4.25).

As a result, we obtain the system of the three linear second order differential equations for the three functions U, V, $\bar{V}$:

$$A_0(\hat{\Delta}V + \hat{\Delta}U + 2\lambda U) + a_1\hat{\Delta}V + \bar{A}_0\hat{\Delta}\,\bar{V} = 0, \tag{4.39}$$

$$-\ell_\rho^2(b_1 + b_4)(\hat{\Delta}V + \hat{\Delta}U + 2\lambda U) - A_1 V - \ell_\rho^2(\bar{b}_1 - \bar{b}_2)\hat{\Delta}\,\bar{V} - \bar{A}_1\bar{V} = 0, \tag{4.40}$$

$$\ell_\rho^2(\bar{b}_1 - \bar{b}_2)(\hat{\Delta}V + \hat{\Delta}U + 2\lambda U) + \bar{A}_1 V - \ell_\rho^2(b_1 + b_2)\hat{\Delta}\,\bar{V} - A_1\bar{V} = 0. \tag{4.41}$$

D. Solution for potentials

Before starting the analysis of solutions, one can notice that the system (4.40) and (4.41) is actually not equivalent to the original equation (4.38). Indeed, by taking the covariant divergence (applying $\hat{D}^A$) and by taking the covariant curl (applying $\eta^{AB}\hat{D}_B$) of (4.38), we derive the pair of equations where on the right-hand sides of (4.40) and (4.41) one finds not zeros but arbitrary functions, say, $\alpha(\sigma, x^A)$ and $\beta(\sigma, x^A)$, which are *harmonic*, in the sense that they both satisfy equations $\hat{\Delta}\alpha = \hat{\Delta}\beta = 0$. However, one then immediately notices that with the help of redefinitions

$$V \to V + v, \qquad \hat{\Delta}v = 0, \tag{4.42}$$

$$\bar{V} \to \bar{V} + \bar{v}, \qquad \hat{\Delta}\,\bar{v} = 0, \tag{4.43}$$

we can always remove these nontrivial right-hand sides and come to the system (4.40) and (4.41).

In other words, a solution of the system (4.39)–(4.41) admits the *gauge* transformation (4.42)–(4.43), under which the potentials V and $\bar{V}$ can be shifted by arbitrary harmonic functions. Such gauge transformed potentials are of course still solutions of the Poincaré gauge field equations (4.37) and (4.38). What is important, however, the curvature and the torsion remain invariant under the redefinition (4.42)–(4.43) of potentials: (4.35) and (4.36) obviously are not affected by the arbitrary harmonic functions.

Now, as a first step, we substitute $(\hat{\Delta}V + \hat{\Delta}U + 2\lambda U)$ from (4.39) into (4.40) and (4.41). The resulting system reads

$$\ell_\rho^2\hat{\Delta}\{a_1(b_1+b_4)V + [-A_0(\bar{b}_1-\bar{b}_2)+\bar{A}_0(b_1+b_4)]\bar{V}\} \\ -A_0A_1V - A_0\bar{A}_1\bar{V} = 0, \tag{4.44}$$

$$\ell_\rho^2\hat{\Delta}\{a_1(\bar{b}_1-\bar{b}_2)V + [A_0(b_1+b_2)+\bar{A}_0(\bar{b}_1-\bar{b}_2)]\bar{V}\} \\ -A_0\bar{A}_1V + A_0A_1\bar{V} = 0. \tag{4.45}$$

After solving this system, we can use the potentials V and $\bar{V}$ to substitute them into (4.39) which then becomes an inhomogeneous differential equation for the metric potential U:

$$A_0(\hat{\Delta}U + 2\lambda U) = -(a_1 + A_0)\hat{\Delta}V - \bar{A}_0\hat{\Delta}\,\bar{V}. \tag{4.46}$$

For the parity-even models with $\bar{a}_I = 0$, $\bar{b}_I = 0$, hence $\bar{A}_0 = 0$ and $\bar{A}_1 = 0$, the system (4.44)–(4.45) reduces to the two uncoupled equations

$$a_1(b_1+b_4)\ell_\rho^2\hat{\Delta}V - A_0A_1V = 0, \tag{4.47}$$

$$(b_1+b_2)\ell_\rho^2\hat{\Delta}\,\bar{V} + A_1\bar{V} = 0, \tag{4.48}$$

recovering the result of [56].

To analyze the system (4.44)–(4.45), let us rewrite it in matrix form

$$\hat{\Delta}\mathcal{V} - M\mathcal{V} = 0, \qquad M := \frac{A_0}{\ell_\rho^2}F, \tag{4.49}$$

where we combined the potentials into a single object, a "2-vector" $\mathcal{V} = \binom{V}{\bar{V}}$, and the 2×2 matrix $F = K^{-1}N$ is constructed from

$$K = \left(\begin{array}{c|c} a_1(b_1+b_4) & \bar{A}_0(b_1+b_4) - A_0(\bar{b}_1-\bar{b}_2) \\ \hline a_1(\bar{b}_1-\bar{b}_2) & A_0(b_1+b_2)+\bar{A}_0(\bar{b}_1-\bar{b}_2)\end{array}\right),$$

$$N = \left(\begin{array}{c|c} A_1 & \bar{A}_1 \\ \hline \bar{A}_1 & -A_1\end{array}\right). \tag{4.50}$$

One immediately notices the simple structure of the matrix N which is manifest in the properties

$$N^2 = (A_1^2+\bar{A}_1^2)\left(\begin{array}{c|c} 1 & 0 \\ \hline 0 & 1\end{array}\right), \qquad \det N = -(A_1^2+\bar{A}_1^2). \tag{4.51}$$

One can solve the matrix differential equation (4.49) by diagonalizing this system. To achieve this, one needs to find the eigenvalues of the matrix M and to construct the corresponding eigenvectors. Let m^2 be an eigenvalue of the matrix M. It is determined from the corresponding characteristic equation $\det(M - m^2) = 0$ which has the meaning of the dispersion relation for the mass:

$$\ell_\rho^4 m^4 \det K + \ell_\rho^2 m^2 A_0 \mathrm{tr}(NK) - A_0^2(A_1^2+\bar{A}_1^2) = 0. \tag{4.52}$$

The coefficients of the quadratic equation (4.52) are constructed from the coupling constants of the gauge gravity model. From (4.50) we have explicitly:

$$\det K = a_1A_0[(b_1+b_4)(b_1+b_2) + (\bar{b}_1-\bar{b}_2)^2], \tag{4.53}$$

$$\mathrm{tr}(NK) = (a_1A_1 + \bar{A}_0\bar{A}_1)(b_1+b_4) - A_0A_1(b_1+b_2) \\ + (a_1\bar{A}_1 - A_0\bar{A}_1 - \bar{A}_0A_1)(\bar{b}_1-\bar{b}_2). \tag{4.54}$$

For the parity-even models with $\bar{a}_I = 0$, $\bar{b}_I = 0$, hence $\bar{A}_0 = 0$ and $\bar{A}_1 = 0$, the dispersion equation (4.52) reduces to

$$[\ell_\rho^2 m^2 a_1(b_1+b_4) - A_0A_1] \\ \times[\ell_\rho^2 m^2 A_0(b_1+b_2) + A_0A_1] = 0, \tag{4.55}$$

and hence we recover the result (4.47)–(4.48).

General case with parity-odd terms in the Lagrangian is more complicated. No obvious simplification of (4.52) is visible.

Having found the eigenvalues m_1^2 and m_2^2 of the mass matrix M as the two roots of the quadratic equation (4.49), one can construct the matrix P that transforms M to its diagonal form. For $M_{12} \neq 0$, the latter reads

$$P = \begin{pmatrix} -M_{12} & -M_{12} \\ M_{11} - m_1^2 & M_{11} - m_2^2 \end{pmatrix}. \tag{4.56}$$

Multiplying Eq. (4.49) by P^{-1}, one then obtains

$$\hat{\Delta}\mathcal{V}' - M'\mathcal{V}' = 0, \tag{4.57}$$

where

$$M' \coloneqq P^{-1}MP = \begin{pmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{pmatrix}, \tag{4.58}$$

and $\mathcal{V}'$ is the eigenvector of M, corresponding to the eigenvalues m_1^2 and m_2^2:

$$\mathcal{V}' = \begin{pmatrix} \mathcal{V}'_1 \\ \mathcal{V}'_2 \end{pmatrix} = P^{-1}\mathcal{V} = \frac{1}{\det P}\begin{pmatrix} (M_{11} - m_2^2)V + M_{12}\bar{V} \\ -(M_{11} - m_1^2)V - M_{12}\bar{V} \end{pmatrix}. \tag{4.59}$$

Recalling $\hat{\Delta} = p^2\underline{\Delta}$, we thus recast the system of the field equations (4.44) and (4.45) into a diagonal form

$$p^2\underline{\Delta}\mathcal{V}'_n - m_n^2\mathcal{V}'_n = 0, \tag{4.60}$$

with $n = 1, 2$. The solutions for $\mathcal{V}'_n$ are given in terms of the hypergeometric functions ${}_2F_1(a, b, c, z)$, see [56]. Similar construction exists in the case $M_{21} \neq 0$.

Now, we can return to (4.46) to find the solution for U. Each solution for $\mathcal{V}'_n$ defines the corresponding solution

$$\mathcal{V} = P\mathcal{V}' \tag{4.61}$$

of (4.49). Inserting these solutions for V and $\bar{V}$ on the right-hand side of (4.46), this equation becomes an inhomogeneous differential equation for U. Its general solution is given as a general solution of the homogeneous equation plus a particular solution of the inhomogeneous equation, $U = U_h + U_p$. Note that U_h coincides with the general vacuum solution of GR, see (4.32). The solution for U obtained by choosing $U_h = 0$ has a very interesting interpretation. Indeed, in that case U reduces just to the particular solution U_p, the form of which is completely determined by the torsion potentials $(V, \bar{V})$. A similar mechanism was found also in the parity even sector [56]. Clearly, there are many other solutions for U_h, and consequently, for U. In each of them, the influence of torsion on the metric is quite clear.

E. Masses of the torsion modes

In order to get a deeper understanding of the role of the torsion in our gravitational wave solution, it is important to examine the mass spectrum of the associated torsion modes. Having found the matrix $F = K^{-1}N$ with the help of (4.50), the solutions of the characteristic equation (4.52) can be conveniently represented in terms of the matrix $f = (\det K)F$ as

$$m_\pm^2 = \frac{A_0}{2\ell^2 \det K}\left(\mathrm{tr} f \pm \sqrt{(\mathrm{tr} f)^2 - 4\det f}\right). \tag{4.62}$$

This is an exact formula for the mass eigenvalues $m_\pm^2$ associated to the gravitational wave. It is worthwhile to notice that $\mathrm{tr} f = -\mathrm{tr}(NK)$, and $\det f = (\det N)(\det K)$.

The particle spectrum of PGT has been calculated only with respect to the Minkowski background [24–29], and never for the (anti)-de Sitter spacetime. Accordingly, we can compare the result (4.62) with those existing in the literature only for the values of $m_\pm^2$ in the limit of the vanishing cosmological constant. In the limit of $\lambda \to 0$, we have

$$\begin{aligned} \mathrm{tr} f &= -[a_1(a_0 + a_1) + (\bar{a}_0 + \bar{a}_1)^2](b_1 + b_4) \\ &\quad + a_0(a_0 + a_1)(b_1 + b_2) + 2a_0(\bar{a}_0 + \bar{a}_1)(\bar{b}_1 - \bar{b}_2), \\ \det f &= -a_0 a_1[(a_0 + a_1)^2 + (\bar{a}_0 + \bar{a}_1)^2] \\ &\quad \times [(b_1 + b_2)(b_1 + b_4) + (\bar{b}_1 - \bar{b}_2)^2], \\ \det K &= a_0 a_1[(b_1 + b_2)(b_1 + b_4) + (\bar{b}_1 - \bar{b}_2)^2]. \end{aligned} \tag{4.63}$$

As a first test, we apply the formula (4.62) to the parity even sector of PGT. One can straightforwardly see that the corresponding values of $m_\pm^2$ coincide with the masses of the spin-$2^\pm$ torsion modes, known from the literature [24]; compare also with [56]. This is consistent with (4.55).

A more complete verification can be done by comparing (4.62) with the recent work of Karananas [30], which presently offers the only existing calculation of the complete mass spectrum for the most general PGT with both parity even and parity odd sectors included. A comparison of the Lagrangian (5) of Ref. [30] with our expression (2.9) is straightforward, although one should be careful since the paper [30] contains numerous misprints. As a result, we establish the following relations between our and Karananas' coupling constants (we use the notation t_0 instead of Karananas' λ to distinguish it from our cosmological term):

$$a_0 = 2\kappa c t_0, \qquad \bar{a}_0 = 0, \tag{4.64}$$

$$a_1 = 2\kappa c(-t_1 - t_0), \tag{4.65}$$

$$a_2 = 4\kappa c(-t_3 + t_0), \tag{4.66}$$

$$a_3 = \kappa c(-t_2 + t_0), \tag{4.67}$$

$$\bar{a}_1 = 4\kappa c t_5, \tag{4.68}$$

$$\bar{a}_2 = \bar{a}_3 = 2\kappa c t_4. \tag{4.69}$$

$$b_1 = 4\rho(-r_1 + r_3), \tag{4.70}$$

$$b_2 = 4\rho(-r_3), \tag{4.71}$$

$$b_3 = 4\rho(-r_2 + r_3), \tag{4.72}$$

$$b_4 = 4\rho(-r_1 + r_3 - r_4), \tag{4.73}$$

$$b_5 = 4\rho(-r_3 - r_5), \tag{4.74}$$

$$b_6 = 4\rho(-r_1 + r_3 - 3r_4), \tag{4.75}$$

$$\bar{b}_1 = \rho(-r_7 + 3r_8), \tag{4.76}$$

$$\bar{b}_2 = \bar{b}_4 = \rho(-r_7 - r_8), \tag{4.77}$$

$$\bar{b}_3 = \bar{b}_6 = \rho(4r_6 - r_7 - r_8), \tag{4.78}$$

$$\bar{b}_5 = \rho(3r_7 - r_8). \tag{4.79}$$

Substituting the expressions for a_I, b_I and $\bar{a}_I$, $\bar{b}_I$ into (4.63), one finds that the resulting values of $m^2_\pm$ in (4.62) exactly reproduce the result (A.3.5) of Karananas' paper [30] (after correcting a number of his misprints), which displays the spin-$2^\pm$ torsion modes.

Thus, we conclude that the massive spin-$2^\pm$ torsion modes turn out to be an essential ingredient of our gravitational wave, in the sense that these massive torsion modes determine the structure of the wave profile encoded in the functions V, $\bar{V}$ and U. This is a remarkable result if one recalls that the particle spectrum of PGT is derived from the linearized equations of motion, whereas our gravitational waves are exact solutions of the full nonlinear field equations.

V. DISCUSSION AND CONCLUSION

In this paper, we have found a family of the exact vacuum solutions of the most general PGT model with all possible parity even and parity odd linear and quadratic invariants in the Lagrangian (2.9), and with a nontrivial cosmological constant $\lambda_0 \neq 0$. This family represents generalized plane waves with torsion, propagating on the (anti)-de Sitter background. The present paper extends the results obtained recently in [55,56].

The underlying construction can be understood as a generalization of the Kerr-Schild-Kundt ansatz from the Riemannian to the Riemann-Cartan geometry of PGT. An essentially new element in this extended formalism is the ansatz for the local Lorentz connection $\Gamma_\alpha{}^\beta$, the radiation piece of which is constructed in terms of the null covector field k. The generalized plane wave ansatz (3.15)–(3.16) ensures that the torsion 2-form T^α and the radiation piece of the curvature 2-form $S^{\alpha\beta} := R^{\alpha\beta} - \lambda\vartheta^\alpha \wedge \vartheta^\beta$ satisfy the radiation conditions

$$k \wedge {}^*T^\alpha = 0, \qquad k \wedge {}^*S^{\alpha\beta} = 0, \tag{5.1}$$

$$k \wedge T^\alpha = 0, \qquad k \wedge S^{\alpha\beta} = 0, \tag{5.2}$$

$$T^\alpha \wedge {}^*T^\beta = 0, \qquad S^{\alpha\beta} \wedge {}^*S^{\rho\sigma} = 0. \tag{5.3}$$

These relations represent an extension of the well-known Lichnerowicz criterion for identifying gravitational waves [68] (see also [32]), based on analogy with the electromagnetic waves, to the framework of the PGT.

In the limit of vanishing torsion, the generalized plane waves with torsion reduce to the family of the Riemannian pp waves on the (anti)-de Sitter background. The pp waves are classified as solutions of Petrov type N, since the corresponding Weyl tensor satisfies the special algebraic condition $k^\alpha C_{\alpha\beta\mu\nu} = 0$, see [34,35]. This criterion can be naturally extended to a Riemann-Cartan geometry of PGT as

$$k^{\alpha\,(1)}R_{\alpha\beta\mu\nu} = 0, \tag{5.4}$$

where ${}^{(1)}R_{\alpha\beta\mu\nu}$ is the first irreducible part of the curvature tensor, see [55,56]. The validity of (5.4) for the generalized plane waves with torsion confirms that they are also of type N.

The spacetime torsion is an essential ingredient of the generalized gravitational wave solution; its dynamical characteristics are described by the two potentials V and $\bar{V}$, satisfying the matrix equation (4.49). The mass matrix M is of particular importance for the physical interpretation of the torsion. We demonstrate that, in the limit of $\lambda \to 0$, the eigenvalues of M coincide with the values of the mass square the spin-$2^\pm$ torsion modes, identified in the work of Karananas [30]. Generically, wave front profile of a generalized plane wave with torsion is thus determined by two spin-2 massive torsion modes and the massless graviton, produced by the third, coframe potential U (which enters the spacetime metric).

It is interesting to note that there exist particular solutions for which the metric potential is completely determined by the torsion. For such solutions, the motion of a spinless test particle is effectively determined by the spacetime torsion.

The results obtained in this work were checked with the help of the computer algebra systems *Reduce* and *Mathematica*.

ACKNOWLEDGMENTS

We thank Friedrich Hehl for the constant support and encouragement and for his contribution to the development of the Poincaré gauge gravity theory. We are grateful to Georgios Karananas for the clarifying comments on his study of the particle spectrum in the general quadratic Poincaré gravity models. Y. N. O. thanks Metin Gurses for the correspondence, kindly informing us about his recent work. This work was partially supported by the Serbian Science Foundation (Grant No. 171031) and by the Russian Foundation for Basic Research (Grant No. 16-02-00844-A).

APPENDIX: IRREDUCIBLE DECOMPOSITION OF THE TORSION AND CURVATURE

The torsion 2-form can be decomposed into the three irreducible pieces, $T^\alpha = {}^{(1)}T^\alpha + {}^{(2)}T^\alpha + {}^{(3)}T^\alpha$, where

$$ {}^{(2)}T^\alpha = \frac{1}{3}\vartheta^\alpha \wedge (e_\nu \rfloor T^\nu), \tag{A1} $$

$$ {}^{(3)}T^\alpha = \frac{1}{3}e^\alpha \rfloor (T^\nu \wedge \vartheta_\nu), \tag{A2} $$

$$ {}^{(1)}T^\alpha = T^\alpha - {}^{(2)}T^\alpha - {}^{(3)}T^\alpha. \tag{A3} $$

The Riemann-Cartan curvature 2-form is decomposed $R^{\alpha\beta} = \sum_{I=1}^{6} {}^{(I)}R^{\alpha\beta}$ into the 6 irreducible parts

$$ {}^{(2)}R^{\alpha\beta} = -{}^*(\vartheta^{[\alpha} \wedge \bar{\Psi}^{\beta]}), \tag{A4} $$

$$ {}^{(3)}R^{\alpha\beta} = -\frac{1}{12}{}^*(\bar{X}\vartheta^\alpha \wedge \vartheta^\beta), \tag{A5} $$

$$ {}^{(4)}R^{\alpha\beta} = -\vartheta^{[\alpha} \wedge \Psi^{\beta]}, \tag{A6} $$

$$ {}^{(5)}R^{\alpha\beta} = -\frac{1}{2}\vartheta^{[\alpha} \wedge e^{\beta]} \rfloor (\vartheta^\gamma \wedge X_\gamma), \tag{A7} $$

$$ {}^{(6)}R^{\alpha\beta} = -\frac{1}{12}X\vartheta^\alpha \wedge \vartheta^\beta, \tag{A8} $$

$$ {}^{(1)}R^{\alpha\beta} = R^{\alpha\beta} - \sum_{I=2}^{6} {}^{(I)}R^{\alpha\beta}, \tag{A9} $$

where

$$ X^\alpha \coloneqq e_\beta \rfloor R^{\alpha\beta}, \qquad X \coloneqq e_\alpha \rfloor X^\alpha, \tag{A10} $$

$$ \bar{X}^\alpha \coloneqq {}^*(R^{\beta\alpha} \wedge \vartheta_\beta), \qquad \bar{X} \coloneqq e_\alpha \rfloor \bar{X}^\alpha, \tag{A11} $$

and

$$ \Psi_\alpha \coloneqq X_\alpha - \frac{1}{4}\vartheta_\alpha X - \frac{1}{2}e_\alpha \rfloor (\vartheta^\beta \wedge X_\beta), \tag{A12} $$

$$ \bar{\Psi}_\alpha \coloneqq \bar{X}_\alpha - \frac{1}{4}\vartheta_\alpha \bar{X} - \frac{1}{2}e_\alpha \rfloor (\vartheta^\beta \wedge \bar{X}_\beta). \tag{A13} $$

Directly from the definitions (A1)–(A3) and (A4)–(A9), one can prove the relations

$$ T^\alpha \wedge {}^{(2)}T_\alpha = T^\alpha \wedge {}^{(3)}T_\alpha = {}^{(2)}T^\alpha \wedge {}^{(3)}T_\alpha, \tag{A14} $$

$$ R^{\alpha\beta} \wedge {}^{(2)}R_{\alpha\beta} = R^{\alpha\beta} \wedge {}^{(4)}R_{\alpha\beta} = {}^{(2)}R^{\alpha\beta} \wedge {}^{(4)}R_{\alpha\beta}, \tag{A15} $$

$$ R^{\alpha\beta} \wedge {}^{(3)}R_{\alpha\beta} = R^{\alpha\beta} \wedge {}^{(6)}R_{\alpha\beta} = {}^{(3)}R^{\alpha\beta} \wedge {}^{(6)}R_{\alpha\beta}, \tag{A16} $$

whereas $T^\alpha \wedge {}^{(1)}T_\alpha = {}^{(1)}T^\alpha \wedge {}^{(1)}T_\alpha$ and $R^{\alpha\beta} \wedge {}^{(1)}R_{\alpha\beta} = {}^{(1)}R^{\alpha\beta} \wedge {}^{(1)}R_{\alpha\beta}$ and $R^{\alpha\beta} \wedge {}^{(5)}R_{\alpha\beta} = {}^{(5)}R^{\alpha\beta} \wedge {}^{(5)}R_{\alpha\beta}$.

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PHYSICAL REVIEW D **96,** 044027 (2017)

Holography in Lovelock Chern-Simons AdS gravity

Branislav Cvetković,[1,*] Olivera Miskovic,[2,†] and Dejan Simić[1,‡]

[1]*Institute of Physics, University of Belgrade, P. O. Box 57, 11001 Belgrade, Serbia*

[2]*Instituto de Física, Pontificia Universidad Católica de Valparaíso, Casilla 4059, Valparaíso, Chile*

(Received 13 June 2017; published 22 August 2017)

We analyze holographic field theory dual to Lovelock Chern-Simons anti–de Sitter (AdS) gravity in higher dimensions using first order formalism. We first find asymptotic symmetries in the AdS sector showing that they consist of local translations, local Lorentz rotations, dilatations and non-Abelian gauge transformations. Then, we compute 1-point functions of energy-momentum and spin currents in a dual conformal field theory and write Ward identities. We find that the holographic theory possesses Weyl anomaly and also breaks non-Abelian gauge symmetry at the quantum level.

DOI: 10.1103/PhysRevD.96.044027

I. INTRODUCTION

The AdS/CFT correspondence [1] relates the fields in $(d+1)$-dimensional asymptotically anti–de Sitter (AAdS) space and correlators in a d-dimensional conformal field theory (CFT). These two theories are dual in the asymptotic sector of gravity, such that the weak coupling regime of one is related to the strong coupling regime of another. For a weak gravitational coupling, the bulk theory is well described by its semiclassical approximation, leading to the form of the duality most often used.

Since its discovery, the correspondence tools have been applied to many strongly coupled systems, giving rise to new insights into their dynamics, for example in hydrodynamics [2] and condensed matter systems such as superconductors [3].

On the other hand, much effort has been invested in analyzing the duality in semiclassical approximation of a bulk theory, with twofold purpose. First, it enables us to test the conjecture itself. Second, it helps us to gain the knowledge about strongly coupled systems which are nonperturbative and not very well understood. However, most of this investigation deals with Riemannian geometry of bulk spacetime, see for example [3–8], while a more general structure based on Riemann-Cartan geometry, where both torsion and curvature determine gravitational dynamics, is mostly underinvestigated. One of the first studies of Riemann-Cartan holography used first order formalism to obtain a holographic dual of Chern-Simons AdS gravity in five dimensions [9]. After that, in three dimensions, holographic dual to the Mielke-Baekler model was analyzed in [10], and to the most general parity-preserving three-dimensional gravity with propagating torsion in [11]. The physical interpretation of torsional degrees of freedom as carriers of a nontrivial gravitational magnetic field in 4D Einstein-Cartan gravity was discussed in [12].

Studying holographic duals of gravity with torsion has many benefits. Since its setup is more general, it also contains the results of torsion-free gravity. One of the very important features is that treating vielbein and spin connection as independent dynamical variables simplifies calculations to great extent. In Ref. [11], it was shown that for three-dimensional bulk gravity conservation laws of the boundary theory take the same form in Riemann-Cartan and Riemannian theory when the boundary torsion is set to zero. Thus, it is possible to treat vielbein and spin connection as independent dynamical variables and reproduce Riemannian results in the limit of zero torsion. In this work, we extend the results of [11] to all odd dimensions in case of holographic theory dual to Lovelock-Chern-Simons AdS gravity, by reproducing the conservation laws with respect to diffeomorphisms, Weyl and local Lorentz symmetry using first order formalism after taking a Riemannian limit.

Working in the framework of gravity with torsion also leads to richer boundary non-Abelian symmetries, as it is explicitly demonstrated for the particular model studied in this paper.

We analyze a holographic structure of Lovelock Chern-Simons AdS Gravity [13,14] in asymptotically AdS spaces. The key feature of this model is that it possesses a unique AdS vacuum, which is multiply degenerate in odd $D \geq 5$ dimensions. Unlike general Lovelock-Lanczos [15] gravity, it contains only two free parameters—gravitational constant κ and the AdS radius ℓ. This theory also features a symmetry enhancement from local Lorentz to AdS gauge symmetry. Degenerate vacuum makes the linear perturbation analysis not applicable around the AdS background. The holographic study in AAdS spacetimes, however, is nonperturbative, because the gravitational fields in a dual theory are not dynamical but they play the role of external sources for the CFT matter. Indeed, the holographic theory will remain fully nonlinear in gravitational fields, which

*cbranislav@ipb.ac.rs
†olivera.miskovic@pucv.cl
‡dsimic@ipb.ac.rs

will be explicitly shown in Sec. IV. On the other hand, these theories couple successfully to external sources [16], which are stable in the framework of Lovelock Chern-Simons (LCS) supergravities [17].

The paper is organized as follows. In Sec. II we introduce the holographic ansatz for the fundamental dynamical variables and we arrive to their radial expansion in the asymptotic sector. Expressed in terms of the metric, it reduces to Fefferman-Graham expansion [18]. We also analyze corresponding residual gauge symmetries which leave this ansatz invariant. In Sec. III we focus to the holographic quantum theory and derive the Noether-Ward identities. In Sec. IV we focus on Chern-Simons–AdS gravity in arbitrary odd dimensions and compute 1-point functions in the corresponding dual theory, which are energy-momentum and spin currents. We show that translational and Lorentz symmetries are present also at the quantum level, but the Weyl anomaly and non-Abelian anomaly arise, breaking the conformal and non-Abelian symmetries quantically, the former being proportional to the Euler density up to a divergence. Our results generalize the ones of [9] from five to arbitrary dimensions. Our calculations are simplified to great extent by using the results of [19]. Section V contains concluding remarks, while appendices deal with some technical details.

Our conventions are given by the following rules. On a $D = d + 1$-dimensional spacetime manifold M, the latin indices $(i, j, k, \ldots)$ refer to the local Lorentz frame, the greek indices $(\mu, \nu, \rho, \ldots)$ refer to the coordinate frame. The symmetric and antisymmetric parts of a tensor X_{ij} are $X_{(ij)} = \frac{1}{2}(X_{ij} + X_{ji})$ and $X_{[ij]} = \frac{1}{2}(X_{ij} - X_{ji})$, respectively. The $d + 1$ decomposition of spacetime is described in terms of the suitable coordinates $x^\mu = (\rho, x^\alpha)$, where ρ is a radial coordinate and x^α are local coordinates on the boundary ∂M. In the local Lorentz frame, this decomposition is expressed by $i = (1, a)$.

II. HOLOGRAPHIC ANSATZ

We are interested in a gravitational theory which possesses a local AdS symmetry. The presence of local spacetime translations and spacetime rotations introduces naturally the vielbein and the spin connection as the fundamental fields. Our goal is to gauge fix this symmetry by imposing a set of conditions on the fundamental fields in a such a way that it singles out a particular coordinate frame which is suitable for a description of a holographically dual theory. This frame should be consistent with the known Fefferman-Graham coordinate choice used on the Riemannian manifold. All the properties that follow from this gauge-fixing are purely kinematical and can be applied to any gravity invariant under local AdS group. To include the dynamics we focus, in particular, on Lovelock-Chern-Simons gravity.

A. AdS gauge transformations

In a theory with local AdS symmetry, the fundamental fields are components of a gauge field (1-form) for the AdS group $SO(D-1, 2)$ (see Appendix A) and is defined by

$$A = \frac{1}{\ell}\hat{e}^A P_A + \frac{1}{2}\hat{\omega}^{AB} J_{AB}, \tag{2.1}$$

where ℓ is the AdS radius. For the sake of simplicity, we set $\ell = 1$. Gauge transformations, parametrized by $\lambda \coloneqq \eta^A P_A + \frac{1}{2}\lambda^{AB} J_{AB}$, act on the gauge field as

$$\delta_0 A = D\lambda = d\lambda + [A, \lambda], \tag{2.2}$$

wherefrom we get the transformation law of the fundamental fields,

$$\begin{aligned} \delta_0 \hat{e}^A &= \hat{\nabla}\eta^A - \lambda^{AB}\hat{e}_B, \\ \delta_0 \hat{\omega}^{AB} &= \hat{\nabla}\lambda^{AB} + 2e^{[A}\eta^{B]}. \end{aligned} \tag{2.3}$$

Here, the $\hat{\omega}$-covariant derivative is $\hat{\nabla}\eta^A \coloneqq d\eta^A + \hat{\omega}^{AB}\eta_B$. The AdS field strength $F = dA + A \wedge A$ has components

$$F = \hat{T}^A P_A + \frac{1}{2}F^{AB} J_{AB}, \tag{2.4}$$

which are the torsion 2-form $\hat{T}^A$ and AdS curvature F^{AB},

$$\begin{aligned} \hat{T}^A &= \frac{1}{2}\hat{T}^A{}_{\mu\nu} dx^\mu \wedge dx^\nu = d\hat{e}^A + \hat{\omega}^{AB} \wedge \hat{e}_B, \\ F^{AB} &= \frac{1}{2}F^{AB}{}_{\mu\nu} dx^\mu \wedge dx^\nu = d\hat{\omega}^{AB} + \hat{\omega}^{AC} \wedge \hat{\omega}_C{}^B + \hat{e}^A \wedge \hat{e}^B. \end{aligned} \tag{2.5}$$

The wedge product sign is going to be omitted for simplicity from now on in the text. The global AdS space is described by a Riemannian manifold ($\hat{T}^A = 0$), whose AdS curvature vanishes ($F^{AB} = 0$), and where the Riemannian curvature $\hat{R}^{AB} = d\hat{\omega}^{AB} + \hat{\omega}^{AC} \wedge \hat{\omega}_C{}^B$ becomes explicitly constant, $\hat{R}^{AB} = -\hat{e}^A \wedge \hat{e}^B$.

B. Radial expansion and residual gauge transformations

We use the radial foliation with the local coordinates $x^\mu = (\rho, x^\alpha)$ and the Lorentz indices decomposed correspondingly as $A = (1, a)$. The asymptotic boundary of the manifold is located at the constant radius $\rho = \rho_0$. For convenience we set $\rho_0 = 0$.

1. Gauge fixing

There are two types of local symmetries, small and large, depending on how they behave asymptotically. Small local symmetries are characterized by the parameters which go to zero at infinity and all other local symmetries are large.

Small gauge symmetries act trivially on boundary fields and must be considered as redundancies in the theory, i.e., they must be gauge fixed. A good gauge choice should fix all small gauge transformations and should lead to a well-posed boundary value problem, meaning that the form of a residual symmetry in the bulk is completely determined by the boundary values of the symmetry parameters. Note that the large gauge transformations do not have to be fixed by a gauge choice. For more details, see Ref. [20].

Local transformations at our disposal are spacetime diffeomorphisms and local AdS transformations. Let us first focus on local AdS symmetry. A good gauge fixing for our purposes is the one where the spacetime is AAdS and where residual gauge transformations contain conformal transformations on the boundary.

The last condition is introduced because we want to have a CFT as a holographic theory. Too strong gauge fixing can overkill all residual transformations and give rise to a trivial holographic theory. Since the bulk theory is gauge invariant only up to boundary terms, different gauge fixings can lead to nonequivalent boundary theories.

Another important observation is that, in the metric formulation of Riemann gravity, according to the theorem of Fefferman-Graham (FG) [18], in any AAdS space there is a coordinate choice so that the metric can be cast in the FG form, that is, with $\hat{g}_{\rho\rho} = 1/(2\rho)^2$, $\hat{g}_{\rho\alpha} = 0$ and $\rho\hat{g}_{\alpha\beta}(\rho, x)$ regular on the boundary $\rho = 0$. Thus, a gauge-fixing choice of the vielbein and spin connection must be such that the corresponding metric acquires the FG form.

The number of gauge parameters of AdS group is $\frac{D(D+1)}{2}$, implying that we need the same number of gauge conditions. We impose the following D conditions on the vielbeins $\hat{e}^A{}_\rho$ and $\frac{D(D-1)}{2}$ conditions on connection $\hat{\omega}^{AB}{}_\rho$:

$$\hat{e}^A{}_\rho = -\frac{1}{2\rho}\delta_1{}^A, \qquad \hat{\omega}^{AB}{}_\rho = 0. \tag{2.6}$$

In the choice of the gauge fixing one has to keep in mind the invertibility of vielbein. Therefore, all $\hat{e}^A{}_\rho$ components cannot be set to zero. Furthermore, although in principle a choice of the radial coordinate is arbitrary, we want to have the Fefferman-Graham coordinate frame, where the metric component $g_{\rho\rho}$ behaves as $1/4\rho^2$, generalized to first order formalism, which implies the above behavior of the radial component of the vielbein.

To find residual transformations, we look at the restrictions on gauge parameters imposed by the gauge conditions (2.6) and we find that they have to satisfy

$$\begin{aligned} &\partial_\rho\eta^1 = 0, \qquad \partial_\rho\eta^a - \frac{1}{2\rho}\lambda^{1a} = 0, \\ &\partial_\rho\lambda^{ab} = 0, \qquad \partial_\rho\lambda^{1a} - \frac{1}{2\rho}\eta^a = 0. \end{aligned} \tag{2.7}$$

The equations in η^1 and λ^{ab} are straightforward to solve. To find η^a and λ^{1a}, we combine the corresponding differential equations and obtain for the parameter η^a

$$\rho^2\partial_\rho^2\eta^a + \rho\partial_\rho\eta^a - \frac{1}{4}\eta^a = 0. \tag{2.8}$$

This is the Euler-Fuchs equation which solution takes the form $\eta^a(\rho) \sim \rho^k$. Hence, from (2.8) we get $k^2 = \frac{1}{4}$ and consequently the general solution is given by

$$\begin{aligned} &\eta^1(\rho, x) = u(x), \qquad \eta^a(\rho, x) = \frac{1}{\sqrt{\rho}}\alpha^a(x) + \sqrt{\rho}\beta^a(x), \\ &\lambda^{ab}(\rho, x) = \lambda^{ab}(x), \qquad \lambda^{1a}(\rho, x) = -\frac{1}{\sqrt{\rho}}\alpha^a(x) + \sqrt{\rho}\beta^a(x). \end{aligned} \tag{2.9}$$

We see that our gauge choice is good, as desired, because symmetry parameters in the whole bulk are determined by a few arbitrary functions u, α^a, β^a and λ^{ab} defined on the boundary. We still have to identify an asymptotic symmetry group defined by these parameters.

The residual gauge parameters which describe asymptotic symmetry group naturally induce a change of the basis in the Lie algebra $J_a^\pm = P_a \pm J_{1a}$, so that the Lie-algebra valued gauge parameter has the form

$$\lambda = u(x)P_1 + \frac{1}{\sqrt{\rho}}\alpha^a(x)J_a^- + \sqrt{\rho}\beta^a(x)J_a^+ + \frac{1}{2}\lambda^{ab}(x)J_{ab}. \tag{2.10}$$

The AdS algebra in terms of the new generators reads

$$\begin{aligned} &[J_a^+, J_b^-] = 2J_{ab} + 2\eta_{ab}P_1, \qquad [J_a^\pm, J_b^\pm] = 0, \\ &[J_{ab}, J_c^\pm] = -\eta_{ac}J_b^\pm + \eta_{bc}J_a^\pm, \qquad [P_1, J_{ab}] = 0, \\ &[P_1, J_a^\pm] = \pm J_a^\pm. \end{aligned} \tag{2.11}$$

2. *Radial decomposition of gauge field and field strength*

Up to now the results are valid for any theory possessing AdS gauge symmetry. From now on we concentrate on Chern-Simons AdS gravity. For holography, one needs to know how the fields evolve along the radial direction and to study their near-boundary behavior. Since the radial components are already fixed by the gauge condition (2.6), now we have to determine the behavior of the spatial components.

To this end, we can use invariance of gravity under general coordinate transformations. In Ref. [21], it was shown that only $D-1$ spatial diffeomorphisms are linearly independent on gauge generators, in a physical system where time evolution was analyzed. In our case, we look at the radial quantization of a Hamiltonian, because we are interested in radial evolution of the fields from the bulk to the boundary. Thus, our independent diffeomorpisms act only in the transversal section of spacetime, that is, as $x^\alpha \to x^\alpha + \xi^\alpha(\rho, x)$. Furthermore, we know that the radial diffeomorphisms are broken by the boundary set at constant radii, so this choice of quantization is natural in our case.

Thus, we have $D-1$ transversal diffeomorphisms to gauge fix. In Ref. [21] it was shown that, in any generic Chern-Simons gauge theory (AdS in our case), there is an on-shell identity $F_{\rho\alpha} = F_{\alpha\beta}N^\beta$, with $D-1$ arbitrary functions N^β related to the transversal diffeomorphisms $\xi^\alpha(\rho, x)$. Therefore, to gauge fix them, we can just set the $D-1$ functions to zero, $N^\beta = 0$. As a consequence, we also get $F_{\rho\alpha} = 0$ or, equivalently, $\hat{T}^A{}_{\rho\alpha} = F^{AB}{}_{\rho\alpha} = 0$. These conditions are particular for Chern-Simons theory and they arise from its dynamics. Interestingly, they can be exactly solved using the gauge fixing (2.6), also written as $A_\rho = -\frac{1}{2\rho}P_1$. Rewriting the AdS Lie-algebra valued condition $F_{\rho\alpha} = 0$ as $(dA + A^2)_{\rho\alpha} = 0$, we get

$$\partial_\rho A_\alpha - \frac{1}{2\rho}\hat{e}^a{}_\alpha J_{a1} + \frac{1}{2\rho}\hat{\omega}^{1a}{}_\alpha P_A = 0.$$

This first order differential equation in $A_\alpha(\rho, x)$ can be exactly solved, given the initial condition

$$A_\alpha(0, x) \equiv e^a{}_\alpha(x)J_a^+ + k^a{}_\alpha(x)J_a^- + \frac{1}{2}\omega^{ab}{}_\alpha(x)J_{ab}. \quad (2.12)$$

The solution is

$$A_\alpha(\rho, x) = \frac{1}{\sqrt{\rho}}e^a{}_\alpha(x)J_a^+ + \sqrt{\rho}k^a{}_\alpha(x)J_a^- + \frac{1}{2}\omega^{ab}{}_\alpha(x)J_{ab}. \quad (2.13)$$

In components, this solution leads to the radial expansion of the gravitational fields expressed in terms of the boundary fields $e^a{}_\alpha$, $k^a{}_\alpha$ and $\omega^{ab}{}_\alpha$,

$$\begin{aligned} \hat{e}^a{}_\alpha &= \frac{1}{\sqrt{\rho}}(e^a{}_\alpha + \rho k^a{}_\alpha), \\ \hat{\omega}^{1a}{}_\alpha &= -\frac{1}{\sqrt{\rho}}(e^a{}_\alpha - \rho k^a{}_\alpha), \\ \hat{\omega}^{ab}{}_\alpha &= \omega^{ab}{}_\alpha. \end{aligned} \quad (2.14)$$

Thus, this is a generalization of the FG expansion of the bulk metric. Indeed, the metric $\hat{g}_{\mu\nu} = \hat{e}^A{}_\mu\hat{e}^B{}_\nu\eta_{AB}$ takes the FG form since the line element can be written as

$$ds^2 = \frac{1}{4\rho^2}d\rho^2 + \frac{1}{\rho}(g_{\alpha\beta} + 2\rho k_{(\alpha\beta)} + \rho^2 k^a{}_\alpha k_{a\beta})dx^\alpha dx^\beta, \quad (2.15)$$

where $g_{\alpha\beta} \coloneqq \eta^{ab}e^a{}_\alpha e^b{}_\beta$ and $k_{\alpha\beta} \coloneqq e_{a\alpha}k^a{}_\beta$. We conclude that the FG expansion is *finite*. Finite FG expansion is typical for Chern-Simons gravity [9] and also for general relativity when the Weyl tensor vanishes [8].

The induced metric $\gamma_{\alpha\beta}$ is defined by $\gamma_{\alpha\beta} = \rho\hat{g}_{\alpha\beta}$. The coefficients in the radial expansion of $\gamma_{\alpha\beta}$ are

$$\begin{aligned} \gamma^{(0)}_{\alpha\beta} &= g_{\alpha\beta}, \qquad \gamma^{(1)}_{\alpha\beta} = 2k_{(\alpha\beta)}, \\ \gamma^{(2)}_{\alpha\beta} &= k^a{}_\alpha k_{a\beta}, \qquad \gamma^{(n)}_{\alpha\beta} = 0, \quad n \geq 3. \end{aligned} \quad (2.16)$$

From the radial expansion of the field strength we get on the boundary

$$\begin{aligned} F^{a1} &= \frac{1}{\sqrt{\rho}}(T^a - \rho\nabla k^a), \qquad \hat{T}^1 = -2e^a k_a, \\ F^{ab} &= R^{ab} + 4e^{[a}k^{b]}, \qquad \hat{T}^a = \frac{1}{\sqrt{\rho}}(T^a + \rho\nabla k^a), \end{aligned} \quad (2.17)$$

where $T^a = \nabla e^a$ and $R^{ab} = d\omega^{ab} + \omega^a{}_c\omega^{cb}$.

Physical interpretation of the boundary fields can be found from their transformation law under the residual (boundary) gauge transformations.

3. Residual gauge transformations

The complete transformation law of the basic dynamical variables in the bulk that include the spacetime diffeomorphisms is given by

$$\begin{aligned} \delta_0\hat{e}^A{}_\mu &= \hat{\nabla}_\mu\eta^A - \lambda^{AB}\hat{e}_{B\mu} - \partial_\mu\xi^\nu\hat{e}^A{}_\nu - \xi^\nu\partial_\nu\hat{e}^A{}_\mu, \\ \delta_0\hat{\omega}^{AB}{}_\mu &= \hat{\nabla}_\mu\lambda^{AB} + 2\hat{e}^{[A}{}_\mu\eta^{B]} - \partial_\mu\xi^\nu\hat{\omega}^{AB}{}_\nu - \xi^\nu\partial_\nu\hat{\omega}^{AB}{}_\mu, \end{aligned} \quad (2.18)$$

where the last two terms of each line are the Lie derivatives with respect to ξ^μ. If we make the following redefinition of parameters,

$$\begin{aligned} \eta^A &\to \eta^A + \xi^\mu\hat{e}^A{}_\mu, \\ \lambda^{AB} &\to \lambda^{AB} + \xi^\mu\hat{\omega}^{AB}{}_\mu, \end{aligned} \quad (2.19)$$

transformations (2.18) take the following form:

$$\begin{aligned} \delta_0\hat{e}^A{}_\mu &= \hat{\nabla}_\mu\eta^A - \lambda^{AB}\hat{e}_{B\mu} + \xi^\nu\hat{T}^A{}_{\mu\nu}, \\ \delta_0\hat{\omega}^{AB}{}_\mu &= \hat{\nabla}_\mu\lambda^{AB} + 2\hat{e}^{[A}{}_\mu\eta^{B]} + \xi^\nu F^{AB}{}_{\mu\nu}. \end{aligned} \quad (2.20)$$

Due to the condition $F_{\rho\alpha} = 0$, the transformation laws (2.20) of $\hat{e}^A{}_\rho$ and $\hat{\omega}^{AB}{}_\rho$ with redefined parameters (2.19) take the *same form* as in the case when diffeomorphisms are absent in the transformation law (2.18). Therefore, introduction of diffeomorphisms *does not effectively change* the result (2.9).

From the transformation law for $\omega^{ab}{}_\alpha$, it follows that ξ^α *does not depend on* ρ. The complete transformation law of the gauge fields under residual transformations reads

$$\begin{aligned}\delta_0 e^a{}_\alpha &= \nabla_\alpha \alpha^a - \lambda^{ab} e_{b\alpha} + u e^a{}_\alpha - \xi^\beta{}_{,\alpha} e^a{}_\beta - \xi^\beta \partial_\beta e^a{}_\alpha,\\ \delta_0 k^a{}_\alpha &= \nabla_\alpha \beta^a - \lambda^{ab} k_{b\alpha} - u k^a{}_\alpha - \xi^\beta{}_{,\alpha} k^a{}_\beta - \xi^\beta \partial_\beta k^a{}_\alpha,\\ \delta_0 \omega^{ab}{}_\alpha &= \nabla_\alpha \lambda^{ab} + 4 e^{[a}{}_\alpha \beta^{b]} + 4 k^{[a}{}_\alpha \alpha^{b]}\\ &\quad - \xi^\beta{}_{,\alpha} \omega^{ab}{}_\beta - \xi^\beta \partial_\beta \omega^{ab}{}_\alpha, \end{aligned} \tag{2.21}$$

with

$$\eta^1 + \frac{\xi^\rho}{2\rho} = u(x), \qquad \xi^\alpha = \xi^\alpha(x). \tag{2.22}$$

Let us note that the residual diffeomorphisms do not change the condition $F_{\rho\alpha} = 0$, as expected. Their form shows that our gauge choice is good.

In holography it is important for the boundary to be orthogonal to the radial direction. That is why we shall impose an additional condition $\hat{e}^1{}_\alpha = 0$, which puts the bulk vielbein in the block-diagonal form with the only one boundary component $e^a{}_\alpha(x)$. The extra condition reduces the asymptotic symmetries because the parameter β^a is not independent any longer,

$$\beta^a = e^{a\alpha}\left(\frac{1}{2}\partial_\alpha u + k^b{}_\alpha \alpha_b\right). \tag{2.23}$$

The generators of the asymptotic group cannot be determined straightforwardly because a change of the basis of the Lie algebra necessary to identify this subgroup is nonlinear, that is, it depends on the point of spacetime. We shall deduce the algebra directly from the action on the fields.

Independent transformations acting on the fields are transversal diffeomorphisms or local translations $\delta_T(\xi)$, local Lorentz rotations $\delta_L(\lambda)$, local Weyl or conformal transformations $\delta_C(u)$ and non-Abelian gauge transformations $\delta_G(\alpha)$. Each transformation can be seen as generated by some generator T_a through the commutator, for example $[\delta_G(\alpha'), \delta_G(\alpha'')] = \alpha'^a \alpha''^b [T_a, T_b]$, and similarly for all other transformations. In that way, the asymptotic algebra closes as

$$\begin{aligned} [\delta_T(\xi'), \delta_T(\xi'')] &= \delta_T([\xi', \xi'']), & [\delta_C(u), \delta_G(\alpha)] &= \delta_C(\alpha \cdot \partial u) - \delta_L(\tilde{\lambda}) - \delta_G(u\alpha),\\ [\delta_T(\xi), \delta_L(\lambda)] &= \delta_L(\xi \cdot \partial \lambda), & [\delta_G(\alpha'), \delta_G(\alpha'')] &= -\delta_C(\tilde{u}) - \delta_L(\Lambda),\\ [\delta_T(\xi), \delta_C(u)] &= \delta_C(\xi \cdot \partial u), & [\delta_L(\lambda, \delta_G(\alpha)] &= \delta_G(\lambda \cdot \alpha),\\ [\delta_T(\xi), \delta_G(\alpha)] &= \delta_G(\xi \cdot \partial \alpha), & [\delta_L(\lambda), \delta_C(u)] &= 0,\\ [\delta_L(\lambda'), \delta_L(\lambda'')] &= \delta_L([\lambda', \lambda'']), & [\delta_C(u'), \delta_C(u'')] &= 0, \end{aligned} \tag{2.24}$$

where $[\xi', \xi'']^\alpha = \xi' \cdot \partial \xi''^\alpha - \xi' \cdot \partial \xi''^\alpha$ is the Lie bracket and $[\lambda', \lambda'']^{ab} = \lambda'^{ac} \lambda''^{b}_c - \lambda''^{ac} \lambda'^{\,b}_c$ is the group commutator. We also introduced the contraction $\xi \cdot \partial = \xi^\beta \partial_\beta$ and the matrix multiplication $(\lambda \cdot \alpha)^a = \lambda^{ab} \alpha_b$, and defined the auxiliary Lorentz parameters $\tilde{\lambda}^{ab} = 2\alpha^{[a} \partial^{b]} u$ and $\Lambda^{ab} = 4k^{c[a}(\alpha'_c \alpha''^{b]} - \alpha''_c \alpha'^{b]})$, as well as the Weyl parameter $\tilde{u} = 4k^{[ab]} \alpha'_a \alpha''_b$.

The above brackets are computed by acting on $e^a{}_\alpha$, but their form is field independent. The boundary diffeomorphisms, Lorentz rotations and Weyl dilatations close in the standard way and they form the Weyl subgroup. Furthermore, the non-Abelian extension is realized nonlinearly, because the parameters Λ and $\tilde{u}$ explicitly depend on the field k^{ab}. To understand better the origin of such non-Abelian transformations, let us note that

$$\delta_G(\alpha) e^a{}_\alpha = (\partial_\alpha \alpha^\beta) e^a{}_\beta + \alpha^\beta \partial_\beta e^a{}_\alpha + \alpha^\beta \omega^{ab} e_{b\alpha} + \alpha^\beta T^a{}_{\alpha\beta}, \tag{2.25}$$

where $\alpha^\beta = \alpha^a e_a{}^\beta$. Therefore, the gauge transformations can be cast in the form

$$\delta_G(\alpha) e^a{}_\alpha = -\delta_T(\alpha^\beta) - \delta_L(\omega^{ab}{}_\beta \alpha^\beta) + \alpha^\beta T^a{}_{\alpha\beta}. \tag{2.26}$$

Shifting the parameters as $\xi^\beta \to \xi^\beta + \alpha^\beta$ and $\lambda^{ab} \to \lambda^{ab} + \omega^{ab}{}_\beta \alpha^\beta$ helps us identify the independent non-Abelian gauge transformations $\delta_G(\alpha) e^a{}_\alpha = \alpha^\beta T^a_{\alpha\beta}$. From (2.25) and the above relation we easily conclude that non-Abelian gauge transformations act on the boundary vielbein independently if and only if torsion is nonvanishing. In the case of vanishing torsion non-Abelian gauge transformations stop to be independent and they can be represented as composition of local translations and local Lorentz rotations with the suitable redefinition of parameters. Similar conclusion holds when one acts on the boundary spin connection because it is an independent field only if the torsion is nonvanishing.

Let us now, for completeness, inspect the action of the transformations (2.21) on the metric $g_{\alpha\beta} = e^a{}_\alpha e_{a\beta}$. We obtain

$$\begin{aligned}\delta_0 g_{\alpha\beta} &= -\xi^\gamma{}_{,\alpha} g_{\gamma\beta} - \xi^\gamma{}_{,\beta} g_{\alpha\gamma} - \xi^\gamma \partial_\gamma g_{\alpha\beta} + 2u g_{\alpha\beta}\\ &\quad + e_{a\beta} \nabla_\alpha \alpha^a + e_{a\alpha} \nabla_\beta \alpha^a.\end{aligned}$$

Similarly, as in the case of vielbein, the action of the non-Abelian gauge transformations on the metric reads

$$\delta_G(\alpha)g_{\alpha\beta} = -\delta_T(\alpha)g_{\alpha\beta} + 2\alpha^\gamma T_{(\alpha\beta)\gamma}. \tag{2.27}$$

Again, we conclude that in the case when torsion vanishes the action of non-Abelian gauge transformations on the metric reduces to local translations with the already mentioned redefinition of parameters [4]. The above transformation law of the metric is not usual in field theories, but is not surprising because we started with local AdS symmetry which mixes vielbein and spin connection.

III. NOETHER-WARD IDENTITIES

The AdS/CFT correspondence between the D-dimensional AdS space and d-dimensional CFT identifies the quantum effective action in CFT with the classical gravitational action in AdS space for given boundary conditions. Thus, let us assume that the *renormalized* effective action in a holographic theory, $I_{\rm ren}[e,\omega]$, has an extremum for Dirichlet boundary conditions on the independent fields, which are the vielbein, $e^a{}_\alpha$, and the spin connection, $\omega^{ab}{}_\alpha$, so that its variation takes the form

$$\delta I_{\rm ren}[e,\omega] = -\int d^dx\left(\tau^\alpha{}_a\delta_0 e^a{}_\alpha + \frac{1}{2}\sigma^\alpha{}_{ab}\delta_0\omega^{ab}{}_\alpha\right). \tag{3.1a}$$

The tensor densities,

$$\tau^\alpha{}_a = -\frac{\delta I_{\rm ren}}{\delta e^a{}_\alpha}, \qquad \sigma^\alpha{}_{ab} = -\frac{\delta I_{\rm ren}}{\delta\omega^{ab}{}_\alpha}, \tag{3.1b}$$

are the energy-momentum and spin currents of our dynamical system.

The holographic theory is invariant under d-dimensional diffeomorphisms with the parameter ξ^α and the local Lorentz transformations with the parameter λ^{ab}. The conservation law of the corresponding Noether current reads

$$e^a{}_\beta\nabla_\alpha\tau^\alpha{}_a + \tau^\alpha{}_a T^a{}_{\alpha\beta} + \frac{1}{2}\sigma^\alpha{}_{ab}R^{ab}{}_{\alpha\beta} + \frac{1}{2}\omega^{ab}{}_\beta(\nabla_\alpha\sigma^\alpha{}_{ab} - 2\tau_{[ab]}) = 0, \tag{3.2a}$$

$$\nabla_\alpha\sigma^\alpha{}_{ab} - 2\tau_{[ab]} = 0, \tag{3.2b}$$

which is also known as the generalized conservation laws of $\tau_\alpha{}^a$ and $\sigma^\alpha{}_{ab}$. Note that if the second Noether identity (3.2b) is fulfilled, the last term in (3.2a) can be omitted. We shall keep this term, however, because it modifies the conservation law in cases when there are quantum anomalies.

The invariance of $I_{\rm ren}$ under Weyl transformations leads to the additional conservation law,

$$\tau - \nabla_\beta\sigma^a{}_a{}^\beta = 0, \tag{3.2c}$$

where $\tau \coloneqq \tau^a{}_a$ is the trace of the energy-momentum tensor.

Finally, invariance under the non-Abelian gauge transformations leads to

$$\nabla_\alpha\tau^\alpha{}_a - 2\sigma^b{}_{bc}k_a{}^c - 2\sigma_{bca}k^{cb} = 0. \tag{3.2d}$$

In Ref. [9], it was proposed that these residual gauge transformations contain the information about the chiral anomaly of the fermions in holographic CFT, encoded in the completely antisymmetric part of the spin current.

Gravitational dynamics in the bulk is described by nonvanishing torsion, but it may happen that some solutions on the boundary are Riemannian. For such solutions, the boundary connection $\omega^{ab}{}_\alpha$ takes its Riemannian value $\tilde\omega^{ab}{}_\alpha = \tilde\omega^{ab}{}_\alpha(e)$ and can be expressed in terms of the vielbein $e^a{}_\alpha$ in the following way:

$$\tilde\omega_{ab\alpha} = \frac{1}{2}(c_{abc} - c_{cab} + c_{bca})e^c{}_\alpha,$$
$$c_{a\alpha\beta} \coloneqq \partial_\alpha e_{a\beta} - \partial_\beta e_{a\alpha}. \tag{3.3}$$

Although boundary connection is no more independent dynamical variable, the Noether-Ward identities keep the form (3.2), but now $\omega_{ab\alpha}$ takes on the Riemannian value $\tilde\omega_{ab\alpha}$.

From the Riemannian renormalized action $\tilde I_{\rm ren} = I_{\rm ren}[e^a{}_\alpha, \tilde\omega_\alpha]$, we get that the related spin current $\Sigma^\alpha \coloneqq -\delta\tilde I_{\rm ren}/\delta\omega_\alpha$ vanishes, while the energy-momentum current $\Theta^\alpha{}_a \coloneqq -\delta\tilde I_{\rm ren}/\delta e^a{}_\alpha$ acquires an additional contribution

$$\Theta^\alpha{}_a = \tilde\tau^\alpha{}_a - \frac{1}{2}\tilde\nabla_\beta(\tilde\sigma^{\beta\alpha}{}_a - \tilde\sigma_a{}^{\beta\alpha} + \tilde\sigma^\alpha{}_a{}^\beta), \tag{3.4}$$

where $\tilde X$ denotes the Riemannian limit of a tensor X. The Noether identities for the action $\tilde I_{\rm ren}$ are found to be

$$e^a{}_\beta\tilde\nabla_\alpha\Theta^\alpha{}_a - \tilde\omega^{ab}{}_\beta\Theta_{[ab]} = 0, \tag{3.5a}$$

$$\Theta_{ab} = \Theta_{ba}, \tag{3.5b}$$

$$\Theta = 0. \tag{3.5c}$$

Let us remind that, as we concluded at the end of the previous section, the non-Abelian gauge transformations are not independent for Riemannian solutions, thus in this case there are only three independent Noether identities (3.5).

When the Lorentz invariance is fulfilled, (3.5a) reduces to the usual form $D_\alpha(e^{-1}\Theta^\alpha{}_\beta) = 0$, where D_α is the

Riemannian covariant derivative. The relations (3.5b) and (3.5c) are the standard Riemannian conditions for the Lorentz and Weyl invariance, respectively.

After using the condition of vanishing torsion, $T_{abc} = 0$, the identity $[\tilde{\nabla}_\alpha, \tilde{\nabla}_\beta] f_a = \tilde{R}_{ab\alpha\beta} f^b$ and the Bianchi identity, $\tilde{R}_{abcd} + \tilde{R}_{acdb} + \tilde{R}_{adbc} = 0$, enable us to write the expressions (3.5) as

$$e^a{}_\beta \tilde{\nabla}_\alpha \tilde{\tau}^\alpha{}_a + \frac{1}{2}\tilde{\sigma}^\alpha{}_{ab}\tilde{R}^{ab}{}_{\alpha\beta} + \frac{1}{2}\tilde{\omega}^{ab}{}_\beta(\nabla_\alpha \tilde{\sigma}^\alpha{}_{ab} - 2\tilde{\tau}_{[ab]}) = 0, \tag{3.6a}$$

$$\tilde{\nabla}_\alpha \tilde{\sigma}^\alpha{}_{ab} - 2\tilde{\tau}_{[ab]} = 0, \tag{3.6b}$$

$$\tilde{\tau} - \tilde{\nabla}_\beta \tilde{\sigma}^a{}_a{}^\beta = 0. \tag{3.6c}$$

Hence, the Riemannian identities (3.5a), (3.5b) and (3.5c) coincide with those obtained from (3.2a), (3.2b) and (3.2c) in the limit $T_{abc} \to 0$, as expected. Therefore, taking torsionless limit and calculating Noether-Ward identities gives an equivalent result as first calculating the Ward identities and taking torsion zero [22]. This is important when we do not know whether the torsion vanishes. Therefore, one may safely work in first order formalism assuming the boundary conditions and gauge fixing presented previously.

IV. LOVELOCK-CHERN-SIMONS GRAVITY

A. Action and equations of motion

The Lovelock-Lanczos gravity [15] in first order formulation is described by the action

$$I_{\mathrm{L}} = \sum_{p=0}^{[D/2]} \alpha_p L_p, \tag{4.1a}$$

where α_p are arbitrary coupling constants and L_p is dimensionally continued Euler density in D dimensions,

$$L_p = \epsilon_{i_1 i_2 \dots i_D} R^{i_1 i_2} \dots R^{i_{2p-1} i_{2p}} e^{i_{2p+1}} \dots e^{i_D}. \tag{4.1b}$$

Here p is the power of the curvature tensor in the polynomial L_p. We omit writing the wedge product for the sake of simplicity.

Lovelock-Lanczos gravity possesses numerous black hole solutions with Riemannean geometry [23–25], although some choices of the coupling constants $\{\alpha_p\}$ exhibit a causality problem in the dual CFT [26], or have instable geometries [27,28]. Generic Lovelock gravity without torsion possesses the same number of degrees of freedom as general relativity [29]. With torsion, or when the parameters take the critical values, the dynamical content of Lovelock-Lanczos gravity might change. Solutions in these cases are known as well, for example the ones with Riemann-Cartan geometry in five-dimensional gravity [30,31] and supergravity [32].

In odd-dimensional case $D = 2n + 1$, the special choice of coefficients $\alpha_p = \frac{\kappa}{2n+1-2p}\binom{n}{p}$ defines theory with the *unique* (degenerate) AdS vacuum, known as LCS AdS gravity. Alternatively, LCS action can be constructed as a Chern density by taking the topological invariant, Chern form $dL_{\mathrm{CS}} = \epsilon_{i_1 j_1 \dots i_n j_n} F^{i_1 j_1} \dots F^{i_n j_n}$, and writing L_{CS} by using holonomy operator [14,33]. Then, an equivalent form of LCS action is given by

$$I_{\mathrm{LCS}} = \kappa \int_{\mathcal{M}} \int_0^1 dt \epsilon_{A_1 B_1 A_2 B_2 \dots A_n B_n C} \times \prod_{k=1}^{n} (\hat{R}^{A_k B_k} + t^2 \hat{e}^{A_k} \hat{e}^{B_k}) \hat{e}^C. \tag{4.2}$$

Dropping the indices for simplicity, the above expression reads

$$I_{\mathrm{LCS}} = \kappa \int_{\mathcal{M}} \int_0^1 dt \epsilon (\hat{R} + t^2 \hat{e}^2)^n \hat{e} = \kappa \int_{\mathcal{M}} \sum_{k=0}^{n} \binom{n}{k} \frac{1}{2k+1} \epsilon \hat{R}^{n-k} \hat{e}^{2k+1}, \tag{4.3}$$

where we used the binomial expansion to perform an integration over t.

Equations of motion are obtained from the variation of the action (4.3) with respect to fundamental variables $\hat{e}^A$ and $\hat{\omega}^{AB}$. Variation with respect to $\hat{e}$ yields

$$C_A \coloneqq \epsilon_{A A_1 B_1 \dots A_n B_n} \prod_{k=1}^{n} F^{A_k B_k} = 0, \tag{4.4}$$

which can be split into 1 and a components,

$$C \coloneqq \epsilon_{1 a_1 b_1 \dots a_n b_n} \prod_{k=1}^{n} F^{a_k b_k} = 0, \tag{4.5a}$$

$$C_a \coloneqq \epsilon_{a 1 b a_2 b_2 \dots a_n b_n} F^{1b} \prod_{k=2}^{n} F^{a_k b_k} = 0. \tag{4.5b}$$

Variation with respect to ω yields

$$C_{AB} \coloneqq \epsilon_{ABA_1 B_1 \dots A_{n-1} B_{n-1} C} \prod_{k=1}^{n-1} F^{A_k B_k} \hat{T}^C = 0, \tag{4.6}$$

and can be split into $[1a]$ and $[ab]$ components,

$$\bar{C}_a \coloneqq \epsilon_{1 a a_1 b_1 \dots a_{n-1} b_{n-1} c} \prod_{k=1}^{n-1} F^{a_k b_k} \hat{T}^c = 0, \tag{4.7a}$$

$$C_{ab} \coloneqq \epsilon_{1 a b a_1 b_1 \dots a_{n-1} b_{n-1}} \times \prod_{k=1}^{n-2} F^{a_k b_k} (F^{a_{n-1} b_{n-1}} \hat{T}^1 + (n-1) F^{1 a_{n-1}} \hat{T}^{b_{n-1}}). \tag{4.7b}$$

Let us note that $\hat{T}^a = 0$ is a particular solution of the equations (4.7) belonging to the Riemannian subclass of all solutions of the theory. Also, the global AdS space ($F^{ab} = 0$) is a particular solution of all equations of motion.

B. 1-point functions

In this section we calculate the renormalized gravitational LCS action in the classical approximation. Then we use the AdS/CFT correspondence to promote it to the quantum effective action in a holographic CFT, and compute the holographic 1-point functions.

The variation of the LCS action reads

$$\delta I_{\rm LCS} = n\kappa \int_{\partial\mathcal{M}} \int_0^1 dt \varepsilon_{ABCA_1B_1\ldots A_{n-1}B_{n-1}} \delta\hat{\omega}^{AB} \hat{e}^C \times \prod_{k=1}^{n-1} (\hat{R}^{A_kB_k} + t^2 \hat{e}^{A_k}\hat{e}^{B_k}). \tag{4.8}$$

To perform a near-boundary expansion of the fields, let us first rewrite the following quantity in terms of the AdS tensor:

$$\hat{R}^{A_kB_K} + t^2 \hat{e}^{A_k}\hat{e}^{B_k} = F^{A_kB_k} + (t^2-1)\hat{e}^{A_k}\hat{e}^{B_k}.$$

The first term in the above expression is *independent* of ρ since on the boundary $\hat{e}^1 = 0$, and therefore the particular components expand as

$$\hat{R}^{a_kb_k} + \hat{e}^{a_k}\hat{e}^{a_k} = F^{a_kb_k},$$
$$\hat{R}^{a_k1} + \hat{e}^{a_k}\hat{e}^1 = \frac{1}{\sqrt{\rho}}(T^{a_k} - \rho\nabla k^{a_k}). \tag{4.9}$$

Plugging these expansions in the variation of the action, we find

$$\delta I_{\rm LCS} = n\kappa \int_{\partial\mathcal{M}} \varepsilon\delta\hat{\omega} \sum_{k=0}^{n-1} \begin{matrix} n-1 \\ k \end{matrix} \frac{(-1)^k(2k)!!}{(2k+1)!!} F^{n-k-1}\hat{e}^{2k+1}, \tag{4.10}$$

where we used the beta function to solve the integral $\int_0^1 dt(t^2-1)^k = \frac{(-1)^k(2k)!!}{(2k+1)!!}$.

Variation (4.10) is divergent on the boundary, that is, in the limit $\rho \to 0$ and extraction of physical quantities requires its renormalization, or removal of divergences. For related work on Riemannian Lovelock gravity, see Ref. [5].

The procedure for obtaining finite results consists in introducing a regulating surface at $\rho = \epsilon$ and adding the counterterms which cancel all divergent contributions as ϵ tends to zero [8,34]. Equivalently said, the divergent terms in a variation of an action have to be represented as total variations of local terms integrated over boundary. In general, the computation of the total variation can be substantially simplified after noting that the conditions for the application of the theorem [19] are fulfilled in our case. For an alternative proof of the theorem [19], see Appendix C. The theorem [19] states that the terms which are asymptotically divergent or zero (when $\rho \to 0$) can always be represented as total variations of local boundary functionals. Therefore, we can discard all ρ^α ($\alpha \neq 0$) terms in the expression (4.10) and keep only the ρ^0-terms. For the form of the ρ^α-terms ($\alpha \neq 0$), see Appendix B. Note that the counterterms can contain arbitrary local finite part which is nonphysical and depends on a renormalization scheme. The divergent counterterms are local and there is finite number of them. They also depend on only one coupling constant κ. Counterterms in Riemannian gemetry were calculated in Ref. [35].

Keeping only the finite terms, we obtain the variation of the regularized action $I_{\rm ren} = I_{\rm LCS} + I_{\rm ct}$ in the form

$$\delta I_{\rm ren} = -2n\kappa\varepsilon\Big[\delta\omega T \sum_{l=0}^{n-2} \binom{n-2}{l} \frac{(-1)^l 2^{2l+1}(n-1)}{l+1} (R+4ek)^{n-2-l} e^l k^{l+1} - \delta e \sum_{l=0}^{n-1} \binom{n-1}{l} \frac{(-1)^l 2^{2l+1}}{l+1} (R+4ek)^{n-1-l} e^l k^{l+1}\Big], \tag{4.11}$$

where $T = \nabla e$ is the boundary torsion tensor. Comparing to (3.1), the spin and energy-momentum currents are given by, respectively,

$$\sigma_{ab} = -n\kappa\varepsilon_{1ab} T \sum_{l=1}^{n-1} \binom{n-1}{l} 4^l R^{n-1-l} e^{l-1} k^l, \tag{4.12}$$

$$\tau_a = \kappa\varepsilon_{1a} \sum_{l=1}^{n} \binom{n}{l} 4^l R^{n-l} e^{l-1} k^l, \tag{4.13}$$

and they correspond to the vacuum expectation values of the quantum CFT operators, the spin current $\mathcal{S}_{ab}$ and the energy-momentum of the conformal matter $\mathcal{T}_a$,

$$\sigma_{ab} = \langle \mathcal{S}_{ab} \rangle_{\rm CFT}, \qquad \tau_a = \langle \mathcal{T}_a \rangle_{\rm CFT}. \tag{4.14}$$

Using these representations of the 1-point functions of the CFT operators, we can study their quantum conservation laws, that is, the Noether-Ward identities.

C. Anomalies

The equations (3.2) describe classical conservation laws in a holographic theory invariant under diffeomorphisms, conformal transformations and non-Abelian gauge transformations. Since now we know the form of the corresponding quantum currents, we can also check the quantum conservation laws. If the law is not satisfied, then the quantum theory possesses a quantum anomaly.

In this section we explore the Ward identities and check for the existence of quantum anomalies: Lorentz anomaly A_{ab}, diffeomorphism anomaly $\bar{A}_a$, conformal anomaly A and gauge anomaly A_a. It is well known that there are two types of non-Abelian anomalies, covariant and consistent. All the anomalies we compute here are covariant, i.e., they transform covariantly under gauge symmetries.

1. Lorentz Ward identity

The conservation law for Lorentz symmetry is given by Eq. (3.2b), so we have to calculate the quantity

$$A_{ab} = \nabla\sigma_{ab} - 2e_{[a}\tau_{b]}. \tag{4.15}$$

Using the expressions (4.12) and (4.13) for the quantum currents, we find

$$A_{ab} = -4n\kappa\varepsilon_{ab}\Bigg[2(n-1)T\nabla k\sum_{l=0}^{n-2}\sum_{m=0}^{n-2-l}\binom{n-2}{l}\binom{n-2-l}{m}4^m(l+m+1)R^{n-2-l-m}e^{l+m}k^{l+m} + e_ck^c\sum_{l=0}^{n-1}\sum_{m=0}^{n-1-l}\binom{n-1}{l}\binom{n-1-l}{m}\frac{(-1)^l2^{2l+2m+1}(l+m+1)}{l+1}R^{n-1-l-m}e^{l+m}k^{l+m}\Bigg].$$

It turns out that A_{ab} can be completely expressed in terms of the field equations, that means that it vanishes,

$$A_{ab} = -4n\kappa C_{ab} = 0. \tag{4.16}$$

Therefore, there is no Lorentz anomaly in the holographic theory because the Lorentz symmetry is conserved also quantically. This is an expected result, since the Lorentz symmetry is usually broken in the actions that are not parity invariant.

2. Ward identity for diffeomorphisms.

The conservation law for local translations has the form (3.2a),

$$\bar{A}_a = \nabla\tau_a - \left(I_aT^b\tau_b + \frac{1}{2}I_aR^{bc}\sigma_{bc}\right), \tag{4.17}$$

where I_a is the contraction operator with the spacetime index projected to the tangent manifold using the inverse vielbein $e_a{}^\alpha$. Plugging in the quantum currents (4.12) and (4.13), one can show that the conservation law is satisfied,

$$\bar{A}_a = 4n\kappa(k^b{}_aC_b - \bar{C}_a) = 0. \tag{4.18}$$

Therefore, there is no gravitational anomaly, as expected.

3. Conformal Ward identity

The conservation law for local Weyl transformations can be read off from Eq. (3.2c) as

$$A = e^a\tau_a + \nabla(e^aI^b\sigma_{ab}), \tag{4.19}$$

where $e^a\tau_a$ is the trace of energy-momentum tensor, so A is also called the trace anomaly. Using the field equations and discarding the total divergence, one can show that the trace anomaly has the form

$$e^a\tau_a = \kappa\varepsilon_{a_1b_1a_2b_2\ldots a_nb_n}R^{a_1b_1}R^{a_2b_2}\ldots R^{a_nb_n} = \kappa\mathcal{E}_n(R). \tag{4.20}$$

Thus, the holographic anomaly is nonvanishing and, up to a divergence, proportional to the Euler density $\mathcal{E}_n(R) = \varepsilon R^n$, as expected in a CFT dual to a higher-dimensional AdS gravity [36]. Since the Weyl anomaly is topological invariant, it is of the type A, according to the general classification of conformal anomalies given in Ref. [37].

4. Ward identity for gauge symmetry

The conservation law for non-Abelian gauge transformations is given by Eq. (3.2d) as

$$A_a = \nabla\tau_a - 2(e^b\sigma_{bc}k_a{}^c + k^b\sigma_{ba}). \tag{4.21}$$

Using (4.12) and (4.13), as well as the equations of motion, we can express it as

$$A_a = -2n\kappa\epsilon_{1b}I_aT^b\sum_{l=0}^{n-1}\binom{n-1}{l}\frac{(-1)^l2^{2l+1}}{l+1}(R+4ek)^{n-1-l}e^lk^{l+1}$$
$$-4n\kappa\epsilon_{1bc}T\left(\frac{1}{2}I_aR^{bc}-2e^bk_a{}^c\right)\sum_{l=0}^{n-2}\binom{n-2}{l}\frac{(-1)^l2^{2l+1}(n-1)}{l+1}(R+4ek)^{n-2-l}e^lk^{l+1}$$
$$+8n\kappa\epsilon_{1a}T\sum_{l=0}^{n-2}\binom{n-2}{l}\frac{(-1)^l2^{2l+1}(n-1)}{l+1}(R+4ek)^{n-2-l}e^lk^{l+2}\neq 0. \qquad (4.22)$$

The above holographic anomaly is in general nonvanishing, but it cancels out when *the torsion is equal to zero,* as expected. Indeed, when $T^a = 0$, the non-Abelian gauge symmetry is not independent, but it can be expressed in terms of the diffeomorphisms, which are conserved at the quantum level. Another derivation of this result is possible by noting that in this particular case the spin tensor vanishes and both Eqs. (3.2a) and (3.2d) reduce to

$$\tilde{\nabla}_\alpha\tilde{\tau}^\alpha{}_\alpha = 0. \qquad (4.23)$$

Again non-Abelian gauge anomaly vanishes since $A_a = 0$.

V. CONCLUDING REMARKS

We analyzed a holographic dual of Lovelock Chern-Simons AdS gravity in an arbitrary odd dimension and calculated corresponding holographic currents and anomalies in the quantum CFT. First part of the work is devoted to the kinematics of gravitational theory with AdS gauge symmetry. After motivating a gauge fixing suitable for a holographc analysis, we calculated residual (asymptotic) symmetries. Then we focused to Chern-Simons AdS gravity. We concluded that the largest asymptotic symmetry consists of local translations and rotations (local Poincaré group), local Weyl rescalings and, in the presence of torsion on the boundary, of non-Abelian gauge symmetry. If the torsion on the boundary is zero, then a non-Abelian symmetry is not independent any longer and reduces to local Poincaré transformations.

We found holographic representations of the energy-momentum and spin tensors in a dual theory, which we identified with the corresponding 1-point functions in CFT, in the presence of sources. We also computed their conservation laws and obtained that some of quantum symmetries are broken, leading to quantum anomalies. Explicitly, we obtained that local translations and rotations are symmetries of the quantum theory, while Weyl rescalings and non-Abelian gauge symmetry are anomalous. Similarly as in five dimensions [9], the trace anomaly is proportional to the Euler density and is therefore of the type A.

Because of nonlinearity of the model and working in higher-dimensional Riemann-Cartan space, the regularization of the action was quite involved. However, with the help of a general renormalization theorem shown in Appendix C, it was possible to circumvent an explicit construction of divergent counterterms and extract directly its finite part. An alternative proof of the theorem is given in Ref. [19].

One of the open questions left for future work is an application on non-Abelian gauge transformations to the calculation of chiral anomaly. Namely, in Ref. [9] it was suggested that the chiral anomaly is related to the completely antisymmetric component of the torsion tensor. Another question would be to find a different gauge fixing of either transversal diffeomorphisms or local AdS symmetry, in order to obtain an *infinite* radial expansion of the fields, and possibly the type B anomaly. This would describe an inequivalent holographic theory. Finally, we are also interested in introducing a gauge fixing which breaks relativistic covariance in an arbitrary Poincaré gauge theory, and is suitable for the formulation of Lifshitz holography. These last topics is the work in progress.

ACKNOWLEDGMENTS

The authors thank Milutin Blagojević for many useful discussions and comments. This work was partially supported by the Serbian Science Foundation under Grant No. 171031, Chilean Fondo Nacional de Desarrollo Científico y Tecnologico (FONDECYT) Project No. 1170765 and the Pontificia Universidad Catolica de Valparaiso (PUCV) Grant No. 039.428/2017.

APPENDIX A: ADS ALGEBRA

The algebra of generators $J_{\bar{A}\bar{B}} = -J_{\bar{B}\bar{A}}$ $(\bar{A},\bar{B} = 0, 1,\ldots,D)$ of AdS group $SO(D-1,2)$ if given by

$$[J_{\bar{A}\bar{B}}, J_{\bar{C}\bar{E}}] = \eta_{\bar{B}\bar{C}}J_{\bar{A}\bar{E}} + \eta_{\bar{B}\bar{C}}J_{\bar{A}\bar{E}} - \eta_{\bar{A}\bar{C}}J_{\bar{B}\bar{E}} - \eta_{\bar{B}\bar{E}}J_{\bar{A}\bar{C}}, \qquad (A1)$$

where $\eta_{\bar{A}\bar{B}} = (-1,\underbrace{1,\ldots,1}_{D-1},-1)$. Introducing the splitting of indices $\bar{A} = (A, D)$ and with

$$P_A = J_{AD},$$
$$J_{AB} = -J_{BA}, \qquad A,B = 0,1,\ldots,D-1, \qquad (A2)$$

the algebra (A1) (after taking into account that $\eta_{DD} = -1$) takes the familiar form

$$
\begin{aligned}
[P_A, P_B] &= J_{AB}, \\
[P_A, J_{BC}] &= \eta_{AB} P_C - \eta_{AC} P_B, \\
[J_{AB}, J_{CE}] &= \eta_{BC} J_{AE} + \eta_{AE} J_{BC} - \eta_{AC} J_{BE} - \eta_{BE} J_{AC}.
\end{aligned} \tag{A3}
$$

APPENDIX B: VARIATION OF LCS ACTION

In this appendix we present the nonvanishing parts of the variation of LCS action given by Eq. (4.10),

$$
\delta I_{\rm LCS} = \sum_{j=0}^{n} \frac{1}{\rho^j} \delta I_j. \tag{B1}
$$

We find the following terms, with $1 \le j \le (n-2)$:

$$
\begin{aligned}
\delta I_n &= \varepsilon_{a1a_1b_1\ldots a_{n-1}b_{n-1}c} \delta e^a e^c K_{-(n-1)}, \\
\delta I_{n-1} &= \varepsilon_{aba_1b_1\ldots d1c} \delta\omega^{ab} e^c \nabla e^d J_{-(n-2)} \\
&\quad + \varepsilon_{a1a_1b_1\ldots a_{n-1}b_{n-1}c} [\delta e^a e^c K_{-(n-2)} + (\delta e^a k^c - \delta k^a e^c) K_{-(n-1)}], \\
\delta I_j &= \varepsilon_{1abcda_1b_1\ldots} \delta\omega^{ab} [e^c \nabla e^d J_{-(j-1)} - (e^c \nabla k^d - k^c \nabla e^d) J_{-j} - k^c \nabla k^d J_{-(j+1)}] \\
&\quad - \varepsilon_{1aca_1b_1\ldots a_{n-1}b_{n-1}} [\delta e^a e^c K_{-(j-1)} + (\delta e^a k^c - \delta k^a e^c) K_{-j} - \delta k^a k^c K_{-(j+1)}], \\
\delta I_0 &= \varepsilon_{1abcda_1b_1\ldots} \delta\omega^{ab} [e^c \nabla e^d J_1 - (e^c \nabla k^d - k^c \nabla e^d) J_0 - k^c \nabla k^d J_{-1}] \\
&\quad - \varepsilon_{1aca_1b_1\ldots a_{n-1}b_{n-1}} [\delta e^a e^c K_1 + (\delta e^a k^c - \delta k^a e^c) K_0 - \delta k^a k^c K_{-1}],
\end{aligned} \tag{B2}
$$

and

$$
\begin{aligned}
K_\alpha &= \sum_{l=0}^{n-1} \binom{n-1}{l} (R + 4ek)^{n-l-1} A_{l\alpha} e^{l-\alpha} k^{l+\alpha}, \\
J_\alpha &= (n-1) \sum_{l=0}^{n-2} \binom{n-2}{l} (R + 4ek)^{n-l-2} A_{l\alpha} e^{l-\alpha} k^{l+\alpha},
\end{aligned} \tag{B3}
$$

where

$$
A_{l\alpha} = \frac{(-1)^l 4^l l!^2}{(2l+1)(l-\alpha)!(l+\alpha)!}. \tag{B4}
$$

APPENDIX C: ALTERNATIVE PROOF OF THE RENORMALIZATION THEOREM

In this appendix we show an alternative derivation of the results of Ref. [19].

Theorem 1 A surface counterterm can be added to an action of any classical field theory in the bulk to cancel all the terms which depend on the radial coordinate in an on-shell variation, if any of the following conditions are satisfied:

(i) The bulk has the topology $\mathbb{R} \times \partial M$;

(ii) The boundary has a finite number of disjoint pieces and near each one the bulk looks like $\mathbb{R} \times \partial M$.

Here, ∂M is any manifold without boundary with the coordinates x^α and the radial coordinate is labeled by ρ. If the fields have asymptotic expansion near the boundary of the form $\phi^i = \sum_n f_n^i(\rho) \phi_n^i(x^\alpha)$, where $f_n^i(\rho)$ are functions that depend only on ρ and $\phi_n^i(x^\alpha)$ are (ρ-independent) boundary fields, then the counterterm is a local functional of the boundary fields.

Let the action in $(D+1)$-dimensional bulk M be defined in language of differential forms as

$$
S = \int_M L. \tag{C1}
$$

A variation of the action (C1) takes the form

$$
\delta S = \int_M \delta L = \int_M e.o.m. + \int_M d_{D+1} L_D^B \tag{C2}
$$

where *e.o.m* are the terms proportional to the equation of motion. Formula (C2) is also valid without integral and it will be used in that form later. By using the Stoke's theorem, we can write the last term in (C2) as

$$
\int_M d_{D+1} L_D^B = \int_{\partial M} L_D, \tag{C3}
$$

where the boundary of M is placed at fixed distance $\rho = \varepsilon$ near (but not equal) zero and $L_D \coloneqq L_D^B|_{\rho=\varepsilon}$. Let ∂M be a boundary at each ρ. The most general D-form L_D near the boundary is

$$
L_D^B = L_D + d\rho \wedge V, \tag{C4}
$$

where V is an arbitrary $(D-1)$-form. The exterior derivative in the bulk can be decomposed near the boundary as

$$
d_{D+1} = \partial_\rho d\rho + d, \tag{C5}
$$

where d is the exterior derivative at the boundary and d_ρ is the derivative along the direction ρ. From Eqs. (C2), (C4) and (C5), we get on-shell

$$\delta L = d\rho \wedge \partial_\rho L_D - d\rho \wedge dV. \tag{C6}$$

Equivalently, this can be rewritten as

$$\partial_\rho L_D = \delta U + dV \tag{C7}$$

where $\delta L = d\rho \wedge \delta U$. Hence, from (C7) it follows that

$$L_D = \delta A + dB + R(x^\alpha) \tag{C8}$$

where $A = \int d\rho U$, $B = \int d\rho V$ and $R(x^\alpha)$ does not depend on ρ. This conclusion is valid under the assumption that the right side of Eq. (C7) is integrable and that the derivative and integral mutually commute. Therefore, L_D is a sum of a total variation, exact form and a function which does not depend on ρ.

Consequently, we get

$$\int_{\partial M} L_D = \delta \int_{\partial M} A + \int_{\partial M} R, \tag{C9}$$

where we used the fact that an integral of the exact form dB vanishes due to the Stoke's theorem and because the boundary of a boundary is an empty set. After substituting (C9) into (C2) we obtain on-shell

$$\delta(S - S_{\mathrm{ct}}) = \int_{\partial M} R, \tag{C10}$$

where $S_{\mathrm{ct}} = \int_{\partial M} A$. Since R is ρ independent, the expression (C10) is well defined at the boundary $\rho = 0$. Thus, all ρ-dependent terms can be eliminated by adding a suitable counterterm. An important observation is that this counterterm is *unique*. Given an asymptotic solution of the field equations, a near-boundary behavior is fixed. Furthermore, the counterterm is obtained from the Lagrangian, thus it depends on the same parameters. In other words, we do not include new parameters in the theory. If the starting Lagrangian has a finite number of parameters, so it does the renormalized Lagrangian.

As the counterterm is obtained as a primitive function of local functions, it is not necessarily local. The near-boundary expansion method is, however, able to determine only local counterterms.

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PHYSICAL REVIEW D **95,** 104018 (2017)

Generalized *pp* waves in Poincaré gauge theory

M. Blagojević[*] and B. Cvetković[†]

Institute of Physics, University of Belgrade, Pregrevica 118, 11080 Belgrade, Serbia

(Received 17 February 2017; published 16 May 2017)

Starting from the generalized pp waves that are exact vacuum solutions of general relativity with a cosmological constant, we construct a new family of exact vacuum solutions of Poincaré gauge theory, the generalized pp waves with torsion. The ansatz for torsion is chosen in accordance with the wave nature of the solutions. For a subfamily of these solutions, the metric is dynamically determined by the torsion.

DOI: 10.1103/PhysRevD.95.104018

I. INTRODUCTION

The principle of gauge symmetry was born in the work of Weyl [1], where he obtained the electromagnetic field by assuming local $U(1)$ invariance of the Dirac Lagrangian. Three decades later, the Poincaré gauge theory (PGT) was formulated by Kibble and Sciama [2]; it is nowadays a well-established gauge approach to gravity, representing a natural extension of general relativity (GR) to the gauge theory of the Poincaré group [3,4]. Basic dynamical variables in PGT are the tetrad field b^i and the Lorentz connection $\omega^{ij} = -\omega^{ji}$ (1-forms), and the associated field strengths are the torsion $T^i = db^i + \omega^i{}_k \wedge b^k$ and the curvature $R^{ij} = d\omega^{ij} + \omega^i{}_k \wedge \omega^{kj}$ (2-forms). By construction, PGT is characterized by a Riemann-Cartan geometry of spacetime, and its physical content is directly related to the existence of mass and spin as basic characteristics of matter at the microscopic level. An up-to-date status of PGT can be found in a recent reader with reprints and comments [5].

General PGT Lagrangian L_G is at most quadratic in the field strengths. The number of independent (parity invariant) terms in L_G is nine, which makes the corresponding dynamical structure rather complicated. As is well known from the studies of GR, exact solutions have an essential role in revealing and understanding basic features of the gravitational dynamics [6–9]. This is also true for PGT, where exact solutions allow us, among other things, to study the interplay between dynamical and geometric aspects of torsion [5].

In the context of GR, one of the best known families of exact solutions is the family of *pp waves*: it describes plane-fronted waves with parallel rays propagating on the Minkowski background M_4; see, for instance, Ehlers and Kundt [6]. There is an important generalization of this family, consisting of the exact vacuum solutions of GR with a cosmological constant (GR_Λ) such that for $\Lambda \to 0$, they reduce to the pp waves in M_4. We will refer to this family as the generalized pp waves, or just pp_Λ waves for short. In contrast to the pp waves in M_4, the wave surfaces of the pp_Λ waves have constant curvature proportional to Λ. The family of the pp_Λ waves belongs to a more general family, known as the Kundt class of type N, labeled $\mathrm{KN}(\Lambda)$. Details on the $\mathrm{KN}(\Lambda)$ spacetimes can be found in the monograph by Griffiths and Podolský [9]; see also Refs. [10–12]. In this paper, we start from the Riemannian pp_Λ waves in GR_Λ and construct a new family of the pp_Λ waves with torsion, representing a new class of exact vacuum solutions of PGT. The torsion is introduced relying on the approach used in our previous paper [13]. The present work is motivated by earlier studies of the exact wave solutions in PGT [14], and is regarded as a complement to them.

The paper is organized as follows. In Sec. II, we give a short account of the Riemannian pp_Λ waves, including the relevant geometric and dynamical aspects, as a basis for their extension to pp_Λ waves with torsion. In Sec. III, we first introduce an ansatz for the new, Riemann-Cartan (RC) connection, the structure of which complies with the wave nature of a RC spacetime. The ansatz is parametrized by a specific 1-form K living on the wave surface, and the related torsion has only one, tensorial irreducible component. Then, we use the PGT field equations to show that the dynamical content of K is described by two torsion modes with the spin-parity values $J^P = 2^+$ and 2^-. In Sec. IV, we find solutions for both the metric function H and the torsion function K, in the spin-2^+ sector and for $\lambda > 0, < 0$ and $= 0$. It is shown that K has a decisive influence on the solution for H, and consequently, on the resulting metric. In Sec. V, we shortly discuss solutions in the spin-2^- sector, which are found to be much less interesting. Section VI concludes the exposition with a few remarks on some issues not covered in the main text, and the Appendices are devoted to certain technical details.

Our conventions are as follows. The latin indices $(i, j, \ldots)$ refer to the local Lorentz (co)frame and run over (0, 1, 2, 3), b^i is the tetrad (1-form), and h_i is the dual basis (frame), such that $h_i b^k = \delta^i_k$. The volume 4-form is $\hat\epsilon = b^0 \wedge b^1 \wedge b^2 \wedge b^3$, the Hodge dual of a form α is ${}^\star\alpha$, with ${}^\star 1 = \hat\epsilon$, and the totally antisymmetric tensor is

[*]mb@ipb.ac.rs
[†]cbranislav@ipb.ac.rs

defined by ${}^\star(b_i \wedge b_j \wedge b_k \wedge b_m) = \varepsilon_{ijkm}$ and normalized to $\varepsilon_{0123} = +1$. The exterior product of forms is implicit, except in Appendix B.

II. RIEMANNIAN pp_Λ WAVES

In this section, we give an overview of Riemannian pp_Λ waves using the tetrad formalism [15], necessary for the transition to PGT.

A. Geometry

The family of pp_Λ waves is a specific subclass of the Kundt spacetimes KN(Λ), labeled by KN(λ)[$\alpha = 1, \beta = 0$]; for the full classification of the KN(Λ) spacetimes, see Refs. [9,10]. In suitable local coordinates $x^\mu = (u, v, y, z)$ (see Appendix A), the metric of the pp_Λ waves can be written as

$$ds^2 = 2\left(\frac{q}{p}\right)^2 du(Sdu + dv) - \frac{1}{p^2}(dy^2 + dz^2), \tag{2.1a}$$

where

$$p = 1 + \frac{\lambda}{4}(y^2 + z^2), \qquad q = 1 - \frac{\lambda}{4}(y^2 + z^2),$$
$$S = -\frac{\lambda}{2}v^2 + \frac{p}{2q}H(u, y, z), \tag{2.1b}$$

with λ being a suitably normalized cosmological constant, and the unknown metric function H is to be determined by the field equations. The coordinate v is an affine parameter along the null geodesics $x^\mu = x^\mu(v)$, and u is retarded time such that $u = \text{const}$ are the spacelike surfaces parametrized by $x^\alpha = (y, z)$. Since the null vector $\xi = \xi(u)\partial_v$ is orthogonal to these surfaces, they are regarded as wave surfaces, and ξ is the null direction (ray) of the wave propagation. The vector ξ is not covariantly constant, and consequently, the wave rays are not parallel and the wave surfaces are not flat. For $\lambda \to 0$, the metric (2.1) reduces to the metric of pp waves on the M_4 background, which explains the term generalized pp waves, or pp_Λ waves.

Next, we choose the tetrad field (coframe) in the form

$$b^0 := du, \qquad b^1 := \left(\frac{q}{p}\right)^2(Sdu + dv),$$
$$b^2 := \frac{1}{p}dy, \qquad b^3 := \frac{1}{p}dz, \tag{2.2a}$$

so that $ds^2 = \eta_{ij} b^i \otimes b^j$, where η_{ij} is the half-null Minkowski metric:

$$\eta_{ij} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}.$$

The corresponding dual frame h_i is given by

$$h_0 = \partial_u - S\partial_v, \qquad h_1 = \left(\frac{p}{q}\right)^2\partial_v,$$
$$h_2 = p\partial_y, \qquad h_3 = p\partial_z. \tag{2.2b}$$

For the coordinates $x^\alpha = (y, z)$ on the wave surface, we have

$$x^c = b^c{}_\alpha x^\alpha = \frac{1}{p}(y, z), \qquad \partial_c = h_c{}^\alpha\partial_\alpha = p(\partial_y, \partial_z),$$

where $c = 2, 3$.

Starting from the general formula for the Riemannian connection 1-form,

$$\omega^{ij} := -\frac{1}{2}\left[h^i \rfloor db^j - h^j \rfloor db^i - (h^i \rfloor h^j \rfloor db^k)b_k\right],$$

one can find its explicit form; for $i < j$, it reads

$$\omega^{01} = \lambda v b^0 - \frac{1}{q}(\lambda y b^2 + \lambda z b^3), \quad \omega^{02} = \frac{\lambda y}{q}b^0, \quad \omega^{03} = \frac{\lambda z}{q}b^0,$$
$$\omega^{12} = \frac{\lambda y}{q}b^1 - \frac{q^2}{p}\partial_y S b^0, \qquad \omega^{13} = \frac{\lambda z}{q}b^1 - \frac{q^2}{p}\partial_z S b^0,$$
$$\omega^{23} = \frac{1}{2}(\lambda z b^2 - \lambda y b^3). \tag{2.3a}$$

Introducing the notation $i = (A, a)$, where $A = 0, 1$ and $a = (2, 3)$, one can rewrite ω^{ij} in a more compact form:

$$\omega^{01} = \lambda v b^1 - \frac{2}{qp}(b^c\partial_c p),$$
$$\omega^{Ac} = -\frac{2}{qp}b^A\partial^c p + k^A\frac{q^2}{p^2}b^0\partial^c S,$$
$$\omega^{23} = -\frac{1}{p}(b^2\partial^3 p - b^3\partial^2 p), \tag{2.3b}$$

where $k^i = (0, 1, 0, 0)$ is a null propagation vector, $k^2 = 0$.

The above connection defines the Riemannian curvature $R^{ij} = d\omega^{ij} + \omega^i{}_m\omega^{mj}$; for $i < j$, it is given by

$$R^{ij} = \begin{cases} -\lambda b^1 b^c + k^1 b^0 Q^c, & \text{for } (i, j) = (1, c) \\ -\lambda b^i b^j, & \text{otherwise,} \end{cases} \tag{2.4a}$$

where Q^c is a 1-form introduced by Obukhov [15],

$$Q_c = -\underline{\nabla}\left[\left(\frac{q}{p}\right)^2 h_c\rfloor \underline{d}S\right] + \left(\frac{q}{p}\right)^3 h_c\rfloor\left[\underline{d}\left(\frac{p}{q}\right)\wedge \underline{d}S\right],$$

and $\underline{d} = dx^\alpha\partial_\alpha$ is the exterior derivative on the wave surface. In more details

$$Q^2 = \frac{q}{2p}[2qp\partial_{yy}S + (q-4)\lambda y\partial_y S - q\lambda z\partial_z S]b^2 + \frac{q}{2}[2q\partial_{yz}S - \lambda z\partial_y S - \lambda y\partial_z S]b^3,$$

$$Q^3 = \frac{q}{2p}[2qp\partial_{zz}S + (q-4)\lambda z\partial_z S - q\lambda y\partial_y S]b^3 + \frac{q}{2}[2q\partial_{yz}S - \lambda z\partial_y S - \lambda y\partial_z S]b^2.$$

As a consequence, R^{ij} can be represented more compactly as

$$R^{ij} = -\lambda b^i b^j + 2b^0 k^{[i}Q^{j]}. \tag{2.4b}$$

The Ricci 1-form $Ric^i := h_m\rfloor Ric^{mi}$ is given by

$$Ric^i = -3\lambda b^i + b^0 k^i Q,$$
$$Q = h_c\rfloor Q^c = \frac{qp}{2}\left[\partial_{yy}H + \partial_{zz}H + \frac{2\lambda}{p^2}H\right], \tag{2.5}$$

and the scalar curvature $R := h_i\rfloor Ric^i$ reads

$$R = -12\lambda. \tag{2.6}$$

B. Dynamics

1. pp_Λ waves in GR_Λ

Starting with the action $I_0 = -\int d^4x(a_0 R + 2\Lambda_0)$, one can derive the GR_Λ field equations in vacuum,

$$2a_0 G^n{}_i - 2\Lambda_0\delta^n_i = 0, \tag{2.7a}$$

where $G^n{}_i$ is the Einstein tensor. The trace and the traceless piece of these equations read

$$\Lambda_0 = 3a_0\lambda, \qquad Ric^i - \frac{1}{4}Rb^i \equiv b^0 k^i Q = 0. \tag{2.7b}$$

As a consequence, the metric function H must obey

$$\partial_{yy}H + \partial_{zz}H + \frac{2\lambda}{p^2}H = 0. \tag{2.8}$$

There is a simple solution of these equations,

$$H_c = \frac{1}{p}(A(u)q + B_\alpha x^\alpha)f(u), \tag{2.9}$$

for which Q^a vanishes. This solution is trivial (or pure gauge), since the associated curvature takes the background form, $R^{ij} = -\lambda b^i b^j$; moreover, it is conformally flat, since its Weyl curvature vanishes. The nontrivial vacuum solutions are characterized by $Q = 0$, but $Q^c \neq 0$; their general form can be found in [10].

2. pp_Λ waves in PGT

To better understand the relation between GR_Λ and PGT, it is interesting to examine whether pp_Λ waves satisfying the GR_Λ field equations in vacuum are also a vacuum solution of PGT. It turns out that a more general version of the problem has been already solved by Obukhov [4]. Studying the PGT field equations for torsion-free configurations, he proved the following important theorem:

T1. In the absence of matter, any solution of GR_Λ is a torsion-free solution of PGT.

It is interesting to note that the inverse statement, that any torsion-free vacuum solution of PGT is also a vacuum solution of GR_Λ, is also true, except for three specific choices of the PGT coupling constants.

III. pp_Λ WAVES WITH TORSION

In this section, we extend the pp_Λ waves that are vacuum solutions of GR_Λ to a new family of the exact vacuum solutions of PGT, characterized by the existence of torsion.

A. Ansatz

The main step in constructing the pp_Λ waves with torsion is to find an ansatz for torsion that is compatible with the wave nature of the solutions. This is achieved by introducing torsion at the level of connection.

Looking at the Riemannian connection (2.3), one can notice that its radiation piece appears only in the ω^{1c} components:

$$(\omega^{1c})^R = \frac{q^2}{p^2}(h^{c\alpha}\partial_\alpha S)b^0.$$

This motivates us to construct a new connection by applying the rule

$$\partial_\alpha S \to \partial_\alpha S + K_\alpha, \qquad K_\alpha = K_\alpha(u, y, z), \tag{3.1a}$$

where K_α is the component of the 1-form $K = K_\alpha dx^\alpha$ on the wave surface. Thus, the new form of $(\omega^{ij})^R$ reads

$$(\omega^{ic})^R := k^i\frac{q^2}{p^2}h^{c\alpha}(\partial_\alpha S + K_\alpha)b^0, \tag{3.1b}$$

whereas all the other nonradiation pieces retain their Riemannian form (2.3).

The geometric content of the new connection is found by calculating the torsion:

$$T^i = \nabla b^i + \omega^i{}_m b^m = k^i \frac{q^2}{p} b^0 (b^2 K_y + b^3 K_z) = k^i \frac{q^2}{p^2} b^0 b^c K_c. \tag{3.2}$$

The only nonvanishing irreducible piece of T^i is ${}^{(1)}T^i$.

The new connection modifies also the curvature, so that its radiation piece becomes

$$(R^{1c})^R = k^1 b^0 \Omega^c, \qquad \Omega^c \coloneqq Q^c + \Theta^c, \tag{3.3a}$$

where the term Θ^c that represents the contribution of torsion is given by

$$\Theta^2 = \frac{q}{2p}[(2qp\partial_y K_y - pK_y\lambda y - qK_z\lambda z)b^2 + (-2qp\partial_z K_y + pK_y\lambda z - qK_z\lambda y)b^3],$$
$$\Theta^3 = \frac{q}{2p}[(2qp\partial_z K_z - pK_z\lambda z - qK_y\lambda y)b^3 + (-2qp\partial_y K_z + pK_z\lambda y - qK_y\lambda z)b^2].$$

The covariant form of the curvature reads

$$R^{ij} = -\lambda b^i b^j + 2b^0 k^{[i}\Omega^{j]}, \tag{3.3b}$$

and the Ricci curvature takes the form

$$Ric^i = -3\lambda b^i + b^0 k^i \Omega, \qquad \Omega \coloneqq h_c \rfloor \Omega^c = Q + \Theta. \tag{3.3c}$$

The torsion has no influence on the scalar curvature:

$$R = -12\lambda. \tag{3.3d}$$

Thus, our ansatz defines a RC geometry of spacetime.

B. PGT field equations

Having adopted the ansatz for torsion defined in Eq. (3.1), we now wish to find explicit form of the PGT field equations and use them to determine dynamical content of our ansatz.

As shown in Appendices B and C, the field equations depend on the structure of the irreducible components of the field strengths. For torsion, we already know that the only nonvanishing irreducible component is ${}^{(1)}T_i = T_i$, defined in Eq. (3.2). As for the curvature, we note that our ansatz yields $X = 0$ and $b^m Ric_m = 0$, where X is defined in (B2b). Then, the irreducible decomposition of the curvature implies (see Appendix B)

$${}^{(3)}R_{ij} = 0, \qquad {}^{(5)}R_{ij} = 0, \tag{3.4}$$

whereas the remaining pieces ${}^{(n)}R^{ij}$ are defined by their nonvanishing components as

$${}^{(2)}R^{1c} = \frac{1}{2}{}^\star(\Psi^1 b^c), \qquad {}^{(4)}R^{1c} = \frac{1}{2}(\Phi^1 b^c),$$
$${}^{(6)}R^{ij} = -\lambda b^i b^j, \qquad {}^{(1)}R^{1c} = b^0\left(\Omega^{(ce)} - \frac{1}{2}\eta^{ce}\Omega\right)b_e, \tag{3.5a}$$

where the 1-forms Φ^i and Ψ^i are given by

$$\Phi^i = k^i b^0 (Q + \Theta), \quad \Theta = qp\left[\partial_y\left(\frac{q}{p}K_y\right) + \partial_z\left(\frac{q}{p}K_z\right)\right],$$
$$\Psi^i = X^i = -k^i b^0 \Sigma, \quad \Sigma = qp\left[\partial_z\left(\frac{q}{p}K_y\right) - \partial_y\left(\frac{q}{p}K_z\right)\right]. \tag{3.5b}$$

Having found ${}^{(1)}T_i$ and ${}^{(n)}R_{ij}$, we apply the procedure described in Appendix C to obtain the following form of the two PGT field Eqs. (C3):

$$\text{(1ST)} \quad \Lambda_0 = 3a_0\lambda, \qquad a_1\Theta - A_0(Q + \Theta) = 0, \tag{3.6a}$$

$$\text{(2ND)} \quad -(b_2 + b_1)(\nabla\Psi^1)b^2 - (b_4 + b_1)(\nabla\Phi^1)b^3 - 2(a_0 - A_1)T^1 b^3 = 0,$$
$$-(b_2 + b_1)(\nabla\Psi^1)b^3 + (b_4 + b_1)(\nabla\Phi^1)b^2 + 2(a_0 - A_1)T^1 b^2 = 0, \tag{3.6b}$$

where $A_0 = a_0 + (b_4 + b_6)\lambda$ and $A_1 = a_1 - (b_6 - b_1)\lambda$ [16].

Leaving (1ST) as is, (2ND) can be given a more clear structure as follows:

(i) use (1ST) to express $\Phi^1 = b^0(Q + \Theta)$ in the form $\Phi^1 = (a_1/A_0)b^0\Theta$;

(ii) multiply (2ND) by A_0/q.

As a result, the previous two components of (2ND) transform into

$$A_0(b_2 + b_1)\partial_z(p\Sigma/q) + a_1(b_4 + b_1)\partial_y(p\Theta/q) + 2A_0(A_1 - a_0)(q/p)K_y = 0, \tag{3.7a}$$

$$-A_0(b_2 + b_1)\partial_y(p\Sigma/q) + a_1(b_4 + b_1)\partial_z(p\Theta/q) + 2A_0(A_1 - a_0)(q/p)K_z = 0. \tag{3.7b}$$

Then, calculating $\partial_y(3.7a) + \partial_z(3.7b)$ and $\partial_z(3.7a) - \partial_y(3.7b)$ yields the final form of (2ND):

$$(\partial_{yy} + \partial_{zz})(p\Theta/q) - m^2_{2^+}\frac{1}{p^2}(p\Theta/q) = 0,$$
$$m^2_{2^+} := \frac{2A_0(a_0 - A_1)}{a_1(b_1 + b_4)}, \quad (3.8a)$$

$$(\partial_{yy} + \partial_{zz})(p\Sigma/q) - m^2_{2^-}\frac{1}{p^2}(p\Sigma/q) = 0,$$
$$m^2_{2^-} := \frac{2(a_0 - A_1)}{b_1 + b_2}. \quad (3.8b)$$

The parameters $m^2_{2^\pm}$ have a simple physical interpretation. In the limit $\lambda \to 0$, they represent masses of the spin-$2^\pm$ torsion modes with respect to the M_4 background [17],

$$\bar{m}^2_{2^+} = \frac{2a_0(a_0 - a_1)}{a_1(b_1 + b_4)}, \qquad \bar{m}^2_{2^-} = \frac{2(a_0 - a_1)}{b_1 + b_2},$$

whereas for finite λ, $m^2_{2^\pm}$ are associated to the torsion modes with respect to the (anti)de Sitter [(A)dS] background.

In M_4, the physical torsion modes are required to satisfy the conditions of no ghosts (positive energy) and no tachyons (positive m^2) [17,18]. However, for spin-2^+ and spin-2^- modes, the requirements for the absence of ghosts, given by the conditions $b_1 + b_2 < 0$ and $b_1 + b_4 > 0$, do not allow for both m^2 to be positive. Hence, only one of the two modes can exist as a propagating mode (with finite mass), whereas the other one must be "frozen" (infinite mass). Although these conditions refer to the M_4 background, we assume their validity also for the (A)dS background, in order to have a smooth limit when $\lambda \to 0$.

One should note that the two spin-2 sectors have quite different dynamical structures.

(i) In the spin-2^- sector, the infinite mass of the spin-2^+ mode implies $\Theta = 0$, whereupon (1ST) yields $Q = 0$, which is exactly the GR_Λ field equation for metric. Thus, the existence of torsion has no influence on the metric.

(ii) In the spin-2^+ sector, the infinite mass of the spin-2^- mode implies $\Sigma = 0$, whereas (1ST) yields that Q is proportional to Θ, with $\Theta \neq 0$. Thus, the torsion function Θ has a decisive dynamical influence on the form of the metric.

In the next section, we focus our attention on the spin-2^+ sector, where the metric appears to be a genuine dynamical effect of PGT.

IV. SOLUTIONS IN THE SPIN-2^+ SECTOR

In this section, we first find solutions of Eq. (3.8a) for the spin-2^+ mode $V = (p/q)\Theta$, and then use that V to find the metric function H and the torsion functions K_α, the quantities that completely define the geometry of the pp_Λ waves with torsion.

A. Solutions for $V = (p/q)\Theta$

The field equation for the spin-2^+ sector can be written in a slightly simpler form as

$$(\partial_{yy} + \partial_{zz})V - \frac{m^2}{p^2}V = 0, \quad (4.1)$$

where $V = (p/q)\Theta$ and $m^2 = m^2_{2^+}$. We have seen in Appendix A that local coordinates (y, z) are well defined in the region where p and q do not vanish, which is an open disk of finite radius, $y^2 + z^2 < 4|\lambda|^{-1}$. Since (4.1) is a differential equation with circular symmetry, it is convenient to introduce polar coordinates, $y = \rho\cos\varphi$, $z = \rho\sin\varphi$, in which Eq. (4.1) takes the form

$$\left(\frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2}{\partial\varphi^2}\right)V - \frac{m^2}{p^2}V = 0. \quad (4.2a)$$

Looking for a solution of V in the form of a Fourier expansion,

$$V = \sum_{n=0}^{\infty} V_n(\rho)(c_n e^{in\varphi} + \bar{c}_n e^{-in\varphi}),$$

we obtain

$$V''_n + \frac{1}{\rho}V'_n - \left(\frac{n^2}{\rho^2} + \frac{m^2}{p^2}\right)V_n = 0, \quad (4.2b)$$

where prime denotes $d/d\rho$.

1. $\lambda/4 \equiv \ell^{-2} > 0$

Let us first consider solutions of the dS type, using a convenient notation:

$$x = \frac{\rho}{\ell}, \qquad \mu = m\ell, \qquad \xi = \frac{1}{2}\left(1 + \sqrt{1 - \mu^2}\right).$$

The general solutions of (4.2b) for $n = 0$ and $n > 0$ are given by

$$V_0 = c_1(1 + x^2)^{1-\xi}{}_2F_1(1 - \xi, 1 - \xi; 2(1 - \xi); -|1 + x^2|) + c_2(1 + x^2)^{\xi}{}_2F_1(\xi, \xi; 2\xi; -|1 + x^2|), \quad (4.3a)$$

$$V_n = c_1(x^2)^{n/2}(1 + x^2)^{\xi}{}_2F_1(\xi, \xi + n; 1 + n, -x^2) + c_2(x^2)^{-n/2}(1 + x^2)^{\xi}{}_2F_1(\xi, \xi - n; 1 - n, -x^2), \quad (4.3b)$$

where $c_n = c_n(u) (n = 1, 2)$ and ${}_2F_1(a, b, c, z)$ is the hypergeometric function [19].

2. $\lambda/4 \equiv -\ell^{-2} < 2$

In the AdS sector, using

$$\bar{\xi} = \frac{1}{2}\left(1 + \sqrt{1+\mu^2}\right),$$

the solutions for $n = 0$ and $n > 0$ take the following forms:

$$V_0 = c_1(1-x^2)^{1-\bar{\xi}}{}_2F_1(1-\bar{\xi}, 1-\bar{\xi}; 2(1-\bar{\xi}); |1-x^2|) + c_2(1-x^2)^{\bar{\xi}}{}_2F_1(\bar{\xi}, \bar{\xi}; 2\bar{\xi}; |1-x^2|), \tag{4.4a}$$

$$V_n = c_1(x^2)^{n/2}(x^2-1)^{\bar{\xi}}{}_2F_1(\bar{\xi}, \bar{\xi}+n; 1+n, x^2) + c_2(x^2)^{-n/2}(x^2-1)^{\bar{\xi}}{}_2F_1(\bar{\xi}, \bar{\xi}-n; 1-n, x^2). \tag{4.4b}$$

These solutions are essentially an analytic continuation by $\ell \to i\ell$ of those in Eq. (4.3).

3. $\lambda = 0$

The general solution of Eq. (4.2b) has the form

$$V_n = c_1 J_n(-im\rho) + c_2 Y_n(-im\rho), \qquad n = 0, 1, 2, \ldots \tag{4.5}$$

where J_n and Y_n are Bessel functions of the first and second kind, respectively.

B. Solutions for the metric function H

For a given Θ, the first PGT field equation $A_0 Q = (a_1 - A_0)\Theta$, with Q defined in (2.5), represents a differential equation for the metric function H:

$$(\partial_{yy} + \partial_{zz})H + \frac{2\lambda}{p^2}H = \frac{2(a_1 - A_0)}{A_0}\frac{1}{p^2}V. \tag{4.6}$$

This is a second order, linear nonhomogeneous differential equation, and its general solution can be written as

$$H = H^h + H^p,$$

where H^h is the general solution of the homogeneous equation, and H^p a particular solution of (4.6). By comparing Eq. (4.6) to Eq. (4.1), one finds a simple particular solution for H:

$$H^p = \sigma V, \qquad \sigma = \frac{2(a_1 - A_0)}{(2\lambda + m^2)A_0}. \tag{4.7a}$$

On the other hand, H^h coincides with the general vacuum solution of GR_Λ; see (2.8). Since our idea is to focus on the genuine torsion effect on the metric, we choose $H^h = 0$ and adopt H^p as the most interesting PGT solution for the metric function H. Thus, we have

$$H_n = \sigma V_n. \tag{4.7b}$$

C. Solutions for the torsion functions K_α

In the spin-2^+ sector, the torsion functions K_α can be determined from Eq. (3.7), combined with the condition $\Sigma = 0$:

$$\partial_y V + m^2 \frac{q}{p} K_y = 0, \qquad \partial_z V + m^2 \frac{q}{p} K_z = 0. \tag{4.8}$$

Going over to polar coordinates,

$$K_y = K_\rho \cos\varphi - \frac{K_\varphi}{\rho}\sin\varphi, \qquad K_z = K_\rho \sin\varphi + \frac{K_\varphi}{\rho}\cos\varphi,$$

the previous equations are transformed into

$$K_\rho = -\frac{1}{m^2}\frac{p}{q}\partial_\rho V, \qquad K_\varphi = -\frac{1}{m^2}\frac{p}{q}\partial_\varphi V, \tag{4.9a}$$

or equivalently, in terms of the Fourier modes,

$$K_{\rho n} = -\frac{1}{m^2}\frac{p}{q}\partial_\rho V_n, \qquad K_{\varphi n} = -\frac{1}{m^2}\frac{p}{q} n V_n, \tag{4.9b}$$

where $K_\varphi = \sum_{n=1}^{\infty}(d_n e^{in\varphi} + \bar{d}_n e^{-in\varphi})$ with $d_n = -ic_n$, and similarly for K_ρ.

D. Graphical illustrations

Here, we illustrate graphical forms of two specific solutions by giving plots of their metric functions H and the typical torsion component $T^1{}_{02}$,

$$H = \sigma V,$$

$$T^1{}_{02} = \frac{q^2}{p^2}K_2 = \frac{q^2}{p}K_y = -\frac{1}{m^2}q(\partial_\rho V \cos\varphi - \rho^{-1}K_\varphi \sin\varphi). \tag{4.10}$$

For $\lambda \neq 0$, it is convenient to use the units in which $\ell = 1$.

In the dS sector (Fig. 1), the zero modes of both H and $T^1{}_{02}(\varphi = 0)$ are regular functions with a clear-cut wavelike behavior in the region $0 < x < 1$. The plots correspond to the pp_Λ geometry for fixed u, and as u increases, the

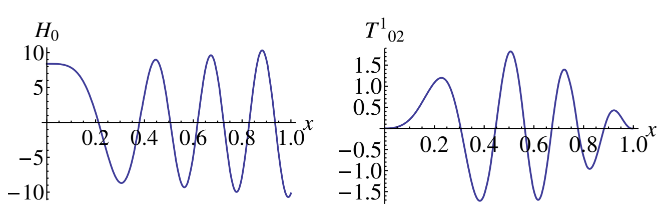

FIG. 1. The plots of a solution in the sector $\lambda > 0$, in units $\ell = 1$, for $n = 0, \mu = 100, c_1 = 1, c_2 = 0, \sigma = 1$. Left: H_0. Right: $T^1{}_{02}(\varphi = 0)$.

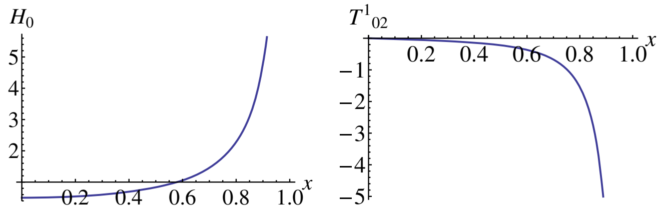

FIG. 2. The plots of a solution in the sector $\lambda < 0$, in units $\ell = 1$, for $n = 0, \mu = \sqrt{8}, c_1 = 0.1, c_2 = 0$. Left: H_0. Right: $T^1{}_{02}(\varphi = 0)$.

pictures change. In the AdS sector (Fig. 2), the solution is singular at $x = 1$, or equivalently at $p = 0$, and it does not have a typical wavelike shape. For a discussion of the singularity at $p = 0$, see Ref. [11]. We also examined a zero mode solution ($n = 0$) in the M_4 sector ($\lambda = 0$); its shape is similar to what we have in Fig. 2, but it remains finite at $x = 1$.

V. SOLUTIONS IN THE SPIN-2^- SECTOR

As we noted at the end of Sec. III, the spin-2^- sector is characterized by $\Theta = 0$ and, as a consequence of (1ST), by $Q = 0$. Equation (3.8b) for Σ reads

$$(\partial_{yy} + \partial_{zz})U - \frac{m^2}{p^2}U = 0, \tag{5.1}$$

where $U = (p/q)\Sigma$ and $m^2 = m^2_{2^-}$. Clearly, the solutions for U coincide with the solutions for $V = (p/q)\Theta$ in Sec. IV A. Furthermore, the metric function H, defined by $Q = 0$, has the GR_Λ form, and the solutions for the torsion functions K_α follow from the two equations

$$\partial_y U + m^2 \frac{q}{p} K_y = 0, \qquad \partial_z U + m^2 \frac{q}{p} K_z = 0, \tag{5.2}$$

the counterparts of those in (4.8).

The fact that the metric of the spin-2^- sector is independent of torsion makes this sector, in general, much less interesting. There is, however, one solution in this sector that should be mentioned: it is the solution with $H = 0$ for which the metric takes the (A)dS$/M_4$ form, and the complete dynamics is carried solely by the torsion. We skip discussing details of this case, as they can be easily reconstructed from the results given in the previous section, following the procedure outlined above.

VI. CONCLUDING REMARKS

In this paper, we found a new family of the exact vacuum solutions of PGT, the family of the pp_Λ waves with torsion. Here, we wish to clarify a few issues that have not been properly covered in the main text.

The essential step in our construction is the ansatz for the RC connection (3.1), which modifies only the radiation piece of the corresponding Riemannian connection (2.3). A characteristic feature of the resulting solution is the presence of the null vector $k^i = (0, 1, 0, 0)$ in the spacetime geometry. The vector field $k^i\partial_i = (p/q)^2\partial_v$ is orthogonal to the spatial surfaces $u = \text{const}$, and is interpreted as the propagation vector of the pp_Λ wave with torsion. Is such an interpretation justifiable?

Although gravitational waves belong to one of the best known families of exact solutions in GR_Λ, a unique covariant criterion for their precise identification is still missing. One of the early criteria of this type was formulated by Lichnerowicz, based on an analogy with methods used to determine electromagnetic radiation; see Zakharov [7]. This criterion can be formulated as a requirement that the radiation piece of the curvature, $S^{ij} = R^{ij} + \lambda b^i b^j$, satisfies the radiation conditions:

$$k^i S_{ij} = 0, \qquad \varepsilon^{ijkn} k_j S_{kn} = 0. \tag{6.1a}$$

However, when applied to a RC geometry, the Lichnerowicz criterion can be naturally extended to include the torsion 2-form:

$$k^i T_i = 0, \qquad \varepsilon^{ijmn} k_m T_n = 0. \tag{6.1b}$$

A direct calculation based on the expressions (3.2) and (3.3b) shows that both sets of the radiation conditions are satisfied. This result gives a strong support to interpreting the pp_Λ waves with torsion as proper wave solutions of PGT.

Looking at the explicit solutions for the pp_Λ waves with torsion, one should note that, in general, the hypergeometric function ${}_2F_1(a, b, c, x)$ is singular at $x = 1$ ($\rho = \ell$) [19]; moreover, local coordinates we are using are singular at both $x = 1$ and $x = 0$ (Appendix A). To test the nature of these singularities, we calculated the following torsion and curvature invariants:

$$\begin{aligned}
&T^i \wedge {}^\star T_i = 0, \\
&R = -12\lambda, \qquad R^{ij\star}R_{i,j} = 12\lambda^2\hat{\varepsilon}, \\
&R^{ij}{}_{kl}R^{kl}{}_{mn}R^{mn}{}_{ij} = -48\lambda^3,
\end{aligned} \tag{6.2}$$

the fourth order invariant is $96\lambda^4$, and so on. All these invariants are well behaved at $x = 1$, 0, which might be a signal that the singularities in question are just the coordinate singularities. However, according to Wald [20], the geometric singularities are not always visible in the field strength invariants. This issue deserves further clarification.

If the curvature R^{ij} is replaced by its radiation piece S^{ij}, all the invariants in (6.2) are found to vanish. According to Bell's second criterion [7], we have here another result that supports the wave interpretation of our pp_Λ solutions.

In GR_Λ, the pp_Λ waves are algebraically special solutions of Petrov type N; this property can be formulated as an algebraic condition on the Weyl curvature: $W_{ijmn}k^m = 0$ [9,21]. However, one cannot use the same criterion for classifying the solutions of PGT, since W_{ijmn} is not an irreducible part of the RC curvature. The problem can be overcome by replacing W_{ijmn} with ${}^{(1)}R_{ijmn}$, which is a genuine PGT generalization of W_{ijmn} [4]. Using the expression for ${}^{(1)}R_{ijmn}$ from Eq. (3.5), one can directly prove the relation

$$ {}^{(1)}R_{ijmn}k^m = 0, \tag{6.3} $$

which is a natural PGT generalization of the Riemannian condition. The condition (6.3) can be considered as a well-founded criterion for a family of PGT solutions to be of type N.

Finally, we wish to stress that a subfamily of the solutions in the spin-2^+ sector reveals an unexpected dynamical aspect of torsion. Namely, although torsion is introduced by a minor modification of the Riemannian connection [see (3.1)], the metric function H in (4.7) is determined solely by the torsion, and consequently, the related metric is a genuine dynamical effect of PGT. More detailed information could be obtained by analyzing the motion of test particles/fields in the RC spacetimes associated to the pp_Λ waves with torsion.

ACKNOWLEDGMENTS

One of us (M. B.) would like to thank Yuri Obukhov for interesting comments and information on his recent work on the subject [22]. This work was partially supported by the Serbian Science Foundation under Grant No. 171031.

APPENDIX A: ON HYPERBOLIC GEOMETRIES

(A)dS space can be simply represented as a 4D hyperboloid H_4 embedded in a 5D Minkowski space M_5 with metric $\eta_{MN} = (+,-,-,-,\sigma)$,

$$ H_4\colon\ X_0^2 - X_1^2 - X_2^2 - X_3^2 - \sigma X_5^2 = -\sigma\ell^2, \tag{A1a} $$

where $\sigma = +1$ for a dS space and $\sigma = -1$ for an AdS space [9,23]. The metric on H_4 reads

$$ ds^2 = dX_0^2 - dX_1^2 - dX_2^2 - dX_3^2 - \sigma dX_5^2, \tag{A1b} $$

and its scalar curvature is $R = -12\sigma/\ell^2$. The group of isometries of the dS/AdS spaces is $SO(1,4)/SO(2,3)$, and the corresponding topologies are $R \times S^3$ for the dS space, and $S^1 \times R^3$ for the AdS space (or R^4 for its universal covering).

Going now back to the generalized pp wave metric (2.1), we note that in the limit $H = 0$, it describes the background (A)dS geometry:

$$ ds^2 = 2\left(\frac{q}{p}\right)^2 du(-2\Lambda v^2 du + dv) - \frac{1}{p^2}(dy^2 + dz^2), $$
$$ p = 1 + \Lambda(y^2 + z^2), \qquad q = 1 - \Lambda(y^2 + z^2). \tag{A2} $$

As we shall see below, Λ is related to ℓ by $4\sigma\Lambda = 1/\ell^2$; moreover, $\Lambda > 0$ for dS and $\Lambda < 0$ for AdS. The two forms of the metric associated to the hyperboloid H_4 are related to each other by a coordinate transformation [11],

$$ \begin{aligned} X_0 &= \frac{q}{2p}(u + v + \Lambda u^2 v), & u &= 2\sigma\ell\frac{X_5 - \sqrt{-\sigma(X_0^2 - X_1^2 - \sigma X_5^2)}}{X_0 - X_1}, \\ X_1 &= \frac{q}{2p}(u - v + \Lambda u^2 v), & v &= \frac{X_0 - X_1}{4\ell\sqrt{-\sigma(X_0^2 - X_1^2 - \sigma X_5^2)}}, \\ X_2 &= \frac{y}{p}, \quad X_3 = \frac{z}{p}, & y &= \frac{2\ell X_2}{\ell + \sqrt{\ell^2 - \sigma(X_2^2 + X_3^2)}}, \\ X_5 &= \frac{1}{2\sqrt{\sigma\Lambda}}\frac{q}{p}(1 + 2\Lambda uv), & z &= \frac{2\ell X_3}{\ell + \sqrt{\ell^2 - \sigma(X_2^2 + X_3^2)}}. \end{aligned} \tag{A3} $$

Indeed, the coordinates X_M in M_4 describe the hyperboloid H_4,

$$ (X_0^2 - X_1^2 - \sigma X_5^2) - X_2^2 - X_3^2 = -\frac{1}{4\Lambda}\frac{q^2}{p^2} - \frac{1}{p^2}(y^2 + z^2) = -\frac{1}{4\Lambda} = -\sigma\ell^2, $$

and the corresponding metric (A1b), followed by the rescaling $v \to 2v$, coincides with (A2).

Since local coordinates $x^\mu = (u, v, x, y)$ are introduced by the parametrization (A3), they are well defined for

$$X_0^2 - X_1^2 - \sigma X_5^2 = -\frac{1}{4\Lambda}\frac{q^2}{p^2} > 0.$$

The limiting value $q = 0$ is not allowed, as it represents the singularity of the local coordinate system (u, v, y, z); this singularity is visible only for $\Lambda > 0$. The same conclusion follows from the fact that the determinant of the metric (A2) vanishes for $q = 0$. Furthermore, an inspection of Eq. (A3) reveals the existence of another singularity, located at $p = 0$; it is visible only for $\Lambda < 0$. Thus, local coordinates (u, v, y, z) are restricted to the region where q and/or p do not vanish: $y^2 + z^2 \leq |\Lambda|^{-1}$. More on the geometric interpretation of these singularities can be found in Ref. [11].

APPENDIX B: IRREDUCIBLE DECOMPOSITION OF THE FIELD STRENGTHS

We present here formulas for the irreducible decomposition of the PGT field strengths in a 4D Riemann-Cartan spacetime [4,24].

The torsion 2-form has three irreducible pieces:

$$\begin{aligned} {}^{(2)}T^i &= \frac{1}{3} b^i \wedge (h_m \rfloor T^m), \\ {}^{(3)}T^i &= \frac{1}{3} h^i \rfloor (T^m \wedge b_m), \\ {}^{(1)}T^i &= T^i - {}^{(2)}T^i - {}^{(3)}T^i. \end{aligned} \tag{B1}$$

The RC curvature 2-form can be decomposed into six irreducible pieces:

$$\begin{aligned} &{}^{(2)}R^{ij} = {}^*(b^{[i} \wedge \Psi^{j]}), && {}^{(4)}R^{ij} = b^{[i} \wedge \Phi^{j]}, \\ &{}^{(3)}R^{ij} = \frac{1}{12} X^*(b^i \wedge b^j), && {}^{(6)}R^{ij} = \frac{1}{12} F b^i \wedge b^j, \\ &{}^{(5)}R^{ij} = \frac{1}{2} b^{[i} \wedge h^{j]} \rfloor (b^m \wedge F_m), && {}^{(1)}R^{ij} = R^{ij} - \sum_{a=2}^{6} {}^{(a)}R^{ij}, \end{aligned} \tag{B2a}$$

where

$$\begin{aligned} &F^i := h_m \rfloor R^{mi} = Ric^i, && F := h_i \rfloor F^i = R, \\ &X^i := {}^*(R^{ik} \wedge b_k), && X := h_i \rfloor X^i, \end{aligned} \tag{B2b}$$

and

$$\begin{aligned} \Phi_i &:= F_i - \frac{1}{4} b_i F - \frac{1}{2} h_i \rfloor (b^m \wedge F_m), \\ \Psi_i &:= X_i - \frac{1}{4} b_i X - \frac{1}{2} h_i \rfloor (b^m \wedge X_m). \end{aligned} \tag{B2c}$$

The above formulas differ from those in Refs. [4,24] in two minor details: the definitions of F^i and X^i are taken with an additional minus sign, but at the same time, the overall signs of all the irreducible curvature parts are also changed.

APPENDIX C: CALCULATING THE PGT FIELD EQUATIONS

The gravitational dynamics of PGT is determined by a Lagrangian $L_G = L_G(b^i, T^i, R^{ij})$ (4-form), which is assumed to be at most quadratic in the field strengths (quadratic PGT) and parity invariant [24]. The form of L_G can be conveniently represented as

$$L_G = -{}^\star(a_0 R + 2\Lambda) + \frac{1}{2} T^i H_i + \frac{1}{4} R^{ij} H'_{ij}, \tag{C1}$$

where $H_i := \partial L_G / \partial T^i$ (the covariant momentum) and H'_{ij} define the quadratic terms in L_G:

$$H_i = 2 \sum_{n=1}^{3} {}^\star(a_n {}^{(n)}T_i), \qquad H'_{ij} := 2 \sum_{n=1}^{6} {}^\star(b_n {}^{(n)}R_{ij}). \tag{C2a}$$

Varying L_G with respect to b^i and ω^{ij} yields the PGT field equations in vacuum. After introducing the complete covariant momentum $H_{ij} := \partial L_G / \partial R^{ij}$ by

$$H_{ij} = -2a_0 {}^\star(b^i b^j) + H'_{ij}, \tag{C2b}$$

these equations can be written in a compact form as [4,24]

$$\begin{aligned} (1ST) \quad \nabla H_i + E_i &= 0, \\ (2ND) \quad \nabla H_{ij} + E_{ij} &= 0, \end{aligned} \tag{C3}$$

where E_i and E_{ij} are the gravitational energy-momentum and spin currents:

$$\begin{aligned} E_i &:= h_i \rfloor L_G - (h_i \rfloor T^m) H_m - \frac{1}{2} (h_i \rfloor R^{mn}) H_{mn}, \\ E_{ij} &:= -(b_i H_j - b_j H_i). \end{aligned} \tag{C4}$$

The above procedure is used in Sec. III B to find the explicit form of the PGT field equations for the pp_Λ waves with torsion, with the result displayed in Eqs. (3.6), (3.7), and (3.8). To simplify calculation of the term $\nabla^{\star(1)} R_{ij}$ in ∇H_{ij}, we used the identity

$$\frac{1}{2} \nabla^\star R_{ij} = \nabla^{\star(2)} R_{ij} + \nabla^{\star(4)} R_{ij}, \tag{C5}$$

that follows from the Bianchi identity $\nabla R^{ij} = 0$ and the double duality properties of the irreducible parts of the curvature.

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PHYSICAL REVIEW D **94,** 084037 (2016)

5D Lovelock gravity: New exact solutions with torsion

B. Cvetković* and D. Simić†

Institute of Physics, University of Belgrade Pregrevica 118, 11080 Belgrade, Serbia

(Received 5 September 2016; published 24 October 2016)

Five-dimensional Lovelock gravity is investigated in the first order formalism. A new class of exact solutions is constructed: the Bañados, Teitelboim, Zanelli black rings with and without torsion. We show that our solution with torsion exists in a different sector of the Lovelock gravity, as compared to the Lovelock Chern-Simons sector or the one investigated by Canfora *et al*. The conserved charges of the solutions are found using Nester's formula, and the results are confirmed by the canonical method. We show that the theory linearized around the background with torsion possesses two additional degrees of freedom with respect to general relativity.

DOI: 10.1103/PhysRevD.94.084037

I. INTRODUCTION

The general theory of relativity introduced a revolution in our understanding of space-time and gravity, the influence of which on modern physics can hardly be emphasized enough—almost all present investigations in high-energy physics are, in certain way, related to it. On one hand, the general theory of relativity has been very successful in explaining experimental results, but on the other, it produced a lot of problems for physicists to solve. The first of them is the problem of singularities, appearing quite often in gravitational solutions; there are theorems which show that singularities must appear under certain physically reasonable assumptions [1]. This situation inspired research in the direction of alternative theories of gravity, with an idea of finding a singularity free theory that reproduces experimental results equally as well as general relativity.

The second problem is quantization of tje general theory of relativity. The inability to quantize general relativity in a standard way, like Yang-Mills theories, motivated physicists to search for alternatives, on one side for a different quantization procedure (loop quantum gravity) and on the other for modifications of the original theory (extra dimensions, supersymmetry, string theory, alternative theories of gravity) [2–5]. In this paper, we shall focus on an alternative theory of gravity with one extra dimension—Lovelock gravity in five dimensions (5D).

Lovelock gravity is one of many generalizations of general relativity, physically appealing because of its similarity to the former. It possesses equations of motion which are the second order differential equations; it is ghost free; etc. But beyond this, most of its basic properties are not well known, and as the old saying says, "The devil is in the details." First, not many solutions are known, and those constructed usually are torsionless or belong to some special point in the parameter space [6–10]. Second, symmetries and local degrees of freedom of the theory are not known for the generic choice of parameters but only for the special case of Lovelock Chern-Simons gravity [11].

In this paper, we shall introduce new solutions with(out) torsion within Lovelock gravity in 5D by using the first order formulation. The most interesting of them are the Bañados, Teitelboim, Zanelli (BTZ) black rings with(out) torsion, the properties of which can be analyzed by using the canonical formalism. The canonical analysis is a powerful tool for studying gauge theories, but it is not limited solely to them. It gives a well-defined procedure for determining symmetries of a theory, construction of the symmetry generators, and for counting the number of local degrees of freedom. Applying the canonical analysis to a theory is extremely rewarding because of the already mentioned results it gives. Note, in particular, that the most reliable approach to conserved charges in gravity is based on the canonical analysis [12,13]. The main aspect of this approach consists in demanding the canonical generators to have well-defined functional derivatives. For a given asymptotic behavior of the fields, this condition usually requires the form of the generators to be improved by adding suitable surface terms.

The paper is organized as follows. Section II contains a short review of the Poincaré gauge theory of gravity and Lovelock gravity. Section III is devoted to the new solutions of 5D Lovelock gravity—the BTZ black rings with(out) torsion. The conserved charges for these solutions are computed by using Nester formula [14]. In Sec. IV, we construct the canonical generator of gauge transformations, local translations, and Lorentz rotations and compute the canonical conserved charges for the solutions constructed in Sec. III, confirming the results obtained in Sec. III. In Sec. V, we investigate the canonical structure of the theory linearized around the solution with torsion and conclude that in this sector the theory exhibits additional degrees of freedom.

Our conventions are given by the following rules: the Latin indices refer to the local Lorentz frame, and the Greek

*cbranislav@ipb.ac.rs
†dsimic@ipb.ac.rs

indices refer to the coordinate frame; the first letters of both alphabets $(a, b, c, \ldots; \alpha, \beta, \gamma, \ldots)$ run over $1, 2, \ldots D-1$, and the middle alphabet letters $(i, j, k, \ldots; \mu, \nu, \lambda, \ldots)$ run over $0, 1, 2, \ldots D-1$; the signature of space-time is $\eta = (+, -, \ldots, -)$; and the totally antisymmetric tensor $\varepsilon^{i_1 i_2 \ldots i_D}$ and the related tensor density $\varepsilon^{\mu_1 \mu_2 \ldots \mu_D}$ are both normalized so that $\varepsilon^{01\ldots D-1} = 1$. The symbol $\wedge$ of the exterior (wedge) product between forms is omitted for simplicity.

II. LOVELOCK GRAVITY

A. PGT in brief

The basic gravitational variables in poincaré gauge theory (PGT) are the vielbein e^i and the Lorentz connection $\omega^{ij} = -\omega^{ji}$ (1-forms). The field strengths corresponding to the gauge potentials e^i and ω^{ij} are the torsion T^i and the curvature R^{ij} (2-forms): $T^i = de^i + \omega^i{}_m \wedge e^m$ $R^{ij} = d\omega^{ij} + \omega^i{}_m \wedge \omega^{mj}$. Gauge symmetries of the theory are local translations and local Lorentz rotations, parametrized by ξ^μ and ε^{ij}.

In local coordinates x^μ, we can expand the vielbein and the connection 1-forms as $e^i = e^i{}_\mu dx^\mu$, $\omega^i = \omega^i{}_\mu dx^\mu$. Gauge transformation laws have the form

$$\begin{aligned} \delta_0 e^i{}_\mu &= \varepsilon^{ij} e_{j\mu} - (\partial_\mu \xi^\rho) e^i{}_\rho - \xi^\rho \partial_\rho e^i{}_\mu =: \delta_{\text{PGT}} e^i{}_\mu, \\ \delta_0 \omega^{ij}{}_\mu &= \nabla_\mu \varepsilon^{ij} - (\partial_\mu \xi^\rho) \omega^{ij}{}_\rho - \xi^\rho \partial_\rho \omega^{ij}{}_\mu =: \delta_{\text{PGT}} \omega^{ij}{}_\mu, \end{aligned} \tag{2.1}$$

and the field strengths are given as

$$\begin{aligned} T^i &= \nabla e^i \equiv de^i + \omega^{ij} \wedge e_j = \frac{1}{2} T^i{}_{\mu\nu} dx^\mu \wedge dx^\nu, \\ R^{ij} &= d\omega^{ij} + \omega^{ik} \wedge \omega_k{}^j = \frac{1}{2} R^{ij}{}_{\mu\nu} dx^\mu \wedge dx^\nu, \end{aligned} \tag{2.2}$$

where $\nabla = dx^\mu \nabla_\mu$ is the covariant derivative.

To clarify the geometric meaning of the above structure, we introduce the metric tensor as a specific, bilinear combination of the vielbeins,

$$g = \eta_{ij} e^i \otimes e^j = g_{\mu\nu} dx^\mu \otimes dx^\nu,$$
$$g_{\mu\nu} = \eta_{ij} e^i{}_\mu e^j{}_\nu, \quad \eta_{ij} = (+, -, -, -, -).$$

Although the metric and connection are in general independent dynamical/geometric variables, the antisymmetry of ω^{ij} in PGT is equivalent to the so-called *metricity condition*, $\nabla g = 0$. The geometry of which the connection is restricted by the metricity condition (metric-compatible connection) is called *Riemann-Cartan geometry*. Thus, PGT has the geometric structure of Riemann-Cartan space.

The connection ω^{ij} determines the parallel transport in the local Lorentz basis. Being a true geometric operation, parallel transport is independent of the basis. This property is incorporated into PGT via the so-called *vielbein postulate*, the vanishing of the total covariant derivative of $e^i{}_\mu$,

$$D_\mu(\omega + \Gamma) e^i{}_\nu := \partial_\mu e^i{}_\nu + \omega^{ij}{}_\mu e_{j\nu} - \Gamma^\rho{}_{\nu\mu} e^i{}_\rho = 0,$$

where $\Gamma^\rho{}_{\nu\mu}$ is the affine connection and the torsion is defined by $T^\rho{}_{\mu\nu} = \Gamma^\rho{}_{\nu\mu} - \Gamma^\rho{}_{\mu\nu}$. The previous relation implies the identity

$$\omega_{ijk} = \Delta_{ijk} + K_{ijk}, \tag{2.3}$$

where Δ is Riemannian (Levi-Civitá) connection and $K_{ijk} = -\frac{1}{2}(T_{ijk} - T_{kij} + T_{jki})$ is the contortion. Latin indices are changed into Greek and vice versa by means of vielbeins (and its inverse). Namely, $X^i = e^i{}_\mu X_\mu$ and $X^\mu = e_i{}^\mu X^i$. For details, see Ref. [13].

B. Lovelock action and equations of motion

Lovelock gravity can also be considered in the framework of PGT. Dimensionally continued Euler density L_p in D dimensions is defined as

$$L_p = \varepsilon_{i_1 i_2 \ldots i_D} R^{i_1 i_2} \ldots R^{i_{2p-1} i_{2p}} e^{i_{2p+1}} \ldots e^{i_D}, \tag{2.4}$$

where p is the number of curvature tensors in Euler density. In the previous relation, we omitted the wedge product for simplicity. The general form of the Lovelock gravity Lagrangian [15] in 5D is a linear combination of all dimensionally continued Euler densities in five dimensions,

$$I = \frac{\alpha_0}{5} I_0 + \frac{\alpha_1}{3} I_1 + \alpha_2 I_2, \tag{2.5a}$$

where

$$\begin{aligned} I_0 &= \int \varepsilon_{ijkln} e^i e^j e^k e^l e^n, \\ I_1 &= \int \varepsilon_{ijkln} R^{ij} e^k e^l e^n, \\ I_2 &= \int \varepsilon_{ijkln} R^{ij} R^{kl} e^n. \end{aligned} \tag{2.5b}$$

C. Field equations

Variation of the action with respect to vielbein e^i and connection ω^{ij} yields the gravitational field equations:

$$\varepsilon_{ijkln}(\alpha_0 e^j e^k e^l e^n + \alpha_1 R^{jk} e^l e^n + \alpha_2 R^{jk} R^{ln}) = 0, \tag{2.6a}$$

$$\varepsilon_{ijkln}(\alpha_1 e^k e^l + 2\alpha_2 R^{kl}) T^n = 0. \tag{2.6b}$$

Let us note that in the generic case the field equations (2.6) imply that torsion can be nonvanishing.

For later convenience, let us present the tensor form of the field equations,

$$\varepsilon^{\mu\nu\rho\sigma\tau}_{ijkln}\left(\alpha_0 e^j{}_\nu e^k{}_\rho e^l{}_\sigma e^n{}_\tau + \frac{1}{2}\alpha_1 R^{jk}{}_{\nu\rho} e^l{}_\sigma e^n{}_\tau + \frac{1}{4}\alpha_2 R^{jk}{}_{\nu\rho} R^{ln}{}_{\sigma\tau}\right) = 0, \tag{2.7a}$$

$$\varepsilon^{\mu\nu\rho\sigma\tau}_{ijkln}(\alpha_1 e^k{}_\nu e^l{}_\rho + \alpha_2 R^{kl}{}_{\nu\rho}) T^n{}_{\sigma\tau} = 0, \tag{2.7b}$$

where $\varepsilon^{\mu\nu\rho\sigma\tau}_{ijkln} \coloneqq \varepsilon^{\mu\nu\rho\sigma\tau}\varepsilon_{ijkln}$.

D. Consequences of field equations

If we take covariant derivative of (2.6a), make use of the Bianchi identities, and multiply (2.6b) with e^j, we get the following system:

$$\varepsilon_{ijkln}(2\alpha_0 e^j e^k e^l + \alpha_1 R^{jk} e^l) T^n = 0,$$
$$\varepsilon_{ijkln}(\alpha_1 e^j e^k e^l + 2\alpha_2 R^{jk} e^l) T^n = 0.$$

In the case $4\alpha_0\alpha_2 - \alpha_1^2 \neq 0$, the previous set of equations reduces to the following conditions,

$$v_i \coloneqq T^j{}_{ji} = 0, \tag{2.8a}$$

$$R^{jk}{}_{ir} T^r{}_{jk} - 2\mathrm{Ric}^j{}_k T^k{}_{ij} = 0, \tag{2.8b}$$

where $\mathrm{Ric}^j{}_k \coloneqq R^{jl}{}_{kl}$ is the Ricci tensor.

Therefore, in the generic case, torsion is *traceless*, and the second irreducible component of torsion ${}^{(2)}T_i$ vanishes. For details on irreducible decomposition of torsion and curvature in PGT, see Ref. [16]. Let us note that the condition $4\alpha_0\alpha_2 - \alpha_1^2 \neq 0$ is violated in the case of Lovelock Chern-Simons gravity.

E. Maximally symmetric solution

The field equation admits the existence of the maximally symmetric Riemannian solution (maximally symmetric Riemannian background) defined by

$$\bar{R}^{ij} = -\Lambda e^i e^j, \qquad \bar{T}^i = 0, \tag{2.9}$$

where Λ is the effective cosmological constant iff

$$\alpha_0 - \alpha_1\Lambda + \alpha_2\Lambda^2 = 0. \tag{2.10}$$

This equation can be solved for Λ:

$$\Lambda_\pm = \frac{\alpha_1 \pm \sqrt{\alpha_1^2 - 4\alpha_0\alpha_2}}{2\alpha_2}. \tag{2.11}$$

The solution is unique for $\alpha_1^2 - 4\alpha_0\alpha_2 = 0$, which is the case in Lovelock Chern-Simons gravity.

Let us note that in terms of $\Lambda_\pm$ equations of motion (2.6) take an elegant form:

$$\varepsilon_{ijkln}(R^{jk} + \Lambda_+ e^j e^k)(R^{ln} + \Lambda_- e^l e^n) = 0, \tag{2.12a}$$

$$\varepsilon_{ijkln}\left(R^{kl} + \frac{\Lambda_+ + \Lambda_-}{2} e^k e^l\right) T^n = 0. \tag{2.12b}$$

In obtaining these equations, we assumed that $\alpha_2 \neq 0$, and this condition will be used in the rest of the paper, because for $\alpha_2 = 0$ the theory reduces to general relativity.

III. NEW CLASS OF SOLUTIONS

The search for a new class of solutions is inspired by Canfora *et al.* [17], who found a solution of the type $AdS_2 \times S_3$ when the coupling constants satisfy the relation

$$\alpha_1^2 = 12\alpha_0\alpha_2, \tag{3.1}$$

which is different from the one satisfied in Lovelock Chern-Simons gravity. We shall now construct another class of solutions of the "complementary" type $\Sigma_3 \times \Gamma_2$, where Σ_3 and Γ_2 are three- and two-dimensional manifolds, determined by solving the equations of motion. We start from the following anzatz for curvature,

$$R^{ab} = Ae^a e^b,$$
$$R^{3a} = R^{4a} = 0,$$
$$R^{34} = Be^3 e^4, \tag{3.2}$$

and torsion,

$$T^a = p\varepsilon^{abc} e_b e_c,$$
$$T^3 = T^4 = 0. \tag{3.3}$$

In the anzatz, we used the notation $a, b, c, \ldots \in \{0, 1, 2\}$ and $\varepsilon^{abc} \coloneqq \varepsilon^{abc34}$, and A, B, and p are some functions restricted by the equations of motion. Note that torsion is totally antisymmetric, and thus only the third irreducible component ${}^{(3)}T^i$ is nonvanishing; see Ref. [16]. Let us now check whether the anzatz solves the equations of motion (2.12). From (2.12b), we obtain

$$\left(B + \frac{\Lambda_- + \Lambda_+}{2}\right) p = 0.$$

Thus, one can have a vanishing torsion for $p = 0$ or a nonvanishing torsion for

$$B = -\frac{\Lambda_- + \Lambda_+}{2}. \tag{3.4}$$

From (2.12a), we obtain

$$A(\Lambda_- + \Lambda_+) + 2\Lambda_-\Lambda_+ = 0, \tag{3.5}$$

$$4\Lambda_-\Lambda_+ + (A+B)(\Lambda_- + \Lambda_+) + 2AB = 0. \tag{3.6}$$

If $\Lambda_- + \Lambda_+ = 0$, which is equivalent to $\alpha_1 = 0$, Eq. (3.5) implies $\alpha_0 = 0$, whereas A remains undetermined; otherwise, for $\alpha_1 \neq 0$, we have

$$A = -\frac{2\Lambda_-\Lambda_+}{\Lambda_- + \Lambda +}. \tag{3.7}$$

Let us first analyze the case with nonvanishing torsion and $\alpha_1 \neq 0$, when A and B are both determined. By combining Eqs. (3.4), (3.5), and (3.6) and using Vieta's formulas, $\Lambda_- + \Lambda_+ = \frac{\alpha_1}{\alpha_2}$ and $\Lambda_-\Lambda_+ = \frac{\alpha_0}{\alpha_2}$, we obtain that the solution exists in the sector:

$$\alpha_1^2 = 8\alpha_0\alpha_2. \tag{3.8}$$

This sector is different from the one in Ref. [17], and the above solution is the first one in this sector. Using Eqs. (3.4), (3.10), and (3.8), we obtain

$$A = \frac{B}{2}. \tag{3.9}$$

Now, we turn to the solution with vanishing torsion and $\alpha_1 \neq 0$. In this case, A is determined, and B is arbitrary, which can be used to insure the validity of (3.6), which takes the form

$$2\frac{\alpha_0}{\alpha_2} + B\left(\frac{\alpha_1}{\alpha_2} - 4\frac{\alpha_0}{\alpha_1}\right) = 0. \tag{3.10}$$

We see that if $\alpha_1^2 - 4\alpha_0\alpha_2 = 0$, which is the Lovelock Chern-Simons gravity, for the validity of (3.10), one must have $\alpha_0 = 0$. These two conditions imply $\alpha_1 = 0$, which is in contradiction with our assumption; hence, the solution does not exist in the Lovelock Chern-Simons case. If $\alpha_1^2 - 4\alpha_0\alpha_2 \neq 0$ and $\alpha_1 \neq 0$ (recall that we are not interested in general relativity, so $\alpha_2 \neq 0$ also), we can choose any value of parameters obeying this conditions and get a solution. So, this class of solutions exists generically i.e. for almost any choice of parameters.

For clarity of the exposure, we devote next few sections to the most interesting solutions which belong to the class derived in this section.

A. BTZ black ring with torsion

For this case, the curvature takes the following form,

$$\begin{aligned} R^{ab} &= q e^a e^b, \\ R^{3a} &= R^{4a} = 0, \\ R^{34} &= -\frac{1}{r_0^2} e^3 e^4, \end{aligned} \tag{3.11}$$

while the torsion is given by

$$\begin{aligned} T^a &= p\varepsilon^{abc} e_b e_c, \\ T^3 &= T^4 = 0. \end{aligned} \tag{3.12}$$

The Bianchi identity implies that p is *constant*, and the Riemannian curvature reads

$$\begin{aligned} \tilde{R}^{ab} &= \left(q + \frac{p^2}{4}\right) e^a e^b, \\ \tilde{R}^{3a} &= \tilde{R}^{4a} = 0, \\ \tilde{R}^{34} &= -\frac{1}{r_0^2} e^3 e^4. \end{aligned} \tag{3.13}$$

Therefore, we can introduce the AdS_3 radius ℓ as

$$\frac{1}{\ell^2} := q + \frac{p^2}{4}.$$

Identity (3.9) implies the following relation:

$$\frac{1}{\ell^2} = -\frac{1}{2r_0^2} + \frac{p^2}{4}. \tag{3.14}$$

In the AdS_3 sector, the anzatz for curvature and torsion is solved by the AdS_3 solution with torsion as well as by the BTZ black hole [18] with torsion. In the latter, physically more appealing case, the 5D vielbein reads

$$\begin{aligned} &e^0 = N dt, \quad e^1 = N^{-1} dr, \quad e^2 = r(d\varphi + N_\varphi dt), \\ &e^3 = r_0 d\theta, \quad e^4 = r_0 \sin\theta d\chi, \end{aligned} \tag{3.15a}$$

where

$$N^2 = -2m + \frac{r^2}{\ell^2} + \frac{j^2}{r^2}, \qquad N_\varphi = \frac{j}{r^2},$$

where m and j are (dimensionless) parameters. The Cartan connection is given by

$$\begin{aligned} \omega^{ab} &= \tilde{\omega}^{ab} - \varepsilon^{abc}\frac{p}{2} e_c, \\ \tilde{\omega}^{01} &= -\frac{r}{\ell^2} dt - \frac{j}{r} d\varphi, \\ \tilde{\omega}^{12} &= N d\varphi, \\ \tilde{\omega}^{20} &= N^{-1}\frac{j}{r^2} dr, \\ \omega^{34} &= \tilde{\omega}^{34} = -\cos\theta d\chi, \end{aligned} \tag{3.15b}$$

where $\tilde{\omega}^{ij}$ is the Riemannian connection. Let us note that the coordinate ranges are

$$\begin{aligned} &-\infty < t < +\infty, \quad 0 \leq r < +\infty, \quad 0 \leq \varphi \leq 2\pi, \\ &0 \leq \theta \leq \pi, \quad 0 \leq \chi \leq 2\pi. \end{aligned}$$

1. Killing vectors

The maximal number of Killing vectors of the solution with field strengths (3.11), (3.12), and (3.13) is $9 = 6 + 3$, since the AdS_3 solution with(out) torsion has six Killing vectors; see Ref. [19]. The solution (3.15) has *five* Killing vectors, since the BTZ solution possesses two Killing vectors. They are given by

$$\xi^{(1)} = \ell \frac{\partial}{\partial t}, \qquad \xi^{(2)} = \frac{\partial}{\partial \varphi}, \qquad \xi^{(3)} = \frac{\partial}{\partial \chi},$$
$$\xi^{(4)} = \sin\chi \frac{\partial}{\partial \theta} + \cot\theta \cos\chi \frac{\partial}{\partial \chi},$$
$$\xi^{(5)} = \cos\chi \frac{\partial}{\partial \theta} - \cot\theta \sin\chi \frac{\partial}{\partial \chi}. \tag{3.16}$$

B. Riemannian BTZ ring

For this case, the curvature (Riemannian) takes the following form,

$$R^{ab} = \frac{1}{\ell^2} e^a e^b,$$
$$R^{3a} = R^{4a} = 0,$$
$$R^{34} = -\frac{1}{r_0^2} e^3 e^4, \tag{3.17}$$

while the torsion equals zero, $T^i = 0$.

Let us note that since torsion is zero there are no further constraints on B, so we can chose $B = -\frac{1}{r_0^2}$. In terms of the action constants, we get

$$\frac{1}{\ell^2} = -\frac{2\alpha_0}{\alpha_1}, \qquad \frac{1}{r_0^2} = \frac{2\alpha_0\alpha_1}{\alpha_1^2 - 4\alpha_0\alpha_2}. \tag{3.18}$$

The solution exists provided that $\alpha_0\alpha_1 < 0$ and $\alpha_1^2 - 4\alpha_0\alpha_2 < 0$. Let us note this solution does not solve equations of motion in Lovelock Chern-Simons gravity.

The vielbein fields and connection take the same form as in (3.15) with $p = 0$, while Killing vectors are identical and given by (3.16).

C. Conserved charges

In order to compute conserved charges, we shall make use of Nester formula. Let us denote the difference between any variable X and its reference value $\bar{X}$ by $\Delta X = X - \bar{X}$. In 5D, the boundary term B is a 3-form. With a suitable set of boundary conditions for the fields, the proper boundary term reads [14]

$$B = (\xi\rfloor b^i)\Delta\tau_i + \Delta b^i(\xi\rfloor\bar{\tau}_i) + \frac{1}{2}(\xi\rfloor\omega^i{}_j)\Delta\rho_i{}^j + \frac{1}{2}\Delta\omega^i{}_j(\xi\rfloor\bar{\rho}_i{}^j), \tag{3.19}$$

where ξ is an asymptotically Killing vector, while τ_i and ρ_{ij} are covariant momenta corresponding to torsion and curvature, respectively. The covariant momenta for the Lovelock action (2.5) are given by

$$\tau_i \coloneqq \frac{\partial L}{\partial T^i} = 0, \tag{3.20}$$

$$\rho_{ij} \coloneqq \frac{\partial L}{\partial R^{ij}} = 2\varepsilon_{ijkln}\left(\frac{\alpha_1}{3} e^k e^l + 2\alpha_2 R^{kl}\right)e^n. \tag{3.21}$$

Consequently, we obtain

$$\rho_{ab} = 4\varepsilon_{abc}\left(\alpha_1 - \frac{2\alpha_2}{r_0^2}\right)e^c e^3 e^4,$$
$$\rho_{a3} = 2\varepsilon_{abc}(\alpha_1 + 2\alpha_2 q)e^b e^c e^4 = \alpha_1\varepsilon_{abc}e^b e^c e^4,$$
$$\rho_{a4} = 2\varepsilon_{abc}(\alpha_1 + 2\alpha_2 q)e^b e^c e^3 = \alpha_1\varepsilon_{abc}e^b e^c e^3,$$
$$\rho_{34} = 2\varepsilon_{abc}\left(\frac{\alpha_1}{3} + 2\alpha_2 q\right)e^a e^b e^c = -\frac{\alpha_1}{3} e^a e^b e^c. \tag{3.22}$$

In our calculations of the boundary integrals, we use the coordinates $x^\mu = (t, r, \varphi, \theta, \chi)$. The background configuration is the one defined by zero values of parameters $m = 0$ and $j = 0$ of the solution (3.15). For the solutions with Killing vectors ∂_t and ∂_φ, the conserved charges are the energy and angular momentum, respectively,

$$E = \int_{\partial\Sigma} B(\partial_t) = \int_{\partial\Sigma} e^i{}_t\Delta\tau_i + \Delta e^i\bar{\tau}_{it} + \frac{1}{2}\omega^{ij}{}_t\Delta\rho_{ij} + \frac{1}{2}\Delta\omega^{ij}\bar{\rho}_{ijt}, \tag{3.23a}$$

$$J = \int_{\partial\Sigma} B(\partial_\varphi) = \int_{\partial\Sigma} e^i{}_\varphi\Delta\tau_i + \Delta e^i\bar{\tau}_{i\varphi} + \frac{1}{2}\omega^{ij}{}_\varphi\Delta\rho_i + \frac{1}{2}\Delta\omega^{ij}\bar{\rho}_{ij\varphi}, \tag{3.23b}$$

where $\partial\Sigma$ is a boundary $S^1 \times S^2$, located at infinity, described by coordinates φ, θ, χ.

Thus, conserved charges for the black ring with torsion and the Riemannian black ring are given by

$$E = 8\pi^2 r_0^2\left(\alpha_1 - \frac{2\alpha_2}{r_0^2}\right)m, \qquad J = 8\pi^2 r_0^2\left(\alpha_1 - \frac{2\alpha_2}{r_0^2}\right)j. \tag{3.24}$$

Let us note that the solution with torsion exists in the sector $\alpha_1^2 = 8\alpha_0\alpha_2$, where both conserved charges vanish.

IV. CANONICAL GAUGE GENERATOR

As an important step in our examination of the asymptotic structure of space-time, we are going to construct the canonical gauge generator, which is our basic tool for studying asymptotic symmetries and conserved charges of 5D Lovelock gravity.

A. Hamiltonian and constraints

The best way to understand the dynamical content of gauge symmetries is to explore the canonical generator, which acts on the basic dynamical variables via the Poisson bracket (PB) operation. To begin the canonical analysis, we rewrite the action (2.5) as

$$
\begin{aligned}
I &= \int d^5x\mathcal{L} \\
\mathcal{L} &= \varepsilon^{\mu\nu\rho\sigma\tau}_{ijkln}\int_{\mathcal{M}} d^5x\left(\frac{\alpha_0}{5}e^i{}_\mu e^j{}_\nu e^k{}_\rho e^l{}_\sigma + \frac{\alpha_1}{6}R^{ij}{}_{\mu\nu}e^k{}_\rho e^l{}_\sigma \right.\\
&\quad\left. + \frac{\alpha_2}{4}R^{ij}{}_{\mu\nu}R^{kl}{}_{\rho\sigma}\right)e^n{}_\tau. \qquad (4.1)
\end{aligned}
$$

1. Primary constraints and canonical Hamiltonian

The basic Lagrangian variables $(e^i{}_\mu, \omega^{ij}{}_\mu)$ and the corresponding canonical momenta $(\pi_i{}^\mu, \pi_{ij}{}^\mu)$ are related to each other through the set of primary constraints:

$$
\begin{aligned}
&\phi_i{}^0 \coloneqq \pi_i{}^0 \approx 0, \qquad \phi_{ij}{}^0 \coloneqq \pi_{ij}{}^0 \approx 0,\\
&\phi_i{}^\alpha \coloneqq \pi_i{}^\alpha \approx 0,\\
&\phi_{ij}{}^\alpha \coloneqq \pi_{ij}{}^\alpha - 2\varepsilon^{0\alpha\beta\gamma\delta}_{ijkln}\left(\frac{\alpha_1}{3}e^k{}_\beta e^l{}_\gamma + \alpha_2 R^{kl}{}_{\beta\gamma}\right)e^n{}_\delta \approx 0. \qquad (4.2)
\end{aligned}
$$

The algebra of primary constraints is displayed in the Appendix.

The canonical Hamiltonian is defined by

$$
\mathcal{H}_c = \pi_i{}^\mu \dot{e}^i{}_\mu + \frac{1}{2}\pi_{ij}{}^\mu\dot{\omega}^{ij}{}_\mu - \mathcal{L}.
$$

Since the Lagrangian is linear in velocities, the canonical Hamiltonian in the formula given above reduces to $\mathcal{H}_c = -\mathcal{L}(\dot{e}^i{}_\mu = 0, \dot{\omega}^{ij}{}_\mu = 0)$. It is linear in unphysical variables:

$$
\begin{aligned}
\mathcal{H}_c &= e^i{}_0\mathcal{H}_i + \frac{1}{2}\omega^{ij}{}_0\mathcal{H}_{ij} + \partial_\alpha D^\alpha,\\
\mathcal{H}_i &= -\varepsilon^{0\alpha\beta\gamma\delta}_{ijkln}\left(\alpha_0 e^j{}_\alpha e^k{}_\beta e^l{}_\gamma e^n{}_\delta + \frac{1}{2}\alpha_1 R^{jk}{}_{\alpha\beta}e^l{}_\gamma e^n{}_\delta \right.\\
&\quad\left. + \frac{1}{4}\alpha_2 R^{jk}{}_{\alpha\beta}R^{ln}{}_{\gamma\delta}\right),\\
\mathcal{H}_{ij} &= -\varepsilon^{0\alpha\beta\gamma\delta}_{ijkln}(\alpha_1 e^k{}_\alpha e^l{}_\beta + \alpha_2 R^{kl}{}_{\alpha\beta})T^n{}_{\gamma\delta},\\
D^\alpha &= \varepsilon^{0\alpha\beta\gamma\delta}_{ijkln}\omega^{ij}{}_0(\alpha_1 e^k{}_\beta e^l{}_\gamma + \alpha_2 R^{kl}{}_{\beta\gamma})e^n{}_\delta. \qquad (4.3)
\end{aligned}
$$

2. Secondary constraints

Going over to the total Hamiltonian,

$$
\mathcal{H}_T = \mathcal{H}_c + u^i{}_\mu\phi_i{}^\mu + \frac{1}{2}u^{ij}{}_\mu\phi_{ij}{}^\mu, \qquad (4.4)
$$

we find that the consistency conditions of the primary constraints $\pi_i{}^0$ and π^0_{ij} yield the secondary constraints:

$$
\mathcal{H}_i \approx 0, \qquad \mathcal{H}_{ij} \approx 0. \qquad (4.5)
$$

Let us note that these constraints reduce to the $\mu = 0$ components of the Lagrangian field equations (2.7).

The consistency of the remaining primary constraints $\phi_i{}^\alpha$ and $\phi_{ij}{}^\alpha$ leads to the relations for multipliers $u^i{}_\beta$ and $u^{ij}{}_\beta$,

$$
\begin{aligned}
&\varepsilon^{0\alpha\beta\gamma\delta}_{ijkln}[\underline{R}^{jk}{}_{0\beta}(\alpha_1 e^l{}_\gamma e^n{}_\delta + \alpha_2 R^{ln}{}_{\gamma\delta})\\
&\quad + (\alpha_1 R^{jk}{}_{\beta\gamma} + 4\alpha_0 e^j{}_\beta e^k{}_\gamma)e^l{}_0 e^n{}_\delta] = 0,\\
&\varepsilon^{0\alpha\beta\gamma\delta}_{ijkln}[\underline{T}^k{}_{0\beta}(\alpha_1 e^l{}_\gamma e^n{}_\delta + \alpha_2 R^{ln}{}_{\gamma\delta})\\
&\quad + \alpha_2\underline{R}^{kl}{}_{0\beta}T^n{}_{\gamma\delta} + \alpha_1 e^k{}_0 e^l{}_\beta T^n{}_{\gamma\delta}] = 0, \qquad (4.6)
\end{aligned}
$$

where $\underline{T}^i{}_{0\alpha} = T^i{}_{0\alpha}(\dot{e}^i{}_\alpha \to u^i{}_\alpha)$ and $\underline{R}^{ij}{}_{0\alpha} = R^{ij}{}_{0\alpha}(\dot{\omega}^{ij}{}_\alpha \to u^{ij}{}_\alpha)$. Using the Hamiltonian equations of motion $\dot{e}^i{}_\alpha = u^i{}_\beta$ and $\dot{\omega}^{ij}{}_\alpha = u^{ij}{}_\alpha$, these relations reduce to the $\mu = \alpha$ components of the Lagrangian field equations (2.7).

3. Further consistency procedure

Some of the relations (4.6) can be solved in terms of the multipliers $u^i{}_\alpha$ and $u^{ij}{}_\alpha$, while the others may lead to ternary constraints, the consistency of which has to be examined as well. However, this procedure is extremely sensitive to the particular sector of the theory as we shall illustrate in the next section (for the pure Lovelock theory, see Ref. [20]). The final form of the total Hamiltonian is given by

$$
\begin{aligned}
\mathcal{H}_T &= \bar{\mathcal{H}}_T + u^i{}_0\pi_i{}^0 + \frac{1}{2}u^{ij}{}_0\pi_{ij}{}^0 + (u\cdot\phi),\\
\bar{\mathcal{H}}_T &= e^i{}_0\bar{\mathcal{H}}_i + \frac{1}{2}\omega^{ij}{}_0\bar{\mathcal{H}}_{ij} + \partial_\alpha\bar{D}^\alpha,\\
\bar{\mathcal{H}}_i &= \mathcal{H}_i + (\bar{u}\cdot\phi),\\
\bar{\mathcal{H}}_{ij} &= \mathcal{H}_{ij} + (\bar{u}\cdot\phi),\\
\bar{D}^\alpha &= D^\alpha + (\bar{u}\cdot\phi), \qquad (4.7)
\end{aligned}
$$

where by $(u\cdot\phi)$ we denoted terms stemming form the *undetermined* multipliers and belonging to the set $(u^i{}_\beta, u^{ij}{}_\beta)$, and by $(\bar{u}\cdot\phi)$ we denoted terms stemming form the *determined* multipliers belonging to the same set.

B. Canonical generator and charges

The sure symmetries of the theory are local translations and local Lorentz rotations. The general form of the canonical generator of the local Poincaré transformations constructed by the Castellani procedure [21] is given by

$$\begin{aligned} G &= -G_1 - G_2, \\ G_1 &= \dot{\xi}^\rho \left(e^i{}_\rho \pi_i{}^0 + \frac{1}{2}\omega^{ij}{}_\rho \pi^0_{ij} \right) \\ &\quad + \xi^\rho \left(e^i{}_\rho \bar{\mathcal{H}}_i + \frac{1}{2}\omega^{ij}{}_\rho \bar{\mathcal{H}}_{ij} + C_{\rm PFC} \right), \\ G_2 &= \frac{1}{2}\dot{\varepsilon}^{ij}\pi_{ij}{}^0 + \frac{1}{2}\varepsilon^{ij}(\bar{\mathcal{H}}_{ij} + C_{\rm PFC}), \end{aligned}$$

where $C_{\rm PFC}$ are terms proportional to sure primary first class constraints $(\pi_i{}^0, \pi_{ij}{}^0)$.

The canonical generator acts on dynamical variables via the PB operation, and hence, it should have well-defined functional derivatives. In order to ensure this property, we have to improve the form of G by adding a suitable surface term Γ, such that $\tilde{G} = G + \Gamma$ is a well-defined canonical generator. In this process, the asymptotic conditions play a crucial role; see for instance Refs. [22,23]. Though we did not construct the exact form of the canonical generator, it still allows us to compute canonical charges for the solutions found in Sec. III. Namely, if we adopt the general principle that the quantities that vanish on shell have an arbitrary fast asymptotic decrease, we obtain that the on-shell variation of the generator takes the following form,

$$\delta G(\xi^t = \ell, \xi^\varphi = 1) \approx \delta\Gamma = -\ell\delta E_c - \delta J_c, \qquad (4.8)$$

where

$$E_c = 8\pi^2 r_0^2 \left(\alpha_1 - \frac{2\alpha_2}{r_0^2}\right) m, \qquad J_c = 8\pi^2 r_0^2 \left(\alpha_1 - \frac{2\alpha_2}{r_0^2}\right) j \qquad (4.9)$$

are the *canonical* conserved charges, which are identical to the expressions (3.24), obtained from the Nester formula.

V. LINEARIZED THEORY

The canonical structure of the full nonlinear theory crucially depends on the relations (4.6), as we already mentioned in the previous section. In order to get a deeper insight into the structure of the Lovelock gravity in the sector $\alpha_1^2 = 8\alpha_0\alpha_2$, we shall consider the theory linearized around the BTZ black ring with torsion (3.15). The linearization is based on the expansion of the basic dynamical variables $(e^i{}_\mu, \omega^{ij}{}_\mu)$ and the related conjugate momenta $(\pi_i{}^\mu, \pi_{ij}{}^\mu)$ denoted shortly by Q_A,

$$Q_A = \bar{Q}_A + \tilde{Q}_A, \qquad (5.1)$$

where $\bar{Q}_A$ refers to the background [solution (3.15) with $m = j = 0$ and $p \neq 0$], while $\tilde{Q}_A$ denotes small excitations.

From the linearized form of the 60 relations (4.6), we conclude that out of $60 = 5 \times 4 + 10 \times 4$ multipliers $(\tilde{u}^i{}_\alpha, \tilde{u}^{ij}{}_\alpha)$ 46 are determined, while among 14 remaining relations, there are 12 new constraints (since two pairs of them are identical), the explicit form of which is given by

$$\alpha_1 \tilde{R}^{24}{}_{r\chi} + \alpha_1 \sin\theta \tilde{R}^{23}{}_{r\theta} + 4\alpha_0 r_0 \sin\theta \tilde{e}^2{}_r \approx 0, \qquad (5.2a)$$

$$\alpha_1 \tilde{R}^{14}{}_{\varphi\chi} + \alpha_1 \sin\theta \tilde{R}^{13}{}_{\varphi\theta} + 4\alpha_0 r_0 \sin\theta \tilde{e}^2{}_r \approx 0, \qquad (5.2b)$$

$$\begin{aligned} &\frac{r^2}{\ell}(\alpha_1 \tilde{R}^{14}{}_{r\chi} + \alpha_1 \sin\theta \tilde{R}^{13}{}_{r\theta} + 2\alpha_0 r_0 \sin\theta \tilde{e}^1{}_r) \\ &\quad - \alpha_1 \tilde{R}^{24}{}_{\varphi\chi} - \alpha_1 \sin\theta \tilde{R}^{23}{}_{\varphi\theta} - 2\alpha_0 r_0 \sin\theta \tilde{e}^2{}_\varphi \approx 0 \end{aligned} \qquad (5.2c)$$

and

$$\tilde{T}^4{}_{r\chi} + \sin\theta \tilde{T}^3{}_{r\theta} \approx 0, \qquad (5.3a)$$

$$p(\alpha_1 r_0(\tilde{e}^4{}_\chi + \sin\theta \tilde{e}^3{}_\theta) + 2\alpha_2 \tilde{R}^{34}{}_{\theta\chi}) \approx 0, \qquad (5.3b)$$

$$\tilde{T}^4{}_{\varphi\chi} + \sin\theta \tilde{T}^3{}_{\varphi\theta} \approx 0, \qquad (5.3c)$$

$$\alpha_1 \frac{r}{\ell} r_0 \sin\theta \tilde{T}^2{}_{r\theta} - 2p(\alpha_1 r_0 \sin\theta \tilde{e}^0{}_\theta + 2\alpha_2 \tilde{R}^{04}{}_{\theta\chi}) \approx 0, \qquad (5.3d)$$

$$\alpha_1 \frac{r}{\ell} r_0 \tilde{T}^2{}_{r\chi} - 2p(\alpha_1 r_0 \sin\theta \tilde{e}^0{}_\chi - 2\alpha_2 \tilde{R}^{03}{}_{\theta\chi}) \approx 0, \qquad (5.3e)$$

$$\alpha_1 r_0 \tilde{T}^1{}_{\varphi\chi} + 2pr(\alpha_1 r_0 \tilde{e}^0{}_\chi - 2\alpha_2 \tilde{R}^{03}{}_{\theta\chi}) \approx 0, \qquad (5.3f)$$

$$\tilde{R}^{03}{}_{r\chi} \approx 0, \qquad (5.3g)$$

$$\tilde{R}^{02}{}_{\theta\chi} \approx 0, \qquad (5.3h)$$

$$\tilde{R}^{01}{}_{\theta\chi} \approx 0. \qquad (5.3i)$$

Let us denote 12 constraints (5.2a) and (5.3a) by $\tilde{\psi}_A$. The consistency conditions of $\tilde{\psi}_A$ leads to the determination of 12 additional multipliers, thus finishing the consistency procedure. Thus, out of 60 multipliers $(\tilde{u}^i{}_\alpha, \tilde{u}^{ij}{}_\alpha)$, 58 are determined, while 2 remain undetermined. By using the PB algebra from the Appendix, we find

$$\begin{aligned} &\{\tilde{\phi}_{12}{}^r, \tilde{\phi}_i{}^\alpha\} \approx 0, \qquad \{\tilde{\phi}_{12}{}^r, \tilde{\phi}_{ij}{}^\alpha\} \approx 0, \\ &\{\tilde{\phi}_{12}{}^r, \tilde{\psi}_A\} \approx 0, \\ &\{\tilde{\phi}_{12}{}^\varphi, \tilde{\phi}_i{}^\alpha\} \approx 0, \qquad \{\tilde{\phi}_{12}{}^\varphi, \tilde{\phi}_{ij}{}^\alpha\} \approx 0, \\ &\{\tilde{\phi}_{12}{}^\varphi, \tilde{\psi}_A\} \approx 0. \end{aligned}$$

TABLE I. Classification of constraints.

	First class	Second class
Primary	$\tilde{\phi}_i{}^0$, $\tilde{\phi}_{ij}{}^0$, $\tilde{\phi}_{12}{}^r$, $\tilde{\phi}_{12}{}^\varphi$	$\tilde{\phi}_i{}^\alpha$, $\tilde{\phi}_{ij}{}^\alpha$ $ij \neq 12 \wedge \alpha \neq r,\varphi$
Secondary	$\tilde{\bar{\mathcal{H}}}_i$, $\tilde{\bar{\mathcal{H}}}_{ij}$	$\tilde{\psi}_A$

The undetermined multipliers correspond to the constraints $\tilde{\phi}_{12}{}^r$ and $\tilde{\phi}_{12}{}^\varphi$ which are first class (FC). The final classification of constraints is given in Table I. In total, there are $N_1 = 32$ FC constraints and $N_2 = 70$ second class (SC) constraints. The number of propagating degrees of freedom in phase space is

$$N^* = 2N - 2N_1 - N_2 = 150 - 64 - 70 = 14.$$

In the configuration space, there are seven degrees of freedom: five of them correspond to general relativity in $D = 5$, and two are additional degrees of freedom. The presence of two primary FC constraints $\tilde{\phi}_{12}{}^r$, $\tilde{\phi}_{12}{}^\varphi$ implies that there is an additional gauge symmetry in the theory, as a consequence of the fact that variables $\tilde{\omega}^{12}{}_r$ and $\tilde{\omega}^{12}{}_\varphi$ do not appear in the linearized equations of motion.

VI. CONCLUSION

In this paper, we found a new class of solutions of Lovelock gravity in 5D, in the first order formalism. The most interesting solutions are the BTZ black rings with(out) torsion. It is shown that the solution with torsion exists provided that the parameters of the theory satisfy the relation $\alpha_1^2 = 8\alpha_0\alpha_2$. This sector of the parameter space is different from the one of Lovelock Chern-Simons gravity, as well as from the sector investigated by Canfora *et al.* [17]. Restricting our attention to the basic properties of the solutions, we calculated the values of conserved charges by using Nester's formula and the canonical method. The canonical structure of the theory linearized around the background with torsion shows that there are two additional degrees of freedom, compared to general relativity.

ACKNOWLEDGMENTS

We thank Milutin Blagojević for useful remarks and suggestions. This work was supported by the Serbian Science Foundation, Serbia, Grant No. 171031.

APPENDIX: ALGEBRA OF CONSTRAINTS

The structure of the PB algebra of constraints is an important ingredient in the analysis of the Hamiltonian consistency conditions. Starting from the fundamental PB $\{e^i{}_\mu, \pi_j{}^\nu\} = \delta^i_j\delta^\nu_\mu\delta(\mathbf{x}-\mathbf{x}')$ and $\{\omega^{ij}{}_\mu, \pi_{kl}{}^\nu\} = 2\delta^{[i}_k\delta^{j]}_l\delta^\nu_\mu\delta(\mathbf{x}-\mathbf{x}')$, we find PB between primary constraints:

$$\begin{aligned}\{\phi_i{}^\alpha, \phi_{jk}{}^\beta\} &= -2\varepsilon^{0\alpha\beta\gamma\delta}_{ijkln}(\alpha_1 e^l{}_\gamma e^n{}_\delta + \alpha_2 R^{ln}{}_{\gamma\delta})\delta,\\ \{\phi_{ij}{}^\alpha, \phi^\beta_{kl}\} &= -8\alpha_2\varepsilon^{0\alpha\beta\gamma\delta}_{ijkln}T^n{}_{\gamma\delta}\delta.\end{aligned} \tag{A1}$$

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PHYSICAL REVIEW D **93,** 044018 (2016)

Conformally flat black holes in Poincaré gauge theory

M. Blagojević[*] and B. Cvetković[†]

Institute of Physics, University of Belgrade, Pregrevica 118, 11080 Belgrade, Serbia

(Received 5 October 2015; published 8 February 2016)

General criteria for the existence of conformally flat Riemannian solutions in three-dimensional Poincaré gauge theory without matter are formulated. Using these criteria, we show that the Oliva-Tempo-Troncoso black hole, a solution of the Bergshoeff-Hohm-Townsend gravity, is also an exact vacuum solution of the Poincaré gauge theory. The related conserved charges, calculated from the Hamiltonian boundary term, are shown to satisfy the first law of black hole thermodynamics. The form of the boundary term is verified by using the covariant Hamiltonian approach.

DOI: 10.1103/PhysRevD.93.044018

I. INTRODUCTION

The use of three-dimensional gravitational models in the Poincaré gauge theory (PGT), the first properly formulated gauge theory of gravity [1–4], started in the early 1990s, when Mielke and Baekler formulated a topological model of three-dimensional gravity with torsion [5]. Studies of different aspects of the model made a significant contribution to a proper understanding of the influence of torsion on the gravitational dynamics; for a recent review, see Blagojević and Hehl [4], chapter 17. But, as time went on, it eventually became clear that transition to the level of quadratic PGT Lagrangians is needed, as the existence of propagating torsion modes offers a more realistic insight into the dynamical role of torsion; for more details, see Helayël-Neto *et al.* [6], Blagojević and Cvetković [7].

It is well known that classical solutions are an important tool for exploring dynamical content of gravitational theories, including the quadratic PGT [4]. Looking at what has been done in three dimensions, one should note that the model can accommodate exact torsion waves [8] and a Vaidya-like solution with torsion [9]. Quite interestingly, the methods used to construct Siklos waves in [8] are recently generalized to four dimensions [10].

In order to properly understand the complex dynamical structure of PGT, powerful Lagrangian and Hamiltonian formalisms have been developed; see Obukhov [3], Chen *et al.* [11], and Refs. [2,4]. This machinery is very useful not only for genuine PGT problems, characterized by a nonvanishing torsion, but also in studying torsion-free solutions of PGT. On the other hand, quite recently [9] we noticed that the issue of conserved charges of the Oliva-Tempo-Troncoso (OTT) black hole [12], a solution of the Bergshoeff-Hohm-Townsend (BHT) massive gravity [13] for the special choice of parameters, is not completely settled in the literature, see [14–16]. Such a situation motivated us to reconsider the OTT black hole as a Riemannian (torsion-free) solution of PGT, and try to find the conserved charges, energy and angular momentum, relying on the full power of the constrained Hamiltonian formalism. The analysis is based on deriving the Hamiltonian boundary term, the values of which correctly reproduce the conserved charges.

The paper is organized as follows. In Sec. II, we use the PGT field equations to study dynamical properties of Riemannian solutions. In particular, we show that (i) for a specific condition on the coupling constants, Riemannian solutions of PGT are conformally flat, and (ii) any conformally flat solution of the BHT gravity is also a solution of PGT. The results are used in Sec. III to prove that the static OTT black hole is a solution of PGT. In Sec. IV, we introduce a set of asymptotic conditions naturally associated to this black hole, and use the constrained Hamiltonian formalism to construct the improved canonical generator $\tilde{G}$, acting on the related phase space [17]. The form of the boundary term in $\tilde{G}$ is shown to be directly related to the OTT asymptotic conditions, and the conserved charges, defined as the values of $\tilde{G}$, are proved to be fully compatible with the first law of black hole thermodynamics. In Sec. V, the same approach is used to analyze the rotating OTT black hole, and in Sec. VI, we summarize our results and verify the form of the boundary term by comparing it to the generalized covariant formula proposed by So [18]. Appendices contain some technical details.

Our conventions are the same as in Ref. [9]: the latin indices $(i,j,k,\dots)$ refer to the local Lorentz frame, the greek indices $(\mu,\nu,\rho,\dots)$ refer to the coordinate frame, b^i is the orthonormal triad (coframe 1-form), ω^{ij} is the Lorentz connection (1-form), the respective field strengths are the torsion $T^i = db^i + \omega^i{}_m \wedge b^m$ and the curvature $R^{ij} = d\omega^{ij} + \omega^i{}_k \wedge \omega^{kj}$ (2-forms), the frame h_i dual to b^j is defined by $h_i \rfloor b^j = \delta^j_i$, the signature of the metric is $(+,-,-)$, the totally antisymmetric symbol ε^{ijk} is normalized to $\varepsilon^{012} = 1$, the Lie dual of the antisymmetric form X^{ij} is $X_i \coloneqq -\varepsilon_{ijk} X^{jk}/2$, the Hodge dual of the form α is ${}^\star\alpha$, and the exterior product of forms is implicit.

[*]mb@ipb.ac.rs
[†]cbranislav@ipb.ac.rs

II. CONFORMALLY FLAT RIEMANNIAN SOLUTIONS IN PGT

The OTT black hole is a vacuum solution of the BHT gravity with a unique AdS ground state [12,14]. Here, based on our earlier experience [8,9], we wish to interpret it as a Riemannian solution of PGT in vacuum. By doing so, we will be able to use the full power of the constrained Hamiltonian formalism to clarify the asymptotic structure and find the conserved charges for both the static and the rotating OTT black hole.

The possibility to interpret the OTT black hole as a Riemannian solution of PGT (a solution with vanishing torsion) is not just a coincidence, it is based on a deep dynamical relation between the PGT sector of Riemannian solutions and the BHT gravity. The content of this relation is expressed by a theorem stating that any conformally flat solution of the BHT gravity is also a Riemannian solution of PGT. This is, in particular, true for the OTT black holes. In three dimensions, the Weyl curvature identically vanishes, and the Cotton 2-form C^i is used to characterize conformal properties of spacetime [19]. It is defined by $C^i \coloneqq \nabla L^i = dL^i + \omega^i{}_m L^m$ where $L^m \coloneqq Ric^m - \frac{1}{4} R b^m$ is the Schouten 1-form. A spacetime is conformally flat when $C^i = 0$.

To prove the above theorem, we note that the BHT gravity action,

$$I_{\rm BHT} = a_0 \int d^3x \sqrt{g}\left(R - \lambda + \frac{1}{m^2} K\right),$$
$$K \coloneqq Ric^{ij} Ric_{ij} - \frac{3}{8} R^2,$$

leads to the field equations [20],

$$G_{ij} - \lambda \eta_{ij} - \frac{1}{2m^2} K_{ij} = 0,$$
$$K_{ij} = K\eta_{ij} - 2L_{ik} G^k{}_j - 2(\nabla_m C_{in})\varepsilon^{mn}{}_j, \qquad (2.1)$$

where $G_{ij} = Ric_{ij} - R\eta_{ij}/2$ is the Einstein tensor, $C_{ij} = h_j \rfloor {}^\star C_i$ is the Cotton and $L_{ij} = h_j \rfloor L_i$ the Schouten tensor. This compact form of the BHT field equations significantly simplifies the analysis of conformally flat solutions.

The Lagrangian dynamics of PGT is expressed in terms of its basic field variables, the triad b^i and the Lorentz connection ω^{ij} (1-forms), the related field strengths are the torsion $T^i \coloneqq db^i + \omega^i{}_m b^m$ and the curvature $R^{ij} \coloneqq d\omega^{ij} + \omega^i{}_m \omega^{mj}$ (2-forms), and the spacetime continuum is described by a Riemann-Cartan geometry. The gravitational Lagrangian $L_G = L_G(b^i, T^j, R^{mn})$ (3-form) is at most quadratic in the field strengths:

$$L_G = -{}^\star(a_0 R + 2\Lambda_0) + T^{i\star}(a_1{}^{(1)}T_i + a_2{}^{(2)}T_i + a_3{}^{(3)}T_i)$$
$$+ \frac{1}{2} R^{ij\star}(b_4{}^{(4)}R_{ij} + b_5{}^{(5)}R_{ij} + b_6{}^{(6)}R_{ij}),$$

where ${}^{(n)}T^i$ and ${}^{(n)}R^{ij}$ are irreducible components of the respective field strengths, and a_0 is normalized by $a_0 = /16\pi G$; for details, see Ref. [7]. Since we are here interested only in Riemannian solutions of PGT, the torsion can be effectively set to vanish, whereas the curvature becomes Riemannian; in three dimensions, it has only two nonvanishing irreducible components,

$${}^{(6)}R^{ij} = \frac{1}{6} R b^i b^j, \qquad {}^{(4)}R_{ij} = R^{ij} - {}^{(6)}R^{ij},$$

whereas the third one vanishes, ${}^{(5)}R_{ij} = 0$. The Riemannian reduction of the general field equations takes the form derived in Appendix A of Ref. [9]:

$$({\rm 1ST}) E_i = 0,$$
$$({\rm 2ND}) \nabla H_{ij} = 0, \qquad (2.2a)$$

where

$$E_i = h_i \rfloor L_G - \frac{1}{2}(h_i \rfloor R^{mn}) H_{mn},$$
$$H_{ij} = -2a_0 \varepsilon_{ijm} b^m + \frac{b_4 + 2b_6}{6} R \varepsilon_{ijk} b^k - 2b_4 \varepsilon_{ij}{}^m L_m. \qquad (2.2b)$$

Let us now note a simple property of (2ND): the vanishing of the second term in H_{ij} implies that the Cotton 2-form $C_m = \nabla L_m$ vanishes. More precisely,

(T1) A Riemannian solution of PGT is conformally flat if and only if $b_4 + 2b_6 = 0$.

Next, to examine the content of (1ST), it is convenient to express it in the tensorial form:

$$a_0 Ric_{ij} + 2\Lambda_0 \eta_{ij} + b_4 L_{im} G^m{}_j = 0.$$

In combination with its trace, $a_0 R + 6\Lambda_0 + b_4 K = 0$, it can be transformed to

$$a_0 G_{ij} - \Lambda_0 \eta_{ij} - b_4 \frac{1}{2}(K\eta_{ij} - 2L_{im} G^m{}_j) = 0. \qquad (2.3)$$

A direct comparison shows that Eq. (2.3) coincides with the BHT field equation (2.1) for $C_{in} = 0$, provided one makes the following identification of parameters:

$$\Lambda_0 = a_0 \lambda, \qquad b_4 = a_0/m^2. \qquad (2.4)$$

This leads to the main result of this section:

(T2) Any conformally flat solution of the BHT gravity is also a Riemannian solution of PGT with $b_4 + 2b_6 = 0$, and vice versa.

An interesting interpretation of the identifications (2.4) is found by using the BHT condition $\lambda = -m^2$ that ensures the existence of the unique maximally symmetric background. For $m^2 = 1/2\ell^2$, the identifications (2.4) are transformed into

$$\Lambda_0 = -a_0/2\ell^2, \qquad b_4 = 2a_0\ell^2. \tag{2.5}$$

Theorems (T1) and (T2) allow us to study conformally flat solutions of the BHT massive gravity relying on the powerful Hamiltonan methods developed in the context of PGT [2,4,11]. In particular, we will use these methods to study boundary terms, conserved charges, and central charges of the OTT black hole. Recently, it was shown by Barnich *et al.* [21] that BHT gravity admits black hole solutions that can be deformed into dynamical "black flowers," a new class of solutions that are no longer spherically symmetric. Since black flowers are conformally flat, they are also solutions of PGT.

Although PGT is used here as a convenient framework for studying conformally flat solutions of the BHT gravity, it is worth mentioning some general dynamical aspects of PGT, expressed through its unitarity properties. In three dimensions, the requirement of unitary propagation of torsion modes leads to certain conditions on the coupling constants, the form of which is given in Eqs. (17) of Ref. [6]. The content of these equations leads to the following conclusions: (a) the condition $b_4 + 2b_6 = 0$ implies that the spin-0^+ mode does not propagate and (b) for a suitable choice of the remaining coupling constants, the propagation of the spin-0^-, spin-1 or spin-2 modes is unitary.

III. STATIC OTT BLACK HOLE

Now, we turn our attention to the static OTT spacetime, described by the metric [12]

$$ds^2 = N^2dt^2 - \frac{dr^2}{N^2} - r^2d\varphi^2, \qquad N^2 := -\mu + br + \frac{r^2}{\ell^2}, \tag{3.1}$$

where μ and b are real parameters. The roots of equation $N^2 = 0$ are

$$r_\pm = \frac{1}{2}(-b\ell^2 \pm \ell\sqrt{4\mu + b^2\ell^2}).$$

The OTT metric defines a static AdS black hole when $\ell^2 > 0$ and at least r_+ is real and positive; for $b = 0$ it reduces to the BTZ black hole [22].

In order to have a suitable geometric description of the OTT black hole in the framework of PGT, we introduce the triad field (1-form),

$$b^0 := Ndt, \qquad b^1 := \frac{dr}{N}, \qquad b^2 := rd\varphi, \tag{3.2a}$$

so that $ds^2 = \eta_{ij}b^i \otimes b^j$, with $\eta = \mathrm{diag}(+1,-1,-1)$, and the corresponding Riemannian connection (1-form),

$$\omega^{01} = -N'b^0, \qquad \omega^{02} = 0, \qquad \omega^{12} = \frac{N}{r}b^2, \tag{3.2b}$$

where $N' := \partial_r N$. The geometric structure introduced in Eqs. (3.2) can now be used to calculate first the curvature 2-form R^{ij} and then the Schouten 1-form:

$$L^0 = \frac{1}{2\ell^2}b^0, \qquad L^1 = \frac{1}{2\ell^2}b^1, \qquad L^2 = \left(\frac{1}{2\ell^2} + \frac{b}{2r}\right)b^2. \tag{3.3}$$

An explicit calculation yields $C^i = \nabla L^i = 0$, and theorem (T2) from Sec. II implies that the static OTT black hole is an exact Riemannian solution of PGT in vacuum.

It is interesting to compare these general arguments with direct calculations based on the PGT field equations (2.2). As shown in [9], the result takes the form of three conditions on the four Lagrangian parameters (a_0, b_4, b_6, Λ):

$$b_4 - 2a_0\ell^2 = 0, \qquad a_0 + 2\ell^2\Lambda_0 = 0, \qquad b_4 + 2b_6 = 0. \tag{3.4}$$

The meaning of these conditions is now quite clear: the third one follows from the conformal flatness of the static OTT black hole, and the first two coincide with the relations (2.5).

IV. ASYMPTOTIC STRUCTURE OF THE STATIC BLACK HOLE

In this section, we use the canonical approach to analyze the asymptotic structure naturally associated to the static OTT black hole. In particular, we wish to calculate the conserved charges and verify their compatibility with the first law of black hole thermodynamics.

A. Asymptotic conditions

The asymptotic state associated to the triad (3.2a) is determined by the asymptotic formula

$$N = \frac{r}{\ell} + \frac{b\ell}{2} - \frac{\ell}{2r}\left(\mu + \frac{b^2\ell^2}{4}\right) + \mathcal{O}_2,$$

and a similar formula for $1/N$. In order to produce a suitable set of the asymptotic states, we act on this particular state by the transformations belonging to the AdS group $SO(2,2)$, as described in Ref. [7]. The family of triads obtained in this way has the AdS asymptotic behavior given by $b^i{}_\mu = \bar{b}^i{}_\mu + B^i{}_\mu$, where

$$\bar{b}^i{}_\mu := \begin{pmatrix} \frac{r}{\ell} & 0 & 0 \\ 0 & \frac{\ell}{r} & 0 \\ 0 & 0 & r \end{pmatrix}, \qquad B^i{}_\mu := \begin{pmatrix} \mathcal{O}_0 & \mathcal{O}_3 & \mathcal{O}_0 \\ \mathcal{O}_1 & \mathcal{O}_2 & \mathcal{O}_1 \\ \mathcal{O}_0 & \mathcal{O}_3 & \mathcal{O}_0 \end{pmatrix}. \tag{4.1}$$

Here, $\bar{b}^i{}_\mu$ refers to an AdS background ($b = \mu = 0$). Note that the presence of the OTT parameter b makes the asymptotic decrease of $B^i{}_\mu$ slower then in the BTZ case. The subset of the local Poincaré transformations that respect these conditions is determined by the parameters $(\xi^\mu, \varepsilon^{ij} = -\varepsilon^{ijk}\theta_k)$, such that

$$\delta_0 b^i{}_\mu \coloneqq \varepsilon^{ijk}\theta_j b_{k\mu} - (\partial_\mu \xi^\rho) b^i{}_\rho - \xi^\rho \partial_\rho b^i{}_\mu = B^i{}_\mu.$$

As a consequence, the asymptotic parameters of local translations and Lorentz rotations are found to be

$$\frac{\xi^t}{\ell} = T + \frac{\ell^4}{2r^2}\partial_t^2 T + \mathcal{O}_3, \qquad \xi^r = -\ell r \partial_t T + \mathcal{O}_0,$$
$$\xi^\varphi = S - \frac{\ell^2}{2r^2}\partial_\varphi^2 S + \mathcal{O}_3, \tag{4.2a}$$

$$\theta^0 = -\frac{\ell^2}{r}\partial_t\partial_\varphi T + \mathcal{O}_2, \qquad \theta^1 = \partial_\varphi T + \mathcal{O}_1,$$
$$\theta^2 = \frac{\ell^3}{r}\partial_t^2 T + \mathcal{O}_2. \tag{4.2b}$$

The functions T and S are such that $\partial_\pm T^\mp = 0$, with $x^\pm \coloneqq t/\ell \pm \varphi$, and $T^\pm \coloneqq T \pm S$. Thus, in spite of a relaxed asymptotic behavior of $B^i{}_\mu$ as compared to the BTZ black hole, the values of the corresponding asymptotic parameters are essentially the same [23].

Similar procedure leads to the asymptotic conditions for the connection. Introducing the Lie dual connection ω^i by $\omega^{ij} = -\varepsilon^{ijk}\omega_k$, one finds $\omega^i{}_\mu = \bar{\omega}^i{}_\mu + \Omega^i{}_\mu$, where

$$\bar{\omega}^i{}_\mu = \begin{pmatrix} 0 & 0 & -\frac{r}{\ell} \\ 0 & 0 & 0 \\ -\frac{r}{\ell^2} & 0 & 0 \end{pmatrix}, \qquad \Omega^i{}_\mu \coloneqq \begin{pmatrix} \mathcal{O}_0 & \mathcal{O}_3 & \mathcal{O}_0 \\ \mathcal{O}_1 & \mathcal{O}_2 & \mathcal{O}_1 \\ \mathcal{O}_0 & \mathcal{O}_3 & \mathcal{O}_0 \end{pmatrix}. \tag{4.3}$$

The asymptotic behavior of the connection does not impose any new restriction on the asymptotic Poincaré parameters (4.2).

For an easier comparison with the literature, we display here the deviation of the metric from its background value:

$$G_{\mu\nu} \coloneqq g_{\mu\nu} - \bar{g}_{\mu\nu} = \begin{pmatrix} \mathcal{O}_{-1} & \mathcal{O}_2 & \mathcal{O}_{-1} \\ \mathcal{O}_2 & \mathcal{O}_3 & \mathcal{O}_2 \\ \mathcal{O}_{-1} & \mathcal{O}_2 & \mathcal{O}_{-1} \end{pmatrix}.$$

Using the composition law of the asymptotic Poincaré parameters (4.2) to leading order, the commutator algebra of the asymptotic symmetry is found to have the form of two independent Virasoro algebras,

$$i[\ell_m^\pm, \ell_n^\pm] = (m-n)\ell_{m+n}^\pm, \tag{4.4}$$

where $\ell_n^\pm = -\delta_0(T^\pm = e^{\pm inx^\pm})$. The respective central charges $c^\pm$ will be determined by the canonical methods.

The condition $T^i = 0$ leads to further asymptotic requirements (Appendix A).

B. Canonical generator and conserved charges

The standard construction of the canonical generator for the quadratic PGT makes use of the existence and classification of all constraints in the theory. The construction can be significantly simplified by going over to the first-order Lagrangian (3-form)

$$L_G = T^i\tau_i + \frac{1}{2}R^{ij}\rho_{ij} - V(b, \tau, \rho);$$

see Refs. [11,24]. Here, τ^m and ρ_{ij} are independent dynamical variables, the covariant field momenta conjugate to b^i and ω^{ij}, and the potential V ensures the on-shell relations $\tau_i = T_i$, $\rho_{ij} = R_{ij}$, which transform L_G into the standard quadratic form.

The first-order formulation of L_G simplifies the construction of the canonical generator G, the form of which can be found in Ref. [7], Eq. (5.7). Since G acts on the basic dynamical variables via the Poisson bracket operation, it must be a differentiable functional. To examine the differentiability of G, one starts from the form of its variation [8,9]:

$$\delta G = -\int_\Sigma d^2x(\delta G_1 + \delta G_2),$$
$$\delta G_1 = \varepsilon^{t\alpha\beta}\xi^\mu(b^i{}_\mu\partial_\alpha\delta\tau_{i\beta} + \omega^i{}_\mu\partial_\alpha\delta\rho_{i\beta}$$
$$+ \tau^i{}_\mu\partial_\alpha\delta b_{i\beta} + \rho^i{}_\mu\partial_\alpha\delta\omega_{i\beta}) + \mathcal{R},$$
$$\delta G_2 = \varepsilon^{t\alpha\beta}\theta^i\partial_\alpha\delta\rho_{i\beta} + \mathcal{R}. \tag{4.5a}$$

Here, Σ is the spatial section of spacetime, the variation is performed in the set of adopted asymptotic states, $\mathcal{R}$ stands for regular (differentiable) terms, and we use ρ^i and ω^i, the Lie duals of $\rho_{mn} = H_{mn}$ and ω_{mn}, to simplify the formulas.

Using the adopted asymptotic conditions, one finds $\delta G_2 = \mathcal{R}$, which implies

$$\delta G = -\int_\Sigma d^2x\varepsilon^{t\alpha\beta}\xi^\mu(b^i{}_\mu\partial_\alpha\delta\tau_{i\beta} + \omega^i{}_\mu\partial_\alpha\delta\rho_{i\beta}$$
$$+ \tau^i{}_\mu\partial_\alpha\delta b_{i\beta} + \rho^i{}_\mu\partial_\alpha\delta\omega_{i\beta}) + \mathcal{R}. \tag{4.5b}$$

Thus, in general, $\delta G \neq \mathcal{R}$ and G is not differentiable. The problem can be corrected by going over to the improved generator $\tilde{G} \coloneqq G + \Gamma$, where the boundary term Γ is constructed so that $\delta\tilde{G} = \mathcal{R}$ [17]. After making a partial integration in δG, one finds that Γ is defined by the variational equation,

$$\delta\Gamma = \int_{\partial\Sigma} \xi^\mu (b^i{}_\mu \delta\tau_i + \omega^i{}_\mu \delta\rho_i + \tau^i{}_\mu \delta b_i + \rho^i{}_\mu \delta\omega_i), \tag{4.6}$$

where $\partial\Sigma$ is the boundary of Σ located at infinity, parametrized by the coordinate φ. Now, restricting our attention to the Riemannian sector with $\tau^i = 0$, we obtain

$$\delta\Gamma = \int_{\partial\Sigma} \xi^\mu (\omega^i{}_\mu \delta\rho_i + \rho^i{}_\mu \delta\omega_i) = \int_0^{2\pi} (\xi^t \delta\mathcal{E} + \xi^\varphi \delta\mathcal{J}) d\varphi, \tag{4.7a}$$

where (after returning to ω_{ij} and H_{ij})

$$\delta\mathcal{E} := \frac{1}{2}(\omega^{ij}{}_t \delta H_{ij\varphi} + H^{ij}{}_t \delta\omega_{ij\varphi}), \tag{4.7b}$$

$$\delta\mathcal{J} := \frac{1}{2}(\omega^{ij}{}_\varphi \delta H_{ij\varphi} + H^{ij}{}_\varphi \delta\omega_{ij\varphi}). \tag{4.7c}$$

In what follows, one should take into account that the form (2.2b) of H_{mn} is simplified after using the restrictions (3.4) on the Lagrangian parameters:

$$H_{ij} = -2a_0 \varepsilon_{ijk} b^k - 4a_0 \ell^2 \varepsilon_{ijk} L^k.$$

Once we find the solutions for $\mathcal{E}$ and $\mathcal{J}$, the boundary term takes the form

$$\Gamma(\xi) = \int_0^{2\pi} (\xi^t \mathcal{E} + \xi^\varphi \mathcal{J}) d\varphi. \tag{4.8}$$

In general, Eqs. (4.7) refer to the fields and their variations belonging to the entire set of asymptotic states, defined by Eqs. (4.1) and (4.3). However, it is instructive to consider first a simpler situation, in which the fields and their variations refer just to a *single asymptotic state*, the static OTT configuration (3.2). In that case, Eq. (4.7b) takes the form

$$\begin{aligned}\delta\mathcal{E} &= \omega^{01}{}_t \delta H_{01\varphi} + H^{12}{}_t \delta\omega_{12\varphi} \\ &= 2a_0 \ell^2 \left(\frac{r}{\ell^2} + \frac{1}{2} b\right) \delta b - 4a_0 N \delta N \\ &= 2a_0 \delta\left(\mu + \frac{1}{4} \ell^2 b^2\right),\end{aligned} \tag{4.9}$$

which is easily integrated to obtain $\mathcal{E}$. In fact, the procedure just described is sufficient to calculate the values of the conserved charges, but only for this particular configuration.

In the next step, we wish to find a solution for $\mathcal{E}$ on the *whole set of asymptotic states*. Using the special result (4.9) as a guide, we find

$$\begin{aligned}\mathcal{E} &= \mathcal{E}_0 - \frac{1}{4}(\Delta\omega^{ij}{}_t \Delta H_{ij\varphi} + \Delta H_{ijt} \Delta\omega^{ij}{}_\varphi), \\ \mathcal{E}_0 &:= \frac{1}{2}(\omega^{ij}{}_t \Delta H_{ij\varphi} + H_{ijt} \Delta\omega^{ij}{}_\varphi),\end{aligned} \tag{4.10a}$$

where $\Delta X := X - \bar{X}$ is the difference between any field X and its boundary value $\bar{X}$. In a similar manner, Eq. (4.7c) leads to

$$\begin{aligned}\mathcal{J} &= \frac{1}{2}\omega^{ij}{}_\varphi H_{ij\varphi} = \mathcal{J}_0 - \frac{1}{2}\Delta H_{ij\varphi} \Delta\omega^{ij}{}_\varphi, \\ \mathcal{J}_0 &:= \frac{1}{2}(\omega^{ij}{}_\varphi \Delta H_{ij\varphi} + H_{ij\varphi} \Delta\omega^{ij}{}_\varphi),\end{aligned} \tag{4.10b}$$

where the first equality follows directly from (4.7c), and the second one from $\bar{H}_{ij\varphi} \bar{\omega}^{ij}{}_\varphi = 0$. With these results for $\mathcal{E}$ and $\mathcal{J}$, the boundary term (4.8) is seen to be a finite phase-space functional that satisfies the variational equation (4.7a) (Appendix B).

The values of the improved generators for time translations ($\xi = \partial_t$) and spatial rotations ($\xi = \partial_\varphi$) are given by the corresponding boundary terms, which define the conserved charges of the system, the energy and the angular momentum, respectively:

$$E = \int_0^{2\pi} d\varphi \mathcal{E}, \qquad J = \int_0^{2\pi} d\varphi \mathcal{J}. \tag{4.11}$$

Calculated on the static OTT configuration, these expressions take the values

$$E = \frac{1}{4G}\left(\mu + \frac{1}{4} b^2 \ell^2\right), \qquad J = 0. \tag{4.12}$$

The expressions (4.10) for $\mathcal{E}$ and $\mathcal{J}$ are obtained by relying on the set of asymptotic configurations (4.1) and (4.3) that contain the static OTT black hole geometry. It is interesting to compare the boundary term (4.8) to the covariant approach of Chen *et al.* [11]. Looking at the Riemannian reduction of their formula (239) and choosing the upper or lower term in each curly bracket separately, one finds that none of the resulting expressions can reproduce our result. To make the argument more clear, consider, for instance, the term $\mathcal{E}_0$ in (4.10a) that corresponds to choosing all the upper terms in (239); the corresponding expression for the energy would be different from (4.12): $E_0 = \frac{1}{4G}(\mu + \frac{1}{2} b^2 \ell^2)$. How do we know that this result is not correct? The answer can be found by noting that the boundary term $\Gamma[\xi]$ has a twofold role: (i) its values define the conserved charges, and (ii) its form ensures the improved generator $\tilde{G} = G + \Gamma$ to be a differentiable functional on the phase space associated with the chosen boundary conditions. Since $\mathcal{E}_0$ does not satisfy the variational equation (4.7b), replacing $\mathcal{E}$ by $\mathcal{E}_0$ would destroy the differentiability of the new canonical generator

$\tilde{G}[\mathcal{E}\to\mathcal{E}_0]$. The way out of this situation can be found in the work of So [18], who proposed a generalized boundary term by introducing "mixed" choices involving a linear combinations of upper and lower term in (239); see footnote "u" in [11]. As discussed in Sec. VI, our boundary term (4.8) is appropriately described by a particular mixed form. The need for using a mixed boundary term stems directly from the slower asymptotic decrease of the OTT dynamical variables as compared to the BTZ case (see Sec. IV A), or equivalently, from the presence of the br term in the OTT metric (3.1).

C. Asymptotic symmetry

The results obtained so far allow us to precisely describe the OTT asymptotic symmetry by the Poisson bracket algebra of the improved canonical generators. Following the procedure described in [8,9], one finds that this algebra, expressed in terms of the Fourier modes $L^\pm_n$ of $\tilde{G}$, is given by a centrally extended form of the commutator algebra (4.4),

$$i[L^\pm_m, L^\pm_n] = (m-n)L_{m+n} + \frac{c^\pm}{12}m^3\delta_{m,-n}, \qquad (4.13)$$

where $c^\pm$ are classical central charges,

$$c^\pm = c, \qquad c = \frac{3\ell}{G}. \qquad (4.14)$$

D. Black hole entropy

As an additional, theoretical test of the validity of our canonical expression for the OTT energy $(4.12)_1$, we propose to verify its exact agreement with the first law of black hole thermodynamics; the same strategy was used, for instance, by Giribet and Leston [15], and by Maeda [25].

The black hole entropy can be calculated from the Cardy formula [26]

$$S = 2\pi\sqrt{\frac{h^-c^-}{6}} + 2\pi\sqrt{\frac{h^+c^+}{6}},$$

where $h^\pm = (\ell E \pm J)/2$. For the static OTT black hole, this formula yields

$$S = 2\pi\ell\sqrt{\frac{E}{G}}. \qquad (4.15)$$

Then, using the expression for the Hawking temperature,

$$T = \frac{1}{4\pi}\partial_r N^2|_{r=r_+} = \frac{1}{\pi\ell}\sqrt{GE}, \qquad (4.16)$$

one can directly verify the first law of the black hole thermodynamics:

$$\delta E = T\delta S. \qquad (4.17)$$

Since the entropy vanishes for $E=0$, the state with $E=0$ can be naturally regarded as the ground state of the OTT family of black holes [14].

V. ROTATING OTT BLACK HOLE

In order to verify to what extent the canonical expressions (4.10) for the boundary terms of the static OTT black hole are general, we now use the same approach to study the rotating OTT black hole.

A. Geometric aspects

The rotating OTT black hole is defined by the metric [14,15]

$$ds^2 = N^2dt^2 - F^{-2}dr^2 - r^2(d\varphi + N_\varphi dt)^2, \qquad (5.1a)$$

where

$$\begin{aligned}
F &= \frac{H}{r}\sqrt{\frac{H^2}{\ell^2} + \frac{b}{2}H(1+\eta) + \frac{b^2\ell^2}{16}(1-\eta)^2 - \mu\eta},\\
N &= AF, \qquad A = 1 + \frac{b\ell^2}{4H}(1-\eta),\\
N_\varphi &= \frac{\ell}{2r^2}\sqrt{1-\eta^2}(\mu - bH),\\
H &= \sqrt{r^2 - \frac{\mu\ell^2}{2}(1-\eta) - \frac{b^2\ell^4}{16}(1-\eta)^2}. \qquad (5.1b)
\end{aligned}$$

The roots of $N=0$ are

$$r_\pm = \ell\sqrt{\frac{1+\eta}{2}}\left(-\frac{b\ell}{2}\sqrt{\eta} \pm \sqrt{\mu + \frac{b^2\ell^2}{4}}\right).$$

The metric (5.1) depends on three free parameters, μ, b and η. For $\eta = 1$, it represents the static OTT black hole, and for $b=0$, it reduces to the rotating BTZ black hole with parameters (m, j), such that $4Gm \coloneqq \mu$ and $4Gj \coloneqq \mu\ell\sqrt{1-\eta^2}$.

Choosing the triad field as

$$b^0 = Ndt, \quad b^1 = F^{-1}dr, \quad b^2 = r(d\varphi + N_\varphi dt), \qquad (5.2a)$$

the Riemannian connection takes the form

$$\omega^{01} = -\alpha b^0 + \beta b^2, \quad \omega^{02} = \beta b^1, \quad \omega^{12} = -\beta b^0 + \gamma b^2, \qquad (5.2b)$$

where $\alpha \coloneqq FN'/N$, $\beta \coloneqq rFN'_\varphi/2N$ and $\gamma = F/r$. These objects define the Riemannian geometry of the rotating OTT black hole in the context of PGT.

Now, based on theorem (T2) from Sec. II, we know that the rotating OTT black hole, being an exact solution of the BHT gravity, is also a solution of PGT provided its Cotton tensor vanishes. Technically, the proof that $C^{ij} = 0$ is not quite simple due to the complicated structure of the metric functions N, F and N_φ. However, relying on the standard computer algebra systems, one easily finds that C^{ij} indeed vanishes.

B. Asymptotic conditions and conserved charges

A direct inspection of the rotating black hole geometry (5.2) shows that it belongs to the same class of asymptotic states as described by Eqs. (4.1) and (4.3). Hence, the results for (i) the boundary term (4.8) and (ii) the classical central charges (4.14) remain valid also in the rotating black hole case.

Applying formulas (4.10) to the rotating OTT geometry (5.2) yields the following conserved charges:

$$E = \frac{1}{4G}\left(\mu + \frac{1}{4}b^2\ell^2\right), \tag{5.3a}$$

$$J = \ell\sqrt{1-\eta^2}E. \tag{5.3b}$$

For $\eta = 1$ the angular momentum vanishes, whereas for $b = 0$ we have the BTZ black hole with $E = m$ and $J = j$; its energy is twice as big as in GR.

C. The first law of black hole thermodynamics

The entropy for the rotating OTT black hole can be calculated in the same manner as for the static one. Using the above expressions for E, J, and the central charges $c^\pm = 3\ell/G$, the Cardy formula yields

$$S = 2\pi\ell\sqrt{\frac{(1+\eta)E}{2G}}. \tag{5.4}$$

The Hawking temperature and the angular velocity at the outer horizon are

$$T = \frac{1}{4\pi}\frac{\partial_r N^2}{A}\bigg|_{r=r_+} = \frac{1}{\pi\ell}\sqrt{\frac{2\eta^2}{1+\eta}}\sqrt{GE},$$
$$\Omega_+ = N_\varphi|_{r=r_+} = \frac{1}{\ell}\sqrt{\frac{1-\eta}{1+\eta}}. \tag{5.5}$$

Then, the first law of black hole thermodynamics is automatically satisfied:

$$T\delta S = \delta E - \Omega_+\delta J. \tag{5.6}$$

VI. DISCUSSION AND CONCLUSIONS

The OTT black hole energy was calculated already in the original paper [12], based on the Deser-Tekin approach [27]. Since the Deser-Tekin formula (37) in [12] does not contain the asymptotic terms produced by the parameter b, the resulting energy $E_{\rm DT} = \mu/4G$ does not depend on b. This result is evidently not compatible with the first law of black hole thermodynamics. Then, Giribet *et al.* [14] found certain arguments, based on interpreting b as a ‘hair’ parameter, to transform $E_{\rm DT}$ into $E = (\mu + \ell^2 b^2/4)/(4G)$, the expression that is fully compatible with the first law [25].

In the next paper, Giribet and Leston [15] tried to find more convincing arguments to derive the above form of E. Their approach was based on the work of Hohm and Tonni [28], who developed a generalized Brown-York approach to the generic form of the BHT gravity. By restricting their considerations to the special value of m^2, where the OTT black hole is admitted as an exact solution, the authors of [15] succeeded to derive the above result for E, but only for the rotating black hole, where certain ambiguity in the derivation disappears. By improving the construction, Kwon *et al.* [16] obtained the conserved charges for both the static and the rotating OTT black hole. Our expressions (5.3) for the conserved charges confirm their final results, given in Eq. (44).

In the approach initiated by Regge and Teitelboim [17], the gravitational conserved charges and the improved canonical generators are closely related to each other. An important progress in understanding essential aspects of this relation has been achieved in the first-order approach, which allows one to find a covariant boundary term and identify its value as a conserved charge; for an early version of the formalism, see Nester [24], and for a comprehensive exposition of this approach, see Chen *et al.* [11]. The covariant approach has been widely used in four-dimensional gauge theories of gravity with a great success [4,11]. Moreover, it was also confirmed on a set of selected three-dimensional solutions [29]. Now, in order to properly understand our results in the context of this approach, we start from a particular choice of the covariant boundary expression (integrand) defined by the upper line in Eq. (234) of [11]:

$$B_{\rm ul}(\xi) := (\xi\rfloor b^i)\Delta\tau_i + \Delta b^i(\xi\rfloor\tau_i) + (\xi\rfloor\omega^i)\Delta\rho_i + \Delta\omega^i(\xi\rfloor\rho_i). \tag{6.1}$$

Here, $\Delta X = X - \bar{X}$ is a difference between a field X and its boundary value $\bar{X}$, and ξ is asymptotically a Killing vector field. The lower line is obtained by replacing the variables $(b^i, \tau_i, \omega^i, \rho_i)$ with their boundary values. One can verify that formula (6.1), taken in the Riemannian limit, is not compatible with our result (4.8). This is, in fact, true for all sixteen versions of $B(\xi)$, obtained from Eq. (234) of [11] by choosing either the upper or lower term in each of the four curly brackets separately. However, the situation is changed by generalizing the construction of $B(\xi)$ in a way proposed by So [18]. According to his prescription, the original Hamiltonian boundary term $B(\xi)$ is modified by replacing each curly bracket by a linear combination of its upper and lower term. Applying this prescription to Eq. (234) of [11], one finds that its Riemannian reduction takes the form

$$\begin{aligned}\tilde{B}(\xi;c_3,c_4) &:= \xi\rfloor[c_3\omega^i+(1-c_3)\bar{\omega}^i]\wedge\Delta\rho_i\\ &\quad+\Delta\omega^i\wedge\xi\rfloor[c_4\rho_i+(1-c_4)\bar{\rho}_i], \end{aligned}\tag{6.2}$$

where c_3 and c_4 are real parameters. For the particular choice $(c_3,c_4)=(1/2,1/2)$, we have

$$\begin{aligned}\tilde{B}(\xi;1/2,1/2) &:= \xi\rfloor\left[\omega^i-\frac{1}{2}\Delta\omega^i\right]\wedge\Delta\rho_i\\ &\quad+\Delta\omega^i\wedge\xi\rfloor\left[\rho_i-\frac{1}{2}\Delta\rho_i\right].\end{aligned}\tag{6.3}$$

A comparison with Eqs. (4.10) shows that the boundary term $\int_{\partial\Sigma}\tilde{B}(\xi;1/2,1/2)$ exactly coincides with our expression $\Gamma(\xi)$, Eq. (4.8).

Clearly, the result (6.3) represents only a Riemannian reduction of a more general So-like formula for the boundary term. With an obvious extension of notation, this more general formula can be represented in the form

$$B(\xi)=B(\xi;c_1,c_2,1/2,1/2).\tag{6.4}$$

Additional information on the general structure of B can be found in Ref. [28], where the conserved charges of several three-dimensional solutions were calculated. However, the results are not sufficiently sensitive to clearly recognize the general structure of a "good" expression for the boundary term in PGT, in three dimensions. Further work in this direction is needed.

In conclusion, we summarize our results as follows:

(a) First, we found general criteria that allow us to study conformally flat Riemannian spacetime configurations as solutions of PGT. These criteria are used to show that the OTT black hole, a solution of the BHT gravity, is a Riemannian solution of PGT.

(b) Then, we constructed a natural set of the asymptotic conditions and calculated the conserved charges of the OTT black hole as the values of the Hamiltonian boundary term. The expressions for the conserved charges coincide with those found by of Kwon *et al.* [24] in the generalized Brown-York approach.

(c) Finally, the obtained results are verified by showing that: (i) the conserved charges are exactly compatible with the first law of black hole thermodynamics, and (ii) our boundary term is in agreement with the generalized covariant formula proposed by So [18]. On the other hand, the OTT black hole appears to be an interesting physical example for the generalized covariant formula.

ACKNOWLEDGMENTS

M. B. would like to thank James Nester for a very instructive discussion on the form of covariant boundary terms. This work was supported by the Serbian Science Foundation under Grant No. 171031. The results are checked using the computer algebra systems Reduce and Mathematica.

APPENDIX A: USEFUL ASYMPTOTIC RELATIONS

In the Riemannian sector of PGT, the condition $T^i=0$, calculated on the asymptotic configurations (4.1) and (4.3), leads to an additional set of asymptotic requirements:

$$\begin{aligned}&\frac{r^2}{\ell^2}B^1{}_r-\ell\Omega^2{}_t=\mathcal{O}_1, \qquad B^1{}_r-\frac{\ell^2}{r^2}\Omega^0{}_\varphi=\mathcal{O}_1,\\ &\frac{r^2}{\ell}\Omega^1{}_r+\Omega^2{}_\varphi=\mathcal{O}_1, \qquad \frac{r^2}{\ell^2}\Omega^1{}_r+\Omega^0{}_t=\mathcal{O}_1,\\ &\frac{B^0{}_\varphi}{\ell}+\Omega^2{}_\varphi+B^2{}_t+\ell\Omega^0{}_t=\mathcal{O}_1.\end{aligned}\tag{A1}$$

Then, relying on the asymptotic form of the Schouten tensor L_{ij},

$$\begin{aligned}L_{00}&=\frac{1}{2\ell^2}-\frac{1}{r\ell}\left(B^0{}_t+\frac{r^2}{\ell^2}B^1{}_r\right)+\mathcal{O}_2,\\ L_{11}&=-\frac{1}{2\ell^2}+\mathcal{O}_2,\\ L_{22}&=-\frac{1}{2\ell^2}+\frac{1}{r\ell}\left(\frac{r^2}{\ell^2}B^1{}_r+\frac{1}{\ell}B^2{}_\varphi\right)+\mathcal{O}_2,\\ L_{02}&=-\frac{1}{\ell^2 r}B^0{}_\varphi+\frac{r}{\ell^2}\Omega^1{}_r+\mathcal{O}_2,\end{aligned}$$

one obtains the asymptotic relations

$$\begin{aligned}\Delta H_{ijt}&=-4a_0\Omega_{ij\varphi}+\mathcal{O}\left(\frac{\Omega_{ij\varphi}}{r}\right),\\ \Delta H_{ij\varphi}&=-4a_0\ell^2\Omega_{ijt}+\mathcal{O}\left(\frac{\Omega_{ijt}}{r}\right).\end{aligned}\tag{A2}$$

APPENDIX B: CONSISTENCY OF THE BOUNDARY TERM

In this appendix, we prove the consistency of the Hamiltonian boundary term (4.8) by showing that it is a finite expression that satisfies the variational equations (4.7a). Using the expressions (4.10) for $\mathcal{E}$ and $\mathcal{J}$, as well as the results of Appendix A, we have

$$\mathcal{E} = 4a_0 \frac{r}{\ell}\left(\Omega^0{}_\varphi - \frac{r^2}{\ell^2} B^1{}_r\right) + \mathcal{O}_0 = \mathcal{O}_0, \tag{B1a}$$

$$\begin{aligned} \mathcal{J} &= 2a_0\omega^i{}_\varphi b_{i\varphi} + 4a_0\ell^2 L_{ij}\omega^i{}_\varphi b^j{}_\varphi \\ &= -4a_0 r\left(\frac{B^0{}_\varphi}{\ell} + \Omega^2{}_\varphi\right) - 4a_0\ell r^2 L_{02} + \mathcal{O}_0, \\ &= -4a_0 r\left(\Omega^2{}_\varphi + \frac{r^2}{\ell}\Omega^1{}_r\right) + \mathcal{O}_0 = \mathcal{O}_0, \end{aligned} \tag{B1b}$$

which completes the proof of finiteness.

In a similar manner,

$$\begin{aligned} \delta\mathcal{E} = &\frac{1}{2}(\omega^{ij}{}_t \delta H_{ij\varphi} + \delta\omega_{ij\varphi} H^{ij}{}_t) \\ &+ \frac{1}{4}(\Delta H_{ij\varphi}\delta\omega^{ij}{}_t - \Delta\omega^{ij}{}_t \delta H_{ij\varphi} - \Delta H_{ijt}\delta\omega^{ij}{}_\varphi \\ &+ \Delta\omega^{ij}{}_\varphi \delta H_{ijt}), \\ = &\frac{1}{2}(\omega^{ij}{}_t \delta H_{ij\varphi} + \delta\omega_{ij\varphi} H^{ij}{}_t) + \mathcal{O}_1, \end{aligned} \tag{B2}$$

whereas the proof for $\delta\mathcal{J}$ is trivial. Thus, the variational equation (4.7a) is satisfied.

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Siklos waves in Poincaré gauge theory

M. Blagojević* and B. Cvetković†
Institute of Physics, University of Belgrade, Pregrevica 118, 11080 Belgrade, Serbia
(Received 14 June 2015; published 29 July 2015)

A class of Siklos waves, representing exact vacuum solutions of general relativity with a cosmological constant, is extended to a new class of Siklos waves with torsion, defined in the framework of the Poincaré gauge theory. Three particular exact vacuum solutions of this type, the generalized Kaigorodov, the homogeneous solution and the exponential solution, are explicitly constructed.

DOI: 10.1103/PhysRevD.92.024047 PACS numbers: 04.50.Kd, 04.20.Jb, 04.30.-w

I. INTRODUCTION

The first complete formulation of the idea of (internal) gauge invariance was given in Weyl's classic paper [1]. Significant progress in this direction was achieved somewhat later by Yang, Mills and Utiyama [2,3]. It opened a new perspective for understanding gravity as a gauge theory, the perspective that was realized by Kibble and Sciama [4] in their proposal of a new theory of gravity, known as the Poincaré gauge theory (PGT). The PGT is a gauge theory of the Poincaré group, with an underlying Riemann-Cartan (RC) geometry of spacetime [5,6]. In this approach, basic gravitational variables are the tetrad field b^i and the Lorentz connection ω^{ij} (1-forms), and the related field strengths are the torsion $T^i = db^i + \omega^i{}_m \wedge b^{mj}$ and the curvature $R^{ij} = d\omega^{ij} + \omega^i{}_m \wedge \omega^{mj}$ (2-forms). At a more physical level, the source of gravity in PGT is matter possessing both the energy-momentum and spin currents. The importance of the Poincaré symmetry in particle physics leads one to consider PGT as a favorable framework for describing the gravitational phenomena.

Based on the experience stemming from Einstein's general relativity, it is known that exact solutions play a crucial role in developing our understanding of the geometric and physical content of a gravitational theory; for a review, see Refs. [7–10]. An important set of these solutions refers to exact gravitational waves, the structure of which has been studied also in PGT [11]. In the present work, we focus on a particular class of the gravitational waves, the class of Siklos waves that are vacuum solutions of general relativity with a cosmological constant (GR_Λ), propagating on the anti–de Sitter (AdS) background [12]. By generalizing the ideas developed in three dimensions [13], we construct here a class of the four-dimensional Siklos waves with torsion as vacuum solutions of PGT.

The paper is organized as follows. In Sec. II, we give a short account of the Siklos waves in the tetrad formulation of GR_Λ. In Sec. III, we show that Siklos waves are torsion-free vacuum solutions of PGT. In Sec. IV, we introduce new vacuum solutions of PGT, the Siklos waves with torsion, by modifying the Siklos geometry in a manner that preserves the radiation nature of the original configuration. That is achieved by an ansatz for the RC connection that produces only the tensorial irreducible mode of the torsion with $J^P = 2^+$. The PGT field equations are simplified and shown to depend only on three parameters, including the mass of the torsion mode. In Secs. V–VII, we describe three different vacuum solutions belonging to the class of Siklos waves with torsion: the generalized Kaigorodov, the homogeneous solution and the exponential solution. Section VII is devoted to concluding remarks, and two appendixes contain some technical details.

Our conventions are as follows. We use the Poincaré coordinates $x^\mu = (u, v, x, y)$ as the local coordinates; the latin indices $(i, j, \dots)$ refer to the local Lorentz (co)frame and run over $(+, -, 2, 3)$, b^i is the tetrad (1-form), and h_i is the dual basis (frame), such that $h_i \rfloor b^k = \delta_i^k$; the volume 4-form is $\hat\epsilon = b^+ \wedge b^- \wedge b^2 \wedge b^3$, the Hodge dual of a form α is $^\star\alpha$, with $^\star 1 = \hat\epsilon$, and the totally antisymmetric tensor is defined by $^\star(b_i \wedge b_j \wedge b_k \wedge b_m) = \varepsilon_{ijkm}$ and normalized to $\varepsilon_{+-23} = 1$; in the rest of the paper, the exterior product of forms is implicit.

II. SIKLOS WAVES IN GR_Λ

Siklos waves were introduced as a class of exact gravitational waves propagating on the AdS background [12]. In the Poincaré coordinates $x^\mu = (u, v, x, y)$, the Siklos metric is given by

$$ds^2 = \frac{\ell^2}{y^2}[2du(Hdu + dv) - dx^2 - dy^2], \tag{2.1}$$

with $H = H(u, x, y)$. It admits the null Killing vector field ∂_v that is not covariantly constant; the wave fronts are surfaces of constant u and v, and the case $H = 0$ corresponds to the AdS background. The metric (2.1) coincides with a special subclass of the Kundt class [9,10], and is obviously conformal to pp waves. The physical interpretation of the Siklos waves was investigated by Podolský [14,15].

*mb@ipb.ac.rs
†cbranislav@ipb.ac.rs

Now we give a short account of the Siklos waves in the tetrad formulation of GR_Λ, which allows for a simpler generalization to PGT. First, we choose the tetrad field in the form

$$\begin{aligned} b^+ &:= \frac{\ell}{y} du, \qquad b^- := \frac{\ell}{y}(H du + dv), \\ b^2 &:= \frac{\ell}{y} dx, \qquad b^3 := \frac{\ell}{y} dy, \end{aligned} \tag{2.2}$$

so that the line element becomes $ds^2 = 2b^+b^- - (b^2)^2 - (b^3)^2 \equiv \eta_{ij} b^i b^j$, where η is the half-null Minkowski metric,

$$\eta_{ij} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}.$$

The dual frame h_i is given by

$$\begin{aligned} h_+ &= \frac{y}{\ell}(\partial_u - H\partial_v), \qquad h_- = \frac{y}{\ell}\partial_v, \\ h_2 &= \frac{y}{\ell}\partial_x, \qquad h_3 = \frac{y}{\ell}\partial_y. \end{aligned} \tag{2.3}$$

Next, we introduce the Riemannian connection ω^{ij} by imposing the condition of vanishing torsion, $\nabla b^i := db^i + \omega^i{}_m b^m = 0$, which yields

$$\begin{aligned} &\omega^{+-}, \omega^{+2} = 0, \qquad \omega^{+3} = \frac{1}{\ell} b^+, \\ &\omega^{23} = \frac{1}{\ell} b^2, \qquad \omega^{-2} = -\frac{y}{\ell}\partial_x H b^+, \\ &\omega^{-3} = \frac{1}{\ell} b^- - \frac{y}{\ell}\partial_y H b^+. \end{aligned} \tag{2.4a}$$

The wave nature of the Siklos wave is clearly seen by rewriting ω^{ij} in the form

$$\omega^{ij} = \bar\omega^{ij} + k^i (h^j \rfloor H) b^+, \tag{2.4b}$$

where $\bar\omega^{ij} = \omega^{ij}(H=0)$ refers to the AdS background, and the second term is the radiation piece, characterized by the null vector $k^i = (k^+, k^-, k^2, k^3) = (0,1,0,0)$.

Now one can calculate the Riemannian curvature:

$$\begin{aligned} R^{+j} &= \frac{1}{\ell^2} b^+ b^j, \qquad R^{23} = \frac{1}{\ell^2} b^2 b^3, \\ R^{-2} &= \frac{1}{\ell^2} b^- b^2 + \frac{1}{\ell^2}(y^2\partial_{xx}H - y\partial_y H) b^+ b^2 \\ &\quad + \frac{1}{\ell^2}(y^2 \partial_{xy} H) b^+ b^3, \\ R^{-3} &= \frac{1}{\ell^2} b^- b^3 + \frac{1}{\ell^2}(y^2\partial_{yy}H - y\partial_y H) b^+ b^3 \\ &\quad + \frac{1}{\ell^2}(y^2 \partial_{xy} H) b^+ b^2, \end{aligned} \tag{2.5}$$

where we use $\partial_{xx} := \partial^2/\partial x^2$ etc. The Ricci curvature $Ric^i = h_m \rfloor R^{mi}$ and the scalar curvature $R = h_i \rfloor Ric^i$ are found to be

$$\begin{aligned} Ric^m &= \frac{3}{\ell^2} b^m, \qquad m = +, 2, 3, \\ Ric^- &= \frac{3}{\ell^2} b^- + \frac{1}{\ell^2}(y^2\partial_{xx}H + y^2\partial_{yy}H - 2y\partial_y H) b^+, \\ R &= \frac{12}{\ell^2}. \end{aligned} \tag{2.6}$$

Dynamical structure of GR_Λ is defined by the action $I_\Lambda = -\int d^4x \sqrt{-g}(a_0 R + 2\Lambda)$. The corresponding *vacuum* field equations can be suitably written in the traceless form as

$$Ric^i - \frac{1}{4} R b^i = 0. \tag{2.7}$$

As a consequence, the metric function H must obey

$$y^2(\partial_{xx}H + \partial_{yy}H) - 2y\partial_y H = 0. \tag{2.8}$$

The profile (u-dependence) of the Siklos wave may be arbitrary.

We display here three special solutions of (2.8) discussed by Siklos [12]:

$H_1 = y^3$, Kaigorodov's solution (1963);

$H_2 = \arctan(x/y) + xy/(x^2+y^2)$, $\tilde H_2 = (x^2+y^2) H_2$;

$H_3 = C_1 e^x (\cos y + y \sin y) + C_2 e^x (\sin y - y\cos y)$.

Note that Defrise's metric (1969), with $H = 1/y^2$, is *not* a vacuum solution of GR_Λ [15].

III. SIKLOS WAVES AS TORSION-FREE SOLUTIONS OF PGT

In this section, we show that the Siklos spacetime of the previous section is an exact Riemannian solution of PGT in vacuum.

Starting from the general PGT dynamics described in Appendix B, one can easily derive its reduced form in the Riemannian sector of PGT, characterized by $T^i = 0$. First, we note that the only nonvanishing irreducible components of the Riemannian curvature are ${}^{(1)}R^{ij}$, ${}^{(4)}R^{ij}$ and ${}^{(6)}R^{ij}$, defined in Appendix A. And second, the condition $T^i = 0$ implies that dynamical evolution of the Riemannian solutions in PGT is described by a reduced form of the general field equations (B3):

$$\begin{aligned}(1ST) \quad & E_i = 0, \\ (2ND) \quad & \nabla H_{ij} = 0.\end{aligned} \tag{3.1a}$$

Here, the Riemannian expressions for E_i and H_{ij} are obtained directly from the corresponding PGT formulas (see Appendix B) in the limit $T^i = 0$:

$$\begin{aligned}H_{ij} &= -2a_0{}^\star(b^i b^j) + 2{}^\star(b_1{}^{(1)}R_{ij} + b_4{}^{(4)}R_{ij} + b_6{}^{(6)}R_{ij}), \\ E_i &:= h_i \rfloor L_G - \frac{1}{2}(h_i \rfloor R^{mn})H_{mn}.\end{aligned} \tag{3.1b}$$

As shown in Ref. [5], the field equations (3.1) are satisfied for any configuration in which the traceless symmetric Ricci tensor vanishes:

$$Ric_{(ij)} - \frac{1}{4}\eta_{ij}R = 0. \tag{3.2}$$

Comparing this result with the GR_Λ field equation (2.7), one concludes that any vacuum solution of GR_Λ is automatically a torsion-free solution of PGT. In particular, this is true for the Siklos metric.

It is useful to explore this general statement in detail. Using the geometry of the Siklos spacetime found in the previous section, the content of Eqs. (3.1a) is found to be

$$\begin{aligned}(1ST) \quad & (b_4 + b_6 - a_0\ell^2)y[y(\partial_{xx}H + \partial_{yy}H) - 2\partial_y H] = 0, \\ & 3a_0 + \ell^2\Lambda = 0, \\ (2ND) \quad & (b_1 + b_4)y^2\partial_x[y(\partial_{xx}H + \partial_{yy}H) - 2\partial_y H] = 0, \\ & (b_1 + b_4)y^2\partial_y[y(\partial_{xx}H + \partial_{yy}H) - 2\partial_y H] = 0.\end{aligned} \tag{3.3}$$

For the generic values of the Lagrangian parameters a_0, b_1, b_4, b_6, dynamical content of these equations is obviously the same as in GR_Λ, since the metric function H must be such that

$$\hat{S}H := y(\partial_{xx}H + \partial_{yy}H) - 2\partial_y H = 0. \tag{3.4}$$

Thus, although PGT has a rather different dynamical structure as compared to GR_Λ, the class of Riemannian Siklos spacetimes is still an exact vacuum solution of PGT.

IV. SIKLOS WAVES WITH TORSION

We are now ready to generalize the previous results by constructing a new, non-Riemannian class of Siklos waves, the Siklos waves with torsion.

A. Geometry of the ansatz

We wish to introduce torsion in a manner that preserves the radiation nature of the Riemannian Siklos waves of GR_Λ, relying on the approach proposed in [13].

We start the construction by assuming that the tetrad field in PGT retains its Riemannian form (2.2). Then, by noting that the radiation piece of the Riemannin connection (2.4) has the form $(\omega^{ij})^R = k^i(h^{j\mu}\partial_\mu H)b^+$, we assume that the new RC connection is given by

$$\omega^{ij} = \bar{\omega}^{ij} + k^i h^{j\mu}(\partial_\mu H + K_\mu)b^+, \tag{4.1a}$$

where the form of K_μ is defined by

$$\begin{aligned}&K_\mu = (0, 0, K_x, K_y), \\ &K_x = K_x(u, x, y), \qquad K_y = K_y(u, x, y).\end{aligned} \tag{4.1b}$$

This ansatz modifies only two components of the Riemannian connection (2.4):

$$\begin{aligned}\omega^{-2} &= -\frac{y}{\ell}(\partial_x H + K_x)b^+, \\ \omega^{-3} &= \frac{1}{\ell}b^- - \frac{y}{\ell}(\partial_y H + K_y)b^+.\end{aligned}$$

The new terms in the connection are related to the torsion of spacetime:

$$T^- = \frac{y}{\ell}(K_x b^+ b^2 + K_y b^+ b^3), \qquad T^+, T^2, T^3 = 0. \tag{4.2}$$

The only nonvanishing irreducible torsion piece is the tensor piece ${}^{(1)}T^i$, with ${}^{(1)}T^- = T^-$.

Denoting the Riemannian curvature found in Sec. II by $\tilde{R}^{ij}$, the new RC curvature is found to have the form

$$\begin{aligned}R^{+j} &= \frac{1}{\ell^2}b^+b^j, \qquad R^{23} = \frac{1}{\ell^2}b^2b^3, \\ R^{-2} &= \tilde{R}^{-2} + \frac{1}{\ell^2}(y^2\partial_x K_x - yK_y)b^+b^2 + \frac{1}{\ell^2}(y^2\partial_y K_x)b^+b^3, \\ R^{-3} &= \tilde{R}^{-3} + \frac{1}{\ell^2}(y^2\partial_y K_y)b^+b^3 + \frac{1}{\ell^2}(y^2\partial_x K_y + yK_x)b^+b^2.\end{aligned} \tag{4.3a}$$

Note that the radiation piece of R^{ij} is proportional to the null vector $k^i = (0, 1, 0, 0)$. The corresponding Ricci and scalar curvatures are

$$Ric^m = \frac{3}{\ell^2} b^a, \qquad m = +,2,3,$$
$$Ric^- = \widetilde{Ric}^- + \frac{1}{\ell^2}(y^2\partial_x K_x + y^2\partial_y K_y - yK_y)b^+,$$
$$R = \frac{12}{\ell^2}. \tag{4.3b}$$

The nonvanishing irreducible components of the curvature are ${}^{(n)}R^{ij}$ for $n = 1,4,6$ (as in GR_Λ) and $n = 2$. Quadratic invariants of the field strengths are regular:

$$R^{ij\star}R_{ij} = \frac{12}{\ell^4}\hat{\epsilon}, \qquad T^{i\star}T_i = 0.$$

B. Field equations

Dynamical content of our ansatz is effectively described by the RC Lagrangian (B1) with nonvanishing parameters $(a_0, \Lambda; a_1, b_1, b_2, b_4, b_6)$ and the associated PGT field equations (B3). Explicit calculation of the second field equation in (B3), denoted shortly by $\mathcal{F}^{ij}$, is shown to have two nontrivial components, $\mathcal{F}^{-2}$ and $\mathcal{F}^{-3}$. After introducing the quantity $\hat{S}H$ as in Eq. (3.4), these components take the respective forms

$$\begin{aligned} &b_1(y\partial_x\hat{S}H + y^2\partial_{xx}K_x + y^2\partial_{yy}K_x - 2y\partial_x K_y)\\ &+ b_2(y^2\partial_{yy}K_x - y^2\partial_{xy}K_y - y\partial_x K_y)\\ &+ b_4(y\partial_x\hat{S}H + y^2\partial_{xx}K_x + y^2\partial_{xy}K_y - y\partial_x K_y)\\ &+ 2(b_6 - b_1 + a_1\ell^2 - a_0\ell^2)K_x = 0, \end{aligned} \tag{4.4a}$$

and

$$\begin{aligned} &b_1(y\partial_y\hat{S}H + y^2\partial_{xx}K_y + y^2\partial_{yy}K_y + 2y\partial_x K_x)\\ &+ b_2(-y^2\partial_{xy}K_x + y^2\partial_{xx}K_y + y\partial_x K_x)\\ &+ b_4(y\partial_y\hat{S}H + y^2\partial_{xy}K_x + y^2\partial_{yy}K_y + y\partial_x K_x)\\ &+ 2(b_6 - b_1 + a_1\ell^2 - a_0\ell^2)K_y = 0. \end{aligned} \tag{4.4b}$$

The content of the first field equation is much simpler. To have the smooth limit for vanishing torsion, we require $3a_0 + \ell^2\Lambda = 0$, whereupon the first equation reads

$$\begin{aligned} &(b_4 + b_6 - a_0\ell^2)\hat{S}H\\ &+ (b_4 + b_6 - a_0\ell^2 + a_1\ell^2)(y\partial_x K_x + y\partial_y K_y - K_y) = 0. \end{aligned} \tag{4.4c}$$

The form of the differential equations (4.4) appears to be rather complicated [16]. However, there exists a suitable reformulation that makes their content much more transparent. To see that, we first rewrite Eq. (4.4c) in the form

$$\hat{S}H = \sigma(y\partial_x K_x + y\partial_y K_y - K_y),$$
$$\sigma := -\left(1 + \frac{a_1\ell^2}{b_4 + b_6 - a_0\ell^2}\right). \tag{4.5a}$$

Then, by substituting the expressions for $y\partial_x\hat{S}H$ and $y\partial_y\hat{S}H$ into (4.4a)–(4.4b), and dividing the resulting equations by $(b_1 + b_4)(\sigma + 1)$, one obtains

$$\begin{aligned} &(y^2\partial_{xx} + \rho y^2\partial_{yy} + 2\ell^2\mu^2)K_x\\ &\quad + [(1-\rho)y^2\partial_{xy} - (1+\rho)y\partial_x]K_y = 0, \end{aligned} \tag{4.5b}$$

$$\begin{aligned} &(y^2\partial_{yy} + \rho y^2\partial_{xx} + 2\ell^2\mu^2)K_y\\ &\quad + [(1-\rho)y^2\partial_{xy} + (1+\rho)y\partial_x]K_x = 0, \end{aligned} \tag{4.5c}$$

where

$$\rho := \frac{b_1 + b_2}{(b_1 + b_4)(\sigma + 1)}, \qquad \mu^2 := \frac{a_1 - a_0 + (b_6 - b_1)/\ell^2}{(b_1 + b_4)(\sigma + 1)}.$$

The final equations (4.5) contain only three independent parameters, σ, ρ and μ^2, which makes it much easier to find some specific solutions for the Siklos waves with torsion.

The parameter μ^2 has a simple physical interpretation. As the linearized PGT analysis shows, possible torsion excitations around the Minkowski background are modes with spin parity $J^P = 0^\pm, 1^\pm, 2^\pm$ [17]. In particular, the spin-2^+ state is associated to the tensorial piece of the torsion, and its mass is

$$\bar{\mu}^2 = \frac{a_0(a_1 - a_0)}{a_1(b_1 + b_4)}.$$

For $1/\ell^2 \to 0$, the coefficient μ^2 tends exactly to $\bar{\mu}^2$, whereas for finite (and positive) ℓ^2, μ^2 is associated to the spin-2^+ torsion excitation with respect to the AdS background.

In what follows, we present three exact solutions of the PGT field equations (4.5), enlightening thereby basic dynamical aspects of the Siklos waves with torsion. All the integration "constants" appearing in these solutions are functions of u.

V. KAIGORODOV-LIKE SOLUTION

Motivated by the form of the Kaigorodov solution of GR_Λ (Sec. II), we consider now a class of PGT configurations for which the functions H, K_x and K_y are x independent. Then the field equations (4.5) take a much simpler form:

$$(\rho y^2\partial_{yy} + 2\mu^2\ell^2)K_x = 0, \tag{5.1a}$$

$$(y^2\partial_{yy} + 2\mu^2\ell^2)K_y = 0, \tag{5.1b}$$

$$y\partial_{yy}H - 2\partial_y H = \sigma(y\partial_y - 1)K_y. \tag{5.1c}$$

The Euler-Fuchs differential equation (5.1a) is solved by the ansatz $K_x = y^\alpha$, where α is restricted by the requirement $\alpha^2 - \alpha + 2\mu^2\ell^2/\rho = 0$, which implies

$$\alpha_\pm = \frac{1}{2} \pm p, \qquad p := \frac{1}{2}\sqrt{1 - 8\mu^2\ell^2/\rho}. \tag{5.2}$$

(a1) For $8\mu^2\ell^2/\rho < 1$ (real p),

$$K_x = \sqrt{y}(A_1 y^p + A_2 y^{-p}). \tag{5.3a}$$

(a2) For $8\mu^2\ell^2/\rho > 1$ (imaginary p, $q := |p|$),

$$K_x = \sqrt{y}[A_3\cos(q\ln y) + A_4\sin(q\ln y)]. \tag{5.3b}$$

(a3) For $8\mu^2\ell^2/\rho = 1$ ($p = 0$),

$$K_x = \sqrt{y}(A_5 + A_6\ln y). \tag{5.3c}$$

Equation (5.1b) follows from (5.1a) in the limit $\rho \to 1$. Hence, using the notation

$$\bar\alpha_\pm = \frac{1}{2} \pm \bar p, \qquad \bar p := \frac{1}{2}\sqrt{1 - 8\mu^2\ell^2}, \qquad \bar q = |\bar p|, \tag{5.4}$$

the solutions for K_y can be obtained from Eqs. (5.3) by the replacements $p \to \bar p$, $q \to \bar q$.

(b1) For $8\mu^2\ell^2 < 1$,

$$K_y = \sqrt{y}(B_1 y^{\bar p} + B_2 y^{-\bar p}). \tag{5.5a}$$

(b2) For $8\mu^2\ell^2 > 1$,

$$K_y = \sqrt{y}[B_3\cos(\bar q\ln y) + B_4\sin(\bar q\ln y)]. \tag{5.5b}$$

(b3) For $8\mu^2\ell^2 = 1$,

$$K_y = \sqrt{y}(B_5 + B_6\ln y). \tag{5.5c}$$

Knowing the form of K_y, one can integrate Eq. (5.1c) to obtain the metric function H. Let us first find a particular solution $H_{(i)}$ of the inhomogeneous equation (5.1c).

(c1) For $8\mu^2\ell^2 < 1$,

$$H_{(i)} = \sigma y^{3/2}\left(\frac{(\bar\alpha_+ - 1)}{(\bar\alpha_+ + 1)(\bar\alpha_+ - 2)}B_1 y^{\bar p} + \frac{(\bar\alpha_- - 1)}{(\bar\alpha_- + 1)(\bar\alpha_- - 2)}B_2 y^{-\bar p}\right). \tag{5.6a}$$

(c2) For $8\mu^2\ell^2 > 1$,

$$H_{(i)} = \frac{2\sigma}{9 + 4\bar q^2}y^{3/2}[(B_3 - 2B_4\bar q)\cos(\bar q\ln y) + (B_4 + 2B_3\bar q)\sin(\bar q\ln y)]. \tag{5.6b}$$

(c3) For $8\mu^2\ell^2 = 1$,

$$H_{(i)} = \frac{2\sigma}{9}y^{3/2}(B_5 - 2B_6 + B_6\ln y). \tag{5.6c}$$

Adding to $H_{(i)}$ the solution of the homogeneous equation (5.1c), that is the Kaigorodov solution H_1 from Sec. II, one obtains the complete solution:

$$H = H_1 + H_{(i)}, \qquad H_1 = Dy^3. \tag{5.7}$$

Thus, the existence of torsion has a direct influence on the form of metric.

The above solutions for K_x, K_y and H define a Kaigorodov wave with torsion as a vacuum solution of PGT.

A. Asymptotic AdS limit

It is interesting to note that the Kaigorodov solution in GR_Λ is asymptotically AdS, as follows from the asymptotic relation $H = \mathcal{O}(y^3)$ for $y \to 0$, and the form of the Riemannian curvature (2.5). In PGT, the presence of torsion makes the situation not so simple. Namely, the condition that the RC curvature R^{ij} in (4.3) has the AdS asymptotics produces two types of requirements: the first one is obtained from the non-Riemannian piece of R^{ij},

$$yK_x \to 0, \qquad yK_y \to 0, \tag{5.8a}$$

$$y^2\partial_y K_x \to 0, \qquad y^2\partial_y K_y \to 0, \tag{5.8b}$$

and the second from the Riemannian piece:

$$y\partial_y H_{(i)} \to 0, \qquad y^2\partial_{yy}H_{(i)} \to 0. \tag{5.8c}$$

Further analysis goes as follows.

(i) In the sector with $8\mu^2\ell^2/\rho \geq 1$ and $8\mu^2\ell^2 \geq 1$, one can directly verify that the solutions for K_x, K_y and $H_{(i)}$ satisfy the requirements (5.8).

(ii) In the complementary sector with $8\mu^2\ell^2/\rho < 1$ and $8\mu^2\ell^2 < 1$, one finds that the requirements (5.8) are valid for $p < 1$ and $\bar p < 1$, or equivalently, for

$$8\mu^2\ell^2/\rho > -1 \quad \text{and} \quad 8\mu^2\ell^2 > -1. \tag{5.9}$$

Continuing with exploring the asymptotic properties of the torsion, we see that (5.8a) implies $T^i \to 0$ for $y \to 0$. Thus, the choice of parameters described in (5.9) ensures that the Kaigorodov-like solution has an AdS asymptotic behavior, with vanishing torsion. Clearly, in the physical sector with $\mu^2 \geq 0$, the second condition in (5.9) is automatically satisfied.

B. Defrise-like solution as a special case

It is interesting to observe that the form of $H_{(i)}$ in (5.6a) allows us to obtain a generalized Defrise solution, defined in Sec. II, as a special case of the Kaigorodov wave with torsion. Namely, by choosing $D = 0$ one eliminates H_1 from H, whereupon the term $H_{(i)}$, specified by $B_1 = 0$ and $\bar{p} = 7/2$, becomes identical to the Defrise metric function:

$$H = H_{(i)} \sim 1/y^2. \tag{5.10}$$

The restriction $\bar{p} = 7/2$ refers to the tachyonic sector of the 2^+ torsion mode, with $\mu^2\ell^2 = -6$. The above result for H, combined with the corresponding expressions for K_x and K_y, defines the Defrise solution with torsion as a *vacuum* solution of PGT. In contrast to that, the corresponding solution in GR_Λ exists only in the presence of *matter*. One should stress that the metric function H originates purely from the torsional term $H_{(i)}$.

VI. HOMOGENEOUS SOLUTION

Let us now look for a solution in which K_x, K_y, H are homogeneous functions of y and x:

$$K_x = f_x(t), \qquad K_y = f_y(t),$$
$$H = h(t), \qquad t := y/x.$$

As a consequence, the field equations (4.5) become

$$(t^4 + \rho t^2)f_x'' + 2t^3 f_x' + 2\mu^2 f_x - (1-\rho)t^3 f_y'' + 2\rho t^2 f_y' = 0, \tag{6.1a}$$

$$(t^2 + \rho t^4)f_y'' + 2\rho t^3 f_y' + 2\mu^2 f_y - (1-\rho)t^3 f_x'' - 2t^2 f_x' = 0, \tag{6.1b}$$

$$\hat{S}H = \sigma(-t^2 f_x' + t f_y' - f_y), \tag{6.1c}$$

where $\hat{S}H = y[2t(t^2-1)h' + (t^4+t^2)h'']$.

The set of equations (6.1) represents a system of ordinary, second-order, linear differential equations. The system is significantly simplified by assuming that the metric function H retains the same form as in GR_Λ, so that $\hat{S}H = 0$. Consequently, the right-hand side of Eq. (6.1c) vanishes, $-t^2 f_x' + t f_y' - f_y = 0$, which implies

$$f_x = \frac{1}{t} f_y + B, \tag{6.2}$$

where $B = B(u)$. Substituting this expression into (6.1a)–(6.1b), one obtains

$$\rho t^2(t^2+1)f_y'' + 2\rho t(t^2-1)f_y' + 2(\rho + \mu^2\ell^2)f_y + 2\mu^2 tB = 0, \tag{6.3a}$$

$$\rho t^2(t^2+1)f_y'' + 2\rho t(t^2-1)f_y' + 2(\rho + \mu^2\ell^2)f_y = 0. \tag{6.3b}$$

Taking the difference of these two equations yields

$$\mu^2 B = 0.$$

Hence, either μ^2 or B has to vanish.

A. Case $\mu^2 = 0$

Assuming $\rho \neq 0$, the set of equations (6.3) reduces to

$$t^2(t^2+1)f_y'' + 2t(t^2-1)f_y' + 2f_y = 0.$$

Hence, the general solution for f_y is given by

$$f_y = C_1 \frac{t}{t^2+1} + C_2 \frac{t^2}{t^2+1}, \tag{6.4}$$

f_x is determined by (6.2), and the metric function has the same form as in GR_Λ:

$$h = C_3\left(-\arctan t + \frac{t}{1+t^2}\right) + C_4. \tag{6.5}$$

As before, all the integration constants are functions of u.

B. Case $B = 0$

In this case, the set of equations (6.3) reduces to

$$t^2(t^2+1)f_y'' + 2t(t^2-1)f_y' + 2\left(1 + \frac{\mu^2\ell^2}{\rho}\right)f_y = 0.$$

(d1) For $8\mu^2\ell^2/\rho \neq 1$,

$$f_y = C_5 t^{\frac{3}{2}-\xi} {}_2F_1\left(\frac{3}{4} - \frac{\xi}{2}, \frac{5}{4} - \frac{\xi}{2}; 1-\xi; -t^2\right) + C_6 t^{\frac{3}{2}+\xi} {}_2F_1\left(\frac{3}{4} + \frac{\xi}{2}, \frac{5}{4} + \frac{\xi}{2}; 1+\xi; -t^2\right) \tag{6.6a}$$

where $\xi = \frac{1}{2}\sqrt{1 - 8\mu^2\ell^2/\rho}$ and ${}_2F_1(a,b;c;z)$ is the hypergeometric function [18].

(d2) For $8\mu^2\ell^2/\rho = 1$,

$$f_y = C_7 t^{3/2} {}_2F_1\left(\frac{3}{4}, \frac{5}{4}; 1; -t^2\right) + C_8 G^{20}_{20}\left(-t^2 \middle| \begin{matrix} 1/2, 1 \\ 3/4, 3/4 \end{matrix}\right), \tag{6.6b}$$

where G^{mn}_{pq} is the Meijer G function [18]. In both cases, the associated solution for f_x is given by $f_x = f_y/t$, see (6.2), and the metric function h remains the same as in (6.5).

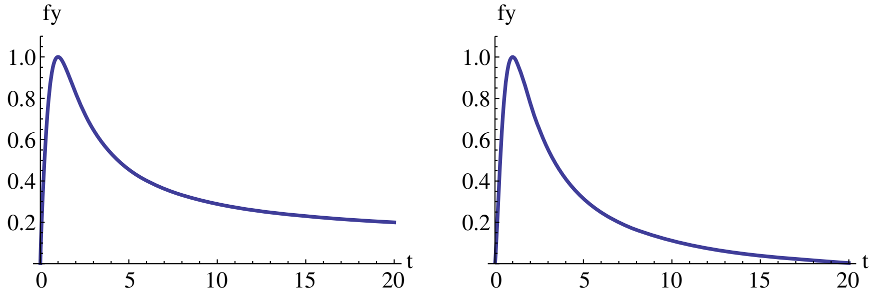

FIG. 1 (color online). The plots of the torsion function f_y in (6.6a), $8\mu^2\ell^2/\rho = -1$, $f_y[1] = 1, f'_y[1] = 0$ (left), and in (6.6b), $f_y[1] = 1$, $f'_y[1] = 0$ (right).

In the above two cases (d1)–(d2), the forms of the corresponding torsion functions f_y are illustrated in Fig. 1.

VII. EXPONENTIAL SOLUTION

In this section, we start with

$$K_x = e^x f_x(y), \qquad K_y = e^x f_y(y), \qquad H = e^x h(y), \tag{7.1}$$

whereupon the field equations (4.5) become

$$(y^2 + \rho y^2\partial_{yy} + 2\mu^2\ell^2)f_x + [(1-\rho)y^2\partial_y - (1+\rho)y]f_y = 0, \tag{7.2a}$$

$$(y^2\partial_{yy} + \rho y^2 + 2\mu^2\ell^2)f_y + [(1-\rho)y^2\partial_y + (1+\rho)y]f_x = 0, \tag{7.2b}$$

$$\hat{S}H = \sigma(yf_x + y\partial_y f_y - f_y), \tag{7.2c}$$

and $\hat{S}H = e^x[y(h + h'') - 2h']$.

As in the previous section, we assume that H coincides with the vacuum solution of GR_Λ, defined by $\hat{S}H = 0$. This imposes an extra condition on f_x and f_y:

$$yf_x + y\partial_y f_y - f_y = 0 \Rightarrow \frac{f_x}{y} + \left(\frac{f_y}{y}\right)' = 0. \tag{7.3}$$

By introducing a change of variables, given by

$$f_x = yg_x, \qquad f_y = yg_y, \tag{7.4a}$$

the condition (7.3) takes a simple form:

$$g_x + g'_y = 0. \tag{7.4b}$$

As a consequence, Eqs. (7.2a)–(7.2b) are transformed into

$$\rho y^2 g_y^{(3)} + 2\rho y g''_y + (\rho y^2 + 2\mu^2\ell^2)g'_y + 2\rho y g_y = 0, \tag{7.5a}$$

$$\rho y^2 g''_y + (\rho y^2 + 2\mu^2\ell^2)g_y = 0. \tag{7.5b}$$

One can note that (7.5a) is equal to the derivative (with respect to y) of (7.5b). The solution of (7.5b) reads

$$g_y = \sqrt{y}[D_1 J_\nu(y) + D_2 Y_\nu(y)], \tag{7.6}$$

where $\nu = \frac{1}{2}\sqrt{1 - 8\mu^2\ell^2/\rho}$, and J_ν, Y_ν are the Bessel functions of the first and second kind, respectively [18]. Hence,

$$f_y = y^{\frac{3}{2}}(D_1 J_\nu(y) + D_2 Y_\nu(y)), \tag{7.7a}$$

and $f_x = -yg'_y$ yields

$$f_x = \sqrt{y}[D_1(yJ_{\nu+1}(y) - (\nu + 1/2)J_\nu(y)) + D_2(yY_{\nu+1}(y) - (\nu + 1/2)Y_\nu(y))]. \tag{7.7b}$$

The forms of the torsion functions (7.7) are illustrated in Fig. 2. They are of the same type as the GR_Λ metric function H_3, defined in Sec. II. Together, they define our third specific Siklos wave with torsion.

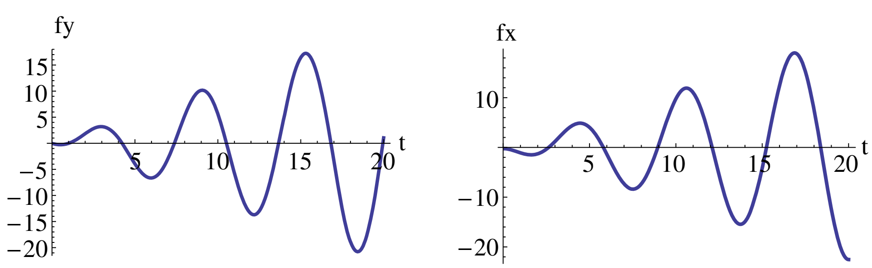

FIG. 2 (color online). The plots of the torsion functions (7.7) for $D_1 = D_2 = 1$, $8\mu^2\ell^2/\rho = -1$.

VIII. CONCLUDING REMARKS

In this paper, we introduced a new class of exact vacuum solutions of PGT, the Siklos waves with torsion. The solution is constructed in a way that respects the radiation nature of the original Siklos configuration in GR_Λ. This is achieved by an ansatz for the RC connection that produces only the tensorial irreducible mode of the torsion, propagating on the AdS background. A compact form of the PGT field equations is used to find three particular vacuum solutions belonging to the class of Siklos waves with torsion; they generalize the Kaigorodov, the homogeneous solution and the exponential solution of GR_Λ.

ACKNOWLEDGMENTS

This work was supported by the Serbian Science Foundation under Grant No. 171031.

APPENDIX A: IRREDUCIBLE DECOMPOSITION OF THE FIELD STRENGTHS

We present here formulas for the irreducible decomposition of torsion and curvature in four-dimensional Riemann-Cartan spacetime [5]; for general D, see [19].

It is convenient to start the exposition with the Bianchi identities:

$$\nabla T^i = R^i{}_m b^m, \qquad \nabla R^{ij} = 0. \tag{A1}$$

The torsion 2-form has three irreducible pieces:

$$\begin{aligned}
{}^{(2)}T^i &= \frac{1}{3} b^i \wedge (h_m \rfloor T^m), \\
{}^{(3)}T^i &= -\frac{1}{3} {}^\star[b^i \wedge {}^\star(T^m \wedge b_m)] = \frac{1}{3} h^i \rfloor (T^m \wedge b_m), \\
{}^{(1)}T^i &= T^i - {}^{(2)}T^i - {}^{(3)}T^i.
\end{aligned} \tag{A2}$$

The RC curvature 2-form can be decomposed into six irreducible pieces:

$$\begin{aligned}
&{}^{(2)}R^{ij} = -{}^*(b^{[i} \wedge \Psi^{j]}), \qquad {}^{(4)}R^{ij} = b^{[i} \wedge \Phi^{j]}, \\
&{}^{(3)}R^{ij} = -\frac{1}{12} X^*(b^i \wedge b^j), \qquad {}^{(6)}R^{ij} = \frac{1}{12} W b^i \wedge b^j, \\
&{}^{(5)}R^{ij} = \frac{1}{2} b^{[i} \wedge h^{j]} \rfloor (b^m \wedge W_m), \\
&{}^{(1)}R^{ij} = R^{ij} - \sum_{a=2}^{6} {}^{(a)}R^{ij},
\end{aligned}$$

where

$$\begin{aligned}
&W^i := h_m \rfloor R^{mi} = Ric^i, \qquad W := h_i \rfloor W^i = R, \\
&X^i := {}^*(R^{ki} \wedge b_k), \qquad X := h_i \rfloor X^i
\end{aligned} \tag{A3}$$

and

$$\begin{aligned}
\Phi_i &:= W_i - \frac{1}{4} b_i W - \frac{1}{2} h_i \rfloor (b^m \wedge W_m), \\
\Psi_i &:= X_i - \frac{1}{4} b_i X - \frac{1}{2} h_i \rfloor (b^m \wedge X_m).
\end{aligned} \tag{A4}$$

The trace and symmetry properties of ${}^{(n)}R_{ij}$ can be found in Ref. [19], page 127. All these properties are satisfied by our ansatz.

For torsion-free solutions, the first Bianchi identity in (A1) implies $X^i = 0$; hence ${}^{(2)}R^{ij}$ and ${}^{(3)}R^{ij}$ vanish. Moreover, $Ric_{[ij]} = 0$ implies ${}^{(5)}R^{ij} = 0$. The remaining three curvature parts, first, fourth and sixth, are the PGT analogues of the irreducible pieces of the Riemannian curvature. In Riemannian geometry, ${}^{(1)}R^{ij}$ coincides with the Weyl (conformal) tensor,

$$C^{ij} := R^{ij} - \frac{1}{2}(b^i Ric^j - b^j Ric^i) + \frac{1}{6} R b^i b^j,$$

but in the RC geometry, ${}^{(1)}R^{ij}$ differs from C^{ij} by the presence of torsion terms. Thus, ${}^{(1)}R^{ij}$ is a true extension of C^{ij} to the RC geometry. The fourth component is defined in terms of the symmetric traceless Ricci tensor,

$$\Phi_i = \left(Ric_{(ij)} - \frac{1}{4}\eta_{ij} R\right) b^j. \tag{A5}$$

The above formulas are taken from Refs. [5,19] with one modification: the definition of W^i is taken with an additional minus sign (Landau-Lifshitz convention), and for consistency, the overall signs of the fourth through sixth curvature parts are also changed.

APPENDIX B: PGT FIELD EQUATIONS

The gravitational dynamics of PGT is determined by a Lagrangian $L_G = L_G(b^i, T^i, R^{ij})$ (4-form), which is assumed to be at most quadratic in the field strengths (quadratic PGT) and parity invariant. The form of L_G can be conveniently represented as

$$L_G = -{}^\star(a_0 R + 2\Lambda) + \frac{1}{2} T^i H_i + \frac{1}{4} R^{ij} H'_{ij}, \tag{B1}$$

where $H_i := \partial L_G / \partial T^i$ (the covariant momentum) and H'_{ij} define the quadratic terms in L_G:

$$H_i = 2 \sum_{n=1}^{3} {}^\star(a_n {}^{(n)}T_i), \qquad H'_{ij} := 2 \sum_{n=1}^{6} {}^\star(b_n {}^{(n)}R_{ij}). \tag{B2a}$$

Varying L_G with respect to b^i and ω^{ij} yields the PGT field equations in vacuum. After introducing the complete covariant momentum $H_{ij} := \partial L_G / \partial R^{ij}$ by

$$H_{ij} = -2a_0{}^{\star}(b^i b^j) + H'_{ij}, \tag{B2b}$$

these equations can be written in a compact form as

$$(1ST) \quad \nabla H_i + E_i = 0,$$
$$(2ND) \quad \nabla H_{ij} + E_{ij} = 0, \tag{B3}$$

where E_i and E_{ij} are the gravitational energy-momentum and spin currents:

$$E_i := h_i \rfloor L_G - (h_i \rfloor T^m) H_m - \frac{1}{2}(h_i \rfloor R^{mn}) H_{mn},$$
$$E_{ij} := -(b_i H_j - b_j H_i). \tag{B4}$$

The general field equations (B3) are used in Sec. IV to describe specific dynamical aspects of the Siklos waves with torsion. In the Riemannian sector with $T^i = 0$, we have $H_i = 0$ and $E_{ij} = 0$, and the field equations (B3) reduce to the form given in Sec. III.

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PHYSICAL REVIEW D **90,** 044006 (2014)

Gravitational waves with torsion in 3D

M. Blagojević[*] and B. Cvetković[†]
University of Belgrade, Institute of Physics, P.O. Box 57, 11001 Belgrade, Serbia
(Received 17 June 2014; published 1 August 2014)

We study gravitational waves with torsion as exact vacuum solutions of three-dimensional gravity with propagating torsion. The new solutions are a natural generalization of the plane-fronted gravitational waves in general relativity with a cosmological constant, in the presence of matter.

DOI: 10.1103/PhysRevD.90.044006 PACS numbers: 04.20.Jb, 04.50.Kd, 04.60.Kz, 04.30.-w

I. INTRODUCTION

Investigations of three-dimensional (3D) gravity have had an important influence on our understanding of both classical and quantum aspects of the realistic gravitational dynamics. In this context, the traditional approach based on *general relativity* has led to a number of outstanding results [1]. However, in the early 1990s, Mielke and Baekler [2] initiated a new approach to 3D gravity, relying on a modern field-theoretic formulation of gravity, the *Poincaré gauge theory* (PGT), proposed in the early 1960s by Kibble and Sciama [3–6]. Compared to general relativity, the dynamical structure of PGT is extended by using both the curvature and the torsion to describe the associated Riemann—Cartan (RC) geometry of spacetime.

The Mielke—Baekler model, like Einstein's general relativity, is a topological theory without propagating degrees of freedom. In PGT, such an unrealistic feature of the gravitational dynamics can be naturally improved by going over to a Lagrangian that is at most quadratic in torsion and curvature (quadratic PGT). Recent investigations reveal elements that indicate a rich dynamical structure of the *quadratic PGT* [7–10]: the theory possesses a number of *propagating torsion modes* (tordions) and black hole solutions, its (anti-)de Sitter [(A)dS] sector is characterized by well-defined conserved charges and central charges, the existence of torsion is compatible with the AdS/CFT correspondence, and the canonical structure shows a close resemblance with the four-dimensional theory.

In the present paper, we continue studying dynamical aspects of the quadratic PGT in three dimensions by looking for *exact wave solutions with torsion*. The weak-field approximation of Einstein's theory around the Minkowski background leads to a simple picture of the wave nature of gravity, which is recognized to have a striking analogy to the electromagnetic phenomena [11,12]. By giving a covariant formulation of this analogy, one can generalize the linearized gravitational wave to the concept of an exact wave solution of general relativity [13–15]. Here, in the context of the quadratic PGT, such generalizations are used to find a class of exact wave solutions with torsion.

A gravitational wave with torsion in three dimensions was first found by Obukhov [16], in the framework of the Mielke—Baekler model [2]. Since the model is defined by a topological action, it was necessary to introduce matter, chosen in the form of a Maxwell field, to have a nontrivial wave solution. On the other hand, our wave solution, being an exact vacuum solution of the quadratic PGT, offers new insight into the wave structure of genuine gravitational degrees of freedom, the propagating torsion modes.

The paper is organized as follows. In Sec. II, we give an overview of the plane-fronted gravitational waves in general relativity without/with a gravitational constant, denoted shortly as $\mathrm{GR}/\mathrm{GR}_\Lambda$, as a basis for further extension to torsion waves in the quadratic PGT. In Sec. III, we start with the GR_Λ form of the metric and introduce a convenient ansatz for the RC connection, or equivalently, for the torsion. The only irreducible component of torsion is taken to be its tensorial piece, parametrized by a single function K. Then, we find the PGT field equations that impose dynamical restrictions on K. A characteristic parameter appearing in these equations is the mass parameter μ^2, associated to the torsion spin-2 mode. In Secs. IV and V, we find a class of exact torsion waves and classify them according to the values of two parameters, μ^2 and λ, the latter one being related to the value of the cosmological constant. In Sec. VI, we discuss criteria that are used to recognize the wave nature of exact solutions and conclude with some specific remarks. Finally, two Appendixes contain useful technical information.

Our conventions are the same as in Ref. [8]: the Latin indices $(i,j,k,\ldots)$ refer to the local Lorentz frame, the Greek indices $(\mu,\nu,\rho,\ldots)$ refer to the coordinate frame, and both run over 0,1,2; the metric components in the local Lorentz frame are $\eta_{ij}=(+,-,-)$; the totally antisymmetric tensor ε^{ijk} is normalized to $\varepsilon^{012}=1$, b^i is the orthonormal triad (coframe 1-form), h_i is the dual basis (frame), the Hodge dual of a form α is ${}^\star\alpha$, and the exterior product of forms is implicit.

[*] mb@ipb.ac.rs
[†] cbranislav@ipb.ac.rs

II. PLANE-FRONTED WAVES IN GENERAL RELATIVITY

In this section, we give a short account of the plane-fronted gravitational waves as exact solutions of Einstein's general relativity.

A. pp waves in GR

A specific class of plane-fronted waves, characterized by having parallel rays (pp waves for short), can be described, in suitable local coordinates, by the metric [13–15]

$$ds^2 = H(u,y)du^2 + 2dudv - dy^2, \tag{2.1}$$

where u is interpreted as the phase of the wave and ∂_v is the covariantly constant null vector field. This metric is a natural generalization of the *linearized* gravitational plane waves propagating on the background Minkowski spacetime [11,12]. General criteria for identifying the wave nature of *exact* solutions will be discussed in Sec. VI.

The explicit form of $H(u,y)$ in Eq. (2.1) can be determined by the general relativity (GR) field equations. Since the only nonvanishing component of the Ricci tensor is $(Ric)_{uu} = H''/2$ (prime means differentiation with respect to y) and the scalar curvature identically vanishes, $R = 0$, the *vacuum* field equations of GR imply

$$H'' = 0 \Rightarrow H = h_1(u) + h_2(u)y, \tag{2.2}$$

where h_1, h_2 are the integration "constants." This solution is in fact trivial since for $H'' = 0$ the Ricci tensor vanishes and, in three dimensions, the full curvature tensor also vanishes. Hence, Eq. (2.2) defines a Minkowski spacetime in nonstandard coordinates.

Thus, in GR, nontrivial pp waves can exist only in the presence of *matter*; see, for instance, Refs. [17–19]. Note, however, that true vacuum waves can exist also in *new dynamical settings*, such as topologically massive gravity or new massive gravity [19–21]. The vacuum waves are an idealization of wave solutions in the region far from matter sources.

B. Plane-fronted waves in GR$_\Lambda$

Now, we turn to a generalized dynamical framework of GR$_\Lambda$ by allowing a nonvanishing cosmological constant. The pp wave (2.1) is not a vacuum solution of GR$_\Lambda$. Indeed, the fact that $R = 0$ for the metric (2.1) implies $\Lambda = 0$. A plane-fronted wave that is compatible with $\Lambda \neq 0$ can be conveniently represented by the metric

$$ds^2 = 2\left(\frac{q}{p}\right)^2 du(Sdu + dv) - \frac{dy^2}{p^2}; \tag{2.3a}$$

see the works by Ozsváth [22] and Obukhov [23], where the functions p, q, and S are chosen as [16]

$$p = 1 + \frac{\lambda}{4}y^2, \qquad q = 1 - \frac{\lambda}{4}y^2, \qquad S = -\frac{\lambda}{2}v^2 + \frac{\sqrt{p}}{2q}H(u,y). \tag{2.3b}$$

Clearly, the limit $\lambda = 0$ returns us back to the pp-wave (2.1). Introducing the ortonormal triad field as

$$\begin{aligned}
b^0 &:= \frac{1}{\sqrt{2}}\left[\left(1 + \frac{q^2}{p^2}S\right)du + \frac{q^2}{p^2}dv\right],\\
b^1 &:= \frac{1}{\sqrt{2}}\left[\left(1 - \frac{q^2}{p^2}S\right)du - \frac{q^2}{p^2}dv\right],\\
b^2 &:= \frac{1}{p}dy,
\end{aligned} \tag{2.4}$$

the metric can be written as $ds^2 = \eta_{ij}b^i \otimes b^j$, with $\eta_{ij} = \mathrm{diag}(+1,-1,-1)$. In the literature, one often uses the light-cone components of the triad:

$$b^+ := du, \qquad b^- := \frac{q^2}{p^2}(Sdu + dv).$$

To verify that the triad (2.4) satisfies the GR$_\Lambda$ field equations,

$$a_0\left((Ric)^i - \frac{1}{2}Rb^i\right) - \Lambda b^i = 0, \qquad a_0 := \frac{1}{16\pi G}, \tag{2.5}$$

we first calculate the Christofell connection; it has the form

$$\begin{aligned}
\Gamma^{01} &= \frac{\lambda y}{q}b^2 - \frac{\lambda v}{\sqrt{2}}(b^0 + b^1),\\
\Gamma^{02} &= \frac{\lambda y}{q}b^0 - \frac{1}{2}(b^0 + b^1)(q^2S'/p),\\
\Gamma^{12} &= \frac{\lambda y}{q}b^1 + \frac{1}{2}(b^0 + b^1)(q^2S'/p),
\end{aligned}$$

or, more compactly,

$$\Gamma^{ij} = \bar{\Gamma}^{ij} + \frac{1}{2}\varepsilon^{ij}{}_m k^m k_n b^n (q^2S'/p). \tag{2.6}$$

Here, the first term, $\bar{\Gamma}^{ij} := \Gamma^{ij}(S' = 0)$, is the piece that describes the "background" (A)dS geometry of spacetime, whereas the second term is the radiation piece, characterized by the null vector $k^i = (1,-1,0)$, $k^2 = 0$, which is not covariantly constant for $\lambda \neq 0$.

Next, we calculate the curvature $R^{ij} = d\Gamma^{ij} + \Gamma^i{}_m\Gamma^{mj}$,

$$R^{ij} = -\lambda b^i b^j + \varepsilon^{ijm}k_m k^n {}^\star b_n p(q^2S'/p)', \tag{2.7a}$$

where ${}^\star b_n = (1/2)\varepsilon_{nrs}b^r b^s$. Note that the radiation piece of R^{ij} is clearly separated from the (A)dS piece. Finally, the form of the Ricci curvature $(Ric)^i = -h_j \rfloor R^{ij}$ and the scalar curvature $R = h_i \rfloor (Ric)^i$,

$$(Ric)^i = -2\lambda b^i + \frac{1}{2}k^i k_m b^m p(q^2 S'/p)',$$
$$R = -6\lambda, \tag{2.7b}$$

implies that the content of the PGT field equations is given by

$$a_0\lambda = \Lambda, \qquad p\left(\frac{q^2}{p}S'\right)' = 0,$$
$$\Rightarrow \frac{\sqrt{p}}{2q}H = \beta_1(u) + \beta_2(u)\frac{y}{q}. \tag{2.8}$$

The function H defines the vacuum solution for the metric (2.3). Since the on-shell value of the curvature is $R^{ij} = -\lambda b^i b^j$, the geometry of the solution (2.8) is fixed: for $\lambda = 0, > 0$, or < 0, it has the Minkowskian, AdS, or de Sitter form, respectively.

Thus, again, for the plane-fronted wave (2.3) to be a nontrivial exact solution, one has to introduce matter. However, by going over to PGT, we expect the new gravitational dynamics to allow for the existence of true wave solutions even in vacuum.

III. DYNAMICS OF TORSION WAVES

In this section, we briefly recapitulate basic aspects of PGT, introduce a geometric extension of the Riemannian plane-fronted waves (2.3) to torsion waves, and discuss their dynamics.

A. Basic aspects of PGT

The PGT is a gauge theory of gravity based on gauging the Poincaré group, with an underlying RC geometry of spacetime [4–6]. Basic gravitational variables are the triad field b^i and the Lorentz connection $A^{ij} = -A^{ji}$ (1-forms), and the corresponding field strengths are the torsion $GT^i = db^i + A^i{}_k b^k$ and the curvature $R^{ij} = dA^{ij} + A^i{}_k A^{kj}$ (2-forms). General dynamics of PGT is defined by the gravitational Lagrangian $L_G = L_G(b^i, T^i, R^{ij})$ (3-form). Varying L_G with respect to b^i and A^{ij} yields the respective gravitational field equations in vacuum [8],

$$\text{(1st)} \qquad \nabla H_i + E_i = 0,$$
$$\text{(2nd)} \qquad \nabla H_{ij} + E_{ij} = 0, \tag{3.1}$$

where

$$H_i \coloneqq \frac{\partial L_G}{\partial T^i}, \qquad H_{ij} \coloneqq \frac{\partial L_G}{\partial R^{ij}}$$

are the covariant field momenta and

$$E_i \coloneqq \frac{\partial L_G}{\partial b^i}, \qquad E_{ij} \coloneqq \frac{\partial L_G}{\partial A^{ij}}$$

are the gravitational energy-momentum and spin currents. We require L_G to be parity invariant and at most quadratic in the field strengths. In that case, H_i and H_{ij} can be expressed linearly in terms of the irreducible pieces of the field strengths (Appendix A),

$$H_i = 2{}^\star(a_1{}^{(1)}T_i + a_2{}^{(2)}T_i + a_3{}^{(3)}T_i),$$
$$H_{ij} = -2a_0\varepsilon_{ijk}b^k + H'_{ij},$$
$$H'_{ij} \coloneqq 2{}^\star(b_4{}^{(4)}R_{ij} + b_5{}^{(5)}R_{ij} + b_6{}^{(6)}R_{ij}), \tag{3.2a}$$

where a_0, a_n, and b_n are coupling constants; moreover, the gravitational Lagrangian takes the form

$$L_G = \frac{1}{2}T^i H_i + R^{ij}(-a_0\varepsilon_{ijk}b^k) + \frac{1}{4}R^{ij}H'_{ij} - \frac{1}{3}\Lambda_0\varepsilon_{ijk}b^i b^j b^k, \tag{3.2b}$$

and the gravitational energy-momentum and spin currents turn out to be

$$E_i = h_i \rfloor L_G - (h_i \rfloor T^m)H_m + \frac{1}{4}(h_i \rfloor R^{mn})H_{mn},$$
$$E_{ij} = -(b_i H_j - b_j H_i). \tag{3.2c}$$

B. Geometry of the ansatz

In our search for the generalized plane-fronted waves, we assume that the form of the triad field, of Eq. (2.4) remains unchanged, whereas the connection is determined by the following rule:

(a) Starting with the Riemannian connection (2.6), (i) we leave its first, (A)dS piece $\bar\Gamma^{ij}$ unchanged (ii) but modify the second, radiation piece in a way that *preserves* the wave nature of the solution.

The instruction (ii) is realized by adopting the following ansatz for the RC connection:

$$A^{ij} = \bar\Gamma^{ij} + \frac{1}{2}\varepsilon^{ij}{}_m k^m k_n b^n G, \tag{3.3a}$$

$$G \coloneqq \frac{q^2}{p}(S' + K). \tag{3.3b}$$

Here, the new term $K = K(u, y)$ describes the effect of torsion, as follows from

$$T^i \coloneqq \nabla b^i = \frac{q^2}{2p}K k^i k_m{}^\star b^m. \tag{3.4}$$

The only nonvanishing irreducible piece of T^i is its tensorial piece (Appendix A):

$${}^{(1)}T^i = T^i.$$

Having chosen the form of the connection, one can now calculate the RC curvatures; they are obtained from Eq. (2.7) by the replacement $S' \to S' + K$:

$$R^{ij} = -\lambda b^i b^j + \varepsilon^{ijm} k_m k^{n\star} b_n p G',$$
$$(Ric)^i = -2\lambda b^i + \frac{1}{2} k^i k_m b^m p G',$$
$$R = -6\lambda. \tag{3.5}$$

The nonvanishing irreducible components of the curvature R^{ij} are (Appendix A)

$${}^{(4)}R^{ij} = \frac{1}{2}\varepsilon^{ijm} k_m k^{n\star} b_n p G', \qquad {}^{(6)}R^{ij} = -\lambda b^i b^j,$$

and the quadratic curvature invariant has the form $R^{ij*}R_{ij} = 6\lambda^2{}^\star 1$.

The geometric configuration defined by the triad field (2.4) and the connection (3.3) represents a generalized gravitational plane-fronted wave of GR_Λ, or the *torsion wave* for short. More details on its wave nature will be given in Sec. VI.

C. Field equations

Having found the expressions for the torsion and the curvature, one can now calculate the covariant momenta H_i, H_{ij}, and the energy-momentum and spin currents E_i, E_{ij}, and obtain the explicit form of the PGT field equations (3.1). The result takes the following form [24]:

$$\text{(1st)} \qquad (a_0 + b_4\lambda + b_6\lambda)pG' - a_1 q(qK)' = 0,$$
$$2\Lambda - 2a_0\lambda + b_6\lambda^2 = 0,$$
$$\text{(2nd)} \qquad b_4(2G''p^3 q + G'\lambda y p^3 + 2G'\lambda y p^2 q)$$
$$+ 2(a_1 - a_0 - b_6\lambda)Kq^3 = 0. \tag{3.6}$$

The second equation in (1st) defines a relation between the parameter λ of the solution and the coupling constants. For $b_6 = 0$, it takes a particularly simple form: $a_0\lambda = \Lambda$. By noting that (2nd) can be rewritten as

$$2b_4 p[pq(pG')' + (pG')\lambda y] + 2(a_1 - a_0 - b_6\lambda)Kq^3 = 0,$$

one finds that the field equations (3.6) can be transformed to a more compact form:

$$\text{(1st)} \qquad pG' = C_0 q\mathcal{K}', \qquad C_0 = \frac{a_1}{a_0 + (b_4 + b_6)\lambda},$$
$$\text{(2nd)} \qquad p(p\mathcal{K}')' + \mu^2\mathcal{K} = 0, \qquad \mu^2 = \frac{a_1 - a_0 - b_6\lambda}{b_4 C_0}, \tag{3.7}$$

with $\mathcal{K} := qK$.

In PGT, the spectrum of excitations around the Minkowski spacetime consists of six independent torsion modes: one scalar, one pseudoscalar, two spin-1, and two spin-2 states [7,8]. Two spin-2 states form a parity invariant multiplet associated to the tensorial piece of the torsion, with equal masses: $m^2 = a_0(a_1 - a_0)/(a_1 b_4)$. Since our ansatz (3.4) reduces torsion just to its tensorial piece, it is not surprising that for $\lambda = 0$, the coefficient μ^2 in Eq. (3.7) reduces exactly to m^2. For $\lambda \neq 0$, μ^2 is associated to the spin-2 excitations around the (A)dS background, and the condition for the absence of tachions requires $\mu^2 \geq 0$.

In what follows, we will solve two dynamical equations (3.7) for the unknown functions $\mathcal{K}$ and G, assuming $\mu^2 \geq 0$; then, we will use of Eq. (3.3b) to find S. The torsion function K and the metric function S, obtained in this way, completely define the solution.

IV. MASSIVE TORSION WAVES

In this section, we classify the solutions of the field equations (3.7) for $\mu^2 > 0$, according to the values of λ.

A. $\lambda = 0$

The simplest form of equations (3.7) is obtained in the limit $\lambda \to 0$:

$$a_0 G' - a_1 K' = 0, \qquad \Lambda = 0,$$
$$K'' + m^2 K = 0, \qquad m^2 = \frac{a_0(a_1 - a_0)}{b_4 a_1}, \tag{4.1}$$

with $G = S' + K$ and $S = H/2$. The solution has a simple form:

$$K = A(u)\cos my + B(u)\sin my,$$
$$\frac{1}{2}H = \frac{a_1 - a_0}{a_0 m}(A\sin my - B\cos my) + h_1(u) + h_2(u)y. \tag{4.2}$$

In Riemannian gravity, one can remove the term $h_1 + h_2 y$ in H by a coordinate transformation. This transformation does not change the form of the metric (2.1), which is the only dynamical variable of the theory in vacuum. In the RC theory, such a coordinate transformation is not particularly useful as it affects the form of the connection. Note, however, that the term $h_1 + h_2 y$ has no influence upon the RC curvature, which depends only on H''. Thus, without loss of generality, we can choose $h_1 = h_2 = 0$.

The vector field $k = \partial_v$ is the Killing vector for both the metric and the torsion; moreover, it is a null and covariantly constant vector field. This allows us to consider the solution (4.2) as a generalized pp wave.

B. $\lambda > 0$

For positive λ, we use the notation

$$\lambda = \frac{1}{\ell^2}, \qquad x = \frac{y}{2\ell}, \qquad \kappa = 2\mu\ell,$$

so that $\int dy = 2\ell \int dx$. Now, having in mind the form of the solution (4.2) for $\lambda = 0$, we use a similar ansatz for the torsion function $\mathcal{K} \equiv qK$:

$$\mathcal{K} = A\cos\alpha + B\sin\alpha, \qquad \alpha = \alpha(y), \tag{4.3a}$$

where $A = A(u), B = B(u)$. Substituting this into (2nd) of Eq. (3.7) produces two conditions on α:

$$p^2(\alpha')^2 - \mu^2 = 0, \qquad p^2\alpha'' - \frac{1}{2}\lambda y p\alpha' = 0.$$

The first condition yields

$$\alpha' = \frac{\mu}{p} = \frac{\mu}{1+x^2} \Rightarrow \alpha = 2\ell\int\frac{\mu}{1+x^2}dx = \kappa\arctan x, \tag{4.3b}$$

whereas the second one is automatically satisfied. In the limit $\lambda \to 0$, we have $\alpha \to \kappa x = my$, and Eq. (4.3) reduces to Eq. (4.2).

In the next step, we use Eq. (4.3) and (1st) to calculate G:

$$G = 2\ell C_0\int\frac{q}{p}\mathcal{K}'dx = D\frac{1}{p}\left[\left(qA - \frac{4x}{\kappa}B\right)\cos\alpha + \left(qB + \frac{4x}{\kappa}A\right)\sin\alpha\right],$$

where $D = C_0\kappa^2/(\kappa^2-4)$. Finally, integrating the relation $S' = (p/q^2)G - K$ yields the metric function H. Using the definition

$$\mathcal{H} := \frac{\sqrt{p}}{2q}H \equiv S + \frac{\lambda}{2}v^2, \tag{4.4}$$

we find

$$\begin{aligned}\mathcal{H} &= \mathcal{H}_1 + \mathcal{H}_2,\\ \mathcal{H}_1 &:= 2\ell\int\frac{p}{q^2}Gdx = 2\ell D\cdot\frac{p}{\kappa q}(A\sin\alpha - B\cos\alpha),\\ \mathcal{H}_2 &:= -2\ell\int Kdx = \frac{2\ell}{\kappa^2-4}\times\left[(B-iA)(2+\kappa)e^{i(2-\kappa)\arctan x}{}_2F_1\left(1,\frac{2-\kappa}{4};\frac{6-\kappa}{4};-e^{4i\arctan x}\right)\right.\\ &\quad\left. -(B+iA)(2-\kappa)e^{i(2+\kappa)\arctan x}{}_2F_1\left(1,\frac{2+\kappa}{4};\frac{6+\kappa}{4};-e^{4i\arctan x}\right)\right],\end{aligned} \tag{4.5}$$

where ${}_2F_1(a,b;c;z)$ is the hypergeometric function [24]. Here, again, the integration term $h_1(u) + h_2(u)y/q$ appearing in $\mathcal{H}$ is removed, as it has no influence upon the RC curvature.

To illustrate the form of the torsion wave, we display here the plots of the torsion function $(q^2/p)K(u,y)$ and the curvature function $pG'(u,y)/2$, for a specific choice of the parameters ℓ, κ, and for fixed amplitudes $A(u)$ and $B(u)$ (see Fig. 1).

C. $\lambda < 0$

In this case, we use the notation

$$\lambda = -\frac{1}{\ell^2}, \qquad x = \frac{y}{2\ell}, \qquad \kappa = 2\ell\mu$$

and find that the torsion function $\mathcal{K}$ is given by

$$\begin{aligned}\mathcal{K} &= A\cos\alpha + B\sin\alpha,\\ \alpha &= \kappa\frac{1}{2}\ln\left|\frac{1+x}{1-x}\right| = \kappa\,\mathrm{arctanh}\,x.\end{aligned} \tag{4.6}$$

Here, $\alpha(x)$ is singular at $x = 1$, but for $\lambda \to 0$, it has the expected limit: $\alpha \to \kappa x = my$. Then, following the same steps as in the previous subsection, we can first calculate G,

$$G = \frac{E}{p}\left[\left(Bq - \frac{4x}{\kappa}A\right)\sin\alpha + \left(Aq + \frac{4x}{\kappa}B\right)\cos\alpha\right],$$

where $E = C_0\kappa^2/(\kappa^2+4)$, and then find the metric function $\mathcal{H}$:

$$\begin{aligned}\mathcal{H} &= \mathcal{H}_1 + \mathcal{H}_2,\\ \mathcal{H}_1 &:= 2\ell\int\frac{p}{q^2}Gdx = 2\ell\frac{E}{\kappa}\frac{p}{q}[A\sin\alpha - B\cos\alpha],\\ \mathcal{H}_2 &:= -2\ell\int Kdx = -\frac{2\ell i}{\kappa^2+4}\times\left[(B-iA)(2+i\kappa)e^{(2-i\kappa)\mathrm{arctanh}\,x}{}_2F_1\left(1,\frac{2-i\kappa}{4};\frac{6-i\kappa}{4};-e^{4\,\mathrm{arctanh}\,x}\right)\right.\\ &\quad\left. -(B+iA)(2-i\kappa)e^{(2+i\kappa)\mathrm{arctanh}\,x}{}_2F_1\left(1,\frac{2+i\kappa}{4};\frac{6+i\kappa}{4};-e^{4\,\mathrm{arctanh}\,x}\right)\right].\end{aligned} \tag{4.7}$$

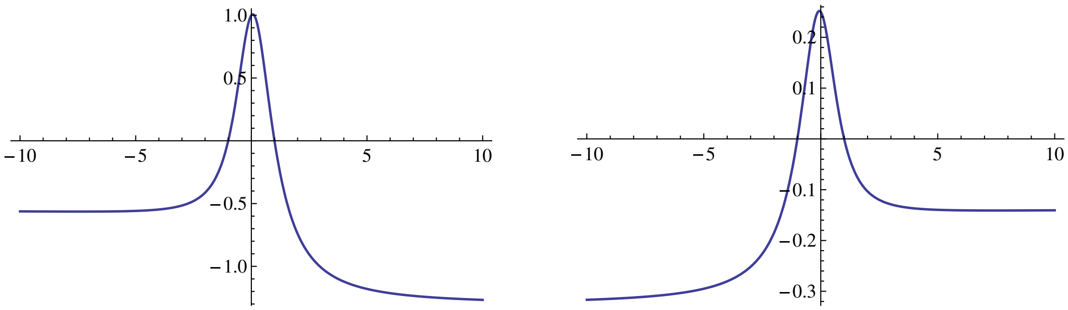

FIG. 1 (color online). The form of the torsion function $(q^2/p)K$ (left) and the curvature function $pG'/2$ (right) for $\mu^2 > 0$, in the region $x \in [-10, 10]$, and for $A(u) = B(u) = 1$, $\ell = 1$, $\kappa = 1/4$.

As before, all the integration terms in $\mathcal{H}$ are removed.

This solution can be obtained from the one for $\lambda > 0$ by the analytic continuation in ℓ:

$$\ell \to i\ell \Rightarrow \kappa \to i\kappa, \qquad x \to \frac{1}{i}x,$$
$$\arctan x \to \frac{1}{i}\operatorname{arctanh} x.$$

For the asymptotic behavior of both massive and massless torsion waves, see Sec. VI and Appendix B.

V. MASSLESS TORSION WAVES

For $\mu^2 = 0$, we have $a_1 - a_0 - b_6\lambda = 0$, and the field equations (3.7) are simplified:

$$pG' = C_0 q\mathcal{K}', \qquad p(p\mathcal{K}')' = 0. \tag{5.1}$$

A. $\lambda = 0$

For vanishing λ, the field equations with $C_0 = 1$ take the form

$$G' - K' \equiv \frac{1}{2}H'' = 0, \qquad K'' = 0, \tag{5.2}$$

so that

$$H = h_1(u) + h_2(u)y, \qquad K = k_1(u) + k_2(u)y. \tag{5.3}$$

This is a rather strange solution: since the metric function H is trivial, the metric takes the Minkowski form, and consequently it is dynamically decoupled from the torsion.

B. $\lambda > 0$

For the positive cosmological constant, with $\lambda \coloneqq 1/\ell^2$ and $x = y/2\ell$, the solution reads

$$\begin{aligned}
\mathcal{K} &= A(u)\arctan x + B(u),\\
G &= A(u)\frac{C_0 x}{p},\\
\mathcal{H}(u,y) &= \ell A(u)\left(\frac{C_0}{q} - \arctan x \cdot \ln\frac{1 - ie^{2i\arctan x}}{1 + ie^{2i\arctan x}}\right)\\
&\quad + \frac{i\ell}{2}A(u)\left[Li_2\left(ie^{2i\arctan x}\right) - Li_2\left(-ie^{2i\arctan x}\right)\right]\\
&\quad - 2\ell B(u)\operatorname{arctanh} x,
\end{aligned} \tag{5.4}$$

where $\mathrm{Li}_2(z)$ is the dilogarithm function [24]. The solution is illustrated in Fig. 2.

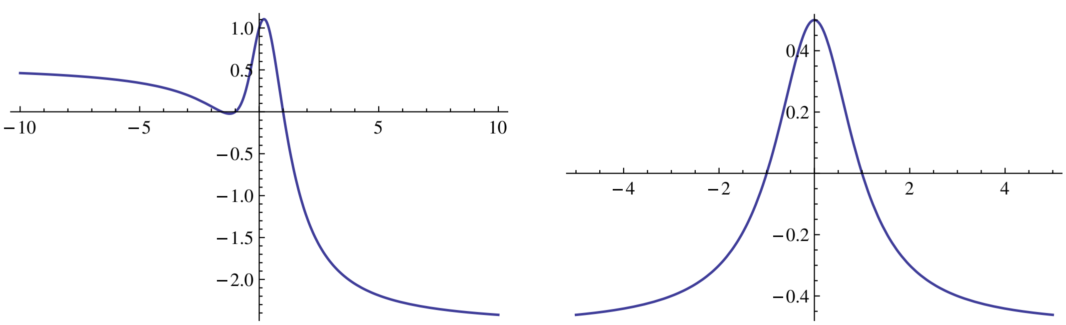

FIG. 2 (color online). The form of the torsion function $(q^2/p)K$ (left) and the curvature function $pG'/2$ (right) for $\mu^2 = 0$, in the region $x \in [-10, 10]$, and for $A(u) = B(u) = 1$ and $\ell = 1$.

C. $\lambda < 0$

Finally, for $\lambda \coloneqq -1/\ell^2$, one finds

$$\mathcal{K} = A(u)\operatorname{arctanh} x + B(u),$$
$$G = A(u)\frac{C_0 x}{p},$$
$$\mathcal{H}(u,y) = \ell A(u)\left(-\frac{C_0}{q} - \frac{i}{2}\operatorname{arctanh} x \cdot \ln\frac{1 - ie^{-2\operatorname{arctanh} x}}{1 + ie^{-2\operatorname{arctanh} x}}\right) + \frac{i\ell}{2}A(u)\left[Li_2\left(ie^{-2\operatorname{arctanh} x}\right) - Li_2\left(-ie^{-2\operatorname{arctanh} x}\right)\right] + 2\ell B(u)\arctan x. \tag{5.5}$$

VI. DISCUSSION AND CONCLUSIONS

In this paper, we derived a new class of exact solutions of 3D gravity with propagating torsion in empty spacetime, the generalized plane-fronted waves, or the torsion waves.

The wave ansatz for the metric, Eqs. (2.4), and the RC connection, Eqs. (3.3), represent a natural generalization of the Riemannian plane-fronted waves with cosmological constant. However, a covariant characterization of the wave nature of an exact solution is a rather complex issue [13–15], which has not been fully clarified for non-Riemannian theories of gravity; for an attempt in this direction, see Ref. [25].

The existence of the null covector $k_i = (1,1,0)$, appearing already in the RC connection, Eqs. (3.3), is an essential element of the geometric structure of a gravitational wave. It can be represented as the 1-form $k_i b^i = \sqrt{2}du$, associated to the wave fronts $u = \text{const}$. The related vector field $k^i\partial_i = \sqrt{2}\partial_v$ is orthogonal to the y direction; moreover, for $\lambda = 0$, k^i is covariantly constant (pp wave).

Based on an analogy with the electromagnetism, Lichnerowicz proposed a covariant criterion for the existence of gravitational waves in general relativity; see Ref. [14]. After separating the radiation piece of the RC curvature, of Eqs. (3.5), $S^{ij} \coloneqq R^{ij} + 2\lambda b^i b^j$, one can verify that it satisfies Lichnerowicz's requirements:

$$k^i S_{ijmn} = 0, \qquad \varepsilon^{ijk} k_i S_{jkmn} = 0. \tag{6.1}$$

Clearly, the above criterion is not sufficient for a RC geometry, where we have one more field strength, the torsion. However, in analogy with electromagnetism, the radiation conditions for torsion are expected to have the form

$$k^m T_{imk} = 0, \qquad \varepsilon^{mnk} k_m T_{ink} = 0. \tag{6.2}$$

A direct verification based on Eq. (3.4) shows that these conditions are also satisfied. The radiation properties, of Eqs. (6.1) and (6.2) strongly support the interpretation of our generalized plane-fronted wave as a genuine PGT extension of the related Riemannian structure.

One should also note that our RC curvature has the same irreducible components as the corresponding Riemannian curvature, and moreover it has all the usual index symmetries of the Riemannian curvature; in particular, $R_{ijmn} = R_{mnij}$. The same properties were found by Pašić and Vassiliev [26] in their pp wave with torsion, constructed in the model with metric-compatible connection and curvature squared Lagrangian. The torsion of their solution is pure tensor, as in our case.

In electrodynamics and in general relativity, exact wave solutions are associated with massless modes of the related fields, so that the appearance of massive torsion waves may seem a bit strange. However, the existence of massive torsion modes is not in conflict with the gauge structure of PGT; it is a generic feature associated to the presence of T^2 terms in the Lagrangian. Massive waves appear also in some Riemannian extensions of GR, such as topologically massive gravity or new massive gravity [19–21].

Asymptotic properties of the torsion waves are defined by the large y limits of the torsion of Eq. (3.4) and the RC curvature of Eq. (3.5). As follows from the results of Appendix B, the generic asymptotic form of the torsion waves does not coincide with the (A)dS geometry.

Our study of exact torsion waves in three dimensions can be considered as a complement to the related results in four dimensions [25–27]. In particular, we wish to place emphasis on the results of Sippel and Goenner [25], who made a significant progress in clarifying the structure of pp waves with torsion: (i) they generalized the Ehlers—Kundt classification of pp waves [13] by relaxing the assumption that the GR field equations hold, and (ii) they introduced a classification of the allowed form of torsion in pp waves. Further advances in this direction would help us to better understand the role of torsion in exact wave solutions.

ACKNOWLEDGMENTS

M. B. would like to thank Yuri Obukhov for useful comments. This work was supported by the Serbian Science Foundation under Grant No. 171031.

APPENDIX A: IRREDUCIBLE DECOMPOSITION

For the sake of completeness, we present here the form of the irreducible components of T^i and R^{ij}, see also Ref. [8], with the wedge product sign explicitly displayed.

Torsion has three irreducible components, the vector, axial, and tensor component:

$$
\begin{aligned}
{}^{(2)}T_i &:= \frac{1}{2} b_i \wedge (h_m \rfloor T^m) = \frac{1}{2}\eta_{ij} V_k b^j \wedge b^k, \\
{}^{(3)}T_i &:= \frac{1}{3} {}^\star [b_i \wedge {}^\star (T^m \wedge b_m)] = \frac{1}{2} A \varepsilon_{ijk} b^j \wedge b^k, \\
{}^{(1)}T_i &:= T_i - {}^{(2)}T_i - {}^{(3)}T_i,
\end{aligned}
\tag{A1}
$$

where $V_k := T^m{}_{mk}$ and $A := \varepsilon_{ijk} T^{ijk}/6$.

The curvature also has three irreducible pieces. Making use of the definitions

$$
\begin{aligned}
A_i &:= \frac{1}{2} h_i \rfloor (b^k \wedge \hat{R}_k) = \hat{R}_{[ik]} b^k, \\
\mathcal{S}_i &:= \hat{R}_i - A_i - \frac{1}{3} R b_i = \hat{R}_{(ik)} b^k - \frac{1}{3} R b_i,
\end{aligned}
$$

where $\hat{R}_i := (Ric)_i$, the irreducible pieces of R_{ij} read

$$
\begin{aligned}
{}^{(4)}R_{ij} &:= b_i \mathcal{S}_j - b_j \mathcal{S}_i, \\
{}^{(5)}R_{ij} &:= b_i A_j - b_j A_i, \\
{}^{(6)}R_{ij} &:= \frac{1}{6} R b_i \wedge b_j.
\end{aligned}
\tag{A2}
$$

Note that in three dimensions, the Weyl curvature vanishes.

APPENDIX B: ASYMPTOTIC GEOMETRY

In this Appendix, we calculate the large y limits of the expressions $(q^2/p)K$ and $pG'/2$; these limits define the respective asymptotic values of the torsion and the radiation piece of the curvature, characterizing the gravitational wave. The formulas for $\lambda = 0$ are omitted, as the related asymptotic behavior can be read off directly from the main text.

1. Case $\mu^2 > 0$

$\lambda > 0$:

$$
\begin{aligned}
\lim_{y\to\pm\infty} \frac{q^2}{p} K &= -\left(A \cos\frac{\kappa\pi}{2} \pm B \sin\frac{\kappa\pi}{2} \right), \\
\lim_{y\to\pm\infty} \frac{1}{2} p G' &= \frac{1}{2} C_0 \mu \left(\pm A \sin\frac{\kappa\pi}{2} - B \cos\frac{\kappa\pi}{2} \right).
\end{aligned}
\tag{B1}
$$

$\lambda < 0$:

$$
\begin{aligned}
\lim_{y\to\infty} \frac{q^2}{p} K &= -A, \\
\lim_{y\to\infty} \frac{1}{2} p G' &= -\frac{1}{2} C_0 \mu B.
\end{aligned}
\tag{B2}
$$

2. Case $\mu^2 = 0$

$\lambda > 0$:

$$
\begin{aligned}
\lim_{y\to\pm\infty} \frac{q^2}{p} K &= \mp A \frac{\pi}{2} - B, \\
\lim_{y\to\infty} \frac{1}{2} p G' &= -\frac{A C_0}{4\ell}.
\end{aligned}
\tag{B3}
$$

$\lambda < 0$:

$$
\begin{aligned}
\lim_{y\to\infty} \frac{q^2}{p} K &= -B, \\
\lim_{y\to\infty} \frac{1}{2} p G' &= -\frac{A C_0}{4\ell}.
\end{aligned}
\tag{B4}
$$

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PHYSICAL REVIEW D 88, 104032 (2013)

Three-dimensional gravity with propagating torsion: Hamiltonian structure of the scalar sector

M. Blagojević* and B. Cvetković†

University of Belgrade, Institute of Physics, P.O. Box 57, 11001 Belgrade, Serbia

(Received 13 September 2013; published 25 November 2013)

We study the Hamiltonian structure of the general parity-invariant model of three-dimensional gravity with propagating torsion, with eight parameters in the Lagrangian. In the scalar sector, containing scalar or pseudoscalar modes with respect to maximally symmetric background, the phenomenon of constraint bifurcation is observed and analyzed. The stability of the Hamiltonian structure under linearization is used to identify dynamically acceptable values of parameters.

DOI: 10.1103/PhysRevD.88.104032 PACS numbers: 04.20.Fy, 04.50.Kd, 04.60.Kz

I. INTRODUCTION

Models of three-dimensional (3D) gravity were introduced to help us in clarifying highly complex dynamical behavior of the realistic four-dimensional general relativity (GR). In the last three decades, they led to a number of outstanding results [1]. However, in the early 1990s, Mielke and Baekler [2] proposed a new, non-Riemannian approach to 3D gravity, based on the Poincaré gauge theory (PGT) [3–6]. In contrast to the traditional GR with an underlying Riemannian geometry of spacetime, the PGT approach is characterized by a Riemann-Cartan geometry, with both the curvature and the torsion of spacetime as carriers of the gravitational dynamics. Thus, PGT allows exploring the interplay between gravity and geometry in a more general setting.

Three-dimensional GR with or without a cosmological constant, as well as the Mielke–Baekler (MB) model, are topological theories without propagating modes. From the physical point of view, such a degenerate situation is certainly not quite realistic. In the context of Riemannian geometry, this limitation is surmounted by two well-known models: topologically massive gravity [7] and the Bergshoeff-Hohm-Townsend massive gravity [8]. On the other hand, including propagating modes in PGT is much more natural: it is achieved simply by using Lagrangians quadratic in the field strengths [9–12].

Since the general parity-invariant PGT Lagrangian in 3D is defined by eight arbitrary parameters [11], it is a theoretical challenge to find out which values of the parameters are allowed in a viable theory. Following the approach of Sezgin and Nieuwenhuizen [13], Helayël-Neto *et al.* [10] used the *weak-field approximation* around the Minkowski background to analyze this issue in a parity-violating version of PGT, and found a number of interesting restrictions on the parameters. However, one should be very careful with the interpretation of these results, since (i) it is not clear how the transition from Minkowski to (anti–)de Sitter [(A)dS] background might influence the perturbative analysis, and (ii) the weak-field approximation does not always lead to a correct identification of the physical degrees of freedom. Regarding (ii), we note that the constrained Hamiltonian method [4,14] is best suited for analyzing dynamical content of gauge field theories, respecting fully their *nonlinear structure*. As noticed by Chen *et al.* [15] and Yo and Nester [16], it may happen, for some ranges of parameters, that the canonical structure of a theory (the number and/or type of constraints) is changed after linearization in a way that affects its physical content, such as the number of physical degrees of freedom. Based on the *canonical stability under linearization* as a criterion for an acceptable choice of parameters, Shie *et al.* [17] were able to define a PGT cosmological model that offers a convincing explanation of dark energy as an effect induced by torsion. Recently, the Bergshoeff-Hohm-Townsend massive gravity is found to be canonically unstable under linearization [18,19].

In this paper, we use the constrained Hamiltonian formalism to study (a) the phenomenon of "constraint bifurcation" and (b) the stability under linearization of the general parity-invariant PGT in 3D [11], in order to find out the parameter values that define consistent models of 3D gravity with propagating torsion. Because of the complexity of the Hamiltonian structure, we restrict our attention to the scalar sector, with $J^P = 0^+$ or 0^- modes, defined with respect to the (A)dS background. Investigation of higher spin modes is left for a future study.

The paper is organized as follows. In Sec. II, we review basic Lagrangian aspects of the parity-invariant PGT in 3D. In Sec. III, we give a brief account of the weak-field approximation around the (A)dS background, restricting our attention to the scalar sector, with $J^P = 0^+$ or 0^-. In Sec. IV, we analyze general aspects of the canonical dynamics of PGT; in particular, we examine how, depending on certain critical values of parameters, some extra primary constraints may appear (if-constraints), leading to a significant effect on the Hamiltonian structure. In Sec. V, we analyze the canonical structure of the spin-0^+ sector, including the "constraint bifurcation" effects. Then, the test of canonical stability under linearization is used to reveal dynamically acceptable values of parameters. In Sec. VI,

*mb@ipb.ac.rs
†cbranislav@ipb.ac.rs

1550-7998/2013/88(10)/104032(15)

the same type of analysis is carried out for the spin-0^- sector. Section VII is devoted to concluding remarks, and appendixes contain technical details.

Our conventions are as follows: the Latin indices $(i, j, k, \ldots)$ refer to the local Lorentz frame, the Greek indices $(\mu, \nu, \lambda, \ldots)$ refer to the coordinate frame, and both run over 0, 1, 2; the metric components in the local Lorentz frame are $\eta_{ij} = (+, -, -)$; totally antisymmetric tensor ε^{ijk} is normalized to $\varepsilon^{012} = 1$.

II. LAGRANGIAN FORMALISM

We begin our considerations by a short account of the Lagrangian formalism for PGT. Assuming parity invariance, the dynamics of 3D gravity with propagating torsion is determined by the gravitational Lagrangian (density) $\tilde{\mathcal{L}}_G = b\mathcal{L}_G$,

$$\mathcal{L}_G = -aR - 2\Lambda_0 + \mathcal{L}_{T^2} + \mathcal{L}_{R^2}, \tag{2.1a}$$

where Λ_0 is a bare cosmological constant, $a = 1/16\pi G$, and the pieces quadratic in the field strengths read

$$\begin{aligned}\mathcal{L}_{T^2} &:= \frac{1}{2}T^{ijk}(a_1{}^{(1)}T_{ijk} + a_2{}^{(2)}T_{ijk} + a_3{}^{(3)}T_{ijk}),\\ \mathcal{L}_{R^2} &:= \frac{1}{4}R^{ijkl}(b_4{}^{(4)}R_{ijkl} + b_5{}^{(5)}R_{ijkl} + b_6{}^{(6)}R_{ijkl}),\end{aligned} \tag{2.1b}$$

where ${}^{(n)}T_{ijk}$ and ${}^{(n)}R_{ijkl}$ are irreducible components of the torsion and the Riemann-Cartan curvature [11]. Since the Weyl curvature vanishes in 3D, one can rewrite these expressions in the form that is more practical for the canonical analysis:

$$\begin{aligned}\mathcal{L}_{T^2} &= T^{ijk}(\alpha_1 T_{ijk} + \alpha_2 T_{kji} + \alpha_3\eta_{ij}V_k),\\ \mathcal{L}_{R^2} &= R^{ij}(\beta_1 R_{ij} + \beta_2 R_{ji} + \beta_3\eta_{ij}R) =: R^{ij}\mathcal{H}_{ij}.\end{aligned} \tag{2.1c}$$

Here, $V_k := T^m{}_{mk}$, $R_{ij} := R^m{}_{imj}$ is the Ricci tensor, R is the scalar curvature, and

$$\begin{aligned}\alpha_1 &= \frac{1}{6}(2a_1 + a_3), & \alpha_2 &= \frac{1}{3}(a_1 - a_3),\\ \alpha_3 &= \frac{1}{2}(a_2 - a_1). & \beta_1 &= \frac{1}{2}(b_4 + b_5),\\ \beta_2 &= \frac{1}{2}(b_4 - b_5), & \beta_3 &= \frac{1}{12}(b_6 - 4b_4).\end{aligned}$$

We also introduce the covariant momenta $\mathcal{H}_{ijk} = \partial\mathcal{L}_{T^2}/\partial T^{ijk}$ and $\mathcal{H}_{ijkl} = \partial\mathcal{L}_{R^2}/\partial R^{ijkl}$:

$$\begin{aligned}\mathcal{H}_{ijk} &= 2(a_1{}^{(1)}T_{ijk} + a_2{}^{(2)}T_{ijk} + a_3{}^{(3)}T_{ijk})\\ &= 4(\alpha_1 T_{ijk} + \alpha_2 T_{[kj]i} + \alpha_3\eta_{i[j}V_{k]}),\\ \mathcal{H}_{ijkl} &= -2a(\eta_{ik}\eta_{jl} - \eta_{jk}\eta_{il}) + \mathcal{H}'_{ijkl},\\ \mathcal{H}'_{ijkl} &= 2(b_4{}^{(4)}R_{ijkl} + b_5{}^{(5)}R_{ijkl} + b_6{}^{(6)}R_{ijkl})\\ &= 2(\eta_{ik}\mathcal{H}_{jl} - \eta_{jk}\mathcal{H}_{il}) - (k \leftrightarrow l).\end{aligned}$$

General field equations for the PGT theory (2.1) are given in [11]. Without matter contribution, these equations, transformed to the local Lorentz basis, take the form

$$\nabla^m\mathcal{H}_{imj} + \frac{1}{2}\mathcal{H}_i{}^{mn}(-T_{jmn} + 2\eta_{jm}V_n) - t_{ij} = 0, \tag{2.2a}$$

$$2aT_{kij} + 2T^m{}_{ij}(\mathcal{H}_{mk} - \eta_{mk}\mathcal{H}) + 4\nabla_{[i}(\mathcal{H}_{j]k} - \eta_{j]k}\mathcal{H}) + \varepsilon_{ijn}\varepsilon^{mr}{}_k\mathcal{H}_{mr}{}^n = 0, \tag{2.2b}$$

where $\mathcal{H} = \mathcal{H}^k{}_k$, and t_{ij} is the energy-momentum tensor of gravity:

$$\begin{aligned}t_{ij} := \eta_{ij}\mathcal{L}_G - T^{mn}{}_i\mathcal{H}_{mnj} + 2a\hat{R}_{ji}\\ - 2(\hat{R}^n{}_i\mathcal{H}_{nj} - \hat{R}_j{}^{nm}{}_i\mathcal{H}_{nm}).\end{aligned}$$

Relying again on the vanishing of the Weyl curvature, one can express Bianchi identities in terms of the Ricci tensor. In the local Lorentz basis, these identities take the form:

$$\begin{aligned}\varepsilon^{mnr}\nabla_m T^i{}_{nr} + \varepsilon^{rsn}T^i{}_{mn}T^m{}_{rs} + 2\varepsilon^{imn}R_{mn} = 0,\\ \nabla_k G^{ki} - V_k G^{ki} = 0,\end{aligned} \tag{2.3}$$

where $G_{ki} := R_{ki} - \frac{1}{2}\eta_{ik}R$.

III. SCALAR EXCITATIONS AROUND (A)DS BACKGROUND

Particle spectrum of 3D gravity with torsion (2.1) around the Minkowski background M_3 is already known [10,11]. Here, we wish to examine the modification of this spectrum induced by transition to the (A)dS background. This will help us to clarify the relation between the canonical stability of the theory under linearization and its M_3 or (A)dS particle spectrum. Our attention is restricted to the scalar sector, with $J^P = 0^+, 0^-$ modes.

Maximally symmetric configuration of 3D gravity with torsion is defined by the set of fields $\bar{\phi} = (\bar{b}^i{}_\mu, \bar{A}^{ij}{}_\mu)$, such that

$$\bar{T}_{ijk} = p\varepsilon_{ijk}, \qquad \bar{R}^{ij}{}_{mn} = -q(\delta^i_m\delta^j_n - \delta^i_n\delta^j_m), \tag{3.1}$$

where the parameters p and q define an effective cosmological constant,

$$\Lambda_{\mathrm{eff}} := q - \frac{p^2}{4}.$$

In order for this configuration to be a solution of the field equations in vacuum, the parameters p and q have to satisfy the following conditions [11]:

$$p(a + qb_6 + 2a_3) = 0, \tag{3.2a}$$

$$aq - \Lambda_0 + \frac{1}{2}p^2 a_3 - \frac{1}{2}q^2 b_6 = 0. \tag{3.2b}$$

In the weak-field approximation around $\bar{\phi}$, the gravitational variables $\phi = (b^i{}_\mu, A^{ij}{}_\mu)$ take the form $\phi = \bar{\phi} + \tilde{\phi}$. We use the convention that indices of the linear excitations $\tilde{\phi}$ are changed by the background triad and/or metric.

The analysis of the particle spectrum is based on the linearized field equations. In the same approximation, the Bianchi identities read:

$$\varepsilon^{kmn}\bar{\nabla}_k \tilde{T}^i{}_{mn} - 2p\tilde{V}^i + 2\varepsilon^{imn}\tilde{R}_{mn} = 0, \tag{3.3a}$$

$$\bar{\nabla}_k \tilde{G}^{ki} - q\tilde{V}^i = 0. \tag{3.3b}$$

A. Spin-0^+ mode

Looking at the particle spectrum of the theory (2.1) on the M_3 background (see Sec. 3 in [11]), one finds that the spin-0^+ mode has a finite mass (and propagates) if

$$a_2(b_4 + 2b_6) \neq 0.$$

In order to study the spin-0^+ mode, we adopt the following, somewhat simplified conditions:

$$a_2, b_6 \neq 0, \qquad a_1 = a_3 = b_4 = b_5 = 0. \tag{3.4a}$$

In fact, this choice is not unique since the existence of a spin-0^+ mode can be realized, for instance, without requiring $b_4 = 0$. However, our "minimal" choice (3.4a) greatly simplifies the calculations, and moreover, one does not expect that any essential dynamical feature of the spin-0^+ mode will be thereby lost; see [15,16]. The corresponding Lagrangian reads

$$\mathcal{L}_G^+ = -aR - 2\Lambda_0 + \frac{1}{2}a_2 V^k V_k + \frac{1}{12}b_6 R^2, \tag{3.4b}$$

and the conditions (3.2) reduce to

$$p(a + qb_6) = 0, \qquad aq - \Lambda_0 - \frac{1}{2}q^2 b_6 = 0. \tag{3.4c}$$

Now, we are going to show that the Minkowskian conditions (3.4a) equally well define the spin-0^+ mode with respect to the (A)dS background (3.1). We start by noting that, under the conditions (3.4a), the linearized field equations (2.2) read

$$(a + qb_6)\tilde{G}_{ji} + a_2\eta_{i[j}\bar{\nabla}^k\tilde{V}_{k]} + \frac{b_6 q}{3}\eta_{ij}\tilde{R} = 0, \tag{3.5a}$$

$$(a + qb_6)\tilde{T}_{ijk} - \frac{pb_6}{6}\varepsilon_{ijk}\tilde{R} + a_2\eta_{i[j}\tilde{V}_{k]} + \frac{b_6}{3}\eta_{i[j}\bar{\nabla}_{k]}\tilde{R} = 0, \tag{3.5b}$$

and their traces are

$$-2a_2\bar{\nabla}_i\tilde{V}^i + (a - qb_6)\tilde{R} = 0, \tag{3.6a}$$

$$(a + qb_6 + a_2)\tilde{V}_k + \frac{b_6}{3}\bar{\nabla}_k\tilde{R} = 0. \tag{3.6b}$$

In the generic case, by combining $\bar{\nabla}_k\bar{\nabla}^k$ of (3.6a) with $\bar{\nabla}^k$ of (3.6b), one obtains

$$(\bar{\nabla}_i\bar{\nabla}^i + m_{0^+}^2)\sigma = 0,$$

$$m_{0^+}^2 = \frac{3(a - qb_6)(a + qb_6 + a_2)}{2a_2 b_6}, \tag{3.7}$$

where $\sigma := \bar{\nabla}_i\tilde{V}^i$. Thus, the field σ can be identified as the spin-0^+ excitation with respect to the (A)dS background, the mass of which is finite. In the limit of vanishing q, $m_{0^+}^2$ reduces to the corresponding Minkowskian expression.

B. Spin-0^- mode

Similar analysis can be applied to the spin-0^- excitation. We start from the Minkowskian condition that the spin-0^- mode has a finite mass (and propagates) [11],

$$(a_1 + 2a_3)b_5 \neq 0.$$

We describe dynamics of the spin-0^- sector by the simplified conditions:

$$a_3, b_5 \neq 0, \qquad a_1 = a_2 = b_4 = b_6 = 0. \tag{3.8a}$$

The related Lagrangian has the form

$$\mathcal{L}_G^- = -aR - 2\Lambda_0 + 3a_3\mathcal{A}^2 + b_5 R_{[ij]}R^{[ij]}, \tag{3.8b}$$

with $\mathcal{A} = \varepsilon^{ijk}T_{ijk}/6$, and the conditions (3.2) reduce to

$$p(a + 2a_3) = 0, \qquad aq - \Lambda_0 + \frac{1}{2}p^2 a_3 = 0. \tag{3.8c}$$

Starting from the linearized field equations,

$$a_3\varepsilon_{ijk}\bar{\nabla}^k\tilde{\mathcal{A}} + a_3 p\eta_{ij}\tilde{\mathcal{A}} + \frac{4a_3}{3}p\varepsilon_{(imn}\tilde{t}_{j)mn} - a_3 p\varepsilon_{ijk}\tilde{V}^k + a\tilde{G}_{ji} + b_5 q\tilde{R}_{[ij]} = 0, \tag{3.9a}$$

$$a\tilde{T}_{ijk} + pb_5\varepsilon^n{}_{jk}\tilde{R}_{[ni]} + b_5\bar{\nabla}_{[j}(\tilde{R}_{k]i} - \tilde{R}_{ik]}) + 2a_3\varepsilon_{ijk}\tilde{\mathcal{A}} = 0. \tag{3.9b}$$

the axial irreducible components of these equations read

$$a_3\bar\nabla^i\tilde{\mathcal{A}} - a_3 p\tilde V^i - \frac{1}{2}(a - qb_5)\varepsilon^{ijk}\tilde R_{jk} = 0,$$
$$(a + 2a_3)\tilde{\mathcal{A}} + \frac{1}{3}b_5\varepsilon^{ijk}\bar\nabla_i\tilde R_{jk} = 0. \tag{3.10}$$

Then, the divergence of the first equation combined with the second one yields

$$a_3\bar\nabla^i\bar\nabla_i\tilde{\mathcal{A}} - pa_3\bar\nabla_i\tilde V^i + \frac{1}{2}(a - qb_5)\frac{3(a + 2a_3)}{b_5}\tilde{\mathcal{A}} = 0. \tag{3.11}$$

Now, using the divergence of the first Bianchi identity (3.3a) and the commutator identity $[\bar\nabla_m, \bar\nabla_n]\tilde X_i = -p\varepsilon_{mnk}\bar\nabla^k\tilde X_i - 2q\eta_{i[m}\tilde X_{n]}$, we find

$$\sigma \equiv \bar\nabla_k\tilde V^k = -\frac{3}{2}p(a + 2a_3)\tilde{\mathcal{A}} = 0,$$

as a consequence of (3.8c). Hence, (3.11) implies

$$(\bar\nabla_k\bar\nabla^k + m^2_{0^-})\tilde{\mathcal{A}} = 0, \qquad m^2_{0^-} = \frac{3(a - qb_5)(a + 2a_3)}{2a_3b_5}. \tag{3.12}$$

Thus, generically, $\tilde{\mathcal{A}}$ can be identified as the spin-0^- excitation with respect to the (A)dS background. For $q = 0$, $m^2_{0^-}$ takes the Minkowskian form.

IV. HAMILTONIAN STRUCTURE

In this section, we analyze general features of the Hamiltonian structure of 3D gravity with propagating torsion, defined by the Lagrangian (2.1); see [4,20].

A. Primary constraints

We begin our study by analyzing the primary constraints. The canonical momenta corresponding to basic dynamical variables $(b^i{}_\mu, A^{ij}{}_\mu)$ are $(\pi_i{}^\mu, \Pi_{ij}{}^\mu)$; they are given by

$$\pi_i{}^\mu := \frac{\partial\tilde{\mathcal{L}}}{\partial(\partial_0 b^i{}_\mu)} = b\mathcal{H}_i{}^{0\mu},$$
$$\Pi_{ij}{}^\mu := \frac{\partial\tilde{\mathcal{L}}}{\partial(\partial_0 A^{ij}{}_\mu)} = b\mathcal{H}_{ij}{}^{0\mu}.$$

Since the torsion and the curvature do not involve the velocities $\partial_0 b^i_0$ and $\partial_0 A^{ij}{}_0$, one obtains the so-called "sure" primary constraints

$$\pi_i{}^0 \approx 0, \qquad \Pi_{ij}{}^0 \approx 0, \tag{4.1}$$

which are always present, independently of the values of coupling constants. If the Lagrangian (2.1) is singular with respect to some of the remaining velocities $\partial_0 b^i{}_\alpha$ and $\partial_0 A^{ij}{}_\alpha$, one obtains further primary constraints. The existence of these primary "if-constraints" (ICs) is determined by the critical values of the coupling constants.

1. The torsion sector

The gravitational Lagrangian (2.1) depends on the time derivative $\partial_0 b^i{}_\alpha$ only through the torsion tensor, appearing in $\mathcal{L}_{T^2}$. It is convenient to decompose T_{ijk} into the parallel and orthogonal components with respect to the spatial hypersurface Σ (see Appendix A),

$$T_{ijk} = T_{i\bar\jmath\bar k} + 2T_{i[\bar\jmath\perp}n_{k]} = \boldsymbol{T}_{ijk} + \mathcal{T}_{ijk},$$

where $\boldsymbol{T}_{ijk} := T_{i\bar\jmath\bar k}$ does not depend on velocities and the unphysical variables $(b^i{}_0, A^{ij}{}_0)$, and n_k is the normal to Σ. Now, by introducing the parallel gravitational momentum $\hat\pi_i{}^{\bar k} = \pi_i{}^\alpha b^k{}_\alpha$ $(\hat\pi_i{}^{\bar k}n_k = 0)$, one obtains

$$\hat\pi_{i\bar k} = J\mathcal{H}_{i\perp\bar k}(T), \tag{4.2a}$$

where $J := \det(b^{\bar\imath}{}_\alpha)$, and

$$\mathcal{H}_{i\perp\bar k} = 2[2\alpha_1 T_{i\perp\bar k} + \alpha_2(T_{\bar k\perp i} - T_{\perp\bar k i}) + \alpha_3(n_i V_{\bar k} - \eta_{i\bar k}V_\perp)].$$

The linearity of $\mathcal{H}_{ijk}(T)$ in the torsion tensor allows us to rewrite (4.2a) in the form

$$\phi_{i\bar k} := \frac{\hat\pi_{i\bar k}}{J} - \mathcal{H}_{i\perp\bar k}(\boldsymbol{T}) = \mathcal{H}_{i\perp\bar k}(\mathcal{T}), \tag{4.2b}$$

where the "velocities" $T_{i\bar\jmath\perp}$ appear only on the right-hand side. This system of equations can be decomposed into irreducible parts with respect to the group of two-dimensional rotations in Σ. Going over to the parameters a_1, a_2, a_3, one obtains

$$\phi_{\perp\bar k} \equiv \frac{\hat\pi_{\perp\bar k}}{J} - (a_2 - a_1)T^{\bar m}{}_{\bar m\bar k} = (a_1 + a_2)T_{\perp\perp\bar k}, \tag{4.3a}$$
$${}^S\phi \equiv \frac{{}^S\hat\pi}{J} = -2a_2 T^{\bar m}{}_{\bar m\perp}, \tag{4.3b}$$
$${}^A\phi_{\bar\imath\bar k} \equiv \frac{{}^A\hat\pi_{\bar\imath\bar k}}{J} - \frac{2}{3}(a_1 - a_3)T_{\perp\bar\imath\bar k} = -\frac{2}{3}(a_1 + 2a_3)T_{[\bar\imath\bar k]\perp}, \tag{4.3c}$$
$${}^T\phi_{\bar\imath\bar k} \equiv \frac{{}^T\hat\pi_{\bar\imath\bar k}}{J} = -2a_1{}^T T_{\bar\imath\bar k\perp}, \tag{4.3d}$$

where ${}^S\phi$, ${}^A\phi_{\bar\imath\bar k}$, and ${}^T\phi_{\bar\imath\bar k}$ are the trace (scalar), antisymmetric, and traceless-symmetric parts of $\phi_{\bar\imath\bar k}$ (Appendix A).

If the critical parameter combinations appearing on the right-hand sides of Eqs. (4.3) vanish, the corresponding expressions ϕ_K become additional primary constraints, the primary ICs. After a suitable reordering, the result of the analysis is summarized as follows:

For $a_2 = 0$, $a_1 + 2a_3 = 0$, $a_1 + a_2 = 0$, and/or $a_1 = 0$, the expressions ${}^S\phi$, ${}^A\phi_{\bar\imath\bar k}$, $\phi_{\perp\bar k}$, and/or ${}^T\phi_{\bar\imath\bar k}$ become primary ICs (see Table I below).

2. The curvature sector

In order to examine how the gravitational Lagrangian depends on the velocities $\partial_0 A^{ij}{}_\alpha$, we start with the following decomposition of the curvature tensor:

$$R_{ijmn} = R_{ij\bar m\bar n} + 2R_{ij[\bar m\perp}n_{n]} = \boldsymbol{R}_{ijmn} + \mathcal{R}_{ijmn},$$

where $\boldsymbol{R}_{ijmn} := R_{ij\bar m\bar n}$ does not depend on the "velocities" $R_{ij\perp\bar k}$ and the unphysical variables. The parallel gravitational momentum $\hat\Pi_{ij}{}^{\bar k} =: \Pi_{ij}{}^\alpha b^k{}_\alpha$ $(\hat\Pi_{ij}{}^{\bar k} n_k = 0)$ is given as

$$\hat\Pi_{ij\bar k} = J\mathcal{H}_{ij\perp\bar k}(R), \tag{4.4a}$$

where

$$\begin{aligned}\mathcal{H}_{ij\perp\bar k} &= -4an_{[i}\eta_{j]\bar k} + 4n_{[i}\mathcal{H}_{j]\bar k} - 4\eta_{[i\bar k}\mathcal{H}_{j]\perp}\\ &= 4n_{[i}\eta_{j]\bar k}(-a + 2\beta_3 R) + 4\beta_1(n_{[i}R_{j]\bar k} - \eta_{[i\bar k}R_{j]\perp})\\ &\quad + 4\beta_2(n_{[i}R_{\bar k j]} - \eta_{[i\bar k}R_{\perp j]}).\end{aligned}$$

Since the velocities $R_{ij\perp\bar k}$ are contained only in $\mathcal{R}$, we rewrite this equation as

$$\Phi_{ij\bar k} := \frac{\hat\Pi_{ij\bar k}}{J} + 4an_{[i}\eta_{j]\bar k} - \mathcal{H}'_{ij\perp\bar k}(\boldsymbol{R}) = \mathcal{H}'_{ij\perp\bar k}(\mathcal{R}). \tag{4.4b}$$

The components of a tensor $X_{\perp\bar\imath\bar\jmath}$ can be decomposed into the trace, antisymmetric, and symmetric-traceless piece (Appendix A). Such a decomposition of (4.4b) yields

TABLE I. Primary if-constraints.

Critical conditions	Primary constraints	J^P
$a_2 = 0$	${}^S\phi \approx 0$	0^+
$b_4 + 2b_6 = 0$	${}^S\Phi_\perp \approx 0$	
$a_1 + 2a_3 = 0$	${}^A\phi_{\bar\imath\bar k} \approx 0$	0^-
$b_5 = 0$	${}^A\Phi_{\perp\bar\imath\bar k} \approx 0$	
$a_1 + a_2 = 0$	$\phi_{\perp\bar k} \approx 0$	1
$b_4 + b_5 = 0$	${}^V\Phi_{\bar k} \approx 0$	
$a_1 = 0$	${}^T\phi_{\bar\imath\bar k} \approx 0$	2
$b_4 = 0$	${}^T\Phi_{\perp\bar\imath\bar k} \approx 0$	

$$\begin{aligned}{}^S\Phi_\perp &\equiv \frac{{}^S\hat\Pi_\perp}{J} + 4a - \frac{2}{3}(b_6 - b_4)R^{\bar k\bar n}{}_{\bar k\bar n}\\ &= \frac{2}{3}(b_4 + 2b_6)R^{\bar k}{}_{\perp\bar k\perp},\end{aligned} \tag{4.5a}$$

$${}^A\Phi_{\perp\bar\imath\bar\jmath} \equiv \frac{{}^A\hat\Pi_{\perp\bar\imath\bar\jmath}}{J} + 2b_5 R^{\bar k}_{[\bar\imath\bar\jmath]\bar k} = 2b_5 R_{[\bar\imath\perp\bar\jmath]\perp}, \tag{4.5b}$$

$$\begin{aligned}{}^T\Phi_{\perp\bar\imath\bar\jmath} &\equiv \frac{{}^T\hat\Pi_{\perp\bar\imath\bar\jmath}}{J} - b_4(2R_{(\bar\imath\bar k\bar\jmath)}{}^{\bar k} - \eta_{\bar\imath\bar\jmath}R^{\bar m\bar n}{}_{\bar m\bar n})\\ &= b_4(2R_{(\bar\imath\perp\bar\jmath)\perp} - \eta_{\bar\imath\bar\jmath}R^{\bar k}{}_{\perp\bar k\perp}).\end{aligned} \tag{4.5c}$$

For a tensor $X_{\bar\imath\bar\jmath\bar k} = -X_{\bar\jmath\bar\imath\bar k}$, the pseudoscalar $(\varepsilon^{\bar\imath\bar\jmath\bar k}X_{\bar\imath\bar\jmath\bar k})$ and the symmetric-traceless piece $(X_{\bar\imath(\bar\jmath\bar k)} - \text{traces})$ identically vanish. Hence, Eq. (4.4b) implies one more relation,

$${}^V\Phi^{\bar\imath} \equiv \frac{{}^V\hat\Pi^{\bar\imath}}{J} - (b_4 - b_5)R_{\perp\bar k}{}^{\bar\imath\bar k} = (b_4 + b_5)R^{\bar\imath\bar k}{}_{\perp\bar k}, \tag{4.5d}$$

where ${}^V X^{\bar\imath} = X^{\bar\imath\bar\jmath}{}_{\bar\jmath}$ (Appendix A).

Thus, when the parameters appearing on the right-hand sides of (4.5) vanish, we have the additional primary constraints Φ_K. Combining these relations with those obtained in the torsion sector, one finds the complete set of primary ICs, including their spin-parity characteristics (J^P), as shown in Table I.

This classification has a noteworthy interpretation: whenever a pair of the ICs with specific J^P is absent, the corresponding dynamical mode is liberated and becomes a *physical degree of freedom* (DoF). Thus, for $a_2(b_4 + 2b_6) \neq 0$, the spin-0^+ ICs are absent, and the related DoF becomes physical. Similarly, $(a_1 + 2a_3)b_5 \neq 0$ implies that the spin-0^- DoF becomes physical. The results obtained here refer to the full nonlinear theory; possible differences with respect to the perturbative analysis (Sec. III) will be discuss in Secs. V and VI.

B. General form of the Hamiltonian

Once we know the complete set of the primary ICs, we can construct first the canonical and then the total Hamiltonian. Being interested only in the gravitational degrees of freedom, we disregard the matter contribution.

1. Canonical Hamiltonian

In the absence of matter, the canonical Hamiltonian (density) is defined by

$$\mathcal{H}_c = \pi_i{}^\alpha \dot b^i{}_\alpha + \frac{1}{2}\Pi_{ij}{}^\alpha \dot A^{ij}{}_\alpha - b\mathcal{L}_G.$$

Using the lapse and shift functions N and N^α, defined in Appendix A, one can rewrite $\mathcal{H}_c$ in the Dirac–Arnowitt-Deser-Misner (DADM) form [4,20],

$$\mathcal{H}_c = N\mathcal{H}_\perp + N^\alpha\mathcal{H}_\alpha - \frac{1}{2}A^{ij}{}_0\mathcal{H}_{ij} + \partial_\alpha D^\alpha, \tag{4.6a}$$

where

$$\begin{aligned}
\mathcal{H}_\perp &= \hat{\pi}_i{}^{\bar{j}} T^i{}_{\perp\bar{j}} + \frac{1}{2}\hat{\Pi}_{ij}{}^{\bar{k}} R^{ij}{}_{\perp\bar{k}} - J\mathcal{L}_G - n^i\nabla_\alpha\pi_i{}^\alpha, \\
\mathcal{H}_\alpha &= \pi_i{}^\beta T^i{}_{\alpha\beta} + \frac{1}{2}\Pi_{ij}{}^\beta R^{ij}{}_{\alpha\beta} - b^i{}_\alpha\nabla_\beta\pi_i{}^\beta, \\
\mathcal{H}_{ij} &= 2\pi_{[i}{}^\alpha b_{j]\alpha} + \nabla_\alpha\Pi_{ij}{}^\alpha, \\
D^\alpha &= b^i{}_\alpha\pi_i{}^\alpha + \frac{1}{2}\Pi_{ij}{}^\alpha A^{ij}{}_\alpha.
\end{aligned} \tag{4.6b}$$

The canonical Hamiltonian is linear in the unphysical variables $(b^i{}_0, A^{ij}{}_0)$, and $\mathcal{H}_\perp$ is the only dynamical part of $\mathcal{H}_c$. The velocities $T^i{}_{\perp\bar{k}}, R^{ij}{}_{\perp\bar{k}}$ appearing in $\mathcal{H}_\perp$ can be expressed in terms of the phase-space variables, using Eqs. (4.3) and (4.5). Explicit calculation is simplified by separating the torsion and the curvature contributions in $\mathcal{H}_\perp$:

$$\begin{aligned}
\mathcal{H}_\perp &= 2\Lambda_0 J + \mathcal{H}_\perp^T + \mathcal{H}_\perp^R, \\
\mathcal{H}_\perp^T &:= \hat{\pi}^{i\bar{j}} T_{i\perp\bar{j}} - J\mathcal{L}_{T^2} - n^i\nabla_\alpha\pi_i{}^\alpha, \\
\mathcal{H}_\perp^R &:= \frac{1}{2}\hat{\Pi}^{ij\bar{k}} R_{ij\perp\bar{k}} - J\mathcal{L}_{R^2} + aJ\boldsymbol{R}.
\end{aligned} \tag{4.7}$$

The torsion piece of $\mathcal{H}_\perp$ turns out to have the form (Appendix A)

$$\mathcal{H}_\perp^T = \frac{1}{2}J\phi^2 - J\mathcal{L}_{T^2}(\boldsymbol{T}) - n^i\nabla_\alpha\pi_i{}^\alpha, \tag{4.8a}$$

$$\begin{aligned}
\phi^2 &:= \frac{\lambda(a_1+a_2)}{a_1+a_2}(\phi_{\perp\bar{\imath}})^2 + \frac{\lambda(a_2)}{2a_2}({}^S\phi)^2 \\
&\quad + \frac{3}{2}\frac{\lambda(a_1+2a_3)}{(a_1+2a_3)}({}^A\phi_{\bar{\imath}\bar{j}})^2 + \frac{\lambda(a_1)}{2a_1}({}^T\phi_{\bar{\imath}\bar{j}})^2,
\end{aligned} \tag{4.8b}$$

where $\lambda(x)$ is the singular function

$$\frac{\lambda(x)}{x} = \begin{cases} \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases},$$

which takes care of the conditions under which ICs become true constraints. Similar calculations for the curvature part yield

$$\mathcal{H}_\perp^R = \frac{1}{4}J\Phi^2 - J\mathcal{L}_{R^2}(\boldsymbol{R}) + aJ\boldsymbol{R}, \tag{4.9a}$$

$$\begin{aligned}
\Phi^2 &:= \frac{\lambda(b_5)}{b_5}({}^A\Phi_{\perp\bar{j}\bar{k}})^2 + \frac{\lambda(b_4)}{b_4}({}^T\Phi_{\perp\bar{j}\bar{k}})^2 \\
&\quad + \frac{3}{2}\frac{\lambda(b_4+2b_6)}{b_4+2b_6}({}^S\Phi_\perp)^2 + 2\frac{\lambda(b_4+b_5)}{b_4+b_5}({}^V\Phi_{\bar{\imath}})^2.
\end{aligned} \tag{4.9b}$$

2. Total Hamiltonian

The total Hamiltonian is defined by the expression

$$\mathcal{H}_{\rm tot} = \mathcal{H}_c + u^k{}_0\phi_k{}^0 + \frac{1}{2}u^{ij}\Phi_{ij}{}^0 + (u\cdot\phi) + (\boldsymbol{v}\cdot\Phi), \tag{4.10a}$$

where u's and $\boldsymbol{v}$'s are arbitrary multipliers and $(u\cdot\phi) + (\boldsymbol{v}\cdot\Phi)$ denotes the contribution of all the primary ICs. Formally, the existence of ICs is regulated by the form of the related multipliers, for instance, $u_{\perp\bar{k}}$ is given as $u_{\perp\bar{k}} := [1-\lambda(a_1+a_2)]u'_{\perp\bar{k}}$, and so on. Using the irreducible decomposition technique, we find

$$\begin{aligned}
(u\cdot\phi) &:= u^{\perp\bar{k}}\phi_{\perp\bar{k}} + {}^T u^{\bar{\imath}\bar{k}T}\phi_{\bar{\imath}\bar{k}} + {}^A u^{\bar{\imath}\bar{k}A}\phi_{\bar{\imath}\bar{k}} + \frac{1}{2}{}^S u^S\phi, \\
(\boldsymbol{v}\cdot\Phi) &:= {}^T\boldsymbol{v}^{\perp\bar{\imath}\bar{k}T}\Phi_{\perp\bar{\imath}\bar{k}} + {}^A\boldsymbol{v}^{\perp\bar{\imath}\bar{k}A}\Phi_{\perp\bar{\imath}\bar{k}} \\
&\quad + \frac{1}{2}{}^S\boldsymbol{v}^{\perp S}\Phi_\perp + {}^V\boldsymbol{v}^{\bar{k}V}\Phi_{\bar{k}}.
\end{aligned} \tag{4.10b}$$

3. Consistency conditions

Having found the form of the total Hamiltonian, we can now apply Dirac's consistency algorithm to the primary constraints, $\dot{\phi}_K = \{\phi_K, H_{\rm tot}\} \approx 0$, where $H_{\rm tot} = \int d^3x\mathcal{H}_{\rm tot}$ and $\{X, Y\}$ is the Poisson bracket (PB) between X and Y; then, the procedure continues with the secondary constraints, and so on [20]. In what follows, our attention will be focused on the scalar sector, with $J^P = 0^+$ or 0^- modes.

V. SPIN-0^+ SECTOR

As one can see from Table I, the absence of two spin-0^+ constraints, ${}^S\phi$ and ${}^S\Phi_\perp$, is ensured by the condition $a_2(b_4+2b_6)\neq 0$, whereby the spin-0^+ degree of freedom becomes physical. To study the dynamical content of this sector, we adopt the relaxed conditions

$$a_2, b_6 \neq 0, \qquad a_1 = a_3 = b_4 = b_5 = 0, \tag{5.1}$$

which define the Lagrangian $\mathcal{L}_G^+$ as in (3.4b).

A. Hamiltonian and constraints

1. Primary constraints

In the spin-0^+ sector (5.1), general considerations of the previous section lead to the following conclusions: the set of primary constraints is given by

$$\begin{aligned}
&\pi_i{}^0 \approx 0, \qquad \Pi_{ij}{}^0 \approx 0, \qquad {}^A\phi_{\bar{\imath}\bar{j}} := \frac{{}^A\hat{\pi}_{\bar{\imath}\bar{j}}}{J} \approx 0, \\
&{}^T\phi_{\bar{\imath}\bar{j}} := \frac{{}^T\hat{\pi}_{\bar{\imath}\bar{j}}}{J} \approx 0, \qquad {}^A\Phi_{\perp\bar{\imath}\bar{j}} := \frac{{}^A\hat{\Pi}_{\perp\bar{\imath}\bar{j}}}{J} \approx 0, \\
&{}^T\Phi_{\perp\bar{\imath}\bar{j}} := \frac{{}^T\hat{\Pi}_{\perp\bar{\imath}\bar{j}}}{J} \approx 0, \qquad {}^V\Phi_{\bar{\imath}} := \frac{{}^V\hat{\Pi}_{\bar{\imath}}}{J} \approx 0,
\end{aligned} \tag{5.2}$$

the dynamical part of the canonical Hamiltonian has the form

$$\begin{aligned}
\mathcal{H}_\perp &= J\left[\frac{1}{2a_2}(\phi_{\perp\bar{k}})^2 + \frac{1}{4a_2}({}^S\phi)^2 + \frac{3}{16b_6}({}^S\Phi_\perp)^2\right] \\
&\quad - J\mathcal{L}_G^+(\boldsymbol{T},\boldsymbol{R}) - n_i\nabla_\alpha\pi^{i\alpha},
\end{aligned} \tag{5.3}$$

where $\phi_{\perp\bar k}$, ${}^S\phi$, and ${}^S\Phi_\perp$ are the "generalized" momentum variables defined in (4.3) and (4.5), and the total Hamiltonian reads

$$\mathcal{H}_{\rm tot} = \mathcal{H}_{\rm c} + {}^A u^{\bar\imath\bar\jmath A}\phi_{\bar\imath\bar\jmath} + {}^T u^{\bar\imath\bar\jmath T}\phi_{\bar\imath\bar\jmath} + {}^A v^{\bar\imath\bar\jmath A}\Phi_{\bar\imath\bar\jmath} + {}^T v^{\bar\imath\bar\jmath T}\Phi_{\bar\imath\bar\jmath} + {}^V v^{\bar\imath V}\Phi_{\bar\imath}. \tag{5.4}$$

2. Secondary constraints

The consistency conditions of the sure primary constraints $\pi_i{}^0$ and $\pi_{ij}{}^0$ produce the secondary constraints

$$\mathcal{H}_\perp \approx 0, \qquad \mathcal{H}_\alpha \approx 0, \qquad \mathcal{H}_{ij} \approx 0, \tag{5.5a}$$

where

$$\begin{aligned}
\mathcal{H}_\alpha &\approx \hat\pi_\perp{}^{\bar\imath} T_{\perp\alpha\bar\imath} - \frac{1}{2}{}^S\hat\pi V_\alpha + \frac{1}{2}{}^S\hat\Pi_\perp R_{\perp\alpha} - b^i{}_\alpha \nabla_\beta \pi_i{}^\beta,\\
\mathcal{H}_{\bar\imath\bar k} &\approx \frac{{}^A\hat\pi_{\bar\imath\bar k}}{J} + \frac{{}^S\hat\Pi_\perp}{2J} T_{\perp\bar\imath\bar k},\\
\mathcal{H}_{\perp\bar k} &\approx \frac{\hat\pi_{\perp\bar k}}{J} - \frac{{}^S\hat\Pi_\perp}{2J} V_{\bar k} + \nabla_{\bar k}\frac{{}^S\hat\Pi_\perp}{2J}.
\end{aligned} \tag{5.5b}$$

Going over to the (eight) primary ICs, $X_M = ({}^A\phi, {}^T\phi, {}^A\Phi, {}^T\Phi, {}^V\Phi)$, we note that the only nonvanishing PBs among them are

$$\begin{aligned}
\{{}^A\Phi_{\perp\bar\imath\bar\jmath}, {}^A\phi^{\bar m\bar n}\} &\approx -\frac{{}^S\hat\Pi_\perp}{2J^2}\delta_i{}^{[\bar m}\delta_j{}^{\bar n]}\delta,\\
\{{}^T\Phi_{\perp\bar\imath\bar\jmath}, {}^T\phi^{\bar m\bar n}\} &\approx \frac{{}^S\hat\Pi_\perp}{2J^2}\delta_{(\bar\imath}{}^{(\bar n}\delta_{\bar\jmath)}{}^{\bar m)}\delta.
\end{aligned} \tag{5.6}$$

As long as ${}^S\hat\Pi_\perp \neq 0$, the constraints $({}^A\phi, {}^T\phi, {}^A\Phi, {}^T\Phi)$ are second class (SC) [4,20], and their consistency conditions fix the values of the corresponding multipliers $({}^Au, {}^Tu, {}^Av, {}^Tv)$ in $\mathcal{H}_{\rm tot}$. On the other hand, ${}^V\Phi$ commutes with all the other primary constraints, but not with its own secondary pair $\chi_{\bar\imath} = \{{}^V\Phi_{\bar\imath}, H_{\rm tot}\}$; see [20]. Using $\chi_{\bar\imath} \approx J^{-1}\{{}^V\hat\Pi_{\bar\imath}, H_{\rm tot}\}$ and

$$\begin{aligned}
&\{{}^V\hat\Pi_{\bar\imath}, \mathcal{H}_{mn}\} \approx 0, \qquad \{{}^V\hat\Pi_{\bar\imath}, \mathcal{H}_\alpha\} \approx 0,\\
&\{{}^V\hat\Pi_{\bar\imath}, \mathcal{H}_\perp\} \approx J\left[\frac{\phi_{\perp\bar\imath}}{a_2}\left(\frac{a_2}{2} - \frac{{}^S\hat\Pi_\perp}{4J}\right) + \frac{a_2}{2}V_{\bar\imath} + \nabla_{\bar\imath}\frac{{}^S\hat\Pi_\perp}{4J}\right],
\end{aligned}$$

one ends up with

$$\chi_{\bar\imath} := \frac{\phi_{\perp\bar\imath}}{a_2}\left(\frac{{}^S\hat\Pi_\perp}{4J} - \frac{a_2}{2}\right) - \frac{a_2}{2}V_{\bar\imath} - \nabla_{\bar\imath}\frac{{}^S\hat\Pi}{4J}. \tag{5.7}$$

The only nonvanishing PB involving $\chi_{\bar\imath}$ is

$$\{\chi_{\bar\imath}, {}^V\Phi_{\bar k}\} = \frac{2}{a_2 J}\eta_{\bar\imath\bar k}\frac{{}^S\hat\Pi_\perp}{4J}\left(\frac{{}^S\hat\Pi_\perp}{4J} - a_2\right)\delta. \tag{5.8}$$

Thus, for ${}^S\hat\Pi_\perp({}^S\hat\Pi_\perp - 4Ja_2) \neq 0$, both $\chi_{\bar\imath}$ and ${}^V\Phi_{\bar k}$ are SC. Consequently, the consistency condition of $\chi_{\bar\imath}$ determines the multiplier ${}^Vv^{\bar\imath}$, which completes the consistency algorithm.

If the kinetic energy density in the Hamiltonian (5.3) is to be positive definite ("no ghosts"), the coefficients of $({}^S\phi)^2$ and $({}^S\Phi_\perp)^2$ should be positive:

$$a_2 > 0, \qquad b_6 > 0. \tag{5.9}$$

On the other hand, $(\phi_{\perp\bar k})^2$ gives a negative definite contribution, but it is an interaction term, as can be seen from (4.3a) and (5.5b).

B. Constraint bifurcation

In the previous discussion, we identified the conditions for which all the ICs, $X'_M = (X_M, \chi)$, are SC. To calculate the determinant of the 10×10 matrix $\Delta^+_{MN} = \{X'_M, X'_N\}$,

$$\Delta^+ \approx \begin{vmatrix}
0 & 0 & \{{}^A\phi, {}^A\Phi\} & 0 & 0 & 0\\
0 & 0 & 0 & \{{}^T\phi, {}^T\Phi\} & 0 & 0\\
-\{{}^A\phi, {}^A\Phi\} & 0 & 0 & 0 & 0 & 0\\
0 & -\{{}^T\phi, {}^T\Phi\} & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & \{{}^V\phi, \chi\}\\
0 & 0 & 0 & 0 & -\{{}^V\phi, \chi\} & 0
\end{vmatrix}$$

we use (5.6) and (5.8), which leads to

$$\Delta^+ \sim \left(\frac{{}^S\Pi_\perp}{4J}\right)^{10}\left(\frac{{}^S\Pi_\perp}{4J} - a_2\right)^4. \tag{5.10}$$

Introducing a convenient notation

$$W := \frac{{}^S\Pi_\perp}{4J}, \tag{5.11}$$

we see that Δ^+ can vanish only on a set (of spacetime points) of measure zero, defined by $W = 0$ or $W - a_2 = 0$. In other words, the condition

$$W(W - a_2) \neq 0 \tag{5.12}$$

is fulfilled *almost everywhere* (everywhere except on a set of measure zero). Thus, our previous discussion can be summarized by saying that all of the ICs are SC almost

TABLE II. Generic constraints in the 0^+ sector.

	First class	Second class
Primary	${\pi_i}^0$, ${\Pi_{ij}}^0$	X_M
Secondary	$\mathcal{H}'_\perp$, $\mathcal{H}'_\alpha$, $\mathcal{H}'_{ij}$	$\chi_{\bar{\imath}}$

everywhere; the related (generic) classification of constraints is shown in Table II.

The Hamiltonian constraints $\mathcal{H}'_\perp$, $\mathcal{H}'_\alpha$, and $\mathcal{H}'_{ij}$ are first class (FC) [4,20]; they are obtained from (5.3) and (5.5b) by adding the contributions containing the determined multipliers. With $N = 18$, $N_1 = 12$, and $N_2 = 10$, the dimension of the phase space is given as $N^* = 2N - 2N_1 - N_2 = 2$. Thus, the theory exhibits a single Lagrangian DoF almost everywhere.

However, the determinant Δ^+, being a field-dependent object, may vanish in some regions of spacetime, changing thereby the number and/or type of constraints and the number of physical degrees of freedom, as compared to the generic situation described in Table II. This effect, known as the phenomenon of *constraint bifurcation*, can be fully understood by analyzing the dynamical behavior of the two factors in (5.12). Although the complete analysis can be carried out in the canonical formalism, we base our arguments on the Lagrangian formalism, in order to simplify the exposition (see Appendix B).

Starting with the second factor,

$$\Omega := W - a_2 \approx -\left(a - \frac{1}{6}b_6 R + a_2\right), \qquad (5.13)$$

where we used (4.5a) to clarify the geometric interpretation, one can prove the relation

$$-\Omega V_k + 2\partial_k \Omega \approx 0, \qquad (5.14)$$

which implies that the behavior of Ω is limited to the following two options (Appendix B):

(a) either $\Omega(x)$ vanishes globally, on the whole spacetime manifold,

(b) or it does not vanish anywhere.

Which of these two options is realized depends upon the initial conditions for Ω; choosing them in accordance with (b) extends the generic behavior of Ω, $\Omega \neq 0$ almost everywhere, to the whole spacetime. This mechanism is the same as the one observed in the spin-0^+ sector of the four-dimensional PGT; compare (5.14) with equation (4.20) in [16].

We now focus our attention to the first factor in (5.12),

$$W \approx -\left(a - \frac{1}{6}b_6 R\right). \qquad (5.15)$$

It is interesting that a solution for the W bifurcation ($W = 0$) can be found by relying on the solution for the Ω bifurcation, which is based on choosing $\Omega \neq 0$ on the initial spatial surface Σ. Indeed, the choice $\Omega > 0$ on Σ implies

$$\Omega > 0 \quad \text{globally}. \qquad (5.16a)$$

Then, since $\Omega = W - a_2$ (a_2 is positive), we find

$$W > a_2 \quad \text{globally}. \qquad (5.16b)$$

Thus, with $\Omega > 0$ and $W > a_2$, the problem of constraint bifurcation simply disappears. Note that geometrically, the condition $W > a_2$ represents a restriction on the Cartan scalar curvature, $b_6 R > 6(a + a_2)$. An equivalent form of this relation is obtained by using the identity $R = \tilde{R} - 2\sigma$, where $\tilde{R}$ is Riemannian scalar curvature.

Thus, with a suitable choice of the initial conditions, one can ensure the generic condition $\Delta^+ \neq 0$ to hold *globally*, so that the constraint structure of the spin-0^+ sector is described exactly as in Table II. Any other situation, with $W = 0$ or $W - a_2 = 0$, would not be acceptable—it would have a variable constraint structure over the spacetime, the property that could not survive the process of linearization.

C. Stability under linearization

Now, we are going to compare the canonical structure of the full nonlinear theory with its linear approximation around a maximally symmetric background.

In the linear approximation, the condition of canonical stability (5.12) is to be taken in the lowest order (zeroth) approximation. Using $\bar{R} = -6q$, it reduces to

$$(a + qb_6)(a + qb_6 + a_2) \neq 0. \qquad (5.17)$$

The three cases displayed in Table III define characteristic sectors of the linear regime (see Appendix C).

(a) When the condition (5.17) is satisfied, the nature of the constraints remains the same as in Table II, and we have a single Lagrangian DoF, the massive spin-0^+ mode.

(b) Here, all ICs become FC, but only six of them are independent. Thus, $N_1 = 12 + 6 = 18$, and with $N_2 = 0$, the number of DoF's is zero: $N^* = 36 - 2 \times 18 = 0$.

(c) In this case, $\tilde{\chi}_{\bar{k}}$ is not an independent constraint, and ${}^V\tilde{\Phi}_{\bar{k}}$ is FC. As compared to (a), the number and type of constraints is changed according to $N_1 \to N_1 + 2$, $N_2 \to N_2 - 4$, but the number of DoF's remains one ($N^* = 2$), corresponding to the massless spin-0^+ mode.

The case when both $a + qb_6$ and $a + qb_6 + a_2$ vanish is not possible, since $a_2 \neq 0$.

To clarify the case (c), we need a more detailed analysis. Consider first the case (a), in which the constraint $\tilde{\chi}_{\bar{\imath}}$, defined in (C3) is replaced by an equivalent expression, $\tilde{\chi}'_{\bar{\imath}} = \tilde{\bar{\pi}}_{\perp\bar{\imath}}/\bar{J}$. Then, the pair of SC constraints $({}^V\tilde{\Phi}_{\bar{k}}, \tilde{\chi}'_{\bar{k}})$,

TABLE III. Canonical stability in the 0^+ sector.

	$a + qb_6$	$a + qb_6 + a_2$	DoF	stability
(a)	$\neq 0$	$\neq 0$	1	stable
(b)	$= 0$	$\neq 0$	0	unstable
(c)	$\neq 0$	$= 0$	1	stable*

with the related Dirac brackets, defines the reduced phase space $\tilde{R}(a)$. Next, consider the case (c), where $\tilde{\chi}_{\bar{\imath}}$ does not exist and ${}^V\tilde{\Phi}_{\bar{k}}$ is FC. Here, we can introduce a suitable gauge condition associated with ${}^V\tilde{\Phi}_{\bar{k}}$, given by $\tilde{\chi}''_{\bar{k}} = \tilde{\hat{\pi}}_{\perp\bar{\imath}}/\bar{J}$. The pair $({}^V\tilde{\Phi}_{\bar{k}}, \tilde{\chi}''_{\bar{k}})$ defines the reduced phase space $\tilde{R}(c)$, which coincides with the reduced phase space $\tilde{R}(a)$, subject to the additional condition $a + qb_6 + a_2 = 0$. Thus, the "massless" nonlinear theory, defined by $a + qb_6 + a_2 = 0$, is essentially (up to a gauge fixing) stable under the linearization. The star symbol in Table III (stable*) is used to remind us of this gauge fixing condition.

For the M_3 background ($p = q = 0$ and $a \neq 0$), the case (b) is not possible.

VI. SPIN-0^- SECTOR

For $(a_1 + 2a_3)b_5 \neq 0$, the constraints ${}^A\phi_{\bar{\imath}\bar{k}}$, ${}^A\Phi_{\perp\bar{\imath}\bar{k}}$ in Table I are absent, and the spin-0^- mode becomes a physical degree of freedom. Here, we study canonical features of the spin-0^- sector by using the specific conditions

$$a_3, b_5 \neq 0, \qquad a_1 = a_2 = b_4 = b_6 = 0, \tag{6.1}$$

which define the Lagrangian $\mathcal{L}_G^-$ as in (3.8b).

A. Hamiltonian and constraints

1. Primary constraints

Applying the conditions (6.1) to the general considerations of Sec. IV, we find the following set of the primary (sure and if-) constraints:

$$\begin{aligned}
&\pi_i{}^0 \approx 0, \qquad \Pi_{ij}{}^0 \approx 0, \qquad {}^S\phi := \frac{{}^S\hat{\pi}}{J} \approx 0,\\
&{}^T\phi_{\bar{\imath}\bar{\jmath}} := \frac{{}^T\hat{\pi}_{\bar{\imath}\bar{\jmath}}}{J} \approx 0, \qquad \phi_{\perp\bar{\imath}} := \frac{\hat{\pi}_{\perp\bar{\imath}}}{J} \approx 0,\\
&{}^S\Phi_\perp := \frac{{}^S\hat{\Pi}_\perp}{J} + 4a \approx 0, \qquad {}^T\Phi_{\perp\bar{\imath}\bar{\jmath}} := \frac{{}^T\hat{\Pi}_{\perp\bar{\imath}\bar{\jmath}}}{J} \approx 0.
\end{aligned} \tag{6.2}$$

The dynamical part of the canonical Hamiltonian has the form

$$\begin{aligned}
\mathcal{H}_\perp = J&\left[\frac{3}{8a_3}({}^A\phi_{\bar{\imath}\bar{\jmath}})^2 + \frac{1}{4b_5}({}^A\Phi_{\perp\bar{\imath}\bar{\jmath}})^2 + \frac{1}{2b_5}({}^V\Phi_{\bar{\imath}})^2\right]\\
&- J\mathcal{L}_G^-(\boldsymbol{T}, \boldsymbol{R}) - n_i\nabla_\alpha\pi^{i\alpha},
\end{aligned} \tag{6.3}$$

where ${}^A\phi_{\bar{\imath}\bar{\jmath}}$, ${}^A\Phi_{\perp\bar{\imath}\bar{\jmath}}$, and $V\Phi_{\bar{\imath}}$ are the "generalized" momentum variables defined in (4.3) and (4.5), and the total Hamiltonian reads

$$\begin{aligned}
\mathcal{H}_T = \mathcal{H}_c &+ \frac{1}{2}{}^Su^S\phi + {}^Tu^{\bar{\imath}\bar{\jmath}T}\phi_{\bar{\imath}\bar{\jmath}} + u^{\perp\bar{\imath}}\phi_{\perp\bar{\imath}}\\
&+ \frac{1}{2}{}^Sv^{\perp S}\Phi_\perp + {}^Tv^{\perp\bar{\imath}\bar{\jmath}T}\Phi_{\perp\bar{\imath}\bar{\jmath}}.
\end{aligned} \tag{6.4}$$

2. Secondary constraints

The consistency conditions of the primary constraints $\pi_i{}^0$ and $\Pi_{ij}{}^0$ produce the usual secondary constraints:

$$\mathcal{H}_\perp \approx 0, \qquad \mathcal{H}_\alpha \approx 0, \qquad \mathcal{H}_{ij} \approx 0, \tag{6.5a}$$

where

$$\begin{aligned}
\mathcal{H}_\alpha &\approx {}^A\hat{\pi}^{\bar{\imath}\bar{\jmath}}T_{\bar{\imath}\alpha\bar{\jmath}} + {}^A\hat{\Pi}_{\perp\bar{\imath}\bar{\jmath}}R_\perp{}^{\bar{\imath}}{}_\alpha{}^{\bar{\jmath}} + R^{\bar{\imath}\bar{\jmath}}{}_{\alpha\bar{\jmath}}{}^V\hat{\Pi}_{\bar{\imath}} - 2aJR_{\perp\alpha}\\
&\quad - b^i{}_\alpha\nabla_\beta\pi_i{}^\beta,\\
\mathcal{H}_{\bar{\imath}\bar{\jmath}} &\approx aT_{\perp\bar{\imath}\bar{\jmath}} + \frac{{}^A\hat{\pi}_{\bar{\imath}\bar{\jmath}}}{2J} + \frac{{}^V\Pi_{\bar{k}}}{2J}T^{\bar{k}}_{\bar{\imath}\bar{\jmath}} + \nabla_{[\bar{\imath}}\frac{{}^V\hat{\Pi}_{\bar{\jmath}]}}{J},\\
\mathcal{H}_{\perp\bar{\imath}} &\approx aV_{\bar{\imath}} + \frac{{}^A\hat{\Pi}_{\perp\bar{m}\bar{n}}}{2J}T^{\bar{m}\bar{n}}{}_{\bar{\imath}} + \frac{{}^V\hat{\Pi}^{\bar{m}}}{2J}T_{\perp\bar{\imath}\bar{m}} + \frac{1}{2}\nabla_{\bar{m}}\frac{{}^A\hat{\Pi}_{\perp\bar{\imath}}{}^{\bar{m}}}{J}.
\end{aligned} \tag{6.5b}$$

Using the PB algebra between the primary ICs $Y_M = ({}^S\phi, {}^T\phi, \phi_{\perp\bar{k}}, {}^S\Phi, {}^T\Phi)$ (Appendix D), one finds that generically, for ${}^A\hat{\pi}_{\bar{\imath}\bar{k}} \neq 0$, they are SC; their consistency conditions result in the determination of the corresponding multipliers $({}^Su, {}^Tu, u_{\perp\bar{k}}, {}^Sv, {}^Tv)$. Moreover, the secondary constraints (6.5a), corrected by the contributions of the determined multipliers, are FC, so that their consistency conditions are trivially satisfied. Thus, in the generic case, the consistency algorithm is completed at the level of secondary constraints.

The first two terms in $\mathcal{H}_\perp$, proportional to the squares of ${}^A\phi_{\bar{\imath}\bar{k}}$ and ${}^A\Phi_{\perp\bar{\imath}\bar{k}}$, describe the contribution of the spin-0^- mode to the kinetic energy density, see Table I. This contribution is positive definite for

$$a_3 > 0, \qquad b_5 > 0. \tag{6.6}$$

At the same time, the contribution of the third term, the square of ${}^V\Phi_{\bar{k}}$, becomes negative definite ("ghost"), which is a *serious problem* for the physical interpretation. As we shall see, this is not the only problem.

B. Constraint bifurcation

Based on the PB algebra of the (eight) primary ICs Y_M, we can now calculate the determinant of the 8×8 matrix $\Delta^-_{MN} = \{Y_M, Y_N\}$ (Appendix D); the result takes the form

$$\Delta^- \sim {}^A\hat{\pi}_{\bar{\imath}\bar{\jmath}}{}^A\hat{\pi}^{\bar{\imath}\bar{\jmath}}\left(\frac{4a^2}{J^2} + \frac{1}{8J^4}{}^A\hat{\Pi}_{\perp\bar{m}\bar{n}}{}^A\hat{\Pi}^{\perp\bar{m}\bar{n}}\right)^2. \tag{6.7}$$

Since the second factor is always positive definite, Δ^- remains different from zero only if

$${}^A\hat{\pi}_{\bar{m}\bar{n}} \neq 0. \tag{6.8}$$

This condition holds everywhere except on a set of measure zero, so that $\Delta^- \neq 0$ almost everywhere. Thus, generically, the eight primary ICs are SC, as shown in

TABLE IV. Generic constraints in the 0^- sector.

	First class	Second class
Primary	${\pi_i}^0$, ${\Pi_{ij}}^0$	Y_M
Secondary	$\mathcal{H}'_\perp$, $\mathcal{H}'_\alpha$, $\mathcal{H}'_{ij}$	

Table IV; the primes in $\mathcal{H}'_\perp$, $\mathcal{H}'_\alpha$, and $\mathcal{H}'_{ij}$ denote the presence of corrections induced by the determined multipliers.

Using $N = 18$, $N_1 = 12$, and $N_2 = 8$, we find $N^* = 2N - 2N_1 - N_2 = 4$. Surprisingly, the theory exhibits *two* Lagrangian DoF: one is the expected spin-0^- mode, and the other is the spin-1 "ghost" mode, represented canonically by ${}^V\Phi_{\bar k}$.

In Appendix E, we analyzed the nature of the critical condition ${}^A\hat\pi_{\bar m \bar n} = 0$. In the region of spacetime where it holds, we found the phenomenom of constraint bifurcation: the number of Lagrangian DoF is changed to zero. Although such a situation is canonically unstable under linearization, it is interesting to examine basic aspects of the linearized theory.

C. Instability under linearization

In the linearized theory, the term ${}^A\bar{\hat\pi}_{\bar j \bar k}$ in the determinant Δ^- takes the form

$$ {}^A\bar{\hat\pi}_{\bar j \bar k} = -2a_3 \varepsilon_{\perp \bar j \bar k} p. \tag{6.9}$$

Hence, the canonical structure of the linearized theory crucially depends on the value of the background parameter p, as shown in Table V.

(α) For $p \neq 0$ (the background with nontrivial torsion, massless spin-0^+ mode), the determinant $\bar\Delta^-$ is positive definite, all the primary ICs are SC, and consequently, $N^* = 4$, as in the generic sector of the full nonlinear theory. However, this is not true in the critical region ${}^A\hat\pi_{\bar i \bar j} = 0$ where $N^* = 0$, and the theory is canonically unstable.

(β) For $p = 0$ (Riemannian background, massive or massless spin-0^- mode), the situation is changed, as discussed in Appendix F. First, the determinant $\bar\Delta^-$ vanishes, which follows from the fact that the primary IC $\tilde\phi_{\perp \bar\imath}$ *commutes* with itself; see (D1). By calculating its consistency condition (which was not needed for $p \neq 0$), one finds its secondary pair $\tilde\chi_{\bar\imath}$. Now, the PB of $\tilde\phi_{\perp\bar\imath}$ with the modified secondary pair $\tilde\chi'_{\bar\imath} = \tilde\chi_{\bar\imath} - \tilde{\mathcal{H}}_{\bar\imath}$ does not vanish.

TABLE V. Canonical stability in the 0^- sector.

		DoF	stability
(α)	$p \neq 0$	2	unstable
(β)	$p = 0$	1	unstable

Thus, there are two SC constraints more than in the case (α) so that $N^* = 2$, and we have the canonical instability under linearization.

Thus, in both cases α and β, the theory is canonically unstable.

VII. CONCLUDING REMARKS

In this paper, we studied the Hamiltonian structure of the general parity-invariant model of 3D gravity with propagating torsion, described by the eight-parameter PGT Lagrangian (2.1). Because of the complexity of the problem, we focused our attention on the scalar sector, containing $J^P = 0^+$ or 0^- modes with respect to a maximally symmetric background. By investigating fully nonlinear "constraint bifurcation" effects as well as the canonical stability under linearization, we were able to identify the set of dynamically acceptable values of parameters, displayed in Tables III and V. Transition from (A)dS to the Minkowski background simplifies the results.

Further analysis involving higher spin sectors is left for future studies.

ACKNOWLEDGMENTS

It is a pleasure to thank Vladimir Dragović for a helpful discussion. This work was supported by the Serbian Science Foundation under Grant No. 171031.

APPENDIX A: THE 1 + 2 DECOMPOSITION OF SPACETIME

To derive the DADM form of the Hamiltonian, it is convenient to pass from the tetrad basis $\boldsymbol{h}_i = {h_i}^\mu \partial_\mu$ to the ADM basis $(\boldsymbol{n}, \boldsymbol{h}_\alpha)$, where $\boldsymbol{n}$ is the unit vector with $n_k = {h_k}^0/\sqrt{g^{00}}$, orthogonal to the vectors $\boldsymbol{h}_\alpha = \partial_\alpha$ lying in the $x^0 = \text{const}$ hypersurface Σ; see [4,20].

Introducing the projectors on $\boldsymbol{n}$ and Σ, $(P_\perp)^i_k = n^i n_k$ and $(P_{||})^i_k = \delta^i_k - n^i n_k$, any vector V_k can be decomposed in terms of its normal and parallel projections:

$$\begin{aligned} V_k &= n_k V_\perp + V_{\bar k}, \\ V_\perp &:= n^k V_k, \\ V_{\bar k} &:= (P_{||})^i_k V_i = {h_{\bar k}}^\alpha V_\alpha. \end{aligned} \tag{A1}$$

The decomposition of $V_0 = {b^k}_0 V_k$ in the ADM basis yields $V_0 = NV_\perp + N^\alpha V_\alpha$, where the lapse and shift functions N and N^α, respectively, are linear in ${b^k}_0$:

$$ N := n_k {b^k}_0, \qquad N^\alpha := {h_{\bar k}}^\alpha {b^k}_0. \tag{A2}$$

The decomposition (A1) can be extended to any tensor field. Thus, a second rank antisymmetric tensor $X_{ik} = -X_{ki}$ can be decomposed as

$$ X_{ik} = X_{\bar\imath \bar k} + (X_{\bar\imath \perp} n_k - X_{\bar k \perp} n_i). \tag{A3}$$

The parallel tensors, like $X_{\bar{\imath}\bar{k}}$, lie in Σ, and can be further decomposed into the irreducible parts with respect to the spatial rotations:

$$X_{\bar{\imath}\bar{k}} = {}^T X_{\bar{\imath}\bar{k}} + {}^A X_{\bar{\imath}\bar{k}} + \frac{1}{2}\eta_{\bar{\imath}\bar{k}}{}^S X, \qquad \text{(A4a)}$$

where

$$\begin{aligned} {}^T X_{\bar{\imath}\bar{k}} &:= X_{(\bar{\imath}\bar{k})} - \frac{1}{2}\eta_{\bar{\imath}\bar{k}} X^{\bar{m}}{}_{\bar{m}}, \\ {}^A X_{\bar{\imath}\bar{k}} &:= X_{[\bar{\imath}\bar{k}]}, \\ {}^S X &:= X^{\bar{m}}{}_{\bar{m}}. \end{aligned}$$

As a consequence, the product $X^{\bar{\imath}\bar{k}} Y_{\bar{\imath}\bar{k}}$ is given by

$$X^{\bar{\imath}\bar{k}} Y_{\bar{\imath}\bar{k}} = {}^T X^{\bar{\imath}\bar{k}T} Y_{\bar{\imath}\bar{k}} + {}^A X^{\bar{\imath}\bar{k}A} Y_{\bar{\imath}\bar{k}} + \frac{1}{2}{}^S X^S Y. \qquad \text{(A4b)}$$

For a tensor $\Phi_{\bar{\imath}\bar{\jmath}\bar{k}} = -\Phi_{\bar{\jmath}\bar{\imath}\bar{k}}$, the pseudoscalar $(\varepsilon^{\bar{\imath}\bar{\jmath}\bar{k}}\Phi_{\bar{\imath}\bar{\jmath}\bar{k}})$ and traceless-symmetric piece $(\Phi_{\bar{\imath}(\bar{\jmath}\bar{k})} - \text{traces})$ identically vanish, so that the only nontrivial piece is the vector ${}^V\Phi_{\bar{\imath}} := \Phi_{\bar{\imath}\bar{k}}{}^{\bar{k}}$:

$$\Phi_{\bar{\imath}\bar{\jmath}\bar{k}} = 2\eta_{[\bar{\jmath}\bar{k}}{}^V\Phi_{\bar{\imath}]}, \qquad \Phi^{\bar{\imath}\bar{\jmath}\bar{k}} Q_{\bar{\imath}\bar{\jmath}\bar{k}} = 2{}^V\Phi^{\bar{\imath}V} Q_{\bar{\imath}}. \qquad \text{(A5)}$$

These results can be now used to find the DADM form of the Hamiltonian. Starting with the torsion sector, we use the formula $T = \boldsymbol{T} + \mathcal{T}$ to rewrite $\mathcal{L}_{T^2}$ in the form

$$\begin{aligned} \mathcal{L}_{T^2} &= \frac{1}{4}\mathcal{H}^{ijk}(\boldsymbol{T})\boldsymbol{T}_{ijk} + \frac{1}{4}\mathcal{H}^{ijk}(\mathcal{T})\boldsymbol{T}_{ijk} \\ &\quad + \frac{1}{4}\mathcal{H}^{ijk}(T)\mathcal{T}_{ijk} = \mathcal{L}_T(\boldsymbol{T}) + \tfrac{\hat{\pi}^{i\bar{\jmath}}}{J} T_{i\perp\bar{\jmath}} - \tfrac{1}{2}\phi^{i\bar{\jmath}} T_{i\perp\bar{\jmath}}, \end{aligned}$$

which yields

$$\mathcal{H}_\perp^T = \frac{1}{2} J\phi^{i\bar{\jmath}} T_{i\perp\bar{\jmath}} - J\mathcal{L}_{T^2}(\boldsymbol{T}) - n^i \nabla_\alpha \pi_i{}^\alpha. \qquad \text{(A6a)}$$

Then, the irreducible decomposition

$$\phi^{i\bar{\jmath}} T_{i\perp\bar{\jmath}} = \phi^{\perp\bar{\jmath}} T_{\perp\perp\bar{\jmath}} + {}^A\phi^{\bar{\imath}\bar{\jmath}} T_{[\bar{\imath}\perp\bar{\jmath}]} + {}^T\phi^{\bar{\imath}\bar{\jmath}} T_{(\bar{\imath}\perp\bar{\jmath})} + \frac{1}{2}{}^S\phi T^{\bar{\imath}}{}_{\perp\bar{\imath}}, \qquad \text{(A6b)}$$

in conjunction with (4.3), leads to (4.8).

Similar calculations for the curvature part yield

$$\mathcal{H}_\perp^R = \frac{1}{4} J\Phi^{ij\bar{k}} R_{ij\perp\bar{k}} - J\mathcal{L}_{R^2}(\boldsymbol{R}) + aJ\boldsymbol{R}. \qquad \text{(A7a)}$$

Then, the irreducible decompositions

$$\begin{aligned} \Phi^{ij\bar{k}} R_{ij\perp\bar{k}} &= 2\Phi^{\perp\bar{\jmath}\bar{k}} R_{\perp\bar{\jmath}\perp\bar{k}} + 2{}^V\Phi^{\bar{\imath}} R_{\bar{\imath}\bar{k}\perp}{}^{\bar{k}}, \\ 2\Phi^{\perp\bar{\jmath}\bar{k}} R_{\perp\bar{\jmath}\perp\bar{k}} &= 2\Big({}^A\Phi^{\perp\bar{\jmath}\bar{k}A} R_{\bar{\jmath}\perp\bar{k}\perp} + {}^T\Phi^{\perp\bar{\jmath}\bar{k}T} R_{\bar{\jmath}\perp\bar{k}\perp} \\ &\quad + \frac{1}{2}{}^S\Phi^\perp R^{\bar{k}}{}_{\perp\bar{k}\perp}\Big), \end{aligned} \qquad \text{(A7b)}$$

combined with (4.5), lead directly to (4.9).

APPENDIX B: CONSTRAINT BIFURCATION IN THE SPIN-0^+ SECTOR

In this appendix, we study the phenomenon of constraint bifurcation in the spin-0^+ sector, determined by the critical condition $\Omega = 0$.

We start our discussion by writing the field equations for the spin-0^+ sector:

$$\begin{aligned} &2a_2\eta_{i[j}\nabla^k V_{k]} + 2\Big(a - \frac{b_6}{6}R\Big)G_{ji} \\ &\quad - \eta_{ij}\Big(\frac{a_2}{2}V^2 + \frac{b_6}{12}R^2 + 2\Lambda_0\Big) = 0, \end{aligned} \qquad \text{(B1)}$$

$$\Big(a - \frac{b_6}{6}R\Big)T_{ijk} + a_2\eta_{i[j}V_{k]} + \frac{b_6}{3}\eta_{i[j}\nabla_{k]}R = 0, \qquad \text{(B2)}$$

where $V^2 = V_k V^k$. The content of these equations can be expressed in terms of their irreducible components. For the first equation, we find

$$-a_2\nabla_{[i}V_{j]} + 2\Big(a - \frac{b_6}{6}R\Big)R_{[ji]} = 0, \qquad \text{(B3a)}$$

$$-a_2\Big(\nabla_{(i}V_{j)} - \frac{1}{3}\eta_{ij}\sigma\Big) + 2\Big(a - \frac{b_6}{6}R\Big)\Big(R_{(ji)} - \frac{1}{3}\eta_{ij}R\Big) = 0, \qquad \text{(B3b)}$$

$$-2a_2\sigma + \frac{3}{2}a_2V^2 + aR + \frac{b_6}{12}R^2 + 6\Lambda_0 = 0, \qquad \text{(B3c)}$$

where $\sigma := \nabla_i V^i$. The irreducible components of the second equation are

$$\left(a - \frac{b_6}{6}R\right)\mathcal{A} = 0, \tag{B4a}$$

$$\left(a - \frac{b_6}{6}R\right)^T T_{ijk} = 0, \tag{B4b}$$

$$\left(a - \frac{b_6}{6}R + a_2\right)V_i + \frac{b_6}{3}\nabla_i R = 0. \tag{B4c}$$

Now, we focus our attention on the factor $\Omega = W - a_2$ in Δ^+. Its dynamical evolution is determined by Eq. (B4c), which can be written in the form

$$-\Omega V_k + 2\partial_k\Omega \approx 0. \tag{B5}$$

Note that this equation is an extension of Eq. (5.13) from Σ to the whole spacetime $\mathcal{M}$.

The spacetime continuum $\mathcal{M}$ on which 3D PGT lives is a differentiable manifold with topology $\mathcal{M} = R \times \Sigma$, where R corresponds to time, and Σ to the spatial section of $\mathcal{M}$. Let us now assume that (i) Ω vanishes at some point $x = a$ in $\mathcal{M}$, (ii) Ω is an infinitely differentiable function on $\mathcal{M}$, and (iii) V_k and all its derivatives are finite at $x = a$. Then, one can notice that (B5) implies $\partial_k\Omega = 0$ at $x = a$. In the next step, we apply the differential operator ∂_{k_1} to (B5) and conclude that $\partial_{k_1}\partial_k\Omega = 0$ at $x = a$. Continuing this procedure, we eventually conclude that for every n, $\partial_{k_n}\dots\partial_{k_1}\partial_k\Omega = 0$ at $x = a$. In general, the behavior of Ω on the whole $\mathcal{M}$ is not determined by its properties at a single point. However, if (iv) Ω is an analytic function on $\mathcal{M}$, its Taylor expansion around $x = a$ implies that $\Omega = 0$ on the whole $\mathcal{M}$.

The result obtained can be formulated in a more useful form: if there is at least one point in $\mathcal{M}$ at which $\Omega \neq 0$, then $\Omega \neq 0$ on the whole $\mathcal{M}$. Thus, by choosing the initial data so that $\Omega \neq 0$ at $x^0 = 0$, it follows that Ω stays nonvanishing for any $x^0 > 0$. In other words, for a suitable choice of initial data, the configuration $\Omega = 0$ is kind of a barrier that the system cannot cross during its dynamical evolution. Moreover, since Ω is a continuous function, it has a definite sign for any $x^0 > 0$.

APPENDIX C: THE LINEARIZED SPIN-0$^+$ SECTOR

In the weak-field approximation, the primary ICs of the spin-0$^+$ sector take the form

$$\begin{gathered}
{}^A\tilde{\phi}_{\bar{\imath}\bar{\jmath}} := \frac{{}^A\tilde{\hat{\pi}}_{\bar{\imath}\bar{\jmath}}}{\bar{J}} \approx 0, \qquad {}^T\tilde{\phi}_{\bar{\imath}\bar{\jmath}} := \frac{{}^T\tilde{\hat{\pi}}_{\bar{\imath}\bar{\jmath}}}{\bar{J}} \approx 0, \qquad {}^A\tilde{\Phi}_{\perp\bar{\imath}\bar{\jmath}} := \frac{{}^A\tilde{\hat{\Pi}}_{\perp\bar{\imath}\bar{\jmath}}}{\bar{J}} - 2(a+qb_6)\tilde{b}_{[\bar{\imath}\bar{\jmath}]} \approx 0, \\
{}^T\tilde{\Phi}_{\perp\bar{\imath}\bar{\jmath}} := \frac{{}^T\hat{\Pi}_{\perp\bar{\imath}\bar{\jmath}}}{\bar{J}} - 2(a+qb_6)^T\tilde{b}_{\bar{\imath}\bar{\jmath}} \approx 0, \qquad {}^V\tilde{\Phi}_{\bar{\imath}} := \frac{{}^V\tilde{\hat{\Pi}}_{\bar{\imath}}}{\bar{J}} - 2(a+qb_6)\tilde{b}_{\perp\bar{\imath}} \approx 0,
\end{gathered} \tag{C1}$$

and the secondary Hamiltonian constraints are given by

$$\tilde{\mathcal{H}}_\perp = \bar{J}\left(2\frac{{}^S\tilde{\Pi}_\perp}{\bar{J}} - (a+qb_6)(\tilde{R}^{\bar{\imath}\bar{\jmath}}{}_{\bar{\imath}\bar{\jmath}} - 4b^{\bar{\imath}}{}_{\bar{\imath}})\right) - \bar{n}_i\bar{\nabla}_\alpha\tilde{\pi}^{i\alpha}, \tag{C2a}$$

$$\tilde{\mathcal{H}}_\alpha = p\varepsilon_{ijk}\bar{b}^j{}_\alpha\tilde{\hat{\pi}}_i{}^{\bar{k}} - \bar{b}^i{}_\alpha\bar{\nabla}_\beta\tilde{\pi}_i{}^\beta - 2(a+b_6q)\tilde{R}^0{}_\beta, \tag{C2b}$$

$$\tilde{\mathcal{H}}_{\bar{\imath}\bar{\jmath}} \approx \frac{{}^A\hat{\pi}_{\bar{\imath}\bar{\jmath}}}{J} - (a+qb_6)\tilde{T}_{\perp\bar{\imath}\bar{\jmath}} + p\varepsilon_{\perp\bar{\imath}\bar{\jmath}}\left(\frac{{}^S\tilde{\hat{\Pi}}_\perp}{4\bar{J}} + \frac{1}{2}(a+qb_6)\tilde{b}^{\bar{k}}{}_{\bar{k}}\right) \approx 0, \tag{C2c}$$

$$\tilde{\mathcal{H}}_{\perp\bar{\imath}} \approx \frac{\tilde{\hat{\pi}}_{\perp\bar{\imath}}}{\bar{J}} + 2(a+qb_6)\tilde{T}^{\bar{k}}{}_{\bar{k}\bar{\imath}} + \nabla_{\bar{\imath}}\left(\frac{{}^S\tilde{\hat{\Pi}}_\perp}{2\bar{J}} + (a+qb_6)\tilde{b}^{\bar{k}}{}_{\bar{k}}\right) \approx 0. \tag{C2d}$$

The consistency condition of ${}^V\tilde{\Phi}_{\bar{\imath}}$ can be expressed in the form

$$\tilde{\chi}_{\bar{\imath}} = \frac{1}{2}\tilde{\mathcal{H}}_{\perp\bar{\imath}} - \frac{a+qb_6+a_2}{a_2}\frac{\tilde{\hat{\pi}}_{\perp\bar{\imath}}}{\bar{J}} \approx \frac{a+qb_6+a_2}{a_2}\frac{\tilde{\hat{\pi}}_{\perp\bar{\imath}}}{\bar{J}}. \tag{C3}$$

For $a + qb_6 \neq 0$ and $a + qb_6 + a_2 \neq 0$, the type and the number of constraints remains the same as in the full nonlinear theory, and we have the canonical stability under linearization.

1. The case $a + qb_6 = 0$

In this case, the analysis depends on the value of p.

(i) For $p \neq 0$, the six secondary constraints $\mathcal{H}_M = (\tilde{\mathcal{H}}_\perp, \tilde{\mathcal{H}}_\alpha, \tilde{\mathcal{H}}_{\bar{\imath}\bar{\jmath}}, \tilde{\mathcal{H}}_{\perp\bar{\imath}})$, in conjunction with ${}^V\tilde{\Phi}_{\bar{\imath}} \approx 0$ take, respectively, the following form:

$${}^S\tilde{\pi} \approx 0, \quad 0 \approx 0, \quad {}^S\tilde{\hat{\Pi}}_\perp \approx 0, \quad \tilde{\hat{\pi}}_{\perp\bar{\imath}} \approx 0. \tag{C4}$$

Thus, $\tilde{\chi}_{\bar{\imath}}$ and $\tilde{\mathcal{H}}_\alpha$ are identically satisfied, and there are no SC constraints, $N_2 = 0$. Hence, the number of FC constraints is $N_1 = 6 + 6 - 2 + 8 = 18$ and consequently, there are no propagating modes: $N^* = 2\times 18 - 2\times 18 - 0 = 0$.

(ii) For $p = 0$, the constraints $\mathcal{H}_M$, in conjunction with ${}^V\tilde{\Phi}_{\bar{\imath}} \approx 0$, read

$$2{}^S\tilde{\Pi}_\perp - \bar{n}_i\bar{\nabla}_\alpha\tilde{\pi}^{i\alpha} \approx 0, \qquad \bar{b}^i{}_\alpha\bar{\nabla}_\beta\tilde{\pi}_i{}^\beta \approx 0,$$
$$0 \approx 0, \qquad \frac{\tilde{\tilde{\pi}}_{\perp\bar{\imath}}}{\bar{J}} + \nabla_{\bar{\imath}}\left(\frac{{}^S\tilde{\tilde{\Pi}}_\perp}{2\bar{J}}\right) \approx 0. \tag{C5}$$

Taking into account the form of $\tilde{\chi}_{\bar{\imath}}$, the set $\mathcal{H}_M$ reduces to

$$\tilde{\tilde{\pi}}_{\perp\bar{\imath}} \approx 0, \qquad {}^S\tilde{\tilde{\pi}} \approx 0, \qquad {}^S\tilde{\tilde{\Pi}}_\perp \approx 0.$$

Thus, we again have $N_1 = 18$, $N_2 = 0$, and $N^* = 0$.

2. The case $a + qb_6 + a_2 = 0$

Compared to the generic case, this condition induces the following change: Eq. (C3) implies that $\tilde{\chi}_{\bar{\imath}}$ is identically satisfied, whereas ${}^V\Phi_{\bar{k}}$ becomes FC. Thus, $N_1 = 6 + 6 + 2 = 14$, $N_2 = 10 - 4 = 6$, and consequently, $N^* = 2$.

APPENDIX D: THE ALGEBRA OF ICS IN THE 0^- SECTOR

The nontrivial PBs between the primary ICs $Y_M = ({}^S\phi, {}^T\phi, \phi_{\perp\bar{k}}, {}^S\Phi, {}^T\Phi)$ in the spin-0^- sector read

$$\{{}^S\phi, {}^S\Phi_\perp\} \approx -\frac{4a}{J}\delta,$$
$$\{{}^T\phi_{\bar{\imath}\bar{\jmath}}, {}^T\Phi_\perp{}^{\bar{m}\bar{n}}\} \approx -\frac{1}{J^2}[\delta_{(\bar{\imath}}{}^{(\bar{m}A}\hat{\Pi}_{\perp\bar{\jmath})}{}^{\bar{n})} - 2a\delta_{(\bar{\imath}}{}^{\bar{m}}\delta_{\bar{\jmath})}{}^{\bar{n}}]\delta,$$
$$\{\phi_{\perp\bar{\imath}}, \phi_{\perp\bar{\jmath}}\} \approx \frac{2}{J^2}{}^A\hat{\pi}_{\bar{\imath}\bar{\jmath}}\delta, \qquad \{\phi_{\perp\bar{\imath}}, {}^S\Phi_\perp\} \approx \frac{1}{J^2}{}^V\hat{\Pi}_{\bar{\imath}}\delta,$$
$$\{\phi_{\perp\bar{\imath}}, {}^T\Phi_{\perp\bar{m}\bar{n}}\} \approx \frac{1}{J^2}\left(\frac{1}{2}\eta_{\bar{m}\bar{n}}{}^V\hat{\Pi}_{\bar{\imath}} - \eta_{\bar{\imath}(\bar{m}}{}^V\hat{\Pi}_{\bar{n})} - 4aJn_{(m}\eta_{\bar{\imath}n)}\right)\delta. \tag{D1}$$

Calculating the determinant of the 8×8 matrix $\Delta^-_{MN} = \{Y_M, Y_N\}$,

$$\Delta^- = \begin{vmatrix} 0 & 0 & -\frac{4a}{J} & 0 & 0 \\ 0 & \{{}^T\phi_{\bar{\imath}\bar{\jmath}}, {}^T\phi_{\bar{m}\bar{n}}\} & 0 & 0 & \{{}^T\phi_{\bar{\imath}\bar{\jmath}}, {}^T\Phi_\perp{}^{\bar{m}\bar{n}}\} \\ 0 & 0 & \{\phi_{\perp\bar{\imath}}, \phi_{\perp\bar{\jmath}}\} & \{\phi_{\perp\bar{\imath}}, {}^S\Phi_\perp\} & \{\phi_{\perp\bar{\imath}}, {}^T\Phi_\perp{}^{\bar{m}\bar{n}}\} \\ \frac{4a}{J} & 0 & -\{\phi_{\perp\bar{\imath}}, {}^S\Phi_\perp\} & 0 & 0 \\ 0 & -\{{}^T\phi_{\bar{\imath}\bar{\jmath}}, {}^T\Phi_\perp{}^{\bar{m}\bar{n}}\} & -\{\phi_{\perp\bar{\imath}}, {}^T\Phi_\perp{}^{\bar{m}\bar{n}}\} & 0 & 0 \end{vmatrix}$$

one obtains the result displayed in Eq. (6.7).

APPENDIX E: ON THE CONDITION ${}^A\hat{\pi}_{\bar{\imath}\bar{\jmath}} = 0$

In this appendix, we wish to clarify the phenomenon of constraint bifurcation in the spin-0^- sector, where the field equations take the form:

$$2a_3\varepsilon_{ijk}\nabla^k\mathcal{A} + \eta_{ij}(a_3\mathcal{A}^2 - b_5R_{[ij]}R^{[ij]} - 2\Lambda_0) + \frac{8a_3}{3}\mathcal{A}\varepsilon_{(imn}t_{j)}{}^{mn} - 2a_3\varepsilon_{ijk}V^k\mathcal{A},$$
$$2aG_{ji} + 2b_5R_{[in]}G_j{}^n = 0. \tag{E1}$$

$$T^i{}_{mn}(a\eta_{ik} + b_5R_{[ik]}) + b_5\nabla_{[m}(R_{n]k} - R_{kn]}) + 2a_3\varepsilon_{kmn}\mathcal{A} = 0. \tag{E2}$$

Condition ${}^A\hat{\pi}_{\bar{\imath}\bar{\jmath}} = 0$ is equivalent to $\mathcal{A} = 0$. Now, the equations of motion take the following form:

$$2aG_{ji} + 2b_5R_{[in]}G_j{}^n - \eta_{ij}(b_5R_{[ij]}R^{[ij]} + 2\Lambda_0) = 0. \tag{E3}$$

$$T^i{}_{mn}(a\eta_{ik} + b_5R_{[ik]}) + b_5\nabla_{[m}(R_{n]k} - R_{kn]}) = 0. \tag{E4}$$

Let us now analyze Eq. (E3). The $\bar{\imath}\bar{\jmath} = \{\perp\perp, \bar{\imath}\perp\}$ components are given by

$$-aR^{\bar{\imath}\bar{\jmath}}{}_{\bar{\imath}\bar{\jmath}} + 2\Lambda_0 - \frac{{}^V\hat{\Pi}^{\bar{\imath}}}{J}R_{\perp\bar{\imath}} - \frac{1}{4b_5J^2}(2{}^V\hat{\Pi}_{\bar{\imath}}{}^V\hat{\Pi}^{\bar{\imath}} + {}^A\hat{\Pi}_{\perp\bar{\imath}\bar{\jmath}}{}^A\hat{\Pi}^{\bar{\imath}\bar{\jmath}}_\perp) = 0, \tag{E5}$$

$$2aR_{\perp\bar{\imath}} + \frac{{}^V\hat{\Pi}_{\bar{\imath}}}{J}G_{\perp\perp} + \frac{{}^A\hat{\Pi}_{\perp\bar{\imath}\bar{\jmath}}}{J}R_\perp{}^{\bar{\jmath}} = 0. \tag{E6}$$

They represent secondary FC constraints $\mathcal{H}_\perp$ and $\mathcal{H}_{\bar{\imath}} := h_{\bar{\imath}}{}^\alpha\mathcal{H}_\alpha$.

The condition ${}^A\hat{\pi}_{\bar{\imath}\bar{\jmath}} = 0$ leads to the appearance of the additional constraints in the theory. Namely, $ij = \{[\bar{\imath}\bar{\jmath}], [\perp\bar{\imath}]\}$ components of the (E3) are given by

$$(2a + b_5R_{\perp\perp})^A\hat{\Pi}_{\perp\bar{\imath}\bar{\jmath}} + 2b_5{}^V\hat{\Pi}_{[\bar{\imath}}R_{\perp\bar{\jmath}]} = 0, \tag{E7}$$

$$2a^V\hat{\Pi}_{\bar{\imath}} - b_5({}^V\hat{\Pi}_{\bar{\imath}}G_{\perp\perp} + {}^A\hat{\Pi}_{\perp\bar{\imath}\bar{\jmath}}R_\perp{}^{\bar{\jmath}} + {}^V\hat{\Pi}_{\bar{\jmath}}G_{\bar{\imath}}{}^{\bar{\jmath}}) = 0. \tag{E8}$$

Since

$$R = \frac{1}{a}\left[\frac{1}{4b_5 J^2}(2{}^V\hat{\Pi}_{\bar{i}}{}^V\hat{\Pi}^{\bar{i}} + {}^A\hat{\Pi}_{\perp\bar{i}\bar{j}}{}^A\hat{\Pi}_{\perp}{}^{\bar{i}\bar{j}}) - 6\Lambda_0\right],$$
$$G_{\perp\perp} = -\frac{1}{2}R^{\bar{i}\bar{j}}{}_{\bar{i}\bar{j}}, \tag{E9}$$

Eq. (E7) is an additional constraint in the theory.

Let us note that $\bar{i}\bar{j}$ components of (E3) read

$$\left(2a\eta_{\bar{i}\bar{k}} + \frac{{}^A\hat{\Pi}_{\perp\bar{i}\bar{k}}}{J}\right)G_{\bar{j}}{}^{\bar{k}} - \eta_{\bar{i}\bar{j}}\left[\frac{1}{4b_5J^2}(2{}^V\hat{\Pi}_{\bar{m}}{}^V\hat{\Pi}^{\bar{m}} + {}^A\hat{\Pi}_{\perp\bar{m}\bar{n}}{}^A\hat{\Pi}_{\perp}{}^{\bar{m}\bar{n}}) + 2\Lambda_0\right] = 0. \tag{E10}$$

Equation (E10) can be solved for $G_{\bar{i}}{}^{\bar{k}}$ since

$$\det\left[2a\eta_{\bar{i}\bar{k}} + \frac{{}^A\hat{\Pi}_{\perp\bar{i}\bar{k}}}{J}\right] = 4a^2 + \frac{{}^A\hat{\Pi}_{\perp\bar{m}\bar{n}}{}^A\hat{\Pi}_{\perp}{}^{\bar{m}\bar{n}}}{2J^2} > 0.$$

Thus, three equations (E7) and (E8) and the condition ${}^A\hat{\pi}_{\bar{i}\bar{j}} = 0$ describe *four* additional SC constraints (if any of these were FC, the number of DoF would be negative). This implies $N^* = 0$, and the two propagating modes of the generic case are eliminated.

APPENDIX F: THE LINEARIZED SPIN-0⁻ SECTOR

In this Appendix, we present the canonical structure of the linearized spin-0^- sector around the maximally symmetric background. We start by noting that

$$\frac{{}^A\bar{\tilde{\pi}}_{\bar{i}\bar{j}}}{J} = -2a_3\varepsilon_{\perp\bar{i}\bar{j}}\bar{\mathcal{A}},$$

where $\bar{\mathcal{A}} = p$. Then, for $p \neq 0$, Eq. (6.7) implies that the determinant $\bar{\Delta}^-$ is positive definite, so that the canonical structure remains the same as before linearization; see Sec. VI. Moreover, in that case the spin-0^- mode is massless; see (3.8c).

To see what happens in the complementary case $p = 0$ (the spin-0^- mode is either massive or massless), we start with

$$\tilde{\pi}_i{}^\alpha = 2a_3\varepsilon^{0\alpha\beta}\bar{b}_{i\beta}\tilde{\mathcal{A}},$$
$$\tilde{\Pi}_{ij}{}^\alpha = -2\varepsilon_{ijk}[a\varepsilon^{0\alpha\beta}\tilde{b}^k{}_\beta + b_5\bar{b}^k{}_\rho\varepsilon^{[\alpha\nu\rho}(\tilde{R}_\nu{}^{0]} - \tilde{R}^{0]}{}_\nu)],$$

and find the following primary ICs:

$$ {}^S\tilde{\phi} := \frac{{}^S\tilde{\tilde{\pi}}}{\bar{J}} \approx 0, \qquad {}^T\tilde{\phi}_{\bar{i}\bar{j}} := \frac{{}^T\tilde{\tilde{\pi}}_{\bar{i}\bar{j}}}{\bar{J}} \approx 0,$$
$$\tilde{\phi}_{\perp\bar{i}} := \frac{\tilde{\tilde{\pi}}_{\perp\bar{i}}}{\bar{J}} \approx 0, \qquad {}^S\tilde{\Phi}_\perp := \frac{{}^S\tilde{\tilde{\Pi}}_\perp}{\bar{J}} + 2a\tilde{b}^{\bar{i}}{}_{\bar{i}} \approx 0,$$
$${}^T\tilde{\Phi}_{\perp\bar{i}\bar{j}} := \frac{{}^T\tilde{\tilde{\Pi}}_{\perp\bar{i}\bar{j}}}{\bar{J}} - 2a{}^T\tilde{b}_{\bar{i}\bar{j}} \approx 0. \tag{F1}$$

The only nontrivial PBs between the primary ICs are

$$\{{}^S\tilde{\Phi}_\perp, {}^S\tilde{\phi}\} \approx \frac{4a}{\bar{J}}\delta,$$
$$\{{}^T\tilde{\Phi}_{\perp\bar{i}\bar{j}}, {}^T\tilde{\phi}^{\bar{m}\bar{n}}\} \approx -\frac{2a}{\bar{J}}\delta_{(\bar{i}}{}^{\bar{m}}\delta_{\bar{j})}{}^{\bar{n}}\delta. \tag{F2}$$

The secondary constraints $\tilde{\mathcal{H}}_i$ and $\tilde{\mathcal{H}}_{ij}$ read

$$\tilde{\mathcal{H}}_\perp = \bar{J}\left[2\frac{{}^S\tilde{\Pi}_\perp}{\bar{J}} - a(\tilde{R}^{\bar{i}\bar{j}}{}_{\bar{i}\bar{j}} - 4b^{\bar{i}}{}_{\bar{i}})\right],$$
$$\tilde{\mathcal{H}}_\alpha = \bar{b}^{\bar{i}}{}_\alpha[-q({}^V\tilde{\tilde{\Pi}}_{\bar{i}} - 2aJ\tilde{b}_{\perp\bar{i}}) - \bar{\nabla}_\beta\tilde{\pi}_i{}^\beta - 2a\bar{J}\tilde{R}_{\perp\bar{i}}],$$
$$\tilde{\mathcal{H}}_{\bar{i}\bar{j}} \approx a\tilde{T}_{\perp\bar{i}\bar{j}} + \frac{{}^A\tilde{\tilde{\pi}}_{\bar{i}\bar{j}}}{2\bar{J}} + \nabla_{[\bar{i}}\left(\frac{{}^V\tilde{\tilde{\Pi}}_{\bar{j}]}}{\bar{J}} - 2a\tilde{b}_{\perp\bar{j}]}\right),$$
$$\tilde{\mathcal{H}}_{\perp\bar{i}} \approx a\tilde{V}_{\bar{i}} + \frac{1}{2}\bar{\nabla}^{\bar{j}}\left(\frac{{}^A\tilde{\tilde{\Pi}}_{\perp\bar{i}\bar{j}}}{\bar{J}} - 2a\tilde{b}_{[\bar{i}\bar{j}]}\right). \tag{F3}$$

Moreover, $\tilde{\mathcal{H}}_\alpha$ can be used to find $\tilde{\mathcal{H}}_{\bar{i}} = \bar{h}_{\bar{i}}{}^\alpha\tilde{\mathcal{H}}_\alpha/\bar{J}$:

$$\tilde{\mathcal{H}}_{\bar{i}} \approx -q\left(\frac{{}^V\tilde{\tilde{\Pi}}_{\bar{i}}}{\bar{J}} - 2a\tilde{b}_{\perp\bar{i}}\right) + \bar{\nabla}^{\bar{j}}\frac{{}^A\tilde{\tilde{\pi}}_{\bar{j}\bar{i}}}{\bar{J}} - 2a\tilde{R}_{\perp\bar{i}}.$$

According to (F2), the consistency of the primary ICs $({}^S\phi, {}^T\phi; {}^S\Phi, {}^T\Phi)$ results in the determination of the multipliers $({}^Su, {}^Tu; {}^Sv, {}^Tv)$, whereas the consistency of $\tilde{\phi}_{\perp\bar{i}}$ yields a new, secondary IC:

$$\tilde{\chi}_{\bar{i}} = \bar{\nabla}^{\bar{j}}\frac{{}^A\tilde{\tilde{\pi}}_{\bar{j}\bar{i}}}{\bar{J}} + \frac{1}{b_5}(a - qb_5)\left(\frac{{}^V\tilde{\tilde{\Pi}}_{\bar{i}}}{\bar{J}} - 2a\tilde{b}_{\perp\bar{i}}\right) \approx 0. \tag{F4}$$

The PB of $\tilde{\phi}_{\perp\bar{i}}$ with its own (modified) secondary pair $\tilde{\chi}'_{\bar{j}} := \tilde{\chi}_{\bar{j}} - \tilde{\mathcal{H}}_{\bar{j}}$ reads

$$\{\tilde{\phi}_{\perp\bar{i}}, \tilde{\chi}'_{\bar{j}}\} = \frac{2a^2}{b_5\bar{J}}\eta_{\bar{i}\bar{j}}\delta. \tag{F5}$$

Thus, the consistency condition of $\tilde{\chi}'_{\bar{j}}$ leads to the determination of the multiplier $u_{\perp\bar{j}}$.

According to Eqs. (F2) and (F5), the ten ICs $\tilde{X}_A = \{{}^S\tilde{\phi}, {}^S\tilde{\Phi}_\perp, {}^T\tilde{\phi}_{\bar{i}\bar{j}}, {}^T\tilde{\Phi}_{\perp\bar{i}\bar{j}}, \tilde{\phi}_{\perp\bar{i}}, \tilde{\chi}'_{\bar{i}}\}$ are SC. Hence, $N = 18$, $N_1 = 12$, $N_2 = 10$, so that $N^* = 2$ (one Lagrangian DoF).

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PHYSICAL REVIEW D **88,** 101501(R) (2013)

"Exotic" black holes with torsion

M. Blagojević,* B. Cvetković,† and M. Vasilić‡

Institute of Physics, University of Belgrade, P.O. Box 57, 11001 Belgrade, Serbia

(Received 7 October 2013; published 11 November 2013)

In the context of three-dimensional gravity with torsion, the concepts of standard and "exotic" Bañados–Teitelboim–Zanelli black holes are generalized by going over to black holes with torsion. This approach provides a unified insight into thermodynamics of black holes, with or without torsion.

DOI: 10.1103/PhysRevD.88.101501 PACS numbers: 04.70.Dy, 04.60.Kz

I. INTRODUCTION

Recently, Townsend and Zhang [1] examined thermodynamics of "exotic" Bañados–Teitelboim–Zanelli (BTZ) black holes—the solutions of a class of three-dimensional (3D) gravity models for which the metric *coincides* with the standard BTZ metric [2] but for which the conserved charges, energy, and angular momentum, are, in a sense, *reversed* (as explained in Sec. III). Their analysis was focused on a simple model of this type, described by the parity-odd gravitational Lagrangian that Witten [3] named exotic. In this framework, the authors discussed basic thermodynamic properties of the exotic BTZ black holes (that is, the standard BTZ black holes viewed as solutions of the exotic model).

In Ref. [1], general relativity with a cosmological constant (GR_Λ) and the exotic gravity are treated as independent models, based on the *Riemannian* geometry of spacetime. In the present paper, we show that these two models can be naturally interpreted as different sectors of a single model—the Mielke–Baekler (MB) model of *3D gravity with torsion* [4]. This approach offers a unified view at GR_Λ and the exotic gravity, revealing a new, "interpolating" role of torsion with respect to Riemannian theories of gravity. In this, more general, setting, standard BTZ black hole solutions can be generalized to BTZ-like *black holes with torsion* [5–7]; see also Ref. [8]. At the same time, their thermodynamic properties [9,10] allow us not only to simplify the considerations presented in Ref. [1] but also to generalize them.

II. 3D GRAVITY WITH TORSION

In the Poincaré gauge theory [11–13], the basic dynamical variables are the triad e^i and the Lorentz connection ω^{ij} (1-forms). Their field strengths, expressed in terms of the Lie dual connection $\omega^i := -\frac{1}{2}\varepsilon^{ijk}\omega_{jk}$, are the torsion $T^i = de^i + \varepsilon^{ijk}\omega_j e_k$ and the curvature $R^i = d\omega^i + \frac{1}{2}\varepsilon^{ijk}\omega_j\omega_k$ (the exterior product sign $\wedge$ is omitted for simplicity). In this framework, the MB model is defined by the Lagrangian

$$L_{\rm MB} = 2ae^iR_i - \frac{\Lambda}{3}\varepsilon_{ijk}e^ie^je^k + \alpha_3 L_{\rm CS}(\omega) + \alpha_4 e^iT_i. \tag{1}$$

Here, $L_{\rm CS}(\omega) := \omega^i d\omega_i + \frac{1}{3}\varepsilon_{ijk}\omega^i\omega^j\omega^k$ is the Chern–Simons Lagrangian for ω^i, and $(a, \Lambda, \alpha_3, \alpha_4)$ are free parameters. In the nondegenerate case $\alpha_3\alpha_4 - a^2 \neq 0$, the variation of $L_{\rm MB}$ with respect to e^i and ω^i leads to the gravitational field equations in vacuum:

$$2T^i = p\varepsilon^i{}_{jk}e^je^k, \qquad 2R^i = q\varepsilon^i{}_{jk}e^je^k, \tag{2}$$

where

$$p = \frac{\alpha_3\Lambda + \alpha_4 a}{\alpha_3\alpha_4 - a^2}, \qquad q = -\frac{(\alpha_4)^2 + a\Lambda}{\alpha_3\alpha_4 - a^2}. \tag{3}$$

Using Eqs. (2) and the formula $\omega^i = \tilde\omega^i + K^i$, where $\tilde\omega^i$ is the Riemannian (torsionless) connection and K^i is the contortion 1-form, defined implicitly by $T_i = \varepsilon_{imn}K^me^n$, one can show [6,14] that the Riemannian piece of the curvature, $\tilde R = R(\tilde\omega)$, reads

$$2\tilde R^i = \Lambda_{\rm eff}\varepsilon^i{}_{jk}e^je^k, \qquad \Lambda_{\rm eff} := q - \frac{1}{4}p^2, \tag{4}$$

where $\Lambda_{\rm eff}$ is the effective cosmological constant.

In the anti-de Sitter (AdS) sector with $\Lambda_{\rm eff} = -1/\ell^2$, the MB model admits a new type of black hole solutions, known as the BTZ-like *black holes with torsion* [5–7]. These solutions can be determined in two steps. First, by combining the form of the BTZ black hole metric,

$$ds^2 = N^2dt^2 - N^{-2}dr^2 - r^2(d\varphi + N_\varphi dt)^2,$$
$$N^2 = \left(-8Gm + \frac{r^2}{\ell^2} + \frac{16G^2j^2}{r^2}\right), \qquad N_\varphi = \frac{4Gj}{r^2},$$

with the relation $ds^2 = \eta_{ij}e^ie^j$, one concludes that the triad field can be chosen in the simple, diagonal form:

$$e^0 = Ndt, \quad e^1 = N^{-1}dr, \quad e^2 = r(d\varphi + N_\varphi dt). \tag{5a}$$

Then, the connection is determined by the first field equation in Eqs. (2):

$$\omega^i = \tilde\omega^i + \frac{p}{2}e^i. \tag{5b}$$

*mb@ipb.ac.rs
†cbranislav@ipb.ac.rs
‡mvasilic@ipb.ac.rs

The pair (e^i, ω^i) determined in this way represents the BTZ-like black hole with torsion [5–7]. The thermodynamic aspects of the new black holes are given as follows.

Energy and angular momentum of the black hole with torsion, defined as the on-shell values of the asymptotic generators for time translations and spatial rotations, have the following form [5,14]:

$$E = 16\pi G\left[\left(a + \frac{\alpha_3 p}{2}\right)m - \frac{\alpha_3}{\ell^2}j\right],$$
$$J = 16\pi G\left[\left(a + \frac{\alpha_3 p}{2}\right)j - \alpha_3 m\right]. \qquad (6)$$

In contrast to GR_Λ, where $E = m$ and $J = j$, the presence of the Chern–Simons term ($\alpha_3 \neq 0$) modifies E and J into linear combinations of m and j.

After choosing the AdS asymptotic conditions, the Poisson bracket algebra of the *asymptotic symmetry* is given by two independent Virasoro algebras with different central charges [6,14]:

$$c^{\mp} = 24\pi\left[\left(a + \frac{\alpha_3 p}{2}\right)\ell \mp \alpha_3\right]. \qquad (7)$$

The partition function of the MB model, calculated in the semiclassical approximation around the black hole with torsion, yields the following expression for the *black hole entropy* [9],

$$S = 8\pi^2\left[\left(a + \frac{\alpha_3 p}{2}\right)r_+ - \alpha_3\frac{r_-}{\ell}\right], \qquad (8)$$

where $r_\pm$ are the outer and inner horizons of the black hole, defined as the zeros of N^2. The gravitational entropy Eq. (8) coincides with the corresponding statistical entropy [10], obtained by combining Cardy's formula with the central charges shown in Eq. (7). The existence of torsion is shown to be in complete agreement with the first law of black hole thermodynamics.

III. SPECIAL CASE: RESULTS OF TOWNSEND AND ZHANG

After clarifying basic thermodynamic aspects of black holes with torsion, the two types of black holes discussed in Ref. [1] can be given a unified treatment by considering the related limiting cases of the MB model.

For $\alpha_3 = \alpha_4 = 0$ and $16\pi G a = 1$, the MB model reduces to GR_Λ, the spacetime geometry is Riemannian ($p = 0$), and formulas (6)–(8) produce the standard expressions for the conserved charges, central charges, and entropy:

$$E = m, \qquad J = j, \qquad c^{\mp} = \frac{3\ell}{2G}, \qquad S = \frac{2\pi r_+}{4G}. \qquad (9)$$

Since $2\pi r_+$ is the length ("area") of the outer horizon, the entropy has the usual Bekenstein–Hawking form.

Similarly, for $a = \Lambda = 0$, the MB model reduces to Witten's exotic gravity with the Riemannian geometry of spacetime. By choosing $16\pi G\alpha_3 = -\ell$, one arrives at the exotic conserved charges, central charges, and entropy,

$$E = \frac{j}{\ell}, \quad J = \ell m, \quad c^{\mp} = \pm\frac{3\ell}{2G}, \quad S = \frac{2\pi r_-}{4G}, \qquad (10)$$

which coincide with those in Ref. [1]. Since $\Lambda_{\mathrm{eff}} = -1/\ell^2$ implies $16\pi G\alpha_4 = -1/\ell$, the corresponding exotic Lagrangian is also the same as in Ref. [1].

These considerations, based on our earlier studies of black holes with torsion, provide a simple way to understand somewhat enigmatic relation between the standard and exotic black hole thermodynamics.

IV. GENERALIZATION: STANDARD AND EXOTIC BLACK HOLES WITH TORSION

In the previous section, the concepts of standard and exotic black holes are used in the context of simple gravitational models with the Riemannian geometry of spacetime. Here, we wish to generalize these concepts by going over to black holes with torsion.

The form of the general results (6)–(8) suggests introducing *standard* black holes with torsion by imposing the following requirements:

$$\alpha_3 = 0, \qquad 16\pi G a = 1. \qquad (11)$$

In this case, the general formulas reduce to the standard form (9), and the corresponding 2-parameter Lagrangian is given by

$$L_{\mathrm{S}} = \frac{1}{8\pi G}e^i R_i - \frac{\Lambda}{3}\varepsilon_{ijk}e^i e^j e^k + \alpha_4 e^i T_i. \qquad (12)$$

The AdS condition,

$$\Lambda_{\mathrm{eff}} = \frac{3}{4}\left(\frac{\alpha_4}{a}\right)^2 + \frac{\Lambda}{a} = -\frac{1}{\ell^2},$$

implies $\Lambda < 0$.

Similar considerations lead to the following definition of *exotic* black holes with torsion:

$$a + \frac{\alpha_3 p}{2} = 0, \qquad 16\pi G\alpha_3 = -\ell, \qquad (13)$$

which implies that the conserved charges, central charges, and entropy take the exotic form (10). The corresponding 2-parameter Lagrangian can be written in the form

$$L_{\mathrm{E}} = \frac{1}{16\pi G}\left[2\beta e^i R_i + \frac{\beta(\beta^2+3)}{3\ell^2}\varepsilon_{ijk}e^i e^j e^k - \ell L_{\mathrm{CS}} - \frac{\beta^2+1}{\ell}e^i T_i\right], \qquad (14)$$

where $\beta := 16\pi G a$ and ℓ are free parameters.

In the limit $p = 0$, L_{S} and L_{E} describe torsionless theories discussed by Townsend and Zhang [1]; thermodynamic aspects of the corresponding black holes are given in

(9) and (10). All the other limits define the standard and exotic gravities *with torsion*. In particular, for the choice $q = 0$ (that is, by taking $(\alpha_4)^2 + \Lambda/16\pi G = 0$ in L_S and $\beta = 1$ in L_E), the geometry of these models becomes *teleparallel* ($R^i = 0$).

ACKNOWLEDGMENTS

M. B. thanks F. W. Hehl for bringing the paper [1] to his attention. We acknowledge the support from Grant No. 171031 of the Serbian Science Foundation.

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IOP Publishing Classical and Quantum Gravity

Class. Quantum Grav. 35 (2018) 055005 (13pp) https://doi.org/10.1088/1361-6382/aaa3a7

A black hole with torsion in 5D Lovelock gravity

B Cvetković and D Simić

Institute of Physics, University of Belgrade, Pregrevica 118, 11080 Belgrade, Serbia

E-mail: cbranislav@ipb.ac.rs and dsimic@ipb.ac.rs

Received 6 November 2017, revised 18 December 2017
Accepted for publication 22 December 2017
Published 31 January 2018

Abstract

We analyze static spherically symmetric solutions of five dimensional (5D) Lovelock gravity in the first order formulation. In the Riemannian sector, when torsion vanishes, the Boulware–Deser black hole represents a unique static spherically symmetric black hole solution for the *generic* choice of the Lagrangian parameters. We show that a *special* choice of the Lagrangian parameters, different from the Lovelock Chern–Simons gravity, leads to the existence of a static black hole solution with *torsion*, the metric of which is asymptotically anti-de Sitter (AdS). We calculate the conserved charges and thermodynamical quantities of this black hole solution.

Keywords: Lovelock gravity, torsion, black holes

1. Introduction

Lovelock gravity [1] represents an intriguing generalization of general relativity, since it is a unique, ghost-free higher derivative extension of Einstein's theory that possesses second order equations of motion. As a higher curvature theory, Lovelock gravity has a considerable number of black hole solutions—see [2–10] and references therein. Many of these possess exotic properties, such as zero mass, peculiar topology of the event horizon etc.

This leads us to an old problem of black hole uniqueness—namely, solutions of general relativity are highly constrained, but the situation changes drastically in the case of higher dimensions. There are new black hole solutions with non-spherical event horizon topology, namely black string, black ring and black brane [11]. Often, these exotic black objects suffer from various instabilities—for example, black strings and branes have Gregory–Laflamme instability [12], and will decay into black holes with spherical horizons. Thus, gravity in higher dimensions represents an interesting area of research, full of surprising discoveries, whose importance stems from its numerous applications.

Lovelock gravity can be also studied within the framework of Poincaré gauge theory (PGT), formulated by Sciamma [13] and Kibble [14] more than half a century ago. PGT is the first modern, gauge-field-theoretic approach to gravity obtained by gauging the Poincaré group of space-time symmetries, the semidirect product of translations and Lorentz transformations. It represents a natural extension of the gauge principle, originally formulated by Weyl within electrodynamics and further developed in the works of Yang, Mills and Utiyama, to the space-time symmetries. The gauge procedure adopted leads directly to a new, Riemann–Cartan geometry of space-time, since torsion and curvature are recovered as the Poincaré gauge field strengths. The Lagrangian in PGT contains a gravitational part, which is a function of the field strengths, the curvature and the torsion, and a suitable matter field Lagrangian.

In the context of Lovelock gravity, this more general setting contains torsionless theory as a limit, and represents a starting point for canonical analysis, coupling with matter fields, supersymmetric extensions of the theory and holographic applications. Interestingly, unlike in the case of Einstein–Cartan theory (first order formulation of general relativity) where all solutions of the equations of motion in vacuum are torsion free, the structure of the vacuum solutions of the Lovelock gravity is more complicated, because there exist solutions with non-vanishing torsion. However, it turns out that exact solutions with torsion are extremely difficult to find, since consistency conditions usually lead to an over-constrained system of equations. Solutions with non-trivial totally antisymmetric torsion have been studied in [8], [15–19]. In this paper, we continue our analysis of the exact solutions of 5D Lovelock gravity solutions with torsion, started in [8], and find a new static, spherically symmetric black hole solution with torsion with zero mass and entropy. The torsion of the solution possesses both tensorial and antisymmetric part. It, unlike the Riemannian Boulware–Deser black hole [20], exists for a specific choice of action parameters. This fine tuning of action parameters was first noticed by Canfora *et al* in their paper [15], and represents a different sector from the highly degenerate Lovelock Chern–Simons gravity.

The paper is organized in the following way. In the second section, we review basics of Poincaré gauge theory and Lovelock gravity in the first order formulation. In section 3 we find the black hole solution of 5D Lovelock gravity with torsion, and analyze its properties. In particular, we find that the quadratic torsional invariant is singular at $r \to 0$. In section 4, we explore the thermodynamics of the previously obtained solution. The appendices contain additional technical details.

We use the following conventions: the Lorentz signature is mostly negative; local Lorentz indices are denoted by the middle letters of the Latin alphabet, while space-time indices are denoted by the letters of the Greek alphabet. Throughout the paper, we mostly use differential forms instead of coordinate notation, and the wedge product is omitted for simplicity.

2. Lovelock gravity

Since the work of Sciamma and Kibble, it has been known that gravity in the first order formulation has the structure of Poincaré gauge theory (PGT)—see [21, 22] for a comprehensive account. For the reader's convenience, we briefly review basics of the PGT.

2.1. PGT in brief

The basic dynamical variables in PGT, playing the role of gauge potentials, are the vielbein e^i 1-form and the spin connection $\omega^{ij} = -\omega^{ji}$ 1-form. In local coordinates x^μ, we can expand the vielbein and the connection 1-forms as $e^i = e^i{}_\mu \mathrm{d}x^\mu$, $\omega^i = \omega^i{}_\mu \mathrm{d}x^\mu$. Gauge symmetries of the theory are local translations (diffeomorphisms) and local Lorentz rotations, parametrized by ξ^μ and ε^{ij} respectively.

From the gauge potentials, we can construct field strengths, namely torsion T^i and curvature R^{ij} (2-forms), which are given as

$$T^i = \nabla e^i \equiv \mathrm{d}e^i + \varepsilon^i{}_{jk}\omega^j \wedge e^k = \frac{1}{2}T^i{}_{\mu\nu}\mathrm{d}x^\mu \wedge \mathrm{d}x^\nu ,$$
$$R^{ij} = \mathrm{d}\omega^{ij} + \omega^{ik} \wedge \omega_k{}^j = \frac{1}{2}R^{ij}{}_{\mu\nu}\mathrm{d}x^\mu \wedge \mathrm{d}x^\nu ,$$

where $\nabla = \mathrm{d}x^\mu \nabla_\mu$ is the exterior covariant derivative.

A metric tensor can be constructed from the vielbein and flat metrics: η_{ij}

$$g = \eta_{ij}e^i \otimes e^j = g_{\mu\nu}\mathrm{d}x^\mu \otimes \mathrm{d}x^\nu ,$$
$$g_{\mu\nu} = \eta_{ij}e^i{}_\mu e^j{}_\nu , \qquad \eta_{ij} = (+,-,-) .$$

The antisymmetry of ω^{ij} in PGT is equivalent to the so-called *metricity condition*, $\nabla g = 0$. A geometry whose connection is restricted by the metricity condition (metric-compatible connection) is called a *Riemann–Cartan geometry*.

The connection ω^{ij} determines the parallel transport in the local Lorentz basis. Because parallel transport is a geometric operation, it is independent of the basis. This property is encoded into PGT via the so-called *vielbein postulate*, which implies

$$\omega_{ijk} = \Delta_{ijk} + K_{ijk} ,$$

where Δ is Levi-Civita connection, and $K_{ijk} = -\frac{1}{2}(T_{ijk} - T_{kij} + T_{jki})$ is the contortion.

2.2. Action and equations of motion

The Lovelock gravity Lagrangian in the first order formulation can be constructed as the linear combination of the dimensionally continued Euler densities L_p, which in D dimensions are defined as

$$L_p = \varepsilon_{i_1 i_2 \dots i_D} R^{i_1 i_2} \dots R^{i_{2p-1} i_{2p}} e^{i_{2p+1}} \dots e^{i_D}.$$

In 5D, there are three Euler densities and the general form of the action of Lovelock gravity [1] is

$$I = \varepsilon_{ijkln} \int \left(\frac{\alpha_0}{5} e^i e^j e^k e^l e^n + \frac{\alpha_1}{3} R^{ij} e^k e^l e^n + \alpha_2 R^{ij} R^{kl} e^n \right) . \tag{2.1}$$

Variation of the action with respect to vielbein e^i and spin connection ω^{ij} yields the gravitational field equations

$$\varepsilon_{ijkln} \left(\alpha_0 e^j e^k e^l e^n + \alpha_1 R^{jk} e^l e^n + \alpha_2 R^{jk} R^{ln} \right) = 0, \tag{2.2}$$

and

$$\varepsilon_{ijkln} \left(\alpha_1 e^k e^l + 2\alpha_2 R^{kl} \right) T^n = 0. \tag{2.3}$$

3. Spherically symmetric solution

3.1. Ansatz

We are looking for a static solution with $SO(4)$ symmetry, which orbits are three-spheres. The most general metric which fulfills these requirements in Schwarzschild-like coordinates $x^\mu = (t, r, \psi, \theta, \varphi)$ is given by

$$ds^2 = N^2 dt^2 - B^{-2} dr^2 - r^2(d\psi^2 + \sin^2\psi d\theta^2 + \sin^2\psi \sin^2\theta d\varphi^2), \tag{3.1}$$

where functions N and B depend solely on r, and $r \in [0, \infty)$, $\psi \in [0, \pi)$, $\theta \in [0, \pi)$ and $\varphi \in [0, 2\pi)$. The metric (3.1) possesses seven Killing vectors (see appendix A).

The vielbeins e^i are chosen in a simple diagonal form

$$\begin{aligned} &e^0 = N dt, \qquad e^1 = B^{-1} dr, \qquad e^2 = r d\psi, \qquad e^3 = r\sin\psi d\theta, \\ &e^4 = r\sin\psi\sin\theta d\varphi. \end{aligned} \tag{3.2}$$

The most general form of the spin connection compatible with Killing vectors (see appendix A) is given by

$$\begin{aligned} &\omega^{01} = A_0 dt + A_1 dr, \qquad \omega^{02} = A_2 d\psi, \\ &\omega^{03} = A_2 \sin\psi d\theta, \qquad \omega^{04} = A_2 \sin\psi \sin\theta d\varphi, \\ &\omega^{12} = A_3 d\psi, \qquad \omega^{13} = A_3 \sin\psi d\theta, \\ &\omega^{14} = A_3 \sin\psi\sin\theta d\varphi, \qquad \omega^{23} = \cos\psi d\theta + A_4 \sin\psi \sin\theta d\varphi, \\ &\omega^{24} = -A_4 \sin\psi d\theta + \cos\psi \sin\theta d\varphi, \qquad \omega^{34} = A_4 d\psi + \cos\theta d\varphi, \end{aligned} \tag{3.3}$$

where A_i are arbitrary functions of radial coordinate.

3.2. Solution

The sector with vanishing torsion equations of motion for spherically symmetric ansatz has a well-known solution, the Boulware–Deser black hole [20], which exists for the generic choice of action parameters. Another solution, which we construct in this paper, possesses non-vanishing torsion and is given by the following anzatz:

$$\begin{aligned} &A_0 \neq 0, \qquad A_1 = A_2 = A_3 = 0, \qquad A_4 \neq 0 \\ &N = B. \end{aligned} \tag{3.4}$$

By using the adopted anzatz we get that the equations (2.2) reduce to

$$i = 0, 1: \qquad 2\alpha_0 r^2 - \alpha_1 + \alpha_1 A_4^2 = 0, \tag{3.5a}$$

$$i = 2, 3, 4: \qquad \left(2\alpha_2 - 2\alpha_2 A_4^2 - \alpha_1 r^2\right) A_0' + 6\alpha_0 r^2 + \alpha_1 (A_4^2 - 1) = 0. \tag{3.5b}$$

The non-vanishing field equations (2.3) take the form

$$ij = 01: \qquad \alpha_1 r^2 + 2\alpha_2 A_4^2 - 2\alpha_2 + 4\alpha_2 r A_4 A_4' = 0, \tag{3.6a}$$

$$ij = 12, 13: \qquad \left(\alpha_1 r^2 + 2\alpha_2 A_4^2 - 2\alpha_2\right)(NN' + A_0) + 2\alpha_1 r N^2 = 0, \tag{3.6b}$$

$$ij = 23, 24, 34: \qquad -2\alpha_2 A_0' + \alpha_1 = 0. \tag{3.6c}$$

From (3.5a) and (3.6c) we get

$$A_4 = \sqrt{1 - \frac{2\alpha_0}{\alpha_1} r^2}, \qquad A_0 = \frac{\alpha_1}{2\alpha_2} r, \tag{3.7}$$

where the integration constant in A_0 is taken to be zero for simplicity. Equation (3.5b) in conjunction with (3.6c) yields to the following constraint between coupling constants:

$$\alpha_1^2 - 12\alpha_0\alpha_2 = 0. \tag{3.8}$$

We consequently get that (3.6*a*) is identically satisfied, while the (3.6*b*) takes the form

$$NN' + \frac{3N^2}{r} - \frac{\alpha_1}{2\alpha_2} r = 0,$$

and can be easily solved for N:

$$N = \sqrt{-\frac{\alpha_1}{8\alpha_2}\left(r^2 - \frac{r_+^8}{r^6}\right)}. \tag{3.9}$$

From (3.8), we conclude that the solution exists in the sector different from the Lovelock Chern–Simons gravity. This is exactly the same fine tuning of parameters found by Canfora *et al* in their paper [15], where the solutions that have the structure of a direct product of a 2D Lorentzian with a 3D Euclidean constant curvature manifold are constructed.

The explicit form of torsion and curvature is given in appendix C. Let us note that both tensorial and antisymmetric part of torsion are non-vanishing unlike in the case of the solution found by Canfora *et al* [16], for which only totally antisymmetric part of torsion is non-vanishing.

Let us now introduce the (anti)-de Sitter ((A)dS) radius ℓ

$$\frac{\alpha_1}{8\alpha_2} = -\frac{\sigma}{\ell^2}, \qquad \sigma = \pm 1. \tag{3.10}$$

By substituting previous relation into (3.7) and (3.9), we get

$$A_4 = \sqrt{1 + \frac{4\sigma r^2}{3\ell^2}}, \ N = \sqrt{\sigma\left(\frac{r^2}{\ell^2} - \frac{r_+^8}{\ell^2 r^6}\right)}. \tag{3.11}$$

Note that for the solution to describe a black hole, the following condition must hold:

$$\frac{\alpha_1}{\alpha_2} < 0 \Leftrightarrow \sigma = +1 \tag{3.12}$$

with an *event horizon* located at $r = r_+$.

From the constraint (3.8), it follows that the sign of the ratio $\frac{\alpha_0}{\alpha_1}$ is the same as the sign of $\frac{\alpha_1}{\alpha_2}$

$$\operatorname{sgn}\left(\frac{\alpha_0}{\alpha_1}\right) = \operatorname{sgn}\left(\frac{\alpha_1}{\alpha_2}\right). \tag{3.13}$$

If the ratio is positive, the expression for A_4 implies that we have the maximum value of the radial coordinate, the so called *cosmological horizon*

$$r_0 = \frac{\ell\sqrt{3}}{2}. \tag{3.14}$$

Meanwhile, if the ratio is negative, we have no restriction on the value of the radial coordinate, except that it is positive, and in maximally extended space-time goes to infinity. In this case, the black hole space-time metric is asymptotically AdS.

3.2.1. Invariants. From expressions for curvature and torsion, given in appendix C, we see that quadratic torsional invariant reads

$$T^i \wedge {}^*T_i = -\frac{12\sigma}{\ell^2}\left(1 - \frac{r_+^8}{r^8}\right)\hat{\epsilon}, \tag{3.15}$$

which is obviously divergent in $r = 0$ for r_+ different from zero. Hence, there is a singularity of torsion at $r \to 0$. Scalar Cartan curvature is constant,

$$R = \frac{16\sigma}{\ell^2}, \tag{3.16}$$

while Riemannian scalar curvature is

$$\tilde{R} = \frac{4\sigma}{\ell^2}\left(5 - \frac{3\sigma\ell^2}{2r^2} - \frac{3r_+^8}{r^8}\right), \tag{3.17}$$

and is divergent for $r \to 0$. The quadratic Cartan and Riemannian curvature invariants both vanish:

$$R_{ij} \wedge {}^*R^{ij} = 0, \quad \tilde{R}_{ij} \wedge {}^*\tilde{R}^{ij} = 0. \tag{3.18}$$

We can conclude that the black hole obtained in this article is not of the regular type, and that it possesses singularity at $r = 0$. It is worth noting that solution [16] also possesses singularity of torsion and Riemannian curvature at $r = 0$.

Solving equations of motion (2.2) and (2.3) with seven arbitrary functions is an extremely tedious task, which is facilitated by Mathematica and xAct packages.

3.3. Conserved charges

Conserved charges can be calculated in a number of ways, we decided to make use of Nester's formula [23], the application of which is quite simple in this particular case. In this section, we shall restrict the analysis to the asymptotically AdS case, which corresponds to the black hole. The covariant momenta stemming from the Lovelock action (2.1) are given by

$$\tau_i := \frac{\partial L}{\partial T^i} = 0, \tag{3.19}$$

$$\rho_{ij} = \frac{\partial L}{\partial R^{ij}} = 2\varepsilon_{ijkln}\left(\frac{\alpha_1}{3}e^k e^l + 2\alpha_2 R^{kl}\right)e^n. \tag{3.20}$$

Let us denote the difference between any variable X and its reference value $\bar{X}$ by $\Delta X = X - \bar{X}$. Reference space-time, in respect to which we measure conserved charges, is given for the zero radius of the event horizon $r_+ = 0$. Conserved charges Q_ξ associated to the Killing vector ξ are given by quasi-local surface integrals

$$Q_\xi = \int_{\partial\Sigma} B,$$

where the boundary $\partial\Sigma$ is located at infinity. With a suitable asymptotic behavior of the fields, the proper boundary term reads [23]

$$B = (\xi \rfloor e^i)\Delta\tau_i + \Delta e^i(\xi \rfloor \bar{\tau}_i) + \frac{1}{2}(\xi \rfloor \omega^i{}_j)\Delta\rho_i{}^j + \frac{1}{2}\Delta\omega^i{}_j(\xi \rfloor \bar{\rho}_i{}^j), \tag{3.21}$$

where $\rfloor$ denotes contraction.

For solution (3.9), by making use of the the results of appendix C, we get the covariant momenta

$$
\begin{aligned}
&\rho_{01} = \frac{4\left(\alpha_1^2 - 12\alpha_0\alpha_2\right)}{\alpha_1} e^2 e^3 e^4 \equiv 0, \qquad \rho_{02} = -\frac{8\alpha_1}{3} e^1 e^3 e^4, \qquad \rho_{03} = \frac{8\alpha_1}{3} e^1 e^2 e^4, \\
&\rho_{04} = -\frac{8\alpha_1}{3} e^1 e^2 e^3, \qquad \rho_{12} = \frac{8\alpha_1}{3} e^0 e^3 e^4 - \frac{4\alpha_1 N}{3A_4} e^0 e^1 e^2, \\
&\rho_{13} = -\frac{8\alpha_1}{3} e^0 e^2 e^4 - \frac{4\alpha_1 N}{3A_4} e^0 e^1 e^3, \qquad \rho_{14} = \frac{8\alpha_1}{3} e^0 e^2 e^3 - \frac{4\alpha_1 N}{3A_4} e^0 e^1 e^4, \\
&\rho_{23} = 0, \qquad \rho_{24} = 0, \qquad \rho_{34} = 0.
\end{aligned} \tag{3.22}
$$

From (3.9), we conclude that the connection takes the same form on the background and for $r_+ \neq 0$, $\omega^{ij} = \bar{\omega}^{ij}$. Therefore, formula (3.21) takes the following simpler form:

$$
B = \frac{1}{2}(\xi \rfloor \omega^i{}_j)\Delta\rho_i{}^j.
$$

For the seven Killing vectors $\xi_{(n)}$ (see appendix A) the conserved charges are given by

$$
\begin{aligned}
Q_{(0)} &= \int_{\partial\Sigma} \omega^{01}{}_t \Delta\rho_{01} = 0, \\
Q_{(1)} &= \int_{\partial\Sigma} -\cot\psi \sin\theta \left(\omega^{23}{}_\theta \Delta\rho_{23} + \omega^{24}{}_\theta \Delta\rho_{24}\right) = 0, \\
Q_{(2)} &= \int_{\partial\Sigma} \cot\psi \cos\theta \cos\varphi \left(\omega^{23}{}_\theta \Delta\rho_{23} + \omega^{24}{}_\theta \Delta\rho_{24}\right) \\
&\quad - \frac{\cot\psi}{\sin\theta} \sin\varphi \left(\omega^{14}{}_\varphi \Delta\rho_{14} + \omega^{23}{}_\varphi \Delta\rho_{23} + \omega^{24}{}_\varphi \Delta\rho_{24} + \omega^{34}{}_\varphi \Delta\rho_{34}\right) = 0, \\
Q_{(3)} &= \int_{\partial\Sigma} \cot\psi \cos\theta \sin\varphi \left(\omega^{23}{}_\theta \Delta\rho_{23} + \omega^{24}{}_\theta \Delta\rho_{24}\right) \\
&\quad + \frac{\cot\psi}{\sin\theta} \cos\varphi \left(\omega^{14}{}_\varphi \Delta\rho_{14} + \omega^{23}{}_\varphi \Delta\rho_{23} + \omega^{24}{}_\varphi \Delta\rho_{24} + \omega^{34}{}_\varphi \Delta\rho_{34}\right) = 0, \\
Q_{(4)} &= \int_{\partial\Sigma} \cos\varphi \left(\omega^{23}{}_\theta \Delta\rho_{23} + \omega^{24}{}_\theta \Delta\rho_{24}\right) \\
&\quad - \cot\theta \sin\varphi \left(\omega^{14}{}_\varphi \Delta\rho_{14} + \omega^{23}{}_\varphi \Delta\rho_{23} + \omega^{24}{}_\varphi \Delta\rho_{24} + \omega^{34}{}_\varphi \Delta\rho_{34}\right) = 0, \\
Q_{(5)} &= \int_{\partial\Sigma} \sin\varphi \left(\omega^{23}{}_\theta \Delta\rho_{23} + \omega^{24}{}_\theta \Delta\rho_{24}\right) \\
&\quad + \cot\theta \cos\varphi \left(\omega^{14}{}_\varphi \Delta\rho_{14} + \omega^{23}{}_\varphi \Delta\rho_{23} + \omega^{24}{}_\varphi \Delta\rho_{24} + \omega^{34}{}_\varphi \Delta\rho_{34}\right) = 0, \\
Q_{(6)} &= \int_{\partial\Sigma} \left(\omega^{14}{}_\varphi \Delta\rho_{14} + \omega^{23}{}_\varphi \Delta\rho_{23} + \omega^{24}{}_\varphi \Delta\rho_{24} + \omega^{34}{}_\varphi \Delta\rho_{34}\right) = 0.
\end{aligned} \tag{3.23}
$$

Therefore, we conclude that conserved charges for the black hole with torsion (3.9) vanish. In particular, conserved charge $Q_{(0)}$, which corresponds to the energy E of the solution, vanishes due to the *specific choice of the parameters* $\alpha_1^2 = 12\alpha_0\alpha_2$.

4. Thermodynamics

By demanding that Euclidean continuation of the black hole has no conical singularity, we obtain the standard formula for the black hole temperature

$$T = \frac{(N^2)'|_{r=r_+}}{4\pi}. \tag{4.1}$$

In the particular case of the solution (3.9) we get

$$T = \frac{2r_+}{\pi\ell^2}. \tag{4.2}$$

The temperature is positive because solution (3.9) describes black hole iff condition (3.12) is satisfied. Let us note that this type of relation between temperature and the radius of the event horizon is unusual for black holes with spherical horizons. The relation (4.2) is standard in the case of planar black holes (black branes) or black holes in three space-time dimensions.

4.1. Euclidean action

Using the equation of motion (2.2), on-shell Euclidean action takes the form

$$I_{\mathrm{E}} = \varepsilon_{ijklm} \int \left(\frac{2\alpha_1}{3} R^{ij} e^k e^l e^m + \frac{4\alpha_0}{5} e^i e^j e^k e^l e^m \right). \tag{4.3}$$

After substituting the solution (3.9), we get

$$I_{\mathrm{E}} = \int_0^\beta \mathrm{d}t \int_{r_+}^\infty \mathrm{d}r \int \mathrm{d}\psi \mathrm{d}\theta \mathrm{d}\varphi \frac{4(\alpha_1^2 - 12\alpha_0\alpha_2)}{\alpha_2} r^3 \sin^2\psi \sin\theta, \tag{4.4}$$

where the integration over time is performed in the interval $[0, \beta := 1/T]$. By using the constraint on the parameters (3.8), we conclude that

$$I_{\mathrm{E}} = 0. \tag{4.5}$$

From the well-known formula for the entropy

$$S = (\beta\partial_\beta - 1)I_{\mathrm{E}}, \tag{4.6}$$

we obtain

$$S = 0. \tag{4.7}$$

This value of entropy is surprising, but it is not uncommon for Lovelock black holes—see for instance [24], where black holes with zero mass and entropy are obtained. From Euclidean action we can, also, calculate the energy

$$E = \partial_\beta I_{\mathrm{E}}, \tag{4.8}$$

and obtain

$$E = 0, \tag{4.9}$$

in accordance with the results of the previous section.

5. Concluding remarks

We have analyzed static spherically symmetric solutions of Lovelock gravity in five dimensions. For the generic values of the Lagrangian parameters, the theory possesses a well-known solution, the Boulware–Deser black hole, while in the sector $\alpha_1^2 = 12\alpha_0\alpha_2$ we have discovered a new black hole solution with torsion.

We analyzed basic properties of the obtained solution, which torsion possesses non-vanishing tensorial and totally antisymmetric part. The solution has a singularity of torsion and Riemannian curvature for $r \to 0$, while the conserved charges, as well as the entropy, vanish.

It is worth stressing that the black hole metric is asymptotically AdS, which is a crucial condition for holographic investigation. The solution that describes the space-time which is asymptotically dS, with the cosmological horizon located at $r_0 = \frac{\alpha_1}{2\alpha_0}$, is not a black hole.

An interesting property of the solution in the asymptotically AdS case is that, in the semi-classical approximation, its entropy is zero. This means that its number of micro-states is 'small' i.e. it is of order one instead of the expected $\mathcal{O}(\frac{1}{G_N})$. It would be interesting to see what kind of consequences this result has on dual interpretation via gauge/gravity duality.

Acknowledgments

This work was partially supported by the Serbian Science Foundation under Grant No. 171031.

Appendix A. Killing vectors for metric (3.1)

In addition to the $\frac{\partial}{\partial t}$ Killing vector static and spherically symmetric metric (3.1) possesses six Killing vectors, due to the $SO(4)$ spherical symmetry. The complete set of Killing vectors $\xi^\mu_{(i)}$ of the metric (3.1) is given by:

$$\begin{aligned}
\xi_{(0)} &= \partial_t, \\
\xi_{(1)} &= \cos\theta\partial_\psi - \cot\psi\sin\theta\partial_\theta, \\
\xi_{(2)} &= \sin\theta\cos\varphi\partial_\psi + \cot\psi\cos\theta\cos\varphi\partial_\theta - \frac{\cot\psi}{\sin\theta}\sin\varphi\partial_\varphi, \\
\xi_{(3)} &= \sin\theta\sin\varphi\partial_\psi + \cot\psi\cos\theta\sin\varphi\partial_\theta + \frac{\cot\psi}{\sin\theta}\cos\varphi\partial_\varphi, \\
\xi_{(4)} &= \cos\varphi\partial_\theta - \cot\theta\sin\varphi\partial_\varphi, \\
\xi_{(5)} &= \sin\varphi\partial_\theta + \cot\theta\cos\varphi\partial_\varphi, \\
\xi_{(6)} &= \partial_\varphi.
\end{aligned} \tag{A.1}$$

The independent Killing vectors are $\xi_{(0)}$, $\xi_{(1)}$, $\xi_{(4)}$ and $\xi_{(6)}$, while the others are obtained as their commutators. The invariance conditions of the vielbein under Killing vectors and local Lorentz transformations with parameters $\epsilon^i{}_j$ are

$$\delta_0 e^i_\mu = L_\xi e^i_\mu + \epsilon^i{}_j e^j_\mu = 0, \tag{A.2}$$

where the Lie derivative with respect to ξ is denoted as L_ξ, giving that the only non-zero parameters of the local Lorentz symmetry are

$$\epsilon^{23} = -\frac{\sin\theta}{\sin\psi},\ \epsilon^{34} = -\frac{\sin\varphi}{\sin\theta}. \tag{A.3}$$

Using this and the transformation law for spin connection,

$$\delta_0 \omega^{ij}_\mu = L_\xi \omega^{ij}_\mu + \epsilon^i{}_k \omega^{kj}_\mu + \epsilon^j{}_k \omega^{ik}_\mu = 0, \tag{A.4}$$

we can derive the most general form of the spherically symmetric spin connection which is given in the main text, formula (3.3).

Appendix B. Irreducible decomposition of the field strengths

We present here formulas for the irreducible decomposition of the PGT field strengths in a 5D Riemann–Cartan space-time [25].

The torsion 2-form has three irreducible pieces:

$$\begin{aligned}
{}^{(2)}T^i &= \frac{1}{4} b^i \wedge (h_m \rfloor T^m)\,, \\
{}^{(3)}T^i &= \frac{1}{3} h^i \rfloor (T^m \wedge b_m)\,, \\
{}^{(1)}T^i &= T^i - {}^{(2)}T^i - {}^{(3)}T^i\,.
\end{aligned} \tag{B.1}$$

The RC curvature 2-form can be decomposed into six irreducible pieces:

$$\begin{aligned}
&{}^{(2)}R^{ij} = -{}^*(b^{[i} \wedge \Psi^{j]})\,, && {}^{(4)}R^{ij} = \tfrac{2}{3} b^{[i} \wedge \Phi^{j]}\,, \\
&{}^{(3)}R^{ij} = -\tfrac{1}{12} X\, {}^*(b^i \wedge b^j)\,, && {}^{(6)}R^{ij} = \tfrac{1}{20} F\, b^i \wedge b^j\,, \\
&{}^{(5)}R^{ij} = \tfrac{1}{3} b^{[i} \wedge h^{j]} \rfloor (b^m \wedge F_m), && {}^{(1)}R^{ij} = R^{ij} - \textstyle\sum_{a=2}^{6} {}^{(a)}R^{ij}\,.
\end{aligned} \tag{B.2a}$$

where

$$\begin{aligned}
&F^i := h_m \rfloor R^{mi} = (Ric)^i\,, \qquad F := h_i \rfloor F^i = R\,, \\
&X^i := {}^*(R^{ik} \wedge b_k)\,, \qquad X := h_i \rfloor X^i,
\end{aligned} \tag{B.2b}$$

and

$$\begin{aligned}
\Phi_i &:= F_i - \frac{1}{4} b_i F - \frac{1}{2} h_i \rfloor (b^m \wedge F_m), \\
\Psi_i &:= X_i - \frac{1}{4} b_i X - \frac{1}{2} h_i \rfloor (b^m \wedge X_m)\,.
\end{aligned} \tag{B.2c}$$

The above formulas differ from those in [25] in two minor details: the definitions of F^i and X^i are taken with an additional minus sign, but at the same time, the overall signs of all the irreducible curvature parts are also changed, leaving their final content unchanged.

Appendix C. Torsion and curvature for the solution (3.9)

In this appendix, we give values of torsion and curvature for the black hole solution.

C.1. Riemannian connection and curvature

The non-vanishing components of the Riemannian connection are given by

$$\begin{aligned}
&\tilde\omega^{01} = -\frac{\sigma}{\ell^2}\left(\frac{r}{N} + \frac{3r_+^8}{Nr^7}\right) e^0, \qquad \tilde\omega^{12} = \frac{N}{r} e^2, \qquad \tilde\omega^{13} = \frac{N}{r} e^3, \\
&\tilde\omega^{23} = \frac{\cot\psi}{r} e^3, \qquad \tilde\omega^{14} = \frac{N}{r} e^4, \qquad \tilde\omega^{24} = \frac{\cot\psi}{r} e^4, \qquad \tilde\omega^{34} = \frac{\cot\theta}{r\sin\psi} e^4.
\end{aligned} \tag{C.1}$$

Riemannian curvature reads

$$\tilde{R}^{01} = \frac{\sigma}{\ell^2}\left(1 - \frac{21r_+^8}{r^8}\right)e^0e^1, \qquad \tilde{R}^{02} = \frac{\sigma}{\ell^2}\left(1 + \frac{3r_+^8}{r^8}\right)e^0e^2,$$
$$\tilde{R}^{03} = \frac{\sigma}{\ell^2}\left(1 + \frac{3r_+^8}{r^8}\right)e^0e^3, \qquad \tilde{R}^{04} = \frac{\sigma}{\ell^2}\left(1 + \frac{3r_+^8}{r^8}\right)e^0e^4,$$
$$\tilde{R}^{12} = \frac{\sigma}{\ell^2}\left(1 + \frac{3r_+^8}{r^8}\right)e^1e^2, \qquad \tilde{R}^{13} = \frac{\sigma}{\ell^2}\left(1 + \frac{3r_+^8}{r^8}\right)e^1e^3,$$
$$\tilde{R}^{14} = \frac{\sigma}{\ell^2}\left(1 + \frac{3r_+^8}{r^8}\right)e^1e^4, \qquad \tilde{R}^{04} = \frac{\sigma}{\ell^2}\left(1 + \frac{3r_+^8}{r^8}\right)e^0e^4,$$
$$\tilde{R}^{23} = \frac{\sigma}{\ell^2}\left(1 - \frac{\sigma\ell^2}{r^2} - \frac{r_+^8}{r^8}\right)e^2e^3, \qquad \tilde{R}^{24} = \frac{\sigma}{\ell^2}\left(1 - \frac{\sigma\ell^2}{r^2} - \frac{r_+^8}{r^8}\right)e^2e^4,$$
$$\tilde{R}^{34} = \frac{\sigma}{\ell^2}\left(1 - \frac{\sigma\ell^2}{r^2} - \frac{r_+^8}{r^8}\right)e^3e^4. \tag{C.2}$$

Riemannian scalar curvature is

$$\tilde{R} = -\frac{4\sigma}{\ell^2}\left(-5 + \frac{3\sigma\ell^2}{2r^2} + \frac{3r_+^8}{r^8}\right). \tag{C.3a}$$

The quadratic Riemannian curvature invariant vanishes

$$\tilde{R}_{ij} \wedge {}^*\tilde{R}^{ij} = 0. \tag{C.3b}$$

C.1.1. Torsion and its irreducible decomposition. The non-vanishing components of torsion are given by

$$T^0 = \frac{3N}{r}e^0e^1, \qquad T^2 = \frac{N}{r}e^1e^2 + \frac{2A_4}{r}e^3e^4,$$
$$T^3 = \frac{N}{r}e^1e^3 - \frac{2A_4}{r}e^2e^4, \qquad T^4 = \frac{N}{r}e^1e^4 + \frac{2A_4}{r}e^2e^3. \tag{C.4}$$

The non-vanishing irreducible components of torsion are

$$^{(1)}T^0 = \frac{3N}{r}e^0e^1,, \qquad ^{(1)}T^2 = \frac{N}{r}e^1e^2,$$
$$^{(1)}T^3 = \frac{N}{r}e^1e^3, \qquad ^{(1)}T^4 = \frac{N}{r}e^1e^4,$$
$$^{(3)}T^2 = \frac{2A_4}{r}e^3e^4, \qquad ^{(3)}T^3 = -\frac{2A_4}{r}e^2e^4, \qquad ^{(3)}T^4 = \frac{2A_4}{r}e^2e^3. \tag{C.5}$$

The 2nd irreducible component of torsion vanishes as in the case of any solution of Lovelock gravity, excluding Lovelock Chern–Simons [8]. Quadratic torsional invariant reads

$$T^i \wedge {}^*T_i = -\frac{12\sigma}{\ell^2}\left(1 - \frac{r_+^8}{r^8}\right)\hat{\epsilon}. \tag{C.6}$$

Non-zero components of the (Cartan) curvature are

$$R^{01} = \frac{4\sigma}{\ell^2}e^0e^1, \qquad R^{23} = \frac{4\sigma}{3\ell^2}\frac{N}{A_4}e^1e^4 + \frac{4\sigma}{3\ell^2}e^2e^3,$$
$$R^{24} = -\frac{4\sigma}{3\ell^2}\frac{N}{A_4}e^1e^3 + \frac{4\sigma}{3\ell^2}e^2e^4, \qquad R^{34} = \frac{4\sigma}{3\ell^2}\frac{N}{A_4}e^1e^2 + \frac{4\sigma}{3\ell^2}e^3e^4. \tag{C.7}$$

Scalar Cartan curvature is constant:

$$R = \frac{16\sigma}{\ell^2}. \tag{C.8}$$

Quadratic Cartan curvature invariant vanishes:

$$R_{ij} \wedge {}^*R^{ij} = 0. \tag{C.9}$$

ORCID iDs

B Cvetković https://orcid.org/0000-0002-9173-8101

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Poincaré gauge theory in 3D: canonical stability of the scalar sector

M. Blagojević and B. Cvetković * †
University of Belgrade, Institute of Physics, P. O. Box 57, 11001 Belgrade, Serbia

June 9, 2017

Abstract

We outline the results of the canonical analysis of the three-dimensional Poincaré gauge theory, defined by the general parity-invariant Lagrangian with eight free parameters [11]. In the scalar sector, containing scalar or pseudoscalar (A)dS modes, the stability of the canonical structure under linearization is used to identify dynamically acceptable values of the parameters.

1 Introduction

Models of three-dimensional (3D) gravity, pioneered by Staruskiewicz [1], were introduced to help us in clarifying highly complex dynamical behavior of the realistic four-dimensional general relativity (GR). In the last three decades, they led to a number of outstanding results [2]. However, in the early 1990s, Mielke and Baekler [3] proposed a new, non-Riemannian approach to 3D gravity, based on the Poincaré gauge theory (PGT) [4]. In PGT, the basic gravitational variables are the triad b^i and the Lorentz connection A^{ij} (1-forms), and their field strengths are the torsion $T^i := db^i + A^i{}_j b^j$ and the curvature $R^{ij} := dA^{ij} + A^i{}_m A^{mj}$ (we omit the exterior product sign for simplicity). In contrast to the traditional GR, with an underlying Riemannian geometry of spacetime, the PGT approach is characterized by a Riemann–Cartan geometry, with both the curvature and the torsion of spacetime as carriers of the gravitational dynamics. Thus, PGT allows exploring the interplay between gravity and geometry in a more general setting.

Three-dimensional GR with or without a cosmological constant, as well as the Mielke–Baekler (MB) model, are *topological* theories without propagating modes. From the physical point of view, such a degenerate situation is certainly not quite realistic. Including the *propagating modes* in PGT is achieved quite naturally by using Lagrangians quadratic in the field strengths [5, 6].

Since the general parity-invariant PGT Lagrangian in 3D is defined by eight free parameters [6], it is a theoretical challenge to find out which values of the parameters are allowed in a viable theory. The simplest approach to this problem is based on the *weak-field approximation* around the Minkowski background [5]. However, one should be very careful with the interpretation of these results, since the weak-field approximation does not always lead to a correct identification of the physical degrees of freedom.

The constrained Hamiltonian method [7, 4] is best suited for analyzing dynamical content of gauge theories of gravity, respecting fully their *nonlinear structure*. However, as noticed by Yo and Nester [8, 9], it may happen, for some ranges of parameters, that the canonical structure of a theory (the number and/or type of constraints) is changed after linearization in a way that affects its physical content, such as the number of physical degrees of freedom. Such an effect is called

*Based on a talk by MB at *New ideas for unsolved problems II*, Divčibare, 22–24 Sep 2013, Serbia.

†Email addresses: mb@ipb.ac.rs, cbranislav@ipb.ac.rs

the phenomenon of *constraint bifurcation*. Based on the *canonical stability under linearization* as a criterion for an acceptable choice of parameters, Shie et al. [10] proposed a PGT cosmological model that offers a convincing explanation of dark energy as an effect induced by torsion.

In this note, we use the constrained Hamiltonian formalism to study (a) the phenomenon of constraint bifurcation and (b) the stability under linearization of the general parity-invariant PGT in 3D, in order to find out the parameter values that define consistent models of 3D gravity with propagating torsion. Because of the complexity of the problem, we restrict our attention to the scalar sector, with $J^P = 0^+$ or 0^- modes, defined with respect to the (A)dS background [11].

The following conventions are of particular importance for our canonical analysis. Let $\mathcal{M}$ be a 3D manifold (spacetime) with local coordinates $x^\mu = (x^0, x^\alpha)$, and $h_i = h_i{}^\mu \partial_\mu$ a Lorentz frame on it. Then, if Σ is a 2D spacelike surface with a unit normal n_k, each tangent vector V_k of $\mathcal{M}$ can be decomposed in terms of its normal and parallel component with respect to Σ:

$$V_k = n_k V_\perp + V_{\bar{k}}\,, \quad \text{where} \quad V_\perp := n^m V_m\,, \quad V_{\bar{k}} = h_k{}^\alpha V_\alpha\,.$$

Note that $V_{\bar{k}}$ does not contain the time component of V_μ.

2 Quadratic PGT and its scalar modes

Assuming parity invariance, the dynamics of 3D gravity with propagating torsion is determined by the gravitational Lagrangian

$$L_G = -a\varepsilon_{ijk} b^i R^{jk} - \frac{1}{3}\Lambda_0 \varepsilon_{ijk} b^i b^j b^k + L_{T^2} + L_{R^2}\,, \tag{1a}$$

where $a = 1/16\pi G$, Λ_0 is a free parameter (bare cosmological constant), the pieces quadratic in the field strengths read

$$\begin{aligned} L_{T^2} &:= T^{i*}\left(a_1{}^{(1)}T_i + a_2{}^{(2)}T_i + a_3{}^{(3)}T_i\right)\,, \\ L_{R^2} &:= \frac{1}{2} R^{ij}\left(b_4{}^{(4)}R_{ij} + b_5{}^{(5)}R_{ij} + b_6{}^{(6)}R_{ij}\right)\,, \end{aligned} \tag{1b}$$

and ${}^{(n)}T_i$ and ${}^{(n)}R_{ij}$ are irreducible components of T^i and R^{ij} [6]. Being interested only in the gravitational degrees of freedom, we disregard the matter contribution.

Particle spectrum of the theory around the Minkowski background M_3 is already known [5, 6]. Restricting our attention to the scalar sector, we display here the masses of the spin-0^+ and 0^- modes:

$$m^2_{0^+} = \frac{3a(a+a_2)}{a_2(b_4+2b_6)}\,, \qquad m^2_{0^-} = \frac{3a(a+2a_3)}{(a_1+2a_3)b_5}\,. \tag{2a}$$

These modes have finite masses and propagate if

$$a_2(b_4+2b_6) \neq 0\,, \qquad (a_1+2a_3)b_5 \neq 0\,, \tag{2b}$$

respectively.

Transition to the (A)dS background is straightforward; it generalizes the mass formulas (2a) by introducing a dependence on the parameter q that measures the strength of the background curvature [11], but the propagation conditions for the scalar modes remain the *same* as in (2b). As we shall see in the next section, the conditions (2b), derived in the *weak-field approximation*, have a critical role also in the canonical analysis of the *full nonlinear theory*.

3 Primary if-constraints

The canonical momenta corresponding to the basic dynamical variables $(b^i{}_\mu, A^{ij}{}_\mu)$ are defined by $\pi_i{}^\mu := \partial\tilde{\mathcal{L}}/\partial(\partial_0 b^i{}_\mu)$ and $\Pi_{ij}{}^\mu := \partial\tilde{\mathcal{L}}/\partial(\partial_0 A^{ij}{}_\mu)$, respectively. Since the torsion and the curvature do not involve the velocities $\partial_0 b^i{}_0$ and $\partial_0 A^{ij}{}_0$, one obtains the primary constraints

$$\pi_i{}^0 \approx 0\,, \qquad \Pi_{ij}{}^0 \approx 0\,, \tag{3}$$

which are always present, independently of the values of coupling constants ("sure" constraints). If the Lagrangian (1) is singular with respect to some of the remaining velocities $\partial_0 b^i{}_\alpha$ and $\partial_0 A^{ij}{}_\alpha$, one obtains further primary constraints, known as the primary "if-constraints" (ICs).

The gravitational Lagrangian (1) depends on the time derivative $\partial_0 b^i{}_\alpha$ only through the torsion tensor, appearing in L_{T^2}. The system of equations defining the parallel gravitational momenta $\hat\pi_i{}^{\bar k} = \pi_i{}^\alpha b^k{}_\alpha$ $(\hat\pi_i{}^{\bar k} n_k = 0)$ can be decomposed into irreducible parts with respect to the group of two-dimensional spatial rotations in Σ:

$$\phi_{\perp\bar k} := \frac{\hat\pi_{\perp\bar k}}{J} - (a_2 - a_1)T^{\bar m}{}_{\bar m\bar k} = (a_1 + a_2)T_{\perp\perp\bar k}\,, \tag{4a}$$

$$^S\phi := \frac{^S\hat\pi}{J} = -2a_2 T^{\bar m}{}_{\bar m\perp}\,, \tag{4b}$$

$$^A\phi_{\bar\imath\bar k} := \frac{^A\hat\pi_{\bar\imath\bar k}}{J} - \frac{2}{3}(a_1 - a_3)T_{\perp\bar\imath\bar k} = -\frac{2}{3}(a_1 + 2a_3)T_{[\bar\imath\bar k]\perp}\,, \tag{4c}$$

$$^T\phi_{\bar\imath\bar k} := \frac{^T\hat\pi_{\bar\imath\bar k}}{J} = -2a_1\,{}^T T_{\bar\imath\bar k\perp}\,, \tag{4d}$$

where the terms depending on the velocities $\partial_0 b^i{}_\alpha$ are moved to the right-hand sides. If the critical parameter combinations appearing on the right-hand sides of Eqs. (4) vanish, the corresponding expressions ϕ_K become additional primary constraints.

Similar analysis can be applied to the equations defining the parallel gravitational momenta $\hat\Pi_{ij}{}^{\bar k} =: \Pi_{ij}{}^\alpha b^k{}_\alpha$ $(\hat\Pi_{ij}{}^{\bar k} n_k = 0)$, leading to an additional set of primary constraints Φ_K. The complete set of primary ICs, including their spin-parity characteristics (J^P), is shown in Table 1.

Table 1. Primary if-constraints

Critical conditions	Primary constraints	J^P
$a_2 = 0$	$^S\phi \approx 0$	0^+
$b_4 + 2b_6 = 0$	$^S\Phi_\perp \approx 0$	
$a_1 + 2a_3 = 0$	$^A\phi_{\bar\imath\bar k} \approx 0$	0^-
$b_5 = 0$	$^A\Phi_{\perp\bar\imath\bar k} \approx 0$	
$a_1 + a_2 = 0$	$\phi_{\perp\bar k} \approx 0$	1
$b_4 + b_5 = 0$	$^V\Phi_{\bar k} \approx 0$	
$a_1 = 0$	$^T\phi_{\bar\imath\bar k} \approx 0$	2
$b_4 = 0$	$^T\Phi_{\perp\bar\imath\bar k} \approx 0$	

This classification has a remarkable interpretation: whenever a pair of the ICs with specific J^P is absent, the corresponding dynamical mode is liberated to become a *physical degree of freedom* (DoF). Thus, for $a_2(b_4 + 2b_6) \neq 0$, the spin-0^+ ICs are absent, and the spin-0^+ mode becomes physical. Similarly, $(a_1 + 2a_34)b_5 \neq 0$ implies that the spin-0^- mode is physical. These results, referring to the full nonlinear theory, should be compared to (2b).

Remark. Once we know the complete set of primary ICs, we can apply Dirac's consistency algorithm to obtain the secondary constraints, and so on.

4 Spin-0^+ sector

As one can see from Table 1, the spin-0^+ degree of freedom propagates for $a_2(b_4+2b_6)\neq 0$. In order to investigate dynamical features of this sector, we adopt somewhat simplified conditions:

$$a_2, b_6 \neq 0\,, \qquad a_1 = a_3 = b_4 = b_5 = 0\,. \tag{5a}$$

While such a "minimal" choice simplifies the calculations, it is not expected to influence any essential aspect of the spin-0^+ dynamics [8, 9].

Generic case

Now, we turn to the canonical analysis. First, the form of the Hamiltonian implies that the kinetic energy density is positive definite (no "ghosts") if

$$a_2 > 0\,, \qquad b_6 > 0\,. \tag{5b}$$

Second, in the simple, *generic* situation, when all of the ICs are second class (their number is $N_2 = 10$), the complete set of constraints is given in Table 2.

Table 2. Generic constraints in the 0^+ sector

	First class	Second class
Primary	${\pi_i}^0$, ${\Pi_{ij}}^0$	${}^V\Phi_{\bar\imath}$; ${}^A\phi$, ${}^A\Phi$, ${}^T\phi$, ${}^T\Phi$
Secondary	$\mathcal{H}'_\perp$, $\mathcal{H}'_\alpha$, $\mathcal{H}'_{ij}$	$\chi_{\bar\imath}$

As always, the Hamiltonian constrains $\mathcal{H}'_\perp$, $\mathcal{H}'_\alpha$ and $\mathcal{H}'_{ij}$ are first class. With $N = 2\times 9$ field components, $N_1 = 2\times 6$ first class constraints and $N_2 = 10$ second class constraints, the dimension of the phase space is $N^* = 2N - 2N_1 - N_2 = 2$, and the theory exhibits a single Lagrangian DoF.

Constraint bifurcation

To clarify the term "generic" used above, we calculate the determinant of the 10×10 matrix $\Delta^+_{MN} = \{X'_M, X'_N\}$, where X'_M is the set of all ICs shown in Table 2. The result is

$$\Delta^+ \sim W^{10}\,(W-a_2)^4 \quad \text{where} \quad W := \frac{{}^S\Pi_\perp}{4J}\,. \tag{6}$$

The generic situation corresponds to $\Delta^+ \neq 0$. However, the determinant Δ^+, being a field-dependent object, may vanish in some regions of spacetime, changing thereby the number and/or type of constraints and the number of physical DoF, as compared to the situation described in Table 2. This phenomenon of *constraint bifurcation* can be fully understood by analyzing dynamical behavior of the critical factors W and $W - a_2$, appearing in Δ^+.

Assuming that W is an analytic function globally, on the whole spacetime manifold $\mathcal{M}$, the analysis of the field equations

$$-\,(W-a_2)V_k + 2\partial_k(W-a_2) \approx 0\,, \tag{7}$$

leads to the following conclusion [11]:

- If there is a point in $\mathcal{M}$ at which $W - a_2 \neq 0$, then $W - a_2 \neq 0$ globally.

Hence, by choosing the initial data so that $W - a_2 \neq 0$ at $x^0 = 0$, it follows that $W - a_2$ stays nonvanishing for any $x^0 > 0$. The surface $W - a_2 = \frac{1}{6}b_6 R - a - a_2 \approx 0$ (on shell) is a dynamical barrier that the spin-0^+ field cannot cross. Moreover, since a_2 is positive, see (5b), we have:

- By choosing $W - a_2 > 0$ at $x^0 = 0$, it follows that $W \neq 0$ globally.

Thus, with a suitable choice of the initial data, one can ensure the generic condition $\Delta^+ \neq 0$ to hold *globally*, whereupon the constraint structure is described exactly as in Table 2. Any other situation, with $W = 0$ or $W - a_2 = 0$, would not be acceptable—it would have a variable constraint structure over the spacetime, the property that could not survive the process of linearization.

Stability under linearization

Now, we compare the canonical structure of the full nonlinear theory with its weak-field approximation around maximally symmetric background. With the background values $\bar{R} = -6q$ and $\bar{W} = \frac{1}{6}b_6\bar{R} - a$, the lowest-order critical factors take the form

$$\bar{W} = -(a + qb_6)\,, \qquad \bar{W} - a_2 = -(a + a_2 + qb_6)\,,$$

which leads to the results shown in Table 3 [11].

Table 3. Canonical stability in the 0^+ sector

	$a + qb_6$	$a + a_2 + qb_6$	DoF	stability
(a)	$\neq 0$	$\neq 0$	1	stable
(b)	$= 0$	$\neq 0$	0	unstable
(c)	$\neq 0$	$= 0$	1	stable*

Based on the conditions (5a), the spin-0^+ mass formula for $q \neq 0$ takes the form:

$$m^2_{0^+} = \frac{3(a - qb_6)(a + a_2 + qb_6)}{2a_2 b_6}\,.$$

Now, a few comments are in order: (a) the nature of constraints remains the same as in Table 2, which implies the stability under linearization; (b) all if-constraints become first class, but only 6 of them remain independent, which leads to $N^* = 0$ (instability); (c) the massless nonlinear theory, defined by the condition $a + a_2 + qb_6 = 0$, is essentially stable under linearization.

5 Concluding remarks

— By investigating fully nonlinear constraint bifurcation effects, as well as the canonical stability under linearization, we were able to identify the set of dynamically acceptable values of parameters for the spin-0^+ sector of PGT, as shown in Table 3.
— On the other hand, the spin-0^- sector is canonically unstable for any choice of parameters; for more details, see Ref. [11].
— Further analysis of higher spin modes is left for future studies.

Acknowledgements

We thank Vladimir Dragović for a helpful discussion This work was supported by the Serbian Science Foundation under Grant No. 171031.

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Vaidya-like exact solutions with torsion

Milutin Blagojević and Branislav Cvetković*

Institute of Physics, University of Belgrade,
P. O. Box 57, 11001 Belgrade, Serbia
**E-mail: cbranislav@ipb.ac.rs*
gravity.ipb.ac.rs/Cvetkovic.html

Starting from the Oliva–Tempo–Troncoso black hole, a solution of the Bergshoeff–Hohm–Townsend massive gravity, a class of the Vaidya-like exact vacuum solutions with torsion is constructed in the three-dimensional Poincaré gauge theory. A particular subclass of these solutions is shown to possess the asymptotic conformal symmetry.

Keywords: 3D gravity; Poincaré gauge theory; Vaidya solution.

1. Introduction

Investigations of Poincar'e gauge theory (PGT)[1] in three-dimensional (3D) spacetime are expected to improve our understanding of both the geometric and dynamical role of torsion. Systematic studies of 3D PGT started with the Mielke–Baekler model[2], introduced in the 1990s. However, this model is, just like GR, a topological theory. In PGT, such an unrealistic dynamical feature can be quite naturally improved by going over to Lagrangians that are quadratic in the field strengths[3].

The exact solutions of a gravitational theory are essential for its physical interpretation. In the context of 3D PGT, exact solutions were first studied in the Mielke–Baekler model. Recently, our research interest moved toward exact solutions in a more dynamical framework of the quadratic PGT. After constructing the Bañados–Teitelboim-Zanelli (BTZ) black hole with torsion[3] we showed that gravitational waves can be naturally incorporated into the PGT dynamical framework[4,5]. The purpose of the present work is to examine a PGT generalization of the Oliva–Tempo–Troncoso (OTT) black hole[6], as well as its Vaidya-like extension[7].

The OTT black hole is an exact solution of the Bergshoeff–Hohm–Townsend (BHT) massive gravity[8]. Generically, the BHT gravity with a cosmological constant admits two distinct maximally symmetric vacua. However, when the coupling constants satisfy a specific critical condition, these two vacua coincide. In this case OTT black hole is a vacuum solution of the BHT gravity. Going a step further, Maeda[7] formulated a Vaidya-like extension of the OTT black hole, assuming the presence of a null dust fluid as a *matter field*. In this paper, we construct a Vaidya-OTT spacetime with torsion as an exact *vacuum solution* of PGT.

We use the following conventions: the Latin indices $(i, j, k, ...)$ refer to the local Lorentz frame, the Greek indices $(\mu, \nu, \rho, ...)$ refer to the coordinate frame, b^i is the triad field (1-form), $\omega^{ij} = -\omega^{ji}$ is a connection 1-form, the respective field strengths are the torsion $T^i = db^i + \omega^i{}_m \wedge b^m$ and the curvature $R^{ij} = d\omega^{ij} + \omega^i{}_k \wedge \omega^{kj}$ (2-forms); the Hodge dual of a form α is ${}^\star\alpha$, and the exterior product is implicit.

2. OTT Black Hole in PGT

Static OTT black hole, a vacuum solution of the BHT gravity with a unique AdS ground state[6], is also a *Riemannian solution* of PGT, in spite of the fact that PGT represents quite a different dynamical framework[3].

Geometric aspects. The metric of the static OTT spacetime is given by

$$ds^2 = N^2 dt^2 - \frac{dr^2}{N^2} - r^2 d\varphi^2 \,, \qquad N^2 := -\mu + Br + \frac{r^2}{\ell^2} \,, \tag{1}$$

where μ and B are integration constants. When at least of the Killing horizons is real and positive, and $\ell^2 > 0$, the OTT metric defines a static and spherically symmetric AdS black hole; for $B = 0$, it reduces to the BTZ black hole.

For $B \neq 0$, the scalar curvature has a singularity at $r = 0$, while the nonvanishing irreducible components of the curvature are ${}^{(6)}R^{ij} = \frac{1}{6}Rb^ib^j$ and ${}^{(4)}R^{ij} = R^{ij} - {}^{(6)}R^{ij}$. In this geometry, the Cotton 2-form $C^i := \nabla L^i$ where $L^i = (Ric)^i - \frac{1}{4}Rb^i$, is vanishing, so that the OTT spacetime is conformally flat.

Riemannian sector of PGT. The general parity-preserving gravitational Lagrangian of PGT is quadratic in the field strengths. In the Riemannian sector of PGT, torsion vanishes, and L_G is expressed only in terms of the curvature:

$$L_G = -{}^\star(a_0 R + 2\Lambda_0) + \frac{1}{2}R^{ij\star}\left(b_4{}^{(4)}R_{ij} + b_6{}^{(6)}R_{ij}\right) \,, \tag{2}$$

and the vacuum PGT field equations produce the following result:

$$b_4 + 2b_6 = 0 \,, \qquad b_4 - 2a_0\ell^2 = 0 \,, \qquad a_0 + 2\ell^2\Lambda_0 = 0 \,. \tag{3}$$

Thus, the OTT black hole is an exact vacuum solution in the Riemannian sector of PGT, provided the four Lagrangian parameters satisfy the above three conditions.

3. Vaidya Extension of the OTT Metric

To obtain a Vaidya extension of the OTT metric, we first make a coordinate transformation from the Schwarzschild-like time coordinate t to a new coordinate u:

$$dt = du + dr/N^2 \,. \tag{4}$$

The physical meaning of u is obtained by noting that $u =$ const. corresponds to a radially outgoing null ray, $dr/dt = N^2$, see Ref. 10. We introduce a Vaidya extension of the OTT black hole by making B a function of u, $B = B(u)$, but leaving μ as a constant. The time dependent spherically symmetric Vaidya–OTT metric reads:

$$ds^2 = N^2 du^2 + 2dudr - r^2 d\varphi^2 \,. \tag{5}$$

In the new coordinates $x^\mu = (u, r, \varphi)$, it is convenient to choose the triad field as

$$b^+ := du \,, \qquad b^- := H du + dr \,, \qquad b^2 := r d\varphi \,, \tag{6}$$

where $H = N^2/2$.

For vanishing torsion, one can use the Riemannian connection

$$\omega^{+-} = -H'b^+ \,, \qquad \omega^{+2} = -\frac{1}{r}b^2 \,, \qquad \omega^{-2} = \frac{1}{r}Hb^2 \,, \tag{7}$$

to calculate the related curvature 2-form R^{ij}. Then, following the procedure described in the previous section, one finds that the PGT field equations imply:

$$b_4 + 2b_6 = 0 \,, \qquad b_4 - 2a_0\ell^2 = 0 \,, \qquad a_0 + 2\ell^2\Lambda = 0 \,, \qquad \underline{\dot{B} := \partial_u B = 0} \,. \tag{8}$$

Thus, the Vaidya–OTT metric l is *not a Riemannian solution of PGT in vacuum.*

To overcome a similar barrier in the BHT gravity, Maeda[7] introduced the Vaidya-OTT solution in the presence of *matter*, represented by a null dust fluid. Based on our experience with exact wave solutions in PGT[4,5], we expect that the presence of torsion could lead to a consistent description of the problem.

4. Vaidya–OTT solution with torsion

4.1. *Geometry of the ansatz*

Following the logic of our approach to exact wave solutions in PGT[4,5], we propose to look for a Vaidya–OTT solution with torsion using the following two assumptions:

(i) The new triad field retains the form (6);

(ii) The RC connection is obtained from the Riemannian expression (7) by the rule $H \to H + K$, where $K = K(u)$:

$$\omega^{+-} = -H'b^+ \,, \qquad \omega^{+2} = -\frac{1}{r}b^2 \,, \qquad \omega^{-2} = \frac{1}{r}(H+K)b^2 \,. \tag{9}$$

The new function K is expected to *compensate* the presence of the problematic $\dot{B}$ term in the Riemannian field equations (8). Geometrically, K defines the torsion of spacetime.

4.2. *Solutions*

We now wish to find the metric function H and the torsion function K as solutions of the vacuum PGT field equations. To ensure a smooth limit to the standard OTT black hole for $B \to$ const., we impose the conditions (3) on the Lagrangian parameters. Then, the field equations read:

$$2\dot{K} + BK = 0 \,, \qquad \dot{B}\ell^2 + 2K = 0 \,. \tag{10}$$

By combining the above two equations, one obtains

$$2K - \frac{1}{4}B^2\ell^2 = -K_0\ell^2 \,, \qquad \dot{B} + \frac{1}{4}B^2 = K_0 \,, \tag{11}$$

where K_0 is an integration constant, the first integral of the field equations (10). Introducing a new constant E by $K_0\ell^2 = 4GE - \mu$, the first equation takes the form

$$4GE = \mu + \frac{1}{4}B^2\ell^2 - 2K \,, \tag{12}$$

where E is recognized as a RC generalization of the gravitational energy. The conservation law of E is defined with respect to the evolution along u, $dE/du = 0$. However, $dt = du + dr/N^2$ implies $t = u + \mathcal{O}_1$, so that asymptotically, one expects E to be conserved also with respect to the Schwarzschild-like time t.

Depending on the value of K_0, there exist three branches of solutions.

1. $K_0 = C_1^2$. Apart from the trivial case $B = 2C_1$, $K = 0$, one finds:

$$B = 2C_1 \tanh \frac{C_1}{2}(u + C_2)\,, \qquad K = -\frac{C_1^2 \ell^2}{2\cosh^2 \frac{C_1}{2}(u + C_2)}\,. \tag{13}$$

2. $K_0 = -C_1^2$. By replacing $C_1 \to iC_1$ in the solution (13), one obtains:

$$B = -2C_1 \tan \frac{C_1}{2}(u + C_2)\,, \qquad K = \frac{C_1^2 \ell^2}{2\cos^2 \frac{C_1}{2}(u + C_2)}\,. \tag{14}$$

3. $K_0 = 0$.

$$B = \frac{4}{u + C_2}\,, \qquad K = \frac{2\ell^2}{(u + C_2)^2}\,. \tag{15}$$

The solutions in branch 1 are perfectly regular, and physically most appealing. Since $B(u)$ and $K(u)$, as well as their derivatives, are bounded functions, the field strengths approach asymptotically to a Riemannian AdS spacetime.

5. Asymptotic Symmetry

In this section, we use the canonical approach to analyze the asymptotic symmetry associated to the Vaidya–OTT solution with torsion in branch 1.

5.1. *AdS asymptotic conditions*

By requiring the *invariance under the AdS group* $SO(2,2)$, one arrives at the following set of the Vaidya–OTT asymptotic states:

$$b^i{}_\mu = \bar{b}^i{}_\mu + B^i{}_\mu\,, \qquad B^i{}_\mu := \begin{pmatrix} \mathcal{O}_1 & \mathcal{O}_3 & \mathcal{O}_1 \\ \mathcal{O}_{-1} & \mathcal{O}_1 & \mathcal{O}_{-1} \\ \mathcal{O}_0 & \mathcal{O}_2 & \mathcal{O}_0 \end{pmatrix}, \tag{16a}$$

$$\omega^i{}_\mu = \bar{\omega}^i{}_\mu + \Omega^i{}_\mu\,, \qquad \Omega^i{}_\mu := \begin{pmatrix} \mathcal{O}_1 & \mathcal{O}_3 & \mathcal{O}_1 \\ \mathcal{O}_{-1} & \mathcal{O}_1 & \mathcal{O}_{-1} \\ \mathcal{O}_0 & \mathcal{O}_2 & \mathcal{O}_0 \end{pmatrix}, \tag{16b}$$

where $\omega^i := -\frac{1}{2}\varepsilon_{ijk}\omega^{jk}$, and $\bar{b}^i{}_\mu$ and $\bar{\omega}^i{}_\mu$ refer to the background configuration with $\mu, B = 0$, representing the massless BTZ black hole. These states are invariant under the set of restricted local Poincaré transformations which commutator algebra is found to have the form of two independent Virasoro algebras.

5.2. *Canonical generators*

In order to examine the canonical structure of the quadratic PGT, we use the *first-order formulation* [11], as it leads to a particularly simple construction of the canonical generator. Since the canonical generator G acts on basic dynamical variables via the Poisson bracket operation, it is required to be a differentiable phase-space functional. For a given set of asymptotic conditions, this property is ensured by adding a suitable surface Γ term to G, such that $\tilde{G} = G + \Gamma$ is *both differentiable and finite* phase-space functional[12]. The improved generator $\tilde{G} = G + \Gamma$ is determined by:

$$\delta\Gamma = \int_0^{2\pi} d\varphi(\xi^t \delta\mathcal{E} + \xi^\varphi \delta\mathcal{M})\,, \tag{17a}$$

$$\delta\mathcal{E} := \frac{1}{2}\left(\omega^{ij}{}_t \delta H_{ij\varphi} + \delta\omega_{ij\varphi} H^{ij}{}_t\right)\,, \tag{17b}$$

$$\delta\mathcal{M} := \frac{1}{2}\left(\omega^{ij}{}_\varphi \delta H_{ij\varphi} + \delta\omega_{ij\varphi} H^{ij}{}_\varphi\right)\,, \tag{17c}$$

where $H_{ij} = -2a_0\varepsilon_{ijk}b^k - 4a_0\ell^2\varepsilon_{ijk}\hat{L}^k$.

The surface term for time translations can be written in the form

$$\mathcal{E} = \frac{1}{2}\big(\omega^{ij}{}_t \Delta H_{ij\varphi} + \Delta\omega^{ij}{}_\varphi \bar{H}_{ijt}\big) - \frac{1}{4}\big(\Delta\omega^{ij}{}_t \Delta H_{ij\varphi} - \Delta\omega^{ij}{}_\varphi \Delta H_{ijt}\big)\,, \tag{18a}$$

where $\Delta X := X - \bar{X}$ is the difference between any form X and its boundary value $\bar{X}$. Equation (17c) leads to a simple surface term for spatial rotations:

$$\mathcal{M} = \frac{1}{2}\omega^{ij}{}_\varphi H_{ij\varphi}\,. \tag{18b}$$

Improved generator is finite phase-space functional.

The boundary terms for $\xi^t = 1$ and $\xi^\varphi = 1$ represent the energy and angular momentum of the system, which for the the Vaidya–OTT configuration are

$$E = \frac{1}{4G}\left(\mu + \frac{1}{4}B^2\ell^2 - 2K\right)\,, \qquad M = 0\,. \tag{19}$$

The form of E confirms the result (12) obtained from the Lagrangian field equations.

The expression for energy defined by equation (18a) consists of two pieces. As shown in Ref. 9, the first piece is sufficient to correctly describe the energy content of a number of solutions in 3D gravity. However, when applied to the (Vaidya–)OTT solution, this piece is not sufficient; in particular, it produces the incorrect coefficient 1/2 for the B^2 term in (19). Thus, our result (18a) represents a generalization of the energy formula used in[9] to the (Vaidya–)OTT case.

5.3. *Canonical algebra of asymptotic symmetries*

The asymptotic symmetry is described by the Poisson bracket algebra of the improved generators. In terms of Fourier modes $L^\pm_n$ of $\tilde{G}$, the PB algebra takes the form of two independent Virasoro algebras,

$$i\{L^\pm_m, L^\pm_n\} = (m-n)L^\pm_{n+m} + \frac{c^\pm}{12}n^3\delta_{m,-n}\,, \tag{20}$$

where the classical central charges are equal to each other, $c^{\pm} = c$, with

$$c = \frac{3\ell}{G}\,. \tag{21}$$

Thus, the value of c is found to be twice the GR value $c_0 = 3\ell/2G$.

6. Concluding remarks

We constructed a Vaidya-like extension of the OTT black hole as an exact solution of the quadratic PGT in vacuum. Firstly, we showed that the OTT black hole is a Riemannian vacuum solution of PGT, provided the coupling constants satisfy certain requirements. Then, following Maeda[7], we introduced a Vaidya-like extension of the OTT black hole; however, this extension is not a Riemannian solution of PGT in vacuum. To overcome this difficulty, we introduced a suitable ansatz for the connection possessing a nontrivial torsion content, making thereby the resulting Vaidya–OTT geometry an exact vacuum solution of PGT. The canonical energy contains a contribution stemming from the torsion and central charges of the asymptotic algebra are the same as in the BHT gravity case.

Acknowledgments

This work was supported by the Serbian Science Foundation, Grant No. 171031.

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Generalized plane waves in Poincaré gauge theory of gravity

A family of exact vacuum solutions, representing generalized plane waves propagating on the (anti-)de Sitter background, is constructed in the framework of Poincaré gauge theory. The wave dynamics is defined by the general Lagrangian that includes all parity even and parity odd invariants up to the second order in the gauge field strength. The structure of the solution shows that the wave metric significantly depends on the spacetime torsion.

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Institute of physics
, University of Belgrade
3D gravity

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Комисија за стицање научних звања**

**Број:660-01-00157/2013-17
25.09.2013. године
Б е о г р а д**

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На основу члана 22. става 2. члана 70. став 6. Закона о научноистраживачкој делатности ("Службени гласник Републике Србије", број 110/05 и 50/06 – исправка и 18/10), члана 2. става 1. и 2. тачке 1 – 4.(прилози) и члана 38. Правилника о поступку и начину вредновања и квантитативном исказивању научноистраживачких резултата истраживача ("Службени гласник Републике Србије", број 38/08) и захтева који је поднео

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Др Бранислав Цвешковић

стиче научно звање
Виши научни сарадник

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утврдио је предлог број 319/2 од 12.03.2013. године на седници научног већа Института и поднео захтев Комисији за стицање научних звања број 319/1 од 13.03.2013. године за доношење одлуке о испуњености услова за стицање научног звања ***Виши научни сарадник***.

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Доношењем ове одлуке именовани стиче сва права која му на основу ње по закону припадају.

Одлуку доставити подносиоцу захтева, именованом и архиви Министарства просвете, науке и технолошког развоја у Београду.

ПРЕДСЕДНИК КОМИСИЈЕ

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Angažovanost u razvoju uslova za naučni rad, obrazovanju i formiranju naučnih kadrova

UNIVERZITET U BEOGRADU

FIZIČKI FAKULTET

Dejan Simić

Lavlokova gravitacija sa torzijom: egzaktna rešenja, kanonska i holografska struktura

doktorska disertacija

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UNIVERSITY OF BELGRADE

FACULTY OF PHYSICS

Dejan Simić

Lovelock gravity with torsion: exact solutions, canonical and holographic structure

Doctoral Dissertation

Belgrade, 2018

Mentor:

- dr Branislav Cvetković, viši naučni saradnik,
  Institut za fiziku, Univerzitet u Beogradu

Članovi komisije:

- dr Branislav Cvetković, viši naučni saradnik,
  Institut za fiziku, Univerzitet u Beogradu
- Prof. dr Maja Burić, redovni profesor, Fizički fakultet,
  Univerzitet u Begradu
- Prof. dr Voja Radovanović, redovni profesor,
  Fizički fakultet, Univerzitet u Beogradu

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Rezime

Tema ovog rada je proučavanje Lavlokove gravitacije sa torzijom u cilju boljeg razumevanja torzije kao i viših stepena krivine. Fokus je na pronalaženju novih rešenja i na ispitivanju holografske strukture. Rad je podeljen na glave, sadržaj pojedinačne glave je dat u daljem tekstu.

Izlaganje počinjemo kratkim uvodom, u kome dajemo pregled poznatih rezultata i problema u fizici gravitacije. Takođe, u uvodu, pravimo pregled pravaca istraživanja, i na kraju dajemo, kratki, osvrt na moderna dostignuća eksperimentalne fizike gravitacije, kao i na njihov značaj za teorijsku fiziku gravitacije i fiziku uopšte.

Nakon toga pristupamo teoriji gravitacije kao teoriji polja. Ideja ove glave je kritički pogled na strukturu gravitacije tako da uvodimo principe efektivne teorije polja, koja predstavlja opšteprihvaćen pogled na teoriju polja. Nakon toga, dajemo pregled konstrukcije teorije gravitacije i problem nerenormalizabilnosti Opšte teorije relativnosti. Ovo će pokazati da potraga za kompletnom teorijom gravitacije nije gotova i daće nam smernice u kom pravcu treba nastaviti potragu. Na kraju, uvodimo gradijentnu teoriju Poenkareove grupe, koja je opšti formalizam za konstrukciju teorije gravitacije i dajemo primer Lavlokove gravitacije.

Treća glava je posvećena kanonskoj analizi, koja je veoma koristan formalizam za ispitivanje strukture teorije. Naime, istom se mogu odrediti gradijentne simetrije i konstruisati njihov generator, kao i broj stepeni slobode.

Naredna glava je skup neophodnih informacija u cilju formulisanja i razumevanja holografske dualnosti. U ovoj glavi, dajemo kratki pregled termodinamike crnih rupa i konformne teorije polja. Uvodimo asimptotske simetrije, koje nas dovode do prvog primera holografse dualnosti. Na kraju, izlažemo Viten-Gubser-Klebanov-Poljakovu preskripciju, koju ilustrujemo na nekoliko elementarnih primera.

Crnim rupama u Lavlokovoj gravitaciji je posvećena peta glava. Prvo pravimo pregled, nekih, poznatih rešenja. Nakon toga, detaljno izlažemo konstrukciju i analizu osobina sferno simetrične crne rupe sa torzijom. Posle toga, konstruišemo crni prsten sa i bez torzije i proučavamo njegove osobine.

U narednoj glavi, ispitujemo holografsku strukturu Lavlok Čern-Sajnmonsove gravitacije. Prvo, detaljno analiziramo fiksiranje gradijentne simetrije u teoriji sa lokalnom AdS simetrijom. Posle toga, izvodimo Vordove identitete. Dalje se fokusiramo na Lavlok Čern-Sajnmonsovu gravitaciju, izvodimo oblik Grinovih funkcija i proveravamo prisustvo anomalija. U finalnom delu ove glave dajemo alternativni dokaz renormalizacione teoreme.

Na kraju sumiramo rezultate, revidiramo status i delimo nekoliko misli kakva je perspektiva teorije gravitacije.

Ključne reči: Alternativne teorije gravitacije, gravitacija u dimenziji različitoj od četiri, crne rupe, holografska dualnost

Naučna oblast: Fizika

Uža naučna oblast: Teorijska fizika visokih energija

UDK broj: 538.9(043.3)

Abstract

Thematics of this work is studding of Lovelock gravity with torsion with the purpose of better understanding of torsion as well as of higher degrees of curvature. The focus is on discovering of new solutions and investigating of holographic structure. The paper is divided in chapters, the content of individual chapter is given in the following text.

We start the exposure with short introduction, in which we review known results and problems in gravitational physics. Also, in introduction, we review directions of research, at the end we look at modern achievements in experimental gravity, as well as their importance for theory of gravity and whole physics. Ovo će pokazati da potraga za kompletnom teorijom gravitacije nije gotova i daće nam smernice u kom pravcu treba da nastavimo potragu. Na kraju, uvodimo gradijentnu teoriju Poenkareove grupe, koja je opšti formalizam za konstrukciju teorije gravitacije i dajemo primer Lavlokove gravitacije.

Afterwards we approach to gravity as a field theory. The idea of this chapter is critical look at structure of gravity to this end we introduce the principles of effective field theory, which represents generally accepted approach to field theory. Next, we review construction of theory of gravity and non-renormalizability of General relativity. This will show that search for the complete theory of gravity is not finished and give us directions in which to continue research. At the end, we introduce gauge theory of Poincare group, which is general formalism for construction of gravitational theory and give example of Lavlock gravity.

Third chapter is devoted to canonical analysis, which is very useful formalism for inspecting properties of a theory. using it we can determine gauge symmetries and construct their generator, as well as number of degrees of freedom.

Following chapter is bundle of necessary informations with the purpose of understanding and formulating holographic duality. in this chapter, we review black hole thermodynamics and conformal field theory. We introduce asymptotic symmetry, which leads us to the first example of holographic duality. In the end, we give Witten-Gubser-Klebanov-Polyakov prescription, which we illustrate on some elementary examples.

Fifth chapter is devoted to black holes in Lavelock gravity. First we review, some, famous solutions. Next, we give construction in detail and analyze properties of spherically symmetric black hole with torsion. After that we construct black ring with and without torsion and inspect its properties

Afterwards, we investigate holographic structure of Lavlock Chern-Simons gravity. First, we analyze in detail gauge fixing in theory with local AdS symmetry. After that, we derive Ward identities. Next, we focus on Lavlock Chern-Simons gravity, we derive Green functions and inspect appearance of anomalies. In the final part of this chapter we give alternative derivation of renormalization theorem.

In the end we sum results, review status and share some thoughts on perspective of theory of gravity.

Key words: Alternative theories of gravity, gravity in dimension other than four, black holes, holographic duality

Scientific field: Physics

Research area: Theoretical high energy physics

UDK broj: 538.9(043.3)

Pri izradi doktorske teze bilo je pomoći, sa mnogih strana, bez koje ovaj rad bi bilo nemoguće privesti kraju. Ovom prilikom bih hteo svima da se zahvalim.

Iskoristio bih priliku da se zahvalim svim članovima grupe za Gravitaciju, kvantna polja i čestice na korisnim komentarima i smernicama.

Zahvaljujem se svojim roditeljima na njihovoj neizmernoj podršci.

Na kraju bih hteo da se zahvalim osobi koja je najviše doprinela izradi ove doktorske disertacije, mentoru dr Branislavu Cvetkoviću. On je bio, u toku doktorata, uvek otvoren za diskusije i spreman da pomogne na svakom koraku. Izbor tematike ove disertacije je, u potpunosti, njegova zasluga, kao i to što je ista uspešno privedena kraju. Bez njegovih uputstava, uvida u prirodu problema i ohrabrivanja put pri izradi doktorske disertacije bio bi neopisivo teži, možda i neizvodljiv.

NASTAVNO-NAUČNOM VEĆU FIZIČKOG FAKULTETA
UNIVERZITETA U BEOGRADU

Pošto smo na VI sednici Nastavno-naučnog veća Univerziteta u Beogradu održanoj 28.03.2018. odredjeni za članove komisije za pregled i ocenu doktorske disertacije Dejana Simiića, diplomiranog fizičara, ,,LAVLOKOVA GRAVITACIJA SA TORZIJOM: EGZAKTNA REŠENJA, KANONSKA I HOLOGRAFSKA STRUKTURA", posle pregleda disertacije podnosimo sledeći

I Z V E Š T A J

1 Biografski podaci

Dejan Simić je rodjen 02.07.1989. u Paraćinu, gde je završio osnovnu školu. Srednju školu, gimnaziju u Paraćinu, završio je 2008. Iste 2008. godine započeo je osnovne studije na Fizičkom fakultetu Univerziteta u Beogradu, smer Teorijska i eksperimentalna fizika, koje je okončao 2012. godine. Master studije na Fizičkom fakultetu završio je 2013. godine odbranom diplomskog master rada pod naslovom ,,*SO(1,2) grupa i nekomutativna geometrija*". Mentor pri izradi master teze je bila prof. dr Maja Burić. Postdiplomske studije na Fizičkom fakultetu, na smeru Kvantna polja, čestice i gravitacija, upisao je 2013. godine.

Školske 2010/2011. bio je stipendista fonda ,,Prof. dr Djordje Živanović", a školske 2011/2012. i 2012/2013. godine stipendista fonda za mlade talente Republike Srbije – Dositeja. U zvanje istraživač saradnik je izabran 2017. godine.

U zimskom semestru školske godine 2012/2013. bio je saradnik u nastavi na Fizičkom fakultetu na predmetu Simetrije u fizici.

Od 07.02.2014. zaposlen je na Institutu za fiziku u Beogradu na projektu osnovnih istraživanja 171031 ,,Fizičke implikacije modifikovanog prostor–vremena", koji je finansiran od strane Ministarstva prosvete, nauke i tehnološkog razvoja vlade Republike Srbije.

Naučna aktivnost Dejana Simića odvija se u oblasti teorijske fizike gravitacije, odnosno preciznije, gradijentnih teorija gravitacije.

2 Opis doktorskog rada

2.1 Tema i ciljevi

Opšta teorija relativnosti (OTR), formulisana pre više od jednog veka, donela je revoluciju u razumevanju gravitacije i strukture prostor–vremena. OTR se pokazala kao veoma uspešna teorija u interpretaciji dosadašnjih eksperimentalnih rezultata ali su njenim zasnivanjem otvorena i neka nerešena pitanja. Jedan od značajnih problema je da su rešenja u OTR generički singularna, što je fizički neprihvatljivo. Ovim je inspirisan pravac istraživanja u smeru alternativnih teorija gravitacije, tj. potraga za teorijom koja reprodukuje eksperimentalne rezultate podjednako dobro ali čija su rešenja fizički prihvatljiva odnosno nesingularna.

Sa razvojem kvantne teorije polja i Standardnog modela koji opisuje elektromagnetnu, slabu i jaku interakciju, logičan korak je bio kvantovanje gravitacije. To je do danas najveći nerešeni problem fizike visokih energija na kome se aktivno radi preko pola veka. Postoje razni pristupi ovom problemu koji mogu grubo da se podele u dve grupe, po tome da li modifikuju gravitaciju ili metod kvantovanja. Najpoznatiji pristup koji modifikuje gravitaciju je teorija struna, u okviru koje se pretpostavlja da je fundamentalni objekat struna, a ne čestica. Poznati pristupi iz ove grupe su, takodje, teorije supergravitacije kao i teorije sa dodatnim (ekstra) dimenzijama. Pristup koji polazi od pretpostavke da je OTR dobra klasična teorija gravitacije, ali da bi trebalo modifikovati način kvantovanja je kvantna gravitacija na petljama (LQG). Cilj doktorske disertacije Dejana Simića je nalaženje i proučavanje osobina egzaktnih rešenja sa torzijom u okviru Lavlokove gravitacije, jedne od alternativnih teorija gravitacije, kao i primena jednog od aspekata teorije struna, holografije, na pomenutu teoriju.

Postoji više teorema koje značajno ograničavaju strukturu crnih rupa u OTR u četiri dimenzije. Tokom vremena, eksplicitnom konstrukcijom rešenja, se uvidelo da ove teoreme ne važe ako razmatramo alternativne teorije gravitacije i/ili broj dimenzija različit od četiri. Medju ovim rešenjima se pojavilo više njih sa različitim topologijama horizonta dogadjaja ili nekim drugim egzotičnim osobinama, dok u OTR-u postoji samo topologija sfere. Relevantnost ovih rešenja je u potencijalnoj detekciji gravitacionih talasa specifične signature i dobijanje informacija o potencijalnoj modifikaciji OTR-a. Rešenja mogu imati zanimljive termodinamičke osobine i relevantna su sa stanovišta ideje primene holografske dualnosti na teorije sa torzijom. Neki od pomenutih rezultata dobijeni su u radovima beogradske grupe[1].

2.2 Sadržaj i rezultati

Doktorska teza ,,Lavlokova gravitacija sa torzijom: egzaktna rešenja, kanonska i holografska struktura” Dejana Simića napisana je na 146 strana, sadrži 7 poglavlja i spisak literature od 108 referenci.

Prvo poglavlje disertacije sadrži opšti fizički uvod. Drugo poglavlje posvećeno je konstrukciji teorije gravitacije sa stanovišta teorije polja, ukratko je izložen problem renormalizabilnosti OTR. Dat je i kratak pregled lokalne Poenkareove teorije i uvedeno je dejstvo za Lavlokovu gravitaciju u formalizmu prvog reda. Izvedene su jednačine kretanja, diskutovane su njihove posledice i egzistencija maksimalno simetričnog rešenja, sa posebnim osvrtom na petodimenzioni slučaj.

U trećem poglavlju izloženi su osnovni elementi kanonske analize sistema sa vezana, kao i Kastelanijeva procedura konstrukcije kanonskog generatora. Poglavlje četiri posvećeno je holografskoj dualnosti. Holografski pogled na gravitaciju ima svoje korene u početku sedamdesetih godina XX veka kada je otkriveno, u okviru OTR, da crne rupe poseduju svojstva slična termodinamičkim sistemima i da poseduju temperaturu, entropiju i ostala termodinamička svojstva. Od svih termodinamičkih osobina crnih rupa najznačajnija je entropija, u OTR-u važi Bekenštajn-Hokingova formula, prema kojoj je entropija proporcionalna površini horizonta dogadjaja, što sugeriše da stepeni slobode nisu rasporedjeni zapreminski, kao što bi se očekivalo, već po površini. Ovo je inspirisalo holografski pogled na gravitaciju, čija gruba formulacija glasi da gravitacija može da se opiše kao teorija u dimenziji manje. Implementacija holografskog principa je postignuta u okviru teorije struna i poznata je pod više imena AdS/CFT, holografija i gauge/gravity dualnost.

U poglavljima pet i šest dati su glavni originalni rezultati ovog rada. Rezultati u poglavlju pet odnose se na egzaktna rešenja petodimenzione Lavlokove gravitacije. Analizirana su statička sferno simetrična rešenja i pokazano je u Rimanovom sektoru, Bulver-Dezerova crna rupa predstavlja jedinstveno rešenje za generički izbor parametara u dejstvu. Za specijalan izbor parametara, različit od Lavlok Čern-Sajmonsovog (LCS) sektora, teorija poseduje statičko rešenje – crnu rupu sa torzijom čija metrika je asimptotski AdS. Izračunati su održani naboji i ispitane termodinamičke osobine dobijenog rešenja. Pronadjeno je još jedno egzaktno rešenje petodimenzione

[1]M. Blagojević, B. Cvetković and M. Vasilić, ”Exotic” black holes with torsion, Phys. Rev. D **88** (2013) 101501 (R); M. Blagojević and B. Cvetković, Black hole entropy in 3D gravity with torsion, Class. and Quantum Gravity **23** , 4781 (2006);M. Blagojević, B. Cvetković, O. Mišković and R. Olea, Holography in 3D AdS gravity with torsion, JHEP **05** (2013) 103.

Lavlokove gravitacije – Banjados, Tajtelbom, Zanelijev (BTZ) crni prsten sa torzijom. Održani naboji izračunati su korišćenjem Nesterove formule i kanonskog metoda. Pokazano je da teorija linearizovana oko ovog rešenja poseduje dva dodatna stepena slobode u odnosu na broj stepeni slobode OTR.

U poglavlju šest analizirana je holografska teorija polja dualna LCS AdS gravitaciji u višim dimenzijama. Nadjene su asimptotske simetrije u AdS sektoru i pokazano je da se one sastoje od lokalnih translacija, lokalnih Lorencovih rotacija, dilatacija i ne-Abelovih lokalnih tranformacija. Izračunate su 1-tačkaste funkcije: struje energije-impulsa i spina u dualnoj konformnoj teoriji polja i zapisani su odgovarajući Vordovi identiteti. Pokazano je da holografska teorija poseduje Vajlovu anomaliju, kao i da je ne-Abelova lokalna simetrija narušena na kvantnom nivou.

Sedmo poglavlje posvećeno je zaključnim razmatranjima.

Rezultati doktorske teze Dejana Simića objavljeni su u tri načna rada u vrhunskim časopisima, dok je četvrti u procesu publikovanja ([1] i [2] su za sajt Fizičkog fakulteta).

2.3 Naučni radovi kandidata vezani za doktorsku disertaciju

[1] B. Cvetković and D. Simić, 5D Lovelock gravity: New exact solutions with torsion, Phys. Rev. D **94**, 084037 (2016), IF=4.568

[2] B. Cvetković, O. Mišković and D. Simić, Holography in Lovelock Chern-Simons AdS gravity, Phys. Rev. D **96**, 044027 (2017), IF=4.568

[3] B. Cvetković and D. Simić, A black hole with torsion in 5D Lovelock gravity, Class. Quantum Grav. **35** (2018) 055005, IF=3.119

Na osnovu prethodne analize doktorskog rada Komisija donosi sledeći

ZAKLJUČAK

Zaključujemo da doktorska teza „Lavlokova gravitacija sa torzijom: egzaktna rešenja, kanonska i holografska struktura" Dejana Simića, diplomiranog fizičara, predstavlja važan i originalan naučni doprinos izučavanju alternativnih teorija gravitacije. Uzimajući u obzir aktuelnost teme doktorata, sadržaj i rezultate teze kao i kvalitet radova koji su iz nje proizašli predlažemo Nastavno-naučnom veću Fizičkog fakulteta Univerziteta u Beogradu da usvoji ovaj izveštaj i odobri javnu odbranu disertacije.

Beograd, 13. 04. 2018.

Prof. Maja Burić
Fizički fakultet, Univerzitet u Beogradu

Prof. Voja Radovanović
Fizički fakultet, Univerzitet u Beogradu

dr Branislav Cvetković, viši naučni saradnik
Institut za fiziku, Univerzitet u Beogradu

Univerzitet u Beogradu
Fizički fakultet

Master rad

Kretanje čestica u polju OTT crne rupe

Student: Marija Tomašević

Mentor: Branislav Cvetković

Septembar, 2017

Sažetak

U ovom radu je razmatrano vakuumsko rešenje BHT gravitacije, nazvano po autorima Bergšof (*Bergshoeff*), Hom (*Hohm*) i Tausend (*Townsend*). Ovo rešenje ima naziv OTT crna rupa, po autorima Oliva (*Oliva*), Tempo (*Tempo*) i Tronkozo (*Troncoso*). Pokazano je da je rešenje koristan model za (3+1)-dimenzionu gravitaciju, kako OTT crna rupa poseduje određene osobine koje se poklapaju sa rešenjem Opšte teorije relativnosti. Posebno se razmatra kretanje čestica u polju statičke i rotirajuće ove crne rupe. Takođe, pokazano je da parametar b, koji razlikuje OTT crnu rupu od BTZ crne rupe, ne predstavlja gravitacioni "čupavi" parametar, kao što se napominje u literaturi.

Abstract

In this paper, we analyze the vacuum solution of the theory of massive gravity in 2+1 dimensions, recently proposed by Bergshoeff, Hohm and Townsend (BHT). The black hole solution, named after its authors Oliva, Tempo and Troncoso (OTT), has been shown to posses certain properties that coincide with the vacuum solution of Einstein's general theory of relativity. We consider the motion of particles in the fields of static and rotating OTT black holes and analyze their potentials. Also, we show that the parameter b is wrongly interpreted as "the gravitational hair parameter" in the previous literature.

Zahvalnica

Htela bih da se zahvalim svom mentoru, dr Branislavu Cvetkoviću, na razumevanju i velikoj pomoći tokom izrade master rada. Zahvaljujem se i kolegama Branislavu Avramovu i Tijani Radenković na korisnim savetima i na pruženoj pomoći pri pisanju rada. Htela bih da se zahvalim i kolegama Gijom Tjamu (Guillaume Thiam), Floriju Sjagliji (Florio Maria Ciaglia) i Radoslavu Simeonovu (Радослав Симеонов), koji su mi pomogli da razrešim svoja pitanja vezana za matematičku strukuru i dublje fizičko značenje. Takođe, zahvaljujem se i svojoj porodici na neizmernoj podršci koju su mi pružali u toku studija. Konačno, zahvaljujem se Stanislavu Miloševiću, bez kojeg ovaj rad ne bi ni postojao.

PONTIFICIA UNIVERSIDAD
CATOLICA
DE VALPARAISO

CERTIFICATE

Dr. René Rojas, Director of the Graduate Course Bachelor in Physics of the Institute of Physics at the Faculty of Science of Pontifical Catholic University of Valparaíso (PUCV), certifies that Miss. **CONSTANZA BELEN CALLENDER OLIVARES,** Chilean ID number **17.978.788-1**, completed her B.Sc. thesis "**Chemistry of three-dimensional black holes in AdS space**" (Original title: "Química de agujeros negros tridimensionales en espacio AdS") in January 2016 under supervision of Prof. Dr. Olivera Mišković (PUCV) and Prof. Dr. Branislav Cvetković (Institute of Physic, Zemun-Belgrade, Serbia), with the **maximal grade 7.0 (seven point zero).**

This Certificate is issued upon request of the interested party to be presented To Whom It May Concern.

VALPARAÍSO, 1st August 2016

Докторске студије ФИЗИКЕ

Ужа научна област: КВАНТНА, МАТЕМАТИЧКА И НАНОФИЗИКА			
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3.	ФИЗДФКН3	Виши курс математичке физике	Татјана Вуковић
4.	ФИЗДФКН4	Квантна информација и заснивање квантне механике	Часлав Брукнер
5.	ФИЗДФКН5	Геометријски методи физике	Раде Живаљевић
6.	ФИЗДФКН6	Симетрија нискодимензионалних система	Иванка Милошевић
7.	ФИЗДФКН7	Нелинеарни динамички системи	Никола Бурић
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9.	ФИЗДФКН9	Квантна механика комплексних система	Таско Грозданов, Иванка Милошевић, Никола Бурић
10.	ФИЗДФКН10	Квантна статистичка теорија светлости	Мирјана Поповић-Божић

Ужа научна област: КВАНТНА ПОЉА, ЧЕСТИЦЕ И ГРАВИТАЦИЈА			
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7.	ФИЗДФПЕ7	Стандардни модел	Ђорђе Шијачки, Марија Димитријевић
8.	ФИЗДФПЕ8	Суперсиметрије	Воја Радовановић
9.	ФИЗДФПЕ9	Теорија струна	Бранислав Саздовић
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2.	ФИЗДФНФ2	Детектори у физици високих енергија	Петар Аџић
3.	ФИЗДФНФ3	Нуклеарна спектроскопија и радијациона физика	Јован Пузовић
4.	ФИЗДФНФ4	Виши курс нуклеарне физике 2	Јован Пузовић
5.	ФИЗДФНФ5	Виши курс физике елементарних честица 2	Петар Аџић, Јован Милошевић
6.	ФИЗДФНФ6	Феноменологија у физици честица	Љиљана Симић
7.	ФИЗДФНФ7	Анализа података у физици високих енергија	Љиљана Симић
8.	ФИЗДФПЕ7	Стандардни модел	Ђорђе Шијачки, Марија Димитријевић

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3.	ФИЗДФАМ3	Теорија расејања	Таско Грозданов
4.	ФИЗДФАМ4	Интеракције електрона са атомским системима	Горан Попарић
5.	ФИЗДФАМ5	Интеракције са површинама	Наташа Недељковић
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8.	ФИЗДФЈП8	Интеракција плазме и ласера са површинама	Јагош Пурић, Срђан Буквић, Иван Дојчиновић
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11.	ФИЗДФЈП11	Кинетичка теорија јонизованих гасова и плазме	Ђорђе Спасојевић

Ужа научна област: ФИЗИКА КОНДЕНЗОВАНЕ МАТЕРИЈЕ И СТАТИСТИЧКА ФИЗИКА			
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9.	ФИЗДФКМ9	Физика полимерних система	Јаблан Дојчиловић, Владимир Ђоковић
10.	ФИЗДФКМ10	Физика суперпроводности	Зоран Радовић
11.	ФИЗДФКМ11	Физика фазних прелаза	Милан Кнежевић
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1.	ФИЗДФПФ1	Изабрана поглавља из медицинске физике	Мирослав Драмићанин
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3.	ФИЗДФПФ3	Изабрана поглавља примењене физике	Иван Белча, Стеван Стојадиновић
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6.	ФИЗДФПФ6	Методе карактеризације наноматеријала	Мирослав Драмићанин

7.	ФИЗДФПФ7	Пројектовање оптичких система	Иван Белча, Бећко Касалица
8.	ФИЗДФПФ8	Пирометарски системи и безконтактне методе мерења температуре	Љубиша Зековић, Иван Белча
9.	ФИЗДФПФ9	Експерименталне методе биофизике	Милош Вићић
10.	ФИЗДФПФ10	Примена плазме у биологији и медицини	Невена Пуач, Зоран Петровић

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2.	ФИЗДФДФ2	Рад са талентованим ученицима	Мићо Митровић
3.	ФИЗДФДФ3	Методологија педагошких истраживања у физици	Андријана Жекић, Јаблан Дојчиловић
4.	ФИЗДФДФ4	Истраживање учења и наставе физике	Јосип Слишко
5.	ФИЗДФДФ5	Методе интерактивне наставе и учења физике	Мирјана Поповић-Божић, Братислав Обрадовић

РАЧУНАРСКИ ПРЕДМЕТИ ЗА ВИШЕ НАУЧНИХ ОБЛАСТИ			
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3.	ФИЗДФВО3	Методи нумеричке симулације у физици јонизованог гаса и плазме	Марија Радмиловић-Рађеновић, Најдан Алексић, Милован Шуваков
4.	ФИЗДФВО4	Нумеричке методе и симулације у квантној оптици	Душан Арсеновић

Координатори смерова:

1. Квантна, математичка и нанофизика: М. Дамњановић, М. Поповић Божић
2. Квантна поља, честице и гравитација: Б. Саздовић, В. Радовановић
3. Физика високих енергија и нуклеарна физика: П. Ацић, Љ. Симић
4. Физика атома и молекула: Т. Грозданов, Н. Недељковић
5. Квантна оптика и ласери: М. Кураица, Љ.Хаџиевски
6. Физика јонизованог гаса и плазме: С. Буквић,З.Петровић
7. Физика кондензоване материје и статистичка физика: Н. Бибић, З. Радовић, М. Кнежевић
8. Примењена физика: М. Драмићанин, И. Белча
9. Настава физике: Ј. Дојчиловић

Rukovodjenje projektima, potprojektima i projekntim zadacima

Potvrda o rukovodjenju potprojektom

Ovim potvrdjujem da dr Branislav Cvetković (za koga se pokreće izbor u zvanje naučni savetnik) u okviru Grupe za gravitaciju, čestice i polja Instituta za fiziku Univerziteta u Beogradu, odnosno u okviru projekta ON171031 "*Fizičke implikacije modifikovanog prostor–vremena*" rukovodi potprojektom – temom "*Gradijentne teorije gravitacije*". Na pomenutom potprojektu su angažovani istraživači: dr Branislav Cvetković, dr Milutin Blagojević, dr Milovan Vasilić i Dejan Simić.

Beograd, 17.04.2018.

Rukovodilac projekta ON171031

Maja Burić

prof. dr Maja Burić

Potvrda o rukovodjenju potprojektom

Ovim potvrdjujem da je dr Branislav Cvetković (za koga se pokreće izbor u zvanje naučni savetnik) u okviru projekta 141036 ,,*Alternativne teorije gravitacije*" u periodu od 2008–2010 rukovodio potprojektom – celinom ,,*Torzija i nemetričnost u gravitaciji i teoriji struna/brana*". Na pomenutom potprojektu bili su angažovani istraživači: dr Branislav Cvetković, dr Milutin Blagojević, dr Branislav Sazdović, dr Milovan Vasilić i dr Bojan Nikolić.

Beograd, 18.04.2018.

Rukovodilac projekta 141036

prof. dr Milutin Blagoejvić

Subject **[e-COST] New position**
From <noreply@cost.eu>
To <cbranislav@ipb.ac.rs>
Date 2015-02-05 09:56

Dear Dr Branislav CVETKOVIC,

This email is sent by the e-COST system to you as a confirmation that you have been nominated as MC Substitute [MP1405 RS].

To complete your nomination, please follow the link below:
https://e-services.cost.eu/?module=user&action=activationCode&userParam[code]=NOMINATION_b85a85369a3e232c54e92e74d430c0dc

Should you have any questions in relation to this nomination and registration of your e-COST profile please send your questions to e-cost@cost.eu.

The COST Association
Avenue Louise 149
1050 Brussels
Belgium
Tel.: +32 2 533 38 00
E-mail: e-cost@cost.eu
Web: www.cost.eu

QSpace

Quantum Structure of Spacetime

European Cooperation in Science and Technology

Management Committee

Overview of management committee members

Name	Country	Title	Affiliation	MC member role	Workgroup(s)	
Caslav Brukner	Austria	Prof	Faculty of Physics, University of Vienna	MC Substitute	WG1, WG3	
Johanna Knapp	Austria	Dr.				
Harold Steinacker	Austria	Dr.	University of Vienna	MC Member	WG1, WG2, WG3	WG3 leader
Pierre Bieliavsky	Belgium	Prof				
Vladimir Dobrev	Bulgaria	Prof	Institute for Nuclear Research and Nuclear Energy,Bulgarian Academy of Sciences	MC Substitute	WG2, WG4	
Emil Nissimov	Bulgaria	Prof	Institute for Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences, Sofia, Bulgaria	MC Member	WG3	
Svetlana Pacheva	Bulgaria	Prof	Institute for Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences, Sofia, Bulgaria	MC Substitute	WG3, WG5	
Larisa Jonke	Croatia	Dr.	Rudjer Boskovic Institute	MC Member	WG2, WG3, WG4	WG4 vice leader
Stjepan Meljanac	Croatia	Dr.				
Kornelija Passek-Kumericki	Croatia	Dr.				
Andjelo Samsarov	Croatia	Dr.				
Josip Trampetic	Croatia	Dr.				
Klaus Bering	Czech Republic	Dr.	Masaryk U	MC Substitute	WG2	
Alfredo Iorio	Czech Republic	Prof	Charles University	MC Substitute		
Branislav Jurco	Czech Republic	Dr.	Charles University	MC Member	WG2	WG2 leader
Rikard von Unge	Czech Republic	Prof	Masaryk university	MC Substitute		
Jan Ambjørn	Denmark	Prof				
Bergfinnur Durhuus	Denmark	Prof				
Ryszard Nest	Denmark	Prof				

Niels Obers	Denmark	Prof	Niels Bohr Institute, University of Copenhagen	MC Member	WG3	
Vincent Rivasseau	France	Prof	Université Paris-Sud	MC Member	WG3	
Carlo Rovelli	France	Prof				
Dorothea Bahns	Germany	Prof		MC Member		
Olaf Lechtenfeld	Germany	Prof	Leibniz University Hannover, Germany	MC Member	WG2, WG3, WG4	WG4 leader
Catherine Meusburger	Germany	Prof	Friedrich-Alexander-Universitaet Erlangen-Nuernberg	MC Substitute	WG3, WG4, WG5	Web moderator WG3
Peter Schupp	Germany	Prof	Jacobs University Bremen	MC Substitute	WG2, WG3, WG4, WG5	WG5 vice leader
Konstantinos Anagnostopoulos	Greece	Prof	National Technical University of Athens	MC Substitute		
George Savvidy	Greece	Prof	National Center for Scientific Research "Demokritos" , Institute of Nuclear and Particle Physics	MC Member	WG1	
George Zoupanos	Greece	Prof		MC Member	WG1	WG1 leader
Gabor Etesi	Hungary	Dr.	Budapest University of Technology and Economics	MC Member	WG2, WG3	
Valentina Giangreco Marotta Puletti	Iceland	Dr.	University of Iceland	MC Substitute	WG3, WG4, WG5	
Thordur Jonsson	Iceland	Prof				
Larus Thorlacius	Iceland	Prof	University of Iceland	MC Member	WG3, WG4, WG5	
Tommy Curran	Ireland	Dr.				
Brian Dolan	Ireland	Dr.	Maynooth University	MC Member	WG1, WG2, WG3	
Denjoe O'Connor	Ireland	Prof	School Of Theoretical Physics, Dublin Institute for Advanced Studies	MC Member	WG1, WG2, WG3	
Paolo Aschieri	Italy	Dr.	Università del Piemonte Orientale	MC member Action Vice-Chair		
Marco Genovese	Italy	Dr.	inrim	MC Member	WG1	
Fedele Lizzi	Italy	Prof	Università di Napoli Federico II	MC Substitute	WG1, WG2, WG5	WG1 vice leader, Web moderator WG1
Patrizia Vitale	Italy	Prof	Universita` di Napoli Federico II & INFN	MC Substitute	WG2, WG3, WG4	Web moderator WG4
Martin Schlichenmaier	Luxembourg	Prof	University of Luxembourg	MC Member	WG2, WG3, WG4	WG2 vice leader
Martin Lukarevski	Macedonia, the former Yugoslav Republic of	Dr.	University "Goce Delcev" - Stip	MC Member	WG1, WG2, WG4	
André Xuereb	Malta	Dr.	University of Malta	MC Member		
Simon Brain	Netherlands	Dr.	Radboud University Nijmegen	MC Substitute	WG1, WG2	
Francesca Vidotto	Netherlands	Dr.	Radboud University Nijmegen	MC Member	WG3, WG5	
Walter van Suijlekom	Netherlands	Dr.	Radboud University Nijmegen	MC Member	WG1, WG2, WG5	Website administrator, Web moderator WG5
Michal Eckstein	Poland	Dr.	Jagiellonian University, Krakow	MC Substitute	WG1, WG2, WG3, WG4	
Wojciech Kaminski	Poland	Dr.			WG3	
Jerzy Lewandowski	Poland	Prof				
Andrzej Sitarz	Poland	Prof				
Orfeu Bertolami	Portugal	Prof	Departamento de Física e Astronomia, Faculdade de Ciências, Universidade do Porto	MC Member	WG1	

José Mourão	Portugal	Prof	Departamento de Matemática, Instituto Superior Técnico, Lisbon University	MC Member	WG2, WG4	
Roger Picken	Portugal	Prof	Instituto Superior Técnico, Universidade de Lisboa	MC Substitute	WG2	
Maja Buric	Serbia	Prof	University of Belgrade, Faculty of Physics	MC Member	WG1, WG3, WG5	
Branislav Cvetkovic	Serbia	Dr.				
Peter Presnajder	Slovakia	Prof	Comenius University in Bratislava, Faculty of Mathematics, Physics and Informatics	MC member	WG2, WG3	
Karin Cvetko-Vah	Slovenia	Prof	University of Ljubljana	MC Member	WG2, WG5	
Martin O'Loughlin	Slovenia	Prof	The University of Nova Gorica	MC Member	WG3	
Manuel Asorey	Spain	Prof	Universidad de Zaragoza	MC Member	WG1, WG2	
José Mariano Gracia-Bondía	Spain	Dr.	Universidad de Zaragoza	MC Substitute	WG1, WG2	
Carmelo Perez Martin	Spain	Prof	Universidad Complutense de Madrid	MC member	WG1, WG2, WG3	
Joakim Arnlind	Sweden	Prof	Linköping University	MC Member	WG2, WG3	
Maxim Zabzine	Sweden	Prof				
Anton Alekseev	Switzerland	Prof				
Alberto Cattaneo	Switzerland	Prof	University of Zurich	MC Member		
Cemsinan Deliduman	Turkey	Prof	Mimar Sinan Fine Arts University	MC Member	WG2, WG3	
John Barrett	United Kingdom	Prof	Universtity of Nottingham	MC Member	WG1, WG2, WG3, WG4, WG5	WG3 vice leader
Mairi Sakellariadou	United Kingdom	Prof	King's College London, Uninersity of London	MC Member	WG1, WG3, WG5	WG5 leader
Richard Szabo	United Kingdom	Prof	Heriot-Watt University	Action Chair		
Apimook Watcharangkool	United Kingdom	MSc.	king's college london		WG1, WG2	

Subject **Пријава пројекта**
From <projekti.minis@mpn.gov.rs>
To <cbranislav@ipb.ac.rs>
Date 2016-04-13 11:48

Поштовани,

Обавештавамо вас да сте задовољили потребне квантитативне услове за руководиоца пројекта. У систему за пријаву пројеката по вашем захтеву је отворен пројекат са евиденционим бројем OI1611010

Унос података о пројекту можете вршити преко веб странице http://minis.mpn.gov.rs/projekti

За приступ страници вашег пројекта користите корисничко име cbranislav@ipb.ac.rs

и лозинку kjrUbtVKXbNR2JLp7fxR

Након првог приступа систему за пријаву пројеката, можете променити вашу лозинку.

С поштовањем,
Андријана Ивановић

Subject **Пријава пројекта**
From <projekti.minis@mpn.gov.rs>
To <branislav.cvetkovic@ipb.ac.rs>
Date 2016-04-18 12:18

Поштовани,

Управо сте ангажовани на пројекту Квантно просторвреме.

Број истраживач месеци је 12.

Руководилац пројекта је Бранислав Цветковић.

С поштовањем,

Aktivnosti u naučnim i naučno– stručnim društvima

webmail

Mail Address Book Settings Logout

Filter: All

Folders
Inbox
Drafts
Sent (3)
Junk (856)
Trash
COST
DFS
Divcibare 2013
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Izbor u zvanje
Jagodina
Karl Sch...rzschild
Komercij...na banka
Konferenc...a Milutin
Kurs Teor...itacije 2
Master Marija
Matemat...imnazija
MG2015
Mini [he...workshop
OTR2015
Postdoc
Projekat (2)
Radovi
Radovi-k...ndencija
Reports
s
saved-messages
Seminari
Sent
Softver
spam-mail
Svadba
Telenor
Trash
Upravi odbor
virus-mail

Subject	From	Date	Size
CQG-104773: This manu...	Classical and Quantum Gravity	2018-04-05 00:43	5 KB
Decision on an article yo...	Classical and Quantum Gravity	2018-03-21 12:08	3 KB
Thank you for your repor...	prd@aps.org	2018-03-20 00:16	3 KB
Referee list for Italian pr...	fedele.lizzi@na.infn.it	2018-03-13 14:39	3 KB
Reminder CVETKOVIC D...	prd@aps.org	2018-03-13 14:13	2 KB
Reminder CVETKOVIC D...	prd@aps.org	2018-03-07 12:07	7 KB
Your review for Classical...	Team Publons	2018-03-06 11:21	17 KB
Thank you for reviewing...	Classical and Quantum Gravity	2018-03-06 11:20	4 KB
Your report for Class. Qu...	Classical and Quantum Gravity	2018-03-04 08:09	5 KB
Referee report overdue f...	Classical and Quantum Gravity	2018-03-01 10:11	5 KB
Your report for Class. Qu...	Classical and Quantum Gravity	2018-02-25 08:07	5 KB
Review_request CVETKO...	prd@aps.org	2018-02-14 18:01	5 KB
CQG-104531: This manu...	Classical and Quantum Gravity	2018-02-13 18:39	5 KB
A request to referee for...	Classical and Quantum Gravity	2018-02-13 11:36	7 KB
Thank you for your repor...	prd@aps.org	2018-01-29 13:16	3 KB
Reminder CVETKOVIC D...	prd@aps.org	2018-01-17 13:29	2 KB
Reminder CVETKOVIC D...	prd@aps.org	2018-01-11 11:35	7 KB
Decision on an article yo...	Classical and Quantum Gravity	2018-01-08 11:12	3 KB
Your review for Classical...	Team Publons	2017-12-26 00:50	17 KB
Thank you for reviewing...	Classical and Quantum Gravity	2017-12-25 13:55	3 KB
Your report for Class. Qu...	Classical and Quantum Gravity	2017-12-24 08:16	4 KB
Review_request CVETKO...	prd@aps.org	2017-12-21 14:46	6 KB
Referee report overdue f...	Classical and Quantum Gravity	2017-12-20 10:59	4 KB
Your report for Class. Qu...	Classical and Quantum Gravity	2017-12-17 08:12	4 KB
To_referee CVETKOVIC...	prd@aps.org	2017-12-05 17:29	2 KB
CQG-104423: This manu...	Classical and Quantum Gravity	2017-12-05 12:46	5 KB
A request to referee for...	Classical and Quantum Gravity	2017-12-05 11:14	5 KB
Thank you for your repor...	prd@aps.org	2017-12-04 22:33	3 KB
Reminder CVETKOVIC D...	prd@aps.org	2017-11-28 14:21	2 KB
Reminder CVETKOVIC D...	prd@aps.org	2017-11-17 11:44	7 KB
Review_request CVETKO...	prd@aps.org	2017-10-27 18:35	5 KB
Your_manuscript DG122...	prd@aps.org	2017-09-26 18:16	3 KB
Decision on an article yo...	Classical and Quantum Gravity	2017-09-15 15:08	3 KB
Your review for Classical...	Publons	2017-09-15 12:13	17 KB
Thank you for reviewing...	Classical and Quantum Gravity	2017-09-15 12:12	3 KB
Decision on an article yo...	Classical and Quantum Gravity	2017-09-08 16:11	3 KB
Request to review revise...	Classical and Quantum Gravity	2017-09-08 12:26	7 KB
Decision on an article yo...	Classical and Quantum Gravity	2017-09-06 13:57	3 KB
Thank you for reviewing...	Classical and Quantum Gravity	2017-09-06 13:47	3 KB
Your review for Classical...	Publons	2017-09-06 13:45	17 KB
Referee report pending f...	Classical and Quantum Gravity	2017-09-04 10:23	4 KB
Thank you for your repor...	prd@aps.org	2017-08-28 13:13	3 KB
Your report for Class. Qu...	Classical and Quantum Gravity	2017-08-28 08:07	4 KB
Reminder CVETKOVIC D...	prd@aps.org	2017-08-22 13:34	2 KB
Your report for Class. Qu...	Classical and Quantum Gravity	2017-08-19 08:09	4 KB
Have you reviewed a ma...	Daniel Johnston	2017-08-17 08:51	28 KB
Reminder CVETKOVIC D...	prd@aps.org	2017-08-14 12:47	7 KB
CQG-103927: This manu...	Classical and Quantum Gravity	2017-08-08 00:22	5 KB
A request to referee for...	Classical and Quantum Gravity	2017-08-07 16:00	6 KB

Select: Threads: Show preview pane: Messages 1 to 50 of 437

Subject **Certificate of evaluation**
From Fondecyt
To cbranislav@ipb.ac.rs
Reply-To eval_lin@conicyt.cl
Date 2010-11-25 16:09
Priority Normal

Fondecyt
UN PROGRAMA CONICYT

Dear Prof. CVETKOVIC,

This is to confirm that our online evaluation system has successfully received your referee report. It has been added to our evaluation database.

At this time, I would like to express my sincere appreciation for the time and effort you have given to our agency's 2010 review process.

Sincerely,

Maria Elena Boisier
Executive Director
FONDECYT Program

Одељење Друштва физичара Србије
за научна истраживања и високо образовање

Одсеци Одељења НИВО ДФС (80+1):
Братислав Обрадовић, председник, obrat@ff.bg.ac.rs

1. Одсек за квантну и математичку физику (7)
Милан Дамњановић, ФФ, председник, yqoq@afrodita.rcub.bg.ac.rs
Татјана Вуковић, ФФ, tanja37@afrodita.rcub.bg.ac.rs
Мирољуб Дугић, ПМФ Крагујевац, dugic@kg.ac.rs
Ненад Милојевић, ПМФ Ниш, nenad81@pmf.ni.ac.rs
Милан Пантић, ПМФ Нови Сад, mpantic@df.uns.ac.rs
Бранислав Цветковић, ИФ, branislav.cvetkovic@ipb.ac.rs
Далибор Чевизовић, Винча, cevizd@vinca.rs

2. Одсек за физику језгра, елементарних честица и основних интеракција (8)
Петар Аџић, ФФ, председник, adzic@ff.bg.ac.rs
Иванка Божовић-Јелисавчић, Винча, ibozovic@vinca.rs
Драгољуб Димитријевић, ПМФ Ниш, ddrag@pmf.ni.ac.rs
Димитрије Малетић, ИФ, dimitrije.maletic@ipb.ac.rs
Јована Николов, ПМФ Нови Сад, jovana.nikolov@df.uns.ac.rs
Воја Радовановић ФФ, rvoja@ff.bg.ac.rs
Светислав Савовић, ПМФ Крагујевац, savovic@kg.ac.rs
Ковиљка Станковић, ЕТФ, kstankovic@etf.bg.ac.rs

3. Одсек за астрономију и астрофизику (9)
Лука Поповић, АО, председник, lpopovic@aob.bg.ac.rs
Весна Борка Јовановић, Винча, vborka@vinca.rs
Драган Гајић, ПМФ Ниш, dgaja@junis.ni.ac.rs
Мирослав Мићић, АО, micic@aob.rs
Тијана Продановић, ПМФ Нови Сад, prodanvc@df.uns.ac.rs
Владимир Срећковић, ИФ, vladimir.sreckovic@ipb.ac.rs
Саша Симић, ПМФ Крагујевац, ssimic@kg.ac.rs
Зорица Цветковић, АО, zcvetkovic@aob.bg.ac.rs
Кристина Чајко, ПМФ Нови Сад,

4. Одсек за физику кондензоване материје и статистичку физику (9)
Антун Балаж, ИФ, председник, antun@ipb.ac.rs , antun.balaz@scl.rs
Наташа Бибић, Винча, natasabi@vinca.rs
Ивица Брадарић, Винча, bradaric@vinca.rs
Владимир Миљковић, ФФ, miljko@ff.bg.ac.rs
Милица Павков Хрвојевић, ПМФ Нови Сад, milica@df.uns.ac.rs
Јована Петровић, ЕТФ, jovana@etf.bg.ac.rs
Ђорђе Спасојевић, ФФ, djordjes@ff.bg.ac.rs
Федор Скубан, ПМФ Нови Сад, skubi@uns.ac.rs
Драган Тодоровић, ПМФ Крагујевац, tosa@kg.ac.rs

5. Одсек за атомску и молекулску физику (7)

Иван Манчев, ПМФ Ниш, председник, mancev@pmf.ni.ac.rs
Душко Борка, Винча, dusborka@vinca.rs
Сава Галијаш, ФФ, galijas@ff.bg.ac.rs
Наташа Недељковић, ФФ, hekata@ff.bg.ac.rs
Владимир Ристић, ПМФ Крагујевац, ristic@kg.ac.rs
Мира Терзић, ПМФ Нови Сад, mira.terzic@df.uns.ac.rs
Сања Тошић, ИФ, sanja.tosic@ipb.ac.rs , seka@ipb.ac.rs

6. Оптика и фотоника (5)

Љупчо Хаџијевски, Винча, председник, ljupcoh@vinca.rs
Бранко Дрљача, ПМФ К. Митровица, brdrljaca@gmail.com
Петар Матавуљ, ЕТФ, matavulj@etf.bg.ac.rs
Јована Петровић, Винча, jovanap@vinca.rs
Михаило Рабасовић, ИФ, mihailo.rabasovic@ipb.ac.rs

7. Одсек за физику плазме и јонизованих гасова (8)

Драгана Марић, ИФ, председник, draganam@ipb.ac.rs
Саша Гоцић, ПМФ Ниш, секретар, gsasa@pmf.ni.ac.rs
Срђан Буквић, ФФ, ebukvic@ff.bg.ac.rs
Теодора Гајо, ПМФ Нови Сад, teodora.gajo@df.uns.ac.rs
Емилија Живановић, Електронски факултет, Emilija.Zivanovic@elfak.ni.ac.rs
Милан Ковачевић, ПМФ Крагујевац, kovac@kg.ac.rs
Братислав Обрадовић, ФФ, obrat@ff.bg.ac.rs
Јован Цветић, ЕТФ, cvetic_j@etf.bg.ac.rs

8. Одсек за примењену и рачунарску физику (10)

Невена Пуач, ИФ, председник, nevena@ipb.ac.rs
Владимир Арсоски, ЕТФ, arsosk@etf.bg.ac.rs
Слободанка Галовић, Винча, bobagal@vinca.rs
Михајло Мудринић, Винча, m.mudrinic@vinca.rs
Никола Нешић, Електронски факултат, Nikola.Nesic@elfak.ni.ac.rs
Зоран Николић, ФФ, nizoran@ff.bg.ac.rs
Зоран Павловић, ПМФ Ниш, zokip@pmf.ni.ac.rs
Миодраг Радовић, ПМФ Ниш, mkradovic@junis.ni.ac.rs
Ненад Стевановић, ПМФ Крагујевац, nstevanovic@kg.ac.rs
Стеван Стојадиновић, ФФ, sstevan@ff.bg.ac.rs

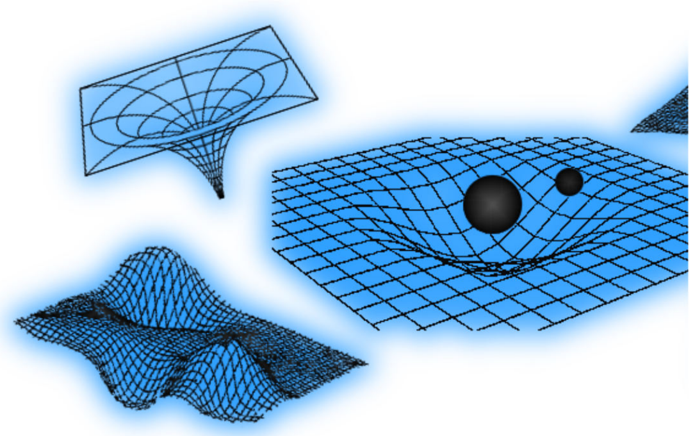
9. Одсек за енергију и заштиту животне средине (9)

Милорад Кураица, ФФ, председник, kuki@ff.bg.ac.rs
Милош Вујисић, ЕТФ, vujsa@etf.bg.ac.rs
Драгана Крстић, ПМФ Крагујевац, dragana@kg.ac.rs
Зоран Мијић, ИФ, zoran.mijic@ipb.ac.rs
Момир Милосављевић, Винча, momirm@vinca.rs
Наташа Тодоровић, ПМФ Нови Сад, natasa.todorovic@df.uns.ac.rs.
Томислав Павловић, ПМФ Ниш, pavlovic@pmf.ni.ac.rs
Горан Попарић, ФФ, goran_poparic@ff.bg.ac.rs
Горан Ристић, Електронски факултет, goran.ristic@elfak.ni.ac.rs

Драгана Штрбац, ФТН Нови Сад,

10. Одсек за наставу, историју и филозофију физике (8)

Љубиша Нешић, ПМФ Ниш, секретар, nesiclj@pmf.ni.ac.rs
Мирјана Поповић-Божић, ИФ, mirjana.bozic@ipb.ac.rs
Имре Гут, ПМФ Нови Сад, imre.gut@df.uns.ac.rs
Стеван Јокић, Винча, sjokic@vin.bg.ac.rs
Илија Марић, ФФ, ilijamaric1@gmail.com
Мићо Митровић, ФФ, mico@ff.bg.ac.rs
Соња Скубан, ПМФ Нови Сад, sogi@uns.ac.rs
Виолета Петровић, ПМФ Крагујевац, vpetrovic@kg.ac.rs



GPF GHSNG2018 Workshop

2018 Workshop on Gravity, Holography, Strings and Noncommutative Geometry

1. February 2018, Belgrade, Serbia

Organization

The Workshop is organized by Group for Gravitation, Particles and Fields (Institute of Physics, University of Belgrade), within the framework of the **national project "Physical implications of modified spacetime"**, number ON 171031, of the Ministry of Education, Science and Technological Development, Serbia.

Scientific committee

Branislav Cvetkovic and Marko Vojinovic

Registration

It is important for all interested participants to register as soon as possible, so that we can reserve an appropriate number of places for lunch in the restaurant. Registration is now closed.

Programme

Lectures were held on Thursday, 1. February 2018, at the Institute of Physics, seminar room 360 (ex-room 300), third floor.

09:50 -- 10:00 --- Opening	**Introduction and opening of the workshop**	
10:00 -- 10:45 --- Lecture 10:45 -- 10:55 --- Discussion	**Speaker:** Rodrigo Olea (lecture slides)	**Title:** Brussels sprouts, black hole mass and pre-holography **Abstract:** We present the first evidence on the fact that topological invariants should renormalize anti-de Sitter gravity with quadratic-curvature corrections. This argument is based on the computation of energy for Einstein black holes in the theory, which appears as an alternative to linearized methods (e.g., Deser-Tekin formula).

10:55 -- 11:40 --- Lecture 11:40 -- 11:50 --- Discussion	**Speaker:** Olivera Miskovic (lecture slides)	**Title:** Thermodynamic instabilities of extremal black holes **Abstract:** We study static, charged extremal black holes in 4D gravity non-linearly coupled to a scalar field. We show that the system can exhibit a phase transition due to electric charge variations only in presence of a cosmological constant and if the scalar is massive. A near-critical analysis reveals that, on one side of the critical point, the hairy black hole has larger entropy than the non-hairy one, thus giving rise to a zero temperature phase transition. Our results are analytical and based on the second law of thermodynamics.
11:50 -- 12:10 --- Break	**Coffee break**	
12:10 -- 12:55 --- Lecture 12:55 -- 13:05 --- Discussion	**Speaker:** Maria Pilar Garcia del Moral (lecture slides)	**Title:** On global aspects of duality invariant theories: M2-brane versus double field theory **Abstract:** In this talk I will discuss the global description of a supermembrane compactified on a $T^2\times M_9$, where T^2 is the 2-dimensional torus and M_9 is a 9-dimensional noncompact spacetime. I will discuss the T-duality transformation of this model and compare global aspects of this construction with that of the double field theory.
13:05 -- 13:50 --- Lecture 13:50 -- 14:00 --- Discussion	**Speaker:** Mihailo Cubrovic (lecture slides)	**Title:** Three tales on boundary action in AdS/CFT: Berry phases, gauge fields and non-canonical Hamiltonians **Abstract:** We argue that AdS/CFT dictionary can be extended by adding new subleading terms to the boundary (surface) part of the AdS action, under the constraint that the bulk equations of motion at semiclassical level remain unchanged. This corresponds to modifying the state space and/or the Poisson structure in dual field theory without additional sources/operator insertions. We give three applications of the general idea. The Berry phase is obtained from a subleading boundary term for the bulk fermion which transforms as a spin on the sphere, hence it is crucial to start from global AdS and subsequently take the planar limit. A phenomenological (bottom-up) description of dynamical gauge fields is encoded in a singleton excitation on the boundary, decoupled from the bulk. The Poisson structure, defined by the time evolution operator in CFT, is modified by sourcing multi-particle states in the bulk from appropriate boundary sources. This opens a way toward constructing the gravity dual of non-canonical Hamiltonians as encountered, e.g., in fluid advection.
14:00 -- 16:00 --- Lunch	**Official workshop lunch at the IPB restaurant**	
16:00 -- 16:20 --- Lecture 16:20 -- 16:25 --- Discussion	**Speaker:** Dejan Simic (lecture slides)	**Title:** Near horizon of the OTT black hole, asymptotic symmetries and soft hair **Abstract:** We study near horizon geometry of extremal (non-)rotating Oliva-Tempo-Troncoso black hole. First we derive the corresponding geometries. Next, we analyze asymptotic structure and determine asymptotic symmetry, which consists of time reparametrization, chiral Virasoro and u(1) Kac-Moody algebra. In the end, interpretation in term of soft hair on the black hole is given.
16:25 -- 16:45 --- Lecture 16:45 -- 16:50 --- Discussion	**Speaker:** Biljana Nikolic (lecture slides)	**Title:** Some geometrical aspects of NC SO(2,3)* gravity **Abstract:** We construct gravity action on the Moyal-Weyl spacetime as a noncommutative SO(2,3)* gauge theory and expand it, using the Seiberg-Witten map, up to the second order in the deformation parameter. After the braking of symmetry of obtained action down to SO(1,3) gauge symmetry, we analyze the low energy sector of the model. We calculate the equations of motion, and discuss the noncommutative corrections as the source of the curvature and torsion. We find one solution: the NC correction to Minkowski spacetime. Using this solution, we explain breaking of the diffeomorphism symmetry as a consequence of working in a particular coordinate system given by the Fermi normal coordinates.
16:50 -- 17:10 --- Break	**Coffee break**	
17:10 -- 17:30 --- Lecture 17:30 -- 17:35 ---	**Speaker:** Dragoljub Gocanin	**Title:** Birefringence property of the Moyal-Weyl noncommutative spacetime in SO(2,3)_* model **Abstract:**

Discussion	(lecture slides)	We demonstrate that flat noncommutative (NC) Moyal-Weyl spacetime acts as a birefringent medium for electrons propagating in it, and we present an action that predicts this "optical" effect. The action is obtained by NC Moyal-Weyl *-product deformation of a certain classical action invariant under local SO(2,3) transformations. After perturbative expansion via Seiberg-Witten map in powers of the deformation parameter θ and a suitable symmetry breaking down to the local Lorentz SO(1,3) symmetry, we get NC deformation of the Dirac action in curved spacetime with various new couplings. One of its significant features is the nonvanishing linear θ-correction which pertains even in flat spacetime. We analyse NC deformation of the Dirac equation and dispersion relation for electrons. The theory predicts Zeeman-like splitting of electron's undeformed (commutative) energy levels due to noncommutativity of the background spacetime. This splitting is helicity-dependent --- electrons with different helicity are affected differently by NC background. NC correction to the electron's energy levels is linear in θ, which brings us closer to the potential observation.
17:35 -- 17:55 --- Lecture 17:55 -- 18:00 --- Discussion	**Speaker:** Dragan Prekrat (lecture slides)	**Title:** Phase transitions on the truncated Heisenberg space **Abstract:** We discuss the phase structure of matrix models on non-commutative spaces. We examine the connection between the geometry of the truncated Heisenberg space, the renormalizability and the striped phase and present the first numerical evidence of the modification of the phase diagram due to the coupling between the matrix field and the curvature.
18:00 -- 18:30 --- Closing	**Final discussion and closing**	

List of participants

- Milutin Blagojevic (Group for Gravitation, Particles and Fields, Institute of Physics Belgrade, Serbia)
- Bojana Brkic (Faculty of Physics Belgrade, Serbia)
- Mihailo Cubrovic (Scientific Computing Laboratory, Institute of Physics Belgrade, Serbia)
- Branislav Cvetkovic (Group for Gravitation, Particles and Fields, Institute of Physics Belgrade, Serbia)
- Ljubica Davidovic (Group for Gravitation, Particles and Fields, Institute of Physics Belgrade, Serbia)
- Aleksandra Dimic (Group for Quantum and Mathematical Physics, Faculty of Physics Belgrade, Serbia)
- Ivan Dimitrijevic (Faculty of Mathematics Belgrade, Serbia)
- Marija Dimitrijevic Ciric (Group for Gravitation, Particles and Fields, Faculty of Physics Belgrade, Serbia)
- Maria Pilar Garcia del Moral (Departamento de Fisica, Universidad de Antofagasta, Chile)
- Dragoljub Gocanin (Group for Gravitation, Particles and Fields, Faculty of Physics Belgrade, Serbia)
- Ilija Ivanisevic (Group for Gravitation, Particles and Fields, Institute of Physics Belgrade, Serbia)
- Filip Jurukovic (Faculty of Physics Belgrade, Serbia)
- Dusko Latas (Group for Gravitation, Particles and Fields, Faculty of Physics Belgrade, Serbia)
- Olivera Miskovic (Instituto de Fisica, Pontificia Universidad Catolica de Valparaiso, Chile)
- Luka Nenadovic (Group for Gravitation, Particles and Fields, College for Medicine, Business and Technology, Sabac, Serbia)
- Biljana Nikolic (Group for Gravitation, Particles and Fields, Faculty of Physics Belgrade, Serbia)
- Bojan Nikolic (Group for Gravitation, Particles and Fields, Institute of Physics Belgrade, Serbia)
- Rodrigo Olea (Departamento de Ciencias Fisicas, Universidad Andres Bello, Santiago, Chile)
- Dragan Prekrat (Group for Gravitation, Particles and Fields, Faculty of Physics Belgrade, Serbia)
- Tijana Radenkovic (Group for Gravitation, Particles and Fields, Institute of Physics Belgrade, Serbia)
- Voja Radovanovic (Group for Gravitation, Particles and Fields, Faculty of Physics Belgrade, Serbia)
- Igor Salom (Group for Gravitation, Particles and Fields, Institute of Physics Belgrade, Serbia)
- Branislav Sazdovic (Group for Gravitation, Particles and Fields, Institute of Physics Belgrade, Serbia)
- Dejan Simic (Group for Gravitation, Particles and Fields, Institute of Physics Belgrade, Serbia)
- Jelena Stankovic (Department of Mathematics, Teacher Education Faculty, Belgrade, Serbia)
- Marko Vojinovic (Group for Gravitation, Particles and Fields, Institute of Physics Belgrade, Serbia)

Last update: 2. February 2018, 22:55:12.

5th MATHEMATICAL PHYSICS MEETING: Summer School and Conference on Modern Mathematical Physics

6 - 17 July 2008, Belgrade, Serbia

Main Page

Latest news

Programme

General Information

Sponsors

Committees

Lecturers/Speakers

Participants

Registration Form

Photo Album

Previous Meetings

Poster

Accommodation

Travel/Visa

Belgrade

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Bojan Nikolic (Inst. of Phys., Belgrade)
Igor Salom (Inst. of Phys., Belgrade)
Dragan Savic (Inst. of Phys., Belgrade)
Marko Vojinovic (Inst. of Phys., Belgrade)

Uvodna predavanja na konferencijama i druga predavanja

UNIVERZITET U BEOGRADU
INSTITUT ZA FIZIKU BEOGRAD
Pregrevica 118, 11080 Zemun – Beograd, Srbija
Tel: +381 11 3713000, Fax: +381 11 3162190, www.ipb.ac.rs
PIB/VAT: 100105980, Matični broj: 07018029, Tekući račun: 205-66984-23

Dr. Branislav Cvetković

Group for Gravitation, Particles and Fields
Institute of Physics Belgrade
Pregrevica 118
11080 Belgrade, Serbia

Belgrade, 25. July 2017.

Dear Dr. Cvetković,

On behalf of the Institute of Physics, University of Belgrade, Serbia, it is my pleasure to invite you to participate in the 9[th] Mathematical Physics Meeting: School and Conference on Modern Mathematical Physics (MPHYS9), which will be held from 18. to 23. September 2017 in Belgrade, Serbia, and give a plenary talk.

Up to date information about this meeting is available at our web page:

`http://www.mphys9.ipb.ac.rs.`

Looking forward to your participation in the conference.

Yours sincerely,

Dr. Igor Salom

Chairman of the MPHYS9 Local Organizing Committee
Institute of Physics, University of Belgrade
Pregrevica 118, 11080 Belgrade, Serbia
e-mail: isalom@ipb.ac.rs

Subject **Re: Fw: Skup u Banja Luci**
From Branislav Cvetkovic <cbranislav@ipb.ac.rs>
To Sinisa Ignjatovic <sinisha@teol.net>
Date 2014-05-02 19:47

Dragi kolega,
Ignjatovicu neka naslov mog predavanja
bude "3D gravity with propagating torsion".
Vidimo se.
Pozdrav Branislav

Institute of Physics Belgrade
Pregrevica 118, 11080 Belgrade, Serbia
http://www.ipb.ac.rs/

On 01 May 2014 16:04, Sinisa Ignjatovic wrote:

> Poštovani kolega,
>
> kako vidite, kolega Blagojević nije u mogućnosti da prisustvuje Skupu 13. septembra u Banja Luci. I ranije sam obaviješten da vi radite sa kolegom Blagojevićem i da biste mogli održati predavanje na istu temu. Ovim vas i formalno pozivam na naš Skup (podaci o Skupu su u attachment-u) kao predavača po pozivu.
>
> Pozdrav,
> Siniša Ignjatović
> ----- Original Message ----- From: "M_Blagojevic" <mb@ipb.ac.rs>
> To: "Sinisa Ignjatovic" <sinisha@teol.net>
> Cc: "bc" <cbranislav@ipb.ac.rs>
> Sent: Wednesday, April 30, 2014 2:27 PM
> Subject: Re: Skup u Banja Luci
>
> Postovani kolega Ignjatovicu,
>
> Hvala vam na pozivu za ucesce na ovom skupu. Posto u tom periodu necu biti slobodan, predlazem vam da umesto mene pozovete Dr. Branislava Cvetkovica (email: cbranislav@ipb.ac.rs), mog mladjeg kolegu i dugogodisnjeg saradnika, koji bi na tom skupu izlozio neke rezultate naseg zajednickog rada na istrazivanju gravitacije.
>
> Uz srdacan pozdrav,
> Milutin Blagojevic
>
> 2014-04-27 22:20 GMT+02:00 Sinisa Ignjatovic <sinisha@teol.net>:
>
>> Poštovani kolega,
>>
>> polovinom septembra Studijski program fizike Prirodno-matematičkog fakulteta u Banja Luci organizovaće skup "Savremena matematička fizika i njene primjene". Radni dio skupa trajaće jedan dan, u subotu 13. septembra 2014. Preliminarni program Skupa, zajedno sa preliminarnom listom učesnika i prijedlogom sastava tijela Skupa, šaljem u prilogu uz ovaj poziv.
>>
>> Ova poruka ujedno predstavlja i poziv predavačima; na Skupu je planirano petnaestak predavanja po pozivu. Skupu bi takođe prisustvovalo petnaestak slušalaca: fizičara sa Univerziteta u Banja Luci i profesora srednjih škola regije Banja Luka. Očekuje se da Skup najvećim dijelom finansira Ministarstvo nauke i tehnologije Republike Srpske. Svim predavačima bi bili plaćeni troškovi puta i smještaja. Prisustvo za ostale zainteresovane kolege je otvoreno i bez plaćanja kotizacije, ali će

predavanja biti samo po pozivu jer se radi o jednodnevnom skupu.

Naučna orijentacija Skupa bi bila bliska skupovima Mathematical Physics Meeting i Balkan Workshop, kojima su do sada bar jednom prisustvovali svi kojima je ovaj poziv upućen. Predavanja bi bila na našem jeziku, a prilozi za Zbornik radova – čije izdavanje se planira za početak 2015. godine – na engleskom.

Jedan od ciljeva Skupa je povećanje "vidljivosti" teorijske i matematičke fizike u našoj sredini. Raznim povodima, u Banja Luci je u posljednjih petnaestak godina održano nekoliko kratkih skupova iz fizike, ali nijedan od njih nije bio posvećen isključivo teorijskoj fizici. Takođe bi ovaj Skup bio i jedan od prvih – vjerovatno i prvi – skup posvećen isključivo teorijskoj fizici u Bosni i Hercegovini.

Nadam se da ćete prihvatiti poziv i vašim predavanjem doprinijeti uspjehu Skupa. Takođe vas pozivam da budete član Naučnog komiteta Skupa. Očekujemo vaš odgovor do 5. 5. kako bismo na vrijeme organizovali Skup.

Srdačno vas pozdravlja

prof. dr Siniša Ignjatović

rukovodilac Studijskog progama fizike

Prirodno-matematički fakultet

Mladena Stojanovića 2

Banja Luka, Bosna i Hercegovina

Raspored predavanja na skupu "Gravity: new ideas for unsolved problems II"

Divčibare, 19-22. septembar 2013.

<table>
<tr><th>Datum</th><th>Vreme</th><th>Naslov predavanja</th></tr>
<tr><td rowspan="7">20.09.

Predsedavaju:
M. Burić
M. Blagojević</td><td>10.45-11.00</td><td>Otvaranje skupa</td></tr>
<tr><td>11.00-11.40</td><td>Lj. Davidović
T-dualnost u slabo zakrivljenom prostoru</td></tr>
<tr><td>11.40.-12.20</td><td>B. Nikolić
Nekomutativnost zatvorene strune</td></tr>
<tr><td colspan="2">Pauza za ručak</td></tr>
<tr><td>16.00-16.40</td><td>D. Latas
Nejednoznačnost Sajberg-Vitenovog preslikavanja i renormalizabilnost nekomutativne kiralne elektrodinamike</td></tr>
<tr><td>16.40-17.20</td><td>I. Salom
Representations and particles of osp(1|2n) generalized conformal supersymmetry</td></tr>
<tr><td>17.20-18.00</td><td>M. Vojinović
Cosine problem and antigravity in EPRL/FK spinfoam model</td></tr>
<tr><td rowspan="6">21.09.
Predsedavaju:
V. Radovanović
B. Sazdović</td><td>10.20-11.00</td><td>M. Burić
Jedan nekomutativni kosmološki model</td></tr>
<tr><td>11.00-11.40</td><td>V. Radovanović
Gravitacija na Mojalovoj ravni</td></tr>
<tr><td>11.40-12.20</td><td>L. Nenadović
One-Loop Structure of GW Model with Gauge Field</td></tr>
<tr><td colspan="2">Pauza za ručak</td></tr>
<tr><td>16.00-16.40</td><td>B. Cvetković
Holografija u 3D gravitaciji sa torzijom</td></tr>
<tr><td>16.40-17.20</td><td>M. Blagojević
3D gravitacija sa propagirajućom torzijom: Hamiltonova struktura skalarnog sektora</td></tr>
<tr><td>22.09.</td><td>10.20</td><td>Projektni sastanak</td></tr>
</table>

Subject **abstrakt**
From Branislav Cvetkovic <cbranislav@phy.bg.ac.yu>
To Olivera Miskovic <olivera_miskovic@yahoo.com>
Date 2009-05-14 14:47

Draga Olivera,
saljem ti abstrakt seminara.

Title: Canonical structure of topologically massive gravity with a cosmological constant

Abstract

We study the canonical structure of three-dimensional topologically massive gravity with a cosmological constant, using the full power of Dirac's method for constrained Hamiltonian systems. It is found that the dimension of the physical phase space is two per spacetime point, which corresponds to a single Lagrangian degree of freedom. The analysis of the AdS asymptotic region reveals a remarkable relation to 3D gravity with torsion: in the limit of vanishing torsion, the conserved charges and asymptotic symmetries of the two theories become identical.

Pozdrav Branislav

Institute of Physics, Belgrade
http://www.phy.bg.ac.yu/

Subject **title and abstract**
From Branislav Cvetkovic <cbranislav@ipb.ac.rs>
To Olivera Miskovic <olivera_miskovic@yahoo.com>, RO <rodrigo.olea@unab.cl>
Date 2015-10-25 02:07

Dear Rodrigo and Olivera,

I am sending you the title and abstract of the seminar.

Title: Siklos wawes in gravity with torsion
Abstract: Starting from the Siklos waves in general relativity with a cosmological constant, interpreted as gravitational waves on the anti-de Sitter background, a new class of exact torsion waves is constructed in the framework of three-dimensional (3D) and four-dimensional (4D) gravity with torsion.
In the 3D case we show than in the asymptotic limit, the geometry of torsion waves takes the anti-de Sitter form. In the sector with massless torsion modes, we found a set of asymptotic conditions that leads to the conformal asymptotic symmetry.
In 4D three particular exact vacuum solutions, the generalized Kaigorodov, the homogeneous solution and the exponential solution, are explicitly constructed.

Best regards
Branislav

P.S.

Institute of Physics Belgrade
Pregrevica 118, 11080 Belgrade, Serbia
http://www.ipb.ac.rs/

Subject **Re: abstract**
From Niklas Johansson <niklasj@hep.itp.tuwien.ac.at>
Sender <hans.niklas.johansson@gmail.com>
To Branislav Cvetkovic <cbranislav@ipb.ac.rs>
Date 2010-03-18 11:24

webmail

Dear Branislav,

Excellent! I will put your abstract on the homepage
immidiately. Sorry for not replying sooner: I was on vacation.

All the best,
Niklas

2010/3/15 Branislav Cvetkovic <cbranislav@ipb.ac.rs>:

> Dear Niklas,
> I am sending you the abstract of my talk.
>
> Title: Conserved charges in 3D gravity
> Abstract: The covariant canonical expression for the conserved charges, proposed by Nester, is tested on several solutions in 3D gravity with or without torsion and topologically massive gravity. In each of these cases, the calculated values of energy-momentum and angular momentum are found to satisfy the first law of black hole thermodynamics.
>
> Sincerely Branislav
>
> -----------------
> Institute of Physics Belgrade
> http://www.ipb.ac.rs/