Numerical study of Faraday waves in binary non-miscible Bose-Einstein condensates*

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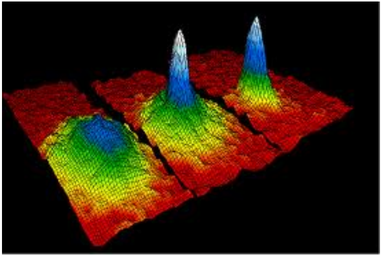
Why ultracold quantum gases are interesting?

- Intensive progress in the field of ultracold atoms has been recognized by Nobel prize for physics in 2001 for experimental realization of Bose-Einstein condensation
- Cold alkali atoms:
  Rb, Na, Li, K . . .
  $T \sim 1\,\mathrm{nK}$, $\rho \sim 10^{14}\mathrm{cm}^{-3}$
- Cold bosons, cold fermions
- Harmonic trap, optical lattice
- Short-range interactions, long-range dipolar interactions
- Spin-orbit-coupled BECs



- Tunable quantum systems concerning dimensionality, type and strength of interactions

Experimental observation of Faraday waves

P. Engels, C. Atherton, M. A. Hoefer, PRL **98**, 095301 (2007)

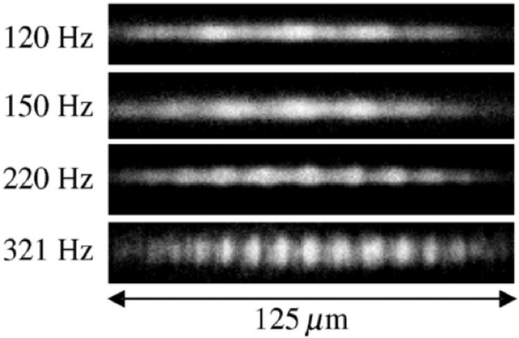


FIG. 1. In-trap absorption images of Faraday waves in a BEC. Frequency labels for each image represent the driving frequency at which the transverse trap confinement is modulated.

Two-component BEC systems

- Experimentally realized with a broad variety of types of atoms and parameters of a system
  - heterogeneous systems: different types of atoms
  - homogeneous systems: same type of atoms, different internal (usually spin) states
- Rich dynamics and interplay of the parameters
- Several possible ground states
- A variety of possible dynamical evolutions
- We focus on the study of Faraday waves and patterns in cigar-shaped two-component ^{87}Rb BECs, with strong radial confinement, which is harmonically modulated
- We also study resonant waves, which appear for specific values of the frequency of radial modulation

Mean-field description of 2C BEC

- The system is described by a coupled system of GP equations:

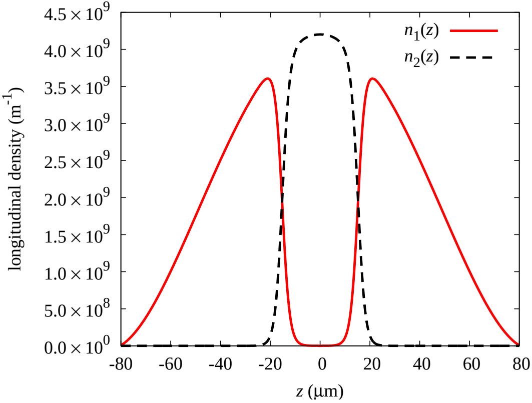
$$i\hbar\frac{\partial\Psi_1(\vec{r},t)}{\partial t} = \left[-\frac{\hbar^2}{2m_1}\triangle + V(\vec{r},t) + g_{11}|\Psi_1(\vec{r},t)|^2 + g_{12}|\Psi_2(\vec{r},t)|^2\right]\Psi_1(\vec{r},t)$$

$$i\hbar\frac{\partial\Psi_2(\vec{r},t)}{\partial t} = \left[-\frac{\hbar^2}{2m_2}\triangle + V(\vec{r},t) + g_{21}|\Psi_1(\vec{r},t)|^2 + g_{22}|\Psi_2(\vec{r},t)|^2\right]\Psi_2(\vec{r},t)$$

  where the couplings are given by:
  $g_{11} = \frac{4\pi\hbar^2 a_1}{m_1}$, $g_{22} = \frac{4\pi\hbar^2 a_2}{m_2}$, $g_{12} = g_{21} = \frac{2\pi\hbar^2 a_{\rm int}}{m_{\rm eff}}$
- Typical experimental values we consider for two hyperfine states of ^{87}Rb:

$$N_1 = 2.5\cdot 10^5\,,\quad N_2 = 1.25\cdot 10^5$$
$$a_1 = 100.4\,a_0\,,\quad a_2 = 98.98\,a_0\,,\quad a_{\rm int} = 100.4\,a_0$$
$$\omega_\rho(t) = \omega_{\rho,0}(1+\epsilon\sin\omega_{\rm m}t)\,,\quad \omega_{\rho,0} = 160\cdot 2\pi\,{\rm Hz}$$
$$\omega_{\rm m} = 250\cdot 2\pi\,{\rm Hz}\,,\quad \epsilon = 0.1\,,\quad \omega_z = 7\cdot 2\pi\,{\rm Hz}$$

Ground state - imaginary-time propagation



Density profile of the converged eigenstate obtained by propagation in the imaginary time. Discretization parameters: $N_\rho = N_z = 2000$, $\varepsilon = 10^{-4}/\omega_z$.

Ground state - experimental realization

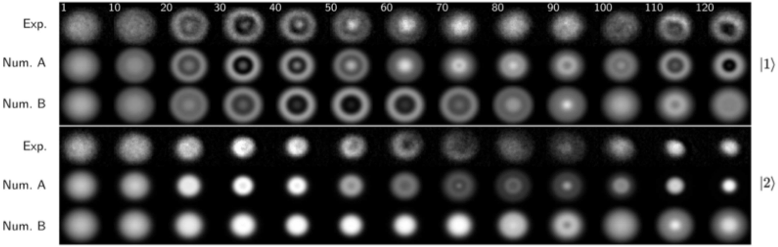
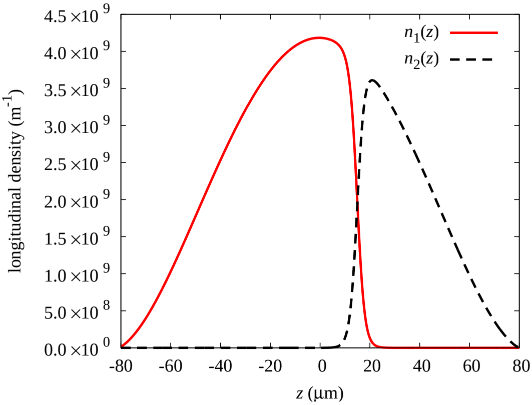


FIG. 1. Top view of a time sequence of experimental and numerical density profiles for $N = 3.50(5) \times 10^5$ ^{87}Rb atoms with equal populations in the $|1\rangle$ and $|2\rangle$ states. The first row shows the measured density profiles for the $|1\rangle$ atoms, while the second and third rows give numerical results including losses and different trap frequencies (Num. A) and without those additional model features (Num. B). A similar arrangement is given for the $|2\rangle$ atoms in the fourth, fifth, and sixth rows. The field of view in all pictures is approximately 100 μm on a side. The evolution time (in ms) for each column is indicated in the top row.

K. M. Mertes, J. W. Merrill, R. Carretero-González, D. J. Frantzeskakis, P. G. Kevrekidis, D. S. Hall, PRL **99**, 190402 (2007)

Segregated state - imaginary-time propagation



First excited eigenstate obtained by imaginary-time propagation.
Discretization parameters: $N_\rho = N_z = 2000$, $\varepsilon = 10^{-4}/\omega_z$.

First excited state - experimental realization

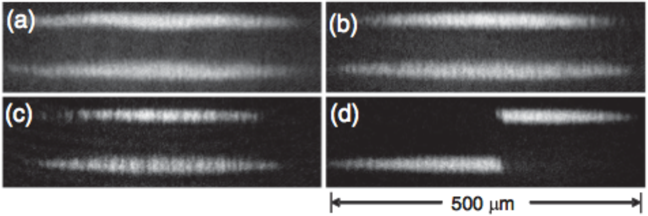
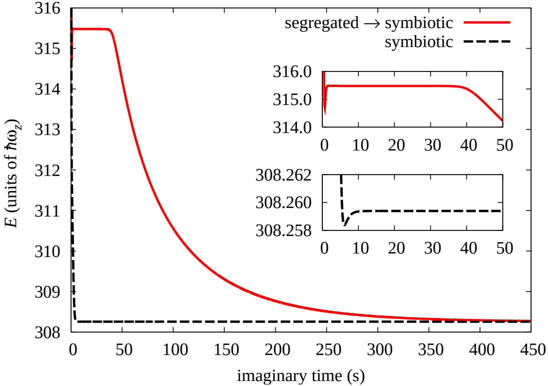


FIG. 1. Time evolution of an initial perfectly overlapped mixture without (a)–(c) and with (d) an applied axial magnetic gradient. Images taken after (a) 100 ms, (b) 1 sec, and (c)–(d) 9 sec of in-trap evolution.

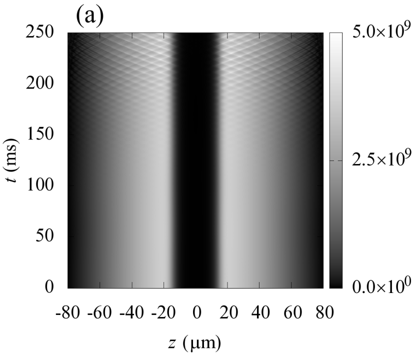
C. Hamner, J. J. Chang, P. Engels, M. A. Hoefer, PRL **106**, 065302 (2011)

Energy: time dependence



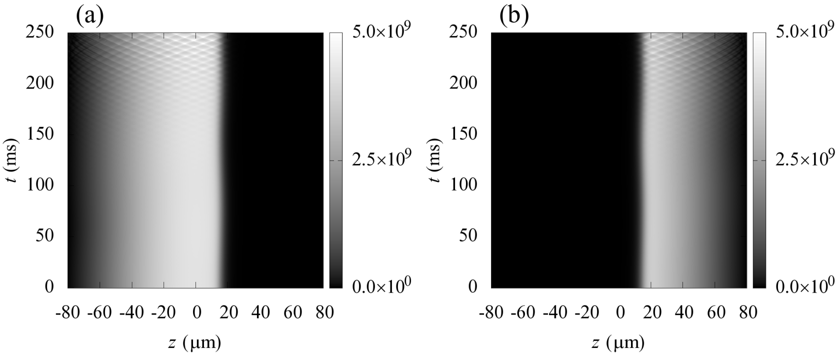
Convergence of the total energy of the system during the imaginary-time propagation for the symbiotic pair and for the segregated state.

Symbiotic pair ground state - Faraday waves



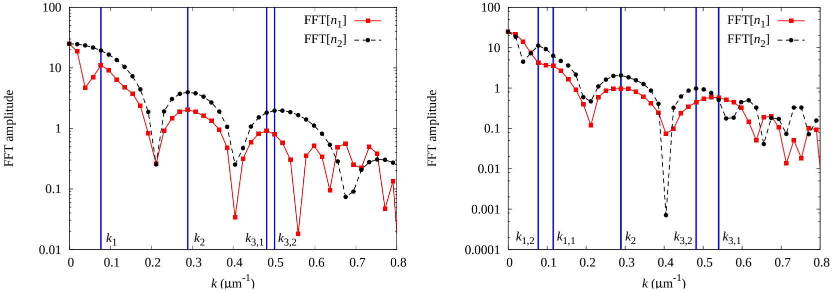
Emergence of Faraday waves as a result of real-time propagation. The radial frequency of the trap is modulated at the non-resonant frequency $\omega_m = 250 \cdot 2\pi$ Hz, $\epsilon = 0.1$.

Segregated state - Faraday waves



Emergence of Faraday waves as a result of real-time propagation. The radial frequency of the trap is modulated at the non-resonant frequency $\omega_m = 250 \cdot 2\pi$ Hz, $\epsilon = 0.1$.

Periods of Faraday waves



FFT of density profiles for the two condensates at $t = 200$ ms. For the symbiotic pair, the periods of waves are found to be 13.0 μm and 12.5 μm, while for the segregated state the periods are 11.6 μm and 13.0 μm.

Variational approach - symbiotic pair (1)

- Variational ansätze for the symbiotic pair wave functions:

$$\psi_1(\rho,z,t)=\mathcal{N}_1\exp\left(-\frac{\rho^2}{2w_\rho^2(t)}+i\rho^2\alpha^2(t)\right)\left[1-\exp\left(-\frac{z^2}{2w_z^2}\right)\right]$$

$$\psi_2(\rho,z,t)=\mathcal{N}_2\exp\left(-\frac{\rho^2}{2w_\rho^2(t)}-\frac{z^2}{2w_z^2}+i\rho^2\alpha^2(t)\right)\left[1+(u(t)+iv(t))\cos kz\right]$$

- ψ_1 is considered to be unperturbed, acting as an additional confinement for ψ_2
- Variational analysis leads to a Mathieu-type equation:

$$\ddot{u}(\tau)+u(\tau)\left[a(k,\omega)+\epsilon b\,(k,\omega)\sin 2\tau\right]=0$$

$$a(k,\omega)=\frac{k^4}{\omega^2}+\frac{k^2}{\omega^2}\Lambda_{\mathrm{sym}}\,,\quad b(k,\omega)=\frac{k^2}{\omega^2}\Lambda_{\mathrm{sym}}\,,\quad \omega t=2\tau$$

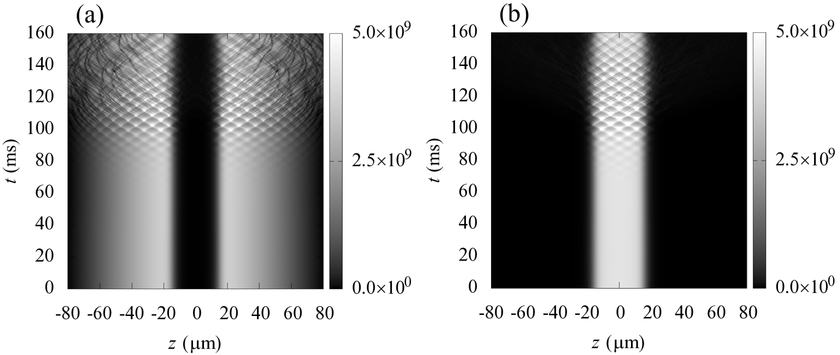
Variational approach - symbiotic pair (2)

- For small positive ϵ and positive $b(k,\omega)$, the Mathieu equation has solutions of the form $\exp(\pm i\mu\tau)\sin\sqrt{a}\tau$ and $\exp(\pm i\mu\tau)\cos\sqrt{a}\tau$, where $\mathrm{Im}[\mu]$ consists of a series of lobes positioned around the solution of the equation $a(k,\omega)=n^2$
- The lobe centered around $a(k,\omega)=1$ is the largest, and yields the most unstable solutions, determined by:

$$k_{\mathrm{F,sym}} = \sqrt{-\frac{\Lambda_{\mathrm{sym}}}{2} + \sqrt{\frac{\Lambda_{\mathrm{sym}}^2}{4} + \omega^2}}$$

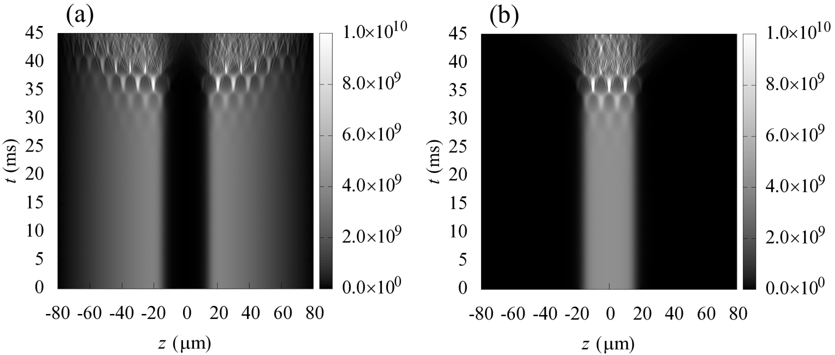
- This dispersion relation yields a period of $12.0\,\mu$m for the Faraday waves, which is in excellent agreement with the numerical results

Self-resonance - symbiotic pair



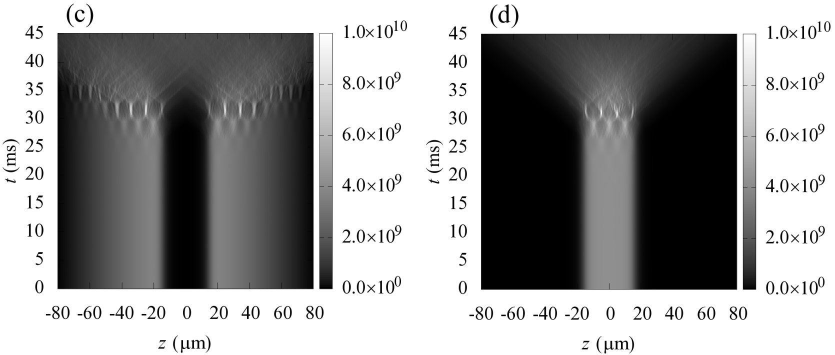
Emergence of Faraday waves as a result of real-time propagation. The radial frequency of the trap is modulated at the resonant frequency $\omega_m = 160 \cdot 2\pi$ Hz, $\epsilon = 0.1$.

Second resonance - symbiotic pair (1)



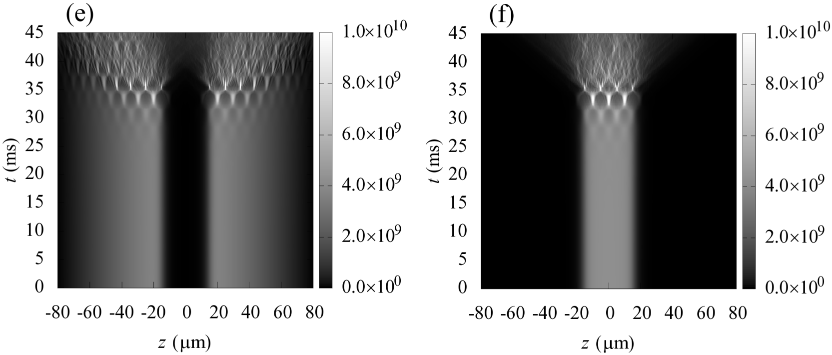
Emergence of resonant waves in a two-component BEC system radially modulated at $\omega_m = 300 \cdot 2\pi$ Hz.

Second resonance - symbiotic pair (2)



Emergence of resonant waves in a two-component BEC system radially modulated at $\omega_m = 320 \cdot 2\pi$ Hz.

Second resonance - symbiotic pair (3)



Emergence of resonant waves in a two-component BEC system radially modulated at $\omega_m = 340 \cdot 2\pi$ Hz.

Conclusions

- We have studied the emergence of surface waves in two-component BEC systems
- First, we have calculated two initial states of interest: ground state or symbiotic pair, and the first excited state or segregated state
- For non-resonant modulation of the radial confinement, the usual Faraday waves are observed, with the similar period in both components
- For the self-resonant modulation of the radial confinement, the expected resonant waves are observed
- For the second-harmonic resonance, much stronger and faster-emerging resonant waves are observed, turning the system from the non-miscible to the miscible state
- A. Balaž and A. Nicolin, PRA **85**, 023613 (2012)

Outlook

- Study of resonances using the Mathieu-type analysis
- Study of miscible two-component systems
- Pancake-shaped two-component systems
- Faraday waves for two-component BEC loaded into an optical lattice