Faraday and resonant waves in Bose-Einstein condensates

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Publications:

- A. I. NICOLIN, *Phys. Rev. E* ***84****, 056202 (2011)*
- A. I. NICOLIN, *Physica A* ***391****, 1062 (2012)*

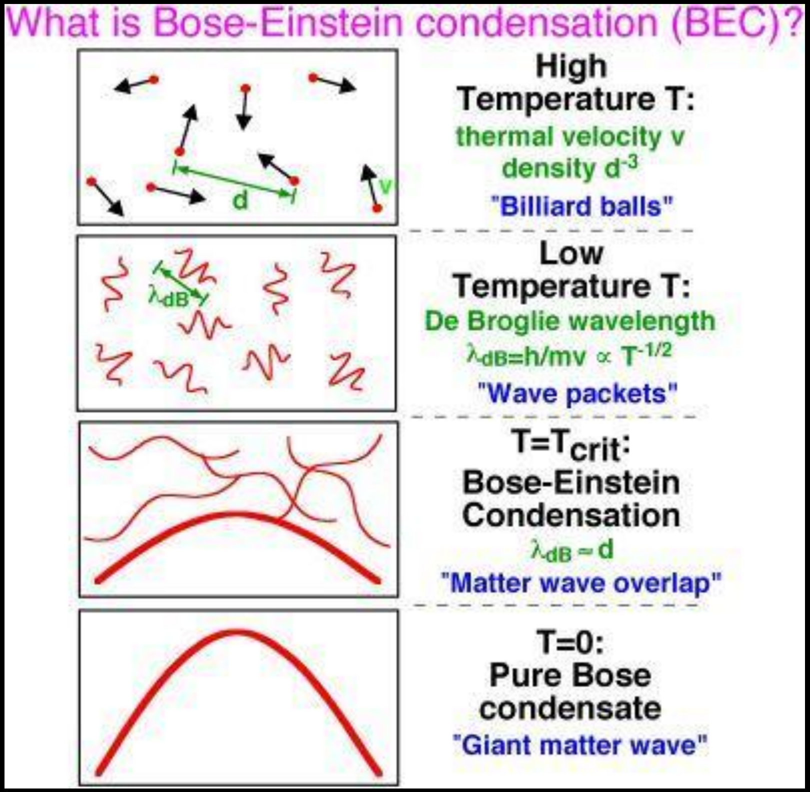
Collaborations:

- Mihaela-Carina RAPORTARU, *NIPNE, Bucharest, Romania*
  Physica A ***389****, 4663 (2010)*
- Antun BALAŽ, *Institute of Physics, Belgrade, Serbia*
  Phys. Rev. A ***85****, 023613 (2012)*

Overview

- Basic theory of Faraday patterns
- Prediction of Faraday patterns in Bose-condensed gases
- Experimental observation of Faraday waves
- Variational treatment of Faraday waves
- Faraday waves in dipolar Bose-condensed gases
- Conclusions

Formation of Bose-Einstein condensates



- For high temperatures (of the order of room temperature) the bosonic gas behaves classically; the behavior is that of "billiard balls"
- For low temperatures atoms can be considered quantum wave packet whose spatial extend is close to the thermal de Broglie wavelength
- When a gas of bosons is cooled below a critical temperature T_c most of the atoms condense into the lowest quantum state
- For T=0 all bosons are in the minimum energy state and form a perfect Bose-Einstein condensate

The Gross-Pitaevskii equation

$$i\hbar\frac{\partial\psi}{\partial t} = \left[-\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{r}) + gN\left|\psi\right|^2\right]\psi$$

Faraday patterns

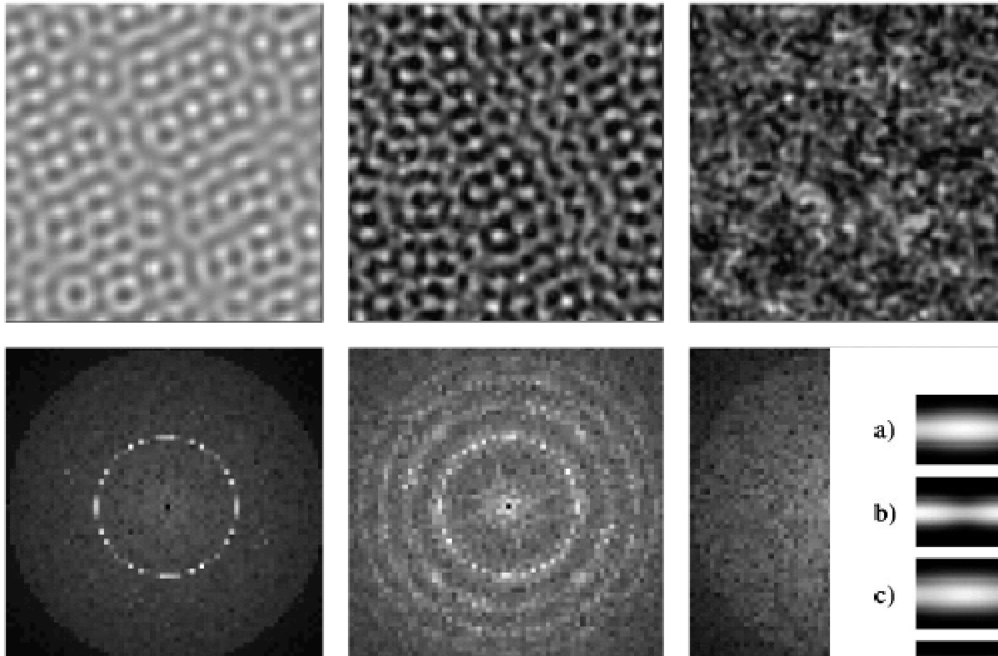
- The first work is due to Faraday. The Appendix of *On a peculiar class of acoustical figures and on certain forms assumed by a group of particles upon vibrating elastic surfaces, Philos. Trans. R. Soc. London* ***121****, 299 (1831)* is now classic:

  "When the upper surface of a plate vibrating so as to produce sound is covered with a layer of water, the water usually presents *a beautifully crispated appearance* […] *Too much water sometimes interferences with the beauty of the appearance*, but the crispation is not incompatible with much fluid." [crispation=undulation]

- Faraday patterns became a standard topic in nonlinear physics due to experiments with liquids and colloids in the 80s

- The main difference between a Faraday pattern and other non-stationary patterns is that *a Faraday pattern has an intrinsic frequency half that of the drive*

Faraday patterns in BECs

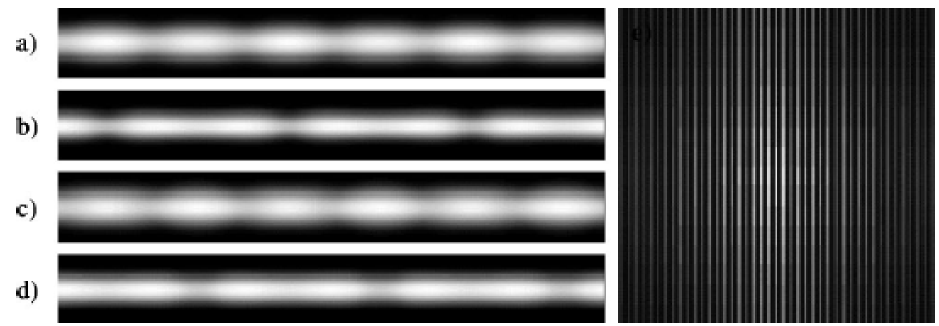


K. Stalinunas *et al.*, Phys. Rev. Lett. **89**, 210406 (2002).

Pancake-shaped condensate

FIG. 2. E... rically ... as obtaine... of Eq. ... potential w... ons, for $\omega = 1.5\pi$. ... ysical s ... row: distri... The sn... take at times: (a) $t = 100$, (b) $t = 200$, (c) $t = 3$... component in momentum space pictures is remov...

K. Staliunas *et al.*, Phys. Rev. A **70**, 011601(R) (2004).



Cigar-shaped condensate

...en at every 1/2 ... e) BEC density in momentum space (density of the spatial Fourier spectrum of the BEC wave function) corresponding to snapshot (a). Plots are obtained by numerical integration of Eq. (2) with periodic boundary conditions in both directions, and with the trapping potential in the vertical (Y) direction. The trap modulation frequency is $\omega = 0.62(\omega_{\text{breath}} \approx 1.77)$. Other parameters are $\alpha = 0.5$, $\gamma = 0.01$, and $\mu = 1.54$. The spatial grid is 256×32 (aspect ratio: 8:1). The size of integration space along the horizontal (X) coordinate is 176. The mode $n = 3$ of periodic boundaries (along the X axis) is excited.

Faraday patterns in BECs

P. Engels *et al.*, Phys. Rev. Lett. **98**, 095301 (2007).

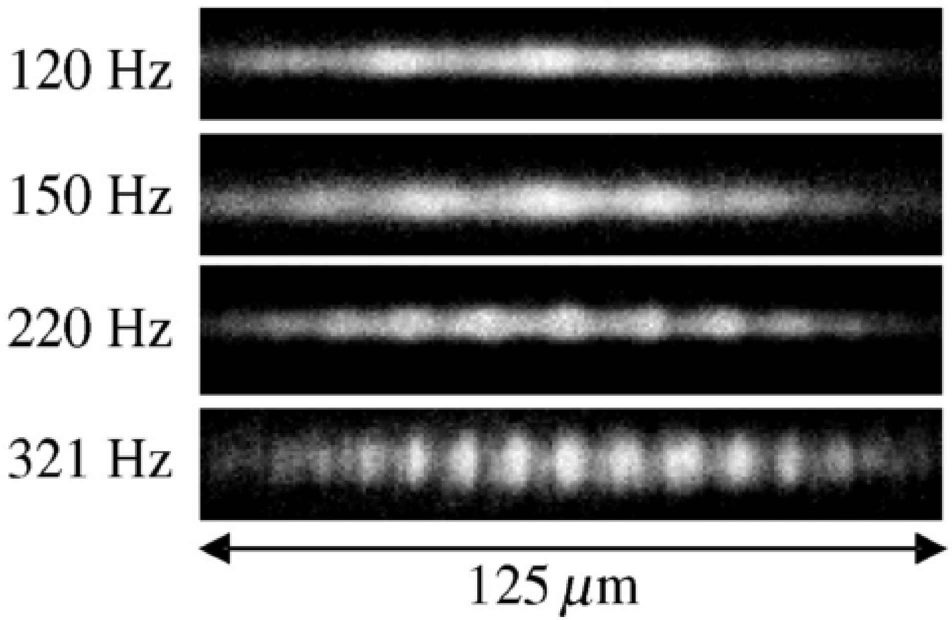


FIG. 1. In-trap absorption images of Faraday waves in a BEC. Frequency labels for each image represent the driving frequency at which the transverse trap confinement is modulated.

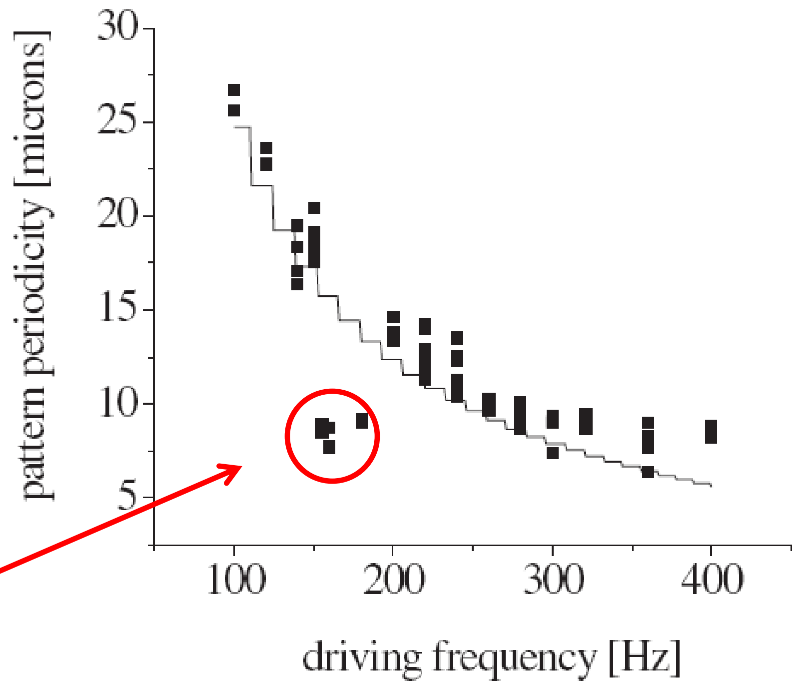


FIG. 2. Average spacing of adjacent maxima of the longitudinal patterns plotted versus the transverse driving frequency. Points are experimental data, while the line shows the theoretical values calculated for the longitudinal modes closest to half the driving frequency.

Resonant waves whose intrinsic frequency is equal to that of the drive

Faraday patterns in BECs

$$i\hbar\frac{\partial\psi}{\partial t} = \left[-\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{r}) + gN\left|\psi\right|^2\right]\psi$$

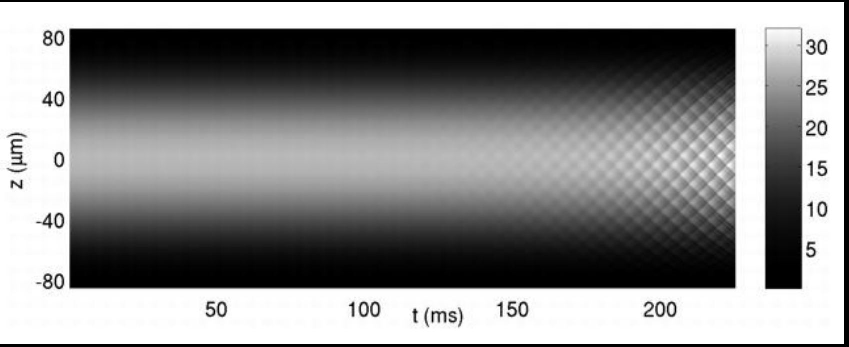
$$V(r,z) = \frac{1}{2}m\omega_r^2 r^2 + U(z)$$

$$\psi(\mathbf{r},t) = \phi(r,t;\sigma(z,t))f(z,t)$$

$$\phi(r,t;\sigma(z,t)) = \frac{\exp\left[-r^2/(2\sigma^2(z,t))\right]}{\sqrt{\pi}\sigma(z,t)}$$

$$i\hbar\frac{\partial f}{\partial t} = \left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2} + \hbar\omega_r\frac{1+3a_sN\left|f\right|^2}{\sqrt{1+2a_sN\left|f\right|^2}}\right]f$$

L. Salasnich, Laser Phys. **12**, 198 (2002); L. Salasnich, *et al.*, Phys. Rev. A **65**, 043614 (2002)



A.I. Nicolin, R. Carretero-Gonzales, and P.G. Kevrekidis, Phys. Rev. A **76**, 063609 (2007)

q-Gaussian based NPSEs:

$$\psi(\mathbf{r},t) = \phi(r,t;a(z,t),q(z,t))f(z,t),$$
$$\phi(r,t;a,q) = c(1-r^2a(1-q))^{1/(1-q)},$$

$$i\hbar\frac{\partial f(z,t)}{\partial t} = \left\{-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2} + 2\hbar\omega_\perp\left[\sqrt{a_s\left|f(z,t)\right|^2 N} - \frac{2^{\frac{1}{3}}}{3}\left(a_s\left|f(z,t)\right|^2 N\right)^{\frac{1}{6}}\right]\right\}f(z,t)$$

A.I. Nicolin and M.C. Raportaru, Physica A **389**, 4663 (2010).

Variational treatment

$$\mathcal{L}(\mathbf{r},t) = \frac{i}{2}\left(\psi\frac{\partial\psi^*}{\partial t} - \psi^*\frac{\partial\psi}{\partial t}\right) + \frac{1}{2}\left|\nabla\psi\right|^2 + V(\mathbf{r},t)\left|\psi\right|^2 + \frac{g(t)N}{2}\left|\psi\right|^4$$

$$\psi(\mathbf{r},t) = f(k, w_z(t), w_r(t), u(t), v(t)) \exp\left(-\frac{r^2}{2w_r^2(t)} + ir^2\beta(t)\right) \times \exp\left(-\frac{z^2}{2w_z^2(t)} + iz^2\alpha(t)\right)(1 + (u(t) + iv(t))\cos kz)$$

Assuming that the period of the wave is smaller than the longitudinal width of the condensate we have:

$$\begin{aligned}
\dot{w}_r(t) &= 2w_r(t)\alpha(t),\\
\dot{w}_z(t) &= 2w_z(t)\beta(t),\\
\dot{\beta}(t) &= -\frac{\Omega_z^2(t)}{2} + \frac{1}{2w_z^4(t)} + \frac{Ng(t)}{4\sqrt{2}\pi^{3/2}w_r^2(t)w_z^3(t)} - 2\beta^2(t)\\
\dot{\alpha}(t) &= -\frac{\Omega_r^2(t)}{2} + \frac{1}{2w_r^4(t)} + \frac{Ng(t)}{4\sqrt{2}\pi^{3/2}w_r^4(t)w_z(t)} - 2\alpha^2(t)
\end{aligned}$$

These are the well-known equations of the collective modes

$$\begin{aligned}
\dot{v}(t) &= -u(t)\left(\frac{k^2}{2} + \frac{Ng(t)}{\sqrt{2}\pi^{3/2}w_r^2(t)w_z(t)}\right)\\
\dot{u}(t) &= \frac{k^2}{2}v(t).
\end{aligned}$$

These are the equations of the surface wave

Variational treatment

Under the assumption that the longitudinal extent of the condesate is constant the amplitude of the surface wave is described by:

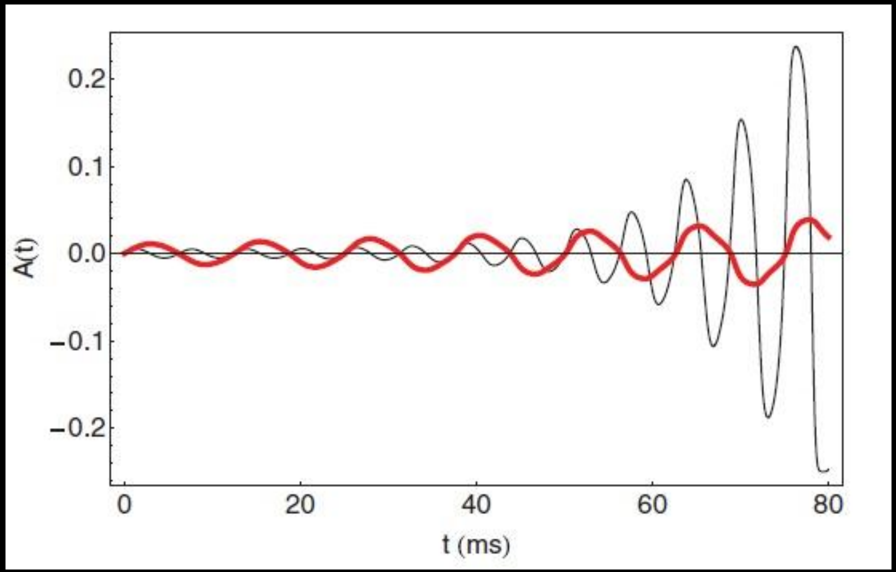
$$\frac{d^2u(\tau)}{d\tau^2} + [a(k,\omega) + b(k,\omega)\sin 2\tau]\,u(\tau) = 0$$

where $a(k,\omega)$ and $b(k,\omega)$ deppend on the geometry of the experimental setup

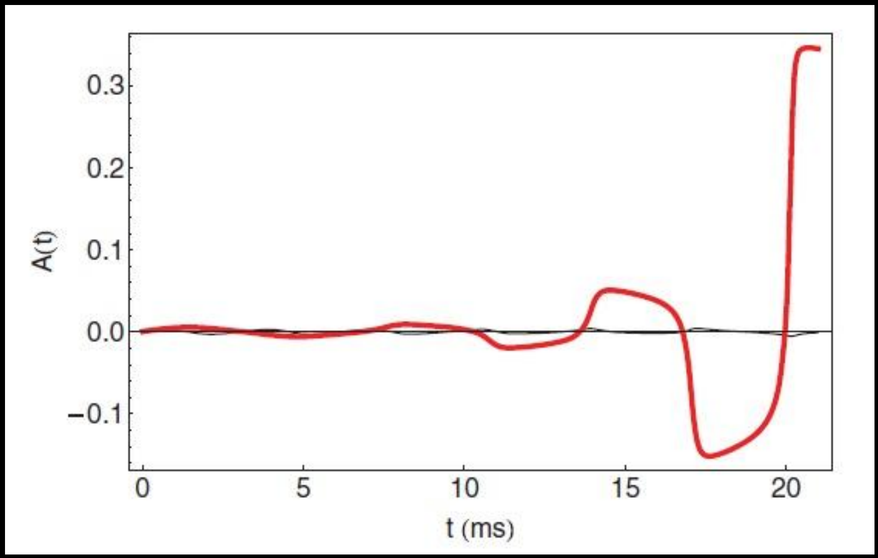
$a(k,\omega) = 1$ corresponds to **Faraday waves**

$a(k,\omega) = 2^2$ corresponds to **resonant waves**

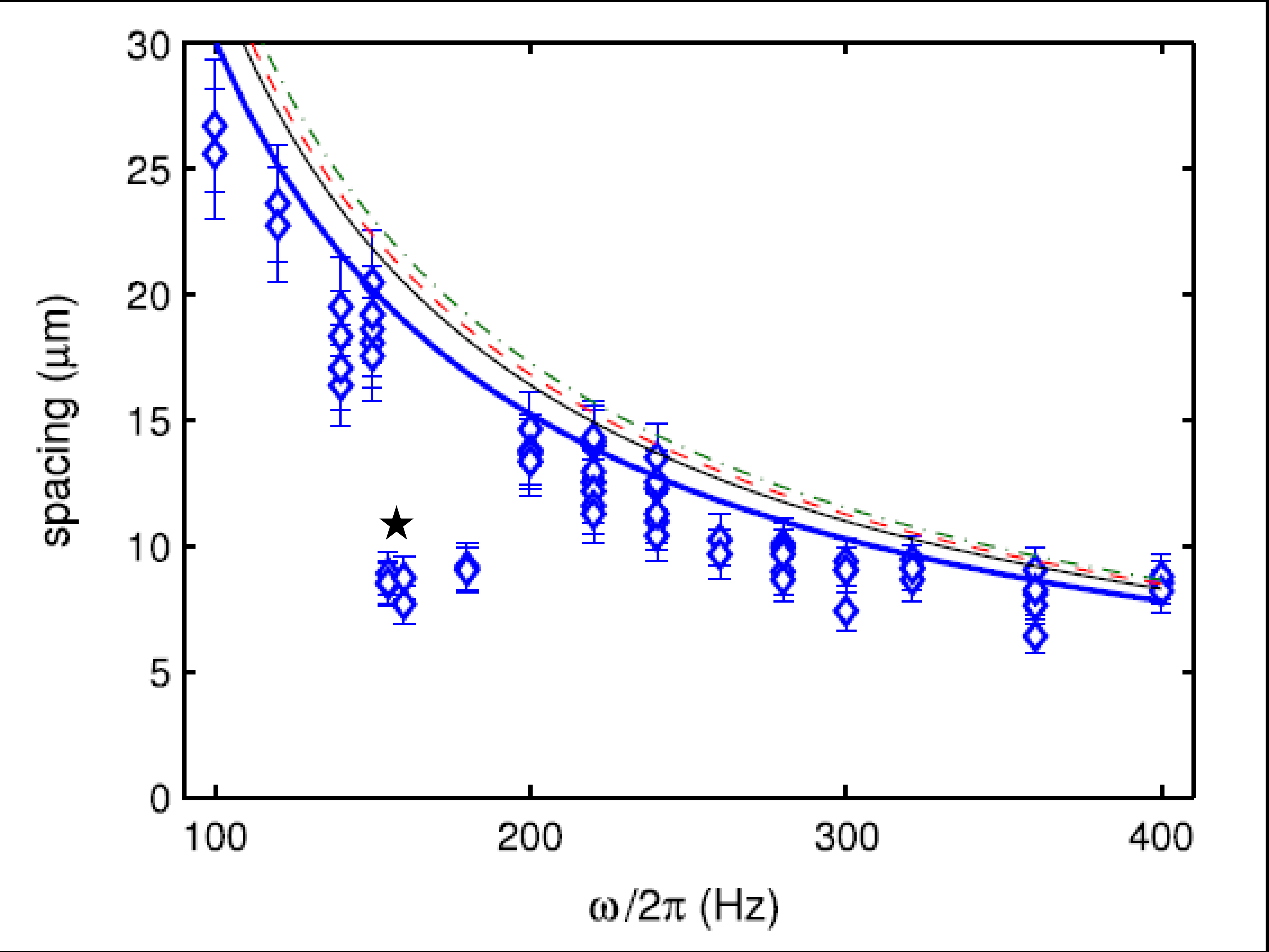
$\omega = \Omega$



$\omega = 2\Omega$



Variational treatment



Dipolar condensates

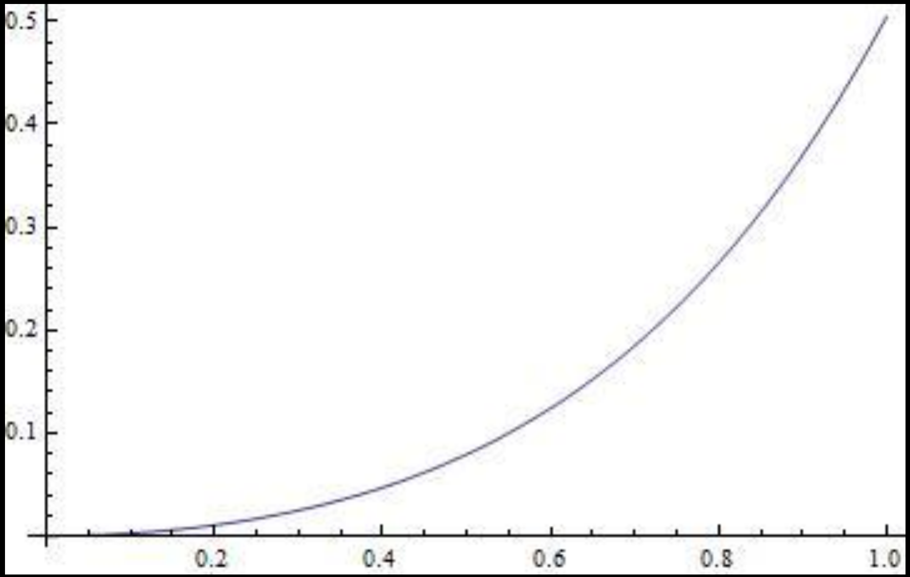
$$i\hbar \frac{\partial}{\partial t}\psi(\mathbf{r},t) = \left\{ -\frac{\hbar^2}{2m}\nabla^2 + \frac{m}{2}(\omega_\rho^2\rho^2 + \omega_z^2 z^2) + g|\psi(\mathbf{r},t)|^2 + d^2 \int d\mathbf{r}' \frac{1-3\cos^2\theta}{|\mathbf{r}-\mathbf{r}'|^3}|\psi(\mathbf{r}',t)|^2 \right\} \psi(\mathbf{r},t)$$

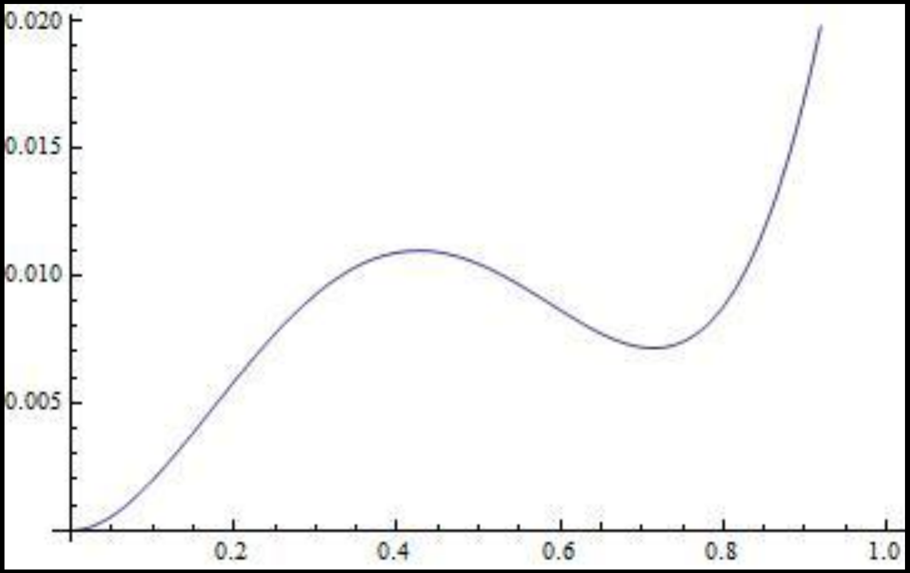
Faraday patterns in coupled one-dimensional dipolar condensates – arXiv:1207.1999v2

$$\frac{d^2u(\tau)}{d\tau^2} + [a(k,\omega) + b(k,\omega)\sin 2\tau]\, u(\tau) = 0$$

$a(k,\omega)$ looks like:

This is done variationally!





Conclusions and outlook

- We have addressed theoretically Faraday waves in one-dimensional one-component BECs
  - We have obtained fully analytical results using the theory of the Mathieu equations
  - We have derived a set of ODEs that describe consistently the dynamics of the bulk of the condensate and that of the Faraday waves
- Future veins of research include
  - Faraday pattens in pancake-shaped two-components condensates (full numerical simulations and effectively 2D NPSEs)
  - Faraday waves/patterns in dipolar BECs