Научном већу Института за физику

Београд, 13. novembar 2017.

Предмет: **Молба за покретање поступка за стицање звања научни сарадник**

С обзиром да испуњавам критеријуме прописане од стране Министарства просвете, науке и технолошког развоја за стицање научног звања научни сарадник, молим Научно веће Института за физику у Београду да покрене поступак за мој избор у наведено звање.

У прилогу достављам:

1. Мишљење коментора са предлогом чланова комисије
2. Биографију са основим и стручним подацима
3. Преглед научне активности
4. Елементе за квалитативну оцену научног доприноса
5. Елементе за квантитативну оцену научног доприноса
6. Списак објављених радова, њихове копије и списак радова у припреми
7. Списак цитата
8. Уверење о одбрани докторске дисертације

С поштовањем,

Др Анђело Мађити
(Dr. Angelo Maggitti)

Научном Већу Института за физику

Београд, 13. новембар 2017.

Предмет: Мишљење коментора за избор др Анђела Мађитија (Dr. Angelo Maggitti) у звање научни сарадник

Анђело Мађити је дипломирао 7. априла 2016. године на физичком факултету Универзитета у Београду., Анђело је докторску дисертацију радио у обалсти теоријске квантне фозике и оптике. У докторској дисертацији "Формирање тамних поларитона и дво-поларитонских везаних стања у низовима атома и оптичких микрорезонатора" успешно је развијао моделе кохерентних интеракција атома и фотона које доводе до формирања квантног стања између светлости и колективног метастаболног стања атома. Значај ових стања, поларитона тамних стања, је значајна јер је основ феномена споре светлости, зауставне светлости, и основ за приемену ових система у квантној информатици.

Анђело Мађити је показао самосталнох у раду и решавању проблема теоријске квантне оптике, наставио је самостално да решава нове, занимљиве и важне, проблеме за сличне атомске системе из те области. Стога предлажем Научном већу Института за физику Београд да подржи избор Анђела Мађитија у звање научни сарадник.

За састав Комисије за избор др Анђела Мађитија (Dr. Angelo Maggitti) у звање научни сарадник предлажем:

1. др Бранислав Јеленковић, научни саветник, Институт за физику,
2. др Милан Радоњић, научни сарадник, Институт за физику,
3. др Едив Добарџић, ванредни порофесор, Физички факултет.

Ко-ментор докторске дисертације

Brane Jelenkovic

др Бранислав Јеленковић

Биографија Др Анђела Мађитија (Dr. Maggitti Angelo)

Анђело Мађити је рођен 4. септембра 1977. године у Базелу, Швајцарска. Завршио је основне студије нанонауке (*Nanoscience*) на факултету за филозофију и природне науке Универзитета у Базелу (*Philosophisch Naturwissenschaftliche Fakultät der Universität Basel*). Након основних студија, наставио је мастер студије такође на Универзитету у Базелу, на смеру нанофизика. Током мастер студија, учествовао је у три научна пројекта од којих је један био основа за мастер рад. Први научни пројекат под називом "Оптимизоване структуре $Mg^{+}(Ne)_{m}$-кластера ($1+m \leq 8$) симулиране методом Фурије интеграла по трајекторијама" [*Fourier Path Integral Simulations and Optimized Structures of* $Mg^{+}(Ne)_{m}$*-Clusters* ($1+m \leq 8$)] припадао је области теоријске квантне хемије. Други научни пројекат о наномеханичним резонаторима на бази јонских замки (*Towards ion trap transducers of nanomechanical resonators*) био је из области теоријске наномеханике. Мастер рад, под називом "О квантованој проводности у двослојном графену" (*Towards quantized conductance in graphene bilayer*) урађен је на Техничком универзитету у Делфту у Холандији под руководством проф. др Ливен Вандерсајпена (*Lieven Vandersypen*) као ментора и коментора проф. др Мартино Пођоа (*Martino Poggio*). За Мастер рад је награђен стипендијом Универзитета у Базелу и учешћем у Еразмус програму.

Анђело Мађити је уписао докторске студије на физичком факултету Универзитета у Београду 2011. године, на смеру Квантна, математичка и нанофизика. Поред докторских студија, Анђело Мађити је учестовао у летњој школи *International Summer Schools on Nanosciences & Nanotechnologies, Organic Electronics and Nanomedicine (ISSON11) 2011,* одржаној у Солуну и на симпозијуму *2nd International Symposium about Quantum Mechanics based on a "Deeper Level Theory": Emergence of Quantum Mechanics,* одржаној у Бечу, у Аустријској академији наука, октобра 2013. године.

Дана 7. априла 2016. године, Анђело Мађити је одбранио докторску дисертацију под називом: *"Формирање тамних поларитона и дво-поларитонских везаних стања у низовима атома и оптичких микрорезонатора" ("Formation of dark-state polaritons and two-polariton bound states in arrays of atoms and optical cavities")* на физичком факултету Универзитета у Београду.

Анђело Мађити је аутор/коаутор 4 радова. Два радова су објављени у међународним, вирхунским часописима а додатних два радова су у припреми за вирхунски часописа. Додатно, презентовао је три постера као аутор на домачим и међународним конференцијама.

Преглед научне активности др Анђела Мађитија (Dr. Angelo Maggitti)

Научно-истраживачки рад др Анђела Мађитија (Dr. Angelo Maggitti) је везан за област теоријске квантне оптике и примена квантна теорија информација у квантној оптици. За време докторских студија у Београду (2011-2016) кандидат се бавио да аспектима унутар теоријске квантне оптике које су

- Проучавање слободних, тамних поларитона у гасу атома са два енергијска нивоа који имају дегенерисане поднивое,
- Реализација дво-поларитонских везаних стања у низовима оптичких микрорезонатора уз увођење модификованог Џенјс-Камингсовог модела.

Резултати оба аспекта су представљени у докторској дисертацији. Докторирао је на теми "*Формирање тамних поларитона и дво-поларитонских везаних стања у низовима атома и оптичких микрорезонатора",* урађеној под руководством др Миланом Радоњићем (главног ментора) и др Браниславом Јеленковићем (Коментора) у Центру за фотонику Института за Физику у Београду.

У модерним истраживањима у физици често се тежи повезивању два истраживачка поља што може бити врло плодотворно. Један пример таквог споја је између физике чврстог стања и квантне оптике. Проучавање поларитона и тамних поларитона као подврсте (*dark-state polaritons*) је веома актуално и представља једно од спона између физике чврстог стања и квантне оптике. Према томе, рве научне активности др Анђела Мађитија (Dr. Angelo Maggitti) су тамне поларитоне и њихово формирање у одређеним атомским системима. Формирање тамних поларитона је могуће у атомским системима који показију кохерентни ефекат електромагнетне индуковане транспаренције (*Eletromagnetically Induced Transparency*). Типичан пример таквог система је атомски систем са три нивоа у тзв. Λ-конфигурацији у којој два ласера спрежу два дугоживећа нивоа са једним побуђеним нивоом. Деструктивна интерференција два начина побуђивања омогућава формирање "тамних стања". Тамна стања представљају специфичне линеарне комбинације дугоживећих стања са особином да се не могу побудити ни једним од ласера и у директној су вези са тамним поларитонима. Тамни поларитони су ниско-енергијске колективне ексцитације атома и електромагнетног поља без доприноса побуђених атомских стања. Као такви, тамни поларитони омогућавају успоравање светлости и складиштење фотонских стања у колективним ексцитацијама атомских система. Шта више, могу имати улогу квантних битова и самим тим бити коришћени у квантној информатици и квантном рачунању. Типични поларитони остварени у квантно-оптичким системима су неинтерагујући.

У првом делу дисертације проучавано је формирање тамних поларитона у ансамблу атома, који поседују два нивоа, основни и побуђени, са дегенерисаним поднивоима. А атомски системи са нивоима са дегенерацијом су у литератури углавном предходно проучани помоћу Морис-Шорове трансформације (*Morris-Shore-Transformation*). Систем је карактерисан спрезањем оба дегенерисана поднивоа помоћу два ласерска поља, тако звано пробно (слабо) поље и контролно (јако) поље. Пробно поље је третирано квантно а контролно поље класично.

За разлику од Морис-Шорове трансформације (*Morris-Shore-Transformation*), развијен је нов и другачији алгоритам за испитивање тамних поларитона који се заснива на решавање микроскопских операторских једначина кретања. До сада, овај метод није примењиван на системе које имају дегенерисане поднивое. Овај алгоритам представља нетривјално проширење познатог алгоритма за не дегенерисане поднивое на поднивое са дегенерацијом. Развијени алгоритам омогућава потпуну анализу формираних тамних поларитона у зависности изабране поларизације примењених ласерских поља. Приказана је примена на експериментално значајне атомске паре рубидијума 87 и дискутовани су могући тамни поларитони. Додатно је разматрано могућност конверзије фреквенције и линеарне поларизације светлости. Ове резултате су објављене у раду [A1].

Интерагујући поларитонски системи су слабо проучавани. Њихово проучавање је од растућег интереса због бројних могућих примена у реализацији квантних логичких кола и симулације јако корелисаних, вишечестичних бозонских система познатих из физике чврстог стања. Један од често коришчених приступа обезбеђивање интеракција међу поларитона, а ефективно преко њих и међу фотонима, је присуство оптичких квантно-електродинамичких микрорезонатора (*optical QED cavities*) који интерагују са атомским системима. Оптички микрорезонатори обезбеђују режим јаког спрезање између фотона и атома. На тај начин се постиже нелинеарност неопходна за реализацију интеракција, као и могућност прецизног и ефикасног контролисања те интеракције. Основни модел за опис интеракције атома са два нивоа и електромагнетног поља оптичког, квантно-електродинамичког микрорезонатора је Ценс-Камингсов модел (*Jaynes-Cummings model*). Физичке особине поменутог модела су детаљно проучене теоријски и проверене експериментално. Квалитативно проширење модела на низове међусобно спрегнутих оптичких микрорезонатора представља Џејнс-Камингс-Харбадов модел (*Jaynes-Cummings-Hubbard-model*) који укључује и могућност размене фотона између суседних микрорезонатора. Физичке особине овог модела су предмет активних истраживања, између осталог и због доступне експерименталне реализације. Интеракција између поларитона у таквом систему доводи до појаве квантних фазних прелаза, нпр. Мот изолатор-суперфлуид, као и до формирања везаних дво-поларитонских стања. Везана дво-поларитонска стања су тек недавно постала предмет истраживања области квантне оптике и физике вишечестичних система. Проучавање физичких особина везаних дво-поларитонских стања може указати на нове фундаменталне основе за будућу реализацију квантних меморија и квантних мрежа. Са друге стране, остварају се значајне могућности у контексту проучавања квантих фазних прелаза, као и реализације фрустрираних, Хајзенбергових спинских система (*Frustrated Heisenberg Spin sistem*), а такође и сасвим нових вишечестичних система.

Из предходно наведених и изложених разлога, у другом делу докторске дисертације Др Анђела Мађитија (Dr. Angelo Maggitti) проучавани су интерагујући поларитони у једнодимензионалном низу еванесцентно спрегнутих оптичких квантно-електродинамичких микро-резонатора од којих сваки интерагује преко једне од својих мода са атомом са три нивоа (Два дугоживећа и једним побуђеним). Додатно, сваки атом је побуђиван спољашњим ласерским пољем тако да је остварен услов дво-фотонске Раманове резонанце у Λ-конфигурацији поља. Показано је да под одређеним условима у оваквом систему Џејнс-Камингс-Хабардовог типа долази до појаве везаних тамних поларитона. Уочено је да везани тамни поларитони показују занимљиву могућност коришћења као квантне меморије за тачно један фотон. Резултате другог дела дисертације су објављене у раду [А2].

У наставку дисертације Анђела Мађитија уведено је контролисано периодично неуређење у једно-димензионалном низу спрегнутих оптичких микрорезонатора преко наизменично променљивог параметра спрезања $J_1 - J_2 - J_1 - J_2 - \ldots$. Показано је да се у таквој конфигурацији појављује специфичан тип везаних тамних поларитона који до сада није виђен. Тај специфичан тип везаних тамних поларитона би дали могућа примена као реализација кјубит система. Ови резултати су у припреми за слање у часопис [А3].

Аспекти проучени у радовима [А1] и [А2] су од знатног како теоријског тако и експерименталног интереса због могућности контролисања понашања система променом параметара спољашњих поља. У раду [А2] је по први пут уведен нов модел у оквиру кога особине везаног пара тамних поларитона могу да се подешавају на поменути начин. Додатно, везани пар тамних поларитона под одређеним условима могу постати основно стање система и бити коришћени као квантна меморија. У раду [А3] је по први пут дискутован утицај наизменичног неуређења на везане парове тамних поларитона.

Научни рад Др Анђела Мађитија карактерише се великом и значајном самосталношћу, оригиналношћу, као и темељитошћу у решавању проблема. Кандидат је самоиницијативно покренуо истраживачки пројекат тј. начинио избор теме и истраживачког правца и успешно их реализовао у сарадњи са менторима.

Након успешне одбране докторске дисертације 7.4.2016, Др Анђело Мађити (Dr. Angelo Maggitti) наставио је своју истраживачку активност у два кључна правца:

- Џејнс-Камингсовог-Хабардово решетке (*Jaynes-Cummings-Hubbard-lattice*) које су дате спрегнутим дводимензионалним квантно-електродинамичким микрорезанторима у којим су смештени ансамбла атома различетим конфигурацијама енергетских нивоа са циљем повезивање везаних тамних поларитона са Фрустрираним-Хеијзенбергових-Спинских системима (*Frustrated-Heisenberg-Spin-System*) и могућим такозваним оптичким тополошким изолаторима.
- Примена методике квантне теорије информације и реализиција интерагујучих квантних шетача у низу једно и дводимензионалних спрегнутих квантно-електродинамичких микрорезонатора на јако корелисаних поларитона где коришчених атома имају различету енергетску конфигурацију.

Елементи за квалитативну анализу рада кандидата

1. Ангажованост у развоју услуга за научни рад, образовању и формирању научних кадрова

Међународна сарадња

Кандидат је учествовао у покретање сарадњу са др Никола Паунковићем са Института телекомуникације Техничког Универзитета у Лисабону.

Назив пројекат: *INTERACTING MULTIPARTICLE QUANTUM WALKS IN ONE AND TWO DIMENSIONAL ARRAYS OF OPTICAL CAVITIES*

2. Квалитет научних резултата

Кандидат је у свом научном раду укупно објавио 2 рада у међународним часописима са ISI листе и додатно 2 рада су у припреми за слање у врхунски међународни часопис. Сва четири радова припадају **категорије М21** (Врхунски међународни часопис).

У категорији М21 кандидат је објавио радова у следећим часописима:
1 рад у *Journal of Laser Physics* (ИФ 2012. године=2.545)
1рад у *Physical Review A* (ИФ 2014. годину=2.808)

У категорији М21 кандидат има радова у припреми за слање у следећим часописима:
2 рада у *Physical Review A* (IФ 2016. годину=2.925)

Укупан импакт фактор радова кандидата у горњим часописима категорија М21 је **11.203.**

Према Science Citation Index-u, научни радови кандидата др Анђела Мађитија (Dr. Angelo Maggitti) цитирани су 3 пута у међународним часописима (не укључујући самоцитате).

Елементи за квантитативну анализу рада кандидата

Др Анђела Мађитија (Dr. Angelo Maggitti) за избор у звање научни сарадник

Остварени резултати у периоду пре избора:

Категорија	М бодова по раду	Број радова	Укупно М бодова
M21	8	2	16
M34	0.5	5	2.5
M71	6	1	6

Поређење са минималним квантитативним условима за избор у звање научни сарадник:

Минималан број М бодова		**Остварено**
Укупно	16	24.5
M10+M20+M31+M32+M33+M34+M41+M42 ≥	10	18.5
M11+M12+M21+M22+M23+M24 ≥	5	16

Списак објављених радова, њихове копије и списак радова у припреми др Анђела Мађитија (Dr. Angelo Maggitti)

Радови у врхунским међународним часописима (Публиковане) (М21)

[A1] **A. Maggitti,** M. Radonjić and B. M. Jelenković,
"Dark-state polaritons in a degenerate two-level system“, Laser Physics **23**, 105202 (2013).
Импакт-фактор часописа **2.545** за 2012. годину

[A2] **A.Maggitti,** M. Radonjić and B. M. Jelenković,
"Dark-polariton bound pairs in the modified Jaynes-Cummings-Hubbard model", Phys. Rev. A **93**, 013835 (2016).
Импакт-фактор часописа **2.808** за 2014. годину

Радови у припреми за врхунске међународне часописе (М21)

[A3] **A.Maggitti,** M. Radonjić and B. M. Jelenković,
"Dark-polariton bound pairs under the effect of staggered disorder“ у припреми за слање у часопис Physical Review A.

[A4] **A.Maggitti,** N. Paunković and B. M. Jelenković,
"Polaritonic guided Loschmidt Echo in coupled QED cavities" у припреми за слање у часопис Physical Review A.

Саопштења са међународних скупова штампана у изводима (М34)

[Д1] **A. Maggitti,** M. Radonjić and B. M. Jelenković,
"Dark-polariton bound pairs in the modified Jaynes-Cummings-Hubbard model“, Proceedings of the PHOTONICA'15 – V International School and Conference on Photonics, 26-30 August 2015, Belgrade.

[Д2] **A.Maggitti,** M. Radonjić and B. M. Jelenković,
"Mott-insulator to superfluid transition by multiple bound dark-state polaritons in Jaynes-Cummings Hubbard lattices" Proceedings of the 19th International Symposium on Condensed Matter of Physics, 06–11 September 2015, Belgrade.

[Д3] **<u>A. Maggitti,</u>** M. Radonjić and B. M. Jelenković,
"Dark-state polaritons in a degenerate two-level system", Proceedings of the PHOTONICA'13–IV International School and Conference on Photonics, 26–30 August 2013, Belgrade.

[Д4] 2nd International Symposium about Quantum Mechanics based on A "Deeper Level Theory": Emergence of Quantum Mechanics, Vienna Austrian Academy of Sciences 7-14 September. Учесник конференције у Бечу, Аустрији.

[Д5] International Summer Schools on Nanosciences & Nanotechnologies, Organic Electronics and Nanomedicine (ISSON11), 10–24 July 2011, Thessaloniki Greece. Учесник летње школе и Конференције у Солун, Грчка.

Одбрањена докторска дисертација (М71)

[Ж1] *Формирање тамних поларитона и дво-поларитонских везаних стања у низовима Атома и оптичких микрорезонатора (Formation of dark-state polaritons and two-polariton bound states in arrays of atoms and optical cavities),* Анђело Мађити (Angelo Maggitti), Физички факултет Универзитета у Београду (2016).

Dark-state polaritons in a degenerate two-level system

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Dark-state polaritons in a degenerate two-level system

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Abstract
We investigate the formation of dark-state polaritons in an ensemble of degenerate two-level atoms admitting electromagnetically induced transparency. Using a generalization of microscopic equation-of-motion technique, multiple collective polariton modes are identified depending on the polarizations of two coupling fields. For each mode, the polariton dispersion relation and composition are obtained in a closed form out of a matrix eigenvalue problem for arbitrary control field strengths. We illustrate the algorithm by considering the $F_g = 2 \to F_e = 1$ transition of the D_1 line in ^{87}Rb atomic vapor. In addition, an application of dark-state polaritons to the frequency and/or polarization conversion, using D_1 and D_2 transitions in cold Rb atoms, is given.

(Some figures may appear in colour only in the online journal)

1. Introduction

At the end of the past century, the novel mechanism of electromagnetically induced transparency (EIT) [1, 2] and its many important applications drew a lot of attention. Nonlinearity of EIT media enables slow, stored and stationary light [3–5]. Mazets and Matisov were the first to introduce the concept of adiabatic Raman polaritons that represent a mixture of photon and collective atomic excitations [6]. Subsequently, Fleischhauer and Lukin further extended the concept to dark-state polaritons (DSPs) in a Λ-type EIT system [7]. They also developed a quantum memory technique [8] in order to transfer quantum states of photon wavepackets onto collective Raman excitations in a loss-free and reversible manner. DSPs in more sophisticated schemes have been studied, e.g. double-Λ [9–11], dual-V [12], inverted-Y [13], four-level [14], tripod [15], M-type [16], cyclic three-level [17] and multi-Λ [18, 19]. Collapses and revivals of the DSP number in an atomic ensemble with ground state degeneracy were found in [20]. Resonance beating of light stored using spinor DSPs in a multilevel-tripod scheme was investigated in [21]. Slow light propagation in a degenerate two-level system was experimentally investigated in [22]. DSPs in these various schemes may find applications in quantum information processing, quantum memory and quantum repeaters. Furthermore, degenerate atomic systems, due to their inherent complexity, could lead to new features of DSPs and building blocks for quantum information and quantum computation.

Most of the works treat DSPs using the perturbative approach to the field operator equations of motion, followed by the adiabatic approximation, which was introduced by Fleischhauer and Lukin. In addition, Zimmer *et al* [12] also used the Morris–Shore transformation [23]. Alternatively, Juzeliunas and Carmichael applied a Bogoliubov-type transformation for exact diagonalization of the model Hamiltonian [24]. Chong and Soljacic [9] elegantly derived the properties of the DSPs in single- and double-Λ systems using the Sawada–Brout technique [25]. In this work, we extend the Sawada–Brout–Chong technique to a degenerate two-level system, having a ground state manifold g and an excited state manifold e, that admits the appearance of EIT, i.e. (multiple) dark states exist within g. We present a general algorithm to identify multiple DSP modes that works for an arbitrary number of degenerate states within manifolds g and e and arbitrary polarizations of two coupling fields. The approach is illustrated by finding DSPs at D_1 line transition $F_g = 2 \to F_e = 1$ in atomic vapor of ^{87}Rb. It is shown

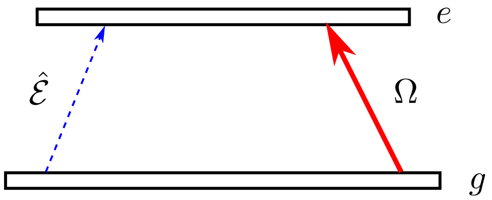

Figure 1. Schematic of a degenerate two-level system, having a ground state manifold g and an excited state manifold e, driven by a strong classical control field (thick line) of Rabi frequency Ω and by a weak quantum probe field $\hat{\mathcal{E}}$ (dashed line) of different polarizations.

that depending on the polarizations of the coupling fields, one or two DSP modes can be determined. In addition, it is shown how DSP modes, originating from different ^{87}Rb transitions, can be utilized for frequency and/or linear polarization conversion.

2. Degenerate two-level system

In this section, we present a general formalism of dark-state polaritons in a degenerate two-level system. It is a generalization of the neat approach of [9]. We consider a gas sample of N atoms, where N is large. Let us denote by $\mathcal{H}_\mathrm{g}$ the Hilbert space of the atomic states in the ground state manifold g and let $\mathcal{H}_\mathrm{e}$ be the Hilbert space of atomic excited states in the manifold e. The corresponding ground- and excited-state energies are denoted by $\hbar\omega_\mathrm{g}$ and $\hbar\omega_\mathrm{e}$, respectively. A strong classical control field of Rabi frequency Ω and a weak quantum probe field $\hat{\mathcal{E}}$, which differ in polarizations and both propagate along the z axis, couple the transition g $\to$ e (see figure 1). The corresponding raising and lowering operators of the control (probe) field, $\hat{V}_\mathrm{c}^\dagger$ and $\hat{V}_\mathrm{c}$ ($\hat{V}_\mathrm{p}^\dagger$ and $\hat{V}_\mathrm{p}$), connect the states in manifold g to the states in manifold e and vice versa. We assume that $\dim\mathcal{H}_\mathrm{g} \geq \dim\mathcal{H}_\mathrm{e}$ holds, so that the system admits EIT [26]. This assures the existence of the Hilbert space $\mathcal{H}_\mathrm{g}^\mathrm{d}$ of the states in manifold g that are dark to the g $\to$ e transition for the control field [27, 28]. Formally, we can view the raising operator $\hat{V}_\mathrm{c}^\dagger$ as a linear mapping $\hat{V}_\mathrm{c}^\dagger\colon \mathcal{H}_\mathrm{g} \to \mathcal{H}_\mathrm{e}$. The space $\mathcal{H}_\mathrm{g}^\mathrm{d}$ is then the null space of the mapping $\hat{V}_\mathrm{c}^\dagger$

$$\mathcal{H}_\mathrm{g}^\mathrm{d} = \{|g\rangle \in \mathcal{H}_\mathrm{g} \mid \hat{V}_\mathrm{c}^\dagger|g\rangle = 0\}. \tag{1}$$

2.1. Model Hamiltonian

We will now present the model Hamiltonian and the dynamics of the lowest energy excitations of the ensemble of degenerate two-level atoms. The free atomic Hamiltonian has the form

$$\hat{H}_\mathrm{at} = \sum_r (\hbar\omega_\mathrm{g}\hat{\mathbb{I}}_\mathrm{g}(r) + \hbar\omega_\mathrm{e}\hat{\mathbb{I}}_\mathrm{e}(r)), \tag{2}$$

where the summation index r counts the atomic positions, while $\hat{\mathbb{I}}_\mathrm{g}$ and $\hat{\mathbb{I}}_\mathrm{e}$ are the projection operators onto the states in the manifolds g and e, respectively. The free photon Hamiltonian, including multiple quantum probe field modes, is

$$\hat{H}_\mathrm{ph} = \sum_k \hbar\omega_k \hat{a}_k^\dagger \hat{a}_k, \tag{3}$$

where $\hat{a}_k^\dagger$ and $\hat{a}_k$ are the creation and annihilation operators of the probe photons with the wavevector k and frequency $\omega_k = c|k| \sim \omega_\mathrm{eg} \equiv \omega_\mathrm{e} - \omega_\mathrm{g}$. The atom interaction with the probe field is given through the minimal coupling Hamiltonian

$$\hat{H}_\mathrm{p} = -\sum_k \sum_r \hbar g_k \hat{a}_k \exp(\mathrm{i}kr)\hat{V}_\mathrm{p}^\dagger(r) + \mathrm{H.c.} \tag{4}$$

with coupling constant $\hbar g_k = \sqrt{\frac{\hbar\omega_k}{2\epsilon_0 V}}\, d_{ge}$, where d_{ge} is the effective electric dipole moment of the g $\to$ e transition, ϵ_0 is the vacuum permittivity and V is the quantization volume. The interaction of the atomic ensemble with the classical control field of the carrier frequency $\omega_\mathrm{c} \sim \omega_\mathrm{eg}$ and the wavevector k_c is of the form

$$\hat{H}_\mathrm{c}(t) = -\sum_r \hbar\Omega \exp[-\mathrm{i}(\omega_\mathrm{c}t - k_\mathrm{c}r)]\hat{V}_\mathrm{c}^\dagger(r) + \mathrm{H.c.} \tag{5}$$

For simplicity, we have used the rotating-wave approximation. In addition, for an atomic operator $\hat{A}(r)$ we define a Fourier-transformed operator $\hat{A}(k) = \sum_r \hat{A}(r)\exp(\mathrm{i}kr)/\sqrt{N}$. Note that $\left(\hat{A}(k)\right)^\dagger = \hat{A}^\dagger(-k)$. Especially, one has $\sum_r \hat{A}(r) = \sqrt{N}\hat{A}(k=0)$. In terms of the Fourier-transformed operators, various Hamiltonian parts are

$$\hat{H}_\mathrm{at} = \hbar\omega_\mathrm{g}\sqrt{N}\,\hat{\mathbb{I}}_\mathrm{g}(k=0) + \hbar\omega_\mathrm{e}\sqrt{N}\,\hat{\mathbb{I}}_\mathrm{e}(k=0), \tag{6a}$$

$$\hat{H}_\mathrm{p} = -\sum_k \hbar g_k\sqrt{N}\,\hat{a}_k\hat{V}_\mathrm{p}^\dagger(k) + \mathrm{H.c.}, \tag{6b}$$

$$\hat{H}_\mathrm{c}(t) = -\hbar\Omega\sqrt{N}\mathrm{e}^{-\mathrm{i}\omega_\mathrm{c}t}\hat{V}_\mathrm{c}^\dagger(k_\mathrm{c}) + \mathrm{H.c.} \tag{6c}$$

The entire Hamiltonian of the ensemble of degenerate two-level atoms interacting with the probe and the control field is $\hat{H}(t) = \hat{H}_\mathrm{at} + \hat{H}_\mathrm{ph} + \hat{H}_\mathrm{p} + \hat{H}_\mathrm{c}(t)$.

2.2. Dark-state polaritons

Now, we focus on the dark-state polaritons in an ensemble of degenerate two-level atoms. Various features of the method in [9], which are obvious *per se* in the case of a simple Λ system, need to be properly adapted to the degenerate two-level system. The additional complexity of the system we investigate also yields some new inherent requirements.

First of all, we remove the time dependence from the Hamiltonian $\hat{H}(t)$ by performing the following unitary gauge transformation:

$$\hat{H}_\mathrm{T} = \hat{U}_\mathrm{c}(t)\hat{H}(t)\hat{U}_\mathrm{c}^\dagger(t) - \hbar\omega_\mathrm{c}\left(\sqrt{N}\hat{\mathbb{I}}_\mathrm{e}(k=0) + \sum_k \hat{a}_k^\dagger\hat{a}_k\right), \tag{7}$$

where

$$\hat{U}_\mathrm{c}(t) = \exp\left[\mathrm{i}\omega_\mathrm{c}t\left(\sqrt{N}\,\hat{\mathbb{I}}_\mathrm{e}(k=0) + \sum_k \hat{a}_k^\dagger\hat{a}_k\right)\right]. \tag{8}$$

Eventually, we restate the time-dependent Schroedinger equation $i\hbar\partial_t|\phi(t)\rangle = \hat{H}(t)|\phi(t)\rangle$ as

$$i\hbar\,\partial_t\big[\hat{U}_c(t)|\phi(t)\rangle\big] = \hat{H}_T\big[\hat{U}_c(t)|\phi(t)\rangle\big]. \qquad (9)$$

Solutions of (9) can be obtained by finding the energy eigenstates of the time-independent Hamiltonian $\hat{H}_T$.

Assume that the atomic ensemble is initially prepared in the collective vacuum state with no probe photons $|\mathbf{g_0}, 0\rangle = |g_0\rangle \otimes |0\rangle \equiv \otimes_r |g_0\rangle_r \otimes |0\rangle$. Analogously with the Λ system case [8, 9], the atomic ground state $|g_0\rangle$ must be dark with respect to the control field, i.e.

$$\hat{V}_c^\dagger|g_0\rangle = 0, \qquad \text{or equivalently} \qquad |g_0\rangle \in \mathcal{H}_g^d. \qquad (10)$$

Additional requirements on the state $|g_0\rangle$ will be specified later.

Dark-state polaritons are particular low energy, single probe photon driven, collective excitations that do not have a contribution of the excited atomic states. To obtain DSPs, we look for a polariton excitation operator $\hat{\phi}_k^\dagger$ such that in the low energy, single excitation case $\hat{\phi}_k^\dagger|\mathbf{g_0}, 0\rangle$ is an eigenstate of $\hat{H}_T$ with the energy $\hbar\omega(k)$. This leads to the following relation:

$$\big[\hat{H}_T, \hat{\phi}_k^\dagger\big] = \hbar\omega(k)\hat{\phi}_k^\dagger + \cdots, \qquad (11)$$

where dots represent the terms that are omitted in the single excitation case and also terms that give zero when acting on the collective vacuum state $|\mathbf{g_0}, 0\rangle$. Note that, for notational simplicity, we keep in mind that all subsequent commutators always act on the state $|\mathbf{g_0}, 0\rangle$. In agreement with [8, 9], we neglect Langevin noise effects, which do not influence the adiabatic evolution of the DSPs.

Collective atomic excitations are driven by the probe photons. Hence, we begin by calculating the commutator

$$\big[\hat{H}_T, \hat{a}_k^\dagger\big] = \hbar(\omega_k - \omega_c)\hat{a}_k^\dagger - \hbar g_k\sqrt{N}\,\hat{V}_p^\dagger(k). \qquad (12)$$

The states that arise from the interaction with the probe field are the pure photon excitation $\hat{a}_k^\dagger|\mathbf{g_0}, 0\rangle$, and the collective atomic excitation $\hat{V}_p^\dagger(k)|\mathbf{g_0}, 0\rangle$, up to a normalization constant. Hence, in addition to $\hat{a}_k^\dagger$ the operator $\hat{V}_p^\dagger(k)$ is also a member of the polariton excitation operator $\hat{\phi}_k^\dagger$. Next, we determine the commutation relation

$$\big[\hat{H}_T, \hat{V}_p^\dagger(k)\big] = \hbar(\omega_{eg} - \omega_c)\hat{V}_p^\dagger(k) - \hbar\Omega^*(\hat{V}_c\hat{V}_p^\dagger)(k - k_c) - \sum_{k'} \hbar g_{k'}^*\hat{a}_{k'}^\dagger(\hat{V}_p\hat{V}_p^\dagger)(k - k'). \qquad (13)$$

Note that $\sqrt{N}\big[\hat{A}_1(k), \hat{A}_2(k')\big] = \big[\hat{A}_1, \hat{A}_2\big](k + k')$ holds for any two atomic operators $\hat{A}_1$ and $\hat{A}_2$. The new operators, $(\hat{V}_c\hat{V}_p^\dagger)(k - k_c)$ and $\hat{a}_{k'}^\dagger(\hat{V}_p\hat{V}_p^\dagger)(k - k')$, appearing in (13) yield the collective states via stimulated emission. The former can readily be included into the polariton excitation operator $\hat{\phi}_k^\dagger$. It creates the spatially dependent coherence among the atomic ground states $|g_0\rangle$ and $\hat{V}_c\hat{V}_p^\dagger|g_0\rangle$, i.e. the ground state coherence wave. When we commute the latter operator with $\hat{H}_T$, we get the operator $\hat{a}_{k''}^\dagger(\hat{V}_p\hat{V}_p^\dagger)(k - k')(\hat{V}_p\hat{V}_p^\dagger)(k' - k'')$. The emergence of such operators of increasing complexity continues and ends with $\hat{a}_{k^{(N)}}^\dagger\prod_{i=1}^N(\hat{V}_p\hat{V}_p^\dagger)(k^{(i)} - k^{(i-1)})$, where $k^{(0)} = k$. This case corresponds to a formidably complex DSP mode that is not tractable. Tractable modes are obtained by imposing one further requirement on the collective vacuum state. Namely, it is crucial that upon action $\hat{V}_p\hat{V}_p^\dagger|g_0\rangle$ we end up with the state $|g_0\rangle$, i.e.,

$$\hat{V}_p\hat{V}_p^\dagger|g_0\rangle = \lambda_p|g_0\rangle, \qquad (14)$$

where $\lambda_p > 0$ is the corresponding eigenvalue. Thus, one obtains $(\hat{V}_p\hat{V}_p^\dagger)(k - k')|\mathbf{g_0}, 0\rangle = \lambda_p\sqrt{N}\delta_{k,k'}|\mathbf{g_0}, 0\rangle$, so that the relation (13) greatly simplifies to

$$\big[\hat{H}_T, \hat{V}_p^\dagger(k)\big] = \hbar(\omega_{eg} - \omega_c)\hat{V}_p^\dagger(k) - \hbar\Omega^*(\hat{V}_c\hat{V}_p^\dagger)(k - k_c) - \hbar g_k^*\lambda_p\sqrt{N}\hat{a}_k^\dagger. \qquad (15)$$

To proceed further, we define the excited atomic state $|e\rangle = \hat{V}_p^\dagger|g_0\rangle/\sqrt{\lambda_p}$ associated with the action of the probe field. Clearly, it has the property $\hat{V}_p|e\rangle = \sqrt{\lambda_p}|g_0\rangle$ and it is an eigenstate of $\hat{V}_p^\dagger\hat{V}_p$, i.e. $\hat{V}_p^\dagger\hat{V}_p|e\rangle = \lambda_p|e\rangle$. The eigenstates $|g_0\rangle$ and $|e\rangle$ are 'tuned' to the polarization of the probe field. These are so-called polarization-dressed states, first introduced and used in [28, 29] for problems of interaction of resonant elliptically polarized light with atomic and molecular energy levels degenerate in angular momentum projections. Next, let us consider the commutators

$$\big[\hat{H}_T, (\hat{V}_c\hat{V}_p^\dagger)(k - k_c)\big] = -\hbar\Omega(\hat{V}_c^\dagger\hat{V}_c\hat{V}_p^\dagger)(k), \qquad (16)$$

and also

$$\big[\hat{H}_T, (\hat{V}_c^\dagger\hat{V}_c\hat{V}_p^\dagger)(k)\big] = \hbar(\omega_{eg} - \omega_c)(\hat{V}_c^\dagger\hat{V}_c\hat{V}_p^\dagger)(k) - \hbar\Omega^*(\hat{V}_c\hat{V}_c^\dagger\hat{V}_c\hat{V}_p^\dagger)(k - k_c) - \sum_{k'} \hbar g_{k'}^*\hat{a}_{k'}^\dagger(\hat{V}_p\hat{V}_c^\dagger\hat{V}_c\hat{V}_p^\dagger)(k - k'). \qquad (17)$$

Similar to the discussion of the relation (13), in order to avoid the appearance of probe photons with all wavevectors, we require that $\hat{V}_p\hat{V}_c^\dagger\hat{V}_c\hat{V}_p^\dagger|g_0\rangle \propto |g_0\rangle$. That can hold provided that

$$\hat{V}_c^\dagger\hat{V}_c\hat{V}_p^\dagger|g_0\rangle = \lambda_c\hat{V}_p^\dagger|g_0\rangle \qquad \text{i.e. } \hat{V}_c^\dagger\hat{V}_c|e\rangle = \lambda_c|e\rangle, \qquad (18)$$

where $\lambda_c > 0$ is the corresponding eigenvalue. Thus, the excited atomic state $|e\rangle$ is a common eigenstate of the operators $\hat{V}_p^\dagger\hat{V}_p$ and $\hat{V}_c^\dagger\hat{V}_c$. Under such a condition, the relation (16) becomes

$$\big[\hat{H}_T, (\hat{V}_c\hat{V}_p^\dagger)(k - k_c)\big] = -\hbar\Omega\lambda_c\hat{V}_p^\dagger(k), \qquad (19)$$

while (17) turns into

$$\big[\hat{H}_T, (\hat{V}_c^\dagger\hat{V}_c\hat{V}_p^\dagger)(k)\big] = \lambda_c\big[\hat{H}_T, \hat{V}_p^\dagger(k)\big], \qquad (20)$$

where the last commutator is found in (15). Hence, under the previous conditions no new components of the polariton excitation operator $\hat{\phi}_k^\dagger$ appear. Stimulated emission, which is driven by the control field, transfers the atoms from the excited state $|e\rangle$ into the ground state $|f\rangle = \hat{V}_c|e\rangle/\sqrt{\lambda_c}$. The states $|g_0\rangle$ and $|e\rangle$ are coupled by the probe field, while the states $|e\rangle$ and $|f\rangle$ are coupled by the control field. Thus, for each eigenvalue pair (λ_p, λ_c) the three states $|g_0\rangle$, $|e\rangle$ and $|f\rangle$ form an independent Λ system that is related to one independent collective DSP mode. The number of such Λ systems, i.e. tractable DSP modes, can be at most equal to the total number of DSP modes, i.e. to the dimensionality of the dark space $\mathcal{H}_g^d$.

Now, we collect the necessary commutation relations

$$[\hat{H}_T, \hat{a}_k^\dagger] = \hbar(\omega_k - \omega_c)\hat{a}_k^\dagger - \hbar g_k\sqrt{N}\hat{V}_p^\dagger(k), \tag{21a}$$

$$[\hat{H}_T, \hat{V}_p^\dagger(k)] = \hbar(\omega_{eg} - \omega_c)\hat{V}_p^\dagger(k) - \hbar g_k^*\lambda_p\sqrt{N}\hat{a}_k^\dagger - \hbar\Omega^*(\hat{V}_c\hat{V}_p^\dagger)(k - k_c), \tag{21b}$$

$$[\hat{H}_T, (\hat{V}_c\hat{V}_p^\dagger)(k - k_c)] = -\hbar\Omega\lambda_c\hat{V}_p^\dagger(k), \tag{21c}$$

so that the polariton excitation operator is of the form

$$\hat{\phi}_{nk}^\dagger = \alpha_{nk}\hat{a}_k^\dagger + \beta_{nk}\frac{\hat{V}_p^\dagger(k)}{\sqrt{\lambda_p}} + \gamma_{nk}\frac{(\hat{V}_c\hat{V}_p^\dagger)(k - k_c)}{\sqrt{\lambda_p\lambda_c}}, \tag{22}$$

where the band index n enumerates the different polariton species. Orthonormal collective excitations $|\boldsymbol{g_0}, 1_k\rangle$, $|\boldsymbol{e}(k), 0\rangle$ and $|\boldsymbol{f}(k - k_c), 0\rangle$ result from the action of the operators $\hat{a}_k^\dagger$, $\hat{V}_p^\dagger(k)/\sqrt{\lambda_p}$ and $(\hat{V}_c\hat{V}_p^\dagger)(k - k_c)/\sqrt{\lambda_p\lambda_c}$ on the collective vacuum state $|\boldsymbol{g_0}, 0\rangle$, respectively,

$$|\boldsymbol{g_0}, 1_k\rangle = \bigotimes_r |g_0\rangle_r \otimes |1_k\rangle, \tag{23a}$$

$$|\boldsymbol{e}(k), 0\rangle = \frac{1}{\sqrt{N}}\sum_r e^{ikr}|e\rangle_r \bigotimes_{r'\neq r}|g_0\rangle_{r'} \otimes |0\rangle, \tag{23b}$$

$$|\boldsymbol{f}(k - k_c), 0\rangle = \frac{1}{\sqrt{N}}\sum_r e^{i(k-k_c)r}|f\rangle_r \bigotimes_{r'\neq r}|g_0\rangle_{r'} \otimes |0\rangle. \tag{23c}$$

Note that the collective states $|\boldsymbol{e}(k), 0\rangle$ and $|\boldsymbol{f}(k - k_c), 0\rangle$ are entangled. This enables the usage of the polariton state

$$|\phi_{nk}\rangle = \alpha_{nk}|\boldsymbol{g_0}, 1_k\rangle + \beta_{nk}|\boldsymbol{e}(k), 0\rangle + \gamma_{nk}|\boldsymbol{f}(k - k_c), 0\rangle \tag{24}$$

as a resource for quantum information processing [2].

We determine the c-numbers α_{nk}, β_{nk} and γ_{nk} by inserting (22) into (11) and make use of (21). This leads to three self-consistency equations that we can represent in the basis $\{|\boldsymbol{g_0}, 1_k\rangle, |\boldsymbol{e}(k), 0\rangle, |\boldsymbol{f}(k - k_c), 0\rangle\}$ as

$$\begin{bmatrix} \omega_k - \omega_c & -\widetilde{g}_k^*\sqrt{N} & 0 \\ -\widetilde{g}_k\sqrt{N} & \omega_{eg} - \omega_c & -\widetilde{\Omega} \\ 0 & -\widetilde{\Omega}^* & 0 \end{bmatrix} \begin{bmatrix} \alpha_{nk} \\ \beta_{nk} \\ \gamma_{nk} \end{bmatrix} = \omega_n(k) \begin{bmatrix} \alpha_{nk} \\ \beta_{nk} \\ \gamma_{nk} \end{bmatrix}, \tag{25}$$

where $\widetilde{g}_k = g_k\sqrt{\lambda_p}$ and $\widetilde{\Omega} = \Omega\sqrt{\lambda_c}$. Our effective Hamiltonian in (25) is similar to the one in [9], but with a major difference. The effective coupling constant $\widetilde{g}_k$ and the effective Rabi frequency $\widetilde{\Omega}$ differ from the corresponding one in [9] because of the inclusion of the eigenvalues λ_p and λ_c. The mentioned difference clearly arises as a consequence of the degenerate two-level atomic system.

The dark-state polaritons are obtained as one of the solutions of the eigenproblem (25). The other two solutions are bright-state polaritons, similarly as in [9]. Exactly at the Raman resonance, $\omega_k = \omega_c$, there is an eigenvector $\propto [-\frac{\widetilde{\Omega}}{\widetilde{g}_k\sqrt{N}}, 0, 1]$. This eigenvector has no contribution of the excited atomic states and represents a stable dark-state polariton that is insensitive to incoherent decay processes acting on the excited atoms. Expansion around the resonance $\omega_k \sim \omega_{eg}$ and $\omega_c \sim \omega_{eg}$ yields a linearized solution for the dark-state polaritons

$$\omega(k) = \frac{|\widetilde{\Omega}|^2}{|\widetilde{g}_k|^2N + |\widetilde{\Omega}|^2}(\omega_k - \omega_c), \tag{26a}$$

$$\alpha_k = -\frac{\widetilde{\Omega}}{\widetilde{g}_k\sqrt{N}}\gamma_k, \qquad \beta_k = -\frac{\widetilde{\Omega}(\omega_k - \omega_c)}{|\widetilde{g}_k|^2N + |\widetilde{\Omega}|^2}\gamma_k. \tag{26b}$$

An interesting property of the DSP solution is that it only depends on the Raman detuning $\omega_k - \omega_c$ of the coupling fields and on the coupling parameters $\widetilde{g}_k$ and $\widetilde{\Omega}$. It does not depend on the energy spacing ω_{eg} of the underlying degenerate two-level system.

The algorithm for finding tractable DSP modes in a degenerate two-level system can be summarized as:

(1) determine the dark space $\mathcal{H}_g^d$ for the operator $\hat{V}_c^\dagger$;
(2) find all states $|g_0\rangle$ from $\mathcal{H}_g^d$ and pairs of eigenvalues (λ_p, λ_c) such that $\hat{V}_p\hat{V}_p^\dagger|g_0\rangle = \lambda_p|g_0\rangle$ and $\hat{V}_c^\dagger\hat{V}_c\hat{V}_p^\dagger|g_0\rangle = \lambda_c\hat{V}_p^\dagger|g_0\rangle$ hold;
(3) for every such pair of eigenvalues obtain DSPs $|\psi_k(\lambda_p, \lambda_c)\rangle$ from (24) and (26).

3. Dark-state polaritons in rubidium vapor

In this section we apply the general formalism to the rubidium vapor. Control and probe fields couple the hyperfine levels $5S_{1/2}, F_g = 2$ and $5P_{1/2}, F_e = 1$ of ^{87}Rb. The atomic lowering operators of the control and probe fields are, respectively,

$$\hat{V}_c = \hat{\mathbf{V}}\cdot\mathbf{e}_c, \qquad \hat{V}_p = \hat{\mathbf{V}}\cdot\mathbf{e}_p, \tag{27}$$

where $\mathbf{e}_c$ and $\mathbf{e}_p$ are polarizations of the fields. The vector operator $\hat{\mathbf{V}}$ is defined by [28, 30, 31]

$$\hat{\mathbf{V}} = (-1)^{F_e+J_g+I+1}\sqrt{(2F_e+1)(2J_g+1)}\begin{Bmatrix} J_e & J_g & 1 \\ F_g & F_e & I \end{Bmatrix} \times \sum_{q=-1}^{1}\sum_{m_g,m_e}\langle F_g, m_g|F_e, m_e; 1, q\rangle|F_g, m_g\rangle\langle F_e, m_e|\mathbf{e}_q^*, \tag{28}$$

where $I = 3/2$ is the nuclear quantum number of ^{87}Rb, $\{:::\}$ is the Wigner $6j$-symbol and $\langle F_g, m_g|F_e, m_e; 1, q\rangle$ is the Clebsch–Gordan coefficient that connects the excited level state $|F_e, m_e\rangle$ to the ground level state $|F_g, m_g\rangle$ via polarization $\mathbf{e}_q^*$,

$$\mathbf{e}_{\pm1} = \mp\frac{1}{\sqrt{2}}(\mathbf{e}_x \pm i\,\mathbf{e}_y), \qquad \mathbf{e}_0 = \mathbf{e}_z, \tag{29}$$

given in some orthonormal basis of polarization vectors. We choose the coordinate system such that the fields propagate along the z axis, and define a basis of Zeeman states relative to this quantization axis. The bases of the individual Hilbert spaces $\mathcal{H}_e$ and $\mathcal{H}_g$ are

$$\mathcal{E} = \{|1, -1\rangle_e, |1, 0\rangle_e, |1, 1\rangle_e\}, \tag{30a}$$

$$\mathcal{G} = \{|2, -2\rangle_g, |2, -1\rangle_g, |2, 0\rangle_g, |2, 1\rangle_g, |2, 2\rangle_g\}. \tag{30b}$$

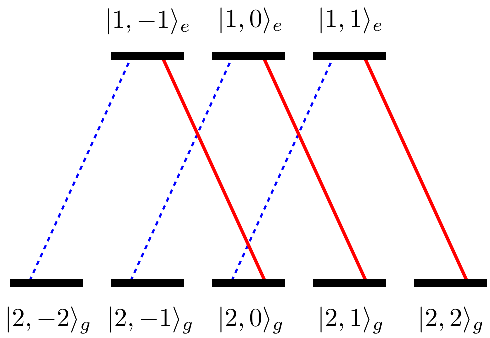

Figure 2. Zeeman sublevel scheme of the transition $F_g = 2 \to F_e = 1$ at the D_1 line of ^{87}Rb. Solid lines denote σ^- transitions coupled by the control field while dashed lines denote σ^+ transitions coupled by the probe field.

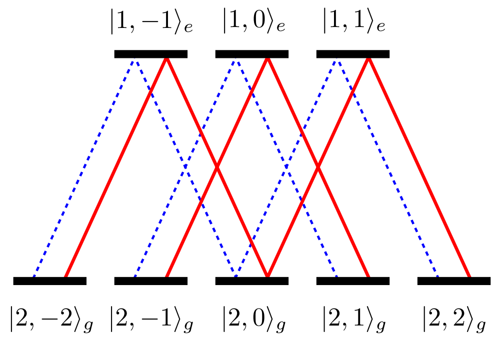

Figure 3. Zeeman sublevel scheme of the transition $F_g = 2 \to F_e = 1$ at the D_1 line of ^{87}Rb. Solid lines denote control field linearly polarized along the y axis while dashed lines denote probe field linearly polarized along the x axis.

We will show that according to the appropriate choice of the polarizations of the coupling fields, one or two DSP modes can be obtained.

3.1. Case of orthogonal circular polarizations

Let the control field couple σ^- transitions, while the probe field couples σ^+ transitions, i.e. $\mathbf{e}_c = \mathbf{e}_{+1}$ and $\mathbf{e}_p = \mathbf{e}_{-1}$ (see figure 2). The lowering operators of the coupling fields, $\hat{V}_c$ and $\hat{V}_p$, are represented in the basis $\mathcal{E} \cup \mathcal{G}$ with the matrices

$$\mathbf{V}_c = \begin{bmatrix} \mathbf{0}_{3,3} & \mathbf{0}_{3,5} \\ \begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{1}{2\sqrt{3}} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} \end{matrix} & \mathbf{0}_{5,5} \end{bmatrix}, \tag{31a}$$

$$\mathbf{V}_p = \begin{bmatrix} \mathbf{0}_{3,3} & \mathbf{0}_{3,5} \\ \begin{matrix} \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2\sqrt{3}} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix} & \mathbf{0}_{5,5} \end{bmatrix}, \tag{31b}$$

where zeros $\mathbf{0}_{m,n}$ denote rectangular $m \times n$ null matrices. Ground level dark space determined from the null space of $\mathbf{V}_c^\dagger$ is

$$\mathcal{H}_g^d = \{|2,-2\rangle_g, |2,-1\rangle_g\}. \tag{32}$$

Both dark states are appropriate as the initial state $|g_0\rangle$. Below we tabulate the corresponding states and eigenvalues of the Λ system:

	$\lvert g_0\rangle$	$\lvert e\rangle$	$\lvert f\rangle$	λ_p	λ_c
I	$\lvert 2,-2\rangle_g$	$\lvert 1,-1\rangle_e$	$\lvert 2,0\rangle_g$	1/2	1/12
II	$\lvert 2,-1\rangle_g$	$\lvert 1,0\rangle_e$	$\lvert 2,1\rangle_g$	1/4	1/4,

that lead to two DSP modes:

$$\omega^{\mathrm{I}}(k) = \frac{|\Omega|^2}{6\,|g_k|^2 N + |\Omega|^2}(\omega_k - \omega_c), \tag{33a}$$

$$|\psi_k^{\mathrm{I}}\rangle \propto -\frac{\Omega}{\sqrt{6}\,g_k\sqrt{N}}|\boldsymbol{g}_0^{\mathrm{I}}, 1_k\rangle + |\boldsymbol{f}^{\mathrm{I}}(k-k_c), 0\rangle$$
$$- \frac{2\sqrt{3}\,\Omega\,(\omega_k - \omega_c)}{6\,|g_k|^2 N + |\Omega|^2}|\boldsymbol{e}^{\mathrm{I}}(k), 0\rangle, \tag{33b}$$

$$\omega^{\mathrm{II}}(k) = \frac{|\Omega|^2}{|g_k|^2 N + |\Omega|^2}(\omega_k - \omega_c), \tag{34a}$$

$$|\psi_k^{\mathrm{II}}\rangle \propto -\frac{\Omega}{g_k\sqrt{N}}|\boldsymbol{g}_0^{\mathrm{II}}, 1_k\rangle + |\boldsymbol{f}^{\mathrm{II}}(k-k_c), 0\rangle$$
$$- \frac{2\,\Omega\,(\omega_k - \omega_c)}{|g_k|^2 N + |\Omega|^2}|\boldsymbol{e}^{\mathrm{II}}(k), 0\rangle. \tag{34b}$$

We see that for orthogonal circular polarizations of the coupling fields, the maximal number of tractable DSP modes exists. This is the generic case, because relevant independent Λ system(s) can be easily recognized.

3.2. Case of orthogonal linear polarizations

Now we analyze the case of the control field polarization along the y axis and the probe field polarization along the x axis, i.e. $\mathbf{e}_c = \mathbf{e}_y$ and $\mathbf{e}_p = \mathbf{e}_x$ (see figure 3). The matrices representing the atomic lowering operators $\hat{V}_c$ and $\hat{V}_p$ in the basis $\mathcal{E} \cup \mathcal{G}$ are

$$\mathbf{V}_c = i \begin{bmatrix} \mathbf{0}_{3,3} & \mathbf{0}_{3,5} \\ \begin{matrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2\sqrt{2}} & 0 \\ \frac{1}{2\sqrt{6}} & 0 & \frac{1}{2\sqrt{6}} \\ 0 & \frac{1}{2\sqrt{2}} & 0 \\ 0 & 0 & \frac{1}{2} \end{matrix} & \mathbf{0}_{5,5} \end{bmatrix}, \tag{35a}$$

$$\mathbf{V}_{\mathrm{p}} = \begin{bmatrix} & \mathbf{0}_{3,3} & & \mathbf{0}_{3,5} \\ \frac{1}{2} & 0 & 0 & \\ 0 & \frac{1}{2\sqrt{2}} & 0 & \\ -\frac{1}{2\sqrt{6}} & 0 & \frac{1}{2\sqrt{6}} & \mathbf{0}_{5,5} \\ 0 & -\frac{1}{2\sqrt{2}} & 0 & \\ 0 & 0 & -\frac{1}{2} & \end{bmatrix}. \qquad (35b)$$

In this case, the ground level dark space is

$$\mathcal{H}_{\mathrm{g}}^{\mathrm{d}} = \Big\{ -\frac{1}{\sqrt{2}}|2,-1\rangle_{\mathrm{g}} + \frac{1}{\sqrt{2}}|2,1\rangle_{\mathrm{g}}, \\ \frac{1}{\sqrt{8}}|2,-2\rangle_{\mathrm{g}} - \frac{\sqrt{3}}{2}|2,0\rangle_{\mathrm{g}} + \frac{1}{\sqrt{8}}|2,2\rangle_{\mathrm{g}}, \Big\}, \qquad (36)$$

but only the first dark state satisfies all necessary conditions for the vacuum state of the tractable mode. The states and eigenvalues of the corresponding Λ system are

$$\begin{aligned} |g_0\rangle &= -\frac{1}{\sqrt{2}}|2,-1\rangle_{\mathrm{g}} + \frac{1}{\sqrt{2}}|2,1\rangle_{\mathrm{g}}, \\ |e\rangle &= |1,0\rangle_{\mathrm{e}}, \\ |f\rangle &= \frac{1}{\sqrt{2}}|2,-1\rangle_{\mathrm{g}} + \frac{1}{\sqrt{2}}|2,1\rangle_{\mathrm{g}}, \\ \lambda_{\mathrm{p}} &= 1/4, \qquad \lambda_{\mathrm{c}} = 1/4. \end{aligned} \qquad (37a)$$

We identify one DSP mode

$$\omega(k) = \frac{|\Omega|^2}{|g_k|^2 N + |\Omega|^2}(\omega_k - \omega_{\mathrm{c}}), \qquad (38a)$$

$$|\psi_k\rangle \propto -\frac{\Omega}{g_k\sqrt{N}}|\boldsymbol{g}_0, 1_k\rangle + |\boldsymbol{f}(k-k_{\mathrm{c}}), 0\rangle \\ -\frac{2\,\Omega(\omega_k - \omega_{\mathrm{c}})}{|g_k|^2 N + |\Omega|^2}|\boldsymbol{e}(k), 0\rangle, \qquad (38b)$$

while the other one is non-tractable.

From the above examples, it can be seen that the choice of the polarization of the coupling fields yields entirely different DSP modes. This is reflected in the composition of the DSP state as well as in the polariton dispersion relation. Note that different polariton dispersion relations would lead to distinct slow light group velocities. In section 4 we outline one possible application of DSP modes in degenerate two-level systems for frequency and/or linear polarization conversion.

4. Frequency and polarization conversion

Let us consider the DSP modes that can be formed from the states within $5S_{1/2}, F_{\mathrm{g}} = 1$ hyperfine level of ^{87}Rb atoms, when the control and the probe field have orthogonal linear polarizations. There are three relevant atomic transitions:

(a) $5S_{1/2},\ F_{\mathrm{g}} = 1 \rightarrow 5P_{1/2},\ F_{\mathrm{e}} = 1$,

(b) $5S_{1/2},\ F_{\mathrm{g}} = 1 \rightarrow 5P_{3/2},\ F_{\mathrm{e}} = 1$,

(c) $5S_{1/2},\ F_{\mathrm{g}} = 1 \rightarrow 5P_{3/2},\ F_{\mathrm{e}} = 0$.

The first belongs to the D_1 line. The last two belong to the D_2 line and can be rendered non-overlapping by using ultracold rubidium atoms.

In the case of orthogonal linear polarizations $\mathbf{e}_{\mathrm{c}} = \mathbf{e}_x$ and $\mathbf{e}_{\mathrm{p}} = \mathbf{e}_y$ of the fields that are resonant to the D_1 line transition (a), we have

$$\begin{aligned} |g_0\rangle &= -\frac{1}{\sqrt{2}}|1,-1\rangle_{\mathrm{g}} + \frac{1}{\sqrt{2}}|1,1\rangle_{\mathrm{g}}, \\ |e\rangle &= |1,0\rangle_{\mathrm{e}}, \\ |f\rangle &= \frac{1}{\sqrt{2}}|1,-1\rangle_{\mathrm{g}} + \frac{1}{\sqrt{2}}|1,1\rangle_{\mathrm{g}}, \\ \lambda_{\mathrm{p}} &= 1/12, \qquad \lambda_{\mathrm{c}} = 1/12. \end{aligned} \qquad (39a)$$

When considering the D_2 line transition (b) with the same polarizations of the coupling fields as in the previous case, $\mathbf{e}_{\mathrm{c}} = \mathbf{e}_x$ and $\mathbf{e}_{\mathrm{p}} = \mathbf{e}_y$, we find

$$\begin{aligned} |g_0\rangle &= -\frac{1}{\sqrt{2}}|1,-1\rangle_{\mathrm{g}} + \frac{1}{\sqrt{2}}|1,1\rangle_{\mathrm{g}}, \\ |e\rangle &= |1,0\rangle_{\mathrm{e}}, \\ |f\rangle &= \frac{1}{\sqrt{2}}|1,-1\rangle_{\mathrm{g}} + \frac{1}{\sqrt{2}}|1,1\rangle_{\mathrm{g}}, \\ \lambda_{\mathrm{p}} &= 5/24, \qquad \lambda_{\mathrm{c}} = 5/24. \end{aligned} \qquad (40a)$$

Finally, for the *swapped linear polarizations*, $\mathbf{e}_{\mathrm{c}} = \mathbf{e}_y$ and $\mathbf{e}_{\mathrm{p}} = \mathbf{e}_x$, of the fields coupling the D_2 line transition (c), we have

$$\begin{aligned} |g_0\rangle &= -\frac{1}{\sqrt{2}}|1,-1\rangle_{\mathrm{g}} + \frac{1}{\sqrt{2}}|1,1\rangle_{\mathrm{g}}, \\ |e\rangle &= |0,0\rangle_{\mathrm{e}}, \\ |f\rangle &= \frac{1}{\sqrt{2}}|1,-1\rangle_{\mathrm{g}} + \frac{1}{\sqrt{2}}|1,1\rangle_{\mathrm{g}}, \\ \lambda_{\mathrm{p}} &= 1/6, \qquad \lambda_{\mathrm{c}} = 1/6. \end{aligned} \qquad (41a)$$

Note, if the polarizations of the fields had not been swapped, the states $|g_0\rangle$ and $|f\rangle$ would have been interchanged.

As can be seen from (39) to (41), the DSP modes are formed from the same states $|g_0\rangle$ and $|f\rangle$ in all three cases, but the considered transitions and polarizations of the coupling fields are different. This provides the possibility for frequency [32, 18] and/or polarization conversion [33] of linearly polarized light. First, one can store a pulse of the probe light polarized along the y axis into the atomic coherence among the states $|g_0\rangle$ and $|f\rangle$ using the transition (a) and the control field polarized along the x axis. The retrieval process, using the transition (b) and the control field polarized along the x axis, would release the pulse at a different frequency, but of the same optical quantum state and polarization along the y axis as the original probe pulse. However, the pulse retrieved using the transition (c) and the control field polarized along the y axis would be in the same optical quantum state as the original probe pulse, but of different carrier frequency and linear polarization along the x axis, i.e. *orthogonal to the original one*. Moreover, this realization does not suffer from losses in the retrieved pulse, since the ratios of the probe and control Clebsch–Gordan coefficients are the same among all three transitions [33].

5. Conclusion

To sum up, we have investigated the formation of dark-state polaritons in an ensemble of degenerate two-level atoms with ground state Hilbert space $\mathcal{H}_g$ and excited state Hilbert space $\mathcal{H}_e$, where $\dim \mathcal{H}_g \geq \dim \mathcal{H}_e$ holds. We elaborated an algorithm, which is a generalization of the Sawada–Brout–Chong approach [9, 25]. Under suitable conditions, the polariton mode dispersion relation and composition can be stated in a closed form. Such DSPs do not depend on the energy spacing of the two-level system, but rather on the Raman detuning of the coupling fields. For each polariton mode, the effective field coupling parameters depend on the appropriate eigenvalues of the atomic operators $\hat{V}_p^\dagger \hat{V}_p$ and $\hat{V}_c^\dagger \hat{V}_c$ that determine the eigenproblem for the polariton species. The application of the general procedure is given for ^{87}Rb atomic transition $F_g = 2 \rightarrow F_e = 1$ of the D_1 line. Two cases of polarizations of the control and probe field are analyzed, when the two fields have orthogonal circular polarizations and when both are linearly polarized in the orthogonal directions. In the former case, two DSP modes are identified, while in the latter case, only one DSP mode can be determined. The formation of the modes as well as their dispersion relation critically depend on the polarizations chosen. Possible application of DSP modes in ultracold ^{87}Rb atoms for frequency and/or linear polarization conversion without losses in the retrieved pulse is presented. Our algorithm can be extended to degenerate systems with more levels and might have applications in quantum information processing as a building block for a preparation and read out schemes with the DSPs as qubit states.

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Dark-polariton bound pairs in the modified Jaynes-Cummings-Hubbard model

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We investigate a one-dimensional modified Jaynes-Cummings-Hubbard chain of N identical QED cavities with nearest-neighbor photon tunneling and periodic boundary conditions. Each cavity contains an embedded three-level atom which is coupled to a cavity mode and an external classical control field. In the case of two excitations and common large detuning of two Raman-resonant fields, we show the emergence of two different species of dark-polariton bound pairs (DPBPs) that are mutually localized in their relative spatial coordinates. Due to the high degree of controllability, we show the appearance of either one or two DPBPs, having the energies within the energy gaps between three bands of mutually delocalized eigenstates. Interestingly, in a different parameter regime with negatively detuned Raman fields, we find that the ground state of the system is a DPBP which can be utilized for the photon storage, retrieval, and controllable state preparation. Moreover, we propose an experimental realization of our model system.

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I. INTRODUCTION

The interaction between light and matter is one of the most fundamental and basic processes in nature, and it represents a milestone in our understanding of a broad range of physical phenomena. The recent experimental success in engineering strong interactions between photons and atoms in high-quality microcavities opens up the possibility to use light-matter systems as quantum simulators for many-body physics [1]. Key examples as first-principles proposals are quantum phase transitions of light in coupled cavities [2–4], quantum fluids of light (see [5]) and the Mott-insulator-to-superfluid phase transition of polaritons in an array of coupled QED cavities [6–11]. Coupled cavities are realized in a variety of physical systems, among them microcavities and nanocavities in photonic crystals [12]. These have paved the way to study strongly correlated phenomena in a controlled way by using such systems. Richness in these systems emerges from the interplay of two main effects. At one side, light-matter interaction inside the cavity leads to a strong effective Kerr nonlinearity between photons. By controlling the atomic level spacings as well as the cavity-mode frequency, it is possible to achieve a photon-blockade regime [13–16] where photon fluctuations are suppressed in each cavity. On the other side, photon hopping between neighboring cavities supports delocalization and competes with the photon blockade.

At the end of the past century, Fleischhauer and Lukin introduced the theoretical concept of dark-state polaritons (DSPs), form-stable coupled excitations of light and matter associated with the propagation of quantum fields in electromagnetically induced transparency (EIT), and showed their potential usage as quantum memories for photons [17,18]. Since then, DSPs have been in the focus of intense theoretical and experimental investigations [19–33]. The first proposal for realization of strong interactions among DSPs and Mott-insulator-to-superfluid phase transition thereof was given by Hartmann *et al.* [9]. They demonstrated the possibility to generate attractive onsite potentials for polaritons yielding highly entangled states and a phase with particles much more delocalized than in superfluids. Moreover, two-polariton bound states, composite excitations of two polaritons that may be spatially confined together, were predicted by Wong and Law [34]. Very recently, two-polariton bound states have been related to spin-orbit interactions by Li *et al.* [35]. Both are features of the systems described by the one-dimensional Jaynes-Cummings-Hubbard model (JCH) and represent an important connection between condensed matter physics and quantum optics. In such systems, it is possible to realize various many-body effects where the particles of interest are photons rather than electrons.

In this paper, we present a scheme based on a modified Jaynes-Cummings-Hubbard model (MJCH) that enables the formation of two different species of spatially, mutually localized dark-polariton bound pairs (DPBPs). Our scheme is based on N identical coupled QED cavities with periodic boundary conditions. Each cavity embeds a single three-level atom. A cavity mode and an external control field, which are in two-photon Raman resonance, drive the transitions from the two atomic ground states to the excited state. We assume that a common single-photon detuning of the fields is large compared to the coupling strengths. Under such conditions, the description of the three-level atoms is effectively reduced to ground-state two-level systems with tunable coupling strength between the ground levels and controllable level Stark shifts. Hence, our model circumvents the drawbacks of the excited-state spontaneous emission and provides a tunable extension of two-polariton bound states of the classical Jaynes-Cummings-Hubbard model [34]. Furthermore, we find that when the common detuning of the coupling fields is negative, the lowest-energy eigenstate of the system becomes a mutually localized DPBP of a new type that may be used as a quantum memory of light. This may find potential use in quantum information processing and controllable state preparation.

This paper is organized as follows. In Sec. II, we recapitulate the standard Jaynes-Cummings model and focus on its spectrum and eigenstates. In Sec. III, we discuss the modified Jaynes-Cummings model where we derive the modified

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Jaynes-Cummings Hamiltonian from a bare model. Further, we analyze the eigenstates and highlight the differences to the standard Jaynes-Cummings model. In Sec. IV, we present the considered model system and extend the modified Jaynes-Cummings model to a modified Jaynes-Cummings-Hubbard model, highlighting that it features the formation of bound states of two dark-polaritons. In Sec. V, we present a detailed discussion of the two-excitation subspace and explain the formation of dark-polariton bound pairs (DPBPs), accentuating their tunability through the control field Stark shift. In Sec. VI, we demonstrate an application of a ground-state DPBP as a quantum memory on which storage and retrieval of a single photon can be performed, while the second photon remains not influenced by the storage and retrieval process. Even though two photons are bound, exactly one photon can be addressed. The state composition of the ground-state DPBP can be tuned by the relative importance of the intercavity photon hopping, e.g., increasing the common single-photon detuning $|\Delta|$. In Sec. VII, we propose an experimental realization of our model system, where we state not only promising candidates to the creation of one-dimensional chains of N-coupled QED cavities, but also name single Λ atoms which can be considered. In addition, we point out that for Cs the measured strong-coupling constant g_m fits very well with our theoretical prediction, where the formation of DPBPs as well as the storage and retrieval process can be seen. Finally, In Sec. VIII we draw our conclusions.

II. STANDARD JAYNES-CUMMINGS MODEL

Within this section, we recapitulate the standard Jaynes-Cummings model (JC). Especially, we focus on its spectrum and eigenstates. In this model, a two-level atom with ground level $|g\rangle$ and excited level $|e\rangle$ having energies ω_g and ω_e interacts with a single mode of an electromagnetic field of frequency ω_0 that couples the transition $|g\rangle \rightarrow |e\rangle$ with the strength g_0. In the (rotating-wave) approximation (RWA), JC Hamiltonian has the form ($\hbar = 1$) [36,37]

$$\hat{H}^{(\mathrm{JC})} = \omega_0 \hat{n} + \delta \hat{\sigma}^+ \hat{\sigma}^- - g_0(\hat{a}\hat{\sigma}^+ + \hat{a}^\dagger \hat{\sigma}^-), \quad (1)$$

where $\hat{c}^\dagger$ ($\hat{c}$) is the photonic creation (annihilation) operator and $\hat{\sigma}^+ = |e\rangle\langle g|$ ($\hat{\sigma}^- = |g\rangle\langle e|$) is the atomic raising (lowering) operator. $\hat{n} = \hat{c}^\dagger\hat{c} + \hat{\sigma}^+\hat{\sigma}^-$ is the number operator of the combined photonic and atomic excitations (polaritons) which is a conserved quantity, i.e., $[\hat{H}^{(\mathrm{JC})},\hat{n}] = 0$. $\delta = \omega_e - \omega_0$ is the detuning. Due to the conservation of $\hat{n}$, $\hat{H}^{(\mathrm{JC})}$ in the subspace $\{|g,n\rangle,|e,n-1\rangle\}$ is represented with the block matrix h_n:

$$h_n = \begin{pmatrix} \omega_0 n & -g_0\sqrt{n} \\ -g_0\sqrt{n} & \omega_0 n + \delta \end{pmatrix}, \quad (2)$$

with $n = 1,2,3,\ldots$ being the total number of excitations. The matrix in (2) is a 2×2 matrix and can be analytically diagonalized. The eigenenergies are given as

$$E_n = \begin{cases} E_{n\pm} = \omega_0 n + \frac{1}{2}[\delta \pm \chi_n(\delta)], & n \geqslant 1 \\ E_0 = 0, & n = 0 \end{cases} \quad (3)$$

with $\chi_n(\delta) = \sqrt{\delta^2 + 4g_0^2 n}$ being the generalized Rabi frequency and $+$ stands for the higher and $-$ for the lower eigenenergy, while the eigenstates are

$$|n,+\rangle := \sin(\theta_n)|g,n\rangle + \cos(\theta_n)|e,n-1\rangle, \quad (4a)$$

$$|n,-\rangle := \cos(\theta_n)|g,n\rangle - \sin(\theta_n)|e,n-1\rangle. \quad (4b)$$

$n = 0$ corresponds to the state of zero polaritons. It takes on the form

$$|0,\pm\rangle \equiv |0,g\rangle = |0\rangle, \quad (5)$$

whereas the occurring mixing angle θ_n is defined as

$$\theta_n = \frac{1}{2} \arctan\left(\frac{2g_0\sqrt{n}}{\delta}\right). \quad (6)$$

The eigenstates (4) are called polaritons. Polaritons are low-energy quasiparticles which are composed of photonic and atomic excitations in superposition. As we change the mixing angle θ_n by a rotation from 0 to $\frac{\pi}{2}$, which basically corresponds to a change of the detuning δ, we tune the polaritons to either pure photonic or pure atomic excitations in a reversible manner. Due to the contribution of the excited atomic state $|e,n-1\rangle$, these polaritons in a more precise way can be called bright polaritons similar to [17–19,33].

III. MODIFIED JAYNES-CUMMINGS MODEL

For the subsequent discussion, we need to derive the modified Jaynes-Cummings (mJC) Hamiltonian which describes an effective interaction of a Λ system with a highly detuned mode of an electromagnetic and classical field. We show that due to the large, common single-photon detuning Δ, i.e., $|\Delta| \gg |g_m|,|\Omega|$, it is possible to circumvent the drawback of the excited-state spontaneous emission that would plague realizations of the JC model by using atoms and optical cavities [37]. Moreover, we focus on the discussion of the eigenstates and eigenspectrum in two specific cases which naturally arise in our case.

A. Derivation of the modified Jaynes-Cummings model Hamiltonian

We consider a single photon in a single-mode QED cavity in which a Λ three-level atom is embedded. The ground levels are $|g\rangle$ and $|f\rangle$ with their level energies ω_g and ω_f, whereas the excited level $|e\rangle$ with level energy ω_e is detuned by a large, common single-photon detuning Δ with respect to two coupling fields. The cavity field with frequency ω_m couples the transition $|g\rangle \rightarrow |e\rangle$ with strength g_m. Further, a classical control field with frequency ω_c and Rabi frequency Ω couples the transition $|f\rangle \rightarrow |e\rangle$. Our bare model Hamiltonian ($\hbar = 1$) has the form

$$\hat{H}_{\mathrm{bare}}(t) = \hat{H}_c + \hat{H}_a + \hat{H}_{\mathrm{int}}(t), \quad (7a)$$

$$\hat{H}_c = \omega_m \hat{c}^\dagger \hat{c}, \quad (7b)$$

$$\hat{H}_a = \omega_g \hat{\sigma}_{gg} + \omega_f \hat{\sigma}_{ff} + \omega_e \hat{\sigma}_{ee}, \quad (7c)$$

$$\hat{H}_{\mathrm{int}}(t) = -(g_m \hat{c}\hat{\sigma}_{eg} + g_m^* \hat{c}^\dagger \hat{\sigma}_{ge} + \Omega e^{-i\omega_c t}\hat{\sigma}_{ef} + \Omega^* e^{i\omega_c t}\hat{\sigma}_{fe}), \quad (7d)$$

where $\hat{H}_c$ denotes the free-field Hamiltonian of the QED cavity, $\hat{H}_a$ stands for the free-atomic Hamiltonian, and $\hat{H}_{\text{int}}(t)$ describes the interaction of the fields with the atom. $\hat{c}^\dagger$ ($\hat{c}$) is the photonic creation (annihilation) operator and $\hat{\sigma}_{\alpha\beta} = |\alpha\rangle\langle\beta|$ ($\alpha,\beta \in \{g,f\}$) are the atomic operators. $\hat{H}_{\text{bare}}(t)$ in (7) satisfies the time-dependent Schrödinger equation

$$i\partial_t|\Psi(t)\rangle = \hat{H}'(t)|\Psi(t)\rangle. \tag{8}$$

We move to a rotating frame in which (7) is time independent. The corresponding gauge transformation [19,33] has the form ($\hbar = 1$)

$$\hat{H}^T = \hat{U}(t)\hat{H}_{\text{bare}}(t)\hat{U}^\dagger(t) + i\partial_t[\hat{U}(t)]\hat{U}^\dagger(t), \tag{9}$$

where $\hat{U}(t)$ is a unitary transformation. Under the gauge (9), $\hat{H}_{\text{bare}}(t)$ reads as

$$\hat{H}^T_{\text{bare}} = \hat{H}_c + \hat{H}_a + \hat{H}_{\text{int}}, \tag{10a}$$

$$\hat{H}_c = \omega_m\hat{c}^\dagger\hat{c}, \tag{10b}$$

$$\hat{H}_a = \omega_g\hat{\sigma}_{gg} + (\omega_f + \omega_c)\hat{\sigma}_{ff} + \omega_e\hat{\sigma}_{ee}, \tag{10c}$$

$$\hat{H}_{\text{int}} = -(g_m\hat{c}\hat{\sigma}_{eg} + g_m^*\hat{c}^\dagger\hat{\sigma}_{ge} + \Omega\hat{\sigma}_{ef} + \Omega^*\hat{\sigma}_{fe}). \tag{10d}$$

$\hat{U}(t) = e^{-i\omega_c t\hat{\sigma}_{ff}}$ has been chosen as the unitary transformation in deriving (10). Assume that the Λ three-level atom is initially prepared in the state $|g,n\rangle = |g\rangle \otimes |n\rangle$. n represents the arbitrary but fixed number of excitations with $n = 1,2,3\ldots$ and $|n\rangle$ the corresponding number state. Under the action of $\hat{H}^T_{\text{bare}}$ onto the state $|g,n\rangle = |g\rangle \otimes |n\rangle$, we get the relations

$$\hat{H}^T_{\text{bare}}|g,n\rangle = (\omega_m n + \omega_g)|g,n\rangle - g_m\sqrt{n}|e,n-1\rangle, \tag{11a}$$

$$\hat{H}^T_{\text{bare}}|e,n-1\rangle = [\omega_m(n-1) + \omega_e]|e,n-1\rangle - g_m^*\sqrt{n}|g,n\rangle - \Omega^*|f,n-1\rangle, \tag{11b}$$

$$\hat{H}^T_{\text{bare}}|f,n-1\rangle = [\omega_m(n-1) + \omega_f + \omega_c]|f,n-1\rangle - \Omega|e,n-1\rangle. \tag{11c}$$

In the subspace $\{|g,n\rangle,|e,n-1\rangle,|f,n-1\rangle\}$, $\hat{H}^T_{\text{bare}}$ has the matrix representation

$$h_{\text{bare}} = \begin{pmatrix} (\omega_m n + \omega_g) & -g_m\sqrt{n} & 0 \\ -g_m^*\sqrt{n} & \omega_m(n-1) + \omega_e & -\Omega^* \\ 0 & -\Omega & (\omega_m(n-1)+\omega_f+\omega_c) \end{pmatrix}. \tag{12}$$

Under Raman resonance condition $\omega_m n + \omega_g = \omega_f + \omega_c = \omega_e - \Delta$, we get

$$h_{\text{bare}} = \begin{pmatrix} (\omega_m n + \omega_g) & -g_m\sqrt{n} & 0 \\ -g_m^*\sqrt{n} & (\omega_m(n-1) + \omega_e) & -\Omega^* \\ 0 & -\Omega & (\omega_m(n-1)+\omega_f+\omega_c) \end{pmatrix}. \tag{13}$$

Under a rotating-wave approximation, (13) is reduced to

$$h^{\text{Raman}}_{\text{bare}} = \begin{pmatrix} 0 & -g_m\sqrt{n} & 0 \\ -g_m^*\sqrt{n} & \Delta & -\Omega^* \\ 0 & -\Omega & 0 \end{pmatrix}. \tag{14}$$

In addition, as we have a far detuned excited state $|e,n-1\rangle$, i.e., $|\Delta| \gg |g_m|,|\Omega|$ [36,38] we can adiabatically eliminate the contribution of the excited state $|e,n-1\rangle$ directly on the level of (14). This yields to

$$h^{(\text{mJC})} = \begin{pmatrix} -\frac{|g_m|^2 n}{\Delta} & -\frac{g_m^*\Omega\sqrt{n}}{\Delta} \\ -\frac{g_m\Omega^*\sqrt{n}}{\Delta} & -\frac{|\Omega|^2}{\Delta} \end{pmatrix}. \tag{15}$$

Equation (15) represents the matrix form of the modified Jaynes-Cummings Hamiltonian (mJC) in the subspace $\{|g,n\rangle,|f,n-1\rangle\}$. The operator form of the modified Jaynes-Cummings Hamiltonian (mJC) reads as

$$\hat{H}^{(\text{mJC})} = \hat{H}_S + \hat{H}_{\text{int}}, \tag{16a}$$

$$\hat{H}_S = -\left(\frac{|g_m|^2}{\Delta}\hat{c}^\dagger\hat{c}\hat{\sigma}_{gg} + \frac{|\Omega|^2}{\Delta}\hat{\sigma}_{ff}\right), \tag{16b}$$

$$\hat{H}_{\text{int}} = -\left(\frac{g_m^*\Omega}{\Delta}\hat{c}^\dagger\hat{\sigma}_{gf} + \frac{g_m\Omega^*}{\Delta}\hat{c}\hat{\sigma}_{\text{fg}}\right). \tag{16c}$$

The term $\hat{H}_S$ incorporates the influence of Stark shifts of the detuned fields, while $\hat{H}_{\text{int}}$ represents the interaction of the cavity field and the atom, where $G = g_m^*\Omega/\Delta$ is the effective atom-photon coupling constant. Hamiltonians $\hat{H}_S$ and $\hat{H}_{\text{int}}$ constitute the modified Jaynes-Cummings Hamiltonian. In the sequel, we are going to discuss the eigenstates of $\hat{H}^{(\text{mJC})}$ and look at the effect of the control field Stark shift.

B. Eigenstates of the modified Jaynes-Cummings model Hamiltonian

In the following, we calculate the eigenenergies and eigenstates of $\hat{H}^{(\text{mJC})}$. We show that dependent on whether one compensates the control field Stark shift by using external fields or not, the eigenenergies, composition of the eigenstates, and the mixing angle θ_n differ significantly. First, we consider the case of noncompensated control field Stark shift. $\hat{H}^{(\text{mJC})}$ of (16) reduces in the subspace $\{|g,n\rangle,|f,n-1\rangle\}$ as

$$h_n^{(m)} = \begin{pmatrix} -\frac{|g_m|^2 n}{\Delta} & -G\sqrt{n} \\ -G^*\sqrt{n} & -\frac{|\Omega|^2}{\Delta} \end{pmatrix}, \tag{17}$$

with $n = 1,2,3,\ldots$ the total number of excitations and corresponding number state $|n\rangle$. The eigenenergies are given as

$$E^{(m)}_{+,n} = 0, \tag{18}$$

$$E^{(m)}_{-,n} = -\left(\frac{|g_m|^2 n}{\Delta} + \frac{|\Omega|^2}{\Delta}\right). \tag{19}$$

The eigenstates to the eigenenergies $E^{(m)}_{+,n}$ and $E^{(m)}_{-,n}$ read as

$$|n,\text{DP}^{(+)}\rangle := \sin(\theta_n)|f,n-1\rangle - \cos(\theta_n)|g,n\rangle, \tag{20a}$$

$$|n,\text{DP}^{(-)}\rangle := \cos(\theta_n)|f,n-1\rangle + \sin(\theta_n)|g,n\rangle \tag{20b}$$

with the occurring mixing angle θ_n which is defined as

$$\theta_n = \frac{1}{2}\arctan\left(\frac{2|g_m|\sqrt{n}}{|\Omega|}\right). \tag{21}$$

However, $|n,\text{DP}^{(\pm)}\rangle$ are called dark-polaritons. A dark-polariton is a quasiparticle which is a superposition of photonic and atomic excitations, where the atomic excitations have only contributions of ground levels $|g\rangle$ and $|f\rangle$ and not the excited level $|e\rangle$. Such dark-polaritons are very similar to the known dark-state polaritons [17,18], but with one major difference. Dark-state polaritons are defined at Raman resonance of two coupling fields and formed independently of the single-photon detuning. Instead, dark-polaritons, which are also defined at Raman resonance, are formed for a large single, common photon detuning Δ of the two coupling fields, i.e., $|\Delta| \gg |g_m|,|\Omega|$. The dependence on Δ enables to tune the eigenstate $|n,\mathrm{DP}^{(\pm)}\rangle$ from an excited to a ground eigenstate. This follows from the eigenenergy $E^{(m)}_{-,n}$ of the dark-polariton $|n,\mathrm{DP}^{(-)}\rangle$. If $\Delta > 0\,(\Delta < 0)$, $|n,\mathrm{DP}^{(+)}\rangle$ is an excited (a ground) eigenstate and $|n,\mathrm{DP}^{(-)}\rangle$ a ground (an excited) eigenstate. Note that $|n,\mathrm{DP}^{(+)}\rangle$ is a degenerate eigenstate because the corresponding eigenenergy $E^{(m)}_{+,n}$ does not depend on the dark-polariton number n. $|n,\mathrm{DP}^{(-)}\rangle$ is a degenerate eigenstate as well for $n \geqslant 2$. Thus, the spectrum is discrete and degenerate in dependence of the dark-polariton number n.

Now, we switch to the case of compensated control field Stark shift. Compensation is achieved by using an additional field, which couples the ground state $|f\rangle$ with some far-off-resonant excited state [39]. Within (17) we set the control field Stark shift $\frac{|\Omega|^2}{\Delta}$ to zero. Hence, the new block-matrix representation $h_n^{(m,\mathrm{comp})}$ in the subspace $\{|g,n\rangle,|f,n-1\rangle\}$ reads as

$$h_n^{(m,\mathrm{comp})} = \begin{pmatrix} -\frac{|g_m|^2 n}{\Delta} & -G\sqrt{n} \\ -G^*\sqrt{n} & 0 \end{pmatrix}, \tag{22}$$

with $n = 1,2,3,\ldots$ the total number of excitations and corresponding number state $|n\rangle$. The block-matrix (22) is a 2×2 matrix and can be analytically diagonalized. The eigenenergies are given as

$$E^{(\mathrm{comp},m)}_{-,n} = -\frac{|g_m|^2 n + |g_m|\sqrt{n}\sqrt{|g_m|^2 n + 4|\Omega|^2}}{2\Delta},$$
$$E^{(\mathrm{comp},m)}_{+,n} = \frac{-|g_m|^2 n + |g_m|\sqrt{n}\sqrt{|g_m|^2 n + 4|\Omega|^2}}{2\Delta}. \tag{23}$$

The respective eigenstates to the eigenenergies $E^{(\mathrm{comp},m)}_{+,n}$ and $E^{(\mathrm{comp},m)}_{-,n}$ are

$$|n,\mathrm{DP}^{(+)}_{\mathrm{comp}}\rangle := \sin(\theta_n)|f,n-1\rangle + \cos(\theta_n)|g,n\rangle, \tag{24a}$$
$$|n,\mathrm{DP}^{(-)}_{\mathrm{comp}}\rangle := \cos(\theta_n)|f,n-1\rangle - \sin(\theta_n)|g,n\rangle, \tag{24b}$$

with the occurring mixing angles θ_n which are defined as

$$\theta_n = \frac{1}{2}\arctan\left[\frac{A(\Omega,n)}{B(g_m,\Omega,n)}\right], \tag{25a}$$
$$A(\Omega,n) = 2\sqrt{2}\times|\Omega|\sqrt{n}, \tag{25b}$$
$$B(g_m,\Omega,n) = \sqrt{C(g_m,\Omega,n)}, \tag{25c}$$
$$C(g_m,\Omega,n) = |g_m|n^2 + 4|\Omega|^2 n + D(g_m,\Omega,n), \tag{25d}$$
$$D(g_m,\Omega,n) = |g_m|n\sqrt{n}\sqrt{|g_m|^2 n + 4|\Omega|^2}. \tag{25e}$$

$|n,\mathrm{DP}^{(\pm)}_{\mathrm{comp}}\rangle$ are dark-polaritons, but of a different type compared to the case of noncompensated control field Stark shift. First of all, the eigenenergies $E^{(\mathrm{comp},m)}_{s,n}$ with $s = +,-$ depend on the generalized Rabi frequency $\xi(n) = \sqrt{|g_m|^2 n + 4|\Omega|^2}$. Second, $|n,\mathrm{DP}^{(\pm)}_{\mathrm{comp}}\rangle$ have a common mixing angle θ_n that depends on the generalized Rabi frequency $\xi(n)$ as well. In addition, the two dark-polariton branches, represented through $|n,\mathrm{DP}^{(\pm)}_{\mathrm{comp}}\rangle$, are separated by the energy amount

$$E^{(\mathrm{comp},m)}_{-,n} - E^{(\mathrm{comp},m)}_{+,n} = \frac{|g_m|\sqrt{n}\sqrt{|g_m|^2 n + 4|\Omega|^2}}{\Delta}. \tag{26}$$

The separation energy is directly dependent on the generalized Rabi frequency $\xi(n)$ and the common single-photon detuning Δ as well. This separation is related to the photon-photon repulsion. It is a consequence of the onsite repulsion $U(n)$ which is a measure of the Kerr nonlinearity [40].

C. Comparison to standard Jaynes-Cummings model

On the level of the individual Hamiltonians, major differences are that at first, in $\hat{H}^{(\mathrm{mJC})}$ the number operator depends on the projection operator $\hat{\sigma}_{gg}$ of the ground level $|g\rangle$ which is not the case in $\hat{H}^{(\mathrm{JC})}$. Second, in $\hat{H}^{(\mathrm{mJC})}$ the atom-cavity field coupling strength $G = g_m\Omega/\Delta$ is rescaled by the common single-photon detuning Δ and the Rabi frequency Ω, where G is chosen to be real. Regarding the eigenstates, a key difference between $\hat{H}^{(\mathrm{mJC})}$ and $\hat{H}^{(\mathrm{JC})}$ is that in the modified Jaynes-Cummings model we have eigenstate dependence on the control field Stark shift. In addition, within the modified Jaynes-Cummings model, we only have a dependence on ground levels, whereas in the standard Jaynes-Cummings model there exists a dependence on the excited level. Hence, these dependencies affect the coherences. Namely, the bright polaritons in the standard Jaynes-Cummings model only consist of optical coherences $\hat{\sigma}_{eg}$ and are explored to spontaneous emission, while in the modified Jaynes-Cummings model, dark-polaritons only consist of spin coherences $\hat{\sigma}_{\mathrm{fg}}$ and no exploration to spontaneous emission is present. This enables the usage of dark-polaritons as a quantum memory for photons over their spin coherences likewise the dark-state polaritons [17–33]. Changing the mixing angles in (21) and (25) over rotations from $0 \to \frac{\pi}{2}$, which corresponds to an adiabatical change of the Rabi frequency Ω, photons are transferred to and stored in the spin coherences in a reversible manner. Optical coherences have shorter coherence times compared to the spin coherences which have longer coherence times. Coherence times of spin coherences are in the range of μs to ms in dark-state polaritons [17,18]. Similar is the case for dark-polaritons. In the sequel, we focus on our model system and state the effective model Hamiltonian which is based on our derivation of the modified Jaynes-Cummings model.

IV. MODEL SYSTEM AND EFFECTIVE MODEL HAMILTONIAN

In the previous sections, we have investigated the standard and modified Jaynes-Cummings model on the level of a single QED cavity. In the subsequent step, we extend the modified Jaynes-Cummings model to a one-dimensional array of coupled QED cavities. This will lead us to the modified Jaynes-Cummings Hubbard model as our effective model Hamiltonian. It includes the hopping between adjacent cavities. First, we state the model system and, second, present the effective model Hamiltonian.

A. Model system

The system we consider consists of a one-dimensional array of N-coupled QED cavities. We assume periodic boundary conditions, i.e., the cavity labeled by $n = N + 1$ corresponds to the cavity $n = 1$. Each cavity embeds a three-level atom with two ground levels $|g\rangle$ and $|f\rangle$, and an excited level $|e\rangle$. The level energies are ω_g, ω_f, and ω_e, respectively, and the excited level $|e\rangle$ is detuned by the common single-photon detuning Δ. In reality, the levels can be either fine or hyperfine levels of alkali-metal atoms. Their D_1 or D_2 line transitions are nowadays easily accessible via available lasers and optical modes of QED cavities. One mode of a tunable cavity [41,42] of frequency ω_m couples the transition $|g\rangle \to |e\rangle$ with the strength g_m, and the classical control field of frequency ω_c and Rabi-frequency Ω couple the transition $|f\rangle \to |e\rangle$. This configuration is known to feature vacuum induced transparency, as first experimentally demonstrated by the group of Vuletić [43]. Both g_m and Ω are typically in MHz range for alkali-metal atoms, which are strongly coupled to QED cavities, and for moderate laser powers.

B. Effective model Hamiltonian

As we consider a one-dimensional chain of N identical coupled QED cavities, the derived modified Jaynes-Cummings model for a single QED cavity is valid for all QED cavities in the one-dimensional chain. Therefore, our effective model Hamiltonian (modified Jaynes-Cummings Hubbard model) ($\hbar = 1$) has the form

$$\hat{H}^{(\mathrm{mJCH})} = \hat{H}^{(\mathrm{mJC})} + \hat{H}_{\mathrm{hop}}, \tag{27a}$$

$$\hat{H}^{(\mathrm{mJC})} = \hat{H}_S + \hat{H}_{\mathrm{int}}, \tag{27b}$$

$$\hat{H}_S = -\sum_{\mu=1}^{N}\left(\frac{g_m^2}{\Delta}\hat{c}_\mu^\dagger\hat{c}_\mu\hat{\sigma}_{gg}^{(\mu)} + \frac{\Omega^2}{\Delta}\hat{\sigma}_{ff}^{(\mu)}\right), \tag{27c}$$

$$\hat{H}_{\mathrm{int}} = -G\sum_{\mu=1}^{N}\left(\hat{c}_\mu^\dagger\hat{\sigma}_{gf}^{(\mu)} + \hat{c}_\mu\hat{\sigma}_{\mathrm{fg}}^{(\mu)}\right), \tag{27d}$$

$$\hat{H}_{\mathrm{hop}} = -J\sum_{\mu=1}^{N}(\hat{c}_{\mu+1}^\dagger\hat{c}_\mu + \hat{c}_\mu^\dagger\hat{c}_{\mu+1}), \tag{27e}$$

where $\hat{c}_\mu^\dagger$ ($\hat{c}_\mu$) is the photonic creation (annihilation) operator and $\hat{\sigma}_{\alpha\beta}^{(\mu)} = |\alpha\rangle_\mu\langle\beta|$ ($\alpha,\beta \in \{g,f\}$) are the atomic operators for the site number μ. The term $\hat{H}_S$ incorporates the influence of Stark shifts of the detuned fields, while $\hat{H}_{\mathrm{int}}$ represents the interaction of the cavity field and the atom, where $G = g_m\Omega/\Delta$ is the effective atom-photon coupling constant which is set to be real. Hamiltonians $\hat{H}_S$ and $\hat{H}_{\mathrm{int}}$ constitute the modified Jaynes-Cummings Hamiltonian. As will be shown in the sequel, the Stark shifts have profound influence on the energy eigenspectrum. $\hat{H}_{\mathrm{hop}}$ describes the photon hopping between adjacent cavities, based on evanescent field coupling, with J as the intercavity photon hopping strength. Similar effective Hamiltonian has been previously used to describe a network of fiber coupled cavities, embedded with three-level atoms [39]. However, while that scheme requires the compensation of the level Stark shifts, here we utilize the individual Stark shifts to achieve tunability. Our effective model Hamiltonian (27) supports the formation of dark-polariton bound pairs. We will see that the different dark-polaritons, which have been discussed in Sec. III, are actually involved in the formation of the energy bands and the bound states. Moreover, we show and discuss that the bound states are formed due to the presence of a force called Kerr nonlinearity which is determined by the onsite repulsion.

V. FORMATION OF DARK-POLARITON BOUND PAIRS

In the following, we discuss the formation of dark-polariton bound pairs in our system. In order to exploit the invariance of the system under cyclic permutations of the sites, we introduce the following operators via discrete Fourier transforms:

$$\hat{b}_k = \frac{1}{\sqrt{N}}\sum_{\mu=1}^{N} e^{-\frac{2\pi i}{N}\mu k}\,\hat{c}_\mu, \tag{28a}$$

$$\hat{s}_{gf}^{(k)} = \frac{1}{\sqrt{N}}\sum_{\mu=1}^{N} e^{-\frac{2\pi i}{N}\mu k}\,\hat{\sigma}_{gf}^{(\mu)}, \tag{28b}$$

where $k = 0,1,\ldots,N-1$ is related to the (discrete) quasimomentum of the excitation. Similarly to [34], we work in the two-excitation subspace that is spanned by the states $|kj\rangle_F \equiv \hat{b}_k^\dagger\hat{b}_j^\dagger|\Phi_0\rangle$, $|k\rangle_F|j\rangle_A \equiv \hat{b}_k^\dagger\hat{s}_{gf}^{(j)\dagger}|\Phi_0\rangle$, and $|kj\rangle_A \equiv \hat{s}_{gf}^{(k)\dagger}\hat{s}_{gf}^{(j)\dagger}|\Phi_0\rangle$. The subscripts F and A stand for the photonic and atomic excitations, respectively. The state $|\Phi_0\rangle = \otimes_{\mu=1}^{N}|g\rangle_\mu|0\rangle_\mu$ is the ground state of the system, where $|0\rangle_\mu$ denotes the vacuum state of the cavity number μ. We note that the excitations (polaritons) are in our case dark in a sense that they do not have the contribution of the excited levels $|e\rangle$ and are not subjected to spontaneous emission. The atomic excitations $|kj\rangle_A$ are in general not orthogonal to each other because of ${}_A\langle k'j'|kj\rangle_A = \delta_{k,k'}\delta_{j,j'} + \delta_{k,j'}\delta_{j,k'} - \frac{2}{N}\delta_{k+j,k'+j'}$. $\hat{b}_k$ and $\hat{b}_j^\dagger$ fulfill the bosonic commutation relation $[\hat{b}_k,\hat{b}_j^\dagger] = \delta_{kj}$, while the atomic operators fulfill the commutation relation $[\hat{s}_{gf}^{(k)},\hat{s}_{gf}^{(j)\dagger}] = -\frac{1}{N}\sum_{\mu=1}^{N} e^{\frac{2\pi i}{N}\mu(j-k)}\hat{\sigma}_z^{(\mu)}$ with $\hat{\sigma}_z^{(\mu)}$ as the Pauli z matrix for the atom in the μth cavity. Under the action of $\hat{H}$ on the states which form the two-excitation subspace, we get the relations

$$\hat{H}|kj\rangle_F = (\omega_k + \omega_j - 2a)|kj\rangle_F - G(|k\rangle_A|j\rangle_F + |j\rangle_A|k\rangle_F), \tag{29a}$$

$$\hat{H}|k\rangle_A|j\rangle_F = (\omega_j - a - b)|k\rangle_A|j\rangle_F - G(|kj\rangle_A + |kj\rangle_F) + \frac{a}{N}\sum_{(k',j')\in S_P}(|k'\rangle_A|j'\rangle_F + |j'\rangle_A|k'\rangle_F) + \frac{2G}{N}\sum_{(k',j')\in S_P}|k'j'\rangle_A, \tag{29b}$$

$$\hat{H}|j\rangle_A|k\rangle_F = (\omega_k - a - b)|j\rangle_A|k\rangle_F - G(|kj\rangle_A + |kj\rangle_F) + \frac{a}{N}\sum_{(k',j')\in S_P}(|k'\rangle_A|j'\rangle_F + |j'\rangle_A|k'\rangle_F) + \frac{2G}{N}\sum_{(k',j')\in S_P}|k'j'\rangle_A, \quad (29c)$$

$$\hat{H}|kj\rangle_A = -G(|k\rangle_A|j\rangle_F + |j\rangle_A|k\rangle_F) - 2b|kj\rangle_A, \quad (29d)$$

where $\omega_l = -2J\cos(\frac{2\pi l}{N})$ for $l \in \{k,j\}$, $a = g_m^2/\Delta$, and $b = \Omega^2/\Delta$. Within Eqs. (29b) and (29c), we have a sum over the set $S_P = \{(k,j) \mid 0 \leqslant k < j \leqslant N-1, k+j \equiv P \,(\mathrm{mod}\, N)\}$ that is determined by the quasimomentum P. From Eqs. (29a)–(29d) we can deduce that the quasimomentum P is a conserved quantity and hence a good quantum number. Apart from the quasimomentum, the total number of excitations (dark-polaritons) $\hat{N} = \sum_{\mu=1}^{N}(\hat{c}_\mu^\dagger\hat{c}_\mu + \hat{\sigma}_{ff}^{(\mu)})$ is a conserved quantity.

We can construct the complete set of eigenvectors by solving the eigenproblem within each of the subspaces $P = 0,1,\ldots,N-1$. Following [34], we restrict the discussion to the case of even N and odd P. A general dark two-polariton eigenvector $|\Psi_P^{(D)}\rangle$ has the form

$$|\Psi_P^{(D)}\rangle = \sum_{(k,j)\in S_P}(\alpha_{kj}|kj\rangle_F + \beta_{kj}|k\rangle_A|j\rangle_F + \beta'_{kj}|j\rangle_A|k\rangle_F + \gamma_{kj}|kj\rangle_A). \quad (30)$$

$|\Psi_P^{(D)}\rangle$ satisfies the time-independent Schrödinger equation $\hat{H}|\Psi_P^{(D)}\rangle = \lambda|\Psi_P^{(D)}\rangle$ which yields within each of the subspaces $P = 1,3,\ldots,N-1$ an eigenproblem that is given by the subsequent set of linear equations

$$\lambda\alpha_{kj} = (\omega_k + \omega_j - 2a)\alpha_{kj} - G(\beta_{kj} + \beta'_{kj}), \quad (31a)$$

$$\lambda\beta_{kj} = -G\alpha_{kj} + (\omega_j - a - b)\beta_{kj} - G\gamma_{kj} + \frac{a}{N}\sum_{(k',j')\in S_P}(\beta_{k'j'} + \beta'_{k'j'}) + \frac{2G}{N}\sum_{(k',j')\in S_P}\gamma_{k'j'} \quad (31b)$$

$$\lambda\beta'_{kj} = -G\alpha_{kj} + (\omega_k - a - b)\beta_{kj} - G\gamma_{kj} + \frac{a}{N}\sum_{(k',j')\in S_P}(\beta_{k'j'} + \beta'_{k'j'}) + \frac{2G}{N}\sum_{(k',j')\in S_P}\gamma_{k'j'}, \quad (31c)$$

$$\lambda\gamma_{kj} = -G(\beta_{kj} + \beta'_{kj}) - 2b\gamma_{kj}, \quad (31d)$$

where λ is the corresponding eigenvalue. As it was demonstrated in [34], for various values of the quasimomentum P the majority of eigenvalues are at most distributed among three bands. When all three bands are well resolved, it was shown that each of the two band gaps contains an eigenenergy of the single two-polariton bound state. For sufficiently large intercavity photon hopping strength J comparing to the strength of the atom-photon interaction, the bands start to overlap.

However, since we are not dealing with the standard JCH model, but rather with a modified one, we find some important differences and new features. Namely, as opposed to [34] there is only one mutually localized DPBP within one of the existing band gaps, while the other one joins the adjacent outer band. The other DPBP can reappear provided that the Stark shift of the control field is compensated. In both cases, when $\Delta < 0$, $g_m \gg \Omega$ and $g_m^2/|\Delta| \gtrsim 1.5\,J$, the ground state of the system is DPBP of a different type than the aforementioned ones. In the sequel, we report on the state composition of the different DPBP types.

The Kerr nonlinearity is a known force in light-atom interactions which depends on the atomic level structure as well as on the coupling strength of light-atom interactions. In our case, the strength of light-atom interaction is described by the effective coupling strength $G = g_m\Omega/\Delta$. Tuning g_m and/or Ω directly affects the Kerr nonlinearity. Compared to [34], we can not only tune and control the Kerr nonlinearity by the cavity-mode coupling strength g_m, but also by the Rabi frequency Ω. This force can be attractive or repulsive [1,13–16]. This force generates the bound state of two dark-polaritons in our case. A measure of the Kerr nonlinearity is the onsite repulsion $U(n)$ which is in general defined as

$$U(n) := (E_+ - E_-)(n+1) - (E_+ - E_-)(n) \quad (32)$$

with $E_\pm$ the eigenenergies of the considered eigenstates. In case of the standard Jaynes-Cummings model, the onsite repulsion $U(n) = \chi(n+1) - \chi(n)$ is determined by the generalized Rabi frequency $\chi(n)$ [3]. This will be different in our case as we will see in the following. In our DPBPs we have bound photons and bound atoms. In [44], they have experimentally shown bound states of atoms in coupled QED cavities, when atoms occupy the same site.

A. Dark-polariton bound pairs in the regime of noncompensated control field Stark shift

We focus on the single DPBP solution of Eqs. (31) which is given in red color within Fig. 1(a) representing the energy eigenspectrum of the model Hamiltonian $\hat{H}$ in dependence of odd values of quasimomentum P. Three energy bands are visible for the used parameter values. We define the gap between the two upper energy bands as the high-energy band gap and in accordance the gap between the two lower-energy bands as the low-energy band gap. The dark-polaritons, which are involved in the formation of energy bands and the single DPBP in Fig. 1(a), are given in (20). This can be seen by solving Eqs. (31) for intercavity hopping $J = 0$. Note that the bands are a consequence of repulsively interacting dark-polaritons of different types with respect to the eigenenergies $E_{\pm,n}^{(m)}$. By different types here, we mean that the dark-polariton with eigenenergy $E_{+,n}^{(m)}$ interacts with the dark-polariton of eigenenergy $E_{-,n}^{(m)}$ in a repulsive way at the same site μ. This is a consequence of the onsite repulsion $U(n)$. On different sites, dark-polaritons with eigenenergies $E_{+,n}^{(m)}$ and $E_{-,n}^{(m)}$ are noninteracting. Instead, the mentioned Kerr nonlinearity, expressed through the onsite repulsion $U(n) = \frac{g_m^2}{\Delta}$, enables the single DPBP state formation by the two dark-polaritons with eigenenergies $E_{-,n}^{(m)}$ which is placed at the same site μ in case of $\Delta > 0$. There is an additional DPBP, formed by the two dark-polaritons with eigenenergies $E_{+,n}^{(m)}$ in case of $\Delta > 0$, but is not visible in the spectrum as it is attached to the central

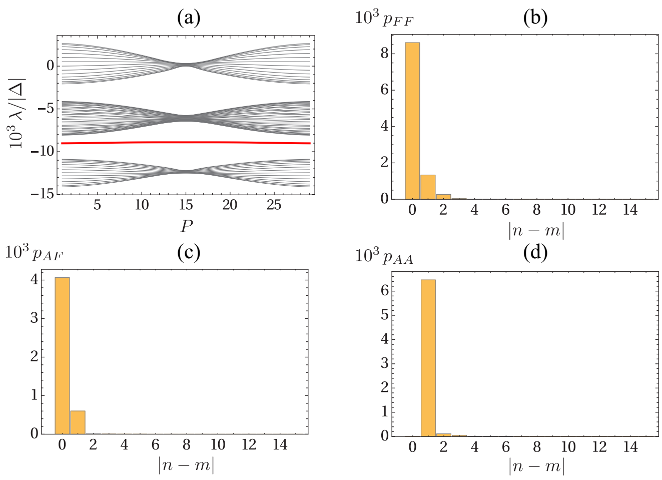

FIG. 1. (a) Normalized eigenvalues dependence on the quasi-momentum P for $N = 30$ cavities. Dark-polariton bound pair state (red curve) appears in the low-energy band gap. The eigenvalues are joined by lines for ease of visualization. (b)–(d) Joint probabilities for different types of double excitations associated to DPBP state for $P = 1$. Used parameters: $\Delta > 0$, $g_m = 0.05\,|\Delta|$, $\Omega = 0.06\,|\Delta|$, and $J = 0.001\,|\Delta|$.

band. On the contrary, formation of single DPBP interchanges for $\Delta < 0$. Our determined $U(n)$ from [3] is mainly affected by the cavity field coupling strength g_m. By increasing g_m we increase the onsite repulsion $U(n)$ which directly enhances the interaction between the two dark-polaritons with eigenenergies $E^{(m)}_{-,n}$ at the same site μ with $\Delta > 0$. Thus, single DPBP is strengthened. Due to the interaction, the single DPBP lies inside the energy band gaps. Depending on the sign of the common single-photon detuning Δ, DPBP lies either in the high- or low-energy band gap. In the case $\Delta > 0$, DPBP lies in the low-energy band gap, whereas in the opposite case it resides within the high-energy band gap. In order to get some information on the inherent state composition of the single DPBP, we calculate, in line with [34], the joint probabilities

$$p_{FF} = \left|\langle\Psi^{(D)}_P|\frac{\hat{c}^\dagger_n\hat{c}^\dagger_m}{\sqrt{1+\delta_{nm}}}|\Phi_0\rangle\right|^2, \tag{33a}$$

$$p_{AF} = \left|\langle\Psi^{(D)}_P|\hat{c}^\dagger_n\hat{\sigma}^{(m)\dagger}_{gf}|\Phi_0\rangle\right|^2, \tag{33b}$$

$$p_{AA} = \left|\langle\Psi^{(D)}_P|\hat{\sigma}^{(n)\dagger}_{gf}\hat{\sigma}^{(m)\dagger}_{gf}|\Phi_0\rangle\right|^2 \tag{33c}$$

of finding pure photonic, photon-atom, and pure atomic excitations, respectively, in cavities at positions n and m. These excitations (pure photonic, pure atomic, and photon-atom) reflect the unique property of dark-polaritons in which the superposition of photonic and collective atomic excitations can be tuned by changing Ω in first place. In our case, we can not only change Ω, but also g_m as we use tunable cavities [41,42]. For a given value of quasimomentum P, all three joint probabilities only depend on the relative distance $|n - m|$ within the cavities.

In Figs. 1(b)–1(d) we present the joint probabilities for the single DPBP state of Fig. 1(a). We have chosen the number of coupled QED cavities to be $N = 30$, single-photon detuning $\Delta > 0$, cavity-mode coupling strength $g_m = 0.05\,\Delta$, the control field Rabi frequency $\Omega = 0.06\,\Delta$, intercavity photon hopping strength $J = 0.001\,\Delta$, and subspace $P = 1$. One can see that the DPBP excitations are well confined together, and all three possible excitation types coexist with roughly equal contributions. The state composition gradually changes by decreasing the contribution of double atomic excitations when P approaches the midrange values. This regime is roughly characterized by $g_m \approx \Omega$ and $(g_m^2 + \Omega^2)/|\Delta| > 5J$. The energy band gaps close when decreasing the ratio of $(g_m^2 + \Omega^2)/|\Delta|$ and J. At the same time, DPBP becomes relatively delocalized, similarly as in [34].

B. Dark-polariton bound pairs in the regime of compensated control field Stark shift

The tunability of our model enables not only the control of the shape of the energy bands, but also the emergence of an additional DPBP state. Namely, if the control field Stark shift is compensated by using an additional field, which couples the ground state $|f\rangle$ with some far-off-resonant excited state [39], another DPBP state appears in the formerly empty energy band gap. Such an add reflects in the removal of the parameter b from Eqs. (31). The energy bands in Fig. 2(a), shown for discrete and distinct quasimomenta P, are formed by the dark-polaritons in (24). This can be seen by solving Eqs. (31) for the intercavity hopping strength $J = 0$ and set the parameter b equal to zero. The onsite repulsion $U(n)$, which ensures the formation of the two DPBPs, is given as $U(n) = \frac{g_m\sqrt{n+1}\sqrt{g_m^2(n+1)+4\Omega^2} - g_m\sqrt{n}\sqrt{g_m^2 n+4\Omega^2}}{\Delta}$ for positive and negative common single-photon detuning Δ. Thus, the onsite repulsion $U(n)$ is invariant under the sign change of Δ. Distinctly to the DPBP formation under noncompensated control field Stark shift, the onsite repulsion $U(n)$ apart from the cavity field coupling strength g_m directly depends on the Rabi frequency Ω. This gives the opportunity to effectively control and enhance the interaction through g_m and Ω. Further, in Fig. 2(a) one can observe that each of the two energy band gaps now contains a single DPBP state (blue and red curves). We used the same parameter values as in Fig. 1, but with compensated control field Stark shift. In Figs. 2(b)–2(d) and Figs. 2(e)–2(g) we characterize the state composition of lower- and higher-energy DPBP states, respectively, by considering the joint probabilities as in the previous subsection. The DPBP in the lower-energy band gap is dominantly composed of two-photon excitation, while in the other DPBP state atom-photon excitation prevails. Moreover, higher-energy DPBP state is further apart from the outer energy band and it is relatively more localized than the lower-energy DPBP state. We checked that the same behavior persists for other values of quasimomentum P. Note that the described situation is for $\Delta > 0$, while it interchanges for $\Delta < 0$.

VI. QUANTUM MEMORY OF LIGHT IN A DARK-POLARITON BOUND PAIR

In the parameter regime where the common single-photon detuning Δ is negative and the cavity-atom coupling strength g_m is significantly larger than the control field Rabi frequency Ω, we have a single DPBP state which is the ground state

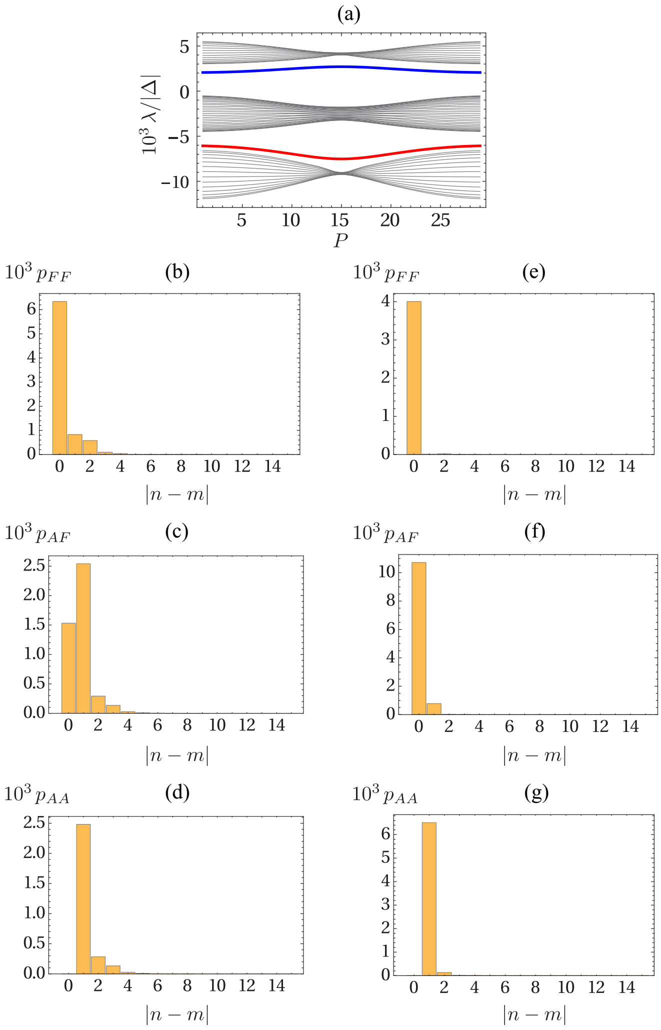

FIG. 2. (a) Normalized eigenvalues dependence on the quasi-momentum P for $N = 30$ cavities. Two dark-polariton bound pair states (blue and red curves) appear in both energy band gaps. The eigenvalues are joined by lines for ease of visualization. (b)–(d) Joint probabilities for different types of double excitations associated to lower-energy DPBP state. (e)–(g) Joint probabilities for different types of double excitations associated to higher-energy DPBP state for $P = 1$. Used parameters: $\Delta > 0$, $g_m = 0.05\,|\Delta|$, $\Omega = 0.06\,|\Delta|$, and $J = 0.001\,|\Delta|$.

of the system. It is well separated from the rest of the energy spectrum when $g_m^2/|\Delta| \gtrsim 1.5\,J$. This is presented in Fig. 3(a). DPBP state composition, given in Figs. 3(b)–3(d) by the corresponding joint probabilities, reveals that the state is dominantly composed of combined atomic and photonic excitations which are localized in their relative spatial coordinates. Note that this DPBP state is of a completely different type than the ones found in the previous section.

It is important that this state also enables the storage of a single photon in the form of a collective atomic spin coherence excitation to which the other photon is closely bound. Namely, when $\Omega \to 0$ adiabatically, a DPBP becomes a pure combination of an atomic and photonic excitation. From this we can deduce that one photon remains attached

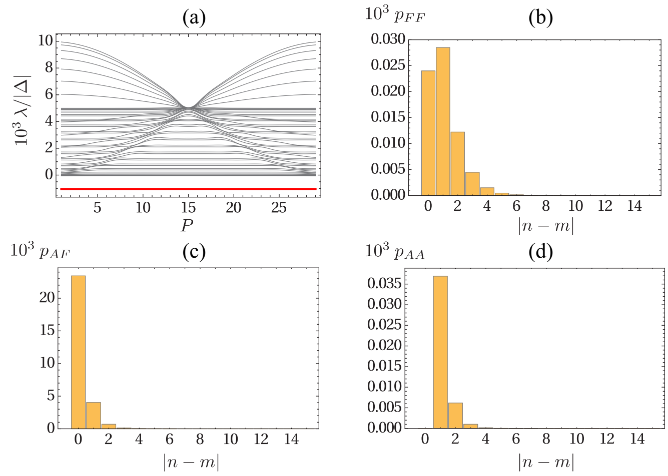

FIG. 3. (a) Normalized eigenvalues dependence on the quasi-momentum P for $N = 30$ cavities. Dark-polariton bound pair state (red curve) appears as the ground state. The eigenvalues are joined by lines for ease of visualization. (b)–(d) Joint probabilities for different types of double excitations associated to DPBP state for $P = 1$. Used parameters: $\Delta < 0$, $g_m = 0.05\,|\Delta|$, $\Omega = 0.001\,|\Delta|$, and $J = 0.00125\,|\Delta|$.

to the atomic spin coherence wave. This is reminiscent of the atom-photon molecule [36].

The state composition can be tuned by increasing the relative importance of the intercavity photon hopping, e.g., by increasing $|\Delta|$. This is achieved gradually for distinct values of quasimomentum, starting from the values $P = 1$, $N - 1$ and proceeding towards the midrange values of P. Figure 4(a) shows the energy spectrum in such a case. For $P \in \{1, 3, N-3, N-1\}$ the DPBP state is predominantly composed of two-photon excitations which become delocalized in their relative

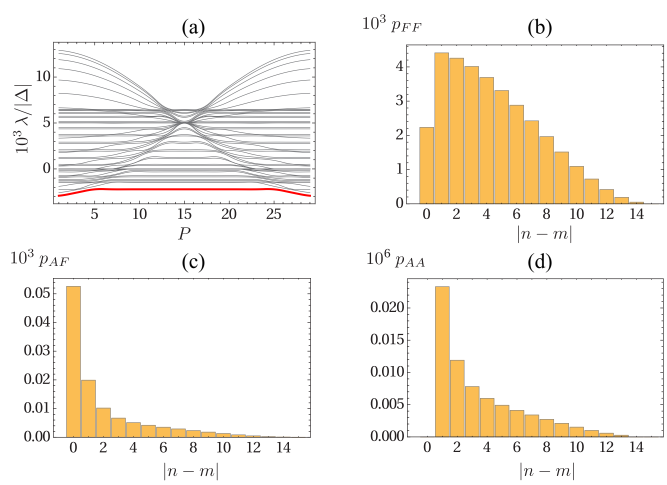

FIG. 4. (a) Normalized eigenvalues dependence on the quasi-momentum P for $N = 30$ cavities. Dark-polariton bound pair state (red curve) appears as the ground state. The eigenvalues are joined by lines for ease of visualization. (b)–(d) Joint probabilities for different types of double excitations associated to DPBP state for $P = 1$. Used parameters: $\Delta < 0$, $g_m = 0.05\,|\Delta|$, $\Omega = 0.001\,|\Delta|$, and $J = 0.002\,|\Delta|$.

spatial positions, as can be seen in Figs. 4(b)–4(d). The reason for such behavior can be traced back to the emergence of the avoided crossings of the ground state and the first excited state near the edges of the quasimomentum zone. The crossings shift towards the P-zone center as the influence of the photon hopping is being increased. For the quasimomentum values between the crossings, the DPBP state remains dominantly of the atom-photon type. In the case when the control field strength adiabatically reduces to zero, the DPBP state becomes of a pure two-photon type. Therefore, this corresponds to the retrieval procedure of the previously stored photon excitation.

VII. EXPERIMENTAL REALIZATION

Our model system is a large, one-dimensional mJCH chain of N-coupled QED cavities. In order to realize it, we need a structure, in which large arrays of coupled QED cavities can be realized. Promising candidates are photonic band-gap cavities [12,45]. It is manageable to produce and position them with high precision and in large numbers. A tempting alternative are photonic crystals as they offer the possibility of fabricating large arrays of QED cavities in one- or two-dimensional lattices as well as networks [46–48]. A third possibility would be the use of toroidal micro-QED cavities that are coupled via tapered optical fibers [49]. Single atoms, embedded in each QED cavity, are three-level atoms where the excited level is far detuned by the common single-photon detuning with respect to the two coupling fields. In real experiments, Cs and ultracold ^{87}Rb atoms have shown to be very suitable [44,50,51]. For Cs in a toroidal micro-QED cavity it has be shown that g_m in the strong-coupling regime reaches the value of ~ 50 MHz [50]. This fits pretty well with our theoretically chosen value for the formation of individual DPBP inside the energy band gaps, but also for the ground DPBP at $\Delta < 0$ with its potential use as a quantum memory for a single photon.

VIII. CONCLUSION

To summarize, we have derived a modified Jaynes-Cummings model from the bare model under two conditions: (i) two-photon Raman resonance of the cavity mode and classical control field, (ii) common single-photon detuning $|\Delta| \gg g_m, \Omega$. We have shown that the eigenstates on one hand depend on the common single-photon detuning and, on the other hand, their composition differs with respect to the control field Stark shift. Moreover, we have extended the modified Jaynes-Cummings model to a modified Jaynes-Cummings-Hubbard model where an array of N-coupled QED cavities, each having an embedded single three-level atom, is considered. The modified Jaynes-Cummings-Hubbard model supports DPBPs. The formation of two different species of spatially localized dark-polariton bound pairs (DPBPs) has been elaborated when there are exactly two excitations in the system. It was shown that the onsite repulsion $U(n)$ as a consequence of the Kerr nonlinearity represents the attractive force between interacting dark-polaritons and enables the existence of DPBP states. Furthermore, it is demonstrated that our model system offers a high degree of tunability that can affect both quantitative and qualitative behavior. In particular, the number of DPBP states can be controlled by (not) compensating the Stark shift due to the control field. Further, in the regime when cavity-atom coupling overwhelms the influence of the control field, and the common single-photon detuning of the fields is negative, we obtained a ground DPBP eigenstate on which the storage and readout of a single photon can be effectively performed. An experimental realization is proposed for our model system. Cs atom has been mentioned as a promising candidate as its value of the cavity-mode coupling strength g_m fits very well with our theoretically chosen and determined one. We expect that future investigations of this kind of system under different settings, i.e., with distinct and alternating hopping strengths between the cavities, in the presence of disorder, or in two-dimensional lattice configurations, may lead to various effects and rich physics.

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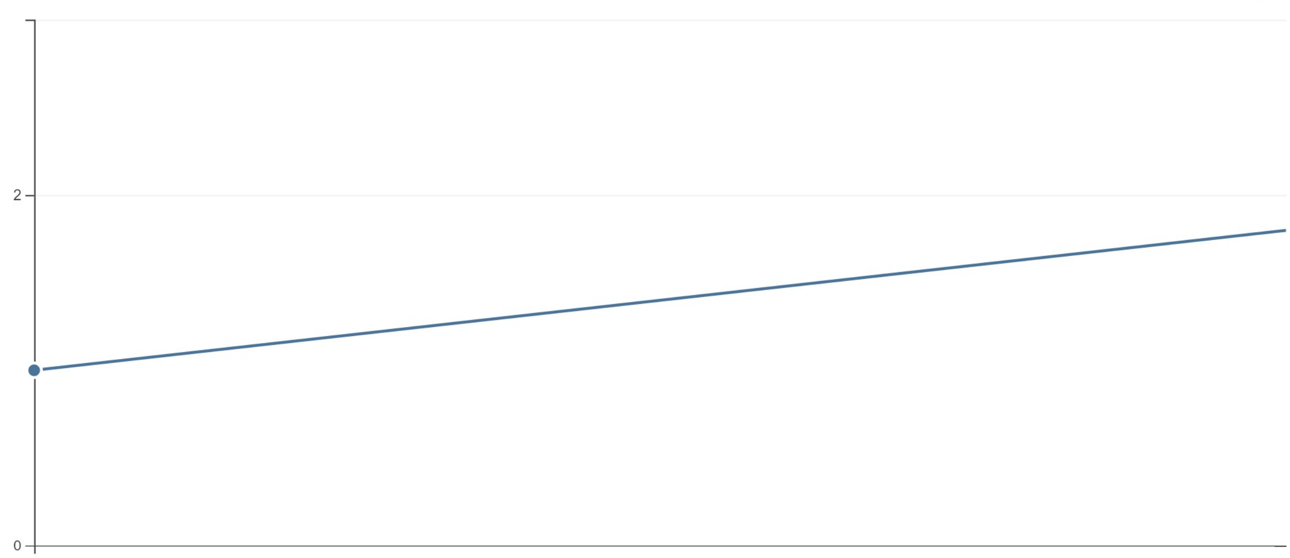

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На основу члана 161 Закона о општем управном поступку («Службени Лист СРЈ» број 33/97 и 31/01), и члана 120 Статута Универзитета у Београду - Физичког факултета, по захтеву АНЂЕЛА МАЂИТИЈА, дипломираног физичара, издаје се следеће

У В Е Р Е Њ Е

АНЂЕЛО МАЂИТИ, дипломирани физичар, дана 7. априла 2016. године, одбранио је докторску дисертацију под називом

„FORMATION OF DARK-STATE POLARITONS AND TWO-POLARITON BOUND STATES IN ARRAYS OF ATOMS AND OPTICAL CAVITIES“ (Формирање тамних поларитона и дво-поларитонских везаних стања у низовима атома и оптичких микрорезонатора)

пред Комисијом Универзитета у Београду - Физичког факултета, и тиме испунио све услове за промоцију у ДОКТОРА НАУКА – ФИЗИЧКЕ НАУКЕ.

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