1. Потписани захтев кандидата за покретање поступка
Никола З Петровић

Научном Већу Института за Физику, Београд

Београд 28.10.2018.

Предмет: Молба за покретање избора у звање виши научни сарадник за Николу З. Петровића

Молим Вас да у складу са Законом о научном раду као и Правилником о поступку, начину вредновања и квантитативном исказивању научноистраживачких резултата истраживача покренете мој избор у звање ВИШИ НАУЧНИ САРАДНИК. Одлука о мом избору у звање научни сарадник донесена је 28. Априла 2014. Године и овај захтев за избор се покреће у тренутку који одговара редовној процедури.

У прилогу:

1. Потписани захтев кандидата за покретање поступка.
2. Мишљење руководиоца пројекта са предлогом комисије која ће писати извештај. Ово мишљење садржи и потврду о руковођењу задатком на пројекту на којем сам ангажован.
3. Стручна биографија кандидата.
4. Преглед научне активности кандидата – преглед главних истраживачких тема и постигнутих резултата са нагласком на период након претходног избора.
5. Елементи за квалитативну анализу рада кандидата разврстани по ставкама у складу са Прилогом 1 Правилника, уз **обавезне доказе** за сваку од наведених ставки.
6. Елементи за квантитативну анализу рада кандидата приказани у виду одговарајућег дела табеле из Прилога 4 Правилника, разврстани у складу са Прилозима 2 и 3 Правилника.
7. Списак објављених радова и других публикација разврстан по важећим категоријама прописаним Правилником. Приликом избора у звања виши научни сарадник и научни саветник, потребно је јасно издвојити радове објављене након претходног избора у звање.
8. Подаци о цитираности кандидата, посебно број цитата без аутоцитата. Приложени су и подаци из базе Web of Science.
9. За кандидате који се бирају у звање научни сарадник или се први пут бирају у звање у Србији потребно је приложити и докторску диплому (или уверење о докторирању), која мора бити нострификована, кад су у питању кандидати који су је стекли у иностранству.
   Није приложено - Ово није први избор у звање!

10. Копије објављених радова и других публикација након претходног избора у звање (верзије из часописа, зборника апстраката, итд.).
11. Решење о претходном избору у звање (за кандидате који већ имају научно звање приликом избора у више звање или реизбора).

др Никола З Петровић,

научни сарадник

Институт за физику, Београд

2. Предмет: Мишљење руководиоца пројекта др Душана Јовановића о покретању избора у звање виши научни сарадник за Николу З. Петровића

2.а Покретање поступка је у редовном року-кандидат је изабран у звање 30. Априла 2014.

Научном већу Института за физику, Београд

Београд 25.11.2018.

Предмет: Мишљење руководиоца пројекта о покретању избора у звање виши научни сарадник за Николу З. Петровића

Др Никола З. Петровић је ангажован на пројекту 171006 под руководством др Душана Јовановића под насловом: "Нелинеарна динамика локализованих самоорганизованих структура у плазми, нано-композитним материјалима, течним и фотоничним кристалима и ултрахладним кондензатима."

Никола Петровић се бави математичком физиком примењеном пре свега на системе у нелинеарној оптици.

На овом пројекту Никола Петровић руководи задатком: ''Аналитичка решења нелинеарних једначина у оптици.''

Мишљења сам да је колега Петровић задовољио све услове прописане Законом о научном раду као и Правилником о поступку, начину вредновања и квантитативном исказивању научноистраживачких резултата истраживача и показао значајан степен самосталности у научном раду те подржавам покретање поступка за његов избор у звање виши научни сарадник.

За састав комисије за избор др Николе З. Петровића у звање виши научни сарадник предлажем:

1. Др Милана Петровића, научног саветника Института за физику Београд
2. Др Најдана Алексића, научног саветника Института за физику Београд
3. Др Жељка Шљиванчанина, научног саветника ИНН Винча

Руководилац пројекта 171006

Душан Јовановић

Проф. др Душан Јовановић,
научни саветник и редовни професор

3. Стручна биографија кандидата Никола З Петровић

Никола Петровић је рођен 12. 03. 1980. године у Београду.

Завршио је Математичку Гимназију 1999. године као ученик генерације са просеком 5.00. У гимназији се такмичио на такмичењима из физике и математике на свим нивоима националних такмичења. На међународним такмичењима је освојио више медаља од којих треба издвојити две сребрне и једну бронзану медаљу на Међународним олимпијадама из математике. Захваљујући тим успесима постао је стипендиста Министарства за науку и технолошки развој.

Дипломирао је физику и математику у јуну 2003. године на Масачусетс институту за технологију (Massachusetts Institute of Technology) са просеком 4.5 (на скали од 0 до 5). Дипломски рад је био на тему кодова за исправљање грешака у квантним компјутерима: "Constructing an Infinite Class of Perfect Codes", са оценом Б (9). Ментор је био проф. Исак Чуанг (Isaac Chuang).

Објавио је са још три коаутора књигу "The IMO Compendium" са свим задацима предложеним на Међународним математичким олимпијадама (Springer Verlag, Berlin, 2006). Дугогодишњи је сарадник Истраживачког центра Петница и члан Државне комисије за такмичења из математике. Као члан комисије за такмичења из физике учествовао је у припреми и оцени задатака на националним такмичењима.

Од 2004. године Никола Петровић је у радном односу са Институтом за Физику у Београду. Његов статус је замрзнут од августа 2005. године када одлази на Тексашки A&M универзитет у Катару (Texas A&M University at Qatar) где је запослен као лабораторијски координатор и ради такође као асистент све до јула 2012. године, када се враћа у Институт за Физику. У септембру 2012. године је изабран у звање истраживача сарадника. Докторску дисертацију под насловом: "Тачна таласна и солитонска решења генералисане нелинеарне Шредингерове једначине" је одбранио 16. октобра 2013. године на Физичком факултету Универзитета у Београду.

У звање научни сарадник изабран је 3. маја 2014. У Београду. Ангажован је на пројекту 171006 под руководством др Душана Јовановића 'Нелинеарна динамика локализованих самоорганизованих структура у плазми, нано-композитним материјалима, течним и фотоничним кристалима и ултрахладним кондензатима'.

Ожењен је са супругом Ташаном и има двоје деце: Бориса и Емилију.

4. Никола З Петровић: Преглед научне активности

Општи подаци о активностима Никола З Петровића

Под руководством ментора проф. др Миливоја Белића, Никола Петровић је почео 2005. године да се бави и истраживањима у нелинеарној оптици.

Никола Петровић се бави проналажењем егзактних решења за широке класе нелинеарних еволутивних парцијалних диференцијалних једначина, углавном користећи се методом експанзије по Јакобијевим елиптичким функцијама. Овај метод примењен је досад на неколико облика нелинеарне Шредингерове једначине, као и на једначину Грос-Питаевског. Такође, досад је користио и такозвану само-сличну методу и уз то радио линеарну анализу стабилности добијених решења. Тренутно је ангажован на пројекту Министарства просвете и науке ОИ171006 под руководством др Душана Јовановића.

Никола Петровић је досад објавио 20 радова у угледним међународним часописима, од којих је један објављен у престижном часопису Physics Review Letters. Укупан број цитата његових радова до сада је 352, према ISI Web of Knowledge.

Важно је поред научних резултата напоменути и вишегодишњи педагошки рад др Николе Петровића као асистента на Тексас A&M универзитету, где је за 7 генерација студената држао вежбе и лабораторију из механике, електромагнетике и модерне физике. У току свог истраживачког рада, др Никола Петровић је такође био и ментор најталентованијим студентима, који су под окриљем проф. др Миливоја Белића добили прилику да учествују у научном истраживању и буду ко-аутори на Николиним радовима.

Активности пре избора у звање научни сарадник и опис његових доприноса

Никола Петровић се у свом раду бавио применом и модификацијом такозване методе развоја по Јакобијевим елиптичним функцијама, у циљу проналажења нових класа егзактних и аналитичких решења мултидимензионих генералисаних нелинеарних Шредингерових једначина, као и других једначина. Кључни допринос кандидата је била генерализација методе Јакобијевих елиптичних функција на нелинеарну Шредингерову једначину са кубичном нелинеарношћу у 3 димензије, која је дотад претходно примењена на 2 димензије, у раду у којем је и Никола Петровић био укључен. Рад у коме су ови резултати презентовани је објављен у Physical Review Letters и постао је високо цитирани рад који је отворио целу једну подобласт математичке физике. Добијени су и тамни и светли солитони, у оба случаја са и без просторног чирпа. Контролишући параметар Јакобијевих функција добија се солитонски талас као гранични случај решења која описују бесконачан низ путујућих таласа. Добијена решења имају велику флексибилност у зависности од параметара једначине – коефицијената дифракције, нелинеарности, и губитака; једино један од три параметра мора бити дефинисан у функцији осталих. За разлику од претходних радова са једначином у 2 димензије, у овом новом раду је улога чирп функције коначно разјашњена.

У наредним радовима је метода Јакобијевих елиптичних функција модификована да би се пронашла решења за случај нормалне дисперзије, који има много ширу физичку примену од случаја аномалне дисперзије и дотад није био урађен. Др Петровић је открио на који начин да се промени облик решења како би се узела у обзир анти-симетрија времена у односу на остале трансферзне варијабле. Иако се физички систем нормалне дисперзије квалитативно знатно разликује од случаја аномалне дисперзије, показало се да се модификацијом само неколико параметара могу добити решења и за овај случај.

Метода Јакобијевих елиптичних функција је затим генералисана на системе са нелинеарношћу вишег степена, пригодном модификацијом степена решења. Уз одређене специфичне услове пронађена су солитонска решења и за кубично-квинтични (qubic-quintic) и за кубично-квинтично-септични (qubic-quintic-septic) модел. Ово истраживање је отворило могућност евентуалног налажења решења са сатурабилном нелинеарношћу.

Потом је Никола Петровић применио методу на једначину Грос-Питаевског (Gross-Pitaevski), која има облик нелинеарне Шредингерове једначине са укључујеним параболичним потенцијалом. Никола Петровић је установио да је најпре потребно решити такозвану Рикатијеву (Riccati) диференцијалну једначину да би се добило решење једначине Грос-Питаевског. С обзиром на то да је Рикатијеву једначину немогуће решити у општем случају, кандидат је истражио случајеве који имају позната решења а од физичког су значаја. За константне вредности параметра дифракције и јачине потенцијала добио је решења која опадају или имају сингуларитет и утврдио да је решења која опадају могуће стабилизовати додатним напајањем енергије (gain) у тачно одређеној мери. Са друге стране, за синусоидни облик параметра дифракције и јачине потенцијала добио је стабилна таласна и солитонска решења. Др Петровић и студент Анас Ал Бастами, коме је Никола био ментор, су утврдили да је могуће за компликованије облике параметара свести Рикатијеву једначину на решиву линеарну једначину другог степена, у ком случају се добија широка класа нових решења једначине Грос-Питаевски, укључив и решења за случај Фешбахове (Feschbach) резонанце. Решења добијена би могла имати широку примену са обзиром на то да се једначина Грос-Питаевски користи у проучавању Боз-Ајнштајнових (Bose-Einstein) кондензата.

Кандидат је даље модификовао методу за случај да потенцијал није параболички него линеаран и у том случају су пронађена решења за константну вредност параметра дифракције и јачине потенцијала, за синусоидалан облик ова два параметара, као и за оба мешана случаја, тј. кад је један од параметара константан а други синусоидалан.

Метода је од стране Др Петровића такође по први пут примењена и на двокомпонентне, тзв. Манаковљеве системе, тачније на пар ко- и контра-пропагирајућих таласа. Пронађена су решења за случај кад је однос укрштено-фазне (cross-phase) и само-фазне (self-phase) модулације једнак 3. Упркос томе што није било могуће овом методом добити општа решења Манаковљевог система, системе са овим односом двеју модулација је могуће направити.

Активности после избора у звање научни сарадник и опис пет истакнутих радова из тог периода

Од претходног избора Никола Петровић је објавио десет радова у међународним часописима од чега 5 у часописима категорије М21а и два у часописима категорије М21. Као посебно значајни истичу се следећих пет радова од којих прва два демонстрирају доминантан удео кандидата те представљају доказ његове самосталности

Радови са доминантном улогом кандидата

1. Др Никола Петровић у овом периоду довршио рад [19] у којем се анализира стабилност многобројних решења које је добио методом развоја по Јакобијевим елиптичним функцијама објавио у неколико високо цитираних радова у престижним часописима у периоду од 2008. до 2011. године. У сарадњи са др Најданом Алексићем и проф. др Миливојем Белићем, урађена је анализа стабилности решења нелинеарне Шредингерове једначине са нормалном и аномалном дисперзијом и једначине Грос-Питајевског. Најпре је урађена трансформација која нелинеарну Шредингерову једначину са дистрибуираним коефицијентима своди на једначину са константним коефицијентима. Затим је конструисан одговарајући Лагранжијан и под претпоставком постојања модулационе нестабилности су добијене једначине за њихову целокупну амплитуду, тј. њен реалан и имагинаран део, у функцији од таласног броја пертурбација. Затим је систем једначина решен да би се добило да ли параметри дивергирају или не и тиме одредило да ли решења имају стабилност. Утврђено је да у свим случајевима решења поседују или апсолутну стабилност или стабилност уз присуство такозваног менажирања дисперзије, тј. алтернирања знака коефицијента дисперзије уз помоћ метаматеријала. Апсолутна стабилност је утврђена у три димензије за тамне солитоне у аномалној дисперзији, и за светле временске солитоне у нормалној дисперзији, док је у две димензије апсолутна стабилност утврђена за све тамне солитоне. Ови резултати су проверени компјутерским симулацијама и добијено

је скоро потпуно слагање у решењима без чирпа и изузетно добро квалитативно слагање које у сваком случају потврђује критеријуме апсолутне стабилности у решењима са чирпом. Др. Никола Петровић је као први аутор учествовао у свим аспектама овог рада осим компјутерских симулација.

2. Др Никола Петровић је у овом периоду написао и рад [18] у коме је једини аутор. Он је генералисао своје методе зна системе нелинеарних Шредингерових једначина где степен нелинеарности није цео број, као и где постоје два члана, један са дупло већим степеном од другог. Ово је урађено помоћу трансформације која је сводила систем на систем са коефицијентима целобројног степена. Посебна пажња је посвећена такозваним кубично-квинтичним системима код којих су нађене велике класе нових решења јер се случај са тим вредностима испоставља као специјалан случај. Добијена су не само решења заснована на Јакобијевој елиптичној функцији, него и решења која садрже такозвани чирп. Наравно, сви прорачуни и резултати у раду су изведени од стране Др. Николе Петровића.

Опис осталих репрезентативних радова

Никола Петровић се у почетку у свом раду бавио применом и модификацијом такозване методе развоја по Јакобијевим елиптичним функцијама, у циљу проналажења нових класа егзактних и аналитичких решења мултидимензионих генералисаних нелинеарних Шредингерових једначина, као и других једначина. Након почетних резултата укључених у његову докторску дисертацију, он је проширио свој домен рада и продубио сарадњу са колегама из Кине које се баве сличном облашћу.

3. У сарадњи са професором Веипинг Жонгом (Wei-Ping Zhong), Никола је учествовао у раду на утврђивању постојања контролисаних параболично-цилиндричних дивљих таласа (rogue waves) [14]. Дивљи таласи су тренутно врло актуелна тема у свету нелинеарне оптике (а и шире) јер настају изненада и имају велики интензитет, те њихово проучавање је јако битно у циљу успешне примене нелинеарних оптичких система. У раду су добијени дивљи таласи чија амплитуда је пропорционална параболично-цилиндричној функцији. Др Никола Петровић је учествовао у налажењу и провери исправности датих решења.

Затим је др Никола Петровић учествовао у дугогодишњој и плодоносној сарадњи са физичарем из Кине Силију Суом (Si-Liu Xu). У серији од неколико радова др Никола Петровић је дао велики допринос у реализацији идеја, провери тачности, прављењу илустрација и писању радова које је заједно са др Суом објавио.

4. У раду [15] је коришћена такозвана самослична трансформација да би се добила решења нелинеарне тродимензионе Шредингерове једначине с четвртим степеном нелинеарности. Добијени су и тамни и светли солитони као решења за неколико различитих математичких облика коефицијента дифракције и проучавано је динамичко понашање светлости у датим срединама.

5. У раду [16] су нађена решења за (3+1)-димензиону нелинеарну Шредингерову једначину са нецелобројним степеном и такозваним ПТ (parity-time) симетричним потенцијалом. Урађена је трансформација сличности и добијене једначине такве да за сваки облик решења постоји одговарајући потенцијал такав да је оригинална нелинеарна Шредингерова једначина испуњена. Ово отвара могућност налажења решења локализованих у свим трансферзним координатама, такозваних светлосних метака.

Опис преосталих радова

У раду [17] из катерогије М21а, нађена су решења нелинеарне Шредингерове једначине четвртог степена у цилиндричним координатама. За параметар везан за амплитуду је добијена конфлуентна хипергеометријска диференцијална једначина чија су решења такозване Сонине функције. Утврђено је да су решења стабилна кад је тополошко наелектрисање мање од 1, а нестабилна кад је веће од 2.

У раду [20] из категорије (М21) нађена су решења у нелинеарној Шредингеровој једначини са ПТ-симетричним потенцијалном и супротстављеним нелинеарностима степена 3 и 2k+1. Добијена су локализована решења у свим координатама на бази хиперболичког секанса.

У раду [25] из категорије (М51) су нађена решења за нелокални и нелинеарни систем, дефинисан двема једначинама, једном за арешење и другом која одређује јачину индекса преламања у датој тачки. Добијена решења се заснивају на Јакобијевим елиптичним функцијама. Најдзад, урађена је основна анализа стабилности и утврђено да су за велике апсолутне вредности коефицијента дифракције решења стабилна, док у малим вредностима настају нестабилности.

У раду [21] из категорије (М22) су нађена решења за двокомпоненту нелинеарну Шредингерову једначину која су заснована на Перегриновим, Акмедијевим и Маовим решењима.

Коначно, у раду [22] из категорије (М23) је др Никола Петровић у сарадњи са својим студентом Моизом Бохром нашао решења заснована на општем облику елиптичне диференцијалне једначине где је квадрат извода једнак општем полиному четвртог степена оригиналне функције, дакле где се за разлику од једначине за Јакобијеву елиптичну функцију укључују чланови првог и трећег степена. Нађена су решења на основу Вајерштрасове елиптичне функције и на основу општих елиптичних функција које нису симетричне у односу на средњу вредност максимума и мимимума функције.

5. ЕЛЕМЕНТИ ЗА КВАЛТИТАТИВНУ ОЦЕНУ НАУЧНОГ ДОПРИНОСА КАНДИДАТА: НИКОЛА З ПЕТРОВИЋ

5.1 Квалитет научних резултата

5.1.1 Научни ниво и значај резултата, утицај научних радова

Др Никола З Петровић је у досадашњој каријери био аутор или коаутор уз давање кључног доприноса у укупно 20 рада и два рада са конференција, објављених у међународним часописима са ISI листе. Од тога је 9 радова у категорији M21a (међународни часописи изузетних вредности), 6 у категорији M21 (врхунски међународни часописи), 3 (1) у категорији M22 и 2 (1) у категорији M23.

У периоду након одлуке Научног већа о предлогу за стицање претходног научног звања, др Никола З Петровић је објавио 9 радова у часописима са ISI листе. Од тога је 5 у часописима категорије M21a (међународни часописи изузетних вредности), 2 у часописима категорије M21 (врхунски међународни часописи), 1 у часописима категорије M22 и 1 у часописима категорије M23. Такође је публиковао и један рад M51 у водећем индијском часопису из оптике.

Утицај научних радова се види и у секцији 5.1.2. кроз приказану цитираност. Одржао је и два предавања по позиву на научним скуповима.

Као најзначајнијих пет радова кандидата могу се узети (бројеви референце су конзистентни са коначном листом радова из секције 7):
[18] N. Z. Petrović, "Spatiotemporal traveling and solitary wave solutions to the generalized nonlinear Schrodinger equation with single-and dual-power law nonlinearity," Nonlinear Dynamics 93 (4), 2389-2397 (2018) IF=4.339 (8/134) SNIP=1.75

[19] N. Z. Petrović, N.B. Aleksić, M. Belić, "Modulation stability analysis of exact multidimensional solutions to the generalized nonlinear Schrödinger equation and the Gross-Pitaevskii equation using a variational approach," Optics Express 23 (8), 10616-10630 (2015) IF=3.148 (14/90) SNIP=1.67

[14] W. P. Zhong, L. Chen, M. Belić, N. Petrović, "Controllable parabolic-cylinder optical rogue wave," Phys. Rev. E 90 (4), 043201 (2014) IF=2.288 (5/54) SNIP=1.14

[15] S. L. Xu, N. Petrović, M. R. Belić, "Exact solutions of the (2+ 1)-dimensional quintic nonlinear Schrödinger equation with variable coefficients," Nonlinear Dynamics 80 (1-2), 583-589 (2015) IF=3.000 (8/135) SNIP=1.47

[16] S. L. Xu, N. Petrović, M. R. Belić, W. Deng, "Exact solutions for the quintic nonlinear Schrödinger equation with time and space," Nonlinear Dynamics 84 (1), 251-259 (2016) IF=3.464 (8/133) SNIP=1.54

Детаљан опис пет одабраних радова који је већ презентиран у одељку 4. а који укључује и два рада који се користе као доказ за самосталност кандидата а истовремено су из групације М21а или М21.

Радови са доминантном улогом кандидата

1. Др Никола Петровић у овом периоду довршио рад [19] (по референцама из комплетне листе радова) у којем се анализира стабилност многобројних решења које је добио методом развоја по Јакобијевим елиптичним функцијама објавио у неколико високо цитираних радова у престижним часописима у периоду од 2008. до 2011. године. У сарадњи са др Најданом Алексићем и проф. др Миливојем Белићем, урађена је анализа стабилности решења нелинеарне Шредингерове једначине са нормалном и аномалном дисперзијом и једначине Грос-Питајевског. Најпре је урађена трансформација која нелинеарну Шредингерову једначину са дистрибуираним коефицијентима своди на једначину са константним коефицијентима. Затим је конструисан одговарајући Лагранжијан и под претпоставком постојања модулационе нестабилности су добијене једначине за њихову целокупну амплитуду, тј. њен реалан и имагинаран део, у функцији од таласног броја пертурбација. Затим је систем једначина решен да би се добило да ли параметри дивергирају или не и тиме одредило д али решења имају стабилност. Утврђено је да у свим случајевима решења поседују или апсолутну стабилност или стабилност уз присуство такозваног менажирања дисперзије, тј. алтернирања знака коефицијента дисперзије уз помоћ метаматеријала. Апсолутна стабилност је утврђена у три димензије за тамне солитоне у аномалној дисперзији, и за светле временске солитоне у нормалној дисперзији, док је у две димензије апсолутна стабилност утврђена за све тамне солитоне. Ови резултати су проверени компјутерским симулацијама и добијено је скоро потпуно слагање у решењима без чирпа и изузетно добро квалитативно слагање које у сваком случају потврђује критеријуме апсолутне стабилности у решењима са чирпом. Др. Никола Петровић је као први аутор учествовао у свим аспектама овог рада осим компјутерских симулација.

2. Др Никола Петровић је у овом периоду написао и рад [18] у коме је једини аутор. Он је генералисао своје методе зна системе нелинеарних Шредингерових једначина где степен нелинеарности није цео број, као и где постоје два члана, један са дупло већим степеном од другог. Ово је урађено помоћу трансформације која је сводила систем на систем са коефицијентима целобројног степена. Посебна пажња је посвећена такозваним кубично-квинтичним системима код којих су нађене велике класе нових решења јер се случај са тим вредностима испоставља као специјалан случај. Добијена су не само решења заснована на Јакобијевој елиптичној функцији, него и решења која садрже такозвани чирп. Наравно, сви прорачуни и резултати у раду су изведени од стране др Николе Петровића.

Опис осталих репрезентативних радова

Никола Петровић се у почетку у свом раду бавио применом и модификацијом такозване методе развоја по Јакобијевим елиптичним функцијама, у циљу проналажења нових класа егзактних и аналитичких решења мултидимензионих генералисаних нелинеарних Шредингерових једначина, као и других једначина. Након почетних резултата укључених у његову докторску дисертацију, он је проширио свој домен рада и продубио сарадњу са колегама из Кине које се баве сличном облашћу.

3. У сарадњи са професором Веипинг Жонгом (Wei-Ping Zhong), Никола је учествовао у раду на утврђивању постојања контролисаних параболично-цилиндричних дивљих таласа (rogue waves) [14]. Дивљи таласи су тренутно врло актуелна тема у свету нелинеарне оптике (а и шире) јер настају изненада и имају велики интензитет, те њихово проучавање је јако битно у циљу успешне примене нелинеарних оптичких система. У раду су добијени дивљи таласи чија амплитуда је пропорционална параболично-цилиндричној функцији. Др Никола Петровић је учествовао у налажењу и провери исправности датих решења.

Затим је др Никола Петровић учествовао у дугогодишњој и плодоносној сарадњи са физичарем из Кине Силију Суом (Si-Liu Xu). У серији од неколико радова др Никола Петровић је дао велики допринос у реализацији идеја, провери тачности, прављењу илустрација и писању радова које је заједно са др Суом објавио.

4. У раду [15] је коришћена такозвана самослична трансформација да би се добила решења нелинеарне тродимензионе Шредингерове једначине с четвртим степеном нелинеарности. Добијени су и тамни и светли солитони као решења за неколико различитих математичких облика коефицијента дифракције и проучавано је динамичко понашање светлости у датим срединама.

5. У раду [16] су нађена решења за (3+1)-димензиону нелинеарну Шредингерову једначину са нецелобројним степеном и такозваним ПТ (parity-time) симетричним потенцијалом. Урађена је трансформација сличности и добијене једначине такве да за сваки облик решења постоји одговарајући потенцијал такав да је оригинална нелинеарна Шредингерова једначина испуњена. Ово отвара могућност налажења решења локализованих у свим трансверзним координатама, такозваних светлосних метака.

• Остале показатеље, подељене у две групе (А и Б), процењује МОФ:

1	А	до 5 изабраних радова - **Приказано 5 радова**
2	А	утицајност (узимајући у обзир и 2.6) **Већина радова из категорије М21а и М21. Два рада имају преко 100 цитата по систему Google Scholar.**

3	Б	додатни библиометријски показатељи* **Приказана табела**
4	Б	истакнутост, самосталност, дужина радова, **Радови у водећим часописима попут Physical Review Letter, радови са преко 100 цитата. Самосталност показана у часописима највишег ранга М21 и М21а, као и у осталим часописима**.
5	Б	примењивост, награде- **Обзиром да је рад претежно теоријског карактера не постоје техничке реализације. Ипак тематика се односи на простирање светлости поред осталог и кроз таласоводе као и реализацију квантно инжењерских система у оквиру оптике попут квантних рачунара па има индиректну примењивост.**

Додатни библиометријски показатељи (тачка 2 **П1П**) су:

	ИФ	М	СНИП
Укупно	24,526	76	11.19
Усредњено по чланку	2.725	7.6	1.24
Усредњено по аутору	10.35	30.75	4.629

5.1.2 Позитивна цитираност научних радова кандидата

Према бази WOS радови кандидата су цитирани укупно 352 пута, док је број цитата без аутоцитата 308. Према истој бази Н–индекс кандидата је 8.

Прилог: подаци о цитираности са интернет странице WOS.

На бази Google Scholar има 515 цитата (што укључује и 54 цитата књиге IMO Compendium) и Н фактор 10.

Није примећена ниједна инстанца негативне цитираности а у бројним случајевима методе из радова за које је основни доприност дао Никола З Петровић су примењиване у другим публикацијама.

5.1.3 Параметри квалитета часописа

И у периоду пре и у периоду после избора кандидат је већином објављивао радове у часописима категорије М21а и М21. Укупан фактор утицаја (збир импакт фактора) радова кандидата је **49,448**, а у периоду након одлуке Научног већа о предлогу за стицање претходног научног звања тај фактор је **24,526**. Кандидат је објављивао радове у најугледнијим часописима из његове области. Посебно се међу њима истичу: Phys. Rev. Lett., Nonlinear Dynamics, Physical Review A и E. и Optics Express.

У категорији M21a, M21, M22 и M23 кандидат је објавио радове у следећим часописима, где су Посебно означени они часописи у којима је кандидат објављивао у периоду након одлуке Научног већа о предлогу за стицање претходног научног звања:

Пре претходног избора:

- 1 рад у Phys. Rev. Lett 2008 M21a,
- 4 рада Phys. Rev. E (2010 3 puta 2011) M21a+ 3M21
- 1 рад Phys Rev A (2008) M21a,
- 1 рад Optics Letters (2009) M21a
- 1 rad Physica Scripta M22
- 1 рад Acta Physica Polonica A 212, 729 (2007) M23,
- 1 рад Electronic J Diff. Equations (2010) M23

Напомена: Импакт фактори за часописе у којима су публиковани радови пре избора у прошло звање су наведени у листи публикација.

После претходног избора (одлуке научног већа)

- 4 рада у Nonlinear Dynamics M21a (2015)(ИФ = 3.000 8/135), 2 x (2016)(ИФ = 3.464 8/133), (2018)(ИФ = 4.339), M21a
- 1 рад у Physical Review E M21 (2014) (ИФ = 2.288 4/54), M21a

- 1 рад у Europhysics Letters M21 (2016) (ИФ = 1.957 23/79), M21
- 1 рад у Optics Express M21 (2015) (ИФ =3.148 14/90),M21

- 1 рад у Journal of Optics M22 (2015) (ИФ = 1.847 36/90), M22
- 1 рад у Optical and Quantum Electronics M23 (2016) (ИФ = 1.055 70/92), M23

5.1.4 Степен самосталности и степен учешћа у реализацији радова у научним центрима у земљи и иностранству

Кандидат је једини аутор у једном раду у часопису M21a категорије а водећи аутор у осам радова (три од прошлог избора). Такође, после претходног избора има рад са студентом додипломских студија у часопису M23 категорије Optical and Quantum Electronics. Пре избора је био водећи аутора на шест радова, а има један рад само са студентом додипломских студија. На свим тим радовима је дао основни допринос реализацији целога рада а значајно је учествовао на свим осталим радовима.

Као други аутор дао је најважнији математички допринос раду публикованом у Phys. Rev. Lett. У свим радовима дао је основни допринос техникама за решавање нелинеарних једначина. Тиме се може закључити да он има јасан домен главних доприноса који су у математичком третману једначина које се решавају прво аналитички како би нумерички проблем самог решавања био трактабилан.

При изради свих публикација кандидат је учествовао у развоју метода и нумеричким симулацијама теоријских модела, као и у завршној анализи нелинеарних феномена и писању радова.

Два рада, према захтевима за избор у звање виши научни сарадник, одабрана да илуструју самосталност кандидата и његове основне доприносе су раније описани детаљније у секцији 4. и у секцији 5.1 . То су:

1. [18] N. Z. Petrović, "Spatiotemporal traveling and solitary wave solutions to the generalized nonlinear Schrodinger equation with single-and dual-power law nonlinearity," Nonlinear Dynamics 93 (4), 2389-2397 (2018)
Рад је реализовао потпуно самостално од идеје, реализације теорије и решавања свих једначина до писања рада и одговора рецензентима.

2. [19] N. Z. Petrović, N.B. Aleksić, M. Belić, "Modulation stability analysis of exact multidimensional solutions to the generalized nonlinear Schrödinger equation and the Gross-Pitaevskii equation using a variational approach," Optics Express 23 (8), 10616-10630 (2015)
IF=3.148 (14/90) SNIP=1.67

У приказу рада кандидата (секција 4) описани су доприноси кандидата и у другим радовима а то је поновљено и у анализи пет радова.

5.1.5 Награде
Није обавезно за тражено звање.

Вредне помена су и бројне награде на такмичењима из математике и физике у средњој школи и на студијама укључујући две сребрне и једну бронзану медаљу на Међународним математичким олимпијадама.

Освојена је похвала на престижном студентском такмичењу из математике Патнам (William Lowell Putnam) у САД.

5.2 Ангажованост у формирању научних кадрова

Током боравка на Универзитету у Катару кандидат је руководио дипломским радовима више студената. У том периоду није било могуће организовати докторске студије на том одсеку Универзитета али о комплексности пројеката којима је руководио радом студената говори и чињеница да је са једним студентом публиковао три рада у часопису Phys Rev E а са другим два рада у часопису Physica Scripta.

Радио је у комисијама Друштва математичара и друштва физичара на припреми задатака за такмичења из физике и математика и на њиховом оцењивању. Учествовао је у припремама младих математичара. Књига решених проблема са Међународних математичких олимпијада је основни уџбеник за припреме за такмичења свуда у свету, била је цитирана (54 према сервису Google scholar) у низу научних радова из области педагогије и наставе математике, као и рада са талентованим студентима. Треба напоменути да се кандидат бави Математичком физиком те да је математика основни алат у његовом раду.

Захваљујући томе да је кандидат био присутан у Дохи на његовом универзитету је организован током неколико година турнир и припреме студената из десетак околних земаља за учешће на Међународним математичким олимпијадама. После одласка назад у Београд постојала је иницијатива да се он ангажује да настави рад на организацији припрема што је реализовано само током једне школске године.

5.3 Нормирање броја коауторских радова, патената и техничких решења

Сви радови кадидата се базирају на интензивној аналитичкој теорији која се после може нумерички обрадити јер решавање нелинеарних проблема није решиво искључиво аналитички. И поред тога већина радова кандидата је са три или мање коаутора а неколико радова који имају четири коаутора су свакако доминантно у домену интензивних нумеричких прорачуна. Због тога нема потребе да се нормализују радови.

5.4 Руковођење пројектима, потпројектима и пројектним задацима

Кандидат руководи пројектним задатком "*Аналитичко решавање нелинеарних једначина у оптици*" у оквиру пројекта ОН171006 "*Нелинеарна динамика локализованих самоорганизованих структура у плазми, нано-композитним материјалима, течним и фотоничним кристалима и ултрахладним кондензатима*" под руководством др Душана Јовановића.

Потврда руководиоца пројекта да Никола Петровић руководи задатком је дата као део мишљења руководиоца пројекта о оправданости покретања поступка за избор кандидата у звање.

5.5 Активност у научним и научно-стручним друштвима и остали показатељи успеха у научно-стручном раду

Кандидат је био члан комисија за такмичење Друштва физичара Србије и Друштва математичара Србије.

Он је био и још увек јесте члан комисије за међународна такмичења из математике. Учествовао је у састављању задатака за све нивое такмичења и у припремама тима за Међународну математичку олимпијаду и друга међународна такмичења из математике.

У оквиру ове секције треба приказати све релевантне показатеље, који су у сврху процене подељени у две групе, А и Б:

5.5 А Рецензије у часописима:

Рецензија радова у истакнутим међународним часописима (наведени су само неки од скорашњих примера):

1. Nonlinear Dynamics

[redacted]

--Manuscript Draft--

Manuscript Number:	[redacted]

Часопис категорије М21а

2. Communications in Nonlinear Science and Numerical Simulation

Manuscript Draft

Manuscript Number: [redacted]

[redacted]

Article Type: Research Paper

Часопис категорије М21а

3. The European Physical Journal Plus

[redacted]

--Manuscript Draft--

Manuscript Number:	[redacted]
Full Title:	[redacted]

Часопис категорије М22

4. Optical and Quantum Electronics

[redacted]

--Manuscript Draft--

Manuscript Number:	[redacted]
Full Title:	[redacted]
Article Type:	Original Research
Keywords:	The SGEM; longitudinal wave equation in a MEE circular rod; complex, hyperbolic, trigonometric function solutions.

Часопис категорије М23

5. **SpringerPlus**

[redacted]

--Manuscript Draft--

Manuscript Number:	[redacted]
Full Title:	[redacted]
Article Type:	**Research**

Часопис категорије М22

6. Review of J. Phys. [redacted]

manuscript: [redacted]

Title: [redacted]

Часопис категорије М21

7. Review of Optics Communications

[redacted]

[redacted]

5.5 Б Предавања по позиву:

Након претходног избора у звање, кандидат је одржао следећа предавања:

1. Nikola Z Petrović ''General analytic solutions to the various forms of the nonlinear Schrödinger equation using the Jacobi elliptic function expansion method'' 6th International Conference on Photonics July 31- August 01, 2017 Milan, Italy (https://optics.physicsmeeting.com/abstract/2017/general-analytic-solutions-to-the-variousforms-of-the-nonlinear-schr-dinger-equation-using-the-jacobi-elliptic-function-expansion-method)

Приложена је преписка са позивом из које се види да је у питању предавање по позиву, веб страница са радом (публиковани су радови са конференције на веб страници и потврде о присуству на конференцији.

2. Nikola Z Petrović ''General analytic solutions to the various forms of the Nonlinear Schrödinger Equation using Jacobi eliptic function expansion method'' 10th Photonics Workshop Kopaonik 26.2-2.3.2017. ISBN978-86-82441-45-8 Institut za fizku Beograd str. 36

Приложена је преписка са позивом из које се види да је у питању предавање по позиву, копија рада и одговарајуће странице из књиге апстраката.

1	А	научни одбори (друштва, часописи)
2	А	**рецензије (часописи, пројекти)**
3	Б	научна тела (МПНТР, држава)
4	Б	научни одбори конференција
5	Б	**предавања по позиву**

5.6 Утицајност научних резултата

Овде понављамо одељак 4.1.3 уз допуну.

У категорији М21а, М21, М22 и М23 кандидат је објавио радове у следећим часописима, где су Посебно означени они часописи у којима је кандидат објављивао у периоду након одлуке Научног већа о предлогу за стицање претходног научног звања:

Пре претходног избора:

• 1 рад у Phys. Rev. Lett 2008 M21a,
• 4 рада Phys. Rev. E (2010 3 puta 2011) M21a+ 3M21
• 1 рад Phys Rev A (2008) M21a,
• 1 рад Optics Letters (2009) M21a
• 1 rad Physica Scripta M22
• 1 рад Acta Physica Polonica A 212, 729 (2007) M23,
• 1 рад Electronic J Diff. Equations (2010) M23

После претходног избора (одлуке научног већа)

• 4 рада у Nonlinear Dynamics M21a (2015)(ИФ = 3.000 8/135), 2 x (2016)(ИФ = 3.464 8/133), (2018)(ИФ = 4.339), M21a
• 1 рад у Physical Review E M21 (2014) (ИФ = 2.288 4/54), M21a

• 1 рад у Europhysics Letters M21 (2016) (ИФ = 1.957 23/79), M21
• 1 рад у Optics Express M21 (2015) (ИФ =3.148 14/90),M21

• 1 рад у Journal of Optics M22 (2015) (ИФ = 1.847 36/90), M22
• 1 рад у Optical and Quantum Electronics M23 (2016) (ИФ = 1.055 70/92), M23

Укупан фактор утицаја (збир импакт фактора) радова кандидата је **49,448**, а у периоду након одлуке Научног већа о предлогу за стицање претходног научног звања тај фактор је **24,526**. Кандидат је објављивао радове у најугледнијим часописима из његове области. Посебно се међу њима истичу: Phys. Rev. Lett., Nonlinear Dynamics, Physical Review E. и Optics Express.

Према бази WOS радови кандидата су цитирани укупно 352 пута, док је број цитата без аутоцитата 308. Према истој бази H–индекс кандидата је 8.

Прилог: подаци о цитираности са интернет странице WOS.

На бази Google Scholar има 515 цитата (што укључује и 54 цитата књиге IMO Compendium) и H фактор 10.
Није примећена ниједна инстанца негативне цитираности а у бројним случајевима методе из радова за које је основни доприност дао Никола З Петровић су примењиване у другим публикацијама.
Најцитиранији радови у целој каријери су према Google Scholar :

1. Analytical light bullet solutions to the generalized (3+ 1)-dimensional nonlinear Schrödinger equation, M Belić, N Petrović, WP Zhong, RH Xie, G Chen, Physical review letters 101 (12), 123904   (2008), цитата: 147

2. Exact spatial soliton solutions of the two-dimensional generalized nonlinear Schrödinger equation with distributed coefficients WP Zhong, RH Xie, M Belić, N Petrović, G Chen, L Yi, Physical Review A 78 (2), 023821 (2008) цитата: 117

3. The IMO Compendium: A Collection of Problems Suggested for the International Mathematical Olympiads: 1959-2009 Second Edition, D Djukić, V Janković, I Matić, N Petrović, Springer Science & Business Media прво издање (2006) друго издање (2011), цитата: 54

4. Exact spatiotemporal wave and soliton solutions to the generalized (3+ 1)-dimensional Schrödinger equation for both normal and anomalous dispersion, NZ Petrović, M Belić, WP Zhong, RH Xie, G Chen, Optics letters 34 (10), 1609-1611 (2009), цитата: 34

5. Spatiotemporal wave and soliton solutions to the generalized (3+ 1)-dimensional Gross-Pitaevskii equation, NZ Petrović, M Belić, WP Zhong, Physical Review E 81 (1), 016610 (2010), цитата: 32

6. Controllable parabolic-cylinder optical rogue wave, WP Zhong, L Chen, M Belić, N Petrović, Physical Review E 90 (4), 043201 (2014), цитата: 21

5.7 Конкретан допринос кандидата у реализацији радова у научним центрима у земљи и иностранству

Сви радови кандидата спадају у домен нелинеарне оптике односно шире посматрано области Оптика из домена физичких наука. Један рад који се бави динамиком Bose Einsten кондензата спада више у домен атомске и молекуларне физике.

Кандидат предводи активности у домену аналитичких решења нелинеарних једначина пре свега у домену нелинеарне оптике али и уз могућност примене на системе у нелинеарној атомској и молекуларној физици и квантним компјутерима.

Кандидат има активну сарадњу и заједничке публикације са истраживачима у области нелинеаре оптике и нелинеарне динамике, као и математичке физике: проф. др Миливој Белић, Универзитет Texas A&M, Доха Катар, Силију Су (Si-Liu Xu), Школа електронског и информационог инжењеринга, Ху-Беи универзитет науке и технологије, Сјенин, Кина, Веипинг Жонг (Wei-Ping Zhong), Шунде политехнички факултет, Шунде, Кина.

5.8 Уводна предавања на конференцијама и друга предавања

Након претходног избора у звање, кандидат је одржао следећа предавања:

1. Nikola Z Petrović ''General analytic solutions to the various forms of the nonlinear Schrödinger equation using the Jacobi elliptic function expansion method'' 6th International Conference on Photonics July 31- August 01, 2017 Milan, Italy (https://optics.physicsmeeting.com/abstract/2017/general-analytic-solutions-to-the-variousforms-of-the-nonlinear-schr-dinger-equation-using-the-jacobi-elliptic-function-expansion-method)

Приложена је преписка са позивом из које се види да је у питању предавање по позиву, веб страница са радом (публиковани су радови са конференције на веб страници и потврде о присуству на конференцији.

2. Nikola Z Petrović ''General analytic solutions to the various forms of the Nonlinear Schrödinger Equation using Jacobi eliptic function expansion method'' 10th Photonics Workshop Kopaonik 26.2-2.3.2017. ISBN978-86-82441-45-8 Institut za fizku Beograd str. 36

Приложена је преписка са позивом из које се види да је у питању предавање по позиву, копија рада и одговарајуће странице из књиге апстраката.

Subject: Fwd: Re: Honorable Speaker at Photonics 2017
From: Nikola Petrovic <nzpetr@ipb.ac.rs>
Date: 27.10.2018. 03.38
To: zoran@phy.bg.ac.rs

-------- Original Message --------
Subject: Re: Honorable Speaker at Photonics 2017
Date: 2017-04-07 18:19
From: Nikola Petrovic <nzpetr@ipb.ac.rs>
To: photonics@conferenceseries.net

Dear Photonics 2017,

I accept your invitation to participate as a speaker.

I just have a couple of questions regarding registration:

Do I select the ordinary registration fee or one as a delegate?
Is there a way to pay via a Credit Card that does not involve Stripe or PayPal? These kinds of services still aren't active in Serbia.
What is the deadline for abstract submission?
Is the 100$ fee only for the poster to be visible online and not for the poster sessions on Day 2?

Regards,
Dr. Nikola Petrović

On 2017-03-22 08:34, Photonics 2017 wrote:

> DEAR DR. NIKOLAZ.PETROVI,
>
> We are pleased to invite you to the "6TH INTERNATIONAL CONFERENCE ON PHOTONICS" which is scheduled to be held on JULY 31- AUGUST 01, 2017 Milan, Italy. The Conference deliberations will be on the theme "NEW RESEARCH HORIZONS: PHOTONICS IN A CHANGING WORLD".
>
> It's a privilege to invite you to participate in this prestigious conference as a speaker/delegate. We believe that your contribution to this field is unparalleled and your presence in the conference will be greatly beneficial for Future growth.
>
> We kindly request you to submit the Invited Talks at Photonics Conference [1]
>
> Should you have any queries, please do not hesitate to drop us a mail.
>
> Kindest Regards,
>
> Elisa Walker
> Photonics 2017
> 57 Ullswater Avenue, West End
> Southampton, Hampshire
> United Kingdom, SO18 3QS
> photonics@conferenceseries.net

> You are receiving this email because of your relationship with the
> sender. To safely unsubscribe or modify your subscription settings
> please click here [2]
>
> Links:
> ------
> [1]
> http://mailmx.maildirectalpha.com/misc/pages
> /link/url:~bnpwZXRyQGlwYi5hYy5yc34xNDkwMTY2NjQ1fjM0MzEzXzQ1MzM1fjIwMTcwM35U~http:
> //photonics.conferenceseries.com/abstract-submission.php
> [2]
> http://mailmx.maildirectalpha.com/misc/pages/subscribe
> /MzQzMTNfNDUzMzV+bnpwZXRyQGlwYi5hYy5yc34wX0N+MTQ5MDE2NjY0NQ==

--
Institute of Physics Belgrade
Pregrevica 118, 11080 Belgrade, Serbia
http://www.ipb.ac.rs/

conferenceseries (https://optics.physicsmeeting.com/2017/)

6th International Conference on **Photonics**

July 31-August 01, 2017 Milan, Italy

Recommended Global PHYSICS Conferences

USA & Americas

Particle Physics 2018, USA (https://optics.physicsmeeting.com/2017/recommended-global-conferences.php#usa)
Fluiddynamics 2019, USA (https://optics.physicsmeeting.com/2017/recommended-global-conferences.php#usa)
Condensed Matter Physics 2019, USA (https://optics.physicsmeeting.com/2017/recommended-global-conferences.php#usa)
()

Europe & Middle East

Astronomy 2019, UK (https://optics.physicsmeeting.com/2017/recommended-global-conferences.php#europe)
Magnetism Conference 2019, UAE (https://optics.physicsmeeting.com/2017/recommended-global-conferences.php#europe)
Bio-Med Meet 2019, UAE (https://optics.physicsmeeting.com/2017/recommended-global-conferences.php#europe)
High Energy Physics 2018, Spain (https://optics.physicsmeeting.com/2017/recommended-global-conferences.php#europe)
()

Asia Pacific

Optics-Laser 2019, Japan (https://optics.physicsmeeting.com/2017/recommended-global-conferences.php#asia)
Optical Fibre 2019, Japan (https://optics.physicsmeeting.com/2017/recommended-global-conferences.php#asia)
()

Search | Search 1000+ Events

Scientific Program

Conference Series Ltd invites all the participants across the globe to attend 6th International Conference on Photonics Milan, Italy .

Submit your Abstract (https://optics.physicsmeeting.com/abstract-submission.php)

or e-mail to

photonics@conferenceseries.net (mailto:photonics@conferenceseries.net)
photonics@physicsconferences.org (mailto:photonics@physicsconferences.org)
photonics@physicsconferences.org (mailto:photonics@physicsconferences.org)

Scientific Program **Day 1** (https://optics.physicsmeeting.com/2017/scientific-program.php?day=1&sid=3584&date=2017-07-31) Scientific Program **Day 2** (https://optics.physicsmeeting.com/2017/scientific-program.php?day=2&sid=3587&date=2017-08-01)

Day 1 : July 31, 2017

Keynote Forum

Dan Botez (https://optics.physicsmeeting.com/speaker/2017/dan-botez-university-of-wisconsin-madison-usa)

University of Wisconsin-Madison, USA

Keynote: High-internal-efficiency quantum cascade lasers: the road to mid-infrared lasers of 40% CW wall-plug efficiency (https://optics.physicsmeeting.com/abstract/2017/high-internal-efficiency-quantum-cascade-lasers-the-road-to-mid-infrared-lasers-of-40-cw-wall-plug-efficiency)

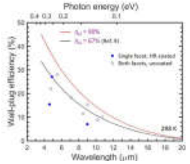
Time : **09:30-10:00**

Biography:

Dan Botez is Philip Dunham Reed Professor in the Department of Electrical and Computer Engineering at University of Wisconsin (UW) - Madison. In 1976, he obtained a PhD degree in Electrical Engineering from University of California, Berkeley. He has carried out and led research in semiconductor lasers at RCA Labs, Princeton, NJ and TRW Research Center, Redondo Beach, CA before joining, in 1993, the faculty at UW-Madison. His research interests lie in three areas of semiconductor-laser physics: high-power, coherent edge-emitting lasers; high-power, coherent grating-coupled surface-emitting lasers; and quantum cascade lasers. The first two are based on one- and two-dimensional, high-index-contrast, photonic-crystal structures, respectively, for insuring both long-range spatial coherence and stable operation under continuous-wave (CW) driving conditions. The third involves electron transitions between the sub-bands of multi-quantum-well structures and is focused on achieving high-efficiency CW operation in the mid-infrared wavelength range: 3-10 microns, via multi-dimensional conduction-band engineering

Abstract:

The internal efficiency h_i of quantum cascade lasers (http://optics.physicsmeeting.com/) (QCLs) is the factor in the expression for the external differential efficiency that encompasses all differential carrier-usage (i.e., the injection efficiency) and lasing-photon-transition efficiencies. For conventional QCLs the h_i values have been found to be rather low: 50-60% in the 4.5-6.0 µm wavelength range and 57-67% in the 7-11 µm wavelength range; with, until recently, no clear explanation why that was the case. With the advent of combining carrier-leakage suppression with fast, efficient carrier extraction out of the active regions of QCLs, the h_i values have steadily increased and are approaching their fundamental upper limit of ~ 90% for mid-infrared (IR)-emitting devices. We will review the developments that led to high h_i values throughout the mid-IR wavelength range. Conduction-band engineering has led to the so-called step-taper active-region (STA) QCLs which have provided h_i values 30-50% higher than in conventional QCLs over both the 4.5-6.0 µm and 7-11 µm wavelength ranges. A record-high, single-facet, continuous-wave (CW) power, for 8.0 µm-emitting QCLs, of 1.0 Watt has been achieved from STA-type QCLs. Furthermore, the recognition that the fundamental limit for h_i (i.e., 90%) is 34% higher than the h_i value employed a decade ago when determining the fundamental limit for the wall-plug efficiency of mid-IR QCLs, has led to the realization that wall-plug efficiencies ≥ 40% can be achieved for 4.5-5.0 µm-emitting QCLs. The practical benefits of achieving such high performance from mid-IR emitting semiconductor lasers (http://optics.physicsmeeting.com/) will be discussed as well.



Wall-plug-efficiency fundamental limits for mid-infrared-emitting QCLs

Recent Publications

1. Kirch J et al (2016) 86% internal differential efficiency from 8-9 μm-emitting, step-taper active-region quantum cascade lasers. Optics Express 24: 24483-24494.
2. Botez D, Chang C-C, Mawst L J (2016) Temperature sensitivity of the electro-optical characteristics for mid-infrared-emitting quantum cascade lasers. J. Phys. D: Applied Physics 49: 043001.
3. Botez D et al. (2013) Multidimensional conduction-band engineering for maximizing the continuous-wave (cw) wallplug efficiencies of mid-infrared quantum cascade lasers. IEEE Journal Selected Topics in Quantum Electronics 19 (4): 1200312.
4. Kirch J et al. (2012) Tapered active-region quantum cascade lasers for suppression of carrier-leakage currents. Electron Lett. 48: 234.

5. Botez D et al. (2010) Temperature dependence of the key electro-optical characteristics for mid-infra-red emitting quantum cascade lasers. Applied Physics Letters 97: 071101.

Keynote Forum

Manyalibo J Matthews (https://optics.physicsmeeting.com/speaker/2017/manyalibo-j-matthews-lawrence-livermore-national-laboratory-usa)

Lawrence Livermore National Laboratory, USA

Keynote: Understanding laser materials processing: the dichotomy between laser damage and laser machining (https://optics.physicsmeeting.com/abstract/2017/understanding-laser-materials-processing-the-dichotomy-between-laser-damage-and-laser-machining)

Time : **10:00-10:30**

Biography:

Manyalibo J Matthews currently serves as Deputy Group Leader in the Optical Materials and Target Science group in MSD. He holds a PhD in Physics from MIT and a BS in Applied Physics from UC Davis. His research interests at LLNL include novel applications in laser-assisted material processing (e.g. metal additive manufacturing, laser-based CVD, nano-coarsening of metal films, non-contact laser polishing of glass), optical damage science, vibrational spectroscopy and *in-situ* optical characterization of transient processes. Prior to LLNL, he was a Member of Technical Staff at Bell Labs and worked on materials characterization of optical devices using novel spectroscopic techniques, stress-induced birefringence management in planar optical devices and research in advanced broadband access networks. He is a Fellow of the Optical Society of America.

Abstract:

In the decades since the invention of the laser, new applications and discoveries in materials science have continued year after year as laser sources evolve and more areas of research exploit them. The transformation of materials using focused, high irradiance laser beams fundamentally involves multiple physical phenomena such as optical absorption (http://optics.physicsmeeting.com/), heat transport, structural mechanics and material phase transitions. For example, nonlinear absorption of nanosecond pulsed laser light can lead to a nano-scale thermal runaway effects and subsequent damage, which can be detrimental in the operation of high power laser (http://optics.physicsmeeting.com/) systems. On the other hand, laser processing of materials often involves ablative removal of material or transformations which rely on efficient coupling of laser energy into a work piece. In both cases, understanding laser-material interactions is essential for the optimization of the high power optical system design (http://optics.physicsmeeting.com/). In this talk, we will present a few examples of high photon flux laser material processing, using both experiment and finite element modeling to understand energy deposition, heat transport and material transformation. Specifically, we will explore the conditions which bring about optical damage in ultraviolet Q-switched laser optics and compare these conditions to those used in typical microscale laser materials (http://optics.physicsmeeting.com/) processing technologies. Among the laser processing techniques discussed, we will focus on microsecond-pulsed, resonant IR laser heating for laser micro-machining and metal powder bed additive manufacturing (3D printing). We will discuss how our results can be used to elucidate material behavior, optimize processing and develop new technologies based on laser modified materials.

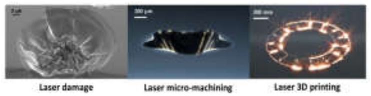


Figure 1: Examples of laser damage (left), laser micro-machining (center) and laser-based 3D printing.

Recent Publications

1. C A R Chapman, L Wang, J Biener, E Seker, M M Biener, and M J Matthews (2016) Engineering on-chip nanoporous gold material libraries via precision photothermal treatment. Nanoscale. 8:785-795.
2. M J Matthews, G Guss, S A Khairallah, A M Rubenchik, P J Depond and W E King (2016) Denudation of metal powder layers in laser powder bed fusion processes. Acta Materialia. 114:33-42.
3. M J Matthews, S T Yang, N Shen, S Elhadj, R N Raman, G Guss, I L Bass, M C Nostrand and P J Wegner (2015) Micro-shaping, polishing, and damage repair of fused silica surfaces using focused infrared laser beams. Advanced Engineering Materials. 17:247-254.
4. J H Yoo, J B In, I Sakellari, R N Raman, M J Matthews, S Elhadj, C Zheng and C Grigoropoulos (2015) Directed dewetting of amorphous silicon film by a donut-shaped laser pulse. Nanotechnology 26: 165303.
5. M J Matthews (2015) Simulating laser-material interactions, Laser Focus World 51: 33-38.

Keynote Forum

Shien-Kuei Liaw (https://optics.physicsmeeting.com/speaker/2017/shien-kuei-liaw-national-taiwan-university-of-science-and-technology-taiwan-1663771806)

National Taiwan University of Science and Technology, Taiwan

Keynote: WDM bidirectional optical wireless communications (https://optics.physicsmeeting.com/abstract/2017/wdm-bidirectional-optical-wireless-communications)

Time : **10:50-11:20**

Biography:

Shien-Kuei Liaw received Double Doctorate from National Chiao-Tung University in Photonics Engineering and from National Taiwan University in Mechanical Engineering, respectively. He joined the Chunghua Telecommunication, Taiwan, in 1993. Since then, he has been working on Optical Communication and Fiber Based Technologies. He joined the Department of Electronic Engineering, National Taiwan University of Science and Technology (NTUST) in 2000. He has ever been Director of the Optoelectronics Research Center and the Technology Transfer Center, NTUST. He was a Visiting Researcher at Bellcore (now Telcordia), USA for six months in 1996 and a visiting Professor at University of Oxford, UK for three months in 2011. He owned six US patents, and authored or coauthored for 250 journal articles and international conference presentations. He earned many domestic honors and international honors. He has been actively contributing for numerous conferences as a conference chair, technical program chair, organizing committee chair, steering committee and/or keynote speaker. He serves as an Associate Editor for *Fiber and Integrated Optics*. Currently, he is a Distinguished Professor of National Taiwan University of Science and Technology (NTUST), Vice President of the Optical Society (OSA) Taiwan Chapter and Secretary-General of Taiwan Photonic Society. His research interests are in Optical Sensing, Optical Communication and Reliability Testing.

Abstract:

In this talk, high-speed free space optics communication (http://optics.physicsmeeting.com/) (FSO) technologies will be reviewed and introduced. Then we will design and demonstrate two proposed FSO schemes. The first scheme is bi-directional short-range free-space optical (FSO) communication with 2x4x10 Gb/s capacity in wavelength division multiplexing (WDM) channels short transmission distance. The single-mode-fiber components are used in the optical terminals for both optical transmitting (http://optics.physicsmeeting.com/) and receiving functions. The measured power penalties for bi-directional, four-channel WDM FSO communication are less than 0.8 dB and 0.2 dB, compared with the back-to-back link and uni-directional transmission system, respectively. The second scheme is hybrid optical fiber (http://optics.physicsmeeting.com/) and FSO link in outdoor environments such as cross bridge or inter-building system. A sensor head is used for monitoring the condition of bridge, and in the case of the bridge being damaged the transmission path could be changed from fiber link to FSO link to ensure data link connectivity. In both cases, the single-mode-fiber (SMF) components are used in the optical terminals for both optical transmitting and receiving functions. The influences of environmental factor including window glasses, air turbulence and rainfall will also be addressed. The colorless and colored window glasses introduce losses under various incident angles, but did not induce substantial power penalties. The air turbulence induces extra transmission loss and instability in the received power. Raindrops are the most influential environmental factor. The bit error rate (BER) test shows that raindrops result in a seriously impaired BER to interrupt the transmission instantaneously. After appropriate performance improvement, these proposed transmission structures show potential applications for outdoor transmission under various natural weather conditions.

Keynote Forum

Carl C Jung (https://optics.physicsmeeting.com/speaker/2017/carl-c-jung-ccj-software-germany)

CCJ Software, Germany

Keynote: Twisted and turned layers – no problem for ITE (Immersion Transmission Ellipsometry) (https://optics.physicsmeeting.com/abstract/2017/twisted-and-turned-layers-no-problem-for-ite-immersion-transmission-ellipsometry)

Time : **11:20-11:50**

Biography:

Carl C Jung has his expertise in finding mathematical models for engineering, physical and physical chemistry questions and implementing them in evaluation and simulation software. His way led from amperometric biosensors (Cambridge University, UK), via biophysics employing florescence (Max Planck Institute, Frankfurt a M) to display technology and ellipsometry (IDM, Berlin and Potsdam). Here the presented topic was generated. Thereafter he returned to biophysics and fluorescence (Bayreuth University), and after one year in research management (Fraunhofer, Munich) he finally performed theoretical and experimental studies on the heating of bond wires used in integrated circuits by electronic engineers (Robert Bosch Center for Power Electronics, Reutlingen).

Abstract:

If looking at optically thin layers or thin films with an anisotropic structure, the main applications of such films are in display technology. There are different ways, such layers can be used: as polarisers, if absorbing, as retarders, if transparent, as photo-alignment films, if very thin and with a specific surface, that can be used to align other attaching films during an annealing step in fabrication. Of course, the optical properties of the resulting display depend on the quality of the layers used to produce it. Therefore, we developed a new method, which can very accurately determine the three-dimensional refractive index and its orientation in a thin layer. Even films, whose properties vary in the direction perpendicular to the film plane, can be studied with success. We employed a combination of transmission in two different media - immersion transmission ellipsometry and reflection ellipsometry at one single wavelength. Ellipsometry (http://optics.physicsmeeting.com/)is the measurement of the alteration of the polarization state of light transmitted or reflected by the layer or film studied. The accuracy of the method was very high compared to conventional reflection ellipsometry in only one medium. If compared to combined transmission and reflection measurements in air, we also reached a drastic improvement. The method of immersion transmission ellipsometry is a significant step forward in the development of non-destructive optical characterization methods for thin films with complex anisotropic structure.

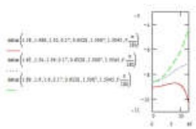


Figure 1: Three normally indistinguishable sets of data can be expanded by immersion transmission ellipsometry. Depicted is the ellipsometric parameter Δ measured in transmission under immersion. The first 3 figures are the refractive indices of the film. Then wavelength in μm, and

immersion and substrate index follow.

1. Jung C, Stumpe J (2015) Immersion transmission ellipsometry (ITE) for the determination of orientation gradients in photoalignment layers. Appl. Phys. B DOI 10.1007/s00340-013-5729-2.
2. Jung C, Stumpe J (2005) Immersion transmission ellipsometry (ITE) – a new method for the precise determination of the 3D indicatrix of thin films. Appl. Phys. B 80:231-238.
3. Jung C, Stumpe J, Peeters E, van der Zande B (2005) A novel way for the full characterisation of splayed retarders using the Wentzel-Kramers-Brillouin (WKB) method. Jpn. J. Appl. Phys. 44: 4000-4005.

Keynote Forum

Fabienne Michelini (https://optics.physicsmeeting.com/speaker/2017/fabienne-michelini-aix-marseille-university-france)

Aix Marseille University, France

Keynote: Energy transfer dynamics in molecular junctions under ultra-short excitation pulses from non-equilibrium Green's function formalism (https://optics.physicsmeeting.com/abstract/2017/energy-transfer-dynamics-in-molecular-junctions-under-ultra-short-excitation-pulses-from-non-equilibrium-green-s-function-formalism)

Time : **11:50-12:20**

Biography:

Fabienne Michelini has worked on the theoretical/numerical building of empirical models within the k·p method to understand the electronic properties of realistic condensed-matter systems. In parallel, she has gained a great expertise in high performance computing for large-scale numerical problems. For the last years, she has investigated the transport properties of opened quantum structures for novel nanodevices using effective methods within the Green function formalism. She is now focusing on time-dependent and non-linear regimes of nanosystems interacting with light for optoelectronic and thermoelectric applications at the nanoscale.

Abstract:

The problem of energy transfer (http://optics.physicsmeeting.com/) is emerging as one of the most crucial issues of our occidental societies. At a fundamental level, how energy flows at the nanometre scale is gaining specific interests due to its implications in both alternative energy production and basics of quantum thermodynamics (http://optics.physicsmeeting.com/). The nature of our work is hence two-fold. In the first part, we provide a definition of energy current operator in the Heisenberg representation, while discussing certain conditions which an operator shall fulfill. The obtained expression is applicable to non-stationary as steady-state situations. We implement this definition to derive time-dependent energy current using non-equilibrium Green's function formalism, which represents a suitable approach for calculating measurable quantities in opened nanosystems. The second part applies these developments to molecular junctions sandwiched in between two thermal reservoirs. Molecular electronic devices are indeed a promising alternative to standard electronic switches due to their fast response on the picosecond time scale. Here, the approach is used for the study of molecular junctions subjected to ultra-short excitation pulses. We thus analyze the electronic energy fluxes across the molecular junction engendered by femtosecond laser pulses. Our numerical implementation enables us to correlate the time-dependent energy current to the underlying intra-molecular dynamics, with special attention paid to the impacts of intra-molecular coupling and incoherence on the energy transfer time-resolved measurables.

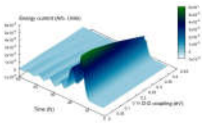


Figure 1: We consider a junction made of two donors (D) that interact with light and an acceptor (A), the whole is in contact with tow thermal reservoirs. Effects of the intra-molecular D-D coupling on the time-resolved energy current flowing from D to A during a 30 fs laser pulse

Recent Publications

1. Michelini F, Crépieux A, Beltako K (2017) Entropy production in photovoltaic-thermoelectric nanodevices from the non- equilibrium Green's function formalism, J. Phys.: Condens. Matter 29: 175301.
2. Beltako K, Cavassilas N, Michelini F (2016) State hybridization shapes the photocurrent in triple quantum dot nanojunctions, Appl. Phys. Lett. 109: 073501.
3. Crépieux A, Michelini F (2015) Mixed, charge and heat noises in thermoelectric nanosystems, J. Phys.: Condens. Matter 27: 015302.
4. Berbezier A, Autran JL, Michelini F (2013) Photovoltaic response in a resonant tunneling wire-dot-wire junction, Appl. Phys. Lett. 103: 041113.
5. Crépieux A, Simkovic F, Cambon B, Michelini F (2011), Enhanced thermopower under a time-dependent gate voltage, Phys. Rev. B 83: 153417.

Keynote Forum

Shao-Wei Wang (https://optics.physicsmeeting.com/speaker/2017/shao-wei-wang-chinese-academy-of-sciences-china)

Chinese Academy of Sciences, China

Keynote: Integrated narrow bandpass filters array for miniature spectrometer (https://optics.physicsmeeting.com/abstract/2017/integrated-narrow-bandpass-filters-array-for-miniature-spectrometer)

Time : **00**

Biography:

Shao-Wei Wang received his Ph.D. (2003) degree in microelectronics and solid state electronics from Shanghai Institute of Technical Physics, Chinese Academy of Sciences, China. He is a professor of the institute and works at National Laboratory for Infrared Physics from 2010. His

research interests include artificial photonic structure and devices, such as interaction between high-Q optical cavity and low-dimensional materials, integrated-cavities for miniature spectrometers, solar selective absorbers, metamaterial polarizers, and optical thin films. He has published more than 50 research papers and authorized one US patent. He got LU JIAXI Young talent award (2009), RAO YUTAI basic optical award (2007) , National Natural science award (2014, 4th principal achiever), National Technological Invention Award (2011, 5th principal achiever), Shanghai Technological Invention Award (2010, 7th principal achiever), Shanghai Natural science award (2007, 5th principal achiever), etc.

Abstract:

Compact, lightweight, and rigid miniature spectrometers without moving parts are needed for a wide variety of applications, including space applications, where every inch of payload counts. Miniaturization increases the portability and paves the way for making in situ spectral measurements for daily life of Food-safety and health etc. It also eases the integration of microspectrometers (http://optics.physicsmeeting.com/)and miniature spectrometers into other technologies, such as microelectronics, and helps to realize lab-on-a-chip devices.

It attracts many research interests in recent years. There are many novel wavelength devision approaches have been proposed for miniature spectrometers (http://optics.physicsmeeting.com/), such as colloidal quantum dot spectrometer and disordered photonic chip. The optical filter array is one of the most important components in wavelength-division multiplexing, multispectral devices, and parallel array optics, which are widely used in communication and electrooptical systems.

We proposed and realized the concept of integrated narrow bandpass filter array from 2004, which can totally match with detectors array with very high spectral resolution and high structure & spectrum flexibility, and resulting in simple structure and small volume with high reliability. We developed the combinatorial etching technique and combinatorial deposition technique for fabrication of such devices. We also demonstrated a concept of a high-resolution miniature spectrometer using an integrated filter array. Such a device has already been succefully used in a multi-spectral luminescence imaging for plant growth research setup of Shijian ten satellite (http://optics.physicsmeeting.com/)launched in 2016.

Recent Publications

1. Xingxing Liu, Shao-Wei Wang, Hui Xia, Xutao Zhang, Ruonan Ji, Tianxin Li, Wei Lu (2016) Interference-aided spectrum fitting method for accurately film thickness determination. Chinese Optics Letters. 14(8):081203.
2. Shao-Wei Wang, et al. (2007) Concept of a high-resolution miniature spectrometer using an integrated filter array. Optics Letters 32(6) 632-634.
3. Shao-Wei Wang, et al. (2007) 128 Channels of integrated filter array rapidly fabricated by using the combinatorial deposition technique. Appl. Phys. B 88(2):281-284.
4. Shao-Wei Wang, et al. (2006) 16 x 1 integrated filter array in the MIR region prepared by using a combinatorial etching technique. Appl. Phys. B 82(4):637-641.
5. Shao-Wei Wang, et al. (2006) Integrated optical filter arrays fabricated by using the combinatorial etching technique. Optics Letters 31(3):332-334.

»Photonics | Nanophotonics | Optical Nanomaterials | Optical Communications and Networking

Location: Brera

Chair

Shien-Kuei Liaw

National Taiwan University of Science and Technology, Taiwan

Co-Chair

V A Belyakov

Landau Institute for Theoretical Physics, Russia

Session Introduction

Alexey Akimov (https://optics.physicsmeeting.com/speaker/2017/alexey-akimov-alexey-akimov-a-m-university-usa)

Alexey Akimov, A&M University, USA

Title: Towards spin-photon interface for NV color center in diamond (https://optics.physicsmeeting.com/abstract/2017/towards-spin-photon-interface-for-nv-color-center-in-diamond)

Time : **12:20-12:45**

Biography:

Alexey Akimov received his PhD degree from Moscow Institute for Physics and Technology in 2003. In 1997, he started working in the Laboratory for Active Media at the Lebedev Physical Institute of the Russian Academy of Sciences. His research was focused on the narrow optical resonances in hot and laser-cooled atoms. During 2006-2012, he was a Visiting Scholar in Misha Lukin's group in Physics Department of Harvard University, where he worked on a number of research projects related to surface plasmons, quantum dots and NV centers in diamond. The main focus of this activity was light-spin interfaces. During 2010-2012, he was the Acting Director of the Russian Quantum Center (RQC). He then assumed a Principal Investigator position at the RQC and conducted research in the fields of cold atoms and solid state spin systems. In October 2015, he joined the Physics Department of Texas A&M University as an Assistant Professor

Abstract:

Nitrogen Vacancy (NV) color centers in diamond attract a lot of attention of quantum optics (http://optics.physicsmeeting.com/) and quantum information community. Due to its long coherence time, possibility of optical readout of electronic spin state and possibility to store information in nearby nuclear spins using this center long quantum memory even at room temperature, long distance quantum entanglement and quantum registers has been demonstrated. Besides, quantum information (http://optics.physicsmeeting.com/) application, this color center is proven to be good high-resolution sensor of magnetic field. Such a sensor is able to combine nanometer resolution with single spin sensitivity. Furthermore, due to its low chemical activity, diamond could be used as *in vivo* sensor. Recently, successful implementation of NV nanodiamonds as temperature sensors for measurement of thermal activation of transient receptor potential was demonstrated. NV color center in diamond could also be used for measurement of electric fields (http://optics.physicsmeeting.com/), tension, rotation or force. This sensor could offer high resolution or cutting edge sensitivity, if bulk sample is used. Also, due to its unique photo stability, this color centers find application in imaging, in particular bio- imaging as well as high resolution imaging such as STED or RESOLFT. In many of these applications, one of the important issues is efficiency, with which light emission of the color center is collected. In this contribution, we present our results on broadband collection of NV color centers emission using optical fiber and nanostructures.

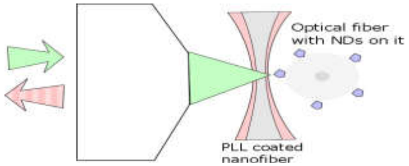


Figure 1: Procedure of single nanocrystal pickup onto optical fiber

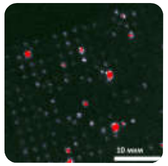


Figure 2: Positioning on nanocrystals on nanostructures

Recent Publications

1. Alexander Sushkov, Nicholas Chisholm, Igor Lovchinsky, Minako Kubo, Pik Kwan Lo, Steven Bennett, David Hunger, Alexey Akimov, Ronald L Walsworth, Hongkun Park, Mikhail D Lukin (2014) All-optical sensing of a single-molecule electron spin. Nano Lett., 14 (11): 6443–6448.
2. Dmitry Sovyk, Victor Ralchenko, Maxim Komlenok, Andrew Khomich, Vladimir Shershulin, Vadim Vorobyev, Igor Vlasov, Vitaly Konov, and Alexey Akimov (2015) Fabrication of diamond microstub photoemitters with strong photoluminescence of SiV color centers:bottom-up approach. Applied Physics A, 118(1).
3. M Y Shalaginov, V V Vorobyov, J Liu, M Ferrera, A V Akimov, A Lagutchev, A N Smolyaninov, V V Klimov, J Irudayaraj, A V Kildishev, A Boltasseva and V M Shalaev (2014) Enhancing the nanodiamond nitrogen-vacancy single-photon source with TiN/AlScN hyperbolic metamaterial superlattice Laser Photonics Rev., 1–8.
4. Vorobyov V V, Soshenko V V, Bolshedvorskii S V, Javadzade J, Lebedev N, Smolyaninov A N, Sorokin V N, Akimov A V (2016) Coupling of single NV center to adiabatically tapered optical single mode fiber. Eur. Phys. J. D 70(12): 269.
5. Vorobyov V V, Kazakov A Y, Soshenko V V, Korneev A A, Shalaginov M Y, Bolshedvorskii S V, Sorokin V N, Divochiy A V, Vakhtomin Y B, et al (2017) Superconducting detector for visible and near-infrared quantum emitters. Opt. Mater. Express 7(2), 513.

Branislav Vlahovic (https://optics.physicsmeeting.com/speaker/2017/branislav-vlahovic-north-carolina-central-university-usa-1699774681)

North Carolina Central University, USA

Title: Optical sensing for dynamics of the localized/delocalized states in binary quantum (https://optics.physicsmeeting.com/abstract/2017/optical-sensing-for-dynamics-of-the-localized-delocalized-states-in-binary-quantum)

Biography:

Branislav Vlahovic is Director of the National Science Foundation Computational Center of Research Excellence, NASA University Research Center for Aerospace Device, and NSF Center Partnership for Research and Education in Materials at North Carolina Central University. In 2004, he was awarded by the Board of Governors of The University System of North Carolina Oliver Max Gardner statewide award for his research and contribution to science. He has published more than 300 papers in peer-reviewed journals. His research interest includes pulsed laser deposition of nanostructures, nonlinear optics, computer simulations of nanostructures, tunneling and charge transfer between nanostructures, detectors and devices based on quantum confinement, nanophotonics, semiconductor structures and photovoltaics.

Abstract:

Weakly coupled binary nano-sized systems demonstrate perspectives for nano-sensor applications. We study electron/hole localization and spectral distributions of localized/delocalized states in binary InAs/GaAs quantum complexes, including quantum wells (QWs) and quantum dots (http://optics.physicsmeeting.com/) (QDs). The InAs/GaAs heterostructures (http://optics.physicsmeeting.com/)are described using the effective potential model. It was shown, that the electron tunneling and spectral distributions of localized/delocalized states in binary system is extremely sensitive on shape symmetry violations. The parameter , which defines delocalized ( ) or localized (0) states of an electron, depends on the energy difference of the spectra in left and right QDs. The difference can be caused by a shape symmetry violation. The sensitivity of the parameter to the small variations of is estimated as . This work focuses on the optical registration of the localized/delocalized states dynamics. Modeling of carrier transfer from a barrier in InAs/GaAs dot-well, tunnel-injection structure is performed. In Fig 1, shown is the electron wave functions of the localized and delocalized states calculated for two spectral levels: E=0.345 eV and E=0.444 eV, respectively. The energy of the delocalized state corresponds to that one for which the tunneling between dot and well occurs. The relation to the PL experiments for such complexes is provided. We model the second pick of the PL spectrum, which corresponds to the carriers tunneling in the dot-well complex, in the terms of the localized/delocalized states. Influence of the variations of geometrical

parameters of QD and QW on the tunneling will be presented.

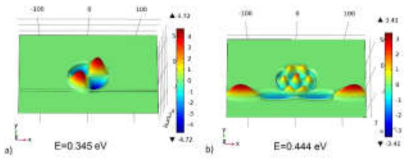


Figure 1. The electron wave functions of the (a) localized and (b) delocalized states in the InGaAs/GaAs dot-well binary complex.

Recent Publications

1. Filikhin I, Karoui A and Vlahovic B (2016) Nanosensing Backed by the Uncertainty Principle. Journal of Nanotechnology. doi:10.1155/2016/3794109.
2. Filikhin I, Suslov V M and Vlahovic B (2006) Modeling of InAs/GaAs quantum ring capacitance spectroscopy in the nonparabolic approximation. Phys. Rev. B 73: 205332-4.
3. Filikhin I, Matinyan S G and Vlahovic B (2015) Electronic structure of quantum dots and rings. Reviews in Theoretical Science 3: 1-22.
4. Filikhin I, Karoui A and B. Vlahovic B (2016) Single electron tunneling in double and triple quantum wells. International Journal of Modern Physics B 30: 1642011-9.
5. Filikhin I, Matinyan S G and Vlahovic B (2015) Localized-delocalized states and tunneling in double quantum dots: effect of symmetry violation. Quantum Matter 4: 1-7.

H C Ong (https://optics.physicsmeeting.com/speaker/2017/h-c-ong-the-chinese-university-of-hong-kong-hong-kong)

The Chinese University of Hong Kong, Hong Kong

Title: Study of the angular momentum of light from plasmonic crystals (https://optics.physicsmeeting.com/abstract/2017/study-of-the-angular-momentum-of-light-from-plasmonic-crystals)

Biography:

H C Ong received his BA and PhD in Materials Science and Engineering from Northwestern University, USA. He currently is an Associate Professor in Physics Department, at the Chinese University of Hong Kong. He has been working on amorphous carbon, diamond, and ZnO for years and his current interest is light-matter interaction focusing on plasmonics. He has published more than 100 technical papers on fluorescence and sensing. He has been serving as an Organizer of international conferences.

Abstract:

In analogy to electron waves, electromagnetic waves (http://optics.physicsmeeting.com/) also carry spin and orbital angular momentum (AM) and this property has been fascinating the world of optical science and engineering for many years. With the rise of nanotechnology (http://optics.physicsmeeting.com/), photonic systems can now be fabricated at the length scale of nanometers, manifesting many intriguing phenomena including the spin-orbit interaction in an observable extent. The polarization, the spatial field distribution, and the propagation direction are no longer treated separately and controlling one with another has become feasible. Plasmonic arrays are one of the most popular nanophotonic (http://optics.physicsmeeting.com/)systems owing to their simplicity and well-defined structures for yielding controllable optical properties. They have been used in extraordinary transmission, fluorescence, photovoltaics, nonlinear optics, sensing, etc. In addition, since surface plasmon polaritons (SPPs) carry transverse spin AM, they should modify the AM of the outgoing radiation under the conservation of angular momentum. Unfortunately, this transverse spin is not properly taken into consideration even though plasmonic research has been carried out for years. Here, I will talk about the AM of light from plasmonic crystals. We have observed substantial polarization conversion and spin-orbital coupling from square lattice circular nanohole arrays, which do not possess intrinsic chirality. We find the transverse spin AM possessed by SPPs play a deterministic role in governing the far-field radiation. The experimental results are supported by finite-difference time-domain simulations and temporal coupled mode theory. Based on the AM study, we propose the AM can be used as a new parameter in surface plasmon resonance (SPR) sensing. As the transverse spin AM of SPPs is strongly dependent on the complex propagation wave vector, which is sensitive to the change of the local refractive index, the change in the AM of light thus reflects the sensing environment. The performance of the spin-SPR will be discussed.

Recent Publications

1. Cao ZL, Yiu LY, Zhang ZQ, Chan CT, Ong HC (2017) Understanding the role of surface plasmon polaritons in two-dimensional achiral nanohole arrays for polarization conversion. Phys. Rev. B (in press).
2. Lin M, Cao ZL, Ong HC (2017) Determination of the excitation rate of quantum dots mediated by momentum resolved Bloch-like surface plasmon polaritons. Opt. Exp. 25: 6092-6103.
3. Cao ZL, Ong HC (2016) Momentum-dependent group velocity of surface plasmon polaritons in two-dimensional metallic nanohole array. Opt. Exp. 24: 12489-12500.
4. Liu C, Chan CF, Ong HC (2016) Direct deconvolution of electric and magnetic responses of single nanoparticles by Fourier space surface plasmon resonance microscopy. Opt. Comm. 378: 28-34.
5. Liu SD, Leong ESP, Li GC, Hou YD, Deng J, Teng JH, Ong HC, Lei DY (2016) Polarization-independent multiple fano resonances in plasmonic nonamers for multimode-matching enhanced multiband second-harmonic generation. ACS Nano 10: 1442-1452.

Biography:

Jian Fu has completed his PhD at Zhejiang University. He is working as Associate Professor at Zhejiang University. He has published more than 45 papers in reputed journals.

Abstract:

The history to simulate quantum states (http://optics.physicsmeeting.com/) using classical optical fields is long. Many researchers utilized classical optical fields to simulate quantum states and quantum computations. However, it is quickly found that the simulation is not efficient and scalable. This is because the classical optical field only supports the product structure but does not support tensor product structure. We proposed a possible scheme to solve the problem that optical fields modulated with pseudorandom phase sequences simulate any state of

Jian Fu (https://optics.physicsmeeting.com/speaker/2017/jian-fu-zhejiang-university-china)

Zhejiang University, China

Title: A possible approach to quantum computation by using classical optical fields modulated with pseudorandom phase sequence (https://optics.physicsmeeting.com/abstract/2017/a-possible-approach-to-quantum-computation-by-using-classical-optical-fields-modulated-with-pseudorandom-phase-sequence)

multiple quantum particles. By using the scheme, we demonstrated optical analogies to many quantum states such as Bell states, GHZ states and W states, and some quantum algorithms (http://optics.physicsmeeting.com/) such as Shor's algorithm, Grove's algorithm and quantum Fourier Transformation. Firstly, we introduced a theoretical framework, a phase ensemble based on pseudorandom phase sequence referring from the concept of quantum ensemble. Then, we represented various quantum states of n particles by using classical fields modulated with n pseudorandom phase sequences and we also demonstrated nonlocal properties of quantum entanglement in the phase ensemble theoretical framework. Finally, we demonstrated some optical implementations to realize some quantum algorithms. We believe these optical implementations are not difficult to implement. After careful analysis and numerical simulation, we can conclude that our scheme provides an efficient approach to quantum computation without exponential classical resources.

Jae Eun Jang (https://optics.physicsmeeting.com/speaker/2017/jae-eun-jang-dgist-korea)

DGIST, Korea

Title: Label free bio detection using nanohole array structure (https://optics.physicsmeeting.com/abstract/2017/label-free-bio-detection-using-nanohole-array-structure)

Biography:

Jae Eun Jang received his PhD degree in Electrical Engineering from the University of Cambridge, UK in 2006. From 2007 to 2011, he was Principal Senior Researcher at Samsung Advanced Institute of Technology, Yongin, Korea. Since 2011, he has been a Professor in Information and Communication Engineering at Daegu Gyeongbuk Institute of Science and Technology (DGIST), Daegu, Korea. He first demonstrated the mechanical nanoswitch and mechanical DRAM concept using vertically aligned carbon nanotubes in 2004 and 2008, respectively. More recently he was involved in nanodevices for ultra-fast driving, biomimic concepts and brain-machine interface. He has authored or co-authored over 200 journal and conference papers, and is an inventor of 100 granted patents.

Abstract:

The different working principles of nano hole array structure from general color can make promising features as a bio-detector (http://optics.physicsmeeting.com/) because the structural color filter (SCF) changes easily, the filtering colors by covering of different biomaterials. Because the nano-hole arrays were designed to present a filtered peak wavelength in the visible light region, filtered color changes caused by different biomolecules were easily observed with a microscope (http://optics.physicsmeeting.com/)or even by the naked eye. Generally, many biomolecules are transparent or colorless in the visible range, so that it is hard to distinguish among them using visible observation. However, their molecular structure and composition induce some differences in the dielectric constant or refractive index, causing a filtered color shift in the nano-hole array structure. Here, the contribution of geometric parameters such as the hole diameter and the spacing between nano-holes for bio-detection was evaluated to maximize the change in color among different biomolecules. A larger hole size and space between the holes enabled the biomolecules to be easily distinguished. Even if the change in color was not distinctive enough by eye in some cases, it was possible to distinguish the change by simple analysis of the 'Hue' values or by the 'Lab' color coordinates obtained from the photo images. Therefore, this skill can have high probability of realization for real-time detection of cells without the use of bio-markers

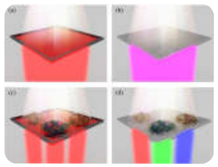


Figure 1: Concept images of bio-detection based on a SCF. (a) Image of a general red CF based on red pigments. Except for the red color component, the other components in white light are absorbed by the red pigments (b) Schematic image of a SCF. Nano-hole arrays induce a color filtering effect (c) Even though three different proteins are dropped on a general CF; there are no color changes due to its transparent optical property. (d) Different transparent biomolecules change the dielectric property of the surface of filters when they are dropped on the SCF. This causes spectral shifts in the SCF

Recent Publications

1. M Ryu, et. al. (2017) Enhancement of interface characteristics of neural probe based on graphene, ZnO nanowires, and conducting polymer PEDOT. ACS Applied Materials & Interfaces. 9 (12): 10577–10586.
2. M Sim et. al. (2016) Structural solution to enhance the sensitivity of a self-powered pressure sensor for an artificial tactile system. IEEE Transactions on Nanobioscience. 15: 804-811.
3. J H Shin et. al. (2016) Ultrafast metal-insulator-multi-wall carbon nanotube tunneling diode employing asymmetrical structure effect. Carbon 102: 172-180.
4. S Kim et. al. (2016) Geometric effects of nano-hole arrays for label free bio-detection. RSC Advances 6: 8935-8940.
5. B O Jun et. al. (2015) Wireless thin film transistor based on micro magnetic induction coupling antenna. Scientific Reports 5:18621.

Biography:

Johan Bauwelinck received his PhD degree in Applied Sciences, Electronics from Ghent University, Belgium in 2005. Since Oct 2009, he is a Professor in the IDLab research group of the Department of Information Technology (INTEC) at the same university where he is leading the Design lab since 2014. He became a Guest Professor at iMinds in the same year, now IMEC since 2016. His research focuses on high-speed, high-frequency (opto) electronic circuits and systems, and their applications on chip and board level, including transmitter and receiver analog front-ends for wireless, wired and fiber-optic communication or instrumentation systems. He is an active person involved in the EU-funded projects GIANT, POWERNET,

Johan Bauwelinck (https://optics.physicsmeeting.com/speaker/2017/johan-bauwelinck-ghent-university-belgium)

Ghent University, Belgium

Title: High-speed transceiver electronics for next-generation optical networks (https://optics.physicsmeeting.com/abstract/2017/high-speed-transceiver-electronics-for-next-generation-optical-networks)

PIEMAN, EuroFOS, C3-PO, Mirage, Phoxtrot, Spirit, Flex5Gware, Teraboard, Streams, WIPE and Optima conducting research on advanced electronic integrated circuits for next generation transport, metro, access, datacenter and radio-over-fiber networks. He has promoted 18 PhDs and co-authored more than 200 publications and 10 patents in the field of High-Speed Electronics and Fiber-Optic Communications.

Abstract:

High-speed electronic integrated circuits (http://optics.physicsmeeting.com/) are essential to the development of new fiber-optic communication systems. Exponentially increasing data consumption is expanding the applications of optical communication and driving the development of faster and more efficient transceivers. Fiber-optic communication networks operate on very different scales from very short interconnects in datacenters to very long links between cities, countries or continents. Optical fibers (http://optics.physicsmeeting.com/) are also increasingly used for access networks (e.g. fiber-to-the-home) and for mobile fronthauling and backhauling. Advances in opto-electronic devices, high-volume manufacturing and packaging technologies are driving numerous developments in these diverse applications. Because of the increasing speeds, close integration and co- design of photonic and electronic devices have become a necessity to realize high-performance sub-systems, while such co-design brings new opportunities as well on the sub-system architecture level to break traditional performance-cost trade-offs. There is no single best solution among electrical and optical technologies due to the different technological constraints in terms of distance, footprint, power consumption, cost, etc. Research is approaching this challenge from different angles, with technological improvements on photonic and electronic devices (http://optics.physicsmeeting.com/) and/or by applying more complex modulation and signal processing. While each application operates on a very different scale (fiber length, number of users) with very different requirements (capacity, signal format, cost, power, etc.), they share one thing, their need for application-specific high-speed electronic transceiver circuits such as driver amplifiers, transimpedance amplifiers, equalizers and clock-and- data recovery circuits. This presentation will illustrate a few recent and ongoing developments from various H2020 projects.

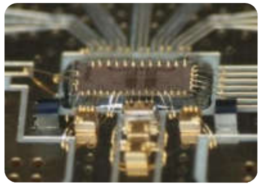


Figure 1: 56Gb/s PAM-4 single-mode VCSEL driver array

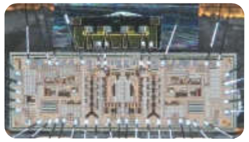


Figure 2: 64Gb/s PAM-4 transimpedance amplifier array

Recent Publications

1. M Vanhoecke et. al. (2017) Segmented optical transmitter comprising a CMOS driver array and an InP IQMZM for advanced modulation formats. J. Lightw. Technol. 35(4):862- 867.
2. J Verbist et. al. (2016) A 40-GBd QPSK/16-QAM integrated silicon coherent receiver. IEEE Photon. Technol. Lett. 28(19):2070-2073.
3. B. Moeneclaey et. al. (2017) 40-Gb/s TDM-PON downstream link with Low-Cost EML transmitter and APD-Based electrical duobinary receiver. J. Lightw. Technol. 35(4):1083-1089.

Nikola Z Petrović (https://optics.physicsmeeting.com/speaker/2017/nikola-z-petrovi--university-of-belgrade-serbia)

University of Belgrade, Serbia

Title: General analytic solutions to the various forms of the nonlinear Schrödinger equation using the Jacobi elliptic function expansion method (https://optics.physicsmeeting.com/abstract/2017/general-analytic-solutions-to-the-various-forms-of-the-nonlinear-schr-dinger-equation-using-the-jacobi-elliptic-function-expansion-method)

Biography:

Nikola Z Petrović received his BSc in Mathematics and in Physics at MIT (Massachusetts Institute of Technology) in 2003 and his PhD in Physics at University of Belgrade in 2013. He was employed as a Teaching Associate and Lab Coordinator at Texas A&M University at Qatar from 2005 to 2012. He is currently an Assistant Research Professor at Institute of Physics, Belgrade. His primary field of expertise is Mathematical Physics applied to nonlinear optics, in particular finding novel exact solutions to the nonlinear Schrodinger equation, the Gross-Pitaevskii equation and other related equations.

Abstract:

The advent of meta-materials has made materials with a negative refractive index possible. This has opened up a possibility of finding stable solutions to various nonlinear equations that naturally occur in the field of nonlinear optics through the use of dispersion management. Finding such stable solutions is invaluable for the field of photonics (http://optics.physicsmeeting.com/)and has many potential practical applications. In our work we use the F-expansion method applied to the Jacobi elliptic function, along with the principle of harmonic balance to find novel solutions to various forms of the Nonlinear Schrödinger equation (NLSE). This approach allowed us to assume a quadratic form for the phase with respect to the longitudinal variable and thus find solutions both with and without chirp. Earlier work done on the NLSE with Kerr

nonlinearity, with both normal and anomalous dispersion, was generalized to nonlinearities of arbitrary polynomial nonlinearity. Stable solutions were also obtained for the Gross-Pitaevskii equation. These solutions were determined to be modulationally stable, either unconditionally or with dispersion management, depending on the signs of various parameters in the original equation. The method was subsequently generalized for functions satisfying an arbitrary elliptic differential equation, including Weierstrass elliptic functions. A relatively new line of research has been finding solutions to the NLSE in a parity-time (PT) conserving potential, i.e. one for which the real part is an even function and the complex part is an odd function. We found a rich new class of exact solutions where the potential resembles the Scarf II potential.

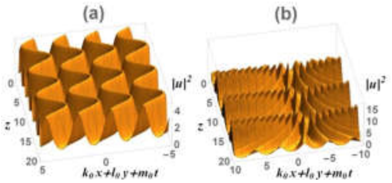


Figure 1: Solution to the NLSE with Kerr nonlinearity using the Weierstrass elliptic function described in: (a) without chirp (b) with chirp

1. S L Xu, Y Zhao, N Z Petrović and M R Belić (2016) Spatiotemporal soliton supported by parity-time symmetric potential with competing nonlinearities. Europhysics Letters. 115: 14006.
2. N Z Petrović and M Bohra (2016) General Jacobi elliptic function expansion method applied to the generalized (3+1)-dimensional nonlinear Schrödinger equation. Optical and Quantum Electronics. 48: 1-8.
3. S L Xu, N Petrović, M R Belić and W Deng (2016) Exact solutions for the quintic nonlinear Schrödinger equation with time and space. Nonlinear Dynamics 84: 251.
4. N Z Petrović, N B Aleksić and M R Belić (2015) Modulational stability analysis of exact multidimensional solutions to the generalized nonlinear Schrödinger equation and the Gross-Pitaevskii equation using a variational approach. Optics Express 23: 10616.
4. W P Zhong, L Chen, M R Belić and N Petrović (2014) Controllable parabolic-cylinder optical rogue wave. Physical Review E. 90: 043201.

V A Belyakov (https://optics.physicsmeeting.com/speaker/2017/v-a-belyakov-landau-institute-for-theoretical-physics-russia)

Landau Institute for Theoretical Physics, Russia

Title: Optics of photonic liquid crystals at frequencies of localized modes (https://optics.physicsmeeting.com/abstract/2017/optics-of-photonic-liquid-crystals-at-frequencies-of-localized-modes)

Biography:

V A Belyakov graduated from Moscow Engineering Institute in 1961 and was a Postgraduate student of I V Kurchatov Atomic Energy Institute during 1961–64. He received Doctor of Science degree in 1974. He was the Head of Laboratory in All-Union Physics-Technical and Radio–Technical Institute from 1964 to 1982 and Surface and Vacuum Research Centre, Moscow during 1982–1995. Since 1995, he is a Senior Researcher in L D Landau Institute for Theoretical Physics. Since 1982, he is a Part-time Professor in Moscow Institute for Physics and Technology; and short term Visiting Professor of some universities: Leuven (Belgium), Tokyo (Japan), Paris Sud (France), Glasgow Thrathclyde (Scotland), etc. He is the Author of the following monographs: Optics of Cholesteric Liquid Crystals, 1982, Optics of Chiral Liquid Crystals, 1989; Diffraction Optics of Complex Structured Periodic Media , 1988, 1992; Optics of Photonic Crystals, Publishing House of Moscow Institute of Physics and Technology, 2013 (Textbook, in Russian). He is honored with the Grants of Russian Foundation for Basic Research (RFBR), Soros Grants, and INTAS Grants. He is a Member of Russian Academy of Natural Sciences, Member of Russian Academy of Metrology, Member of International Liquid Crystal Society and Liquid Crystal Society of CIS (member Governing body 1982).

Abstract:

Recently great attention was paid to the localized optical modes in photonic crystals, in particular, in photonic liquid crystals due to their efficient application in the linear and nonlinear optics. Here a brief survey of the publications and original theoretical results on the localized optical modes in photonic liquid crystals in connection with explanation of the corresponding experimental observations are presented. Theoretical studies were performed for the certainty, as the example of chiral liquid crystals (CLCs). The chosen model (absence of dielectric interfaces in the studied structures) allows one to get rid off the polarization by mixing at the surface of the CLC layer and the defect structure (DMS) to reduce the corresponding equations to equations for the light of diffraction in the CLC polarization, to obtain an analytic description of localized edge (EM) and defect (DM) modes. The dispersion equations determining connection of the EM and DM frequencies with the CLC layer parameters and other parameters of the DMS are obtained. Analytic expressions for the transmission and reflection coefficients of the DMS are presented and analyzed. Specific cases were considered, as DMS with an active (i.e. transforming the light intensity or polarization) defect layer, CLC layer of local anisotropic absorption and conic-helical director structures. It is shown that the active layer (excluding an amplifying one) reduces the DM life-time (and increase the lasing threshold) in comparison with the case of DM at an isotropic defect layer. The case of CLC layers with an anisotropic local absorption is also analyzed and, in particular, shown that due to the Borrmann effect the EM life-times for the EM frequncies at the opposite stop-bands edges may be signifinately different and so in the experiment optimization of it should be taken into account. The experimentally observed enhancement of some optical effects in photonic liquid crystals at the EM and DM frequencies (lowering of the lasing threshold, abnormally strong absorption, etc.) are in good agreement with the presented theory. Options of experimental observations of the new theoretically revealed phenomena are discussed. It is emphasized that the presented localized modes in CLC results are of a general nature and are qualitatively applicable for the localized modes in other structures.

REFERENCES

1. Liquid Crystals Microlasers, Eds. L.M.Blinov, R.Bartolino, Transword Research Network, 2010.
2. V. I. Kopp, Z.-Q. Zhang, and A. Z. Genack, Prog. Quantum Electron. **27**, 369 (2003).
3. V.A.Belyakov, S.V,Semenov, JETP , **118**, 798 (2014).
4. V. A. Belyakov, Mol. Cryst. Liquid Cryst. **612**, 81 (2015).

Enayet Rahman (https://optics.physicsmeeting.com/speaker/2017/enayet-rahman-university-of-london-uk)

University of London, UK

Title: Optical waveguide properties of myelinated and unmyelinated nerve axons from ultraviolet to NIR wavelengths (https://optics.physicsmeeting.com/abstract/2017/optical-waveguide-properties-of-myelinated-and-unmyelinated-nerve-axons-from-ultraviolet-to-nir-wavelengths)

Biography:

Abstract:

Statement of the Problem: Infrared Nerve Stimulation (INS) is becoming popular because of its potential to provide targeted stimulation. Recently it was claimed that myelin sheath can guide light (200 nm – 1300 nm), however propagation characteristics were not reported for wavelengths l > 1500 nm, common in INS. We present them here for l up to 2000 nm for both myelinated and unmyelinated nerve fibers.

Methodology & Theoretical Orientation: Modal analysis was performed on the cross-section of the nerve fiber by solving Maxwell's equations. The effective index (n_eff) of the first three modes was determined and the single mode operating wavelength range was determined for both myelinated and unmyelinated nerve fibers, using a 4-um diameter axon. The overall diameter of the myelinated fiber was 6.66 um. The refractive indices of the fiber cytoplasm, the myelin sheath, and the outside medium were set as 1.34, 1.44 and 1.38 respectively.

Findings: The optical power propagating through unmyelinated fiber is confined by the index of the fiber's cytoplasm (1.38) being higher than its surrounding (1.34). The effective indices of the first three propagating modes were determined and plotted in for 200nm ≤ l ≤ 2000nm. It was found that the unmyelinated fiber is single-mode for l > 1700 nm. In the myelinated fiber, optical power is confined within the myelin sheath (1.44). The effective indices of the myelinated fiber indicate that it supports more modes than the unmyelinated one and the myelin sheath operates in a single-mode condition for wavelengths longer than 1980 nm. This article determines light propagation characteristics of nerve fibers for a range of wavelengths, making it very useful for future INS designs. This study can also be useful in the field of interfacing brain using light.

(https://www.conferenceseries.com/south-korea-meetings)
Taiwan (https://www.conferenceseries.com/taiwan-meetings)
Thailand (https://www.conferenceseries.com/thailand-meetings)
UAE (https://www.conferenceseries.com/uae-meetings)
Romania (https://www.conferenceseries.com/romania-meetings)

/italy-meetings)
Luxembourg (https://www.conferenceseries.com/luxembourg-meetings/)
Netherlands (https://www.conferenceseries.com/netherlands-meetings)
Norway (https://www.conferenceseries.com/norway-meetings)
Poland (https://www.conferenceseries.com/poland-meetings)
Portugal (https://www.conferenceseries.com/portugal-meetings)
Romania (https://www.conferenceseries.com/romania-meetings)
Russia (https://www.conferenceseries.com/russia-meetings)
Slovenia (https://www.conferenceseries.com/slovenia-meetings)
South Africa (https://www.conferenceseries.com/south-africa-meetings)
Spain (https://www.conferenceseries.com/spain-meetings)
Sweden (https://www.conferenceseries.com/sweden-meetings)
Switzerland (https://www.conferenceseries.com/switzerland-meetings)
Turkey (https://www.conferenceseries.com/turkey-meetings)
UK (https://www.conferenceseries.com/uk-meetings)

(https://www.conferenceseries.com/pediatrics-meetings)
Pharma Marketing & Industry (https://www.conferenceseries.com/pharma-marketing-industry-meetings)
Physics (https://www.conferenceseries.com/physics-meetings)
Pulmonology (https://www.conferenceseries.com/pulmonology-meetings)
Toxicology (https://www.conferenceseries.com/toxicology-meetings)

/pharmaceutical-sciences-meetings)
Psychiatry (https://www.conferenceseries.com/psychiatry-meetings)
Surgery (https://www.conferenceseries.com/surgery-meetings)
Vaccines (https://www.conferenceseries.com/vaccines-meetings)

Mail us at

Drop us an email for Program enquiry.
photonics@conferenceseries.net (mailto:photonics@conferenceseries.net)
Sponsors / Exhibiting / Advertising.
photonics@physicsconferences.org (mailto:photonics@physicsconferences.org)
General Queries.
photonics@physicsconferences.org (mailto:photonics@physicsconferences.org)

Be a member and support us

- **Join Our Mailing List (join_our_mailing_list.php)**
- **Suggest A Speaker (suggest_speaker.php)**
- **Suggestions (suggestions.php)**
- **Invite Proposals (invite_proposals.php)**

Highlights from last year's Convention!

- **View Past Conference Report (http://www.conferenceseries.com/Past_Reports/photonics-2016-past)**
- **View Conference gallery (http://www.conferenceseries.com/Past_Reports/photonics-2016-gallery)**
- **View Proceedings (http://www.omicsgroup.org/journals/Archive]LOP/photonics-and-laser-technology-2016-proceedings.php)**

Subject: Fwd: Re: Poziv
From: Nikola Petrovic <nzpetr@ipb.ac.rs>
Date: 27.10.2018. 03.46
To: zoran@phy.bg.ac.rs

-------- Original Message --------
Subject: Re: Poziv
Date: 2017-02-26 23:35
From: Nikola Petrovic <nzpetr@ipb.ac.rs>
To: Brana Jelenkovic <branaj@ipb.ac.rs>

Da li možete samo da mi javite da li mi je potvrđena rezervacija sobe?

Puno pozdrava,
Nikola

On 2017-02-23 08:36, Brana Jelenkovic wrote:

> Ima soba, jednokrevetna, cak mozes da biras, Konaci ili Angela. Javi sto pre.
>
> Pozdrav,
>
> Brana
>
> ---
> Brana Jelenkovic
> Photonics Center
> Institute of Physics
> University of Belgrade
>
> On 21.02.2017 10:51, Nikola Petrovic wrote:
>
>> Dolazim sam, tako da mi treba singl. Nažalost, neću biti u mogućnosti
>> da dođem u nedelju.
>>
>> Stižem u ponedeljak, vraćam se nazad u sredu uveče (kad se završe
>> predavanja krećem kući), tako da mi trebaju samo ponedeljak i utorak
>> uveče.
>>
>> Doći ću svojim kolima.
>>
>> Pozdrav,
>> Nikola
>>
>> On 2017-02-21 08:50, Brana Jelenkovic wrote:
>>
>>> Pozivni predavaci su oslobodjeni kotizacije.
>>>
>>> Nisi se javio da ti rezervisemo sobu u Konacima, da li si sam negde
>>> rezervisao. Ako nisi, reci da li ides sam ili sa porodicom. odn kolika
>>> ti soba treba. Nece biti lako ali mogu da pokusam da rezervisem joe
>>> jednu sobu.
>>>
>>> conference bus krece u nedelju u 7:30 ispred IF-a, reci i da li hoces
>>> da putujes sa nama.

Pozdrav,

Brana

Brana Jelenkovic
Photonics Center
Institute of Physics
University of Belgrade

On 21.02.2017 06:50, Nikola Petrovic wrote:

Šaljem apstrakt. Javite mi ako treba nešto da se ispravi što se formata tiče.

Koliko vidim smeštaj je u sopstvenoj režiji, ali pretpostavljam da su
svi predavači automatski učesnici konferencije i mogu prisustvovati
svim predavanjima.

Koliko para treba da se uplati u žiro račun spomenut u templetu?

Puno pozdrava,
Nikola

On 2017-02-20 23:27, Brana Jelenkovic wrote:

Nikola,

nisi poslao naslov i apstrakt predavanja na Radionici. Ako nisi
odustao od puta na Kopaonik, posalji to da bi bilo u programu i knjizi
apstrakta, sto pre, znaci do sutra u podne.

Pozdrav,
Brana

Brana Jelenkovic
Photonics Center
Institute of Physics
University of Belgrade

On 26.11.2016 19:24, Nikola Petrovic wrote:

On 2016-11-23 17:22, Brana Jelenkovic wrote:

Dragi Nikola,

imas poziv od Organizacionog odbora 10. Radionice iz fotonike da budes
predavac. Predavanja traju 25 min. Radionica se odrzava od 26 februara
do 2 marta na Kopaoniku, u hotelu Konaci,

Verujemo da imas znacajne radove iz nelinearne optike, jedne od tema
Radionice.

Nadamo se da ces da prihvatis poziv.

Pozdrav,
u ime Organizacionog komiteta

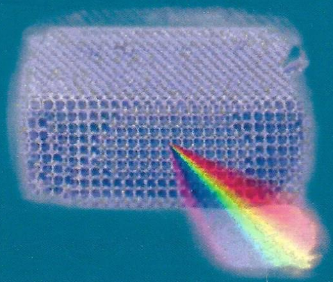
Brana Jelenkovic

> > > > Sa zadovoljstvom Vam saopštavam da prihvatam poziv da održim predaavanje.
> > > >
> > > > puno pozdrava,
> > > > Nikola
> > > >
> > > > --
> > > > Institute of Physics Belgrade
> > > > Pregrevica 118, 11080 Belgrade, Serbia
> > > > http://www.ipb.ac.rs/
> > >
> > > --
> > > Institute of Physics Belgrade
> > > Pregrevica 118, 11080 Belgrade, Serbia
> > > http://www.ipb.ac.rs/
> >
> > --
> > Institute of Physics Belgrade
> > Pregrevica 118, 11080 Belgrade, Serbia
> > http://www.ipb.ac.rs/

--
Institute of Physics Belgrade
Pregrevica 118, 11080 Belgrade, Serbia
http://www.ipb.ac.rs/

УНИВЕРЗИТЕТ У БЕОГРАДУ

Институт за физику Београд



Конференција

Десета радионица фотонике (2017)

Зборник апстраката

10th Photonics Workshop
Book of Abstracts
Kopaonik, 26.2-2.3 2017

Копаоник, 26.2–2.3 2017.

Конференција **Десета радионица фотонике (2017)**

Зборник апстраката

Копаоник 26.2.-2.3.2017.

Издаје

Институт за физику Универзитета у Београду

За издавача

др Александар Богојевић, директор

Уредник

др Драган Лукић

Тираж

100 примерака

ИСБН 978-86-82441-45-8

Штампа

Развојно-истраживачки центар,
Технолошко-металуршког факултета у Београду

Карнегијева 4, Београд

CIP - Каталогизација у публикацији
Народна библиотека Србије, Београд

CIP

535(048)
681.7(048)
66.017/.018(048)

PHOTONICS Workshop (10 ; 2017 ; Kopaonik)
Book of Abstracts / 10th Photonics Workshop, Kopaonik, 26.2-2.3.2017. = Зборник апстраката / Конференција Десета радионица фотонике (2017), Копаоник, 26.2-2.3.2017. ; [urednik Dragan Lukić]. - Београд : Институт за физику Универзитета, 2017 (Београд : Развојно-истраживачки центар графичког инжењерства ТМФ). - X, 46 str. : ilustr. ; 25 cm

Тираж 100. - Реч уредника: str. VII. - Registar.

ISBN 978-86-82441-45-8

a) Оптика - Апстракти b) Оптоелектроника - Апстракти c) Технички материјали - Апстракти
COBISS.SR-ID 229745420

General analytic solutions to the various forms of the Nonlinear Schrödinger Equation using the Jacobi elliptic function expansion method

Nikola Petrović[1]

(1) *Institute of Physics, 118 Pregrevica, 11080 Belgrade, Serbia*

Contact: Nikola Petrović (nzpetr@ipb.ac.rs)

Abstract. In recent years there have been great developments towards finding exact solutions to various forms of the Nonlinear Schrödinger equation (NLSE). In particular, the combination of several ideas, most notably the F-expansion method, the principle of harmonic balance and the use of the Jacobi elliptic function (JEF) for the expansion function, has yielded a rich new class of solutions for a wide range of parameters of the NLSE. Thanks to the mathematical properties of JEFs, both solitary wave and traveling wave solutions can be realized and the effect of chirp can be added to all the solutions [1].

The fundamental ansatz for the solution to the basic NLSE with distributed coefficients, anomalous dispersion and Kerr nonlinearity was described in [1]. This ansatz was subsequently modified in order to find the analytical solution to the NLSE with the normal dispersion in [2] and was also modified to account for nonlinearities of an arbitrary polynomial order [3]. Further adaptation has lead to the analytical solutions of the Gross-Pitaevskii equation [4,5]. Here, it turned out to be necesssary to solve the Ricatti differential equation for the chirp function, which gave rise to very complex forms of solutions. The stability analysis performed in [6] revealed that in the majority of solutions obtained we had either unconditional stability or stability achievable through the use of dispersion management. Other systems that were covered in subsequent papers include the NLSE with linear potential, pairs of co- or counter-propatating beams in a Kerr medium and solutions involving the Weierstrass elliptic function. Current topics of interest include the study of vortex clusters in the NLSE and finding analytic solutions for NLSEs with a non-integer degree of nonlinearity.

REFERENCES

[1] M. Belić, N. Z. Petrović, W. P. Zhong, R. H. Xie and G. Chen, Phys. Rev. Lett.101, 0123904 (2008)
[2] N. Z. Petrović, M. Belić, W. P. .Zhong, R. H. Xie and G. Chen, Opt. Lett.34, 1609 (2009).
[3] N. Z. Petrović, M. Belić and W. P .Zhong, Phys. Rev. E 83, 026604 (2011).
[4] N. Z. Petrović, M. Belić and W. P .Zhong, Phys. Rev. E 81, 016610 (2010).
[5] N. Z. Petrović, N. Aleksić, A. Al Bastami and M. Belić, Phys. Rev. E 83, 036609 (2011).
[6] N. Z. Petrović, N. Aleksić and M. Belić, Optics express 23 (8), 10616-10630 (2015).

10th Photonics Workshop (2017)

Program

Sunday, February 26, 2017

from	to	activity	carried by	type
16:10	16:30	*OPENING*	B. Jelenković	
		chairperson	B. Jelenković	
16:30	17:00	Dynamic interactions between glutamate-mediated plateau potentials and backpropagating action potentials in dendrites of cortical pyramidal neurons	S. Antić	1
17:00	17:30	Towards low-loss metamaterials for nanophotonics and plasmonics	Z. Jakšić	1
17:30	17:40	*BREAK*		
17:40	18:10	Scattering enhanced absorption in biophotonic structures	D. Pantelić	1
18:10	18:30	Luminescence and structural properties of Eu^{3+}doped Sr_2CeO_4 nanopowders	D. Šević	2
18:30	20:00	*DINNER BREAK*		
		chairperson		
20:00	20:20	Proposal for efficient atom localization scheme using Zeeman coherences in degenerate two-level atomic system	J. Dimitrijević	2
20:20	20:40	Effects of water adsorption on thin films of graphene and tungsten disulfide as active components for biochemical sensors	R. Panajotović	2
20:40	21:00	Electromagnetic wave propagation through terahertz chiral metamaterials	D. Stojanović	2

21:00	21:10	*BREAK*		
21:10	21:30	The adsorption of gases during LIPSS formation on thin metal films with femtosecond beam	A. Kovačević	2
21:30	21:50	Comparison of the securities of two-state and four-state quantum bit-commitment protocols	D. Popović	2
21:50	22:10	Light localization in two-dimensional Lieb lattices with alternating spacings and Kerr nonlinearity	P. Beličev	2
22:10	22:30	Laser treatment of multilayered Ti/Ta thin film structures (pdf)	M. Obradović	2

Monady, February 27, 2017

from	to	activity	carried by	type
16:00	16:30	*REFRESHMENT*		
		chairperson		
16:30	17:00	Building your own Light Sheet Fluorescence Microscope	A. Kranz	1
17:00	17:30	H2020-CARDIALLY – Noninvasive capturing and quantitative analysis of multi-scale multi-channel diagnostic data of the cardiovascular system	Lj. Hadžievski	1
17:30	17:40	*BREAK*		
17:40	18:10	General analytic solutions to the various forms of the Nonlinear Schrödinger Equation using the Jacobi elliptic function expansion method	N. Petrović	1
18:10	18:30	Electro-optic techniques and THz time domain spectroscopy (pdf)	R. Pan	2
18:30	20:00	*DINNER BREAK*		
20:00	20:20	FTIR Spectrometer for spectral characterization of THz undulator at FLASH1 (pdf)	E. Zapolnova	2
20:20	20:40	Raman spectroscopy: a tool for the characterization	I. Petrović	2

		of antioxidant components of mature tomato fruits		
20:40	21:00	Imaging of functional and structural alterations in primary cortical astrocytes isolated from the transgenic rat model of ALS	S. Stamenković	2
21:00	21:10	*BREAK*		
21:10	21:30	Deformation of Fermi Surface for Ultracold Dipolar Fermi Gases	V. Veljić	2
21:30	21:50	Benefits of implementing online Dynamic Bandwidth Allocation algorithm in energy efficient WDM EPON	B. Pajčin	2
21:50	22:10	Performances of BB84 and B92 QKD authentication protocols analyzed by proposed physical model	N. Miljković	2
22:10	22:30	Ejection fraction calculation using multiparametric cardiac measurement system	M. Miletić	2
22:30	22:50	G-Protein Coupled Receptors structure prediction by Bayesian probabilistic approach principle	M. Mudrinić	2

Tuesday, February 28, 2017

from	to	activity	carried by	type
16:00	16:30	*REFRESHMENT*		
		chairperson		
16:30	17:00	Quantum Control on an Atom Chip	F. Cataliotti	1
17:00	17:30	Imaging the state of the blood-brain barrier, cellular and molecular markers of inflammation in hSOD1 G93A rat model of ALS	P. Andjus	1
17:30	17:40	*BREAK*		
17:40	18:10	Putting some light on the membrane physiology of filamentous fungi	M. Živić	1
18:10	18:30	Four way mixing in potassium vapor with large photon amplification	B. Jelenković	2

18:30	20:00	*DINNER BREAK*		
		chairperson		
20:00	20:20	Destruction of organophosphate pollutants in water using atmospheric pressure plasma sources	N. Škoro	2
20:20	20:40	Fabrication of fluorescent probe for cell bioimaging by bi-conjugation of gold nanoparticles with riboflavin and tryptophan biomolecules	V. Djoković	2
20:40	21:00	Nonlinear localized flat-band modes in pseudo-spinor diamond chain	G. Gligorić	2
21:00	21:10	*BREAK*		
21:10	21:30	Analysis of Transmission Line Coupled with Antisymmetric Split Ring Resonators	V. Milošević	2
21:30	21:50	Comparative analysis of two porous carbon materials based on similar type of precursors	A. Kalijadis	2
21:50	22:10	Assesment of connexin protein distribution in the human fetal cortex using confocal laser scanning microscopy	D. Kočović	2
22:10	22:30	Quantum Droplets in a Strongly Dipolar Bose-Einstein condensate	D. Vudragović	2
22:30	22:50	Stokes Eigenvectors of Anisotropic Medium	V. Merkulov	2

Wednesday, March 1, 2017

from	to	activity	carried by	type
16:00	16:30	*REFRESHMENT*		
		chairperson		
16:30	17:00	Maximally efficient symmetry based diagonalization of biophysical Hamiltonians	I. Milošević	1
17:00	17:30	Application of non-equilibrium plasmas in treatments of seeds and plant cells	N. Puač	1

17:30	18:00	Harmonic Generation via Excitation of Surface States Formed from Spatially Separated Electrons and Holes in Nanocomposites	O. Khasanov	1
18:00	18:10	*BREAK*		
		chairperson		
18:10	18:30	Photonic density of states near a semi-infinite metallodielectric superlattice	G. Isić	2
18:30	18:50	Hemoglobin imaging using two photon excitation fluorescence microscopy	A. Krmpot	2
18:50	19:10	Spatio-Temporal Localization of Powerful Femtosecond Pulses in Kerr Solids	O. Fedotova	2
19:10	19:20	*CLOSING*		
20:30	?	*WORKSHOP DINNER*		

Workshop Program: Monday, February 27, 2017

Final program

The schedule of lectures can be downloaded in pdf format (pdf, ~500 kB).

Monday, February 27, 2017

from	to	activity	carried by	type
16:00	16:30	*REFRESHMENT*		
		chairperson		
16:30	17:00	Building your own Light Sheet Fluorescence Microscope	A. Kranz	1
17:00	17:30	H2020-CARDIALLY – Noninvasive capturing and quantitative analysis of multi-scale multi-channel diagnostic data of the cardiovascular system	Lj. Hadžievski	1
17:30	17:40	*BREAK*		
17:40	18:10	General analytic solutions to the various forms of the Nonlinear Schrödinger Equation using the Jacobi elliptic function expansion method	N. Petrović	1
18:10	18:30	Electro-optic techniques and THz time domain spectroscopy (pdf)	R. Pan	2
18:30	20:00	*DINNER BREAK*		
20:00	20:20	FTIR Spectrometer for spectral characterization of THz undulator at FLASH1 (pdf)	E. Zapolnova	2
20:20	20:40	Raman spectroscopy: a tool for the characterization of antioxidant components of mature tomato fruits	I. Petrović	2
20:40	21:00	Imaging of functional and structural alterations in primary cortical astrocytes isolated from the transgenic rat model of ALS	S. Stamenković	2
21:00	21:10	*BREAK*		
21:10	21:30	Deformation of Fermi Surface for Ultracold Dipolar Fermi Gases	V. Veljić	2
21:30	21:50	Benefits of implementing online Dynamic Bandwidth Allocation algorithm in energy efficient WDM EPON	B. Pajčin	2
21:50	22:10	Performances of BB84 and B92 QKD authentication protocols analyzed by proposed physical model	N. Miljković	2
22:10	22:30	Ejection fraction calculation using multiparametric cardiac measurement system	M. Miletić	2
22:30	22:50	G-Protein Coupled Receptors structure prediction by Bayesian probabilistic approach principle	M. Mudrinić	2

IMPORTANT DATES

1st announcement: Nov 14, 2016
Registration (early): Dec 31, 2016
Registration (late): Feb 26, 2017
Abstract upload: ~~Jan 28, 2017~~
Feb 15, 2017

Workshop start: Feb 26, 2017

SPONSORS

ORGANIZERS

Institute of Physics, University of Belgrade ♦ Pregrevica 118 ♦ 11080 Belgrade, Serbia ♦ phone +381 (11) 3713-000 ♦ fax +381 (11) 3162-190 ♦ PIB 100105980 ♦ MB 07018029

6. Елементи за квантитативну анализу рада

Остварени резултати у периоду након одлуке Научног већа о предлогу за стицање претходног научног звања

Категорија	М бодова по раду	Број радова	Укупно М бодова
М21а	10	5	50
М21	8	2	16
М22	5	1	5
М23	3	1	3
М31	3,5	0	0
М32	1,5	2	3
М51	2	1	2
Укупно			79

Поређење са минималним квантитативним условима за избор у звање виши научни сарадник

М категорије	Услов	Остварено
Укупно	50	**79**
М10+М20+М31+М32+М33+М41+М42	40	**77**
М11+М12+М21+М22+М23	30	**74**

И у периоду пре и у периоду после избора кандидат је већином објављивао радове у часописима категорије М21а и М21. Укупан фактор утицаја (збир импакт фактора) радова кандидата је **49,448**, а у периоду након одлуке Научног већа о предлогу за стицање претходног научног звања тај фактор је **24,526**.
Додатни библиометријски показатељи за период после избора у садашње звање (тачка 2 **П1П**) су:

	ИФ	М	СНИП
Укупно	24,526	76	11.19
Усредњено по чланку	2.725	7.6	1.24
Усредњено по аутору	10.35	30.75	4.629

Према бази WOS радови кандидата су цитирани укупно 352 пута, док је број цитата без аутоцитата 308. Према истој бази H–индекс кандидата је 8.
Прилог: подаци о цитираности са интернет странице WOS.

На бази Google Scholar има 515 цитата (што укључује и 54 цитата књиге IMO Compendium) и H фактор 10.
Конкретне вредности Импакт Фактора и рангирања часописа су наведене за све радове у листи радова. Том приликом није коришћено правило три године осим у случају када још нису објављени подаци (2018) или када због истека претплате нема података у бази Кобсон.

7. Списак објављених радова и других публикација
Никола З Петровић

СПИСАК РАДОВА У ЧАСОПИСИМА ДО ПРЕТХОДНОГ ИЗБОРА У ЗВАЊЕ

[1] D. Jović, M. Petrović, D. Arsenović, S. Prvanović, M. Belić, N. Z. Petrović, “Counterpropagating beams in photorefractive media and optically induced photonic lattices’’, Asian J. Phys. 15, 283 (2006). M24

[2] W.P. Zhong, R.-H. Xie, M. Belić, N. Z. Petrović, G. Chen and L. Yi, “Exact spatial soliton solutions of the two-dimensional generalized nonlinear Schrödinger equation with distributed coefficients,’’ Phys. Rev. A 78,023821 (2008). IF 2.908 (6/64) M21a

[3] M. Belić, N. Z. Petrović, W.-P. Zhong, R. H. Xie and G. Chen, “Analytical Light Bullet Solutions to the Generalized (3+1)-Dimensional Nonlinear Schrödinger Equation,’’ Phys. Rev. Lett. 101, 0123904 (2008). IF 7.180 (5/68) M21a

[4] N. Z. Petrović, M. Belić, W.-P. Zhong, R.-H. Xie and G. Chen, “Exact spatiotemporal wave and soliton solutions to the generalized (3+1)-dimensional Schrödinger equation for both normal and anomalous dispersion,’’ Opt. Lett. 34, 1609 (2009). IF 3.059 (6/71) M21a

[5] N. Z. Petrović, M. Belić and W.-P. Zhong, “Spatiotemporal wave and soliton solutions to the generalized (3+1)-dimensional Gross-Pitaevskii equation,’’ Phys. Rev. E 81, 016610 (2010). IF 2.352 (4/54) M21a

[6] N. Z. Petrović, M. Belić and W.-P. Zhong, “Exact traveling-wave and spatiotemporal soliton solutions to the generalized (3+1)-dimensional Schrödinger equation with polynomial nonlinearity of arbitrary order,’’ Phys. Rev. E 83, 026604 (2011). IF 2.255 (6/55) M21 *(napomena u ovom materijalu nije korišćeno pravilo najboljih rezultata u tri godine već samo u tekućoj godini, ukoliko bi se to pravilo primenilo ovo bi bio rad u kategoriji M21a)

[7] N. Z. Petrović, N. Aleksić, A. Al Bastami and M. Belić, “Analytical traveling-wave and solitary solutions to the generalized Gross-Pitaevskii equation with sinusoidal time-varying diffraction and potential,’’ Phys. Rev. E 83, 036609 (2011). IF 2.255 (6/55) M21 *

[8] A. Al Bastami, N. Z. Petrović and M. R. Belić, “Special solutions of the Ricatti Equation with applications to the Gross-Pitaevskii nonlinear PDE,’’ Electron. J. Diff. Eqs., Vol. 2010, No. 66, 1 (2010). IF 0.427 (198/245 podaci za 2011) M23

[9] A. Al Bastami, M. R. Belić, D. Milović and N. Z. Petrović, “Analytical chirped solutions to the (3+1)-dimensional Gross-Pitaevskii equation for various diffraction and potential Functions,’’ Phys. Rev. E 84, 016606 (2011). IF 2.255 (6/55) M21 *

[10] N. Z. Petrović, H. Zahreddine and M. Belić, “Exact spatiotemporal wave and soliton solutions to the generalized (3 + 1)-dimensional nonlinear Schrödinger equation with linear potential,’’ Phys. Scr. 83, 065001 (2011). 1.204 (35/84) M22

[11] S. Xu, N. Z. Petrović and M. Belić, "Vortex solitons in the (2+1)-dimensional nonlinear Schrödinger equation with variable diffraction and nonlinearity coefficients,'' Phys. Scr. 87, 045401 (2013). IF 1.296 (40/78) M22

Предавања на конференцијама публикована у целини у часописима

[12] M. R. Belić, M. S. Petrović, D. M. Jović, A. I. Strinić, D. D. Arsenović, S. Prvanović, R. D. Jovanović, N. Z. Petrović, "Dancing Light: Counterpropagating, Beams in Photorefractive Crystals,'' Acta Physica Polonica A 212, 729 (2007). M33 (M23 as a journal)

[13] N. Z. Petrović and H. Zahreddine, "Exact traveling wave solutions to coupled generalized nonlinear Schrödinger equations,'' Phys. Scr. T149, 014039 (2012). M33 (IF 1.032 (48/83) M22 as a journal)

РАДОВИ ПУБЛИКОВАНИ ПОСЛЕ ПРЕТХОДНОГ ИЗБОРА У ЗВАЊЕ

Радови у међународним часописима изузетних вредности M21a

[14] W. P. Zhong, L. Chen, M. Belić, N. Petrović, "Controllable parabolic-cylinder optical rogue wave," Phys. Rev. E 90 (4), 043201 (2014) IF=2.288 (5/54) SNIP=1.14

[15] S. L. Xu, N. Petrović, M. R. Belić, "Exact solutions of the (2+ 1)-dimensional quintic nonlinear Schrödinger equation with variable coefficients," Nonlinear Dynamics 80 (1-2), 583-589 (2015) IF=3.000 (8/135) SNIP=1.47

[16] S. L. Xu, N. Petrović, M. R. Belić, W. Deng, "Exact solutions for the quintic nonlinear Schrödinger equation with time and space," Nonlinear Dynamics 84 (1), 251-259 (2016) IF=3.464 (8/133) SNIP=1.54

[17] S. L. Xu, N. Petrović, M. R. Belić, Z. L. Hu, "Light bullet supported by parity-time symmetric potential with power-law nonlinearity," Nonlinear Dynamics 84 (4), 1877-1882 (2016) IF=3.464 (8/133) SNIP=1.54

[18] N. Z. Petrović, "Spatiotemporal traveling and solitary wave solutions to the generalized nonlinear Schrodinger equation with single-and dual-power law nonlinearity," Nonlinear Dynamics 93 (4), 2389-2397 (2018) IF=4.339 (8/134) SNIP=1.75

Радови у врхунским међународним часописима M21

[19] N. Z. Petrović, N.B. Aleksić, M. Belić, "Modulation stability analysis of exact multidimensional solutions to the generalized nonlinear Schrödinger equation and the Gross-Pitaevskii equation using a variational approach," Optics Express 23 (8), 10616-10630 (2015) IF=3.148 (14/90) SNIP=1.67

[20] S. L. Xu, Y. Zhao, N. Z. Petrović, M. R. Belić, "Spatiotemporal soliton supported by parity-time symmetric potential with competing nonlinearities," EPL (Europhysics Letters) 115 (1), 14006 (2016) IF=1.957 (23/79) SNIP=0.60

Радови у истакнутим међународним часописима М22

[21] S. L. Xu, G. P. Zhou, N. Petrović, M. R. Belić, " Nonautonomous vector matter waves in two-component Bose-Einstein condensates with combined time-dependent harmonic-lattice potential," Journal of Optics 17 (10), 105605 (2015) IF=1.847 (36/90) SNIP=0.87

Радови у међународним часописима М23

[22] N. Z. Petrović, M. Bohra, "General Jacobi elliptic function expansion method applied to the generalized (3+ 1)-dimensional nonlinear Schrödinger equation," Optical and Quantum Electronics 48 (4), 268 (2016) IF=1.055 (70/92) SNIP=0.61

Предавања по позиву на међународним скуповома М32

[23] Nikola Z Petrović ''General analytic solutions to the various forms of the nonlinear Schrödinger equation using the Jacobi elliptic function expansion method'' 6th International Conference on Photonics July 31- August 01, 2017 Milan, Italy (https://optics.physicsmeeting.com/abstract/2017/general-analytic-solutions-to-the-variousforms-of-the-nonlinear-schr-dinger-equation-using-the-jacobi-elliptic-function-expansion-method)

[24] Nikola Z Petrović ''General analytic solutions to the various forms of the Nonlinear Schrödinger Equation using Jacobi eliptic function expansion method'' 10th Photonics Workshop Kopaonik 26.2-2.3.2017. ISBN978-86-82441-45-8 Institut za fizku Beograd str. 36

Радови у водећим националним часописима М51

[25] S. L. Xu, N. Petrović, M. R. Belić, "Two-dimensional dark solitons in diffusive nonlocal nonlinear media," Journal of Optics 44 (2), 172-177 (2015) IF=1.847 (36/90) SNIP=0.87

8. Подаци о цитираности радова
Никола З Петровић

Према бази WOS радови кандидата су цитирани укупно 352 пута, док је број цитата без аутоцитата 308. Према истој бази Н–индекс кандидата је 8.

Прилог: подаци о цитираности са интернет странице WOS.

На бази Google Scholar има 515 цитата (што укључује и 54 цитата књиге IMO Compendium) и H фактор 10.

Приложене су странице са прегледом цитираности кандидата у бази WOS.

Citation report for 20 results from Web of Science Core Collection between 1996 and 2019 Go

You searched for: **From Marked List: ...More**

This report reflects citations to source items indexed within Web of Science Core Collection. Perform a Cited Reference Search to include citations to items not indexed within Web of Science Core Collection.

Export Data: Save to Excel File

Total Publications

20 Analyze

h-index

8

Average citations per item

17.6

Sum of Times Cited

352

Without self citations

308

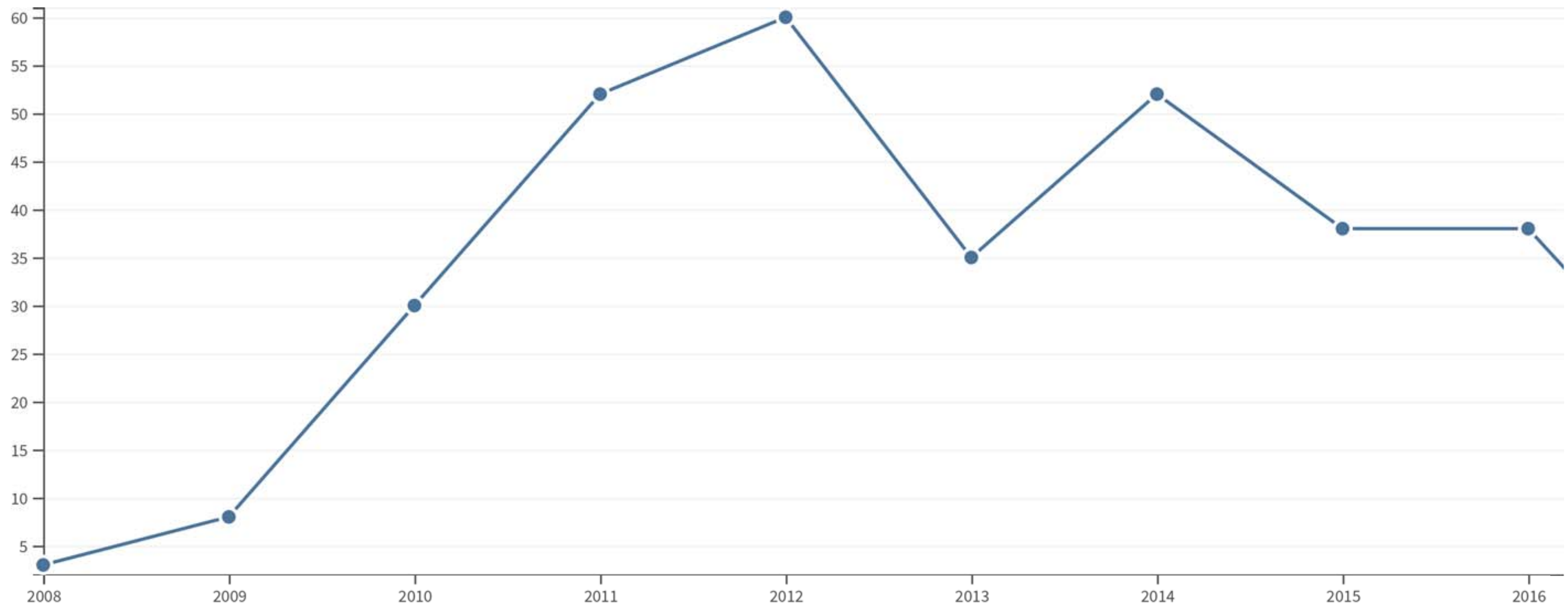
Citing articles

223 Analyze

Without self citations

206 Analyze

Sum of Times Cited per Year



Sort by: Times Cited Date More

Page 1 of 2

Use the checkboxes to remove individual items from this Citation Report

2015	2016	2017	2018	2019	Total	Average Citations per Year
38	38	17	19	0	352	32.00

or restrict to items published between 1996 and 2019 Go

1. **Analytical light bullet solutions to the generalized (3+1)-dimensional nonlinear schrodinger equation** By: Belic, Milivoj; Petrovic, Nikola; Zhong, Wei-Ping; et al. PHYSICAL REVIEW LETTERS Volume: 101 Issue: 12 Article Number: 123904 Published: SEP 19 2008	13	10	4	5	0	124	11.27
2. **Exact spatial soliton solutions of the two-dimensional generalized nonlinear Schrodinger equation with distributed coefficients** By: Zhong, Wei-Ping; Xie, Rui-Hua; Belic, Milivoj; et al. PHYSICAL REVIEW A Volume: 78 Issue: 2 Article Number: 023821 Part: B Published: AUG 2008	6	7	4	3	0	86	7.82
3. **Exact spatiotemporal wave and soliton solutions to the generalized (3+1)-dimensional Schrodinger equation for both normal and anomalous dispersion** By: Petrovic, Nikola Z.; Belic, Milivoj; Zhong, Wei-Ping; et al. OPTICS LETTERS Volume: 34 Issue: 10 Pages: 1609-1611 Published: MAY 15 2009	4	4	1	1	0	33	3.30
4. **Spatiotemporal wave and soliton solutions to the generalized (3+1)-dimensional Gross-Pitaevskii equation** By: Petrovic, Nikola Z.; Belic, Milivoj; Zhong, Wei-Ping PHYSICAL REVIEW E Volume: 81 Issue: 1 Article Number: 016610 Part: 2 Published: JAN 2010	1	2	1	1	0	26	2.89
5. **Exact traveling-wave and spatiotemporal soliton solutions to the generalized (3+1)-dimensional Schrodinger equation with polynomial nonlinearity of arbitrary order** By: Petrovic, Nikola Z.; Belic, Milivoj; Zhong, Wei-Ping	2	3	0	1	0	19	2.38

	PHYSICAL REVIEW E Volume: 83 Issue: 2 Article Number: 026604 Part: 2 Published: FEB 28 2011							
6.	**Controllable parabolic-cylinder optical rogue wave** By: Zhong, Wei-Ping; Chen, Lang; Belic, Milivoj; et al. PHYSICAL REVIEW E Volume: 90 Issue: 4 Article Number: 043201 Published: OCT 6 2014	7	5	2	3	0	17	3.40
7.	**Analytical traveling-wave and solitary solutions to the generalized Gross-Pitaevskii equation with sinusoidal time-varying diffraction and potential** By: Petrovic, Nikola Z.; Aleksic, Najdan B.; Al Bastami, Anas; et al. PHYSICAL REVIEW E Volume: 83 Issue: 3 Article Number: 036609 Part: 2 Published: MAR 30 2011	3	0	1	0	0	12	1.50
8.	**Exact spatiotemporal wave and soliton solutions to the generalized (3+1)-dimensional nonlinear Schrodinger equation with linear potential** By: Petrovic, Nikola Z.; Zahreddine, Hussein; Belic, Milivoj R. PHYSICA SCRIPTA Volume: 83 Issue: 6 Article Number: 065001 Published: JUN 2011	1	1	0	0	0	8	1.00
9.	**Nonautonomous vector matter waves in two-component Bose-Einstein condensates with combined time-dependent harmonic-lattice potential** By: Xu, Si-Liu; Zhou, Guo-Peng; Petrovic, Nikola; et al. JOURNAL OF OPTICS Volume: 17 Issue: 10 Article Number: 105605 Published: OCT 2015	0	1	2	2	0	5	1.25
10.	**Analytical chirped solutions to the (3+1)-dimensional Gross-Pitaevskii equation for various diffraction and potential functions** By: Al Bastami, Anas; Belic, Milivoj R.; Milovic, Daniela; et al. PHYSICAL REVIEW E Volume: 84 Issue: 1 Article Number: 016606 Part: 2 Published: JUL 22 2011	0	0	0	0	0	5	0.63

☐ Select Page

Save to Excel File

Sort by: Times Cited Date More

Page 1 of 2

20 records matched your query of the 42,271,213 in the data limits you selected.

Citation report for 20 results from Web of Science Core Collection between 1996 and 2019 Go

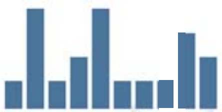
You searched for: **From Marked List: ...More**

This report reflects citations to source items indexed within Web of Science Core Collection. Perform a Cited Reference Search to include citations to items not indexed within Web of Science Core Collection.

Export Data: Save to Excel File

Total Publications

20 Analyze



1997 2016

h-index

8

Average citations per item

17.6

Sum of Times Cited

352

Without self citations

308

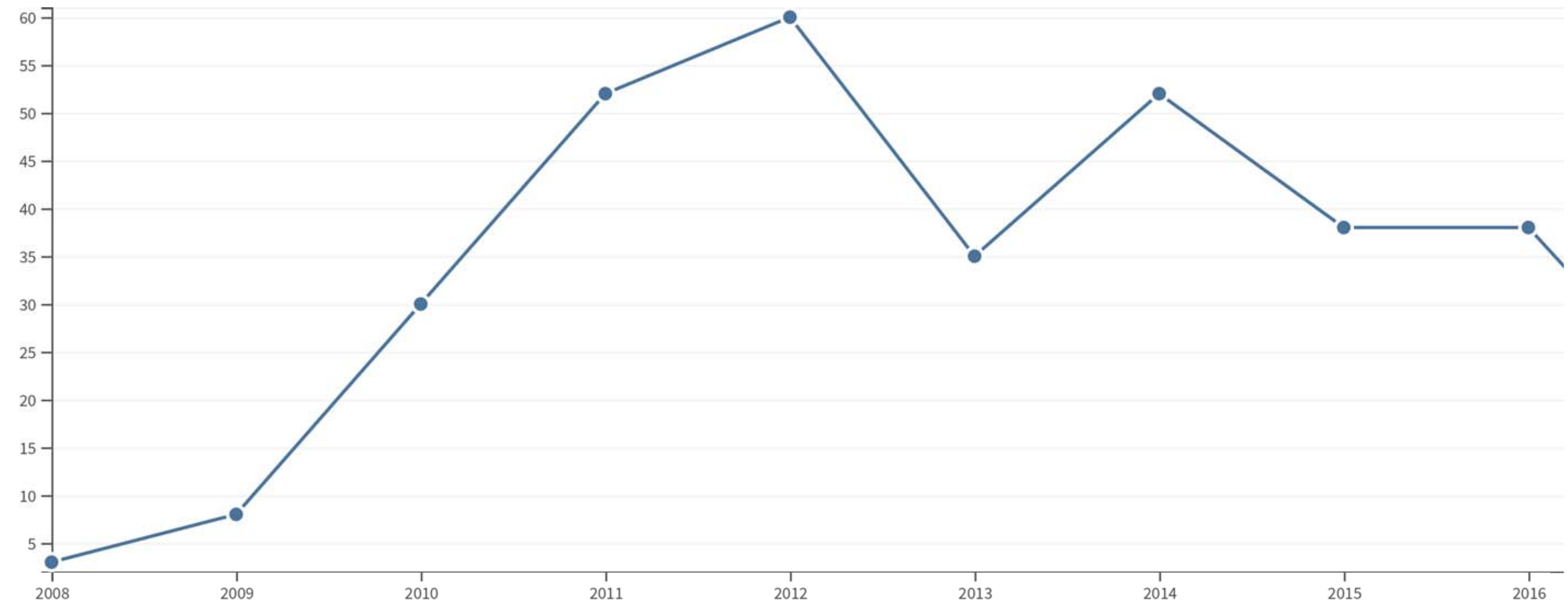
Citing articles

223 Analyze

Without self citations

206 Analyze

Sum of Times Cited per Year



Sort by: Times Cited Date More

Page 2 of 2

	2015	2016	2017	2018	2019	Total	Average Citations per Year
Use the checkboxes to remove individual items from this Citation Report	38	38	17	19	0	352	32.00

or restrict to items published between 1996 and 2019 Go

11. **Modulation stability analysis of exact multidimensional solutions to the generalized nonlinear Schrodinger equation and the Gross-Pitaevskii equation using a variational approach**

By: Petrovic, Nikola Z.; Aleksic, Najdan B.; Belic, Milivoj
OPTICS EXPRESS Volume: 23 Issue: 8 Pages: 10616-10630 Published: APR 20 2015

0	3	0	1	0	4	1.00

12. **Vortex solitons in the (2+1)-dimensional nonlinear Schrodinger equation with variable diffraction and nonlinearity coefficients**

By: Xu, Siliu; Petrovic, Nikola Z.; Belic, Milivoj R.
PHYSICA SCRIPTA Volume: 87 Issue: 4 Article Number: 045401 Published: APR 2013

1	2	0	0	0	4	0.67

13. **Spatiotemporal soliton supported by parity-time symmetric potential with competing nonlinearities**

By: Xu, Si-Liu; Zhao, Yuan; Petrovic, Nikola Z.; et al.
EPL Volume: 115 Issue: 1 Article Number: 14006 Published: JUL 2016

0	0	1	1	0	2	0.67

14. **General Jacobi elliptic function expansion method applied to the generalized (3+1)-dimensional nonlinear Schrodinger equation**

By: Petrovic, Nikola Z.; Bohra, Moiz
OPTICAL AND QUANTUM ELECTRONICS Volume: 48 Issue: 4 Article Number: 268 Published: APR 2016

0	0	1	1	0	2	0.67

15. **Exact spatial soliton solutions of the two-dimensional generalized nonlinear Schrodinger equation with distributed coefficients (vol 78, art no 023821, 2008)**

By: Zhong, Wei-Ping; Xie, Rui-Hua; Belic, Milivoj; et al.

0	0	0	0	0	2	0.18

PHYSICAL REVIEW A Volume: 78 Issue: 3 Article Number: 039906 Published: SEP 2008							
16. **Exact traveling wave solutions to coupled generalized nonlinear Schrodinger equations** By: Petrovic, Nikola; Zahreddine, Hussein Conference: 3rd International School and Conference on Photonics Location: Belgrade, SERBIA Date: AUG 29-SEP 02, 2011 PHYSICA SCRIPTA Volume: T149 Article Number: 014039 Published: APR 2012	0	0	0	0	0	1	0.14
17. **SPECIAL SOLUTIONS OF THE RICCATI EQUATION WITH APPLICATIONS TO THE GROSS-PITAEVSKII NONLINEAR PDE** By: Al Bastami, Anas; Belic, Milivoj R.; Petrovic, Nikola Z. ELECTRONIC JOURNAL OF DIFFERENTIAL EQUATIONS Article Number: 66 Published: MAY 8 2010	0	0	0	0	0	1	0.11
18. **Dancing light: Counterpropagating beams in photorefractive crystals** By: Belic, M. R.; Petrovic, M. S.; Jovic, D. M.; et al. Conference: International School and Conference on Optics and Optical Materials Location: Belgrade, SERBIA Date: SEP 03-07, 2007 Sponsor(s): Univ Belgrade, Inst Phys, Fac Phys; Vinca Inst Nucl Sci & Fac Elect Engn ACTA PHYSICA POLONICA A Volume: 112 Issue: 5 Pages: 729-736 Published: NOV 2007	0	0	0	0	0	1	0.08
19. **Two-dimensional dark solitons in diffusive nonlocal nonlinear media** By: Xu, Si-Liu; Petrovic, Nikola; Belic, Milivoj R. JOURNAL OF OPTICS-INDIA Volume: 44 Issue: 2 Pages: 172-177 Published: JUN 2015	0	0	0	0	0	0	0.00
20. **Exact spatial soliton solutions of the two-dimensional generalized nonlinear Schrodinger equation with distributed coefficients (vol 78, art no 023821, 2008)** By: Zhong, Wei-Ping; Xie, Rui-Hua; Belic, Milivoj; et al.	0	0	0	0	0	0	0.00

PHYSICAL REVIEW A Volume: 78 Issue: 2 Article Number: 029905 Part: B Published: AUG 2008

☐ Select Page

Save to Excel File

Sort by: Times Cited Date More

Page 2 of 2

20 records matched your query of the 42,271,213 in the data limits you selected.

Close

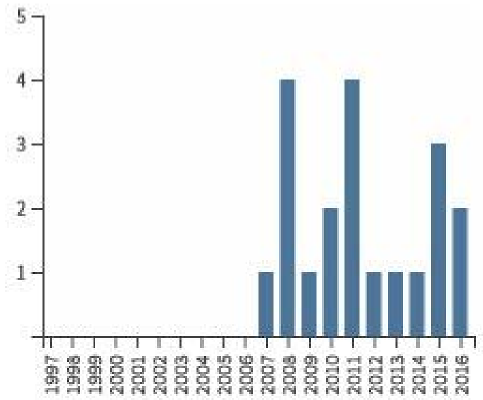
Web of Science
Page 1 (Records 1 -- 10)
[1 | 2]

Print

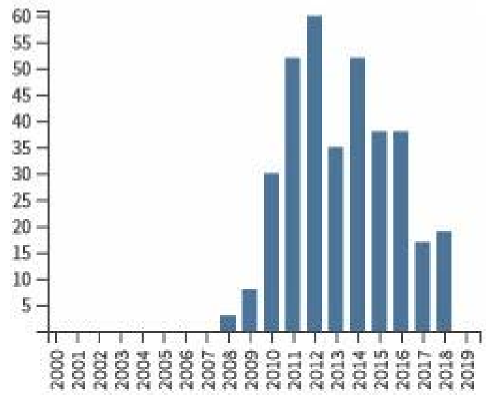
From Marked List:

Timespan=All years. Indexes=SCI-EXPANDED, ESCI, A&HCI, SSCI, CPCI-SSH, CPCI-S.

Total Publications by Year



Sum of Times Cited by Year



Results found: 20
Sum of the Times Cited: 352
Average Citations per Item: 17.60
h-index: 8

		2015	2016	2017	2018	2019	Total	Average Citations per Year
		38	38	17	19	0	352	32.00

		2015	2016	2017	2018	2019	Total	Average Citations per Year
		38	**38**	**17**	**19**	**0**	**352**	**32.00**
1.	**Title:** Analytical light bullet solutions to the generalized (3+1)-dimensional nonlinear schrodinger equation **By:** Belic, Milivoj; Petrovic, Nikola; Zhong, Wei-Ping; et al. **Source:** PHYSICAL REVIEW LETTERS **Volume:** 101 **Issue:** 12 **Article Number:** 123904 **Published:** SEP 19 2008	13	10	4	5	0	124	11.27
2.	**Title:** Exact spatial soliton solutions of the two-dimensional generalized nonlinear Schrodinger equation with distributed coefficients **By:** Zhong, Wei-Ping; Xie, Rui-Hua; Belic, Milivoj; et al. **Source:** PHYSICAL REVIEW A **Volume:** 78 **Issue:** 2 **Article Number:** 023821 **Part:** B **Published:** AUG 2008	6	7	4	3	0	86	7.82
3.	**Title:** Exact spatiotemporal wave and soliton solutions to the generalized (3+1)-dimensional Schrodinger equation for both normal and anomalous dispersion **By:** Petrovic, Nikola Z.; Belic, Milivoj; Zhong, Wei-Ping; et al. **Source:** OPTICS LETTERS **Volume:** 34 **Issue:** 10 **Pages:** 1609-1611 **Published:** MAY 15 2009	4	4	1	1	0	33	3.30
4.	**Title:** Spatiotemporal wave and soliton solutions to the generalized (3+1)-dimensional Gross-Pitaevskii equation **By:** Petrovic, Nikola Z.; Belic, Milivoj; Zhong, Wei-Ping **Source:** PHYSICAL REVIEW E **Volume:** 81 **Issue:** 1 **Article Number:** 016610 **Part:** 2 **Published:** JAN 2010	1	2	1	1	0	26	2.89
5.	**Title:** Exact traveling-wave and spatiotemporal soliton solutions to the generalized (3+1)-dimensional Schrodinger equation with polynomial nonlinearity of arbitrary order **By:** Petrovic, Nikola Z.; Belic, Milivoj; Zhong, Wei-Ping **Source:** PHYSICAL REVIEW E **Volume:** 83 **Issue:** 2 **Article Number:** 026604 **Part:** 2 **Published:** FEB 28 2011	2	3	0	1	0	19	2.38

		2015	2016	2017	2018	2019	Total	Average Citations per Year
		38	38	17	19	0	352	32.00
6.	**Title:** Controllable parabolic-cylinder optical rogue wave **By:** Zhong, Wei-Ping; Chen, Lang; Belic, Milivoj; et al. **Source:** PHYSICAL REVIEW E **Volume:** 90 **Issue:** 4 **Article Number:** 043201 **Published:** OCT 6 2014	7	5	2	3	0	17	3.40
7.	**Title:** Analytical traveling-wave and solitary solutions to the generalized Gross-Pitaevskii equation with sinusoidal time-varying diffraction and potential **By:** Petrovic, Nikola Z.; Aleksic, Najdan B.; Al Bastami, Anas; et al. **Source:** PHYSICAL REVIEW E **Volume:** 83 **Issue:** 3 **Article Number:** 036609 **Part:** 2 **Published:** MAR 30 2011	3	0	1	0	0	12	1.50
8.	**Title:** Exact spatiotemporal wave and soliton solutions to the generalized (3+1)-dimensional nonlinear Schrodinger equation with linear potential **By:** Petrovic, Nikola Z.; Zahreddine, Hussein; Belic, Milivoj R. **Source:** PHYSICA SCRIPTA **Volume:** 83 **Issue:** 6 **Article Number:** 065001 **Published:** JUN 2011	1	1	0	0	0	8	1.00
9.	**Title:** Nonautonomous vector matter waves in two-component Bose-Einstein condensates with combined time-dependent harmonic-lattice potential **By:** Xu, Si-Liu; Zhou, Guo-Peng; Petrovic, Nikola; et al. **Source:** JOURNAL OF OPTICS **Volume:** 17 **Issue:** 10 **Article Number:** 105605 **Published:** OCT 2015	0	1	2	2	0	5	1.25
10.	**Title:** Analytical chirped solutions to the (3+1)-dimensional Gross-Pitaevskii equation for various diffraction and potential functions **By:** Al Bastami, Anas; Belic, Milivoj R.; Milovic, Daniela; et al. **Source:** PHYSICAL REVIEW E **Volume:** 84 **Issue:** 1 **Article Number:** 016606 **Part:** 2 **Published:** JUL 22 2011	0	0	0	0	0	5	0.63

Close

Web of Science
Page 1 (Records 1 -- 10)

Print

Close

Web of Science
Page 2 (Records 11 -- 20)

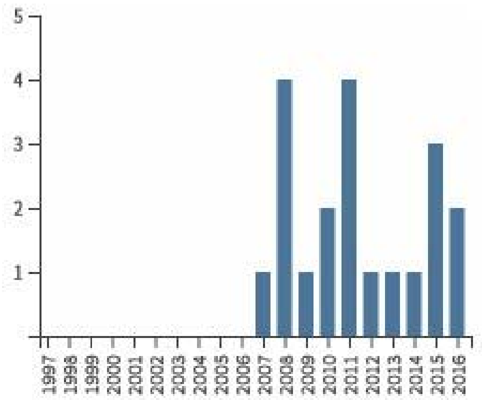
[1 | 2]

Print

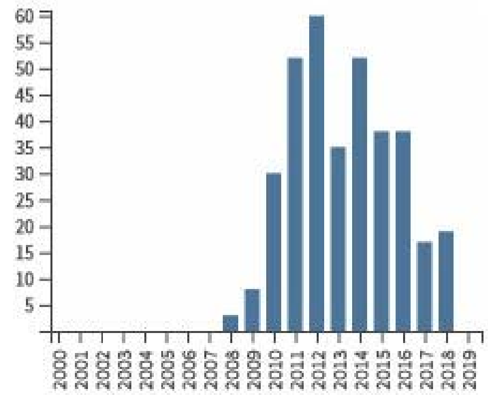
From Marked List:

Timespan=All years. Indexes=SCI-EXPANDED, ESCI, A&HCI, SSCI, CPCI-SSH, CPCI-S.

Total Publications by Year



Sum of Times Cited by Year



Results found: 20
Sum of the Times Cited: 352
Average Citations per Item: 17.60
h-index: 8

		2015	2016	2017	2018	2019	Total	Average Citations per Year
		38	38	17	19	0	352	32.00

		2015	2016	2017	2018	2019	Total	Average Citations per Year
		38	38	17	19	0	352	32.00
11.	**Title:** Modulation stability analysis of exact multidimensional solutions to the generalized nonlinear Schrodinger equation and the Gross-Pitaevskii equation using a variational approach **By:** Petrovic, Nikola Z.; Aleksic, Najdan B.; Belic, Milivoj **Source:** OPTICS EXPRESS **Volume:** 23 **Issue:** 8 **Pages:** 10616-10630 **Published:** APR 20 2015	0	3	0	1	0	4	1.00
12.	**Title:** Vortex solitons in the (2+1)-dimensional nonlinear Schrodinger equation with variable diffraction and nonlinearity coefficients **By:** Xu, Siliu; Petrovic, Nikola Z.; Belic, Milivoj R. **Source:** PHYSICA SCRIPTA **Volume:** 87 **Issue:** 4 **Article Number:** 045401 **Published:** APR 2013	1	2	0	0	0	4	0.67
13.	**Title:** Spatiotemporal soliton supported by parity-time symmetric potential with competing nonlinearities **By:** Xu, Si-Liu; Zhao, Yuan; Petrovic, Nikola Z.; et al. **Source:** EPL **Volume:** 115 **Issue:** 1 **Article Number:** 14006 **Published:** JUL 2016	0	0	1	1	0	2	0.67
14.	**Title:** General Jacobi elliptic function expansion method applied to the generalized (3+1)-dimensional nonlinear Schrodinger equation **By:** Petrovic, Nikola Z.; Bohra, Moiz **Source:** OPTICAL AND QUANTUM ELECTRONICS **Volume:** 48 **Issue:** 4 **Article Number:** 268 **Published:** APR 2016	0	0	1	1	0	2	0.67
15.	**Title:** Exact spatial soliton solutions of the two-dimensional generalized nonlinear Schrodinger equation with distributed coefficients (vol 78, art no 023821, 2008) **By:** Zhong, Wei-Ping; Xie, Rui-Hua; Belic, Milivoj; et al. **Source:** PHYSICAL REVIEW A **Volume:** 78 **Issue:** 3 **Article Number:** 039906 **Published:** SEP 2008	0	0	0	0	0	2	0.18

		2015	2016	2017	2018	2019	Total	Average Citations per Year
		38	38	17	19	0	352	32.00
16.	**Title:** Exact traveling wave solutions to coupled generalized nonlinear Schrodinger equations **By:** Petrovic, Nikola; Zahreddine, Hussein **Conference:** 3rd International School and Conference on Photonics **Location:** Belgrade, SERBIA **Date:** AUG 29-SEP 02, 2011 **Source:** PHYSICA SCRIPTA **Volume:** T149 **Article Number:** 014039 **Published:** APR 2012	0	0	0	0	0	1	0.14
17.	**Title:** SPECIAL SOLUTIONS OF THE RICCATI EQUATION WITH APPLICATIONS TO THE GROSS-PITAEVSKII NONLINEAR PDE **By:** Al Bastami, Anas; Belic, Milivoj R.; Petrovic, Nikola Z. **Source:** ELECTRONIC JOURNAL OF DIFFERENTIAL EQUATIONS **Article Number:** 66 **Published:** MAY 8 2010	0	0	0	0	0	1	0.11
18.	**Title:** Dancing light: Counterpropagating beams in photorefractive crystals **By:** Belic, M. R.; Petrovic, M. S.; Jovic, D. M.; et al. **Conference:** International School and Conference on Optics and Optical Materials **Location:** Belgrade, SERBIA **Date:** SEP 03-07, 2007 **Sponsor(s):** Univ Belgrade, Inst Phys, Fac Phys; Vinca Inst Nucl Sci & Fac Elect Engn **Source:** ACTA PHYSICA POLONICA A **Volume:** 112 **Issue:** 5 **Pages:** 729-736 **Published:** NOV 2007	0	0	0	0	0	1	0.08
19.	**Title:** Two-dimensional dark solitons in diffusive nonlocal nonlinear media **By:** Xu, Si-Liu; Petrovic, Nikola; Belic, Milivoj R. **Source:** JOURNAL OF OPTICS-INDIA **Volume:** 44 **Issue:** 2 **Pages:** 172-177 **Published:** JUN 2015	0	0	0	0	0	0	0.00
20.	**Title:** Exact spatial soliton solutions of the two-dimensional generalized nonlinear Schrodinger equation with distributed coefficients (vol 78, art no 023821, 2008) **By:** Zhong, Wei-Ping; Xie, Rui-Hua; Belic, Milivoj; et al.	0	0	0	0	0	0	0.00

		2015	2016	2017	2018	2019	Total	Average Citations per Year
		38	38	17	19	0	352	32.00
	Source: PHYSICAL REVIEW A **Volume:** 78 **Issue:** 2 **Article Number:** 029905 **Part:** B **Published:** AUG 2008							

Close

Web of Science
Page 2 (Records 11 -- 20)

[1 | 2]

Print

Никола З Петровић

9. За кандидате који се бирају у звање научни сарадник или се први пут бирају у звање у Србији потребно је приложити и докторску диплому.

Ово није први избор у звање. Кандидат је докторирао у Србији. Копија дипломе ће бити приложена

10. Копије објављених радова и других публикација након претходног избора у звање (верзије из часописа, зборника апстраката, итд.).

Приложене су копије радова публиковане после претходне одлуке научног већа/избора у звања.

11. Решење о претходном избору у звање (за кандидате који већ имајунаучно звање приликом избора у више звање или реизбора).

Решење је приложено.

Република Србија
МИНИСТАРСТВО ПРОСВЕТЕ, НАУКЕ И ТЕХНОЛОШКОГ РАЗВОЈА
Комисија за стицање научних звања
Број:660-01-00194/463
30.04.2014. године
Б е о г р а д

ИНСТИТУТ ЗА ФИЗИКУ

ПРИМЉЕНО: 04-06-2014			
Рад. јед.	Број	Арх. шифра	Прилог
0801	669/1		

На основу члана 22. става 2. члана 70. став 5. Закона о научноистраживачкој делатности ("Службени гласник Републике Србије", број 110/05 и 50/06 – исправка и 18/10), члана 2. става 1. и 2. тачке 1 – 4.(прилози) и члана 38. Правилника о поступку и начину вредновања и квантитативном исказивању научноистраживачких резултата истраживача ("Службени гласник Републике Србије", број 38/08) и захтева који је поднео

Институт за физику у Београду

Комисија за стицање научних звања на седници одржаној 30.04.2014. године, донела је

ОДЛУКУ
О СТИЦАЊУ НАУЧНОГ ЗВАЊА

Др Никола Петровић

стиче научно звање

Научни сарадник

у области природно-математичких наука - физика

О Б Р А З Л О Ж Е Њ Е

Институт за физику у Београду

утврдио је предлог број 1712/1 од 24.12.2013. године на седници научног већа Института и поднео захтев Комисији за стицање научних звања број 1719/1 од 26.12.2013. године за доношење одлуке о испуњености услова за стицање научног звања ***Научни сарадник***.

Комисија за стицање научних звања је по претходно прибављеном позитивном мишљењу Матичног научног одбора за физику на седници одржаној 30.04.2014. године разматрала захтев и утврдила да именовани испуњава услове из члана 70. став 5. Закона о научноистраживачкој делатности ("Службени гласник Републике Србије", број 110/05 и 50/06 – исправка и 18/10), члана 2. става 1. и 2. тачке 1 – 4.(прилози) и члана 38. Правилника о поступку и начину вредновања и квантитативном исказивању научноистраживачких резултата истраживача ("Службени гласник Републике Србије", број 38/08) за стицање научног звања ***Научни сарадник***, па је одлучила као у изреци ове одлуке.

Доношењем ове одлуке именовани стиче сва права која му на основу ње по закону припадају.

Одлуку доставити подносиоцу захтева, именованом и архиви Министарства просвете, науке и технолошког развоја у Београду.

ПРЕДСЕДНИК КОМИСИЈЕ
Др Станислава Стошић-Грујичић,
научни саветник
С. Стошић-Грујичић

МИНИСТАР
Др Срђан Вербић
Вербић

Controllable parabolic-cylinder optical rogue wave

Wei-Ping Zhong,[1,2,3] Lang Chen,[1,*] Milivoj Belić,[3,4] and Nikola Petrović[4]
[1]*Department of Physics, South University of Science and Technology, Shenzhen 518055, China*
[2]*Department of Electronic and Information Engineering, Shunde Polytechnic, Guangdong Province, Shunde 528300, China*
[3]*Texas A&M University at Qatar, P.O. Box 23874, Doha, Qatar*
[4]*Institute of Physics, University of Belgrade, P.O. Box 57, 11001 Belgrade, Serbia*
(Received 14 May 2014; published 6 October 2014)

We demonstrate controllable parabolic-cylinder optical rogue waves in certain inhomogeneous media. An analytical rogue wave solution of the generalized nonlinear Schrödinger equation with spatially modulated coefficients and an external potential in the form of modulated quadratic potential is obtained by the similarity transformation. Numerical simulations are performed for comparison with the analytical solutions and to confirm the stability of the rogue wave solution obtained. These optical rogue waves are built by the products of parabolic-cylinder functions and the basic rogue wave solution of the standard nonlinear Schrödinger equation. Such rogue waves may appear in different forms, as the hump and paw profiles.

DOI: 10.1103/PhysRevE.90.043201 PACS number(s): 05.45.Yv, 42.65.Tg, 42.65.Sf, 02.30.Ik

I. INTRODUCTION

The $(1+1)$-dimensional (1D) nonlinear Schrödinger equation (NLSE) with constant coefficients is an integrable model which, among many solutions, supports also the ones that reproduce well the qualitative characteristics of rogue waves [1,2]. This equation describes diverse physical systems, such as nonlinear optical fibers [3], Bose-Einstein condensates (BECs) [4], and others. The relevance of the equation for the study of rogue waves was lately established in various experiments carried out in different physical contexts [5,6]. It should be noted that these solutions—specifically, the Peregrine soliton and Akhmediev and Ma breathers—by themselves are not the proper rogue waves but can be used to model them. On the other hand, the two-dimensional (2D) NLSE with constant coefficients and external potentials may also support propagation of different nonlinear wave packets. These wave packets display many new properties [7–9], such as self-compression [7] and the generation of vortex-ring beams [9].

The elucidation of mechanisms underlying the formation and dynamics of rogue waves (also called the freak or extreme waves) is currently subject to fundamental scientific scrutiny. They occur in many fields, such as oceanography [10], nonlinear optics [11], and BECs [4]. A comprehensive recent review of rogue waves can be found in [12,13]. An explicit rogue wave solution of the standard NLSE was derived in 1983 [14]; after the author, the solution was called the Peregrine breather or an algebraic breather. Other related rogue wave solutions of the standard NLSE were found by Ma [15] in 1979; these are solutions that breathe temporally but are localized spatially, for example, along a fiber. Akhmediev found a new kind of solutions, now called Akhmediev breathers [16,17], which were qualitatively different from the Ma breathers. Akhmediev breathers oscillate spatially but are localized in time. In other words, Akhmediev breathers are the exact pulse solutions of the standard NLSE that extend transversely and may arise from the transverse modulation instability of a plane wave [17]. On the other hand, the Peregrine breather is a localized solution in both space and time [14], and can thus be seen as the limit of both the Ma and the Akhmediev breathers.

Recently, the first-order and second-order Peregrine rogue wave solutions—indeed, solutions all the way up to the fifth order—have been observed in a water wave tank [18,19]. Also, rogue waves described by the Peregrine rational solution have been generated in optics [20] and magnetoplasma [21,22]. Thus the validity of the simplest rogue wave solutions has been confirmed experimentally. A direct approach to finding multi-rogue-wave solutions of the standard NLSE, based on the modified Darboux transformation, is presented in [23]. It is worth mentioning that the rogue wave solutions were exhibited in the inhomogeneous NLSE with variable coefficients. Periodic and hyperbolic wave functions may display the dynamical behavior of roguelike wave phenomena. The profiles of the first-order and second-order rogue wave solutions of the inhomogeneous NLSE with variable coefficients can be controlled by a number of parameters [24]. A common characteristic of all these waves is that they ride on a finite background.

In this paper, we demonstrate that a class of parabolic-cylinder optical rogue waves can exist in inhomogeneous media described by the varying coefficients in NLSE. Such solutions, which are constructed by means of the similarity transformation method as products of the parabolic-cylinder function and the basic rogue wave solutions of the standard nonlinear Schrödinger equation, form relatively stable rogue wave patterns while propagating. These controllable profiles of the optical rogue waves can be realized by selecting different orders of the parabolic-cylinder function. Since in general the media exhibiting rogue waves that can be controlled—such as nonlinear optics and BECs—are inhomogeneous and can be better described by the NLSE with varying coefficients, it is expected that the solutions obtained in this paper will have a greater influence on the quest for finding changeable but feasible rogue waves in experimentally controlled environments [12,13].

The paper is organized as follows. In Sec. II, we introduce the generalized nonlinear Schrödinger equation with spatially modulated coefficients and a special external potential, and construct an explicit form of the rogue wave solution of the model through the similarity transformation. We then

*chenlang@sustc.edu.cn

elaborate on the method of deriving parabolic-cylinder rogue waves. In Sec. III, we discuss the patterns of first-order, second-order, and third-order rogue wave solutions mentioned above. Section III is also devoted to a numerical study of two solutions in order to compare our numerical simulations with the analytical predictions, and also to confirm the stability of localized solutions. Finally, Sec. IV presents our conclusions.

II. THE MODEL AND THE SIMILARITY TRANSFORMATION

We consider nonlinear optical systems ruled by the generalized NLSE with spatially modulated coefficients and a special external potential which can be written in the following dimensionless form:

$$i\frac{\partial u}{\partial z}+d(x)\frac{\partial^2 u}{\partial x^2}+2N(z,x)|u|^2u+U(z,x)u=0, \quad (1)$$

where $u(z,x)$ represents the complex optical wave envelope, the beam propagates along the z axis, and x is the transverse coordinate. Here, $d(x)$ is the diffraction coefficient, $N(z,x)$ is the nonlinearity coefficient, and $U(x)$ is the external potential. We choose the potential as $U(x)=d(x)(ax^2+b)$, where a and b are the two real constants to be determined below. Hence, the external potential is just a simple quadratic potential, modulated by the diffraction coefficient. In this manner, we try to stay close to the physically relevant situations, so when $d(x)$ is constant, the equation reduces to the Gross-Pitaevskii equation of BECs with harmonic potential. All of the parameters of the equation can be controlled and manipulated by the choice of medium. The nonlinearity coefficient $N(z,x)$ may possess different expressions in Eq. (1); thus it may include many special cases of nonlinear optics and BECs. When $N(z,x)=N(z)$ and $d(x)=1$ in Eq. (1), we have obtained bright and dark soliton solutions by means of the F-expansion method in [25]; the first-order and the second-order rogue waves were also obtained, and the dynamical behavior of those waves was discussed in our previous work [24]. However, the important controllable behavior of rogue waves in [24] has not been investigated, not even for $N(z,x)=N(x)$, and also the third-order rogue waves have not been analyzed at all. We focus in this paper on spatially localized solutions for which $N(z,x)=N(x)$.

In order to find rogue wave solutions of Eq. (1), we presume a relation between $u(z,x)$ and the solution $V(z,Y)$ of the NLSE with constant coefficients, Eq. (3), by utilizing the similarity transformation,

$$u(z,x)=A(X)V(z,Y), \quad (2)$$

$$i\frac{\partial V}{\partial z}+\frac{\partial^2 V}{\partial Y^2}+2|V|^2V=0, \quad (3)$$

where $A(X)$ is the amplitude, assumed to be a real function. Here we introduce two similarity variables $X=X(x)$ and $Y=Y(x)$ to be determined [26,27]. In general, the rogue wave solutions of Eq. (3) have the following basic structure [1]:

$$V_n(z,Y)=\left[(-1)^n+\frac{G_n(z,Y)+iH_n(z,Y)}{F_n(z,Y)}\right]e^{2iz}, \quad (4)$$

where $n\,(=1,2,3,\ldots)$ is a positive integer. The polynomial $F_n(z,Y)$ should have no zeros in the region of interest, to ensure that the solution $V_n(z,Y)$ is finite everywhere. The first-order ($n=1$), going to the third-order ($n=3$) rogue wave solutions of Eq. (3), can be found by the direct integration method (see Appendix A).

Substituting Eq. (2) into Eq. (1) leads to Eq. (3), provided that a system of differential equations for X, Y, and $A(X)$ is satisfied:

$$\frac{2}{A}\frac{\partial A}{\partial X}\frac{\partial X}{\partial x}\frac{\partial Y}{\partial x}+\frac{\partial^2 Y}{\partial x^2}=0, \quad (5a)$$

$$\frac{d}{A}\left[\frac{\partial^2 A}{\partial X^2}\left(\frac{\partial X}{\partial x}\right)^2+\frac{\partial A}{\partial X}\frac{\partial^2 X}{\partial x^2}\right]+U=0, \quad (5b)$$

and the following two relations hold:

$$N(x)=\frac{1}{A^2(x)}, \quad \text{and} \quad d(x)=\int\frac{1}{A^2(x)}dx. \quad (5c)$$

These relations establish a connection of the nonlinearity and diffraction coefficients with the presumed amplitude of the solution in Eq. (2) and thus can be considered as constraint conditions on Eq. (1) for solution by the present similarity transformation method. By assuming the simplest possibility $X=x$, one finds the following relation between Y and A:

$$Y(x)=\int A^{-2}dx, \quad (6a)$$

and a simple differential equation for A:

$$\frac{d^2A}{dx^2}+(ax^2+b)A=0. \quad (6b)$$

Equation (6b) is just the Schrödinger equation for the quadratic potential, with the well-known solutions. It is a linear second-order ordinary differential whose general solution can be expressed in terms of many different special functions. We opt for the ones with clear physical relevance and a convenient parameter that allows an easy classification of solutions. More specifically, if we choose $a=-1/4$ and $b=m+1/2$, where m is a non-negative integer, differential equation (6b) is transformed into the canonical form of the parabolic-cylinder differential equation [28] (PCDE), namely,

$$\frac{d^2A}{dx^2}+\left(m+\frac{1}{2}-\frac{1}{4}x^2\right)A=0. \quad (6c)$$

The general solution to PCDE (6c), found by considering the standard Weber differential equation, is

$$A=k\left[c_1D_m(x)+c_2D_{-m-1}(ix)\right], \quad (6d)$$

where c_1 and c_2 ($c_1c_2>0$) are the two integration constants that should be chosen so as to avoid introducing singularities in $Y(x)$. Here, $D_m(x)$ is the parabolic-cylinder function and $k=\sqrt{1/\sqrt{2\pi}m!}$ is the normalization constant. It should be noted that $A(x)$ is real, although the argument of the second parabolic-cylinder function is complex (see Appendix B). Obviously, when $|x|\to\infty$ for any non-negative integer m, $D_m(x)$ tends to a constant.

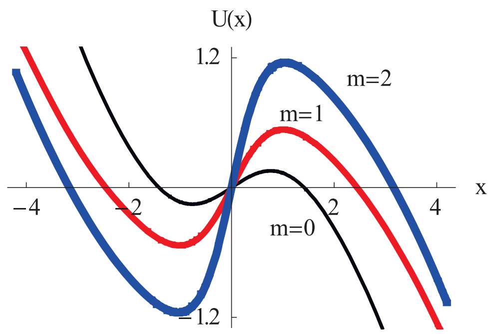


FIG. 1. (Color online) External potential for parameters $c_1 = c_2 = 1$ and different m.

Collecting all these partial solutions together, we obtain an analytical solution of Eq. (1):

$$u(z,x) = \sqrt{\frac{1}{\sqrt{2\pi m!}}}[c_1 D_m(x) + c_2 D_{-m-1}(ix)]V_n(z,Y), \quad (7)$$

which we will refer to as the parabolic-cylinder rogue waves. Since $|u(z,x)|$ vanishes at $|x| \to \infty$, Eq. (7) represents localized wave packets. Here, $Y(x) = k^{-2}\int [c_1 D_m(x) + c_2 D_{-m-1}(ix)]^{-2} dx$ and $V_n(z,Y)$ is defined by Eq. (4). The novel optical rogue waves from Eq. (7) can be conveniently classified by the two integer parameters, n and m. Based on the values of n and m, we can obtain new families of controllable parabolic-cylinder optical rogue waves.

III. CONTROLLABLE ROGUE WAVES

In this section, we consider the cases when m is a non-negative integer and discuss the profiles of the first-order ($n = 1$), the second-order ($n = 2$), and the third-order ($n = 3$) optical rogue waves. We then check the stability of exact solutions to Eq. (1) as given by Eq. (7) with the quadratic potential coefficient $a = -1/4$, by numerical integration of Eq. (1) with appropriate initial conditions.

A. External potential

When $a = -1/4$ and $b = m + 1/2$, the external potential for different values of parameter m becomes $U_m(x) = \sqrt{2\pi m!}(-\frac{1}{4}x^2 + m + \frac{1}{2})\int \frac{dx}{(c_1 D_m(x) + c_2 D_{-m-1}(ix))^2}$ and is shown Fig. 1. Even though the formula for these optical potentials is complicated, the potential barriers of such form are readily realizable experimentally and theoretically [29,30]. Although the potential includes an explicit quadratic dependence, upon modulation it resembles more a cubic polynomial function. An external potential of this form can serve as an anharmonic trapping potential in BECs that includes a tunneling mechanism; one should keep in mind that interest in the field of BECs is mostly confined to harmonic and liner potentials [30,31]. Additionally, anharmonic potentials may arise in the dynamics of waveguides with specially crafted transverse profiles of the refractive index [32]. A discussion of the influence of polynomial external potentials on NLSE is provided in [33]. Our interest here is confined to providing exact rogue wave solutions to the generalized NLSE that ride on complicated-looking external potentials which display benign-looking profiles. Note that the parameter m is connected with the potential, while the parameter n is connected with the order of the rogue wave solution.

In general, we have a large degree of freedom in choosing n and m. However, when n is greater than 3, $V_n(z,x)$ in Eq. (4) becomes quite complex. Thus, in this paper we only study low-order rogue wave packets. We present the optical amplitude $[I_n(z,x) = \sqrt{|u(z,x)|^2}]$ distributions and their contour plots for specific values of the two parameters n and m. Clearly, the optical intensity can be manipulated by the choice of parabolic-cylinder functions and the order of rogue wave solutions.

B. First-order rogue waves

The simplest case in this family of optical rogue wave solutions given by Eq. (7) is obtained when $n = 1$. In Fig. 2, analytical solutions of the first-order ($n = 1$) rogue waves

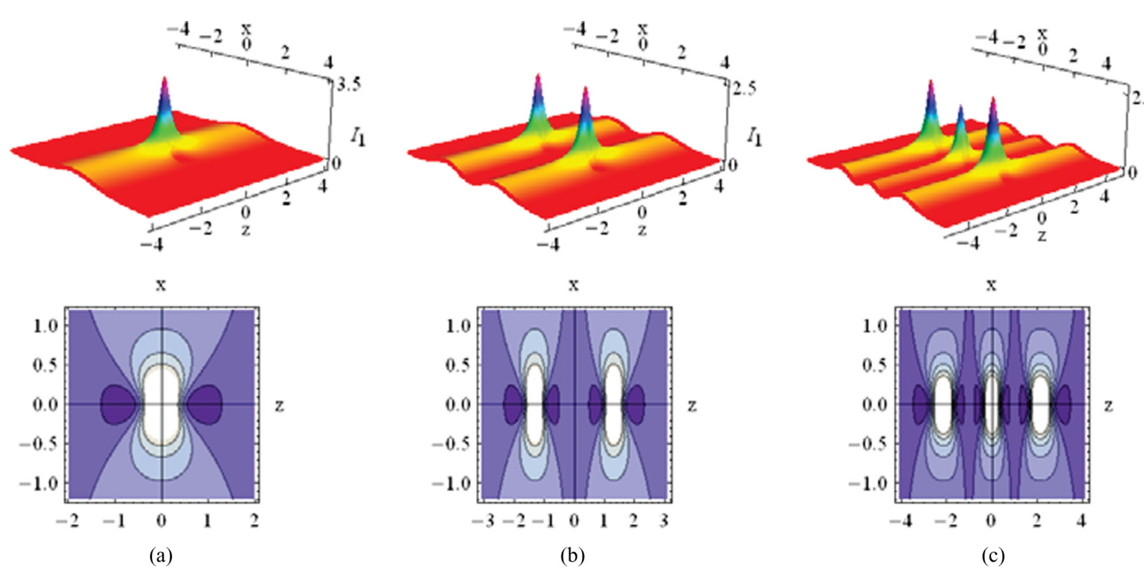


FIG. 2. (Color online) First-order rogue waves with $n = 1$, shown for $m = 0,1,2$ from left to right. Top row shows the intensity distributions, the bottom row the corresponding contour plots.

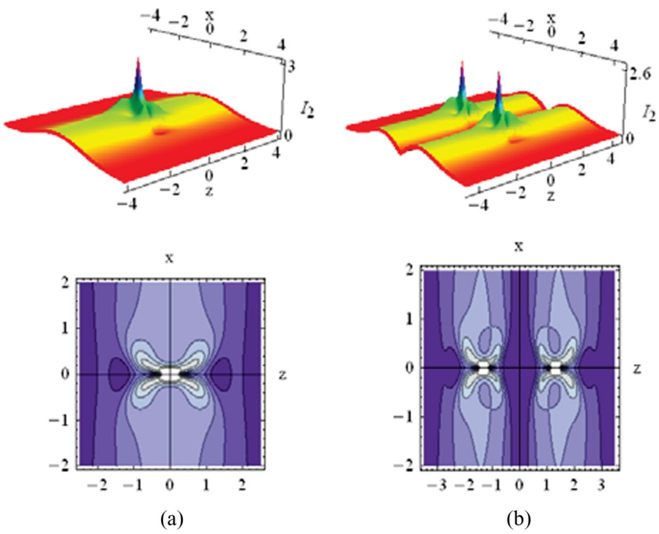


FIG. 3. (Color online) Profiles of the fundamental second-order rogue waves with the parameters $\alpha_1 = \beta_1 = 0$: (a) $m = 0$ and (b) $m = 1$.

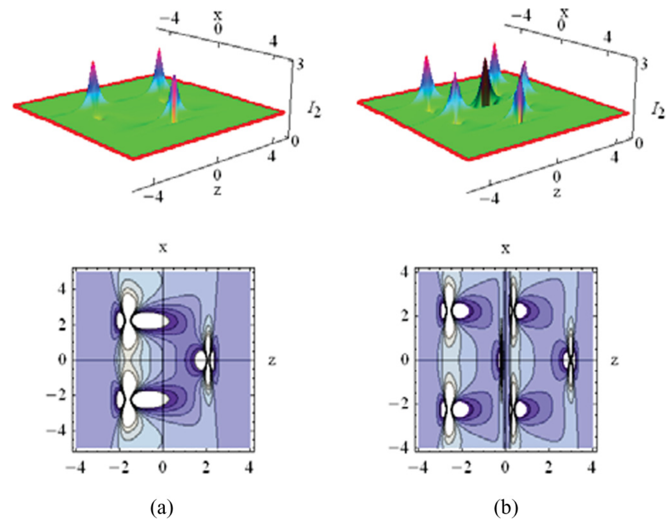


FIG. 4. (Color online) Multiple four-claw structures for the second-order rogue waves with the parameters $\alpha_1 = 0$, $\beta_1 = -10$: (a) $m = 0$ and (b) $m = 1$. The setup is as in Fig. 3.

are depicted for different m with two integral constants: $c_1 = c_2 = 1$. For simplicity, we keep the constants $c_1 = c_2 = 1$ throughout. Figure 2(a) depicts the distribution when $m = 0$; the profile displays a single peak with two dips. The peak is located at $(z,x) = (0,0)$ and there are two transverse valleys around the peak, akin to the Peregrine soliton. By increasing m to 1, the localized structure with two humps is generated, as in Fig. 2(b). Next, when $m = 2$, Fig. 2(c) shows three profiles for this localized wave packet. We can see from Fig. 2(c) that the smaller hump appears at the central position, while the maximum value is attained at the peaks of the two side humps. In general, for the parabolic-cylinder rogue wave with different m, we find $m + 1$ humps. Furthermore, we find that for even m there is a hump at the central position, which is the smallest, and $m/2$ additional humps on each side of the central position, whereas for odd m there is no hump at the central position.

C. Second-order rogue waves

For $n = 2$, there exist two types of parabolic-cylinder rogue wave families, namely, the fundamental rogue waves with the parameters $\alpha_1 = \beta_1 = 0$ and the excited rogue waves with the real parameters α_1 and β_1, of which at least one is not zero. The second-order parabolic-cylinder rogue waves exhibit pawlike patterns, with "four-claw" symmetrical structures around the central peak. Figure 3(a) shows the intensity of these wave packets for $m = 0$. For $m = 1$, these rogue waves form more complex structures. Two four-claw profiles along the transverse direction are seen in Fig. 3(b).

Another case is obtained for parameters $\alpha_1 = 0$ and $\beta_1 = -10$. Figure 4(a) depicts a typical example in which three similar peaks are located at the vertices of an equilateral triangle, for $m = 0$. For $m = 1$, the pattern of the rogue wave displays six four-claw structures, as seen in Fig. 4(b).

D. Third-order rogue waves

To describe their form, the general third-order rogue wave necessitates four real parameters, α_1, β_1, α_2, and β_2. Therefore the third-order rogue wave displays a more complex structure than the second-order wave. The choice of four parameters can be made in many ways, but we consider the three special cases. In the first case $\alpha_1 = \alpha_2 = \beta_1 = \beta_2 = 0$, which describes the fundamental third-order rogue wave. The remaining two cases are given by $\beta_1 = \beta_2 = 0$, and $\alpha_1 \neq 0$, $\alpha_2 \neq 0$ and by $\alpha_1 = \alpha_2 = 0$, and $\beta_1 \neq 0$, $\beta_2 \neq 0$. In general, we find that many different profiles of rogue waves can be obtained by different combinations of these four parameters.

First, we construct the fundamental third-order rogue wave for $\alpha_1 = \alpha_2 = \beta_1 = \beta_2 = 0$. For $m = 0$ there exist two valleys and a high peak surrounded by six small claws, as seen in Fig. 5(a). Note that the peak is located at the origin $(z,x) = (0,0)$. Figure 5(b) illustrates the rogue wave for $m = 2$. There now exist three "six-claw" patterns, and the intensity is smaller at the central peak than at the two side peaks.

Next, we consider the parameters $\beta_1 = \beta_2 = 0$, $\alpha_1 = \alpha_2 = 50$, and $m = 0$. Figure 6(a) exhibits the second-order

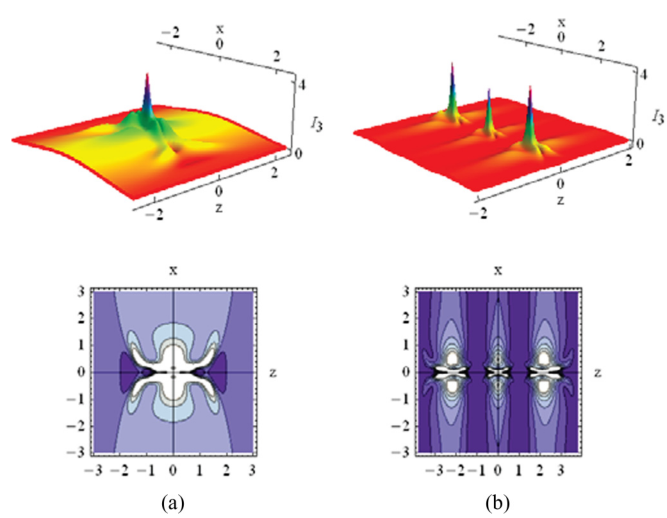
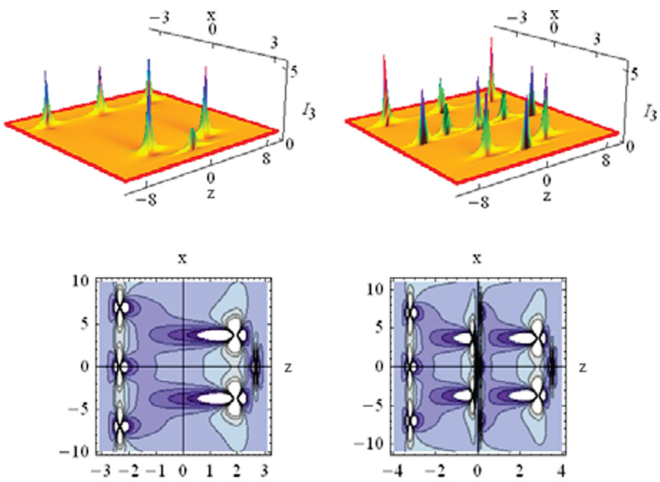


FIG. 5. (Color online) Intensity of the fundamental third-order rogue waves: (a) $m = 0$ and (b) $m = 2$. The setup is as in Fig. 3.



(a) (b)

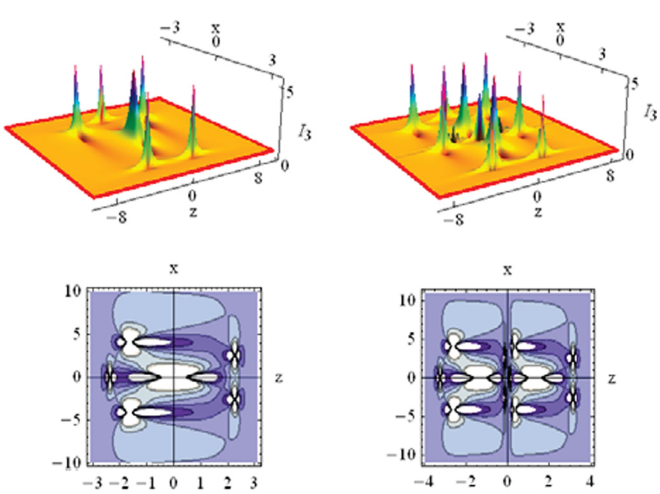
FIG. 6. (Color online) General third-order rogue waves with the parameters $\beta_1 = \beta_2 = 0$ and $\alpha_1 = \alpha_2 = 50$: (a) $m = 0$ and (b) $m = 1$. The setup is as in Fig. 5.

rogue wave with six high peaks. If a larger parameter m is selected, the structure of the rogue wave is significantly more complicated, as shown in Fig. 6(b) for $m = 1$.

Finally, we choose four parameters: $\alpha_1 = 0$, $\alpha_2 = 0$, $\beta_1 = 1$, and $\beta_2 = 5000$. To display the characteristics of this peculiar third-order solution of Eq. (7), the evolution of the rogue wave is plotted in Fig. 7. Complicated patterns are obtained.

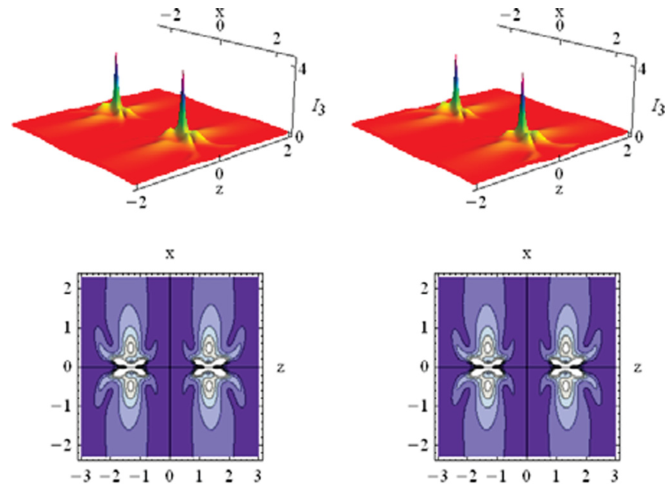
E. Numerical simulation

In the end, we briefly test the stability of the rogue wave solutions found above. We take the fundamental third-order rogue wave solution (7) as an initial wave perturbed by a random noise to perform numerical simulation of Eq. (1) with a special external potential parameter $a = -1/4$. The simulations should also confirm the validity of analytical solutions



(a) (b)

FIG. 7. (Color online) Intensities of the third-order solution of Eq. (7) with the parameters $\alpha_1 = 0$, $\alpha_2 = 0$, $\beta_1 = 1$, and $\beta_2 = 5000$: (a) $m = 0$ and (b) $m = 1$. The setup is as in Fig. 5.



(a) (b)

FIG. 8. (Color online) Comparison of the analytical solution with the numerical simulations for the fundamental third-order rogue waves at $z = 80$: (a) analytical solution of Eq. (7) and (b) numerical simulation of Eq. (1).

(7) by comparing them to their numerical counterparts. In order to do so, we add white noise to the initial pulse $u\,(0,x)$ in the amount of 5% amplitude random noise so that the perturbed pulse is written as $u_{\text{pert}} = u\,(0,x)\,[1 + 0.05\text{random}(x)]$. Figure 8 compares the analytical solution of Eq. (7) with the numerical simulation of Eq. (1) for $\alpha_1 = \beta_1 = 0$ by using the split-step beam-propagation method [34–36]. Here, we keep the same parameters as in Fig. 5, but the parabolic-cylinder order is chosen as $m = 1$. As expected, the rogue wave can propagate in a stable manner for a while under the initial perturbation of white noise and is in good agreement with the analytical solution. Although here we have demonstrated the results of the stability only for an example in Eq. (1), similar conclusions hold for other solution cases as well, provided propagation distances are kept within reasonable values. One should keep in mind that rogue waves riding on a finite background often suffer from modulational instabilities.

IV. CONCLUSIONS

In conclusion, we have presented analytical rogue wave solutions of the generalized NLSE with spatially modulated coefficients and a special external potential. By utilizing the similarity transformation, we have demonstrated that a class of parabolic-cylinder optical rogue waves can exist in specific inhomogeneous media. Our results show that these controllable patterns of the optical rogue waves can be realized by selecting different orders of the parabolic-cylinder function and of the basic rogue wave solutions of NLSE with constant coefficients. A numerical simulation is performed to compare with the analytical solution and to confirm the relative stability of localized solutions. Since the understanding of rogue waves is important in the (2+1)-dimensional models, which characterize the more realistic evolution in the transverse (x, y) plane, we plan to extend our study to multidimensional NLSE models.

ACKNOWLEDGMENTS

This work was supported by the National Natural Science Foundation of China under Grants No. 61275001 and No. 11474146. Work at the Texas A&M University at Qatar is supported by Project No. NPRP 6-021-1-005 with the Qatar National Research Fund (a member of the Qatar Foundation). Work in Serbia was done under Project No. OI 171006 of the Ministry of Education and Science of Serbia.

APPENDIX A: EXACT ROGUE WAVE SOLUTIONS

We use the direct integration method to obtain rogue wave solutions of Eq. (3). To find the first-order rogue wave solution, we assume the solution of Eq. (3) in the form

$$V_1(z,Y)=\left[-1+\frac{G_1(z,Y)+iH_1(z,Y)}{F_1(z,Y)}\right]e^{2iz}, \quad \text{(A1)}$$

with $G_1(z,Y)=g_0$, $H_1(z,Y)=h_0+h_1z+h_2Y$, $F_1(z,Y)=f_0+f_1z^2+f_2Y^2$, where g_j, h_j, and f_j $(j=0,1,2)$ are real constants and the factor e^{2iz} is a seed solution of Eq. (3). Substituting Eq. (A1) into Eq. (3) and setting all coefficients of $z^kY^j(k,j\geqslant 0)$ to zero, for $f_j\neq 0$ to avoid singularities in F_1, we obtain a system of algebraic equations, which are then solved, to obtain $g_0=1$, $h_0=0$, $h_1=4$, $h_2=0$, $f_0=1/4$, $f_1=4$, $f_2=1$, namely,

$$G_1(z,Y)=1,\quad H_1(z,Y)=4z,\quad F_1(z,Y)=\tfrac{1}{4}+4z^2+Y^2.$$

We further apply the direct method to find the second-order rogue wave solution as follows:

$$V_2(z,Y)=\left[1+\frac{G_2(z,Y)+iH_2(z,Y)}{F_2(z,Y)}\right]e^{2iz}, \quad \text{(A2)}$$

with

$$\begin{aligned}G_2(z,Y)&=g_0+g_1z+g_2zY+g_3Y+g_4z^2+g_5z^2Y^2+g_6Y^2+g_7z^4+g_8Y^4,\\ H_2(z,Y)&=h_0+h_1z+h_2Y+h_3z^2+h_4zY+h_5Y^2+h_6z^5+h_7z^3+h_8zY^2+h_9z^3Y^2+h_{10}zY^2+h_{11}zY^4,\\ F_2(z,Y)&=f_0+f_1z+f_2Y+f_3z^2+f_4Y^2+f_5z^3+f_6zY^2+f_7z^2Y+f_8Y^3+f_9z^4+f_{10}z^2Y^2+f_{11}Y^4\\ &\quad+f_{12}z^6+f_{13}z^2Y^4+f_{14}z^4Y^2+f_{15}Y^6.\end{aligned}$$

Again, substituting Eq. (A2) into Eq. (3) and using *Mathematica*, we obtain the following relations:

$$\begin{aligned}&g_0=36,\quad g_1=-115\beta_1,\quad g_2=0,\quad g_3=-576\alpha_1,\quad g_4=-3456,\quad g_5=-4608,\quad g_6=-288,\\ &g_7=-15360,\quad g_8=-192;\\ &h_0=144\beta_1,\quad h_1=720,\quad h_2=0,\quad h_3=-2304\beta_1,\quad h_4=-2304\alpha_1,\quad h_5=576\beta_1,\quad h_6=-12288,\\ &h_7=-1636,\quad h_8=1152,\quad h_9=-6144,\quad h_{10}=1152,\quad h_{11}=-768;\\ &f_0=9+144\left({\alpha_1}^2+{\beta_1}^2\right),\quad f_1=864\beta_1,\quad f_2=144\alpha_1,\quad f_3=1584,\quad f_4=108,\quad f_5=1536\beta_1,\\ &f_6=-1152\beta_1,\quad f_7=2304\alpha_1,\quad f_8=-192\alpha_1,\quad f_9=6912,\quad f_{10}=-1152,\quad f_{11}=48,\quad f_{12}=4096,\\ &f_{13}=768,\quad f_{14}=3072,\quad f_{15}=64;\end{aligned}$$

where α_1 and β_1 are two arbitrary real constants. Thus, $G_2(z,Y),H_2(z,Y)$, and $F_2(z,Y)$ can be written as follows:

$$\begin{aligned}G_2(z,Y)&=36-115\beta_1z-576\alpha_1Y-3456z^2-4608z^2Y^2-288Y^2-15\,360z^4-192Y^4,\\ H_2(z,Y)&=144\beta_1+720z-2304\beta_1z^2-2304\alpha_1zY+576\beta_1Y^2-12\,288z^5-1636z^3\\ &\quad+1152zY^2-6144z^3Y^2+1152zY^2-768zY^4,\\ F_2(z,Y)&=9+144\left({\alpha_1}^2+{\beta_1}^2\right)+864\beta_1z+144\alpha_1Y+1584z^2+108Y^2+1536\beta_1z^3-1152\beta_1zY^2\\ &\quad+2304\alpha_1z^2Y-192\alpha_1Y^3+6912z^4-1152z^2Y^2+48Y^4+4096z^6+768z^2Y^4+3072z^4Y^2+64Y^6.\end{aligned}$$

In a similar procedure, we obtain the following third-order rogue wave solutions of Eq. (3):

$$V_3(z,Y)=\left[-1+\frac{G_3(z,Y)+iH_3(z,Y)}{F_3(z,Y)}\right]e^{2iz}. \quad \text{(A3)}$$

The expressions for G_3, H_3, and F_3 are given as follows:

$$\begin{aligned}G_3(z,Y)&=16200+3600\left(\alpha_1^2+\beta_1^2\right)+144\left(\alpha_2^2+\beta_2^2\right)+G_3^{(1)}(Y)+\sum_{l=1}^{10}g_l(Y)z^l,\\ H_3(z,Y)&=-16200\beta_1-2400\beta_1\left(\alpha_1^2+\beta_1^2\right)+960\alpha_1\alpha_2\beta_1-1080\beta_2+480\beta_2\left(\beta_1^2-\alpha_1^2\right)+H_3^{(1)}(Y)+\sum_{l=1}^{11}h_l(Y)z^l,\\ F_3(z,Y)&=2025+2700\left(\alpha_1^2+\beta_1^2\right)+400\left(\alpha_1^4+\beta_1^4\right)+360\left(\alpha_1\alpha_2+\beta_1\beta_2\right)+36\left(\alpha_2^2+\beta_2^2\right)+800\alpha_1^2\beta_1^2\\ &\quad+F_3^{(0)}+F_3^{(1)}(Y)+\sum_{l=1}^{12}f_l(Y)z^l,\end{aligned}$$

where

$$G_3^{(1)}(Y) = 24576Y^{10} + 92160Y^8 + 322560Y^6 + (230400\alpha_1 - 13824\alpha_2)\,Y^5 + \left(57600\alpha_1^2 - 19200\beta_1^2 - 172800\right)Y^4$$
$$- (115200\alpha_1 + 11520\alpha_2)Y^3 - \left(64800 + 28800\alpha_1^2 + 28800\beta_1^2\right)Y^2 - \left(43200\alpha_1 + 9600\alpha_1^3 + 4320\alpha_2 + 9600\alpha_1\beta_1^2\right)Y,$$

$$g_{10}(Y) = 276824064, \quad g_9(Y) = 0, \quad g_8(Y) = 283115520Y^2 + 778567680, \quad g_7(Y) = -47185920\beta_1,$$

$$g_6(Y) = 110100480Y^4 + 165150720Y^2 + 55050240\alpha_1 Y + 215285760,$$

$$g_5(Y) = -29491200\beta_1 - 1032192\beta_2,$$

$$g_4(Y) = 19660800Y^6 - 14745600Y^4 + 19660800\alpha_1 Y^3 + 165888000Y^2 + (1843200\alpha_2 - 25804800\alpha_1)\,Y$$
$$- 47001600 + 2150400\alpha_1^2 + 921600\beta_1^2,$$

$$g_3(Y) = 4915200\beta_1 Y^4 + (1105920\beta_2 - 36864000\beta_1)\,Y^2 + 2457600\alpha_1\beta_1 Y + 10137600\beta_1 - 645120\beta_2,$$

$$g_2(Y) = 1474560Y^8 - 1474560Y^6 + 1474560\alpha_1 Y^5 + 2764800Y^4 + (9216000\alpha_1 - 184320\alpha_2)\,Y^3$$
$$+ \left(1382400\beta_1^2 - 460800\alpha_1^2 - 20736000\right)Y^2$$
$$+ (414720\alpha_2 - 2764800\alpha_1)\,Y - 345600\alpha_1^2 + 46080\alpha_1\alpha_2 - 806400\beta_1^2 + 46080\beta_1\beta_2 - 777600,$$

$$g_1(Y) = 491520\beta_1 Y^6 + 46080\beta_2 z Y^4 - 614400\alpha_1\beta_1 Y^3 + (1382400\beta_1 + 69120\beta_2)\,Y^2$$
$$+ (23040\alpha_1\beta_2 - 23040\alpha_2\beta_1 + 230400\alpha_1\beta_2)Y + 19200\alpha_1^2\beta_1 + 19200\beta_1^3 - 8640\beta_2,$$

$$H_3^{(1)}(Y) = 30720\beta_1 Y^8 + (92160\beta_1 - 7680\beta_2)\,Y^6 + 30720\alpha_1\beta_1 Y^5 + (57600\beta_1 - 5760\beta_2)\,Y^4$$
$$+ (38400\alpha_1\beta_1 - 3840\alpha_2\beta_1 + 3840\alpha_1\beta_2)\,Y^3 - \left(86400\beta_1 + 9600\beta_1\alpha_1^2 + 9600\beta_1^3 + 12960\beta_2\right)Y^2$$
$$+ 2880\alpha_2\beta_1 Y - 2880\alpha_1\beta_2 Y,$$

$$h_{11}(Y) = 100663296, \quad h_{10}(Y) = 0, \quad h_9(Y) = 125829120Y^2 + 157286400, \quad h_8(Y) = -23592960\beta_1,$$

$$h_7(Y) = 62914560Y^4 - 94371840Y^2 + 31457280\alpha_1 Y - 342097920, \quad h_6(Y) = 3932160\beta_1 - 688128\beta_2,$$

$$h_5(Y) = 15728640Y^6 + 82575360Y^4 + 15728640\alpha_1 Y^3 + 168099840Y^2 + (1474560\alpha_2 - 56033280\alpha_1)\,Y$$
$$+ 1720320\alpha_1^2 + 737280\beta_1^2 - 236666880,$$

$$h_4(Y) = 4915200\beta_1 Y^4 + (1105920\beta_2 - 51609600\beta_1)\,Y^2 + 2457600\alpha_1\beta_1 Y + 35942400\beta_1 - 460800\beta_2,$$

$$h_3(Y) = 1966080Y^8 - 13762560Y^6 + 1966080\alpha_1 Y^5 - 11059200Y^4 + (12288000\alpha_1 - 245760\alpha_2)\,Y^3$$
$$+ \left(1843200\beta_1^2 - 38707200 - 614400\alpha_1^2\right)Y^2 + (184320\alpha_2 - 7372800\alpha_1)\,Y - 1382400\alpha_1^2 + 61440\alpha_1\alpha_2$$
$$- 1996800\beta_1^2 + 61440\beta_1\beta_2 - 3801600,$$

$$h_2(Y) = 983040\beta_1 Y^6 + 92160\beta_2 Y^4 - 1228800\alpha_1\beta_1 Y^3 + (2764800\beta_1 - 138240\beta_2)\,Y^2 + (460800\alpha_1\beta_1 - 46080\alpha_2\beta_1$$
$$+ 46080\alpha_1\beta_2)Y + 38400\beta_1\alpha_1^2 + 38400\beta_1^3 + 190080\beta_2,$$

$$h_1(Y) = 98304Y^{10} - 368640Y^8 - 921600Y^6 + 184320\alpha_1 Y^5 - 55296\alpha_2 Y^5 + \left(230400\alpha_1^2 - 76800\beta_1^2 - 2073600\right)Y^4$$
$$+ (46080\alpha_2 - 1382400\alpha_1)\,Y^3 + \left(115200\alpha_1^2 + 115200\beta_1^2 + 1814400\right)Y^2 - \left(172800\alpha_1 + 38400\alpha_1^3 + 86400\alpha_2\right.$$
$$\left. + 38400\alpha_1\beta_1^2\right)Y + 72000\alpha_1^2 - 23040\alpha_1\alpha_2 + 576\alpha_2^2 + 72000\beta_1^2 - 23040\beta_1\beta_2 + 576\beta_2^2 + 453600,$$

$$F_3^{(1)}(Y) = 4096Y^{12} + 6144Y^{10} - 10240\alpha_1 Y^9 + 34560Y^8 + (1536\alpha_2 - 15360\alpha_1)\,Y^7 + \left(8960\beta_1^2 + 3840\alpha_1^2 + 149760\right)Y^6$$
$$+ (69120\alpha_1 - 3456\alpha_2)\,Y^5 + \left(33600\alpha_1^2 - 1920\alpha_1\alpha_2 + 14400\beta_1^2 - 1920\beta_1\beta_2 + 54000\right)Y^4 + (3200\alpha_1\beta_1^2 - 14400\alpha_1$$
$$+ 3200\alpha_1^3 - 4320\alpha_2)Y^3 + \left(144\beta_2^2 + 3600\alpha_1^2 - 28800\alpha_1\alpha_2 + 144\alpha_2^2 + 3600\beta_1^2 - 2880\beta_1\beta_2 + 48600\right)Y^2$$
$$+ \left(16200\alpha_1 + 2400\alpha_1^3 + 1080\alpha_2 - 480\alpha_1^2\alpha_2 + 2400\alpha_1\beta_1^2 + 480\alpha_2\beta_1^2 - 960\alpha_1\beta_1\beta_2\right)Y,$$

$$F_3^{(0)} = -55115529584640\beta_1 + 35055206400\beta_1\alpha_1^2 - 491520000\alpha_1^4\beta_1 - 4600627200\alpha_1\alpha_2\beta_1 + 196608000\alpha_1^3\alpha_2\beta_1$$
$$+ 1730740224000\beta_1^3 - 6881280000\alpha_1^2\beta_1^3 + 196608000\alpha_1\alpha_2\beta_1^3 - 6389760000\beta_1 + 317973921792\beta_2$$
$$- 66355200\alpha_1^2\beta_2 + 265420800\alpha_1\alpha_2\beta_2 - 54632448000\beta_1^2\beta_2$$
$$+ 196608000\alpha_1^2\beta_1^2\beta_2 + 196608000\beta_1^4\beta_2 + 265420800\beta_1\beta_2^2,$$

$$f_{12}(Y) = 16777216, \quad f_{11}(Y) = 0, \quad f_{10}(Y) = 25165824Y^2 + 132120576, \quad f_9(Y) = -5242880\beta_1,$$

$$f_8(Y) = 15728640Y^4 + 70778880Y^2 + 7864320\alpha_1 Y + 244776960, \quad f_7(Y) = 0,$$
$$f_6(Y) = 5242880Y^6 + 3932160Y^4 + 5242880\alpha_1 Y^3 + 221184000Y^2 + (491520\alpha_2 - 2949120\alpha_1)Y + 573440\alpha_1^2 + 245760\beta_1^2 + 62668800,$$
$$f_5(Y) = 1966080\beta_1 Y^4 + (442368\beta_2 - 26542080\beta_1)Y^2 + 983040\alpha_1\beta_1 Y + 7004160\beta_1 - 626688\beta_2,$$
$$f_4(Y) = 983040Y^8 - 2949120Y^6 + 983040\alpha_1 Y^5 - 5529600Y^4 + (15974400\alpha_1 - 122880\alpha_2)Y^3 + \left(921600\beta_1^2 - 307200\alpha_1^2 + 80179200\right)Y^2 + (829440\alpha_2 - 11059200\alpha_1)Y + 3840,$$
$$f_3(Y) = 655360\beta_1 Y^6 + (61440\beta_2 + 2457600\beta_1)Y^4 - 819200\alpha_1\beta_1 Y^3 + (276480\beta_2 - 16588800\beta_1)Y^2 + (1536000\alpha_1\beta_1 - 30720\alpha_2\beta_1 + 30720\alpha_1\beta_2)Y,$$
$$f_2(Y) = 98304Y^{10} - 368640Y^8 + 552960Y^6 + (184320\alpha_1 - 55296\alpha_2)Y^5 + \left(230400\alpha_1^2 - 76800\beta_1^2 + 3456000\right)Y^4 + (2304000\alpha_1 + 46080\alpha_2)Y^3 + \left(1036800\beta_1^2 + 115200\alpha_1^2 - 2332800\right)Y^2 + \left(190080\alpha_2 - 172800\alpha_1 - 38400\alpha_1^3 - 38400\alpha_1\beta_1^2\right)Y + 302400\alpha_1^2 - 23040\alpha_1\alpha_2 + 576\beta_2^2 + 576\alpha_2^2 + 532800\beta_1^2 - 23040\beta_1\beta_2 + 1490400,$$
$$f_1(Y) = 61440\beta_1 Y^8 + (61440\beta_1 - 15360\beta_2)Y^6 + (34560\beta_2 - 284160\alpha_1\beta_1)Y^4 + (7680\alpha_1\beta_2 - 7680\alpha_2\beta_1 - 230400\alpha_1\beta_1)Y^3 + \left(172800\beta_1 - 19200\alpha_1^2\beta_1 - 19200\beta_1^3 + 43200\beta_2\right)Y^2 + (17280\alpha_1\beta_2 - 17280\alpha_2\beta_1)Y - 118800\beta_1 - 24000\beta_1\alpha_1^2 + 1920\alpha_1\alpha_2\beta_1 - 24000\beta_1^3 - 10800\beta_2 - 960\alpha_1^2\beta_2 + 960\beta_1^2\beta_2.$$

It should be emphasized that the first-order rogue wave solution has no free parameters, the second-order rogue wave solution has a pair of free parameters (two real numbers: α_1 and β_1), while the third-order rogue wave solution has two pairs of free parameters (four real numbers: α_1, β_1, α_2, and β_2).

APPENDIX B: THE RELATION BETWEEN PARABOLIC-CYLINDER AND CONFLUENT HYPERGEOMETRIC FUNCTION

The parabolic-cylinder differential equation (PCDE) has two standard forms:

$$\frac{d^2y}{dx^2} - \left(\frac{1}{4}x^2 + \Omega\right)y = 0, \tag{B1}$$

$$\frac{d^2y}{dx^2} + \left(\frac{1}{4}x^2 - \Omega\right)y = 0. \tag{B2}$$

For a general Ω, the even and odd solutions to (B1) and (B2) are

$$y_1(x) = e^{-\frac{1}{4}x^2}{}_1F_1\left(\frac{1}{2}\Omega + \frac{1}{4}; \frac{1}{2}; \frac{1}{2}x^2\right), \tag{B3}$$

$$y_2(x) = xe^{-\frac{1}{4}x^2}{}_1F_1\left(\frac{1}{2}\Omega + \frac{3}{4}; \frac{3}{2}; \frac{1}{2}x^2\right), \tag{B4}$$

where ${}_1F_1$ is a confluent hypergeometric function. When $\Omega = m + \frac{1}{2}$, from (B3) we easily find

$$D_m(x) = e^{-\frac{1}{4}x^2}{}_1F_1\left(\frac{m+1}{2}; \frac{1}{2}; \frac{1}{2}x^2\right), \tag{B5}$$

$$D_{-m-1}(ix) = e^{\frac{1}{4}x^2}{}_1F_1\left(-\frac{m}{2}; \frac{1}{2}; -\frac{1}{2}x^2\right), \tag{B6}$$

both functions being real. Thus Eq. (6d) can be simplified as

$$A(x) = \sqrt{\frac{1}{\sqrt{2\pi}m!}}\left[c_1 e^{-\frac{1}{4}x^2}{}_1F_1\left(\frac{m+1}{2}; \frac{1}{2}; \frac{1}{2}x^2\right) + c_2 e^{\frac{1}{4}x^2}{}_1F_1\left(-\frac{m}{2}; \frac{1}{2}; -\frac{1}{2}x^2\right)\right].$$

[1] N. Akhmediev, A. Ankiewicz, and M. Taki, Phys. Lett. A **373**, 675 (2009).
[2] N. Akhmediev, A. Ankiewicz, and J. M. Soto-Crespo, Phys. Rev. E **80**, 026601 (2009).
[3] K. Hammani, B. Kibler, C. Finot, P. Morin, J. Fatome, J. M. Dudley, and G. Milot, Opt. Lett. **36**, 112 (2011).
[4] L. Wen, L. Li, Z. D. Li, S. W. Song, X. F. Zhang, and W. M. Liu, Eur. Phys. J. D **64**, 473 (2011).
[5] A. Chabchoub, N. P. Hoffmann, and N. Akhmediev, Phys. Rev. Lett. **106**, 204502 (2011).
[6] H. Bailung, S. K. Sharma, and Y. Nakamura, Phys. Rev. Lett. **107**, 255005 (2011).
[7] L. Berge, Phys. Plasmas **4**, 1227 (1997).
[8] L. Berge, V. K. Mezentsev, J. J. Rasmussen, P. L. Christiansen, and Yu. B. Gaididei, Opt. Lett. **25**, 1037 (2000).
[9] T. J. Alexander and L. Berge, Phys. Rev. E **65**, 026611 (2002).
[10] C. Kharif, E. Pelinovsky, and A. Slunyacv, *Rogue Waves in the Ocean, Theories and Modeling* (Springer-Verlag, New York, 2009).
[11] D. R. Solli, C. Ropers, P. Koonath, and B. Jalali, Nature (London) **450**, 1054 (2007).
[12] M. Onorato, S. Residori, U. Bortolozzo, A. Montinad, and F. T. Arecchi, Phys. Rep. **528**, 47 (2013).

[13] N. Akhmediev, J. M. Dudley, D. R. Solli, and S. K. Turitsyn, J. Opt. **15**, 060201 (2013).
[14] D. H. Peregrine, J. Austral. Math. Soc. Ser. B **25**, 16 (1983).
[15] Y. C. Ma, Stud. Appl. Math. **60**, 43 (1979).
[16] N. Akhmediev, V. M. Eleonskii, and N. Kulagin, Zh. Eksp. Teor. Fiz. **89**, 1542 (1985).
[17] N. Akhmediev and V. I. Korneev, Theor. Math. Phys. **69**, 1089 (1986).
[18] A. Chabchoub, N. P. Hoffmann, M. Onorato, and N. Akhmediev, Phys. Rev. X **2**, 011015 (2012).
[19] A. Chabchoub, N. Hoffmann, M. Onorato, A. Slunyaev, A. Sergeeva, E. Pelinovsky, and N. Akhmediev, Phys. Rev. E **86**, 056601 (2012).
[20] M. Erkintalo, K. Hammani, B. Kibler, C. Finot, N. Akhmediev, J. M. Dudley, and G. Genty, Phys. Rev. Lett. **107**, 253901 (2011).
[21] P. Shukla and W. Moslem, Phys. Lett. A **376**, 1125 (2012).
[22] R. Sabry, W. M. Moslem, and P. K. Shukla, Phys. Rev. E **86**, 036408 (2012).
[23] L. Ling and L.-C. Zhao, Phys. Rev. E **88**, 043201 (2013).
[24] W. P. Zhong, M. R. Belić, and T. Huang, Phys. Rev. E **87**, 065201 (2013); W. P. Zhong, J. Nonlinear Opt. Phys. Mater. **21**, 1250026 (2012).
[25] W. P. Zhong and M. R. Belić, Phys. Rev. E **81**, 056604 (2010).
[26] M. Belic and W. P. Zhong, Eur. Phys. J. D **53**, 97 (2009).
[27] W. P. Zhong and M. Belic, Phys. Lett. A **373**, 296 (2009).
[28] D. Zwillinger, *Handbook of Differential Equations*, 3rd ed. (Academic Press, Boston, 1997).
[29] J. R. Tayor, *Optical Solitons—Theory and Experiment* (Cambridge University Press, Cambridge, UK, 1992).
[30] P. G. Kevrekidis, D. J. Frantzeskakis, and R. Carretero-Gonzales, *Emergent Nonlinear Phenomena in BECs: Theory and Experiment* (Springer, Berlin, 2007).
[31] R. Carretero-Gonzales, D. J. Frantzeskakis, and P. G. Kevrekidis, Nonlinearity **21**, R139 (2008).
[32] Y. S. Kivshar and G. Agrawal, *Optical Solitons: From Fibers to Photonic Crystals* (Academic Press, London, 2003).
[33] S. A. Morgan, R. J. Ballagh, and K. Burnett, Phys. Rev. A **55**, 4338 (1997).
[34] M. Belić, N. Petrović, W. P. Zhong, R.-H. Xie, and G. Chen, Phys. Rev. Lett. **101**, 123904 (2008).
[35] E. A. Sziklas and A. E. Siegman, Appl. Opt. **14**, 1874 (1975).
[36] W. P. Zhong, M. R. Belić, G. Assanto, B. A. Malomed, and T. Huang, Phys. Rev. A **83**, 043833 (2011).

Nonlinear Dyn (2015) 80:583–589
DOI 10.1007/s11071-014-1891-2

ORIGINAL PAPER

Exact solutions of the (2+1)-dimensional quintic nonlinear Schrödinger equation with variable coefficients

Si-Liu Xu · Nikola Petrović · Milivoj R. Belić

Received: 10 December 2014 / Accepted: 30 December 2014 / Published online: 7 January 2015

Abstract Using the self-similarity transformation, we find analytical spatial bright and dark self-similar solitons, i.e., the similaritons, of the generalized (2+1)-dimensional quintic nonlinear Schrödinger equation with varying diffraction, nonlinearity, and gain. Characteristic examples with physically relevant behavior of these similaritons are studied, and the stability of these solutions is verified with numerical integration.

Keywords Nonlinear optics · Spatial solitons · Self-similarity transformation

1 Introduction

The construction of exact soliton solutions for a large variety of nonlinear partial differential equations describing a diverse array of systems such as shallow water waves, DNA excitations, matter waves in Bose–Einstein condensates, and ultrashort pulses (or laser beams) in nonlinear optics is one of the most important tasks in nonlinear science.

One important class of exact solutions is the so-called "self-similar solution", i.e., a similariton [1]. Dynamics induced by self-similar solutions has attracted considerable attention recently in various areas of nonlinear optics, such as the evolution of nonlinear waveguides [2], propagation of pulses in fibers [3], and compression of parabolic solitons [4]. Self-similar optical waves appear when the variable nonlinearity, diffraction, and gain conspire to evolve as a beam with a self-similar shape. The self-similar beams may be useful for various applications in optical telecommunications and waveguides, since they maintain their overall shape but allow the amplitude and width to change with the changing system parameters such as dispersion, nonlinearity, and gain. Several attempts have been made to find exact similaritons [2–7], usually in (1+1)-dimensional [(1+1)D] systems, although solutions have also been found in a higher number of dimensions [8]. The interaction of multiple similaritons was studied in Refs. [9,10].

In recent years, there has been an increased interest in generalizations of the standard NLSE equation with space-modulated diffraction and nonlinearities. It aroused in different physical contexts, such as nonlinear optics and dynamics of Bose–Einstein condensates (BECs). In BEC applications, matter waves are the natural outcome of the mean-field description [11] and

S.-L. Xu (✉)
The School of Electronic and Information Engineering, HuBei University of Science and Technology, Xianning 437100, China
e-mail: xusiliu1968@163.com

N. Petrović
Institute of Physics, University of Belgrade, P.O. Box 68, 11001 Belgrade, Serbia

M. R. Belić
Science Program, Texas A&M University at Qatar, P.O. Box 23874, Doha, Qatar

have attracted a great deal of attention [12]. The possibility of using Feschbach resonances to control the nonlinearities (see e.g., [13,14]) has lead to the proposal of many different nonlinear phenomena induced by the manipulation of the scattering length either in time [15,16] or in space [17].

Many of these works deal with the cubic nonlinear Schrödinger equation (CNLSE), as this equation models a wealth of physical phenomena. However, when the incident fields become stronger, non-Kerr nonlinearities should be considered [18,19]. The cubic–quintic nonlinearity occurs due to an intrinsic nonlinear resonance in the material, which also gives rise to the strong two-photon absorption [20,21]. The quintic nonlinear Schrödinger (QNLS) equation, in contrast to the CNLSE, has received less attention, in spite of the considerable relevance of this equation, both from the mathematical point of view and that of nonlinear physics.

In recent years, many influential papers have been devoted to constructing exact analytical solutions of the QNLSE, such as the pioneering work of Serkin et al. [22]. Hao et al. [23] constructed exact solitary wave solutions by assuming an ansatz solution, Senthilnathan et al. [19,24] investigated the evolution of nonlinear optical pulses in cubic–quintic nonlinear media and Dai et al. [25] constructed chirped and chirp-free self-similar cnoidal and solitary wave solutions of the QNLSE.

In this paper, we go beyond the previous work on the QNLS equation and study explicit solutions of the two-dimensional generalized QNLS equation with space-modulated diffraction and nonlinearity coefficients. We use the multivariate self-similarity transformation to transform this model into the standard QNLS equation and find exact solutions. Although our method is suitable for arbitrary modulations of the coefficients, in this work, we investigate in more detail the generalized QNLS equation, as well as the dynamics of the corresponding nonlinear solitary waves.

The present paper is organized as follows. In Sect. 2, we extend the similarity method given in Refs. [26,27] to Eq. (1), in order to reduce the generalized QNLS equation with variable coefficients to the standard QNLS equation and present exact solitary wave solutions. In Sect. 3, the cases of chirp and chirp-free solitary waves are investigated when the solution parameters are chosen appropriately. Numerical simulations and comparison with the analytical results are also performed. In Sect. 4, the conclusions to the paper are outlined briefly.

2 Similarity transformation and exact solitons

An optical beam propagation in media with power-law nonlinearity can be investigated with the help of the generalized QNLS equation with variable coefficients, written in the following form:

$$i\frac{\partial u}{\partial z}+\frac{\beta(z)}{2}\nabla_{\perp}+\chi(z)|u|^4u=i\gamma(z)u, \tag{1}$$

where $u(z,x,y)$ is the complex envelope of the electric field, z is the coordinate in the propagation direction, and $\nabla_{\perp}=\partial_x^2+\partial_y^2$ is the transverse Laplacian. Here, $\beta(z)$ is the diffraction coefficient, $\chi(z)$ is the nonlinearity coefficient, and $\gamma(z)$ the linear gain ($\gamma>0$) or the loss ($\gamma<0$) coefficient.

To obtain exact analytical solutions of Eq. (1), we introduce a self-similar transformation of the solution [28,29]:

$$u(z,x,y)=A(z)V(Z,X)\mathrm{e}^{iB(z,x,y)}, \tag{2}$$

where $A(z)$ is the amplitude, $Z=Z(z)$ is the transformed propagation variable, $V=V(Z,X)$ is the complex function, $X=X(z,x,y)$ is the multivariate self-similar variable and $B(z,x,y)$ is the phase of the wave. All functions except V are assumed to be real. Writing the amplitude of u as a product of two auxiliary functions allows for more freedom in the treatment of Eq. (1). The goal is to transform Eq. (1) into the QNLS equation with constant coefficients but with transformed variables. Substituting Eq. (2) into Eq. (1) transforms Eq. (1) into the following standard QNLS:

$$i\frac{\partial V}{\partial Z}+\frac{1}{2}\frac{\partial^2 V}{\partial X^2}+\sigma|V|^4V=0, \tag{3}$$

provided that the following two equations are satisfied:

$$\frac{\beta}{2}\left(\frac{\partial^2 X}{\partial x^2}+\frac{\partial^2 X}{\partial y^2}\right)-\sigma\frac{\partial Z}{\partial z}=0, \tag{4a}$$

$$\chi A^4=\frac{\partial Z}{\partial z}, \tag{4b}$$

where $\sigma=\pm1$. Now we can use the fundamental soliton solutions of Eq. (3), namely the bright fundamental (single-peak) soliton in the focusing nonlinear medium for $\sigma=+1$,

$$V(X,Z)=\frac{\eta}{\cosh^{1/2}(\mu X)}\mathrm{e}^{i\frac{Z}{2}}, \tag{5a}$$

where $E = \frac{\mu^2}{4}$, $\sigma = \frac{3\mu^2}{4\eta^4}$ and E is positive constant, and the dark fundamental soliton in the defocusing nonlinear medium for $\sigma = -1$,

$$V(X,Z) = \frac{\sqrt{2}V_\infty \tanh(V_\infty^2 X)}{\sqrt{3-\tanh^2(V_\infty^2 X)}}\mathrm{e}^{-iZ}, \tag{5b}$$

where $E = -\sigma V_\infty$; V_∞ is the value of V when $X \to \infty$. We use these solutions to construct novel soliton solutions of Eq. (1). Assuming Eq. (3), substituting Eq. (2) into Eq. (1), separating its real and imaginary parts, and requiring the coefficients next to V and V_X to be zero, we obtain a system of partial differential equations for X, Z, A, and B:

$$\frac{\partial A}{\partial z} + \frac{\beta}{2}A\left(\frac{\partial^2 B}{\partial x^2} + \frac{\partial^2 B}{\partial y^2}\right) - \gamma A = 0, \tag{6a}$$

$$\frac{\partial X}{\partial z} - \beta\left(\frac{\partial X}{\partial x}\frac{\partial B}{\partial x} + \frac{\partial X}{\partial y}\frac{\partial B}{\partial y}\right) = 0, \tag{6b}$$

$$\frac{\partial B}{\partial z} + \frac{\beta}{2}\left[\left(\frac{\partial B}{\partial x}\right)^2 + \left(\frac{\partial B}{\partial y}\right)^2\right] = 0, \tag{6c}$$

$$\frac{\partial^2 X}{\partial x^2} + \frac{\partial^2 X}{\partial y^2} = 0, \tag{6d}$$

After some algebra, one obtains the simplest particular solutions of system (6) for the given parameters:

$$A = A_0\alpha \exp[G(z)], \tag{7a}$$

$$Z = \frac{(k^2+l^2)\alpha D(z)}{W_0^2}, \tag{7b}$$

$$X = \frac{\alpha}{W_0}\left[kx + ly + (r_0 + \omega_0 s_0)\, D(z) - \omega_0\right], \tag{7c}$$

$$B = -\frac{s_0\alpha}{2}\left(x^2+y^2\right) - \frac{r_0\alpha}{2}\left(\frac{x}{k}+\frac{y}{l}\right) - \frac{r_0^2\left(k^2+l^2\right)\alpha D(z)}{8k^2l^2} + \varphi_0, \tag{7d}$$

where $G(z) = \int_0^z \gamma(\nu)\mathrm{d}\nu$ and $D(z) = \int_0^z \beta(\nu)\mathrm{d}\nu$ represent gain/loss and the accumulated dispersion, respectively, and $\alpha = [1 - s_0 D(z)]^{-1}$ is the chirp function, which is related to the wave front curvature and represents a measure of the phase chirp. The subscript '0' denotes the initial values of the corresponding parameters at distance $z = 0$; $s_0, r_0, A_0, \omega, W_0, k, l$, are all constants, the parameters s_0 and r_0 being the initial curvature and position, A_0 and ω_0 are the initial amplitude and position of the beam center, W_0 is related to the initial beam width and k and l are group velocity parameters. Moreover, the function $D(z)$ influences the form of the amplitude, the width, the phase, and the effective propagation distance of the solution.

The transformation of Eq. (1) into the traceable form (3) imposes a constraint on $\chi(z)$, *i.e.*,

$$\chi(z) = \frac{\beta(z)(k^2+l^2)}{W_0^2 A_0^4 \alpha^2}\mathrm{e}^{-4G(z)}. \tag{8}$$

This constraint can be viewed as an integrability condition placed on Eq. (1) for solution by the present method. Thus, starting with Eq. (1), one casts its solution into a convenient form that depends on the self-similarity variable X (which depends on the original variables x, y and z) and two arbitrary parameters $\beta(z)$ and $\gamma(z)$. According to condition (8), solitons or similaritons transmit stably without the distortion of shape based on the exact balance between the diffraction, nonlinearity, and the gain/loss. Only two of the three coefficients $\beta(z)$, $\chi(z)$ and $\gamma(z)$ in Eq. (1) are free parameters. For example, if $\beta(z)$ and $\gamma(z)$ are freely chosen, then $\chi(z)$ should be determined from Eq. (8). In this way, different solitons or similaritons are obtained, depending on the two real-independent functions.

We have thus obtained explicit analytical solutions that involve the modulated diffraction and nonlinearity. Equation (5a) leads to the bright solitary wave solution:

$$u_B(z,x,y) = \frac{\eta A_0}{[1 - s_0 D(z)]\cosh^{1/2}(X)} \times \mathrm{e}^{i\left[\frac{1}{2}Z(z) + B(z,x,y)\right] + G(z)}, \tag{9a}$$

whereas Eq. (5b) leads to the dark solitary wave solution:

$$u_D(z,x,y) = \frac{\sqrt{2}A_0}{1 - s_0 D(z)}\frac{u_\infty \tanh(u_\infty^2 X)}{\sqrt{3-\tanh^2(u_\infty^2 X)}} \times \mathrm{e}^{i[-Z(z)+B(z,x,y)]+G(z)}, \tag{9b}$$

where $D(z)$, $G(z)$, $Z(z)$ and $B(z,x,y)$ are parameters already mentioned, and u_∞ is the value of u when $X \to \infty$. From Eqs. (9a) and (9b), as long as we appropriately choose $\chi(z)$, we obtain analytical solitary wave solutions to the generalized (2+1)D QNLS equation with variable spatially-modulated diffraction and nonlinearity coefficients.

3 Bright and dark similariton solitons

From the solutions presented above, we can conclude that both bright and dark solitary wave solutions could propagate either in normal diffraction media or in media with spatially-modulated diffraction. Figure 1

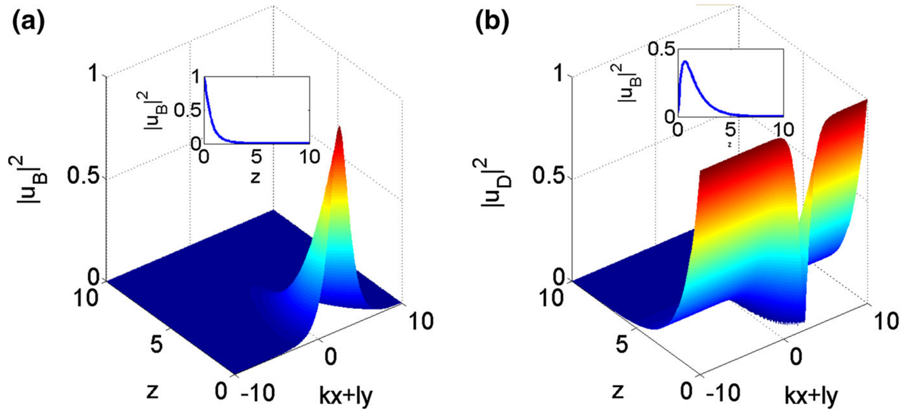


Fig. 1 Decay of the bright soliton (**a**) and the dark soliton (**b**) in the framework of Eq. (1). Here, the diffraction coefficient $\beta(z) = 1$, and the gain/loss coefficient $\gamma(z) = 0$. (*Inset*) The decay of the amplitude. Other parameters: $A_0 = W_0 = s_0 = r_0 = \eta = k = l = 1$, $\omega_0 = 0$

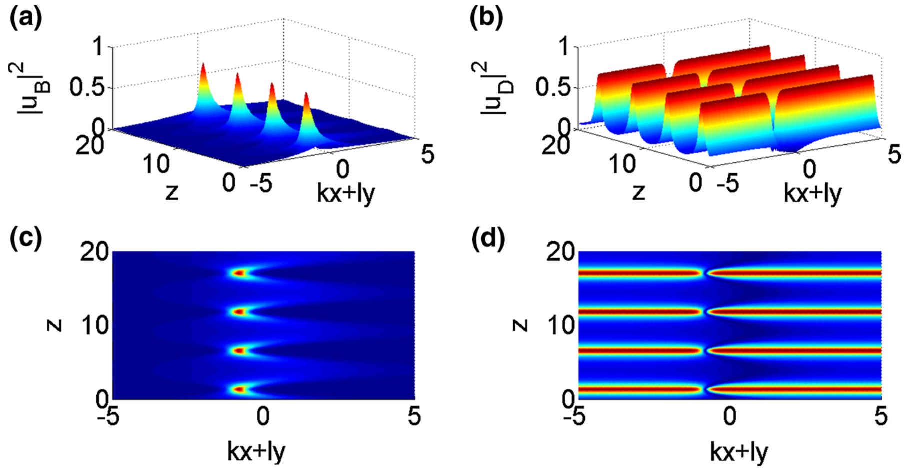


Fig. 2 Evolution of the bright soliton and the dark soliton with chirp $s_0 = 1$. The diffraction coefficient $\beta(z) = \cos(2z)$, and the gain/loss coefficient $\gamma(z) = 0$; other parameters are the same as in Fig. 1

shows the evolution of this kind of bright soliton or dark soliton. Letting $\beta = 1$, $\gamma = 0$, $A_0 = W_0 = s_0 = r_0 = \eta = k = l = 1$, and $\omega_0 = 0$, from the condition (8) one obtains $\chi(z) = (1 - z)^2$. Obviously, the light beam begins to decay after about several propagation distance units (diffraction lengths), and hence the bright soliton and the dark soliton are both unstable.

This situation will be greatly changed if one chooses the proper distributed coefficients in Eq. (1). In the following examples, the control and management of bright/dark chirp and chirp-free solitons is realized. For example, consider the cosine-distributed coefficients in the periodic distributed system [30,31]. Figure 2 shows the evolution of bright and dark chirped solitons with the diffraction coefficient $\beta(z) = \cos(2z)$, the gain/loss $\gamma(z) = 0$, and the nonlinearity coefficient $\chi(z) = \cos(2z)[1 - 0.5 * \cos(2z)]$, with the chirp parameter $s_0 = 1$. From Eq. (7), one finds that the transformed propagation distance $Z = \cos(2z)/[1 - 0.5 * \cos(2z)]$ is a periodic function of the original propagation distance z. In Fig. 2, one can see that as the beam propagates in the medium, the decay and the recovery of the beam's intensity repeat with the period $\pi/2$. Thus, one obtains breathers.

On the other hand, if we chose $s_0 = 0$, the chirp-free bright and dark soliton intensity distributions are shown in Fig. 3. From the figure, one can note that the profile of the chirp-free solitary waves does not change while propagating, although its position oscillates periodically (these oscillations were called "snakelike" in Ref. [32]). Therefore, from Figs. 2 and 3, one can conclude that the spatial chirp is not an essential feature of the self-similar waves and that chirp-free self-similar waves (with $s_0 = 0$) really exist, which is similar to the results reported in Ref. [25]. These results show that the shape of both the bright and dark solitary solutions does not change even if the group velocity and the amplitude is changed due to the presence of the parameter s_0, which might be useful in the application

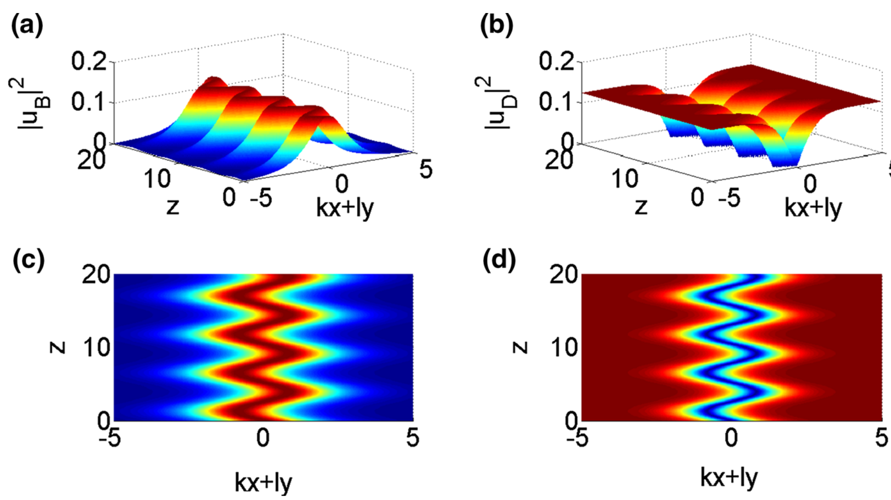


Fig. 3 Evolution of the bright soliton and the dark chirp-free solitons, with $s_0 = 0$. The diffraction coefficient $\beta(z) = \cos(1.2z)$ and the gain/loss coefficient $\gamma(z) = 0$. Other parameters are the same as in Fig. 1

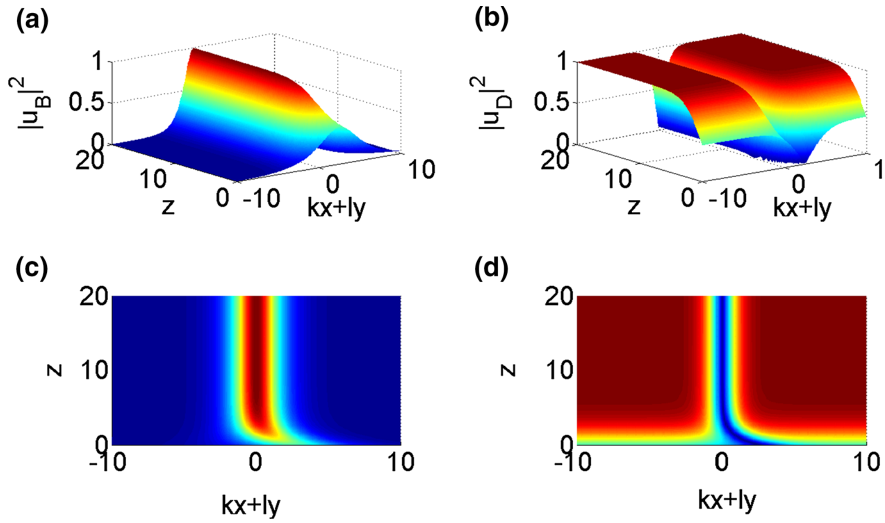


Fig. 4 Evolution of the bright soliton and the dark soliton with chirp, for $s_0 = 1$. The diffraction coefficients $\beta(z) = \exp(-0.8z)$, and the gain/loss $\gamma(z) = 0$; other parameters are the same as in Fig. 1

of self-similar solitary wave for long-distance optical communications.

Next, let us consider the compression problem of the laser beam in a dispersion-decreasing optical medium, by choosing for solution (8) a proper group velocity dispersion coefficient, as given in Ref. [33], $\beta(z) = \exp(-0.8z)$ and the nonlinearity $\chi(z) = \exp[-0.8z]\{1 + 1/0.8\exp[-0.8z]\}$. The evolution plots of the bright soliton and dark soliton with chirp (for $s_0 = 1$) are presented in Fig. 4. It is seen that the intensities of the beam increase gradually with the increase in transmission distance at the start, but at around $z = 2.8$, these intensities reach saturation and remain constant. As the width of the solitary waves is given by $1/\alpha = 1 + s_0\beta_0/\sigma \exp[-\sigma z]$, the width of the beam can thus be compressed quite effectively in propagation along optical medium to any degree that is desired, which can have useful applications in experiments.

To confirm the validity of solutions in Eq. (9) and to test their stability, we performed a direct numerical integration and compared analytical solutions with the numerical ones. We performed numerical simulations by using the split-step beam propagation method [34] and added 9% of white noise to the initial condition. We considered the input pulse from Eq. (9). Figure 5 shows the comparison of the exact bright and dark chirp-free solutions given by Eqs. (9a) and (9b) with the result of numerical simulation. We see that the analytical solution is consistent with the numerical simulation, and the noise does not cause the collapse or the diffraction of the solitary wave.

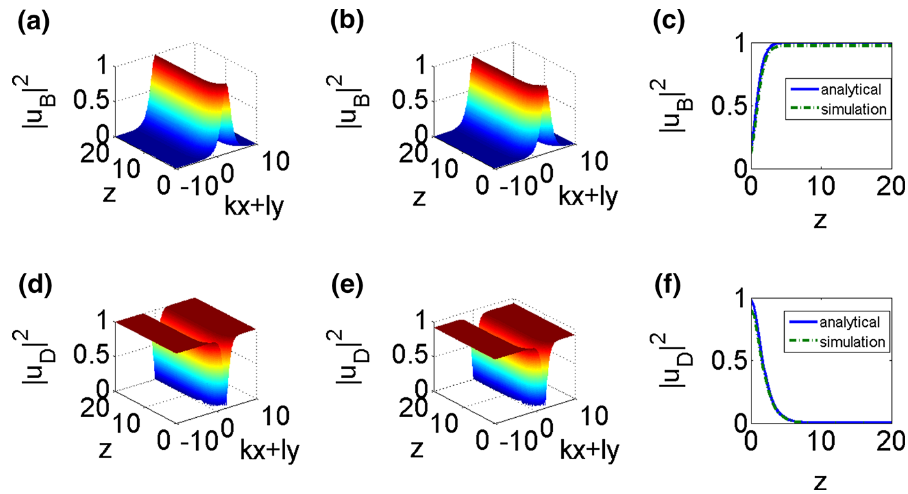


Fig. 5 Comparing the analytical solution with the numerical simulation of the bright and dark chirp-free solitons, for $s_0 = 0$. Other parameters are the same as in Fig. 1

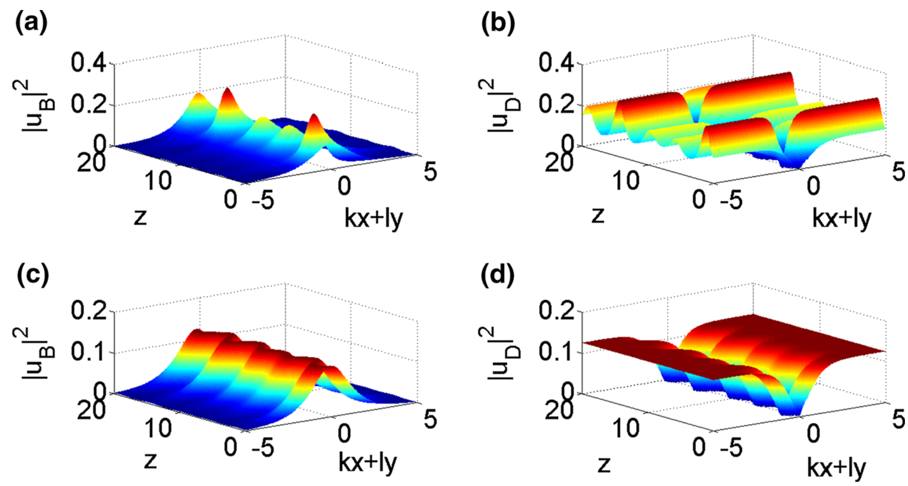


Fig. 6 Evolution of the bright and dark solitons with chirp (**a**),(**b**) and chirp-free (**c**),(**d**); The diffraction coefficient is $\beta(z) = 0.5{*}\cos(\sqrt{2}z){+}0.2{*}\cos(z)$; other parameters are the same as in Fig. 1

Finally, by choosing $\beta(z) = 0.5 * \cos(\sqrt{2}z) + 0.2 * \cos(z)$ and taking $\chi(z)$ as in Eq. (8), we derive quasiperiodic solutions of Eq. (9), provided that the dispersion coefficient $\beta(z)$ has two incommensurable frequencies. Solution (9) in this case shows a quasiperiodic behavior in both the width and the amplitude of the solution, as can be seen in Fig. 6.

4 Conclusion

In this paper, we have used similarity transformation to find exact solutions of the quintic nonlinear Schrödinger equation with spatially-modulated dispersion and nonlinearity. We have explicitly calculated the chirp and chirp-free bright and dark soliton solutions. By appropriately choosing the relations between distributed coefficients, rich self-similar solutions are found. The dynamical behavior along the propagation direction of 2D bright and dark solitons in a periodic and exponentially distributed diffractive system with constant gain is discussed. The chirp parameter $\alpha(z)$ influences the intensity and the width of the beam. Our solutions provide explicit periodic and quasiperiodic bright and dark soliton solutions in a parametrically modulated two-dimensional QNLSE.

Acknowledgments This work is supported in China by the Young teachers corporate action plan Project (XD2012358) and the National college students' innovative entrepreneurial training program at local colleges, under Grant 201210927052, in China. Work in Qatar is supported by the NPRP 6-021-1-005 Project with the Qatar National Research Fund (a member of the Qatar Foundation). Work in Serbia has been supported by the Project OI 171006 with the Serbian Ministry of Education and Science.

References

1. Barenblatt, G.I.: Scaling, Self-Similarity, and Intermediate Asymptotics. Cambridge University Press, Cambridge (1996)
2. Ponomarenko, S.A., Agrawal, G.P.: Do solitonlike self-similar waves exist in nonlinear optical media? Phys. Rev. Lett. **97**, 013901 (2006)
3. Finot, C., Millot, G.: Synthesis of optical pulses by use of similaritons. Opt. Expr. **12**, 5104 (2004)
4. Billet, C., Dudley, J.M., Joly, N., Knight, J.C.: Intermediate asymptotic evolution and photonic bandgap fiber compression of optical similaritons around 1550 nm. Opt. Express **13**, 3236 (2005)
5. Liang, Yueqian, Wei, Guangmei, Li, Xiaonan: Painlevé integrability, similarity reductions, new soliton and soliton-like similarity solutions for the (2+1)-dimensional BKP equation. Nonlinear Dyn. **62**, 195 (2010)
6. Zhong, W.P., Belic, M., Petrovic, M.S.: Solitary and extended waves in the generalized sinh-Gordonequation with a variable coefficient. Nonlinear Dyn. **76**, 717 (2014)
7. Cardoso, W.B., Avelar, A.T., Bazeia, D., Hussein, M.S.: Solitons of two-component Bose–Einstein condensates modulated in space and time. Phys. Lett. A **374**, 2356 (2010)
8. Ozeki, Y., Inoue, T.: Stationary rescaled pulse in dispersion-decreasing fiber for pedestal-free pulse compression. Opt. Lett. **31**, 1606 (2006)
9. Xu, Siliu, Liang, Jiangchu, Yi, Lin: Self-similar solitary waves in Bessel optical lattices. J. Opt. Soc. Am. B **27**, 99 (2010)
10. Ponomarenko, S.A., Agrawal, G.P.: Nonlinear interaction of two or more similaritons in loss- and dispersion-managed fibers. J. Opt. Soc. Am. B **25**, 983 (2008)
11. Pitaevskii, L.P., Stringari, S.: Bose–Einstein Condensation. Oxford University Press, Oxford (2003)
12. Kevrekidis, P.G., Frantzeskakis, D.J., Carretero-González, R. (eds.): Emergent Non-linear Phenomena in Bose–Einstein Condensates: Theory and Experiment, vol. 45. Springer, Berlin (2008)
13. Inouye, S., Andrews, M.R., Stenger, J., et al.: Observation of Feshbach resonances in a Bose–Einstein condensate. Nature **392**, 151 (1998)
14. Theis, M., Thalhammer, G., Winkler, K., Hellwig, M., Ruff, G., Grimm, R., Den-schlag, J.H.: Tuning the scattering length with an optically induced Feshbach resonance. Phys. Rev. Lett. **93**, 123001 (2004)
15. Malomed, B.A.: Soliton Management in Periodic Systems. Springer, New York (2006)
16. Abdullaev, FKh, Kamchatnov, A.M., Konotop, V.V., Brazhnyi, V.A.: Tuning the scattering length with an optically induced Feshbach resonance. Phys. Rev. Lett. **90**, 230402 (2003)
17. Rodas-Verde, M.I., Michinel, H., Pérez-García, V.M.: Controllable soliton emission from a Bose–Einstein condensate. Phys. Rev. Lett. **95**, 153903 (2005)
18. Kivshar, Y.S., Agrawal, G.P.: Solitons: Fom Fibers to Photonic Crystals. Academic Press, San Diego (2003)
19. Senthilnathan, K., Qian, Li: Robust pedestal-free pulse compression in cubic–quintic nonlinear media. Phys. Rev. A **78**, 033835 (2008)
20. Boudebs, G., Cherukulappurath, S., Leblond, H., Troles, J., Smektala, F., Sanchez, F.: Multistable solitons in the cubic–quintic discrete nonlinear Schrödinger equation. Opt. Commun. **219**, 427 (2003)
21. Bemonte-Beitia, J., Cuevas, J.: Exact solutions for the quintic nonlinear Schrödinger equation with inhomogeneous nonlinearity. J. Phys. A **42**, 165201 (2009)
22. Serkin, V.N., Belyaeva, T.L., Alexandrov, I.V., Melchor, G.M.: Topological quasi-soliton solutions for the nonlinear cubic-quintic Schrödinger equation model. In: Proceedings of SPIE. International Society for Optical Engineering, vol. 4271, p. 292 (2001)
23. Hao, R.Y., Li, L., Li, Z.H., Yang, R.C., Zhou, G.S.: A new way to exact quasi-soliton solutions and soliton interaction for the cubic–quintic nonlinear Schrödinger equation with variable coefficients. Opt. Commun. **245**, 383 (2005)
24. Senthilnathan, K., Li, Qian, Nakkeeran, K., Wai, P.K.A.: Robust pedestal-free pulse compression in cubic–quintic nonlinear media. Phys. Rev. A **78**, 033835 (2008)
25. Dai, C.Q., Wang, Y.Y., Yan, C.J.: Chirped and chirp-free self-similar cnoidal and solitary wave solutions of the cubic–quintic nonlinear Schrödinger equation with distributed coefficients. Opt. Commun. **283**, 1489 (2010)
26. Belmonte-Beitia, J., Calvo, G.F.: Exact solutions for the quintic nonlinear Schrödinger equation with time and space modulated nonlinearities and potentials. Phys. Lett. A **373**, 448453 (2009)
27. Xu, Siliu, Belić, M.R., Zhong, Wei-Ping: Three-dimensional spatio-temporal vector solitary waves in coupled nonlinear Schrödinger equations with variable coefficients. J. Opt. Soc. Am. B **30**, 113 (2013)
28. Serkin, V.N., Hasegawa, A., Belyaeva, T.L.: Nonautonomous matter-wave solitons near the Feshbach resonance. Phys. Rev. A **81**, 023610 (2010)
29. Belmonte-Beitia, J., Pérez-García, V.M., Vekslerchik, V., Konotop, V.V.: Localized nonlinear waves in systems with time- and space-modulated nonlinearities. Phys. Rev. Lett. **100**, 164102 (2008)
30. Serkin, V.N., Hasegawa, A.: Novel soliton solutions of the nonlinear Schrödinger equation model. Phys. Rev. Lett. **85**, 4502 (2000)
31. Petrović, N.Z., Belić, M.R., Zhong, W.P.: Exact traveling-wave and spatiotemporal soliton solutions to the generalized (3+1)-dimensional Schrödinger equation with polynomial nonlinearity of arbitrary order. Phys. Rev. E **83**, 026604 (2011)
32. Yang, Zhan-Ying, Zhang, Li-Chen: Snakelike nonautonomous solitons in a graded-index grating waveguide. Phys. Rev. A **81**, 043826 (2010)
33. Serkin, V.N., Hasegawa, A., Belyaeva, T.L.: Comment on "exact self-similar solutions of the generalized nonlinear Schrödinger equation with distributed coefficients". Phys. Rev. Lett. **92**, 199401 (2004)
34. Tomlinson, W.J., Hawkins, R.J., Weiner, A.M., et al.: Dark optical solitons with finite-width background pulses. J. Opt. Soc. Am. B **6**, 329 (1989)

Nonlinear Dyn (2016) 84:251–259
DOI 10.1007/s11071-015-2426-1

ORIGINAL PAPER

Exact solutions for the quintic nonlinear Schrödinger equation with time and space

Si-Liu Xu · Nikola Petrović · Milivoj R. Belić · Wenwu Deng

Received: 1 September 2015 / Accepted: 24 September 2015 / Published online: 9 December 2015

Abstract Using similarity transformations, the analytical solutions to the quintic nonlinear Schrödinger equation with potentials and nonlinearities depending both on time and space are constructed. The stability of resonance solitons is examined by direct numerical simulation. It is found that stable fundamental soliton states with $m = 0$ and low-order soliton states with $m = 1$ can be supported both by self-focusing and self-defocusing materials. Higher-order solitons are found unstable, however, displaying quasi-stable propagation over prolonged distances.

Keywords Nonlinear optics · Spatial solitons · Quintic nonlinear Schrödinger equation

S.-L. Xu (✉) · W. Deng
The School of Electronic and Information Engineering, HuBei University of Science and Technology, 437100 Xianning, China
e-mail: xusiliu1968@163.com

N. Petrović
Institute of Physics, University of Belgrade, P.O. Box 68, Belgrade 11001, Serbia

M. R. Belić
Science Program, Texas A&M University at Qatar, P.O. Box 23874, Doha, Qatar

1 Introduction

A spatial soliton is a stable self-trapped wave packet propagating in a nonlinear (NL) medium, in which diffraction is exactly balanced by the nonlinearity [1]. One of the most often used models to describe two-dimensional (2D) optical spatial solitons propagating in Kerr media is the nonlinear Schrödinger (NLS) Eqs. [1, 2]. In two transverse dimensions, spatial solitons have been identified and can self-trap in different physical systems [2]. However, their stability is still an open problem. It can be improved with different methods, such as employing soliton management techniques [3] or including nonlocality into the analysis [4–8]. In both 2D and 3D physical settings, various types of robust soliton clusters have been constructed by many authors [9–11]. Recent review work [12] lists a variety of exact solutions of the 2D NLS with the trapping potential.

Many of these works deal with the cubic nonlinear Schrödinger equation (CNLSE), as this equation models a wealth of physical phenomena. However, when the incident fields become stronger, non-Kerr nonlinearities should be considered [13, 14]. The cubic-quintic nonlinearity occurs due to an intrinsic nonlinear resonance in the material, which also gives rise to the strong two-photon absorption [15, 16]. In contrast to the CNLSE, the quintic nonlinear Schrödinger (QNLS) equation, has received less attention, in spite of the considerable relevance of this equation, both from the mathematical point of view and that of nonlinear physics.

In recent years, many influential papers have been reporting exact analytical solutions of the QNLSE, such as the pioneering work of Serkin et al. [17] and Hao et al. [18], which constructed exact solitary wave solutions by assuming an ansatz solution. K. Senthilnathan et al. [14,19] investigated the evolution of nonlinear optical pulses in cubic-quintic nonlinear media, and Dai et al. [20] constructed chirped and chirp-free self-similar cnoidal and solitary wave solutions of the QNLSE.

In this paper, we go beyond the previous work on the QNLS equation and study explicit solutions of the two-dimensional QNLS equation with space-modulated diffraction and nonlinearity coefficients. We use the self-similarity transformation to transform this model into the standard QNLS equation and find exact solutions.

The present paper is organized as follows. In Sect. 2, extending the similarity method given in Refs. [21,22] to Eq. (1), we reduce the generalized QNLS equation with variable coefficients to the standard QNLS equation and present exact solitary wave solutions. In Sect. 3, the resonance solitary waves are investigated when the solution parameters are chosen appropriately. Numerical simulations and comparison with the analytical results are also performed. In Sect. 4, the conclusions to the paper are outlined briefly.

2 The model and the soliton solutions

We utilize a model for the propagation of optical electromagnetic field in a bulk nonlinear medium with the quintic nonlinearity, in the presence of a space-modulated photonic lattice [23]

$$i\frac{\partial\psi}{\partial z}+\frac{1}{2}\nabla\psi+\chi(z,r)|\psi|^4\psi+V(z,r)\psi=0. \quad (1)$$

Here, z is the propagation coordinate, and r and φ are the polar coordinates in the transverse plane. The function $\chi(z,r)$ stands for the variable nonlinearity coefficient. The diffraction coefficient in the second term in Eq. (1) has been normalized. Here, $\nabla = \frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\varphi^2}$ is the transverse 2D Laplacian with the transverse radial coordinate $r=\sqrt{x^2+y^2}$, φ is the azimuthal angle and $V(z,r)$ describes a real external potential, which is to be specified.

We search for the axisymmetric cylindrical-beam solutions of Eq. (1) in the form $\psi(z,r,\varphi)=u(z,r)\Phi(\varphi)$. We assume that the azimuthal part of the solution is of the form $\Phi(\varphi)=\cos(m\varphi)+iq\sin(m\varphi)$[24], where m is a non-negative real constant, the so-called topological charge (TC). The parameter $q\in[0,1]$ determines the modulation depth of the beam intensity. Note that the azimuthal part is only an approximate solution of Eq. (1), valid for weak nonlinearities or for large values of q (close to 1). This is because the $|u|^4$ term in the nonlinearity retains the φ-dependence and spoils the assumed separation of variables. Inserting the ansatz for ψ into Eq. (1), integrating over φ from 0 to 2π, and when m is an integer or half-integer is, one readily derives an averaged equation for u:

$$i\frac{\partial u}{\partial z}+\frac{1}{2}\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}-\frac{m^2}{r^2}u\right)+\chi_1(z,r)|u|^4u \\ +V(z,r)u=0. \quad (2)$$

Here $\chi_1(z,r)=\left(3+2q^2+3q^4/8\right)\chi(z,r)$.

In order to transform Eq. (2) into the standard QNLS equation [25] with constant nonlinearity coefficient, we use the following form of the equation with constant coefficients:

$$EU=-U_{RR}+G|U|^4U, \quad (3)$$

where both $U=U(R)$ and $R=R(z,r)$ are real functions to be determined, E denotes the eigenvalue of the nonlinear equation and G is a constant. Henceforth, we explore the cases $G=-1$, the so-called focusing or attractive nonlinearity and $G=1$, the so-called defocusing or repulsive nonlinearity.

Next, we construct the bright soliton solution of Eq. (3), for the case of negative eigenvalue $E=-\frac{\kappa}{4}$ and $G=-\frac{3\kappa^2}{4\eta^4}=-1$,

$$U(R)=\frac{\eta}{\cosh^{1/2}(ER)}, \quad (4a)$$

and for the case of positive eigenvalue $E=Gu_\infty^4$ and $G=1$, where u_∞ is the value of u when $r\to\infty$, the dark soliton solution,

$$U(R)=\frac{\sqrt{2}u_\infty\tanh\left(u_\infty^2\sqrt{G}R\right)}{\sqrt{3-\tanh^2\left(u_\infty^2\sqrt{G}R\right)}}, \quad (4b)$$

To connect solutions of Eq. (2) with this of Eq. (3), we use the following similarity transformation:

$$u(z,r) = A(z,r)U[R(z,r)]e^{i\Theta(z,r)}, \tag{5}$$

where the amplitude $A(z,r)$ and the phase $\Theta(z,r)$ are real functions of z and r [26]. It should be emphasized that we require $U(R)$ to satisfy Eq. (3) and $u(z,r)$to be a solution of Eq. (2). Substituting Eq. (5) into Eq. (2), one obtains:

$$\frac{\mathrm{d}R}{\mathrm{d}z} + \frac{\mathrm{d}R}{\mathrm{d}r}\frac{\mathrm{d}\Theta}{\mathrm{d}r} = 0, \tag{6a}$$

$$\chi_1 A^4 = GR_r^2, \tag{6b}$$

$$2\frac{\mathrm{d}A}{\mathrm{d}r}\frac{\mathrm{d}R}{\mathrm{d}r} + A\frac{\mathrm{d}^2R}{\mathrm{d}r^2} + \frac{A}{r}\frac{\mathrm{d}R}{\mathrm{d}r} = 0, \tag{6c}$$

$$-A\frac{\mathrm{d}\Theta}{\mathrm{d}z} + \frac{1}{2}\left[\frac{\mathrm{d}^2A}{\mathrm{d}r^2} - A\left(\frac{\mathrm{d}\Theta}{\mathrm{d}r}\right)^2 + \frac{1}{r}\frac{\mathrm{d}A}{\mathrm{d}r} - A\frac{m^2}{r^2}\right] + AV - EA\left(\frac{\mathrm{d}R}{\mathrm{d}r}\right)^2 = 0, \tag{6d}$$

$$\frac{\mathrm{d}A}{\mathrm{d}z} + \frac{1}{2}\left(A\frac{\mathrm{d}^2\Theta}{\mathrm{d}r^2} + 2\frac{\mathrm{d}A}{\mathrm{d}r}\frac{\mathrm{d}\Theta}{\mathrm{d}r} + \frac{A}{r}\frac{\mathrm{d}\Theta}{\mathrm{d}r}\right) = 0. \tag{6e}$$

To find exact solutions of Eqs. (6a)–(6e), we introduce another self-similar transformation [4,5] and a few auxiliary functions:

$$A(z,r) = \frac{k_1}{w(z)}F(\theta), \tag{7a}$$

$$\Theta(z,r) = a(z)r^2 + b(z). \tag{7b}$$

Here, k_1 is the normalization constant, $w(z)$ is the beam width, $\theta(z,r)$ is the similarity variable to be determined, $a(z)$ is the wave front curvature, and $b(z)$ represents the phase offset. These variables vary with propagation distance z. Inserting Eqs. (7) into Eqs. (6a)–(6e), we obtain the following expressions for $\theta(z,r) = \frac{r^2}{w^2(z)}$, the wave front curvature $a(z) = \frac{1}{2w}\frac{\mathrm{d}w}{\mathrm{d}z}$, and:

$$R_r = \frac{1}{rF^2(\theta)}, \tag{8a}$$

$$\chi(z,r) = -\frac{8G}{(3+2q^2+3q^4)A^4}\left(\frac{\mathrm{d}R}{\mathrm{d}r}\right)^2. \tag{8b}$$

The amplitude $A(z,r)$ is found from Eq. (6d), which is transformed into the following NL differential equation for $F(\theta)$

$$\theta\frac{\mathrm{d}^2F}{\mathrm{d}\theta^2} + \frac{\mathrm{d}F}{\mathrm{d}\theta} - \left(\theta\frac{w^3}{4}\frac{\mathrm{d}^2w}{\mathrm{d}z^2} + \frac{m^2}{4\theta}\right)F - \frac{w^2F}{2}\frac{\mathrm{d}b}{\mathrm{d}z} - \frac{w^2}{2}\left[V + 2E\left(\frac{\mathrm{d}R}{\mathrm{d}r}\right)^2\right]F = 0, \tag{9}$$

We choose the special trapping potential as follows:

$$V = sr^2 - 2E\left(\frac{\mathrm{d}R}{\mathrm{d}r}\right)^2, \tag{10}$$

where s is a positive constant, and after a variable transformation $F(\theta) = \theta^{\frac{m}{2}}e^{-\frac{\theta}{2}}f(\theta)$, from Eq. (10) one obtains:

$$\theta\frac{\mathrm{d}^2f}{\mathrm{d}\theta^2} + (m+1-\theta)\frac{\mathrm{d}f}{\mathrm{d}\theta} - nf = 0, \tag{11a}$$

with:

$$-\frac{w^2}{2}\frac{\mathrm{d}b}{\mathrm{d}z} - \frac{m+1}{2} = n \tag{11b}$$

and

$$-\frac{w}{2}\frac{\mathrm{d}w^2}{\mathrm{d}z^2} - sw^2 + \frac{1}{2w^2} = 0. \tag{11c}$$

Here, n is a non-negative integer. The differential equation (11a) is known as the confluent hyper-geometric differential equation and its solutions are the Sonine functions [27], namely $f(\theta) = S_n^m(\theta)$ with:

$$S_n^m(\theta) = \sum_{k=0}^{n}(-1)^k\frac{1}{k!(n-k)!}\frac{(m+n)!}{(m+k)!}\theta^k. \tag{12}$$

Taking $w(z)|_{z=0} = w_0$ and $\frac{\mathrm{d}w(z)}{\mathrm{d}z}|_{z=0} = 0$, where the subscript '0' denotes the value of the corresponding quantity at $z = 0$, and integrating Eq. (11c) yields:

$$w(z)^2 = w_0^2\left[1 + (\lambda-1)\sin^2\left(2sw_0^2z\right)\right], \tag{13a}$$

where $\lambda = \frac{1}{2s^2w_0^4}$. Hence, from Eqs. (13a), (11b) and from the definition of $a(z) = \frac{1}{2w}\frac{\mathrm{d}w}{\mathrm{d}z}$, we obtain

$$a(z) = \frac{sw_0^2(\lambda-1)\sin\left(4sw_0^2z\right)}{1+\lambda-(\lambda-1)\cos\left(4sw_0^2z\right)}, \tag{13b}$$

$$b(z) = b_0 - \frac{(2n+m+1)\tan^{-1}\left[\sqrt{\lambda}\tan\left(2sw_0^2z\right)\right]}{2s\sqrt{\lambda}w_0^4}, \tag{13c}$$

Collecting all these partial solutions together, we finally obtain the analytical solution of Eq. (1):

$$\psi(z,r,\varphi)=[\cos(m\varphi+iq\sin(m\varphi)]\,\frac{k_1}{w}\left(\frac{r}{w}\right)^m e^{-\frac{r^2}{2w^2}}$$
$$\times S_n^m\left(\frac{r^2}{w^2}\right)U[R(z,r)]e^{i[a(z)r^2+b(z)]}, \tag{14a}$$

Here $w(z)$, $a(z)$, $b(z)$, $U(R)$, $R(z,r)$, $S_n^m(\frac{r^2}{w^2})$ are determined by Eqs. (13a)–(13c), (4), (8a), (12), and $k_1=\sqrt{\frac{n!}{\Gamma(n+m+1)}}$.

The localized solution in Eq. (14a) has the pulse width, wave front curvature, phase, and other characteristics changing with the propagation distance. Thus, it does not represent a shape-invariant spatial soliton. However, from Eq. (13a) one can see that for $\lambda=1$, i.e., $w=w_0$, the beam diffraction is exactly balanced by the nonlinearity and hence the beam becomes an exact soliton. The other parameters in this case are: $s=\frac{1}{\sqrt{2}w_0^2}$, $a(z)=0$ and $b(z)=b_0-(2n+m+1)\frac{z}{w_0^2}$. Plugging all the results into Eq. (1), we obtain the following analytical resonance soliton solution:

$$\psi(z,r,\varphi)=[\cos(m\varphi)+iq\sin(m\varphi)]\,\frac{k_1}{w_0}\left(\frac{r}{w_0}\right)^m e^{-\frac{r^2}{2w_0^2}}$$
$$\times S_n^m\left(\frac{r^2}{w_0^2}\right)U[R(r)]e^{i\left[b_0-\frac{(2n+m+1)}{w_0^2}z\right]}. \tag{14b}$$

In order to study the linear stability of this soliton, we assume perturbations to Eq. (1) in the form [28]:

$$\psi(r,\varphi,z)=\mathrm{e}^{\mathrm{i}\mu z}\{\bar{\psi}(r,\varphi)+\varepsilon[g(r,\varphi)+h(r,\varphi)]\mathrm{e}^{i\delta z}\}, \tag{15}$$

where $\bar{\psi}(r,\varphi)$ is a soliton solution of Eq. (1), ε is an infinitesimal amplitude, $g(r,\varphi)$ and $h(r,\varphi)$ are the real and imaginary parts of the perturbation and δ denotes the perturbation growth rate. Substituting the perturbed $\psi(x,y)$ into Eq. (1) and then linearizing it around the unperturbed solution (to the first order of ε), one obtains the following eigenvalue equations:

$$\delta g=\frac{1}{2r}g_r+\frac{1}{2}g_{rr}+\frac{1}{2r^2}g_{\theta\theta}+3\chi\bar{\psi}^4h+Vh-\mu h, \tag{16a}$$

$$\delta h=\frac{1}{2r}h_r+\frac{1}{2}h_{rr}+\frac{1}{2r^2}h_{\theta\theta}+3\chi\bar{\psi}^4g+Vg-\mu g, \tag{16b}$$

We rewrite this eigenvalue problem as:

$$\begin{pmatrix}\varsigma_1 & \varsigma_2\\ \varsigma_3 & \varsigma_4\end{pmatrix}\begin{pmatrix}g\\ h\end{pmatrix}=\delta\begin{pmatrix}g\\ h\end{pmatrix}, \tag{16c}$$

where:

$$\varsigma_1=\varsigma_4=\frac{1}{2r}\frac{\partial}{\partial r}+\frac{1}{2}\frac{\partial^2}{\partial r^2}+\frac{1}{2r^2}\frac{\partial^2}{\partial\theta^2}, \tag{16d}$$

$$\varsigma_2=\varsigma_3=3\chi\bar{\psi}^4+V-\mu \tag{16e}$$

This eigenvalue problem can be solved by the Newton-conjugate-gradient method for individual discrete eigenvalues [29]. If any eigenvalue δ has a negative imaginary part, the perturbed solution would grow exponentially with z, and thus, the corresponding solitons would become linearly unstable. Otherwise, solutions can be completely stable, if all imaginary parts of δ are positive or equal to zero.

3 The characteristic distributions of solitons

From Eq. (14b), it is seen that the novel solitons are characterized by three parameters: the mode number n, the TC m and the modulation depth q. Based on the values of these parameters, the new 2D resonance solitons are introduced. It is seen that the auxiliary function R, the nonlinearity coefficient χ, and the trapping potential V depend only on the radial variable r. Hence, to obtain the shape of invariant solitons by the present method, it is necessary to precisely model the forms of the external potentials and nonlinearity coefficients. Their forms are demonstrated by Eqs. (8) and (10), which present a drawback in the procedure and in the applicability of the method. The distributions of the nonlinearity coefficient $\chi(r)$ and the external potential $V(r)$ with respect to the radial coordinate r are shown in Fig. 1. In Eq. (14b), since $\lim_{|r|\to\infty}\psi(z,r,\varphi)=0$, the solutions are localized. Without loss of generality, in this paper we will study the case when n and m are different non-negative integers with an initial condition $w_0=1$.

First, we begin by analyzing the resonance soliton and select the lower-order vortex-shaped distribution of the beams for $m=1$ with $n=0,1,2$, and $q=0.5$. Such 2D resonance soliton can exist as a spatially localized excitation. It is evident that the solitons are composed of two symmetrical half-moon shapes seen in the top row of in Fig. 2 ($G=1$) and Fig. 3 ($G=-1$). The number of rings is $n+1$, the optical intensity in the center is zero, and the intensity of rings surrounding the center decreases with the increasing radial distance. Futhermore, one can see that the intensity of soliton in the self-defocusing materials is larger than that in the

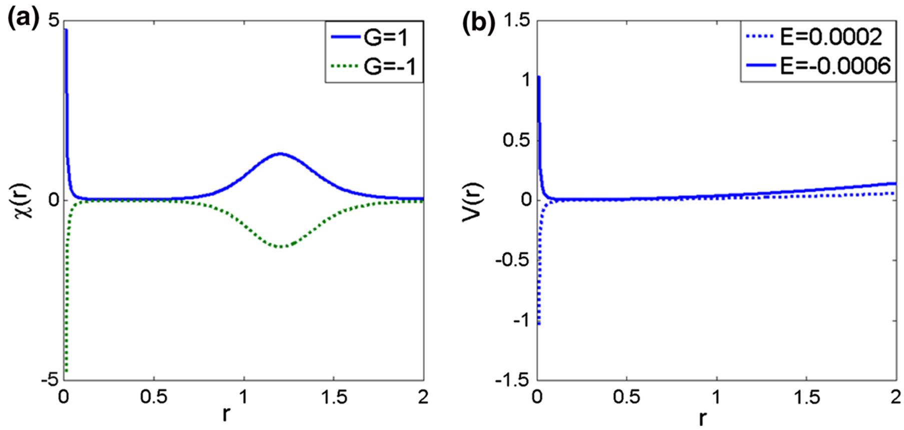


Fig. 1 Distributions of the nonlinearity coefficient $\chi(r)$ and the external potential $V(r)$ for the resonance soliton from Eq. (14b). Parameters: $w_0 = 1, n = 1, m = 0$

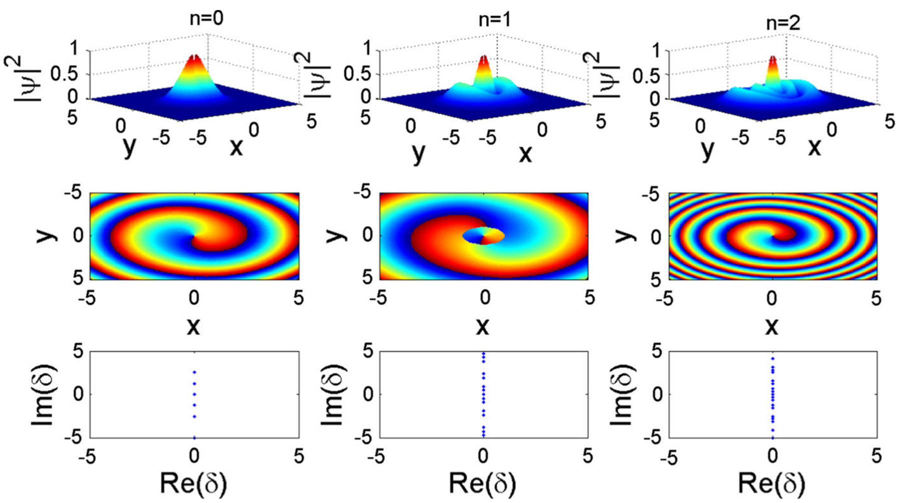


Fig. 2 Intensity profiles of resonance solitons for the self-focusing materials with different n (*top row*); Phase distributions (*middle row*); Linear-stability spectra (*bottom row*). The parameters are: $G = m = 1$, $q = 0.5$, $n = 0, 1, 2$ from left to right, the other parameters are the same as Fig. 1

self-focusing materials. The middle rows in Figs. 2 and 3 depict the phase distributions of soliton, respectively. The bottom rows in Figs. 2 and 3 display the stability analysis of the solitons. It is seen that the eigenvalue δ is pure imaginary; hence some of the solutions are stable, some are unstable.

Figures 4 and 5 display the intensity profiles of resonance solitons for the self-focusing materials and self-focusing with different m (top row). The vortex solitons exhibit spatially modulated patterns, and the amplitude and phase distributions of multipeaked vortex solitons are very similar to those of azimuthons [30]. The profiles of vortex solitons possess several amplitude peaks covering on a constitutive ring-like substrate. The number of amplitude peaks is also determined by the azimuthal index m. Such a localized solution displays a necklace-type self-trapped structure, the number of "petals" is $2m$ in each necklace ring and the total intensity distribution exhibits similar vortex profile. As in Figs. 2 and 3, the intensity of rings surrounding the center decreases with the increasing radial distance. Furthermore, a noticeable difference is that the intensity in self-defocusing materials is reduced faster and is strongly localized in the transverse plane. In the middle row of Figs. 4 and 5, we depict the phase distributions of the soliton solution. Similar to the vortex soliton, the phase pattern changes in the course of the decay, developing a spiral form. One of the interesting properties to note is a wider extent of the resonant solitons in the self-defocusing materials than those in the self-focusing materials, under the same conditions, because of the self-defocusing effect. The linear-stability spectra are displayed in the bottom row of Figs. 4 and 5.

For a fixed n and m, increasing the modulation depth q from 0 to 1, we obtain an angularly-modulated vortex ring and a ring-shaped beam, respectively (see

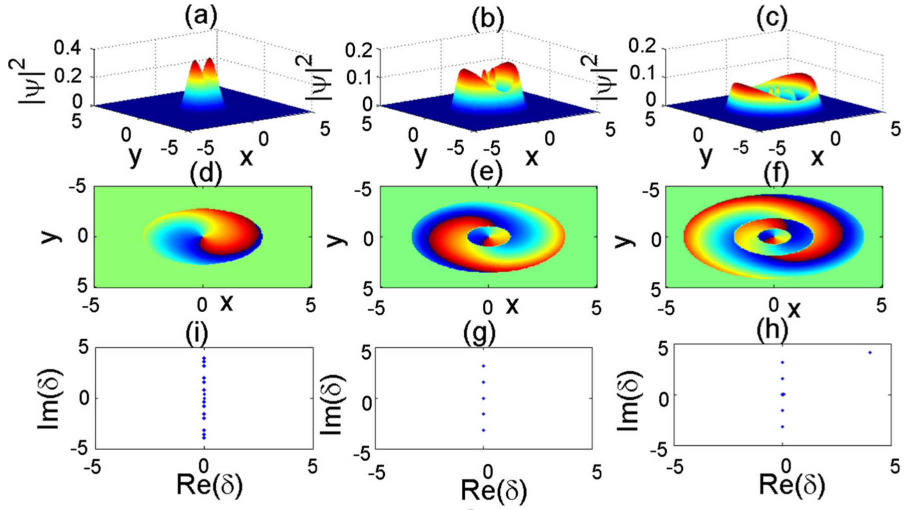


Fig. 3 Same as Fig. 2, but for $G = -1$

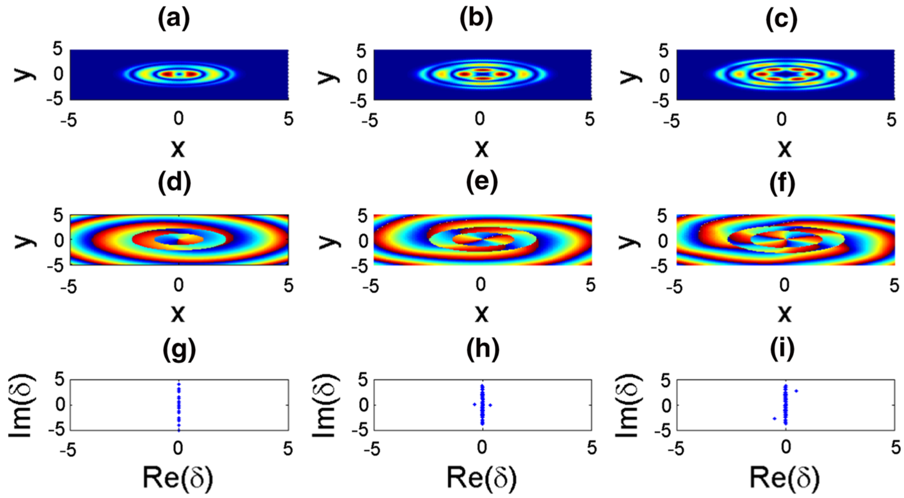


Fig. 4 Same as Fig. 2, but for $n = 2$, $m = 1, 2, 3$ from left to right

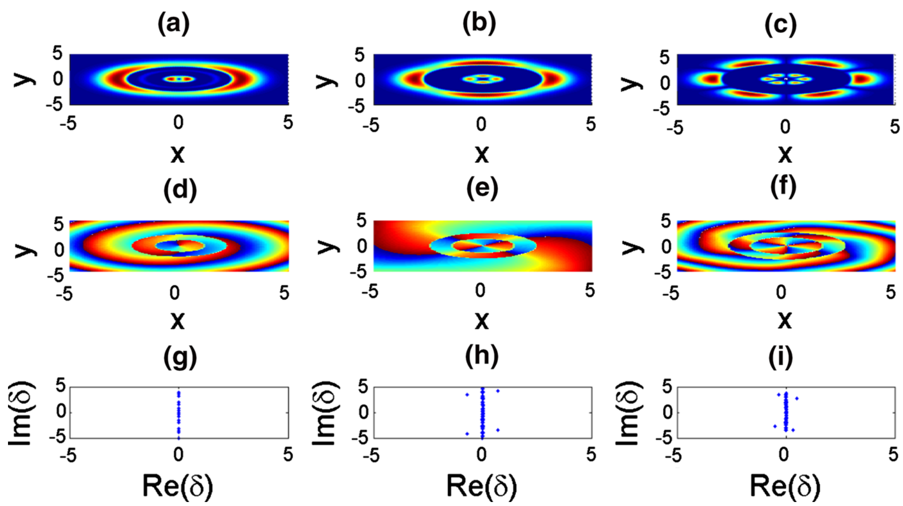


Fig. 5 Same as Fig. 4, but for $G = -1$

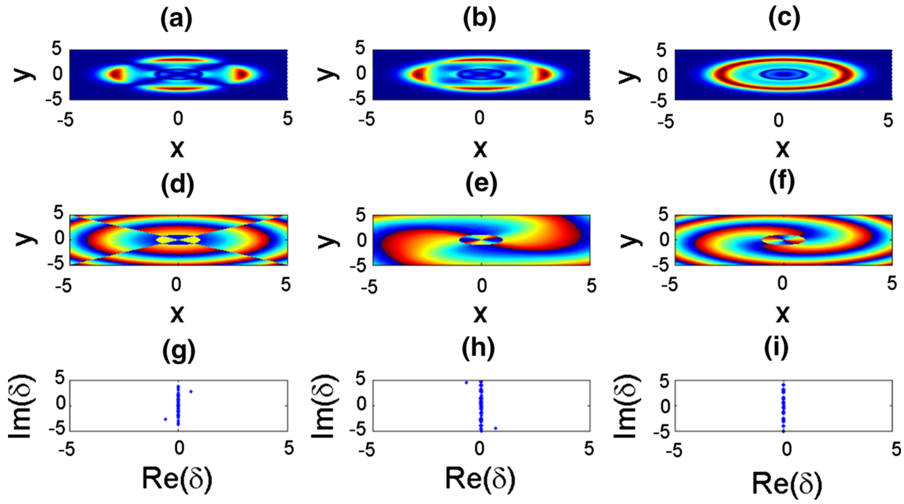


Fig. 6 Same as Fig. 2, but for $G = n = 1, m = 2$, $q = 0, 0.5, 1$ from left to right

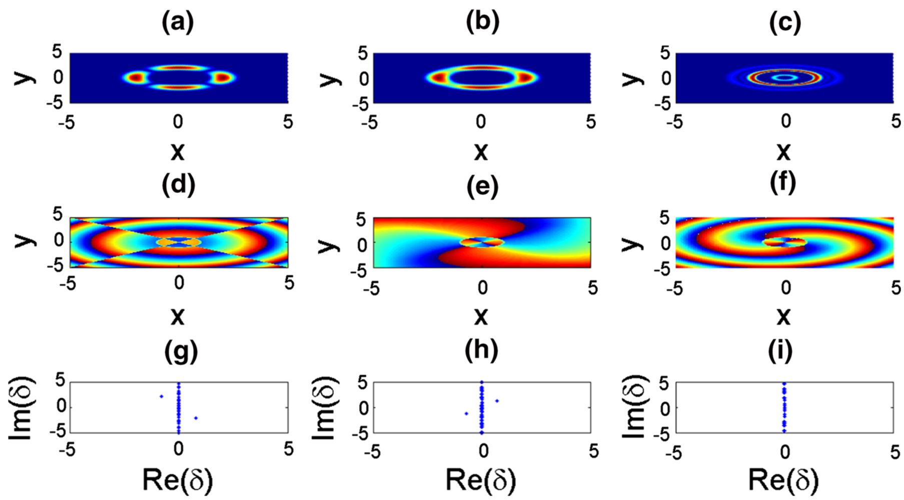


Fig. 7 Same as Fig. 6, but for $G = -1$

the top row of Figs. 6 and 7). It is shown that the beam is modulated by the modulation depth q. Increasing q, the distance between the petals decreases and multi-TC vortices change into the vortex rings. Different from the self-focusing materials, for solitons in the self-defocusing materials the outer soliton intensity reduces to zero, and only a single layer exists. Further, the phase pattern changes with increasing q, so that an angular gradient-changing form evolves into a spiral one (see the middle rows of Figs. 6 and 7). The linear-stability spectra are displayed in the bottom rows of Figs. 6 and 7. It is seen that the larger the value of q, the more stable the solitons.

In order to check the stability of the soliton, we perform direct numerical simulations using the split-step Fourier method [31] and solve Eq. (1) by taking the analytical solution (14b) at $z = 0$ as an initial condition. In Fig. 8, we present the comparison of analytical (the first and the third rows) and numerical (the second and the fourth rows) intensity distribution contour plots in $x - y$ plane, with parameters $n = 2$, $q = 0.96$ and $m = 0, 1, 2, 3$ from left to right. It is seen that only when the topological charge is $m = 0, 1$, the numerical solution is stable against perturbation with an initial Gaussian noise level of 6 %. But when the topological charge is $m \geq 2$, the soliton solution (14b) is unstable in propagation and splits into necklace ring-shaped structures. Furthermore, we find that the stability of solution (14b) in the self-defocusing material is better than that in the self-focusing material. In fact, when $q \to 1$, the stability of resonance soliton solutions of Eq. (14b)

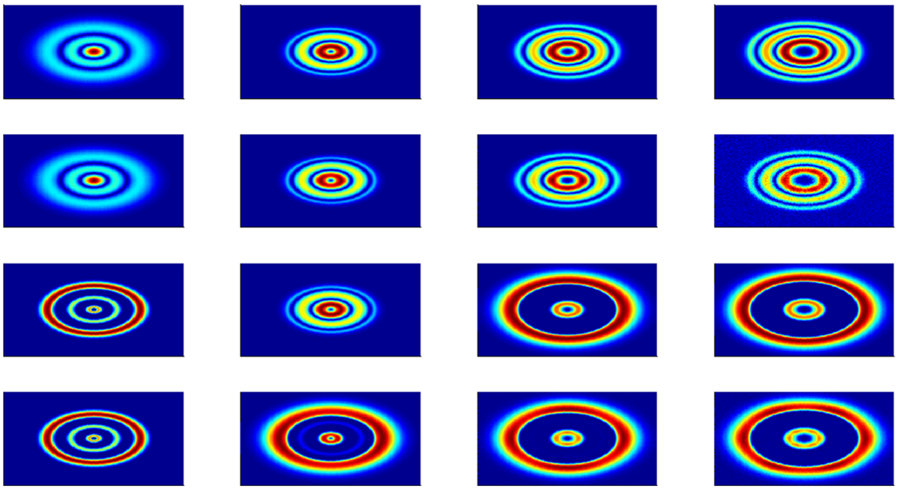


Fig. 8 (Color online) Comparison of analytical (the *first* and the *third rows*) and numerical (the *second* and the *fourth* rows) intensity distribution contour plots in $x - y$ plane; The top two rows are for the self-focusing material, the bottom two rows are for the self-defocusing material. The parameters are $n = 2$, $q = 0.96$ and $m = 0, 1, 2, 3$ from left to right, the other parameters and the coordinate scale range are the same as Figs. 6 and 7

improves, in that it propagates with little change for very long.

4 Conclusion

In summary, the dynamics of azimuthally modulated resonance solitons in self-focusing and self-defocusing materials are investigated. Under the same parameters, the intensity of resonance solitons in the self-defocusing material is smaller than that in the self-focusing material. The stability of the solitons is checked by direct numerical simulation. Our results show that the resonance solitons with $m \leq 1$ are stable, and for higher topological charges ($m \geq 2$), the opposite holds. We find that the stability of resonance solitons in defocusing material is better than in the focusing material, and the stability improves as $q \to 1$. Our approach can be applied to other problems, e.g., Bose–Einstein condensates and light propagation in plasmas.

Acknowledgments This work is supported by the Natural Science Foundation of Hubei Province of China (Grant No. 2015CFB661), the Educational Commission of Hubei Province of China (Grant No. D20142803) and the PhD research startup foundation of Hubei University of Science and Technology (Grant No. BK1410), in China. The work in Serbia is supported by the Serbian Ministry of Education and Science under project OI 171006.

References

1. Hasegawa, A., Kodama, Y.: Solitons in Optical Communications. Oxford University, Oxford (1995)
2. Dalfovo, F., Giorgini, S., Pitaevskii, L.P., Stringari, S.: Theory of Bose–Einstein condensation in trapped gases. Rev. Mod. Phys. **71**, 463–512 (1999)
3. Pitaevski, L.P., Stringari, S.: Bose–Einstein Condensation. Oxford University, Oxford (2003)
4. Baizakov, B.B., Filatrella, G., Malomed, B.A., Salerno, M.: Double parametric resonance for matter-wave solitons in a time-modulated trap. Phys. Rev. E **71**, 036619–036625 (2005)
5. Malomed, B.A.: Soliton Management in Periodic Systems. Springer, Berlin (2006)
6. Lederer, F., Darmanyan, S., Kobyakov, A.: In: Trillo, S., Torruellas, W. (eds.) Discrete solitons in Spatial Solitons, p. 269. Springer, New York (2001)
7. Eisenberg, H., Silberberg, Y., Morandotti, R., Boyd, A.R., Aitchison, J.: Discrete spatial optical solitons in waveguide arrays. Phys. Rev. Lett. **81**, 3383–3386 (1998)
8. Morandotti, R., Peschel, U., Aitchison, J., Eisenberg, H., Silberberg, Y.: Experimental observation of linear and nonlinear optical Bloch oscillation. Phys. Rev. Lett. **83**, 4756–4759 (1999)
9. Dalfovo, F., Giorgini, S., Pitaevskii, L.P., Stringari, S.: Theory of Bose–Einstein condensation in trapped gases. Rev. Mod. Phys. **71**, 463–512 (1999)
10. Serkin, V.N., Hasegawa, A.: Novel soliton solutions of the nonlinear Schrödinger equation model. Phys. Rev. Lett. **85**, 4502–4505 (2000)
11. Zhong, W.P., Xie, R.H., Belić, M., Petrović, N., Chen, G.: Exact spatial soliton solutions of the two-dimensional generalized nonlinear Schrödinger equation with distributed coefficients. Phys. Rev. A **78**, 023821–023821 (2008)
12. Zhong, W.P., Belic, M.R., Mihalache, D., Malomed, B.A., Huang, T.W.: Varieties of exact solutions for the (2+1)–dimensional nonlinear Schrodinger equation with the trapping potential. Rom. Rep. Phys.**64**, Suppl, 1399–1412 (2012)
13. Kivshar, Y.S., Agrawal, G.P.: Solitons: From Fibers to Photonic Crystals. Academic Press, San Diego (2003)
14. Senthilnathan, K., Li, Q., Nakkeeran, K., Wai, P.K.A.: Robust pedestal-free pulse compression in cubic-quintic nonlinear media. Phys. Rev. **78**, 033835–033837 (2008)

15. Boudebs, G., Cherukulappurath, S., Leblond, H., Troles, J., Smektala, F., Sanchez, F.: Experimental and theoretical study of higher-order nonlinearities in chalcogenide glasses. Optics Communication **219**, 427–433 (2003)
16. Bemonte-Beitia, J., Cuevas, J.: Journal of Physics A: Solitons for the cubic–quintic nonlinear Schrödinger equation with time- and space-modulated coefficients. Mathematical and Theoretical **42**, 165201–165206 (2009)
17. Serkin, V.N., Belyaeva, T.L., Alexandrov, I.V., Melchor, G.M.: Proceedings of SPIE. International Society for Optical Engineering **4271**, 292 (2001)
18. Hao, R.Y., Li, L., Li, Z.H., Yang, R.C., Zhou, G.S.: A new way to exact quasi-soliton solutions and soliton interaction for the cubic-quintic nonlinear Schrödinger equation with variable coefficients. Optics Communications **245**, 383–390 (2005)
19. Senthilnathan, K., Li, Qian, Wai, P.K.A., Nakkeeran, K.: Chirped self-similar pulse propagation in cubic-quintic media. Piers Online, **3**, 531-538 (2007)
20. Dai, C.Q., Wang, Y.Y., Yan, C.J.: Chirped and chirp-free self-similar cnoidal and solitary wave solutions of the cubic-quintic nonlinear Schrödinger equation with distributed coefficients. Opt. Commun. **83**, 1489–1494 (2010)
21. Zhong, Wei-Ping, Belić, Milivoj R.: Resonance solitons produced by azimuthal modulation in self-focusing and self-defocusing materials. Nonlinear Dyn. **73**, 2091–2102 (2013)
22. Xu, Si-Liu, Petrović, N., Belić, Milivoj R.: Exact solutions of the (2+ 1)-dimensional quintic nonlinear Schrödinger equation with variable coefficients. Nonlinear Dyn. **80**, 583–589 (2015)
23. Belmonte-Beitia, J., Calvo, G.F.: Exact solutions for the quintic nonlinear Schrödinger equation with time and space modulated nonlinearities and potentials. Phys. Lett. A **373**, 448–453 (2009)
24. Berry, M.V.: Optical vortices evolving from helicoidal integer and fractional phase steps. J. Opt. A **6**, 259–265 (2004)
25. Lee, W.M., Yuan, X.C., Dholakia, K.: Experimental observation of optical vortex evolution in a Gaussian beam with an embedded fractional phase step. Opt. Commun. **239**, 129 (2004)
26. Belmonte-Beitia, J., Perez-Garcia, V.M., Vekslerchik, V., Konotop, V.V.: Lie symmetries and solitons in nonlinear systems with spatially inhomogeneous nonlinearities. Phys. Rev. Lett. **98**, 064102–064105 (2007)
27. Zwillinger, D.: Handbook of Differential Equations, 3rd edn. Academic Press, Boston (1997)
28. Yang, J.K., Lakoba, T.I.: Stud. Appl. Math. **118**, 153 (2007). Kindly provide Article title for the reference [28]
29. Yang, J.K.: Nonlinear Waves in Integrable and Nonintegrable Systems. SIAM, Philadelphia (2010)
30. Desyatnikov, A.S., Sukhorukov, A.A., Kivshar, Y.S.: Azimuthons: spatially modulated vortex solitons. Phys. Rev. Lett. **95**, 203904–203907 (2005)
31. Belic, M., Petrović, N., Zhong, W.P., Xie, R.H., Chen, G.: Analytical light bullet solutions to the generalized (3+ 1)-dimensional nonlinear Schrodinger equation. Phys. Rev. Lett. **101**, 123904–123907 (2008)

Nonlinear Dyn
DOI 10.1007/s11071-016-2612-9

ORIGINAL PAPER

Light bullet supported by parity-time symmetric potential with power-law nonlinearity

Si-Liu Xu · Nikola Petrović · Milivoj R. Belić · Zheng-Long Hu

Received: 17 November 2015 / Accepted: 9 January 2016

Abstract Using a similarity transformation, we find the light bullet solution of $(3+1)$-dimensional nonlinear Schrödinger equation with parity-time (PT) symmetric potential. The diffraction/dispersion and nonlinearity coefficients are chosen as longitudinally inhomogeneous functions. We demonstrate how intensity, width, phase, and chirp of the solution are modulated by the variation in diffraction/dispersion and by the choice of PT-potential. Dynamic characteristics of light bullets in media described by exponentially decreasing diffraction/dispersion and periodically modulated systems are illustrated.

Keywords Light bullet · Parity-time symmetry · Nonlinear Schrödinger equation

S.-L. Xu (✉) · Z.-L. Hu
The School of Electronic and Information Engineering, HuBei University of Science and Technology, Xianning 437100, China
e-mail: xusiliu1968@163.com

N. Petrović · M. R. Belić
Texas A&M University at Qatar, P.O. Box 23874, Doha, Qatar

M. R. Belić
Institute of Physics, University of Belgrade, P.O. Box 68, Belgrade 11001, Serbia

1 Introduction

Solitons in spatially inhomogeneous media have attracted great attention in the past decade, owing to numerous applications in many areas of physics such as photonic devices, nonlinear plasmas, fluid dynamics, and Bose–Einstein condensation (BEC) [1–5]. Recently, the propagation of localized optical beams in complex nonlinear media featuring parity-time (PT) symmetry has become a subject of intense study [6–10].

The PT-symmetry became important in quantum mechanics when Bender and Boettcher in 1998 showed that Hamiltonians with such symmetry can have an entirely real spectrum, although the Hamiltonians are non-Hermitian [11]. Complex PT-symmetric potentials require that the real part of the potential must be an even function of position, whereas the imaginary part should be odd. Another important property of PT-symmetric system is the existence of a sudden phase transition known as the spontaneous PT-symmetry breaking, above which the spectrum ceases to be real.

Nowadays, the properties of solitons in the form of ultrashort and strong laser pulses are quite well known [12–14]. Furthermore, in the long-distance communications and all-optical ultrafast switching devices, many spatiotemporal localized structures such as optical solitons [15], similaritons [16], and light bullets (LBs) [17] have been displayed in nonlinear optics. However, spatiotemporal localized structures in PT-symmetric potentials have not been discussed much. Especially, the 3D solitons in PT-symmetric potentials

Published online: 27 January 2016

with power-law nonlinearities are hardly reported [18]. This study is undertaken in this paper.

The plan of this paper is as follows. In Sect. 2, we briefly introduce the general model and obtain a distinct type of soliton solution. In Sect. 3, the dynamic characteristics of light bullets (LBs), such as their intensity, width, phase, and chirp in specially designed media, are studied. Numerical simulations and comparison with the analytical results are performed in the same section. In Sect. 4, the conclusion to the paper is outlined briefly.

2 The model and the soliton solutions

We present here analytical LB solutions to the general (3 + 1)-dimensional nonlinear Schrödinger equation (NLSE)

$$i\partial_z u + \frac{\beta(z)}{2}(\nabla_\perp u + \partial_t^2 u) + \chi(z)|u|^{2m}u + [v(z,r) + iw(z,r)]u = 0 \quad (1)$$

with the power-law nonlinearity and a PT-symmetric potential. Here, $\nabla_\perp = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the transverse Laplacian, $r \equiv (x, y, t)$ is the position vector, and $u(z, r)$ is the complex envelope of the electric field, normalized with $(k_0 w_0)^{-1}(n_2/n_0)^{-1/2}$. The longitudinal z, transverse x, y coordinates, and the comoving time t are, respectively, scaled by the diffraction length $L_D \equiv k_0 w_0^2$ (with the wave number $k_0 \equiv 2\pi n_0/\lambda$), the typical input spatial width w_0, and the temporal pulse width. Functions $\beta(z)$ and $\chi(z)$ denote the diffraction/dispersion (DD) and the nonlinearity coefficients, respectively. An even function $v(z, r) \equiv k_0^2 w_0^2 n_R(z, r)$ and an odd function $w(z, r) \equiv k_0^2 w_0^2 n_I(z, r)$, to be specified subsequently, are the real and imaginary components of the complex PT-symmetric potential, corresponding to the index-guiding and the gain or loss distribution of the optical potential.

To obtain exact analytical solutions of Eq. (1), we introduce a self-similar transformation of the solution sought [16,17]:

$$u(z,r) = A(z)U[X(z,x), Y(z,y), T(z,t), Z(z)]e^{i\varphi(z,r)}. \quad (2)$$

where $A(z)$ is the amplitude; $X = X(x, z)$, $Y = Y(z, y)$, and $T = T(z, t)$ are the formal self-similar variables; $Z = Z(z)$ is the effective propagation distance; and $\varphi(z, r)$ is the phase of the wave, all assumed to be real functions. Substituting Eq. (2) into Eq. (1), one obtains the following standard NLSE with constant nonlinearity coefficient χ_0:

$$i\frac{\partial U}{\partial Z} + \frac{1}{2}\left[\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} + \frac{\partial^2 U}{\partial T^2}\right] + \chi_0|U|^{2m}U + [V(X,Y,T) + iW(X,Y,T)]U = 0, \quad (3)$$

with the requirements that:

$$\chi_0 = \frac{A_0^{2m}\chi(z)}{\beta(z)[1 - s_0\int\beta(z)\mathrm{d}z]^{3m-2}}, \quad (4a)$$

$$V(X,Y,T) = \frac{[1 - s_0\int\beta(z)\mathrm{d}z]^2}{\beta(z)}v(z,r), \quad (4b)$$

$$W(X,Y,T) = \frac{[1 - s_0\int\beta(z)\mathrm{d}z]^2}{\beta(z)}w(z,r), \quad (4c)$$

where A_0 and s_0 are arbitrary real constants. The requirement that the new nonlinearity coefficient χ_0 is constant enforces a relation between the nonlinearity and the DD coefficient, expressed by Eq. (4a). Thus, in order for the method of solution to be valid and the prescribed LB solutions obtained, an integrability condition on the method must be imposed, in the form of Eq. (4a). After some algebra, the simplest particular solutions are obtained:

$$A = A_0[1 - s_0\int\beta(z)\mathrm{d}z]^{3/2},$$

$$X(z,x) = \frac{x}{1 - s_0\int\beta(z)\mathrm{d}z}, \quad (5a)$$

$$Y(z,y) = \frac{y}{1 - s_0\int\beta(z)\mathrm{d}z},$$

$$T(z,t) = \frac{t}{1 - s_0\int\beta(z)\mathrm{d}z}, \quad (5b)$$

$$Z(z) = \frac{\int\beta(z)\mathrm{d}z}{1 - s_0\int\beta(z)\mathrm{d}z},$$

$$\varphi(z,r) = -\frac{s_0\left(x^2 + y^2 + z^2\right)}{2[1 - s_0\int\beta(z)\mathrm{d}z]}. \quad (5c)$$

Solutions of Eq. (3) can be considered as seeds which generate various solutions of Eq. (1) via relations (4) under conditions (5). Therefore, if we substitute solutions of Eq. (3) into transformation (2), nonautonomous solitons of Eq. (1) can be obtained.

Here, we investigate the localized modes supported by a 3D PT-symmetric complex potential $V_{\mathrm{PT}}(X, Y, T)$ whose real and imaginary parts are given by:

$$V = V_0(X^2 + Y^2 + T^2) - V_1 \mathrm{e}^{-2a^2(X^2+Y^2+T^2)} + V_2(\mathrm{e}^{-2a^2X^2} + \mathrm{e}^{-2a^2Y^2} + \mathrm{e}^{-2a^2T^2}) \tag{6a}$$

and:

$$W = W_0(X\mathrm{e}^{-a^2X^2} + Y\mathrm{e}^{-a^2Y^2} + T\mathrm{e}^{-a^2T^2}), \tag{6b}$$

which satisfy the properties of PT-symmetry: $V(X, Y, T) = V(-X, -Y, -T)$ and $W(X, Y, T) = -W(-X, -Y, -T)$.

We seek a solution of 3D NLSE (3) in the form:

$$U(Z, X, Y, T) = \psi(X, Y, T)\mathrm{e}^{i\delta z + \theta(X,Y,T)}. \tag{7}$$

Here, the real-valued functions of phase $\theta(X, Y, T)$ and amplitude $\psi(X, Y, T)$ satisfy the following differential equations:

$$\nabla^2\psi - |\nabla\theta|^2\psi + V(r)\psi + \chi_0\psi^{2m+1} = \delta\psi, \tag{8a}$$

$$\psi\nabla^2\theta + 2\nabla\theta\cdot\nabla\psi + W(r)\psi = 0. \tag{8b}$$

For potential (6), the above equations possess closed-form localized solutions that satisfy $\psi(X, Y, T) \to 0$ when $(X, Y, T) \to \pm\infty$. Thus, for the amplitude, we obtain:

$$\psi(X, Y, T) = \left|\frac{V_1}{\chi_0}\right|^{\frac{1}{2m}} \mathrm{e}^{-\frac{a^2(X^2+Y^2+T^2)}{m}} \tag{9a}$$

while the phase $\theta(X, Y, T)$ is given by:

$$\theta(X, Y, T) = \frac{mW_0\sqrt{\pi}}{4a^3(m+2)}[\mathrm{Erf}(aX) + \mathrm{Erf}(aY) + \mathrm{Erf}(aT)]|, \tag{9b}$$

where $\mathrm{Erf}(X, Y, Z)$ is the error function, $V_0 = -4a^4/m^2$, $V_2 = -m^2W_0^2/4a^2(m+2)^2$, and $\delta = -4a^4/m$.

From expression (2), the components of the complex PT-potential are given as:

$$v = \frac{\beta(z)}{[1 - s_0\int\beta(z)]^2}[V_0(X^2 + Y^2 + T^2) - V_1\mathrm{e}^{-2a^2(X^2+Y^2+T^2)} + V_2(\mathrm{e}^{-2a^2X^2} + \mathrm{e}^{-2a^2Y^2} + \mathrm{e}^{-2a^2T^2})] \tag{10a}$$

and

$$w = \frac{W_0\beta(z)}{[1 - s_0\int\beta(z)\mathrm{d}z]^2}(X\mathrm{e}^{-a^2X^2} + Y\mathrm{e}^{-a^2Y^2} + T\mathrm{e}^{-a^2T^2}). \tag{10b}$$

The soliton solution of Eq. (1) is thus:

$$u(z, r) = \frac{A_0}{[1 - s_0\int\beta(z)\mathrm{d}z]^{3/2}}\left|\frac{V_1}{\chi_0}\right|^{\frac{1}{2m}} \mathrm{e}^{-\frac{a^2(X^2+Y^2+T^2)}{m} + i\theta(X,Y,T) + i\varphi(z,r) + i\delta z}, \tag{11}$$

where X, Y, T, and $\varphi(t, x, y, z)$ satisfy Eqs. (5a)–(5c), respectively. Here, the phase is made up of the phase $\theta(X, Y, T)$ in solution (9b) and of the chirped phase $\varphi(z, r)$, expressed by Eq. (5c).

3 The characteristic distributions of solitons

To illustrate the characteristics of the analytic solution (11), we present the corresponding system management schemes in a DD medium (DDM) with decreasing $\beta(z) = \beta_0\exp(-\omega z)$ [19] and in the periodically modulated medium (PMM), $\beta(z) = \beta_0\cos(z)$ [20], the choice of which leads to the controlled development of nonautonomous waves.

From Eqs. (10a) and (10b), one finds that the complex PT-symmetric potential satisfies $v(x, y, t) = v(-x, -y, -t)$ and $w(x, y, t) = -w(-x, -y, -t)$. Thus, the index-guiding and the gain or loss distributions are even and odd functions with regard to x, y, and t. For the complex PT-symmetric potential, the even and odd properties of v and w are depicted in Fig. 1a, c and b, d, respectively.

Figure 2 presents the isosurface plots and the intensity distributions of LBs in the x–y plane for DDM, at different longitudinal distances. One can see that the LB exhibits a spherical distribution. Moreover, it is found that the LB profiles are self-similar and that the radii and the intensities of the pulse become slowly bigger as the propagation distance increases.

Figures 3a–c show the real part, the imaginary part, and the phase of the field distribution of the LB in the x–y plane, with $m = 1$. One sees that the real part

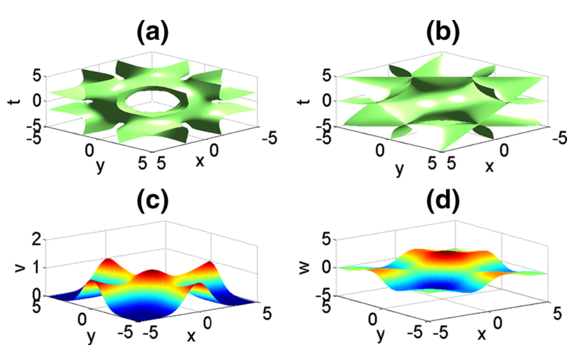


Fig. 1 Even function v and odd function w of the PT-symmetric potential, with **a**, **b** isosurface plots and **c**, **d** distributions in the x–y plane, at $z = 120$, $t = 1$. The remaining parameters are $\beta_0 = 0.2$, $W_0 = 0.1$, $V_1 = 2$, $\omega = 0.15$, and $s_0 = 0.4$

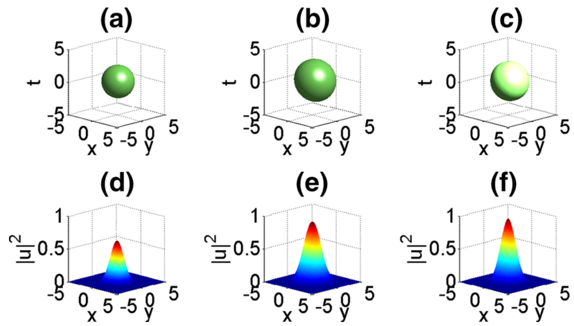


Fig. 2 Isosurface plots and intensity distributions of LBs in the x–y plane, for DDM, at different propagation distances: **a**, **d** $z = 10$, **b**, **e** $z = 100$, and **c**, **f** $z = 160$. The potential used is depicted in Fig. 1

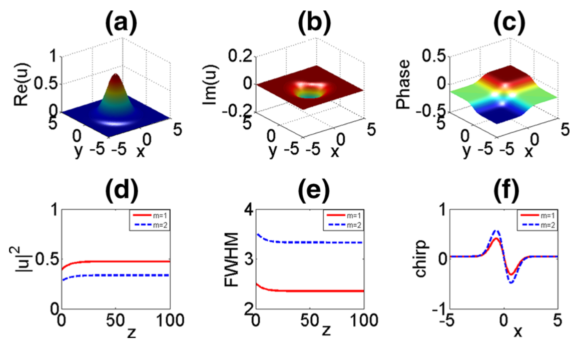


Fig. 3 **a**–**c** Real part, the imaginary part, and the phase of the field distribution of LBs in the x–y plane, with $m = 1$. **d**–**f** Intensity, half width, and the chirp of LBs, for two different values of m. Other parameters are as in Fig. 2

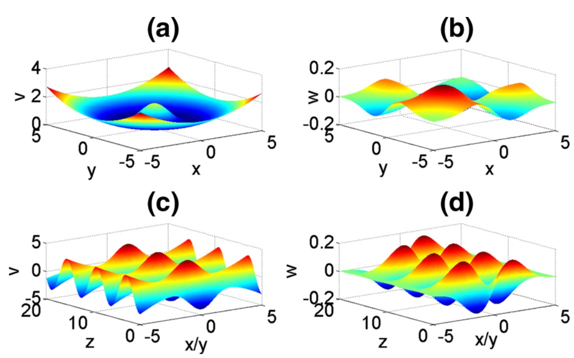


Fig. 4 **a**, **c** Even function v and **b**, **d** odd function w of the PT-symmetric potential in the PMM at **a**, **b** $z = 120$, $t = 1$ and **c**, **d** $t = 1$. The setup and other parameters are the same as in Fig. 1

of the field distribution is positive, while the imaginary part is negative and has a far lower magnitude than the real part. In Fig. 3c, an abrupt phase change is seen. The phase of the LB, as expressed in Eq. (11),

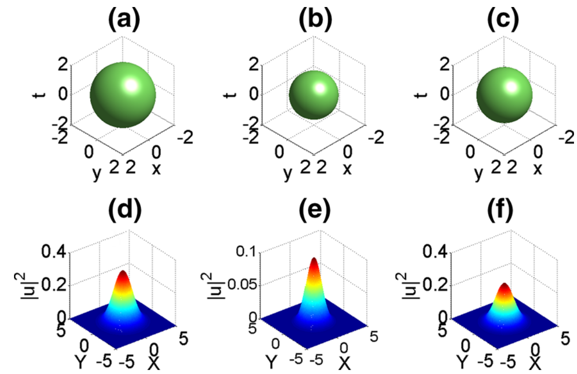


Fig. 5 Isosurface plots and the intensity distribution of LBs in x–y plane for PMM at different program distances **a**, **d** $z = 10$, **b**, **e** $z = 100$, and **c**, **f** $z = 160$. The PT-potential used is depicted in Fig. 4

is a result of the superposition of the original form, the abrupt phase change $\theta(z, r) + \delta z$ in solution (9b), and the parabolic shape $\varphi(z, r)$ in Eq. (5). Thus, the phase shows an abrupt gradient change on the parabolic background, which can be clearly seen from the top of the parabolic shape in Fig. 3c. In Fig. 3d–f, the intensity, half width, and the chirp of the LB with different m are displayed. We see that the intensity of the LB increases slightly until about $z = 15$ and afterward there is little change. On the other hand, the width decreases at first and then remains constant along the propagation distance. From Fig. 3f, it is apparent that the chirp of the LB displays an odd-symmetric property about the origin and at ± 0.75 achieves a maximum and a minimum value, respectively. Moreover, the larger the m, the smaller the intensity. However, in the case of width, the opposite holds.

Figure 4 exhibits a periodic structure of the complex PT-potential along the propagation distance z in PMM. Similar to Fig. 1, the even and odd properties of v and w are displayed in Fig. 4a, c and b, d. Along the propagation distance, the characteristics of the periodic oscillation change as shown in Fig. 4c, d.

Figure 5 presents the isosurface plot and the intensity distribution of LB in the x–y plane for PMM, at different propagation distances. Similar to Fig. 2, a spherical distribution is seen. It is shown that the LB exhibits a periodic propagation along the distance z. This property can be verified from Fig. 6d. Figures 6a–c describe the real part, the imaginary part, and the phase of the field distribution of the LB in the x–y plane, with $m = 1$. Similar to the previous case, one can see that the real

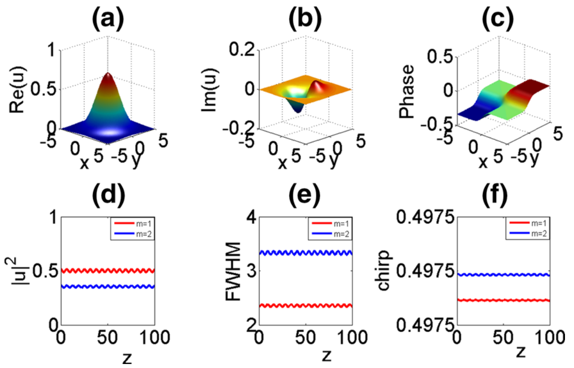


Fig. 6 **a**–**c** Real and imaginary parts, and the phase of field distributions of LBs in x–y plane, with $m = 1$. **d**–**f** Intensity, half width, and the chirp of LBs with two different values of m. The parameters are the same as in Fig. 2

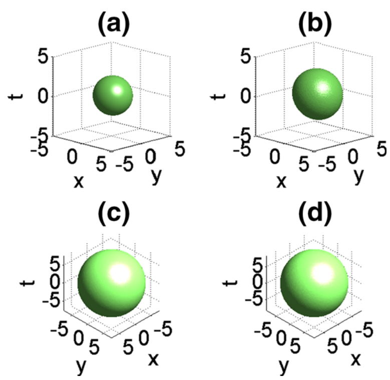


Fig. 7 Numerical simulation of LBs for DDM (**a**, **b**) and PMM (**c**, **d**) at different distances **a**, **c** $z = 100$, and **b**, **d** $z = 160$. A 5 % white noise is added to the initial field. The parameters are the same as in the analytical plots

part of the field distribution is even and that the imaginary is close to being odd and is far less in magnitude than the real part. In the same way, the abrupt phase transition is shown in Fig. 6c. When the DD parameter $\beta(z)$ is a cosine function, the periodic structure of the soliton intensity, width, and the chirp is clearly seen along the propagation direction in Fig. 5d, e. Similarly to Fig. 3, it can be seen that with an increase in m, the width and the chirp increase, while the intensity decreases.

Figure 7 shows the direct numerical integration of Eq. (1) for DDM (Fig. 7a, b) and PMM (Fig. 7a, b) at different propagation distances. We use a 3D split-step FFT beam propagation technique and consider an initial field whose form is given by Eq. (11) at $z = 0$. It is seen that the numerical calculations indicate no collapse, and stable propagation over tens of diffraction lengths is observed, except for some small oscillations. Moreover, the LB is more stable in the PMM than in the DDM. Thus, based on these results, there is strong indication that the dispersion/diffraction management of the type considered can prolong the life of LBs significantly.

4 Conclusion

In summary, we discovered LBs supported by a parity-time symmetric potential with a power-law nonlinearity. We established ways in which intensity, width, phase, and the chirp of these LBs can be modified by the variation in the diffraction/dispersion and the nonlinearity coefficients. Our results indicate that the behavior of 3D LBs in PT-lattice is considerably different from the behavior of the dissipative and the ground state solitons in the 3D case.

Acknowledgments This work is supported in China by the Natural Science Foundation of Hubei Province in China (Grant No. 2013CFB38), the Project in Hubei Province Department of Education, under Grant Q20142805, in China. Work in Qatar is supported by the NPRP 6-021-1-005 project with the Qatar National Research Fund, and additionally in China by the Natural Science Foundation of Guangdong Province, under Grant No. 1015283001000000. The work in Serbia is supported by the Serbian Ministry of Education and Science under project OI 171006. MRB acknowledges support by the Al Sraiya Holding Group.

References

1. Christodoulides, D.N., Lederer, F., Silberberg, Y.: Discretizing light behaviour in linear and nonlinear waveguide lattices. Nature **424**, 817–823 (2003)
2. Lederer, F., Stegeman, G.I., Christodoulides, D.N., Assanto, G., Segev, M., Silberberg, Y.: Discrete solitons in optics. Phys. Rep. **463**, 1–126 (2008)
3. Zhu, H.P.: Nonlinear tunneling for controllable rogue waves in two dimensional graded-index waveguides. Nonlinear Dyn. **72**, 873–882 (2013)
4. Batchelor, G.K.: An Introduction to Fluid Dynamics. Cambridge University Press, Cambridge (2000)
5. Khaykovich, L., Schreck, F., Ferrari, G., Bourdel, T., Cubizolles, J., Carr, L.D., Castin, Y., Salomon, C.: Formation of a matter-wave bright soliton. Science **296**, 1290–1293 (2002)
6. Abdullaev, FKh, Kartashov, Y.V., Konotop, V.V., Zezyulin, D.A.: Solitons in PT-symmetric nonlinear lattice, solitons in PT-symmetric nonlinear lattices. Phys. Rev. A **83**, 041805(R) (2011)

7. Alexeeva, N.V., Barashenkov, I.V., Sukhorukov, A.A., Kivshar, Y.S.: Optical solitons in PT-symmetric nonlinear couplers with gain and loss. Phys. Rev. A **85**, 063837 (2012)
8. Mayteevarunyoo, T., Malomed, B.A., Reoksabutr, A.: Solvable model for solitons pinned to a parity-time-symmetric dipole. Phys. Rev. E **88**, 022919 (2013)
9. Midya, B., Roychoudhury, R.: Nonlinear localized modes in PT-symmetric Rosen–Morse potential wells. Phys. Rev. A **87**, 045803 (2013)
10. Midya, B., Roychoudhury, R.: Nonlinear localized modes in PT-symmetric optical media with competing gain and loss. Ann. Phys. **341**, 12 (2014)
11. Bender, C.M., Boettcher, S.: Real spectra in non-hermitian Hamiltonians having PT-symmetry. Phys. Rev. Lett. **80**, 5243–5246 (1998)
12. Belyaeva, T.L., Serkin, V.N., Agüero, M.A., Hernandez-Tenorio, C., Kovachev, L.M.: Hidden features of the soliton adaptation law to external potentials: optical and matterwave 3D nonautonomous soliton bullets. Laser Phys. **21**, 258–263 (2011)
13. Xu, S.L., Petrović, N., Belić, M.R.: Exact solutions of the $(2 + 1)$-dimensional quintic nonlinear Schrödinger equation with variable coefficients. Nonlinear Dyn. **80**, 583–589 (2015)
14. Zhong, W.P., Belić, M.R., Huang, T.W.: Two-dimensional accessible solitons in PT-symmetric potentials. Nonlinear Dyn. **70**, 2027–2034 (2012)
15. Dai, C.Q., Wang, Y.Y., Zhang, J.F.: Analytical spatiotemporal localizations for the generalized $(3 + 1)$-dimensional nonlinear Schrodinger equation. Opt. Lett. **35**, 1437–1439 (2010)
16. Baizakov, B.B., Filatrella, G., Malomed, B.A., Salerno, M.: Double parametric resonance for matter-wave solitons in a time-modulated trap. Phys. Rev. E **71**, 036619 (2005)
17. Chen, S.H., Dudley, J.M.: Spatiotemporal nonlinear optical self-similarity in three dimensions. Phys. Rev. Lett. **102**, 233903 (2009)
18. Dai, C.Q., Wang, X.G., Zhou, G.Q.: Stable light-bullet solutions in the harmonic and parity-time-symmetric potentials. Phys. Rev. A **89**, 013834 (2014)
19. Soljačić, M., Segev, M., Menyuk, C.R.: Self-similarity and fractals in soliton-supporting systems. Phys. Rev. E **61**, R1048–R1051 (2000)
20. Serkin, V.N., Hasegawa, A., Belyaeva, T.L.: Solitary waves in nonautonomous nonlinear and dispersive systems: nonautonomous solitons. J. Mod. Opt. **57**, 1456–1472 (2010)

Nonlinear Dyn (2018) 93:2389–2397
https://doi.org/10.1007/s11071-018-4331-x

ORIGINAL PAPER

Spatiotemporal traveling and solitary wave solutions to the generalized nonlinear Schrödinger equation with single- and dual-power law nonlinearity

Nikola Z. Petrović

Received: 23 November 2017 / Accepted: 28 April 2018 / Published online: 21 May 2018

Abstract We generalize previously obtained solutions to the generalized nonlinear Schrödinger equation (NLSE) with cubic-quintic nonlinearity and distributed coefficients to obtain spatiotemporal traveling and solitary wave solutions for the NLSE with a general p-$2p$ dual-power law nonlinearity, where p is an arbitrary positive real number (the cubic-quintic model being a special case for $p = 2$). In addition, it is possible to eliminate the lower exponent, producing spatiotemporal traveling and solitary wave solutions to the NLSE with a single power law nonlinearity of arbitrary positive real power, which models many important systems including superfluid Fermi gas.

Keywords Nonlinear · Schrödinger · Cubic-quintic · Dual-power

1 Introduction

The generalized nonlinear Schrödinger equation (NLSE) is a generic model that is very important in nonlinear optics since it describes the propagation of a wave in a medium whose index of refraction is dependent on the intensity of light under the paraxial approximation [1–4]. Recently, there has been a huge development in obtaining stable spatiotemporal soliton solutions for a higher number of transverse dimensions [4,5] using the Jacobi elliptic function (JEF) expansion method and the principle of harmonic balance [6,7] and the stability of these solutions was analyzed in [8]. Recent work [9,10] has allowed generalization of these methods to cubic-quintic and other higher-order nonlinearities. In particular, the transformation in [10] has the potential to reduce many problems with non-integer exponents into problems with integer exponents and thus render a large class of equations applicable to various standard techniques.

The cubic-quintic nonlinear Schrödinger equation (CQNLSE) is a special case of the NLSE with dual-power law nonlinearity for $p = 2$. It is particularly important as the competing nonlinearities in many situations increase the likelihood of finding stable solutions to the NLSE [11]. Various exact solutions to the CQNLSE have been found [12,13]. The most common approach is using the self-similar method [14–16] which involves reducing an CQNLSE with distributed coefficients to one with constant coefficients. Other approaches are possible, such as assuming an ansatz solution [17,18].

Power law and dual-power law nonlinearities have been studied for various systems of nonlinear differential equations, such as the KdV equation [19], the dispersive NLSE [20] and the Biswas–Milović equation [21]. The NLSE with a dual-power law nonlinearity has been extensively studied [22] and solitary wave solutions have been found based on the hyper-

N. Z. Petrović (✉)
Institute of Physics, University of Belgrade, P.O. Box 68, Belgrade 11080, Serbia
e-mail: nzpetr@ipb.ac.rs

bolic secant [23]. The interaction of these waves has been studied in [24]. These solutions were analyzed using the Vaikhitov–Kolokolov criterium in Ref. [22] and in many circumstances were found to be stable. Chirped solitons were found in [25]. The G'/G-expansion method was used in [26] to find topological solitons, and a wide range of differing forms for the solutions were obtained in [27] using the modified simple equation method. Finally, solutions with a single power law nonlinearity have been found in [28,29] and solutions of all forms were found in [30].

These solutions found so far, however, are only a relatively small class of possible solutions and in most cases only contain a linear phase dependence. The goal of this paper is to extend the range of solutions found in [23–30] using the methods developed in [6,10].

2 Method

In this paper, we generalize the work done in [10] to find new spatiotemporal traveling wave solutions to the NLSE with the dual-power law (DPL) nonlinearities (hereafter abbreviated as (DPLNLSE). We consider the standard form of the (1+1)-dimensional ((1+1)D) DPLNLSE [31]:

$$iu_z + \frac{\beta(z)}{2}u_{xx} + \chi_1(z)|u|^p u + \chi_2(z)|u|^{2p}u = i\gamma(z)u, \tag{1}$$

which describes evolution of a slowly varying wavepacket envelope $u(x, y, z, t)$ in a diffractive nonlinear Kerr medium with anomalous dispersion, in the paraxial approximation. Here, z is the propagation coordinate, x the transverse coordinate, and t is the reduced time, i.e., time in the frame of reference moving with the wavepacket. All coordinates are made dimensionless by the choice of coefficients and the indices represent partial derivatives. The functions β and γ stand for the diffraction/dispersion and gain coefficients, respectively. The functions χ_1 and χ_2 represent the nonlinearity coefficients corresponding to the $(p+1)$-th and $(2p+1)$-th order of u, respectively. The special case $p = 2$ corresponds to the cubic-quintic system and, as mentioned before, was extensively covered in Ref. [10]. The conclusions in this paper can be easily generalized to a higher number of transverse variables [6].

Generalizing the results in [10], we define u as follows:

$$u(z, x) = v(z, x)^{1/p} \exp(iB(z, x)). \tag{2}$$

Inserting Eq. (2) into Eq. (1) and separating the real and imaginary part, we obtain the following coupled equations:

$$v_z + \beta v_x B_x + \frac{\beta p}{2} v B_{xx} - p\gamma v = 0, \tag{3}$$

$$p^2 v^2 B_z + \frac{\beta(p-1)}{2} v_x^2 - \frac{\beta p}{2} v v_{xx} + \frac{\beta p^2}{2} v^2 B_x^2 - p^2\chi_1 v^3 - p^2\chi_2 v^4 = 0. \tag{4}$$

We now assume the following form of the solutions:

$$v = f_1(z) + f_2(z)F(\theta) + f_3(z)F^{-1}(\theta), \tag{5}$$

$$\theta = k(z)x + \omega(z), \tag{6}$$

$$B = a(z)x^2 + b(z)x + e(z), \tag{7}$$

where f_i $(i = 1, 2, 3)$, k, ω, a, b and e are parameter functions to be determined. We will assume F to be the solution of the differential equation:

$$\left(\frac{\mathrm{d}F}{\mathrm{d}\theta}\right)^2 = c_0 + c_2F^2 + c_4F^4, \tag{8}$$

satisfied by the JEFs. Here, c_0, c_2 and c_4 are coefficients which depend on the parameter of the JEF M. The sets of all values of c_0, c_2 and c_4 for each of the functions F we used are given in [32].

Applying the F-expansion method and the principle of harmonic balance [5], we obtain the following system of algebraic and first-order differential equations for f_i $(i = 1, 2, 3)$, a, b, k and ω:

$$f_{iz} + a\beta f_i - p\gamma f_i = 0, \quad i = 1, 2, 3, \tag{9}$$

$$k_z + 2ka\beta = 0, \tag{10}$$

$$a_z + 2\beta a^2 = 0, \tag{11}$$

$$b_z + 2\beta ab = 0, \tag{12}$$

$$\omega_z + \beta kb = 0. \tag{13}$$

For χ_1 and χ_2, we obtain four equations:

$$f_2\left(\beta k^2 c_4(p+1) + 2\chi_2 f_2^2 p^2\right) = 0, \tag{14}$$

$$f_3\left(\beta k^2 c_0(p+1) + 2\chi_2 f_3^2 p^2\right) = 0, \tag{15}$$

$$f_2\left(\beta k^2 c_4 f_1 + \chi_1 f_2^2 p + 4\chi_2 f_1 f_2^2 p\right) = 0, \tag{16}$$

$$f_3\left(\beta k^2 c_0 f_1 + \chi_1 f_3^2 p + 4\chi_2 f_1 f_3^2 p\right) = 0. \tag{17}$$

For the equation for e, we obtain three equations instead of one:

$$e_z f_1 p^2 - \frac{1}{4}\beta c_2 f_1 k^2 p + \frac{1}{2}\beta b^2 f_1 p^2 - \frac{3}{2}\chi_1 f_1^2 p^2 - 2\chi_2 f_1^3 p^2 - \frac{3}{2}\chi_1 f_2 f_3 p^2 - 6\chi_2 f_1 f_2 f_3 p^2 = 0, \tag{18}$$

$$f_2\left(e_z f_2 p^2 - \beta c_4 f_3 k^2(2p-1) - \frac{1}{2}\beta c_2 f_2 k^2 + \frac{1}{2}\beta b^2 f_2 p^2 - 3\chi_1 f_1 f_2 p^2 - 6\chi_2 f_1^2 f_2 p^2 - 4\chi_2 f_3 f_2^2 p^2\right) = 0, \tag{19}$$

$$f_3\left(e_z f_3 p^2 - \beta c_0 f_2 k^2(2p-1) - \frac{1}{2}\beta c_2 f_3 k^2 + \frac{1}{2}\beta b^2 f_3 p^2 - 3\chi_1 f_1 f_3 p^2 - 6\chi_2 f_1^2 f_3 p^2 - 4\chi_2 f_2 f_3^2 p^2\right) = 0. \tag{20}$$

We will require two matching conditions imposed on functions f_i $(i = 1, 2, 3)$ for all three equations to be satisfied simultaneously.

3 Results

Equations (9)–(13) are solved as in [6] to obtain:

$$f_i(z) = \alpha^{1/2} f_{i0} \exp\left(p\int_0^z \gamma \mathrm{d}z\right), \tag{21}$$

$$k(z) = \alpha k_0, \tag{22}$$

$$\omega(z) = \omega_0 - \alpha k_0 b_0 \int_0^z \beta \mathrm{d}z, \tag{23}$$

$$a(z) = \alpha a_0, \tag{24}$$

$$b(z) = \alpha b_0, \tag{25}$$

where

$$\alpha = 1/\left(1 + 2a_0 \int_0^z \beta \mathrm{d}z\right) \tag{26}$$

is the chirp function. From solving Eqs. (14)–(20), we obtain three distinct cases.

3.1 Case I: $p \neq 2$

We first assume that $p \neq 2$ and $\chi_1 \neq 0$. The matching conditions between Eqs. (18) and (19), and between Eqs. (19) and (20), respectively, give the following formulas:

$$f_1 = f_2\sqrt{3\epsilon\sqrt{\frac{c_0}{c_4}} + \frac{c_2}{2c_4}}, \tag{27}$$

$$f_3 = f_2 \epsilon \sqrt{\frac{c_0}{c_4}}, \tag{28}$$

where $\epsilon = \pm 1$. Equation (28) is also applied to the other pairs for Eqs. (14), (15) and (16), (17). One of the three differential equations for e is solved to obtain:

$$e(z) = e_0 - \alpha\left(\frac{b_0^2}{2} - \frac{k_0^2}{p^2}\left(6\epsilon\sqrt{c_0 c_4} + \frac{c_2}{2}(p+2)\right)\right) \int_0^z \beta \mathrm{d}z, \tag{29}$$

Equations (14)–(17) are solved to obtain:

$$\chi_1 = \frac{\beta c_4 k^2 (p+2)}{f_2 p^2}\sqrt{3\epsilon\sqrt{\frac{c_0}{c_4}} + \frac{c_2}{2c_4}}, \tag{30}$$

$$\chi_2 = -\frac{\beta c_4 k^2 (p+1)}{2 f_2^2 p^2}. \tag{31}$$

3.2 Case II: $p = 2$

We now assume that $p = 2$ and $\chi_1 \neq 0$, which corresponds to the cubic-quintic (CQ) nonlinearity.

Using the ansatz in this paper, the case for $p = 2$ was first covered in [10], albeit incompletely. The key difference in comparison with Case I is that it turns out that for specifically $p = 2$ the matching condition between Eqs. (18) and (19) is automatically satisfied. Hence, f_1 is independent from f_2 and f_3. In [10], the authors suggest a relationship between f_1 and f_2; however, it turns out to not be necessary. The matching condition between f_3 and f_2 is the same as in Case I:

$$f_3 = f_2 \epsilon \sqrt{\frac{c_0}{c_4}}, \tag{32}$$

where $\epsilon = \pm 1,\ 0$. Unlike in Case I, $\epsilon = 0$, i.e., $f_3 = 0$, is admissible. An equation for e is solved to obtain:

$$e(z) = e_0 - \alpha\left(\frac{b_0^2}{2} - \frac{k_0^2}{4}\left(3c_4\frac{f_{10}^2}{f_{20}^2} - 3\epsilon\sqrt{c_0 c_4} + \frac{c_2}{2}\right)\right) \int_0^z \beta \mathrm{d}z. \tag{33}$$

Equations (14)–(17) are solved to obtain:

$$\chi_1 = \frac{\beta c_4 k^2 f_1}{f_2^2}, \tag{34}$$

$$\chi_2 = -\frac{3\beta c_4 k^2}{8 f_2^2}. \tag{35}$$

3.3 Case III: $\chi_1 = 0$

In the third case, we will assume that $\chi_1 = 0$. This corresponds to the Z law (SPL) nonlinearity of fractional degree $2p$. From Eq. (16), it follows that $f_1 = 0$. Equation (18) is thus trivially satisfied. As in Case II, the matching condition gives:

$$f_3 = f_2 \epsilon \sqrt{\frac{c_0}{c_4}}, \tag{36}$$

where $\epsilon = \pm 1, 0$. The equation for e is solved to obtain:

$$e(z) = e_0 - \alpha \left(\frac{b_0^2}{2} - \frac{k_0^2}{p^2} \left(\frac{c_2}{2} - 3\epsilon \sqrt{c_0 c_4} \right) \right) \int_0^z \beta \mathrm{d}z. \tag{37}$$

Finally, from Eq. (14), we have:

$$\chi_2 = -\frac{\beta c_4 k^2 (p+1)}{2 f_2^2 p^2}. \tag{38}$$

For $p = 2$, we obtain the quintic NLSE [33]. For $p = \frac{2}{3}$, we obtain the equation for the superfluid Fermi gas in the crossover from the weak-coupling Bardeen–Cooper–Schrieffer (BCS) regime [34]. All of the results in Case III reduce to those obtained in [6] for the special case $p = 1$.

4 Solutions

In this section, we present the exact analytic solutions for the DPLNLSE (and SPLNLSE) obtained for all three cases. The solutions were plotted using the program Mathematica. We present only the qualitatively most distinct cases and those that do not have any singularities, being that a large number of functions satisfying Eq. (8) can be used for each case, as can be seen in Appendix of Ref. [32].

We note that there is a large degree of control of the waveforms when it comes to scaling and propagating behaviors. The parameters k_0 and b_0 determine the width of the waveforms, while ω_0 determines the off-set. A constant γ introduces exponential growth or decay, while sinusoidal forms of γ introduce oscillations in amplitude. The parameter a_0 introduces chirp into the system. In general, non-chirped solutions are periodic along the transverse variable and exhibit a constant maximum amplitude as a function of the longitudinal variable, while chirped solutions exhibit a stretching effect and the maximum amplitude is modulated.

4.1 Case I: $p \neq 2$

For Case I, Eqs. (27), (28) give us the following conditions:

$$3\epsilon \sqrt{\frac{c_0}{c_4}} + \frac{c_2}{2c_4} > 0, \tag{39}$$

$$\frac{c_0}{c_4} > 0. \tag{40}$$

An additional constraint is that the JEF used must not have a value of 0 since its inverse will then be infinite. As long as these conditions are satisfied, the JEF is usable for the solution. We find that an appropriate choice is $F = \mathrm{dn}(\cdot|M)$ or $F = \mathrm{nd}(\cdot|M)$, where $M < 0.971$. Thus, for $F = \mathrm{dn}$ and also $F = \mathrm{nd}$ only traveling wave solutions are admissible. As of present, no form of the function has been found which matches all conditions for $M = 1$ and is non-singular. Solutions for $\epsilon = -1$ are more difficult to find, since the whole expression for v must remain positive when p is not an odd integer.

We see in Fig. 1 that the chirp function modulates the intensity of the waveform and also periodically compresses and expands it in the transverse direction. The solutions can also be found for negative p and are displayed in Fig. 2. In Fig. 2c, we see that the chirp now compresses and expands oppositely with respect to Fig. 1c.

In [23,24], the solutions in this paper are only based on the sech function and the phase does not contain chirp. In [25], one only obtains constant chirp for the special case of the DPLNLSE. In [26], one has a large selection of functions, but again the phase is a linear function of the transverse variable. In [27], only a DPLNLSE with constant coefficients is analyzed. Thus, we can say that in comparison with the most

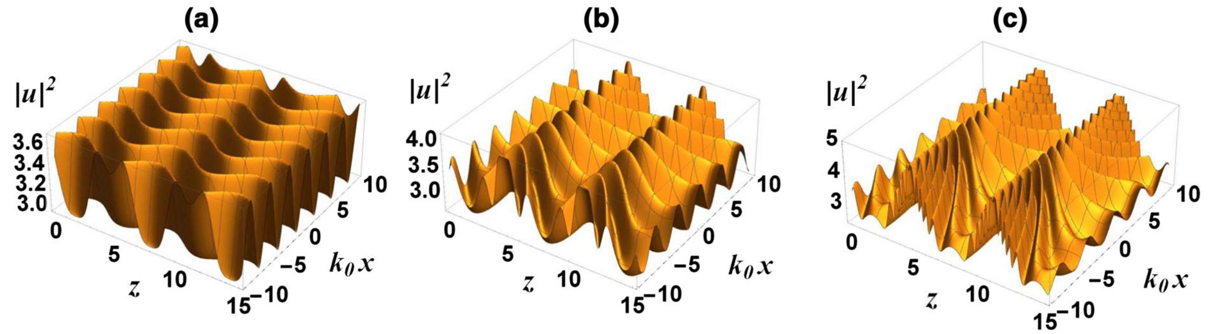


Fig. 1 (Color online) Traveling wave solutions for $F = \text{nd}$ as functions of time. Intensity $|u|^2$ is presented as a function of k_0x and z for $p = 3$, $\beta(z) = \beta_0 \cos \Omega z$ and: **a** $a_0 = 0$, **b** $a_0 = 0.15$ and **c** $a_0 = 0.3$. Other parameters are: $M = 0.97$, $b_0 = 1$, $e_0 = 0$, $k_0 = 1$, $\omega_0 = 0$, $\Omega = 1$, $\beta_0 = 1$ and $\epsilon = 1$

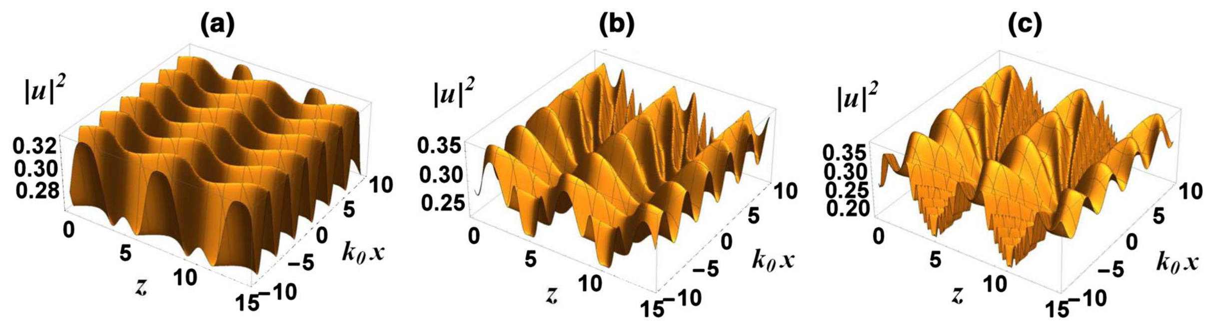


Fig. 2 (Color online) Traveling wave solutions for $F = \text{nd}$ as functions of time. Intensity $|u|^2$ is presented as a function of k_0x and z for $p = -3$, $\beta(z) = \beta_0 \cos \Omega z$ and: **a** $a_0 = 0$, **b** $a_0 = 0.15$ and **c** $a_0 = 0.3$. Other parameters are the same as in Fig. 1

recent papers that have found analytic solutions to the DPLNLSE, the solutions in this paper are novel.

4.2 Case II: $p = 2$

Case II allows for a much wider range of solutions, being that there is no longer a constraining relationship between f_1 and f_2. Also, most importantly, it is now possible to have $\epsilon = 0$. All of the solutions for Case I also apply to Case II for $p = 2$, plus many other forms, being that the condition (39) is no longer necessary. The big difference from Case I is that now we obtain solitary wave equations. The only thing to have in mind is that v must not be negative. We will show in this section only solutions that do not apply to Case I.

In Fig. 3, we see that we can also obtain solutions using $F = \text{sn}$ for Case II, by combining the appropriate value of f_{10} and f_{20}. We see that the waves become wider and flatter for increasing M until for $M = 1$ we effectively obtain a step function in Fig. 3c, i.e., a topological soliton. We can see how only the nonzero half is moderated by chirp in Fig. 3f.

In Fig. 4, we see the effect of f_{10} on the chirp in the case of a solitary wave for $F = \text{dn}$. By switching on f_{10}, we modulate the whole background in the presence of chirp as can be seen in Fig. 4b, c. Without the chirp, adding a nonzero value to the parameter f_{10} essentially amounts to adding a constant term to the background, without much qualitative change to the solitary wave.

Finally, in Fig. 5, we see solutions for a more complicated function chosen for F, namely $F = M\text{sd}+\text{nd}$, and their dependence on the choice of parameter ϵ. We find in Fig. 5b that with an appropriate choice of coefficients one can produce traveling wave solutions with waves of alternating height. We also present in Fig. 5c solutions for $\epsilon = -1$. The chirped solutions are shown in Fig. 5d–f.

We compare our results to those derived in other papers. In [14] solutions with chirp are present, but the

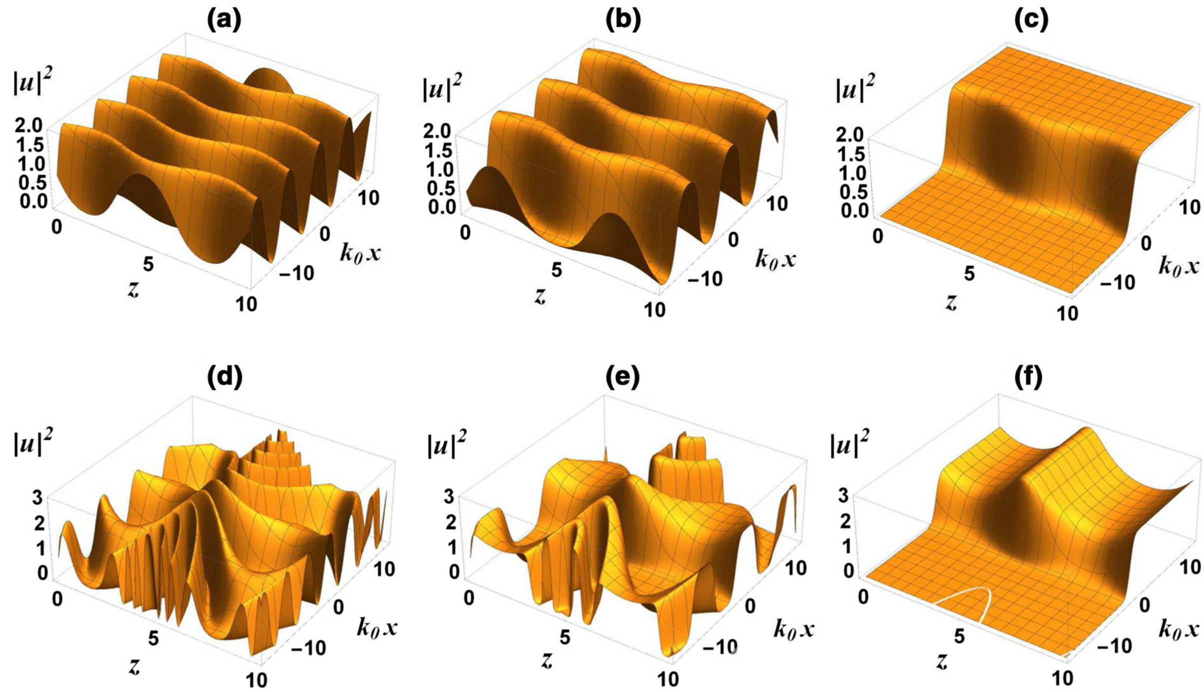


Fig. 3 (Color online) Traveling and solitary wave solutions for $F = \mathrm{sn}$ as functions of time. Intensity $|u|^2$ is presented as a function of k_0x and z for $p = 2$, $\beta(z) = \beta_0 \cos \Omega z$ and: **a** $a_0 = 0$, $M = 0.5$, **b** $a_0 = 0$, $M = 0.9$, **c** $a_0 = 0$, $M = 1$, **d** $a_0 = 0.3$, $M = 0.5$, **e** $a_0 = 0.3$, $M = 0.9$ and **f** $a_0 = 0.3$, $M = 1$. Other parameters are: $b_0 = 1$, $e_0 = 0$, $k_0 = 1$, $\omega_0 = 0$, $f_{10} = f_{20} = 1$, $\Omega = 1$, $\beta_0 = 1$ and $\epsilon = 0$

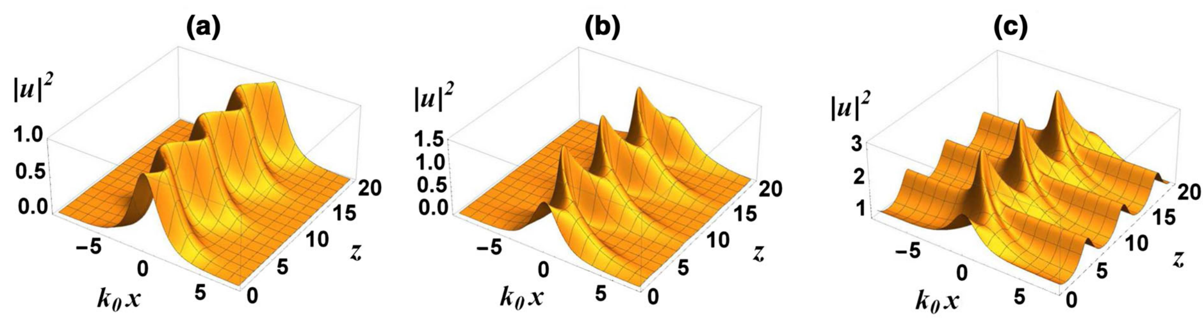


Fig. 4 (Color online) Solitary wave solutions for $F = \mathrm{dn}$ as functions of time. Intensity $|u|^2$ is presented as a function of k_0x and z for $p = 2$, $M = 1$, $\beta(z) = \beta_0 \cos \Omega z$ and: **a** $a_0 = 0$, $f_{10} = 0$, $f_{20} = 1$, **b** $a_0 = 0.3$, $f_{10} = 0$, $f_{20} = 1$ and **c** $a_0 = 0.3$, $f_{10} = f_{20} = 1$. Other parameters are the same as in Fig. 3

external potential and the fourth-order nonlinearity are dependent on all other parameters. One obtains solutions based on the few elementary JEFs and hyperbolic functions. Qualitatively, novel solutions in our paper are those in Figs. 4c and 5b, e. In [15], complicated solutions are obtained where all parameters depend on the values of two functions. The forms for the bright and dark solitons are fixed and one can obtain various forms of the chirp function, including sinusoidal. In [16], one obtains various novel solutions most of whom do not resemble the solutions in this paper. In [17], the interactions of the basic bright soliton solu-

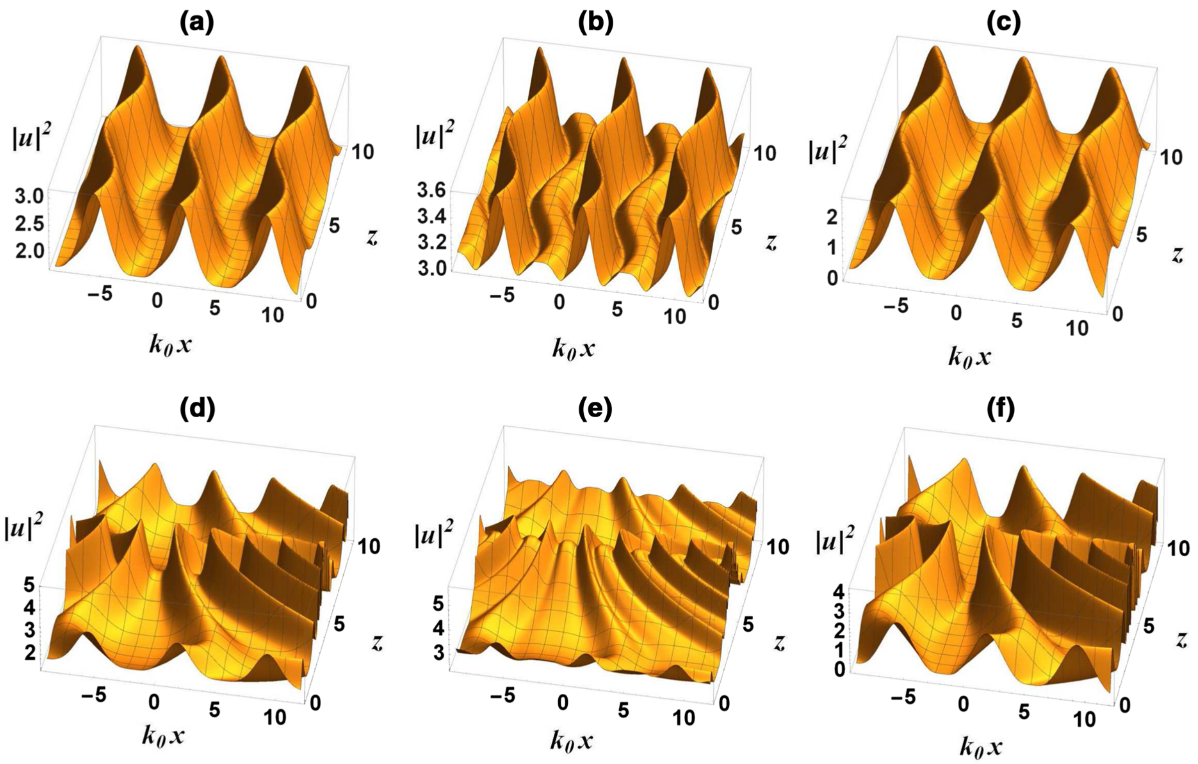


Fig. 5 (Color online) Traveling wave solutions for $F = M\mathrm{sd} + \mathrm{nd}$ as functions of time. Intensity $|u|^2$ is presented as a function of $k_0 x$ and z for $p = 2$, $M = 0.5$, $\beta(z) = \beta_0 \cos \Omega z$ and: **a** $a_0 = 0$, $\epsilon = 0$, **b** $a_0 = 0$, $\epsilon = 1$, **c** $a_0 = 0$, $\epsilon = -1$, **d** $a_0 = 0.3$, $\epsilon = 0$, **e** $a_0 = 0.3$, $\epsilon = 1$ and **f** $a_0 = 0.3$, $\epsilon = -1$. Other parameters are the same as in Fig. 3

tions are studied. Finally, in [18] the Ricatti equation method is used to obtain forms of the amplitude different from that in this paper. Again, we have found novel solutions to the CQNLSE.

4.3 Case III: $\chi_1 = 0$

In this case, all solutions that would normally be allowed for the NLSE in [6] are allowed, provided v is always positive when p is not an odd integer. These solutions for the SPLNLSE largely resemble those found in [6]. In Fig. 6, we see the solutions for dark solitary and traveling waves for $p = 3$. Taking the solutions to the root of p has a narrowing effect on the dark soliton which only gets more pronounced with the increase in p.

The solutions in [28] are only based on $\mathrm{sech}^{1/p}$ and contain no chirp. In [29,30], one obtains complicated solutions based on $\mathrm{sech}^{1/p}$ and $\mathrm{csch}^{1/p}$. These solutions coincide with the solutions in this paper for $F = \mathrm{sech}, \mathrm{csch}$. Still, one can choose other forms of F to obtain novel solutions.

5 Conclusion

In this paper, we demonstrated a large class of new solutions for the NLSE with dual- and fractional power law nonlinearities. The ansatz used allows for a large number of free parameters and thus gives us a very diverse profile of qualitative behavior. For a general dual-power law nonlinearity, we obtain traveling wave solutions, with and without chirp. For the cubic-quintic nonlinearity, we also obtain solitary wave solutions, without chirp, with chirp and with chirp and a modulated background. We also obtain traveling wave solutions where two sets of waves alternate in height, with and without chirp. For the single power law nonlinearity, when p is a positive odd integer we obtain equivalent forms to

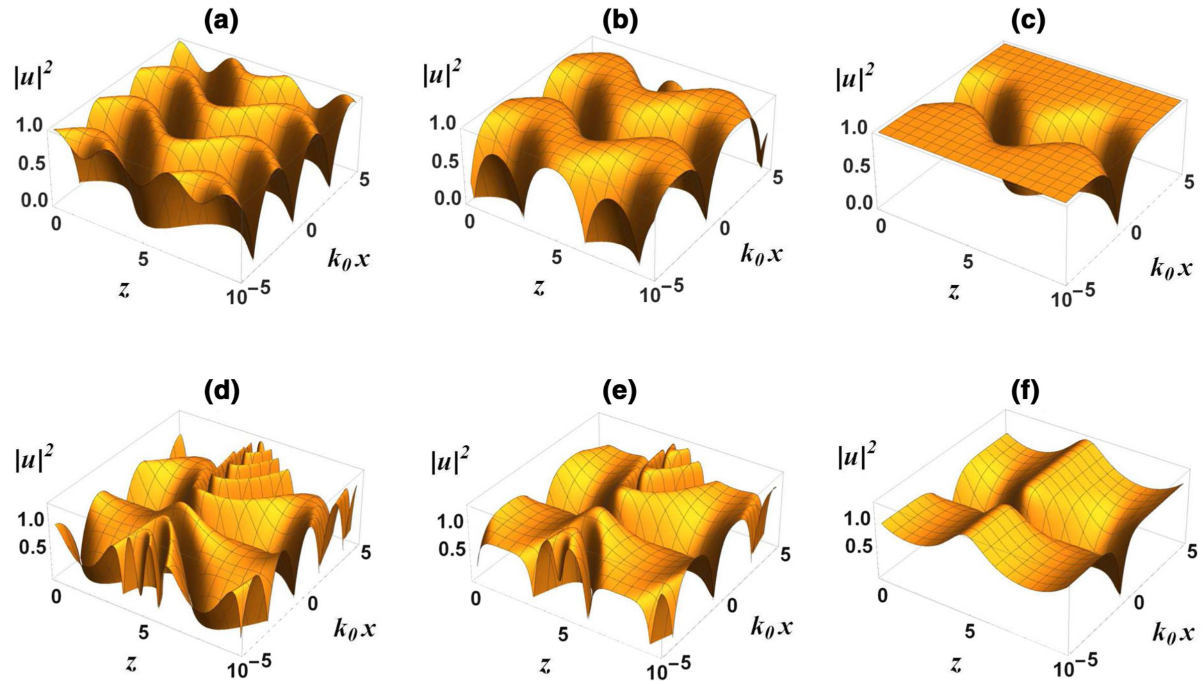


Fig. 6 (Color online) Traveling and solitary wave solutions for $F = \text{sn}$ and $\chi_1 = 0$ as functions of time. Intensity $|u|^2$ is presented as a function of k_0x and z for $p = 3$, $\epsilon = 0$, $\beta(z) = \beta_0 \cos \Omega z$ and: **a** $a_0 = 0$, $M = 0.25$, **b** $a_0 = 0$, $M = 0.9$, **c** $a_0 = 0$, $M = 1$, **d** $a_0 = 0.3$, $M = 0.25$, **e** $a_0 = 0.3$, $M = 0.9$ and **f** $a_0 = 0.3$, $M = 1$. Other parameters are the same as in Fig. 3

all solutions found in [6]. Otherwise, we obtain equivalent forms to all solutions found in [6] for which the parameter v is positive.

The solutions can also adapt to any function F derived from the Jacobi Elliptic functions satisfying Eq. (8), which is the main advantage of this method. The solutions allow a gradual transition from traveling waves to a solitary wave due to the limit of the JEFs as $M \to 1$. Finally, the chirp can be added to each of the solutions which has a profound effect on their form. These solutions have potential practical applications for systems which use a potential with a dual- or fractional power law, such as in nonlinear optics or in the study of superfluid Fermi gases.

Acknowledgements Work at the Institute of Physics is supported by Project OI 171006 of the Serbian Ministry of Education and Science.

Compliance with ethical standards

Conflict of interest The author declares that he has no conflict of interest.

References

1. Akhmediev, N., Ankiewicz, A.: Solitons. Chapman and Hall, London (1997)
2. Kivshar, Y., Agrawal, G.: Optical Solitons, from Fibers to Photonic Crystals. Academic, New York (2003)
3. Hasegawa, A., Matsumoto, M.: Optical Solitons in Fibers. Springer, New York (2003)
4. Malomed, B.: Soliton Management in Periodic Systems. Springer, New York (2006)
5. Zhong, W.P., et al.: Exact spatial soliton solutions of the two-dimensional generalized nonlinear Schrödinger equation with distributed coefficients. Phys. Rev. A **78**, 023821 (2008)
6. Belić, M., et al.: Analytical light bullet Solutions to the generalized (3+1)-dimensional nonlinear Schrödinger equation. Phys. Rev. Lett. **101**, 0123904 (2008)
7. Petrović, N., et al.: Exact spatiotemporal wave and soliton solutions to the generalized (3+1)-dimensional Schrödinger equation for both normal and anomalous dispersion. Opt. Lett. **34**, 1609 (2009)
8. Petrović, N., et al.: Modulation stability analysis of exact multidimensional solutions to the generalized nonlinear Schrödinger equation and the Gross-Pitaevskii equation using a variational approach. Opt. Exp. **23**, 10616 (2015)
9. Petrović, N., et al.: Exact traveling-wave and spatiotemporal soliton solutions to the generalized (3+1)-dimensional

Schrödinger equation with polynomial nonlinearity of arbitrary order. Phys. Rev. E **83**, 026604 (2011)
10. Hong-Yu, W., et al.: Self-similar solutions of variable-coefficient cubic-quintic nonlinear Schrdinger equation with an external potential. Commun. Theor. Phys. (Beijing, China) **54**, 55 (2010)
11. Towers, I., et al.: Stability of spinning ring solitons of the cubicquintic nonlinear Schrdinger equation. Phys. Lett. A **288**, 292 (2001)
12. Schürmann, H.W.: Traveling-wave solutions of the cubic-quintic nonlinear Schrdinger equation. Phys. Rev. E **54**, 4313 (1996)
13. Liu, X.B., et al.: Exact self-similar wave solutions for the generalized (3+1)-dimensional cubic-quintic nonlinear Schröinger [*sic*] equation with distributed coefficients. Opt. Commun. **285**, 779 (2012)
14. Dai, C., et al.: Chirped and chirp-free self-similar cnoidal and solitary wave solutions of the cubic-quintic nonlinear Schrödinger equation with distributed coefficients. Opt. Commun. **283**, 1489 (2010)
15. Belmonte-Beitia, J., Cuevas, J.: Solitons for the cubic-quintic nonlinear Schrödinger equation with time- and space-modulated coefficients. J. Phys. A. **42**, 165201 (2009)
16. He, J.R., Li, H.M.: Analytical solitary-wave solutions of the generalized nonautonomous cubic-quintic nonlinear Schrödinger equation with different external potentials. Phys. Rev. E **83**, 066607 (2011)
17. Hao, R., et al.: A new approach to exact soliton solutions and soliton interaction for the nonlinear Schrödinger equation with variable coefficients. Opt. Commun. **236**, 79 (2004)
18. Zhou, Q., et al.: Optical solitons in media with time-modulated nonlinearities and spatiotemporal dispersion. Nonlinear Dyn. **80**, 983 (2015)
19. Biswas, A.: Solitary wave solution for KdV equation with power-law nonlinearity and time-dependent coefficients. Nonlinear Dyn. **58**, 345 (2009)
20. Biswas, A., Khalique, C.M.: Stationary solutions for nonlinear dispersive Schrdingers equation. Nonlinear Dyn. **63**, 623 (2011)
21. Eslami, M., Mirzazadeh, M.: Optical solitons with Biswas-Milović equation for power law and dual-power law nonlinearities. Nonlinear Dyn. **83**, 731 (2016)
22. Micallef, R., et al.: Optical solitons with power-law asymptotics. Phys. Rev. E **54**, 2936 (1996)
23. Biswas, A.: 1-soliton solution of (1+2)-dimensional nonlinear Schrödinger equation in dual-power law media. Phys. Lett. A **372**, 5941 (2008)
24. Biswas, A.: Soliton-soliton interaction with dual-power law nonlinearity. Appl. Math. Comput. **198**, 605 (2008)
25. Bouzida, A., et al.: Chirped optical solitons in nano optical fibers with dual-power law nonlinearity. Optik **142**, 77 (2017)
26. Mirzazadeh, M., et al.: Topological solitons of resonant nonlinear Schödinger's equation with dual-power law nonlinearity by G/G-expansion technique. Optik **125**, 5480 (2014)
27. Ali, A., et al.: Soliton solutions of the nonlinear Schrödinger equation with the dual power law nonlinearity and resonant nonlinear Schrödinger equation and their modulation instability analysis. Optik **145**, 79 (2017)
28. Biswas, A.: Optical solitons with time-dependent dispertion, nonlinearity and attenuation in a power-law media. Commun. Nonlinear Sci. Numer. Simulat. **14**, 1078 (2009)
29. Wazwaz, A.: Reliable analysis for nonlinear Schrödinger equations with a cubic nonlinearity and a power law nonlinearity. Math. Comput. Model. **43**, 178 (2006)
30. Mirzazadeh, M., et al.: Soliton solutions to resonant nonlinear Schrödinger's equation with time-dependent coefficients by trial solution approach. Nonlinear Dyn. **81**, 277 (2015)
31. Malomed, B.A., et al.: Spatio-temporal optical solitons. J. Opt. B **7**, R53 (2005)
32. Koonprasert, S., Punpocha, M.: More exact solutions of Hirota–Ramani partial differential equations by applying F-Expansion method and symbolic computation. Glob. J. Pure Appl. Math. **12**(3), 1903 (2006)
33. Xu, S.L., et al.: Exact solutions of the (2+1)-dimensional quintic nonlinear Schrdinger equation with variable coefficients. Nonlinear Dyn. **80**, 583 (2015)
34. Adhikari, S.: Nonlinear Schrödinger equation for a superfluid Fermi gas in the BCS-BEC crossover. Phys. Rev. A **77**, 045602 (2008)

Modulation stability analysis of exact multidimensional solutions to the generalized nonlinear Schrödinger equation and the Gross-Pitaevskii equation using a variational approach

Nikola Z. Petrović,[1,*] **Najdan B. Aleksić,**[1] **and Milivoj Belić**[2]

[1] *Institute of Physics, P.O. Box 68, University of Belgrade, Serbia*
[2] *Texas A & M University at Qatar, P.O. Box 23874 Doha, Qatar*

[*] *nzpetr@ipb.ac.rs*

Abstract: We analyze the modulation stability of spatiotemporal solitary and traveling wave solutions to the multidimensional nonlinear Schrödinger equation and the Gross-Pitaevskii equation with variable coefficients that were obtained using Jacobi elliptic functions. For all the solutions we obtain either unconditional stability, or a conditional stability that can be furnished through the use of dispersion management.

OCIS codes: (190.6135) Spatial solitons; (000.3860) Mathematical methods in physics; (000.4430) Numerical approximation and analysis.

References and links

1. G. P. Agrawal, *Nonlinear Fiber Optics*, 4th ed. (Academic, 2007).
2. L. Ostrovskii, "Propagation of wave packets and space-time self focusing in a nonlinear medium," Sov. Phys. JETP **24**, 797–800 (1969).
3. Y. S. Kivshar and M. Peyrard, "Modulational instabilities in discrete lattices," Phys. Rev. A **46**, 3198–3205 (1992).
4. N. Akhmediev and A. Ankiewicz, *Solitons* (Chapman and Hall, 1997).
5. Y. S. Kivshar and G. Agrawal, *Optical Solitons*, *from Fibers to Photonic Crystals* (Academic, 2003).
6. A. Hasegava and M. Matsumoto, *Optical Solitons in Fibers* (Springer, 2003).
7. B. Malomed, *Soliton Management in Periodic Systems* (Springer US, 2006).
8. Z. Rapti, P. G. Kevrekidis, A. Smerzi, and A. R. Bishop, "Variational approach to the modulational instability," Phys. Rev. E **69**, 017601 (2004).
9. Y. S. Kivshar and M. Salerno, "Modulational instabilities in the discrete deformable nonlinear Schrodinger equation," Phys. Rev. E **49**, 3543–3546 (1994).
10. P. Shukla and J. Rasmussen, "Modulational instability of short pulses in long optical fibers," Opt. Lett. **11**, 171–173 (1986).
11. M. Saha and A. Sarma, "Modulation instability in nonlinear metamaterials induced by cubicquintic nonlinearities and higher order dispersive effects," Opt. Commun. **291**, 321–325 (2013).
12. H. Sakaguchi and Y. Kageyama, "Modulational instability and breathing motion in the two-dimensional nonlinear Schrodinger equation with a one-dimensional harmonic potential," Phys. Rev. E **88**, 053203 (2013).
13. M. Potasek, "Modulation instability in an extended nonlinear Schrdinger equation," Opt. Lett. **12**, 921–923 (1987).
14. M. Centurion, M. A. Porter, Y. Pu, P. G. Kevrekidis, D. J. Frantzeskakis, and D. Psaltis, "Modulational instability in a layered Kerr medium: theory and experiment," Phys. Rev. Lett. **97**, 234101 (2006).
15. S. Wen, Y. Wang, W. Su, Y. Xiang, X. Fu, and D. Fan, "Modulation instability in nonlinear negative-index material," Phys. Rev. E **73**, 036617 (2006).

#229339 - $15.00 USD Received 15 Dec 2014; revised 27 Mar 2015; accepted 28 Mar 2015; published 16 Apr 2015

16. S. Wen, Y. Xiang, W. Su, Y. Hu, X. Fu, and D. Fan, “Role of the anomalous self-steepening effect in modulation instability in negative-index material,” Opt. Express **14**, 1568–1575 (2006).
17. Y. Xiang, S. Wen, X. Dai, Z. Tang, W. Su, and D. Fan, “Modulation instability induced by nonlinear dispersion in nonlinear metamaterials,” J. Opt. Soc. Am. B **24**, 3058–3063 (2007).
18. Y. Xiang, X. Dai, S. Wen, and D. Fan, “Modulation instability in metamaterials with saturable nonlinearity,” J. Opt. Soc. Am. B **28**, 908–916 (2011).
19. X. Dai, Y. Xiang, S. Wen, and D. Fan, “Modulation instability of copropagating light beams in nonlinear metamaterials,” J. Opt. Soc. Am. B **26**, 564–571 (2009).
20. A. Sarma and M. Saha, “Modulational instability of coupled nonlinear field equations for pulse propagation in a negative index material embedded into a Kerr medium,” J. Opt. Soc. Am. B **28**, 944–948 (2011).
21. N. Lazarides and G. P. Tsironis, “Coupled nonlinear Schrödinger field equations for electromagnetic wave propagation in nonlinear left-handed materials,” Phys. Rev. E **71**, 036614 (2005).
22. I. Kourakis and P. K. Shukla, “Nonlinear propagation of electromagnetic waves in negative-refraction-index composite materials,” Phys. Rev. E **72**, 016626 (2005).
23. I. Kourakis, N. Lazarides, and G. P. Tsironis, “Self-focusing and envelope pulse generation in nonlinear magnetic metamaterials,” Phys. Rev. E **75**, 067601 (2007).
24. W. P. Zhong, R. H. Xie, M. Belić, N. Petrović, G. Chen, and L. Yi, “Exact spatial soliton solutions of the two-dimensional generalized nonlinear Schrdinger equation with distributed coefficients,” Phys. Rev. A **78**, 023821 (2008).
25. M. Belić, N. Petrović, W. P. Zhong, R. H. Xie, and G. Chen, “Analytical light bullet solutions to the generalized (3+ 1)-dimensional nonlinear Schrdinger equation,” Phys. Rev. Lett. **101**, 123904 (2008).
26. N. Petrović, M. Belić, W. Zhong, R. Xie, and G. Chen, “Exact spatiotemporal wave and soliton solutions to the generalized (3+ 1)-dimensional Schrdinger equation for both normal and anomalous dispersion,” Opt. Lett. **34**, 1609–1611 (2009).
27. N. Z. Petrović, M. Belić, and W. P. Zhong, “Spatiotemporal wave and soliton solutions to the generalized (3+ 1)-dimensional Gross-Pitaevskii equation,” Phys. Rev. E **81**, 016610 (2010).
28. N. Z. Petrović, N. B. Aleksić, A. A. Bastami, and M. R. Belić, “Analytical traveling-wave and solitary solutions to the generalized Gross-Pitaevskii equation with sinusoidal time-varying diffraction and potential,” Phys. Rev. E **83**, 036609 (2011).
29. Y. B. Zhou, M. L. Wang, and Y. M. Wang, “Periodic wave solutions to a coupled KdV equations with variable coefficients,” Phys. Lett. A **308**, 31–36 (2003).
30. Y. B. Zhou, M. L. Wang, and T. D. Miao, “The periodic wave solutions and solitary wave solutions for a class of nonlinear partial differential equations,” Phys. Lett. A **323**, 77–88 (2004).
31. G. P. Agrawal, “Modulation instability induced by cross-phase modulation,” Phys. Rev. Lett. **59**, 880–883 (1987).
32. I. Towers and B. A. Malomed, “Stable (2+ 1)-dimensional solitons in a layered medium with sign-alternating Kerr nonlinearity,” J. Opt. Soc. Am. **19**, 537–543 (2002).
33. S. A. Ponomarenko and G. P. Agrawal, “Interactions of chirped and chirp-free similaritons in optical fiber amplifiers,” Opt. Express **15**, 2963–2973 (2007).
34. C. Dai, Y. Wang, and J. Zhang, “Analytical spatiotemporal localizations for the generalized (3+1)-dimensional nonlinear Schrödinger equation,” Opt. Lett. **35**, 1437–1439 (2010).
35. F. II Ndzana, A. Mohamadou, and T. C. Kofane, “Modulational instability in the cubicquintic nonlinear Schrödinger equation through the variational approach,” Opt. Commun. **275**, 421–428 (2007).
36. E. Wamba, A. Mohamadou, and T. C. Kofane, “A variational approach to the modulational instability of a Bose-Einstein condensate in a parabolic trap,” J. Phys. B: At. Mol. Opt. Phys. **41**, 225403 (2008).
37. N. Petrović, H. Zahreddine, and M. Belić, “Exact spatiotemporal wave and soliton solutions to the generalized (3+ 1)-dimensional nonlinear Schrödinger equation with linear potential,” Phys. Scripta **83**, 065001 (2011).
38. Y. Castin, *Coherent Atomic Matter Waves*, R. Kaiser, C. Westbrook and F. David, eds. (Les Houches Session LXXII, Springer, 2001).

1. Introduction

Modulation instability (MI) is a general phenomenon occurring in solutions of the nonlinear wave equation wherein small perturbations to the solution grow exponentially over time, often producing singularities [1]. Determining whether MI occurs is of prime interest in the field od non-linear optics, where many different forms of wave equation naturally occur [2,3]. In particular, the generalized nonlinear Schrödinger equation (NLSE) is an important generic model that is of great use in NL optics [4–7]. The MI in the NLSE with cubic nonlinearity has been studied in [8], as well as in various related systems: the discrete NLSE [9], the NLSE with loss and a derivative term [10], the cubic-quintic NLSE [11], and others [12,13]. Experimental confirmation of MI in the NLSE with varying coefficients was given in [14]. MI has also been extensively studied in metamaterials, especially in materials with negative index of refraction [15–23]. In particular, it was determined that metamaterials have different stability properties from ordinary materials [15]. The discovery of metamaterials has opened up the possibility of forming stable spatial solutions through the management of the sign of refraction.

Recently, a number of exact solutions have been found for the various forms of NLSE [24–26] and for the Gross-Pitaevskii (GP) equation in Bose-Einstein condensates [27,28], using the F-expansion technique and the principle of harmonic balance [29, 30]. Two types of solutions were found: travelling wave solutions and solitary wave (SW) solutions. The solutions developed in [25] and [26] were for the (3+1)-dimensional ((3+1)D) NLSE with anomalous and normal dispersion, respectively. The solutions developed in [24] were for the (2+1)D NLSE. Meanwhile, the solutions in [28] describe stable spatiotemporal SWs under the influence of a sinusoidal diffraction/dispersion parameter, while the solutions in [27] required external gain in order to maintain the amplitude. Reference [28] briefly addressed the stability of obtained solutions, while [25] and [26] left the stability analysis to be performed in future work. It is the goal of this paper to provide a complete stability analysis of the obtained solutions in all of these papers. While this may look like a restricted goal aimed at specific solutions, it is nonetheless useful because it might be applicable to multidimensional solutions found in other equations that are of questionable stability. To the best of our knowledge, this is the first time MI analysis of this form has been applied to these systems.

The stability of exact soliton solutions to the NLSE in various forms is an important question that requires careful and thoughtful answer [31]. In (1+1)D, bright and dark soliton solutions of the NLSE in Kerr medium with cubic nonlinearity are unconditionally stable for the, respectively, self-focusing and self-defocusing nonlinearity [4]. However, in homogenous bulk media with a self-focusing cubic Kerr nonlinearity one cannot have unconditionally stable solutions of the NLSE in two and three dimensions. Nevertheless, great interest has been generated when it was suggested that (2+1)D generalized NLSE with varying coefficients may lead to stable 2D solitons [32]. The stabilizing mechanism has been the sign-alternating Kerr nonlinearity in a layered medium. The solutions obtained in [24–26, 28] all resulted in the alternating sign of Kerr nonlinearity and therefore it is worth investigating whether those solutions are also stable. In addition, there is strong indication that the SW solutions can be combined into multiple solutions using the self-similar method [33, 34] and that the individual components can interact with each other without affecting each other's form [34], which is a defining characteristic of solitons. Still, we will only use the term "solitary wave solution" to describe these solutions throughout our paper.

We will use a variational approach described in [35,36] to explore the modulation instability of the solitary wave solutions obtained in [24–26, 28]. Localized two- and three-dimensional solutions of the cubic nonlinear Schrödinger equation in [24–26] are extended one dimensional solutions. Namely, the intensity of solutions is homogenous in two out of three spatial dimensions for solutions obtained in [25, 26]. In the case of solutions obtained in [24] it is homogeneous in one out of two dimensions. It is in these homogenous directions, due to the nonlinearity, that modulation instability can develop. For this reason, of particular interest is the analysis of modulation stability of solutions in the direction of homogeneity.

2. Generalized nonlinear Schrödinger equation

We confine our analysis to the (2+1) and (3+1)-dimensional NLSEs considered in [24–26], and use the notation introduced there. We consider the generalized (3+1)D NLSE with varying coefficients and Kerr nonlinearity, developed in [25, 26]

$$i\frac{\partial u}{\partial z}+\frac{\beta(z)}{2}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+s\frac{\partial^2 u}{\partial t^2}\right)+\chi(z)\left|u\right|^2 u=i\delta(z)u. \tag{1}$$

The functions β, χ, and δ stand for the diffraction, nonlinearity, and the gain or loss coefficients, respectively. Our goal is to verify whether the solutions to Eq. (1) developed in [25, 26] are modulationally stable (MS) or modulationally unstable (MU). In Eq. (1), $s=-1$ for the normal dispersion and $s=1$ for the anomalous dispersion. For $s=0$ we have the two-dimensional time independent NSLE studied in [24], which will also be covered in our stability analysis.

The solution to Eq. (1) described in [26] is given as:

$$\begin{aligned} u \;&=\; (\alpha)^{3/2} f_0 e^{\int_0^z \delta dz}\left(F(\theta)+\varepsilon\sqrt{\frac{c_0}{c_4}}\frac{1}{F(\theta)}\right)\cdot \\ &\quad \exp\left(i\left(a(z)\left(x^2+y^2+st^2\right)+b(z)\left(x+y+t\right)+e(z)\right)\right), \end{aligned} \tag{2}$$

where F is a Jacobi elliptic function (JEF), α is the chirp function:

$$\alpha=\frac{1}{1+2a_0\int_0^z \beta dz} \tag{3}$$

and:

$$\theta=k(z)x+l(z)y+m(z)t+\omega(z), \tag{4}$$

for nonlinearity $\chi(z)$ given as:

$$\chi(z)=-c_4\frac{\beta(z)}{\alpha f_0^2}\chi_0\exp(-2\int_0^z \gamma dz), \tag{5}$$

where we define:

$$\chi_0=(k_0^2+l_0^2+sm_0^2). \tag{6}$$

The parameters a, b, k, l, m, ω and e are:

$$a = \alpha a_0, \qquad b=\alpha b_0, \tag{7}$$

$$k = \alpha k_0, \qquad l=\alpha l_0, \qquad m=\alpha m_0, \tag{8}$$

$$\omega = \omega_0-\alpha(k_0+l_0+sm_0)b_0\int_0^z \beta dz, \tag{9}$$

$$e = e_0+(\alpha/2)\left(c\chi_0-(2+s)b_0^2\right)\int_0^z \beta dz, \tag{10}$$

where $c=c_2-6\varepsilon\sqrt{c_0c_4}$ and c_0, c_2 and c_4 are parameters related to the Jacobi elliptic function parameter M [24]. We will limit our attention to the cases where $\varepsilon=0$, hence $c=c_2$. Throughout this paper, we also take $\omega_0=e_0=0$. For convenience, we define the parameter $\overline{e}$ as follows:

$$\overline{e}=e-\frac{c}{2}\int_0^z \beta\alpha^2 dz. \tag{11}$$

We now make the following gauge transformation on the solution and the coordinates:

$$\begin{aligned} u\to G \;&=\; u\exp\left(-\int_0^z \delta dz\right)\cdot \\ &\quad \exp\left(-i(a(x^2+y^2+st^2)+b(x+y+t)+\overline{e})\right)/(f_0\alpha^{3/2}|\chi_0 c_4|^{1/2}), \end{aligned} \tag{12}$$

#229339 - $15.00 USD Received 15 Dec 2014; revised 27 Mar 2015; accepted 28 Mar 2015; published 16 Apr 2015

$$
\begin{aligned}
x \to x' &= \alpha(x-\varsigma), && (13)\\
y \to y' &= \alpha(y-\varsigma), && (14)\\
t \to t' &= \alpha(t-s\varsigma), && (15)\\
z \to z' &= \int_0^z \alpha^2 \beta dz, && (16)
\end{aligned}
$$

where $\varsigma(z) = b_0 \int_0^z \beta dz$ is the solution to the equation:

$$\frac{\partial \varsigma(z)}{\partial z} = \beta(z)(2a(z)\varsigma(z) + b(z)), \tag{17}$$

to obtain the following equation:

$$i\frac{\partial G}{\partial z'} + \frac{1}{2}\left(\frac{\partial^2 G}{\partial x'^2} + \frac{\partial^2 G}{\partial y'^2} + s\frac{\partial^2 G}{\partial t'^2}\right) + \sigma |G|^2 G = 0, \tag{18}$$

where $\sigma = \mathrm{sgn}(-c_4\chi_0)$. Equation (18) is much more suitable for stability analysis than Eq. (1) because all the coefficients next to G and its derivatives are constant. Also, the wave propagation now necessarily happens along a straight line, unlike in solutions shown in [25, 26].

The stationary solutions $F = G\exp\left(-ic_2 \int_0^z \beta\alpha^2 dz/2\right)$, where G is the solution of Eq. (18), contain the whole range of solutions from [25] and [26], for different values of c_2 and c_4, in the direction (k_0, l_0, m_0). Since $\varepsilon = 0$, F will be equal to some JEF. Adjusting parameters (k_0, l_0, m_0) to these values corresponds to the rotation and re-scaling of the coordinate system (x', y', t'). Without loss of generality, we can put $f_0 = 1$ and also $k_0 = 0$, $l_0 = 0$, $m_0 = 1$, for temporal solutions, and $k_0 = 0$, $l_0 = 1$, $m_0 = 0$, for spatial solutions. As mentioned, the amplitude $|G|$ is homogenous in two of the three spatial/temporal dimensions (i.e. in the plane perpendicular to the direction (k_0, l_0, m_0) of inhomogeneity.) It is in this plane that, owing to nonlinearity, the modulation instability can develop. Hence, it is of interest to analyze MI of perturbations in the plane of homogeneity of $|G|$. We chose x' as the perpendicular direction of modulation. In this case, $\chi_0 = 1$ for spatial SWs, whereas $\chi_0 = s$ for temporal SWs. If the SW is spatial, perturbations along the temporal axis t' are also possible. In that case, we will consider the x' and t' to be swapped, so that x' always denotes the axis of perturbation.

3. Variational approach to the modulation stability

We now use the variational approach to examine MI, following [8]. We analyze two periodic traveling wave planar solutions: $F = \mathrm{sn}$ and $F = \mathrm{cn}$, which reduce in the limit of $M = 1$ to two solitary wave solutions: tanh (the dark SW) and sech (the bright SW), respectively. We study spatial and temporal periodic solutions for both cases: the normal and the anomalous dispersion.

The idea of the variational approach is to introduce a perturbation in the solutions and analyze the behavior of the perturbation. To this end, we assume:

$$G = G_0\left(1 + U(z)\cos\left(Kx'\right)\right), \tag{19}$$

where G_0 is the unperturbed solution of Eq. (18), $U(z) = U_r(z) + iU_i(z)$ is the total amplitude perturbation, K is the wave number of the perturbation and x is the direction in which the perturbation occurs, as described in the previous section. One then constructs, according to standard procedure, the corresponding Lagrangian to Eq. (18):

$$L = \frac{i}{2}\left(G\frac{\partial G^*}{\partial z'} - G^*\frac{\partial G}{\partial z'}\right) + \frac{1}{2}|\nabla G|^2 - \sigma |G|^4, \tag{20}$$

#229339 - $15.00 USD Received 15 Dec 2014; revised 27 Mar 2015; accepted 28 Mar 2015; published 16 Apr 2015

where G^* is the complex conjugate of G, and $|\nabla G|^2 = |\partial G/\partial x'|^2 + |\partial G/\partial y'|^2 + s\,|\partial G/\partial t'|^2$. One next performs an averaging of the Lagrangian over all three transverse coordinates, to obtain:

$$\langle L\rangle = \int \left(\frac{i}{2}\left(G\frac{\partial G^*}{\partial z'} - G^*\frac{\partial G}{\partial z'} \right) + \frac{1}{2}\,|\nabla G|^2 - \sigma\,|G|^4 \right) dx'\,dy'\,dt'. \tag{21}$$

Note that the Lagrangian is averaged over one period of perturbation in the direction of perturbation and in the direction of the SW it has been averaged from $-2K(M)$ to $2K(M)$ for $F = \mathrm{cn}$ and from $-K(M)$ to $3K(M)$ for $F = \mathrm{sn}$ (these boundaries converge to $-\infty$ and ∞ for $M = 1$, i.e. to solitary waves). Here $K(M) = \int_0^{\pi/2}(1 - M\sin^2 t)^{-1/2}dt$ is the complete elliptic integral of the first kind and M is the parameter of JEFs. The total action is now defined as:

$$\Lambda = \int_{-\infty}^{\infty} \langle\, L\rangle dz. \tag{22}$$

It remains invariant in the transformation of coordinates from u to G.

Substituting the new formula for G in Eq. (19) into the effective Lagrangian given in Eq. (22), we vary U_r and U_i in the standard procedure [8], to obtain Euler-Lagrange differential equations for $U_r = U_r(z)$ and $U_i = U_i(z)$, as follows:

$$\frac{\partial}{\partial z}U_r = \frac{1}{2}K^2\alpha^2\beta\,(\kappa U_i)\,, \tag{23}$$

$$\frac{\partial}{\partial z}(\kappa U_i) = -\frac{1}{2}\left(K^2 - \kappa\sigma d\right)\alpha^2\beta U_r, \tag{24}$$

where the parameter κ is defined as follows: $\kappa = 1$ for the perturbations along the spatial coordinates (spatial perturbations) and $\kappa = s$ for the temporal perturbation of spatial SWs. The parameter d is defined as:

$$d = d_{cn}^{(M)} = \frac{8}{3}\,\frac{(2M-1)(E(M) - E(am(5K(M)|M)|M)) - 2(2-5M+3M^2)K(M)}{E(M) - E(am(5K(M)|M)|M)) - 4(M-1)K(M))} \tag{25}$$

for $F = cn(\cdot|M)$,

$$d = d_{dn}^{(M)} = \frac{8}{3}\left(2 - M + (1-M)\frac{K(M)}{2E(M)}\right) \tag{26}$$

for $F = dn(\cdot|M)$ and

$$d = d_{sn}^{(M)} = \frac{8}{3}\,\frac{(M+1)E(am(4K(M)|M)|M) - 2(2+M)K(M)}{(E(am(4K(M)|M)|M) - 4K(M))} \tag{27}$$

for $F = sn(\cdot|M)$. Here $K(M) = F(\pi/2|M)$ and $E(M) = E(\pi/2|M)$ are the complete elliptic integrals of the first and second kind, respectively; $F(u|M) = \int_0^u(1 - M\sin^2 t)^{-1/2}dt$ and $E(u|M) = \int_0^u(1 - M\sin^2 t)^{1/2}dt$ are the incomplete elliptic integrals of the first and second kind, respectively, and $am(u,M) = F^{-1}(u,M)$ is the amplitude of the Jacobi elliptic functions. The dependence of coefficients $d_{cn}^{(M)}$, $d_{dn}^{(M)}$ and $d_{sn}^{(M)}$ on the elliptic parameter M ($0 < M < 1$) is shown in Fig. 1(a). For bright SWs $d_{cn}^{(M=1)} = d_{dn}^{(M=1)} = 8/3$ and for dark SWs $d_{sn}^{(M=1)} = 4$.

The solution to Eqs. (23) and (24) can now be written as:

$$U_r = U_0\cosh\left(\frac{\gamma\xi}{1+2a_0\xi}\right), \tag{28}$$

$$U_i = U_0\frac{2\gamma}{K^2}\sinh\left(\frac{\gamma\xi}{1+2a_0\xi}\right), \tag{29}$$

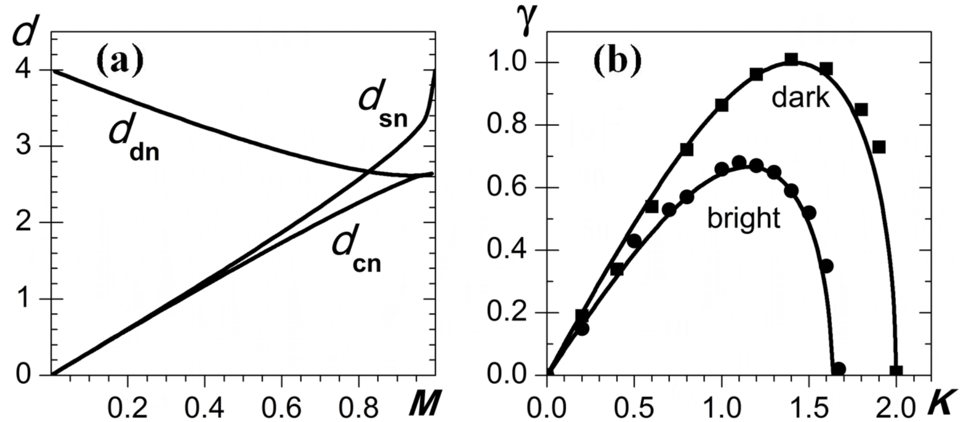


Fig. 1. (a) Nonlinearity parameter d for solutions cn, sn and dn. (b) The growth rate parameter γ for dark an bright SWs, as a function of K for the case $\kappa\sigma = 1$. Modulational instability occurs for values of K depicted in the respective graphs. The solid lines represent the theoretical calculation of K using Eq. (30), and the square and circle dots are values of γ measured using numerical simulations, in which the dark and bright SWs, respectively, were perturbed by a small wave of the given wave number K.

where:

$$\gamma = K\sqrt{(\sigma\kappa d - K^2)}/2 \tag{30}$$

and

$$\xi = \int_0^z \beta dz. \tag{31}$$

For small values of ξ, i.e. $\xi << 1$, the modulus of the perturbation amplitude can be approximated to within second order of ξ to be:

$$|U| = U_0\left(1 + \sigma\kappa\frac{d\gamma^2}{2K^2}(1 - 4a_0\xi)\,\xi^2\right). \tag{32}$$

3.1. Case without chirp

In the case without chirp, i.e. for $a_0 = 0$, the solutions given in Eqs. (28) and (29) to Eqs. (23) and (24) become:

$$U_r = U_0\cosh(\gamma\xi), \tag{33}$$

$$U_i = U_0\frac{2\gamma}{K^2}\sinh(\gamma\xi). \tag{34}$$

The dynamics of the overall evolution of the total perturbation U are determined by the growth rate parameter γ given in Eq. (30), also known as the modulation-instability growth rate [4], and the function $\beta(z)$. For $\kappa\sigma = -1$, the growth rate parameter $\gamma = i\overline{\gamma} = iK\sqrt{K^2+d}/2$ is imaginary for all values of K, and, consequently, the solutions G are modultionally unconditionally stable for any function $\beta(z)$. This case occurs for temporal cn-SWs with normal dispersion $(s = -1)$ and temporal sn-SWs with anomalous dispersion $(s = 1)$ in the self-defocusing media.

In the opposite case, $\kappa\sigma = 1$, which holds for the temporal or spatial cn-solutions for anomalous dispersion $s = 1$ or temporal sn-solutions if $s = -1$ in the focusing media, more interesting

#229339 - $15.00 USD Received 15 Dec 2014; revised 27 Mar 2015; accepted 28 Mar 2015; published 16 Apr 2015

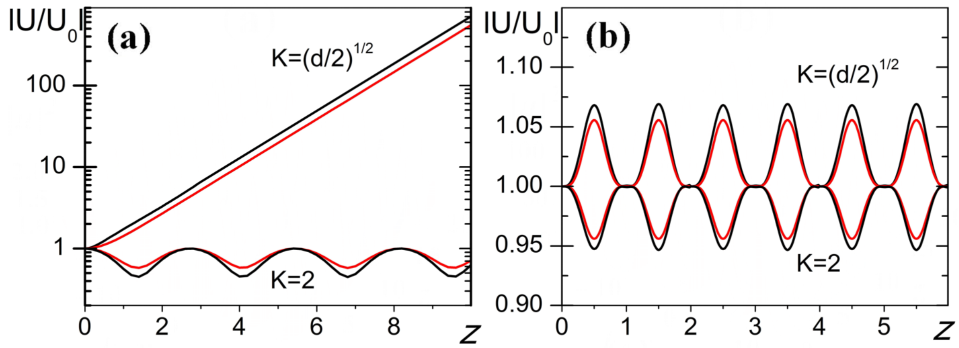


Fig. 2. Perturbation amplitude growth for $\kappa\sigma = 1$, $a_0 = 0$ and $d = 8/3$ as a function of propagation distance z. Black curves are numerical results, while the red curves are analytical results: (a) $\beta_0 = 1$, $\beta_1 = 0$, top to bottom: numerical results for $K = \sqrt{d/2}$, analytic results for $K = \sqrt{d/2}$, analytical results for $K = 2$, numerical results for $K = 2$, (b) $\beta_0 = 0$, $\beta_1 = 1$, $Z = 1$, top to bottom: numerical results for $K = \sqrt{d/2}$, analytical results for $K = \sqrt{d/2}$, analytical results for $K = 2$, numerical results for $K = 2$.

dynamics of perturbations occur. For the analysis of these dynamics, let us assume β to be of the following form: $\beta(z) = \beta_0 + \beta_1 \sin(2\pi z/Z)$, where $Z = 2\pi/\Omega$ is the wavelength of β. For the spatial perturbation with wavenumber $K > \sqrt{d}$ the growth rate is zero (since $\gamma = i\overline{\gamma}$ is imaginary) and the perturbation amplitude has an oscillatory solution in the following range:

$$\sqrt{1 - \frac{d}{K^2}\sin^2\left(\overline{\gamma}\beta_1 Z/\pi\right)} \le |U/U_0| \le 1. \tag{35}$$

In Fig. 2 we see that the mode corresponding to $K = 2 > \sqrt{8/3}$ is stable for both a constant and a sinusoidal form of β (plots (a) and (b), respectively).

If $K < \sqrt{d}$, then $|U|$ grows exponentially at a rate of $\gamma\beta_0$, as seen in Fig. 2(a). Consequently, if $\beta_0 \neq 0$ the solution is unstable. In Fig. 1(b) we plot γ as a function of wavenumber K for bright $(d = 8/3)$ and dark $(d = 4)$ SWs in the case without dispersion management: $\beta(z) = \beta_0 = 1$. In both cases, γ has a maximum $\gamma = \Gamma = d/4$ for modes $K = \sqrt{d/2}$. Numerical simulations of Eq. (18) confirm the analytical prediction for the growth rate γ. It also follows that the amplitude of $|U|$ can be made to be stable if the mean value of the management function is zero $\beta_0 = 0$, and the period of oscillations of β is small (i.e. $Z << 1$). This is due to the finite limit imposed on ξ under these conditions. The variation of the perturbation amplitude in this case is:

$$1 \le |U/U_0| \le \sqrt{1 + \frac{d}{K^2}\sinh^2\left(\overline{\gamma}\beta_1 Z/\pi\right)}. \tag{36}$$

3.2. *Case with chirp*

In the case where $a_0 > 0$ we analyze the evolution of perturbation modes with maximal growth rates as functions of z. Expanding the solution for large ξ to within the first order of $1/\xi$ we find:

$$|U| = U_0\sqrt{1 + \sigma\kappa\frac{d}{K^2}\sinh^2\left(\frac{\gamma}{2a_0}\right)}\left(1 - \frac{C}{a_0\xi}\right), \tag{37}$$

#229339 - $15.00 USD Received 15 Dec 2014; revised 27 Mar 2015; accepted 28 Mar 2015; published 16 Apr 2015

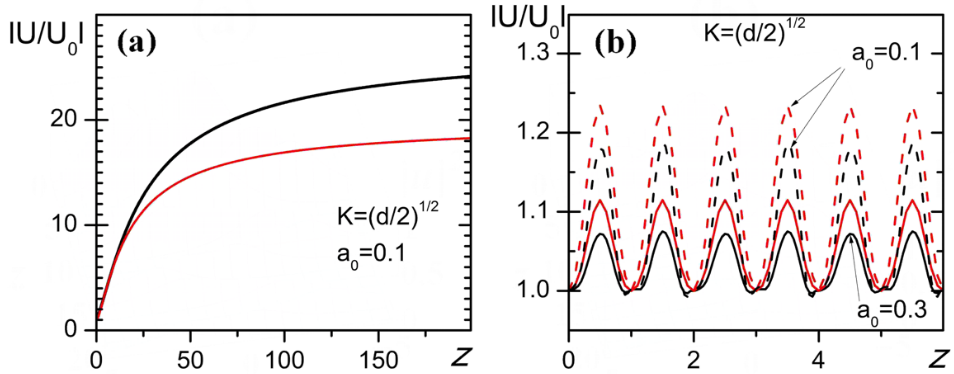


Fig. 3. Perturbation amplitude growth for $\kappa\sigma = 1$, $d = 8/3$ and $K = \sqrt{d/2}$ as a function of propagation distance z for systems with chirp. Black curves are numerical results, while the red curves are analytical results: (a) $\beta_0 = 1$, $\beta_1 = 0$, $a_0 = 0.1$, top: numerical results, bottom: analytical results, (b) $\beta_0 = 0$, $\beta_1 = 1$, $Z = 1$, dashed lines represent plots for $a_0 = 0.1$, top to bottom: analytical results for $a_0 = 0.1$, numerical results for $a_0 = 0.1$, analytical results for $a_0 = 0.3$, numerical results for $a_0 = 0.3$.

where:

$$C = \frac{\gamma}{8a_0} \frac{\sigma\kappa \frac{d}{K^2} \sinh\left(\frac{\gamma}{a_0}\right)}{1 + \sigma\kappa \frac{d}{K^2} \sinh^2\left(\frac{\gamma}{2a_0}\right)}. \tag{38}$$

This implies saturation after linear growth for constant β, i.e. $\beta_0 \neq 0$ and $\beta_1 = 0$. Indeed, some sort of saturation of the perturbation amplitude is to be expected in this case, since the solution dissipates due to the form of the chirp function [25]. The analytical results agree well with the numerical results, as can be seen in Fig. 3(a).

The overall behavior of MIs is otherwise similar to the case without chirp. Typical behavior of $|U|$ is presented in Fig. 3, for different values of parameters. In the case of dispersion management $\beta_0 = 0$, the perturbation amplitude has oscillatory behavior with a maximum variation that depends on the period Z of the management function β. Stabilization can be achieved by reducing the period of the management function. The dependence of the maximum perturbation amplitude on z, d and a_0 is given in Fig. 4. While an increase in parameter d in general increases the amplitude of $|U|$, as is seen in Fig. 4(a), an increase in a_0 reduces the amplitude of $|U|$, as seen in Fig. 4(b). This result is in agreement with results obtained in Fig. 3(b) where values $a_0 = 0.3$ and $a_0 = 0.1$ were compared.

The entire stability analysis is presented in Table 1. We see that depending on the choice of s, σ and whether the SW is spatial or temporal we have eight distinct cases for examining the stability of our solutions. In the case of spatial SWs, perturbations can occur in both the spatial and temporal directions, whereas in the case of temporal SWs they can only occur in spatial directions. For a SW to be stable we must have $\kappa\sigma = -1$ in all directions of perturbation, otherwise it is conditionally stable, i.e. only for $\beta_0 = 0$. A SW is dark if the direction of the SW and the nonlinear term, i.e. σ are of the opposite sign, otherwise the SW is bright. In the 2D time-independent case we no longer need to consider temporal SWs and distortions.

4. Numerical simulations

We now use computer simulations to simulate the behavior of our solutions when a small perturbation is introduced. Observing the rate of change of the amplitude of G in our simulations,

#229339 - $15.00 USD Received 15 Dec 2014; revised 27 Mar 2015; accepted 28 Mar 2015; published 16 Apr 2015

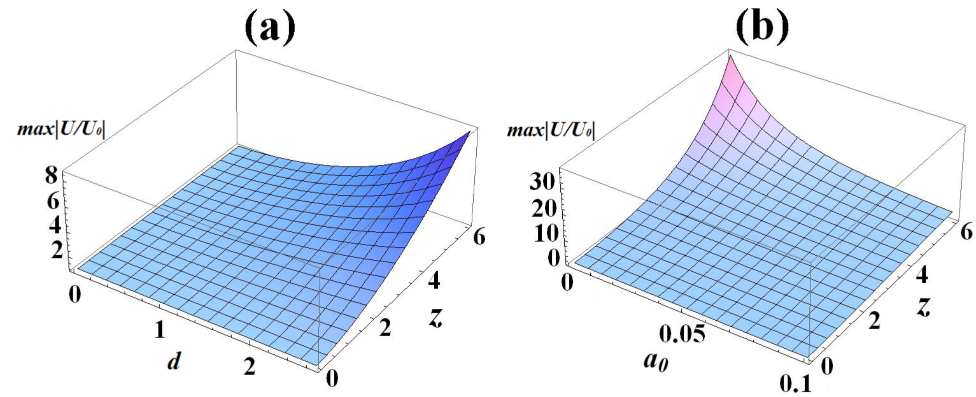


Fig. 4. Maximum amplitude of perturbation for $K=\sqrt{d/2}$ plotted against $z=\beta_1 Z/\pi$ and: (a) d for $a_0=0.05$, (b) a_0 for $d=8/3$.

Table 1. Stability cases

	s	σ	SW	pert.	κ	type	stability (3D)	stability (2D)
1	1	1	S-spatial	S T	1 1	cn	CS-Conditionally Stable	CS
2	1	1	T-temporal	S	1	cn	CS	–
3	1	−1	S	S T	1 1	sn	S-stable	S
4	1	−1	T	S	1	sn	S	–
5	−1	1	S	S T	1 −1	cn	CS	CS
6	−1	1	T	S	1	sn	CS	–
7	−1	−1	S	S T	1 −1	sn	CS	S
8	−1	−1	T	S	1	cn	S	–

we can then measure the value of γ and compare it with the theoretical expectation given in Eq. (30). The split-step FFT simulations produced the points on the plot in Fig. 1(b), which agree well with theoretical expectations denoted by the continuous line. Most importantly, the solutions cease to exponentially increase at precisely the values predicted by the theory of MI.

We see in Figs. 5–7 the main results of our simulations in scenarios involving instability. Starting from the initial form of the solution (Figs. 5–6(a)), we can see that the perturbation rapidly increases (Figs. 5–6(b)) and ultimately completely abandons the original form of the solution (Figs. 5–6(c)). Figure 5 depicts the time evolution of a bright SW, while Fig. 6 depicts the time evolution of a dark SW. Owing to the difficulty that arises from the boundary conditions for the dark SW, due to a change in the sign of u, we have instead run a simulation of two dark SWs with periodic boundary conditions, as is the standard practice. Periodic solutions show a similar pattern of instability formation as seen in Fig. 7.

5. Analysis of the stability of the Gross-Pitaevskii equation

In this section we apply the results and methods of Sec. 2 to provide a stability analysis of our solutions to the Gross-Pitaevskii equation (GPE), obtained in [28]. This section expands upon

#229339 - \$15.00 USD Received 15 Dec 2014; revised 27 Mar 2015; accepted 28 Mar 2015; published 16 Apr 2015

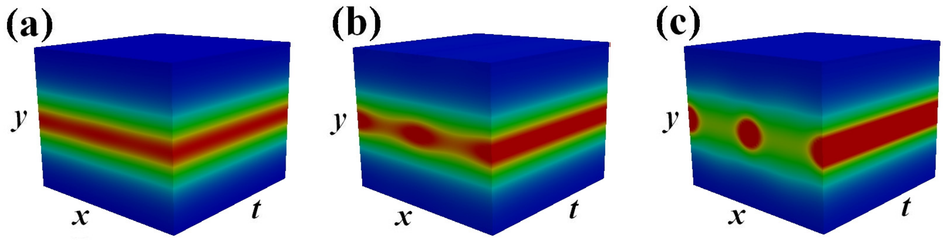


Fig. 5. Development of modulation instability for the bright SW for three different values of z. Here, x is the direction of perturbation, y is the direction of the SW and t is the remaining transverse direction. Bright colors, i.e. towards the color red (the center in Fig. 5(a)), indicate a higher value of $|u|^2$.

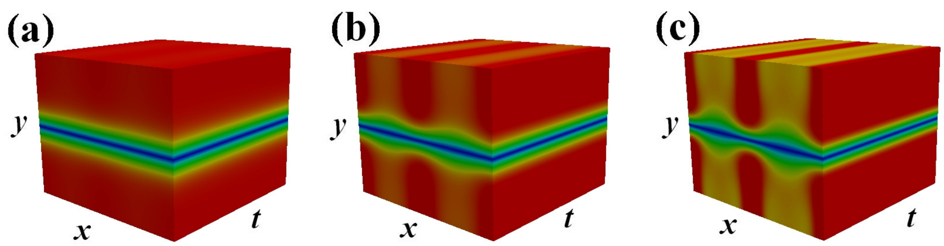


Fig. 6. Development of modulational instability for the dark SW for three different values of z. Here, x is the direction of perturbation, y is the direction of the SW and t is the remaining transverse direction. Red color (away from the center in Fig. 6(a)), indicates a higher value of $|u|^2$.

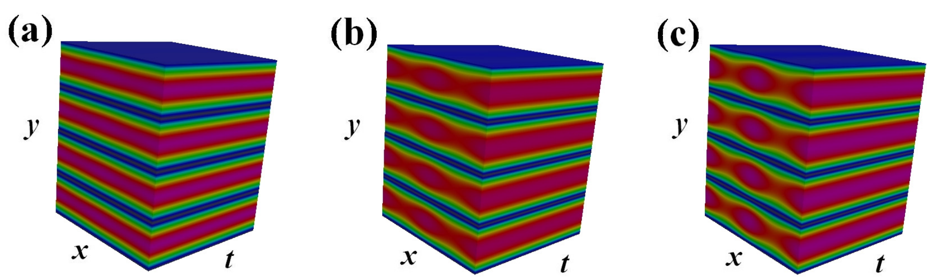


Fig. 7. Development of modulational instability for the dark traveling wave ($F = \mathrm{sn}$) for three different values of z. Here, x is the direction of perturbation, y is the direction of the traveling wave and t is the remaining transverse direction. Parameters are $M = 0.5$ and $K = \sqrt{d/2}$. Blue color (at the top, bottom and the three central stripes in Fig. 7(a)), indicates a lower value of $|u|^2$.

the results briefly summarized in [28] and provides additional data. We now examine the GPE, i.e.

$$i\partial_t u + \frac{\beta(t)}{2}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) + \chi(t)|u|^2 u + \eta(t)(x^2+y^2+z^2)u = i\delta(t)u. \tag{39}$$

The coefficient η refers to the strength of the quadratic potential. The main part of our analysis will be to transform the starting Eq. (39) into a form more amenable to stability analysis [38]. The propagation variable is now t instead of z.

The key differences between Eq. (39) and Eq. (1), apart from the addition of the quadratic potential, is the change in the longitudinal direction from z to t. Hence, there is no longer a distinction between normal and anomalous dispersion. This greatly simplifies the stability analysis. We restrict out attention to SWs found in [28], as the solutions found in [27] do not have a stable amplitude when they are not artificially maintained with a nonzero gain. The solutions in [28] are found under the condition that β and η are proportional trigonometric functions of the same sign. We will call their amplitudes β_0 and η_0. The parameters b, k, l and m are:

$$b = pb_0, \tag{40}$$

$$k = pk_0, \qquad l = pl_0, \qquad m = pm_0, \tag{41}$$

$$\omega = \omega_0 - q(k_0 + l_0 + m_0)b_0. \tag{42}$$

The parameters p and q, as well as the chirp function a, are defined to be:

$$p = \sqrt{\frac{\eta_0}{\eta_0 - 2a_0^2\beta_0}}\operatorname{sech}(\tau(t)+\tau_0), \tag{43}$$

$$q = \frac{\sqrt{\eta_0\beta_0}}{\sqrt{2}(\eta_0 - 2a_0^2\beta_0)}\left(\tanh(\tau(t)+\tau_0) - \tanh\tau_0\right), \tag{44}$$

$$a = \sqrt{\frac{\eta_0}{2\beta_0}}\tanh(\tau(t)+\tau_0), \tag{45}$$

where:

$$\tau_0 = \operatorname{arctanh}\left(a_0\sqrt{\frac{2\beta_0}{\eta_0}}\right) \tag{46}$$

and:

$$\tau(t) = \sqrt{\frac{2\alpha_0}{\beta_0}}\int_0^t \beta(t)dt. \tag{47}$$

Formula (5) now becomes:

$$\chi(t) = -c_4\frac{\beta(t)}{pf_0^2}\chi_0\exp(-2\int_0^t \delta dt), \tag{48}$$

where $\chi_0^2 = k_0^2 + l_0^2 + m_0^2$ and p is defined in Eq. (43). For convenience we set $\chi_0 = 1$, $f_0 = 1$ and $\beta_0 = 1$. We also define:

$$\bar{e} = e - \frac{qc}{2}, \tag{49}$$

where q is given in Eq. (44) and $c = c_2 - 6\varepsilon\sqrt{c_0c_4}$. Again, for $\varepsilon = 0$ we have $c = c_2$.

#229339 - $15.00 USD Received 15 Dec 2014; revised 27 Mar 2015; accepted 28 Mar 2015; published 16 Apr 2015

We now define:

$$G = \frac{u}{p^{3/2} f_0 \sqrt{|c_4 \chi_0|}} \exp\left(-\int_0^t \delta dt\right) \exp\left(-ia(x^2+y^2+z^2) - ib(x+y+z) - i\vec{e}\right) \tag{50}$$

and employ similar changes of coordinates as in Sec. 2:

$$x \rightarrow x' = p(x-\zeta) \tag{51}$$
$$y \rightarrow y' = p(y-\zeta) \tag{52}$$
$$z \rightarrow z' = p(z-\zeta) \tag{53}$$
$$t \rightarrow t' = \int_0^t p^2 \beta dt, \tag{54}$$

where p is defined in Eq. (43) and $\frac{\partial \zeta(t)}{\partial t} = \beta(t)(2a(t)\zeta(t) + b_0 p(t))$. The form of the function $\zeta(t)$ in the transformation is of no immediate interest, other than the fact that it depends on $p(t)$ and $\tau(t)$, which are given in Eq. (43) and Eq. (47), respectively. The transformation gives us the following equation for G:

$$i\frac{\partial G}{\partial t'} + \left(\frac{\partial^2 G}{\partial x'^2} + \frac{\partial^2 G}{\partial y'^2} + \frac{\partial^2 G}{\partial z'^2}\right) - \sigma |G|^2 G = 0, \tag{55}$$

where $\sigma = -\mathrm{sgn}(c_4)$. Qualitatively, this is the same equation as Eq. (18), except for the change of the longitudinal variable from z to t. The new variable t', which only depends on t, involves an integral over β that can change sign. This will be important in the analysis of Eq. (55). Just as in the case of NLSE, we can place without the loss of generality the z' axis in the direction of inhomogeneity of our extended solitary solutions, i.e. assume $k_0 = l_0 = 0$ and $m_0 = 1$, and put x' as the axis of perturbation.

Equation (55) is the usual (3+1)D nonlinear Schrödinger equation with constant coefficients, which is prone to instabilities and the wave function collapse. Instabilities in G translate into instabilities of the general solution u. This would bode disaster for the stability of exact traveling wave and solitary solutions found, were it not for the possibility of diffraction and nonlinearity management [7] in Eq. (55), thanks to the form of the primed variables. We find that, for the choice of coefficients $\alpha(t)$ and $\beta(t)$ made in [28], the typical extended SW solutions of Eq. (55) do not collapse when perturbed, but keep oscillating in a typical breathing behavior.

We now consider the perturbation of G in this plane for the two fundamental solutions, the dark $F = \mathrm{sn}$ and the bright $F = \mathrm{cn}$ SWs, where $F = G\exp(-iqc_2/2)$ in the form:

$$G = G_0\left(1 + U(t)\cos(Kx')\right), \tag{56}$$

where $U(t) = U_r(t) + iU_i(t)$ is the complex amplitude, and K is the wavenumber of the perturbation in the direction perpendicular to z'. In a standard linear stability analysis, as was already done for the NLSE in Sec. 2, the perturbation is substituted into Eq. (55) and linear first-order differential equations for U_r and U_i are obtained. Plugging in the perturbation, one obtains:

$$\frac{\partial U_r}{\partial t} = \frac{1}{2}K^2 p^2 \beta U_i, \tag{57}$$
$$\frac{\partial U_i}{\partial t} = -\frac{1}{2}\left(K^2 - \sigma d\right) p^2 \beta U_r, \tag{58}$$

where d is defined as in Eqs. (25)–(27). The solutions of Eqs. (57) and (58) determine the dynamics of the modulational instability. Equations (57) and (58) can be solved analytically to

#229339 - $15.00 USD Received 15 Dec 2014; revised 27 Mar 2015; accepted 28 Mar 2015; published 16 Apr 2015

yield:

$$U_r(t) = U_0 \cosh(\gamma q(\tau)), \tag{59}$$

$$U_i(t) = U_0 \frac{\gamma}{K^2} \sinh(\gamma q(\tau)), \tag{60}$$

where:

$$\gamma = K\sqrt{(d-K^2)}/2. \tag{61}$$

A similar analysis as in Sec. 3.1 can be performed in this case, and one obtains a much more simplified analysis with respect to Table 1 since there is no case $s = 1$, and there are no temporal perturbations. One obtains that the dark SWs are always stable, and the bright SWs are always conditionally stable.

We now restrict our attention to the bright SWs, i.e. $\sigma = 1$. The the modulus of the perturbation amplitude is given as:

$$|U| = U_0 \left(1 + \frac{\sigma d}{K^2} \sinh^2(\gamma q(\tau))\right)^{1/2}. \tag{62}$$

A graph and a detailed analysis describing the behavior of perturbations were given in [28]. Here we focus our attention on the dependence of the maximum perturbation on parameters d, η_0 and a_0. It is worth noting that unlike in the case of NLSE the parameter q is limited assuming a reasonable choice of η_0 and a_0. Thus assuming η and β are proportional and of equal sign, finding stable solutions is likely even in the absence of dispersion management.

To achieve the maximum amplitude of $|U|$ we take $t \to \infty$ and $K = \sqrt{d/2}$. In that case, along with $\beta_0 = 1$, we obtain

$$\gamma q = -\frac{d}{4\sqrt{2}(\sqrt{\eta_0} - \sqrt{2}a_0)}. \tag{63}$$

Since $\sigma d/K^2 = 2$ and sinh is an increasing function it follows that growth of $|U|$ follows the growth of the magnitude of γq. In other words, d and a_0 contribute to an increase in $|U|$ while η_0 contributes to a decrease in $|U|$. As solutions approach the singularity threshold $a_0 = \sqrt{\eta_0/2}$, the amplitude of perturbations also blows up.

6. Conclusion

In this paper, we have analyzed the stability of solutions of the (3+1)D NLSE with normal or anomalous dispersion and the (2+1)D time-independent NLSE. For the (2+1)D solutions, we obtained stability for dark solitary waves and conditional stability for bright solitary waves, meaning we need to apply dispersion management to keep the solitary waves stable, i.e. the diffraction/dispersion coefficient must oscillate around 0. The management function dynamically stabilizes the nonlinear structure of the transversal perturbation of the soliton if its mean value is zero. Reducing the period of the management function can be achieved by arbitrary limitations of the perturbation level. If, however, the mean value is different from zero, the amplitude modulation perturbation exponentially increases in the case of a solution without chirp and linearly with saturation in the case with chirp. In the former case we have Lyapunov instability. In the latter case the saturation is an indirect consequence of the dissipation of the solution.

For the (3+1)D case we obtain stability for the temporal bright solitons for normal dispersion and dark solitons for anomalous dispersion. All other types of solitons are conditionally stable. For the Gross-Pitaevskii equation we obtain that dark solitons are always stable and bright solitons are always conditionally stable. The obtained results are verified using computer simulations.

#229339 - $15.00 USD Received 15 Dec 2014; revised 27 Mar 2015; accepted 28 Mar 2015; published 16 Apr 2015

Acknowledgments

This publication was made possible by NPRP Grants #5-674-1-114 and #6-021-1-005 from the Qatar National Research Fund (a member of the Qatar Foundation). Work at the Institute of Physics in Belgrade was supported by the Ministry of Education and Science of the Republic of Serbia under the projects OI 171006 and III 46016.

#229339 - $15.00 USD Received 15 Dec 2014; revised 27 Mar 2015; accepted 28 Mar 2015; published 16 Apr 2015

Spatiotemporal soliton supported by parity-time symmetric potential with competing nonlinearities

2016 EPL 115 14006

(http://iopscience.iop.org/0295-5075/115/1/14006)

A LETTERS JOURNAL EXPLORING THE FRONTIERS OF PHYSICS

July 2016

EPL, **115** (2016) 14006
doi: 10.1209/0295-5075/115/14006

www.epljournal.org

Spatiotemporal soliton supported by parity-time symmetric potential with competing nonlinearities

SI-LIU XU[1(a)], YUAN ZHAO[2], NIKOLA Z. PETROVIĆ[3] and MILIVOJ R. BELIĆ[4]

[1] *The School of Electronic and Information Engineering, HuBei University of Science and Technology Xianning 437100, China*
[2] *School of Biomedical Engineering, Hubei University of Science and Technology - Xianning 437100, China*
[3] *Institute of Physics, University of Belgrade - P.O. Box 68, 11001 Belgrade, Serbia*
[4] *Science Program, Texas A&M University at Qatar - P.O. Box 23874 Doha, Qatar*

received 22 February 2016; accepted in final form 8 July 2016
published online 10 August 2016

PACS 42.65.Tg – Optical solitons; nonlinear guided waves
PACS 05.45.Yv – Solitons

Abstract – We construct explicit spatiotemporal or light bullet (LB) solutions to the $(3+1)$-dimensional nonlinear Schrödinger equation (NLSE) with inhomogeneous diffraction/dispersion and nonlinearity in the presence of parity-time (PT) symmetric potential with competing nonlinearities. The solution is based on the similarity transformation, by which the initial inhomogeneous problem is reduced to the standard NLSE with constant coefficients but with redefined variables and potential. Transmission characteristics of LB solutions, such as the phase change, half width and chirp, are studied in the media with exponentially decreasing diffraction/dispersion and with periodic modulation. Our outcomes demonstrate that diffraction/dispersion and nonlinearity management can prolong the stability of LBs in a PT potential.

Introduction. – Solitons are formed by an exact balance of dispersion, diffraction, and nonlinearity. In nonlinear science, one of the most essential tasks is constructing exact soliton solutions to a large variety of nonlinear partial differential equations, describing diverse systems such as shallow water waves, DNA excitations, matter waves in Bose-Einstein condensates (BECs), and laser beams in nonlinear optics [1,2].

The PT symmetry was introduced in quantum mechanics in 1998 [3], when Bender and Boettcher offered the first indication that a class of non-Hermitian but parity and time-reversed (PT) symmetric Hamiltonians may possess a real bound-state spectrum. It was soon reported that balancing gain and loss is an interesting possibility for experimental realization of PT-symmetric Hamiltonians in arrays of waveguides [4,5]. Against the background of experimental findings, various types of stable spatial solitons in PT-symmetric potentials have been reported [6–11]. In optics, the complex PT-symmetric potentials can be realized in the most straightforward way by combining the spatial modulation of the refractive index with properly placed gain and loss [12]. Pioneering theoretical works [12,13] stimulated recent experimental studies and led to the observation of PT symmetry breaking in both active [14] and passive [15] optically coupled systems.

Spatiotemporal solitons, also called "light bullets" (LBs), originate from the simultaneous balance of diffraction and dispersion by the nonlinear self-focusing [16]. They have flourished into a separate research area of great importance and broad interest in various fields, from optics, plasma physics to BECs [17–19].

LBs are described by the spatiotemporal (3+1)D paraxial wave equation or the nonlinear Schrödinger equation (NLSE) [20,21]. It is long known that multi-dimensional solitons in Kerr media are unstable against wave collapse. Therefore, the search for suitable media for the generation of stable 3D LBs remains a viable topic [22]. The same stability problem impedes the creation of multidimensional solitons in self-attractive BECs. Different schemes to stabilize solitons in BECs and cubic media have been proposed in refs. [23–30]. However, spatiotemporal localized structures in PT-symmetric potentials with competing nonlinearities are less often reported [31].

In this paper, we go beyond previous work on the NLSE and study spatiotemporal solitons supported by

(a) E-mail: xusiliu1968@163.com (corresponding author)

a PT-symmetric potential with competing nonlinearities. We utilize the similarity transformation to transform this model into the standard NLS equation with constant coefficients and a PT-symmetric potential, and find exact solutions.

The paper is organized as follows. In the second section, we introduce the model and obtain different types of spatio-temporal soliton solutions. In the third section, the transmission characteristics of the LBs found, such as the phase change, the half width and the chirp are studied in media with periodic modulation and with exponentially decreasing diffraction/dispersion. Numerical simulations and comparison with analytical results are also performed. In the last section, our conclusions are briefly outlined.

The model and the soliton solutions. – We consider propagation of a light beam along the z-axis in paraxial approximation, in the presence of a PT-symmetric potential with competing nonlinearities. In this case the beam dynamics is governed by a generalized $(3+1)$D nonlinear Schrödinger model:

$$i\partial_z u + \frac{\beta(z)}{2}(\nabla_\perp u + \partial_t^2 u) + \chi_1(z)|u|^2 u + \chi_2(z)|u|^{2k}u + [v(z,r) + iw(z,r)]u = 0, \tag{1}$$

where $\nabla_\perp = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the transverse Laplacian, $r \equiv (x, y, t)$ is the spatiotemporal "transverse" position vector, and the complex envelope of the electrical field $u(z,r)$ is normalized by $(k_0 w_0)^{-1}(n_2/n_0)^{-1/2}$. The longitudinal coordinate z, transverse coordinates x, y and the co-moving time t are scaled to the diffraction length $L_D \equiv k_0 w_0^2$ (with the wave number $k_0 \equiv 2\pi n_0/\lambda$ and the input wavelength λ), the input beam width w_0, and $\sqrt{L_D}$, respectively. Function $\beta(z)$ is the diffraction/dispersion coefficient, and $\chi_1(z)$ and $\chi_2(z)$ are the cubic and $(k+1)$-th–order nonlinearity coefficients, respectively. Thus, the model contains competing cubic and higher-order power nonlinearity. The power k is any real number larger than 1. An even function $v(z,r) \equiv k_0^2 w_0^2 n_R(z,r)$ and an odd function $w(z,r) \equiv k_0^2 w_0^2 n_I(z,r)$ are the real and imaginary components of the complex PT-symmetric potential that is considered in this paper; they correspond to the index guiding and the gain or loss distribution of the optical potential, respectively. Their form will be specified later.

To find analytical solutions of eq. (1), we write the field as [13,31]:

$$u(z,r) = A(z)U[X(z,x), Y(z,y), T(z,t), Z(z)]e^{i\varphi(z,r)}, \tag{2}$$

where $A(z)$ is the amplitude, $X = X(x,z)$, $Y = Y(z,y)$, $T = T(z,t)$ are the self-similar variables, $Z = Z(z)$ is an effective propagation distance, and $\varphi(z,r)$ is the phase of the wave, all assumed to be real functions. Substituting eq. (2) into eq. (1), we aim to obtain the standard NLSE with two constant coefficients χ_{10} and χ_{20}:

$$i\frac{\partial U}{\partial Z} + \frac{1}{2}\left[\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} + \frac{\partial^2 U}{\partial T^2}\right] + \chi_{10}|U|U + \chi_{20}|U|^{2k}U + [V(X,Y,T) + iW(X,Y,T)]U = 0. \tag{3}$$

Requiring that

$$\chi_{10} \equiv \frac{A_0^2 \chi_1(z)}{\beta(z)\left(1 - s_0\int_0^z \beta(z)\mathrm{d}z\right)}, \tag{4a}$$

$$\chi_{20} \equiv \frac{A_0^{2k} \chi_2(z)}{\beta(z)\left(1 - s_0\int_0^z \beta(z)\mathrm{d}z\right)^{3k-2}}, \tag{4b}$$

$$V(X,Y,T) \equiv \frac{\left(1 - s_0\int_0^z \beta(z)\mathrm{d}z\right)^2 v(X,Y,T)}{\beta(z)}, \tag{4c}$$

$$W(X,Y,T) \equiv \frac{\left(1 - s_0\int_0^z \beta(z)\mathrm{d}z\right)^2 w(X,Y,T)}{\beta(z)}, \tag{4d}$$

after some algebra one obtains the following expressions for the self-similar variables and the amplitude and phase:

$$X(z,x) \equiv \frac{x}{1 - s_0\int_0^z \beta(z)\mathrm{d}z}, \quad Y(z,y) \equiv \frac{y}{1 - s_0\int_0^z \beta(z)\mathrm{d}z}, \tag{5a}$$

$$T(z,t) \equiv \frac{t}{1 - s_0\int_0^z \beta(z)\mathrm{d}z}, \qquad Z(z) \equiv \frac{\int_0^z \beta(z)\mathrm{d}z}{1 - s_0\int_0^z \beta(z)\mathrm{d}z}, \tag{5b}$$

$$A(z) \equiv \frac{A_0}{\left(1 - s_0\int_0^z \beta(z)\mathrm{d}z\right)^{-3/2}}, \quad \varphi(z,x,y,t) = -\frac{s_0\left(x^2 + y^2 + t^2\right)}{2\left(1 - s_0\int_0^z \beta(z)\mathrm{d}z\right)}, \tag{5c}$$

where s_0 is an arbitrary real constant. Solutions of eq. (3) can be used as seeds for the generation of various solutions of eq. (1) via relations (4), under conditions (5). Therefore, substituting solutions of eq. (3) into (2) will lead to the nonautonomous solitons of eq. (1).

We seek the solution of 3D NLSE (3) in the form

$$U(Z,X,Y,T) = \psi(X,Y,T)e^{i[\delta Z + \theta(X,Y,T)]}, \tag{6}$$

where, δ is an arbitrary constant. Here, the real-valued amplitude $\psi(X,Y,T)$ and the phase $\theta(X,Y,T)$ satisfy the following differential equations:

$$\nabla^2\psi - |\nabla\theta|^2\psi + V\psi + \chi_{10}\psi^3 + \chi_{20}\psi^{2k+1} = \delta\psi, \tag{7a}$$

$$\psi\nabla^2\theta + 2\nabla\theta\nabla\psi + W\psi = 0, \tag{7b}$$

The model admits different solutions, provided the above-mentioned conditions are met. Here, we are interested in two distinct types of soliton solutions, denoted as Case 1 and Case 2 solutions, arrived at by suitable chooses of $V(X,Y,T)$ and $W(X,Y,T)$. It follows that for an arbitrary even function ψ and odd function θ, there is an even potential V and an odd potential W for which eqs. (7) are satisfied.

Case 1 solution. If the PT-symmetric potential is chosen in the following form:

$$\begin{aligned} V(X,Y,T) &= \delta - 3 + \left(2 + \frac{W_0^2}{9}\right) \\ &\times \left[\sec h^2(X) + \sec h^2(Y) + \sec h^2(T)\right] \\ &- \psi_0^2 \chi_{10} \sec h^2(X) \sec h^2(Y) \sec h^2(T) \\ &- \psi_0^{2k} \chi_{20} \sec h^{2k}(X) \sec h^{2k}(Y) \sec h^{2k}(T), \end{aligned} \tag{8a}$$

$$\begin{aligned} W(X,Y,T) &= W_0 \sec h(X) \tanh(X) \\ &+ \sec h(Y) \tanh(Y) + \sec h(T) \tanh(T), \end{aligned} \tag{8b}$$

the potential obviously satisfies the properties of a PT-symmetric potential: $V(X,Y,T) = V(-X,-Y,-T)$ and $W(X,Y,T) = -W(-X,-Y,-T)$, and we obtain a closed-form localized solution (that satisfies the condition $\psi(X,Y,T) \to 0$ as $X,Y,T \to \pm\infty$):

$$\psi(X,Y,T) = \psi_0 \sec h(X) \sec h(Y) \sec h(T), \tag{9a}$$

with the phase given by

$$\begin{aligned} \theta(X,Y,T) \;=\; & \frac{W_0}{3}[\arctan(\sinh X) + \arctan(\sinh Y) \\ & + \arctan(\sinh T)], \end{aligned} \tag{9b}$$

and two arbitrary constants W_0 and ψ_0. Note that this soliton solution is valid irrespective of whether χ_{10} and/or χ_{20} are positive or negative.

The potentials in eqs. (8) resemble the Scarff II potential commonly used in the study of PT potentials [11]. The only additional terms are the two terms with products of hyperbolic secants: $\sec h^2(X) \sec h^2(Y) \sec h^2(T)$ and $\sec h^{2k}(X) \sec h^{2k}(Y) \sec h^{2k}(T)$, which decay much faster than $\sec h^2(X) + \sec h^2(Y) + \sec h^2(T)$ far from the origin. Therefore, it remains plausible that such PT potentials can be experimentally realized.

From eqs. (4), under the given PT potential, one obtains

$$\begin{aligned} v_I(X,Y,T) &= \frac{\beta(z)}{\left(1 - s_0 \int_0^z \beta(z) \mathrm{d}z\right)^2} \\ &* \Bigg[\delta - 3 + \left(2 + \frac{W_0}{9}\right) \left(\sec h^2(X) + \sec h^2(Y) + \sec h^2(T)\right) \\ &- \psi_0^2 \chi_{10} \sec h^2(X) \sec h^2(Y) \sec h^2(T) \\ &- \psi_0^{2k} \chi_{20} \sec h^{2k}(X) \sec h^{2k}(Y) \sec h^{2k}(T)\Bigg] \end{aligned} \tag{10a}$$

and

$$\begin{aligned} w(X,Y,T) &= \frac{W_0 \beta(z)}{\left(1 - s_0 \int_0^z \beta(z) \mathrm{d}z\right)^2} [\sec h(X) \tan h(X) \\ &+ \sec h(Y) \tan h(Y) + \sec h(T) \tan h(T)]. \end{aligned} \tag{10b}$$

The soliton solution of eq. (1) is thus given by

$$\begin{aligned} u(z,x,y,t) &= \pm \frac{A_0 \psi_0}{\left(1 - s_0 \int_0^z \beta(z) \mathrm{d}z\right)^{3/2}} \\ &\times \left[\sec h(X) \sec h(Y) \sec h(T)\right] e^{i[\theta(X,Y,T) + \delta z + \varphi(z,x,y,t)]}. \end{aligned} \tag{11}$$

where X, Y, T, $\varphi(z,r)$ and $\theta(X,Y,T)$ satisfy eqs. (5a)–(5c) and (9b), respectively. Here the phase is made up of the phase $\theta(X,Y,T)$ in solution (9b) and the chirped phase $\varphi(z,r)$, expressed by eq. (5c).

Case 2 solution. If instead, the PT potential is chosen as

$$\begin{aligned} V(X,Y,T) &= \delta - \frac{3}{k^2} + \left[\frac{1+k}{k^2} + \frac{k^2 W_0^2}{(k+2)^2}\right] \\ &\times \left[\sec h^2(X) + \sec h^2(Y) + \sec h^2(T)\right] \\ &- \psi_0^2 \chi_{10} \sec h^{2/k}(X) \sec h^{2/k}(Y) \sec h^{2/k}(T) \\ &- \psi_0^{2k} \chi_{20} \sec h^2(X) \sec h^2(Y) \sec h^2(T), \end{aligned} \tag{12}$$

leaving W equal to eq. (8b), one obtains the following amplitude:

$$\psi(X,Y,T) = \psi_0 \sec h^{1/k}(X) \sec h^{1/k}(Y) \sec h^{1/k}(T), \tag{13a}$$

with the phase given by

$$\begin{aligned} \theta(X,Y,T) \;=\; & \frac{kW_0}{k+2}[\arctan(\sinh X) + \arctan(\sinh Y) \\ & + \arctan(\sinh T)], \end{aligned} \tag{13b}$$

From eqs. (4), the complex PT potential is of the form

$$\begin{aligned} v_{II}(X,Y,T) &= \frac{\beta(z)}{\left(1 - s_0 \int_0^z \beta(z) \mathrm{d}z\right)^2} \\ &* \Bigg[\delta - \frac{3}{k^2} + \left[\frac{1+k}{k^2} + \frac{k^2 W_0}{(1+k)^2}\right] \\ &\times \left(\sec h^2(X) + \sec h^2(Y) + \sec h^2(T)\right) \\ &- \psi_0^2 \chi_{10} \sec h^{2/k}(X) \sec h^{2/k}(Y) \sec h^{2/k}(T) \\ &- \psi_0^{2k} \chi_{20} \sec h^2(X) \sec h^2(Y) \sec h^2(T)\Bigg], \end{aligned} \tag{14}$$

whereas w is of the same form as in eq. (10b). Finally, u is equal to

$$\begin{aligned} u(z,x,y,t) \;=\; & \pm \frac{A_0 \psi_0}{\left(1 - s_0 \int_0^z \beta(z) \mathrm{d}z\right)^{3/2}} \\ & * \left[\sec h^{1/k}(X) \sec h^{1/k}(Y) \sec h^{1/k}(T)\right] \\ & \times e^{i[\theta(X,Y,T) + \delta z + \varphi(z,x,y,t)]}, \end{aligned} \tag{15}$$

where X, Y, T satisfy eqs. (5a)–(5c), respectively. Here, again, the phase is made up of the phase $\theta(X,Y,T)$ in solution (13b) and the chirped phase $\varphi(z,r)$ expressed by eq. (5c). Note that the solutions found depend in a crucial way on the diffraction/dispersion coefficient $\beta(z)$. In the next section we make specific choices for the function $\beta(z)$ and discuss the properties of the corresponding solutions.

Characteristic distributions of solitons. – We next discuss dynamics of LBs found for the two special cases

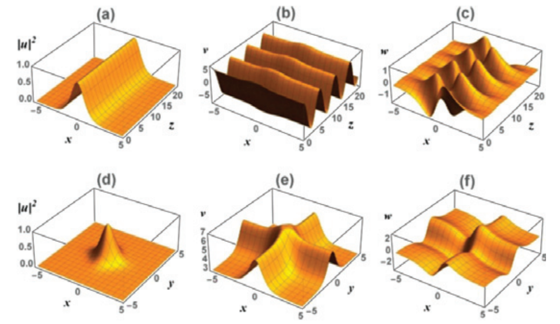


Fig. 1: (Colour online) $|u|^2$, v and w in a PMS for Case 1. (a)–(c) Plots as functions of x and z for $y = t = 0$. (d)–(f) Plots as functions of x and y for $z = t = 0$. Other parameters are $\omega = 1$, $s_0 = 0$, $A_0 = 1$, $k = 2$, $\chi_{10} = 1$, $\chi_{20} = 1$, $\beta_0 = 1$, $W_0 = 3$, $\delta = 3$ and $\psi_0 = 1$.

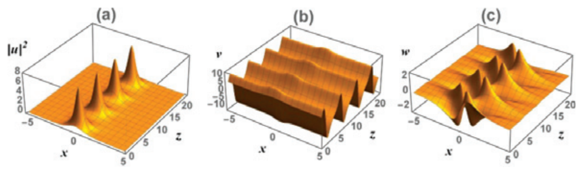


Fig. 2: (Colour online) $|u|^2$, v and w in a PMS with chirp as a function of x and z for $y = t = 0$. Parameters are the same as in fig. 1, except for $s_0 = 0.5$.

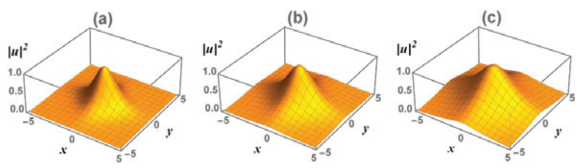


Fig. 3: (Colour online) $|u|^2$ as a function of x and y in Case 2, for various values of k: (a) $k = 2$, (b) $k = 3$ and (c) $k = 5$. Here, $z = t = 0$. The remaining parameters are the same as in fig. 1.

of the diffraction/dispersion coefficient $\beta(z)$: the periodically modulated system (PMS) $\beta(z) = \beta_0 \cos(\omega z)$ [32,33], and the diffraction/dispersion-decreasing medium (DDM) $\beta(z) = \beta_0 e^{\rho z}$ [32], where β_0 is the initial diffraction, ω is the frequency of modulation in the PMS case, and ρ is the exponential rate of growth/decay in the DDM case. One can obtain increased modulation stability of solutions using such distributed coefficients [33,34].

In fig. 1, we display the basic results for the PMS system without chirp. We see in fig. 1(a) that the basic sech profile propagates in the z direction without change. From eqs. (10a), (10b), and (14), it is seen that the complex PT-symmetric potential satisfies $v(x, y, t) = v(-x, -y, -t)$ and $w(x, y, t) = -w(-x, -y, -t)$. Thus, the index guiding and the gain or loss distribution are respectively even and odd functions with regards to x, y, and t. We observe this in figs. 1(b) and (c). In fig. 1(d), we see that the solution is localized in all transverse dimensions, while figs. 1(e) and (f) again confirm the PT symmetry.

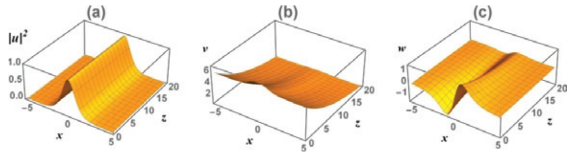


Fig. 4: (Colour online) $|u|^2$, v and w as functions of x and z in a DDM for Case 1 without chirp. We have $y = t = 0$ and $\rho_0 = -0.1$. Other parameters are the same as in fig. 1.

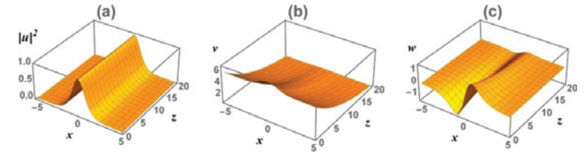


Fig. 5: (Colour online) $|u|^2$, v and w as functions of x and z in a DDM for Case 1, with chirp. Parameters are the same as in fig. 4, except $s_0 = -0.5$.

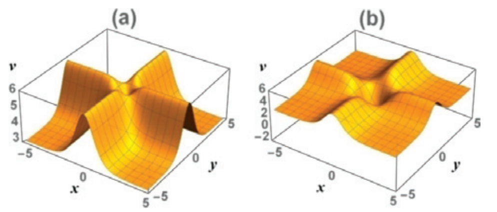


Fig. 6: (Colour online) v in a PMS for different values of χ_{10}. Parameters are the same as in fig. 1(e), except for (a) $\chi_{10} = 3$ and (b) $\chi_{10} = 10$.

Figure 2 displays the same plots as in 1(a), (b) and (c), with the addition of chirp. Hence, one can see that the chirp modulates the intensity of our solution in fig. 2(a). Somewhat surprisingly, the chirp produces only a relatively modest effect on the PT potential, as can be seen if we compare figs. 2(a) and (b) with figs. 1(a) and (b). This is because the modulation is already present in v and w via the parameter β.

Figures 3(a)–(c) show the effect of changing k for Case 2. We find that the increase in parameter k broadens our solution, as can be expected from eq. (13a).

Figures 4 and 5 give the results for Case 1 in DDM. In fig. 5 there is the addition of chirp. A negative number was chosen for s_0, given that the chirp blows up for positive s_0 at some point. One can see from fig. 4(a) that fundamentally the solution without chirp is the same, however with the addition of chirp in fig. 5(a), an exponential decay of the solution is observed, down to some small value. From figs. 4(b), (c) and figs. 5(b), (c) one can see that the potentials also ultimately decay to 0.

In fig. 6 we display the effect of increasing the terms associated with χ_{10}. Given a large enough value, they produce a potential well in the center of v. A similar effect can be found if instead, one increases χ_{20}.

In fig. 7 we observe the effect of an increase in k on the potential, for both the PMS (a) and the DDM (b). One

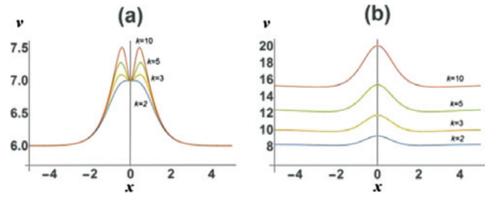


Fig. 7: (Colour online) v in a PMS (a) and DDM (b) for various values of k. The remaining parameters are the same as in figs. 1 and 4.

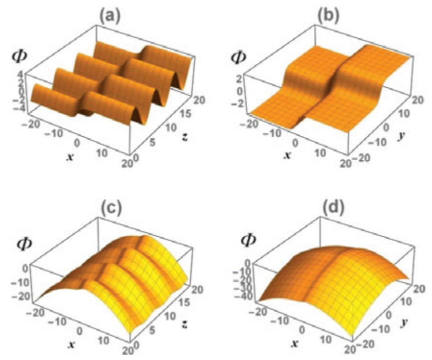


Fig. 8: (Colour online) The total phase Φ in PMS as a function of x and z for $y = t = 0$ in (a) and (c) and of x and y for $z = t = 0$ in (b) and (d). In (a) and (b) $s_0 = 0$, whereas in (c) and (d) $s_0 = 0.1$. Other parameters are the same as in fig. 1.

can see that the two effects are drastically different. For the PMS, an increase in k leads to the formation of a wall, similar to when χ_{10} is increased, as shown in fig. 6. Since the term in which k appears decays rapidly away from the origin, there is no change in the baseline value of v. For the DDM, on the other hand, there is a general increase of the potential, both in the baseline value and in the peak.

Finally, in fig. 8 we present the overall phase $\Phi(z, x, y, t) = \theta(X, Y, T) + \delta z + \varphi(z, x, y, t)$ of the solution. One notes in figs. 8(a) and (b) that in the case of no chirp the only term present is θ, whereas in the presence of chirp the phase is dominated by φ, which is a parabolic function.

The formal stability analysis of our solutions is a formidable problem that will not be addressed in this paper. In principle, these multi-dimensional LBs —as most of other LBs— are unstable, but can propagate stably over prolonged distances. In order to test the stability of our solutions, we perform numerical integration of eq. (1), with a white noise of variance $\sigma = 0.15$ added to the input, for various distances. We used a 3D split-step Fourier technique and considered the initial conditions corresponding to the formulas for u in eqs. (11) and (15), with $z = 0$. It was found that the numerical calculations indicate no collapse, and stable propagation over tens of diffraction/dispersion lengths is observed, except for some small oscillations. Moreover, a LB is more stable in the PMS than in the DDM, and the larger the power k, the larger the instabilities. Thus, as it is seen, the diffraction management of the type considered here can prolong the life of these LBs significantly.

Conclusion. – In summary, we have found light bullets supported by specific parity-time symmetric potentials with competing nonlinearities and have analyzed their properties. We established that the dynamic characteristics of LBs, such as the intensity, phase, and chirp, can be modified by the variation of the diffraction/dispersion parameter and the strength of nonlinearities. These results may provide additional potential applications in the field of PT-symmetric systems.

* * *

This work is supported in China by Natural Science Foundation of Hubei Province (2015CFC779 and 2013CFC011), and by the Natural Science Foundation of Guangdong Province, under Grant No. 1015283001000000. The work in Qatar is supported by the NPRP6-021-1-005 project with the Qatar National Research Fund, a member of the Qatar Foundation. The work in Serbia is supported by the Serbian Ministry of Education and Science, under project OI 171006.

REFERENCES

[1] Barenblatt G. I., *Scaling, Self-similarity, and Intermediate Asymptotics* (Cambridge University Press, Cambridge, UK) 1996.

[2] Ye F. W., Kartashov Y. V. and Torner L., *Phys. Rev. A*, **74** (2006) 063616; Sakaguchi H. and Malomed B. A., *Phys. Rev. A*, **75** (2007) 013609.

[3] Bender C. M. and Boettcher S., *Phys. Rev. Lett.*, **80** (1998) 5243.

[4] El-Ganainy R., Makris K. G., Christodoulides D. N. and Musslimani Z. H., *Opt. Lett.*, **32** (2008) 2632.

[5] Musslimani Z. H., Makris K. G., El-Ganainy R. and Christodoulides D. N., *Phys. Rev. Lett.*, **103** (2009) 030402.

[6] Jisha C. P., Alberucci A., Brazhnyi V. A. and Assanto G., *Phys. Rev. A*, **89** (2014) 013812.

[7] He Y., Zhu X., Mihalache D., Liu J. and Chen Z., *Phys. Rev. A*, **85** (2012) 013831.

[8] Moreira F. G., Abdullaev F. Kh., Konotop V. V. and Yulin A., *Phys. Rev. A*, **86** (2012) 053815.

[9] Moreira F. G., Konotop V. V. and Malomed B. A., *Phys. Rev. A*, **87** (2013) 013832.

[10] Li K., Zezyulin D. A., Kevrekidis P. G., Konotop V. V. and Abdullaev F. Kh., *Phys. Rev. A*, **88** (2013) 053820.

[11] Musslimani Z. H., Makris K. G., El-Ganainy R. and Christodoulides D. N., *Phys. Rev. Lett.*, **100** (2008) 030402.

[12] Makris K. G., El-Ganainy R., Christodoulides D. N. and Musslimani Z. H., *Int. J. Theor. Phys.*, **50** (2011) 1019.

[13] Makris K. G., El-Ganainy R., Christodoulides D. N. and Musslimani Z. H., *Phys. Rev. Lett.*, **100** (2008) 103904.
[14] Ruter C. E., Makris K. G., El-Ganainy R., Christodoulides D. N., Segev M. and Kip D., *Nat. Phys.*, **6** (2010) 192.
[15] Guo A., Salamo G. J., Duchesne D., Morandotti R., Volatier-Ravat M., Aimez V., Siviloglou G. A. and Christodoulides D. N., *Phys. Rev. Lett.*, **103** (2009) 093902.
[16] Malomed B. A., Mihalache D., Wise F. and Torner L., *J. Opt. B Quantum Semiclass. Opt.*, **81** (2005) R53.
[17] Mihalache D., *Proc. SPIE*, **5518** (2004) 564.
[18] Xu S. L. and Belić M. R., *J. Mod. Opt.*, **62** (2015) 683.
[19] Xu S. L. and Belić M. R., *J. Opt. Soc. Am. B*, **30** (2013) 2715.
[20] Lobanov V. E., Kartashov Y. V. and Torner L., *Phys. Rev. Lett.*, **105** (2010) 033901.
[21] Bergé L., Skupin S., Nuter R., Kasparian J. and Wolf J. P., *Rep. Prog. Phys.*, **70** (2007) 1633.
[22] Blagoeva A. B., Dinev S. G., Dreischuh A. A. and Naidenov A., *IEEE J. Quantum Electron.*, **27** (1991) 2060.
[23] Burgess I. B., Peccianti M., Assanto G. and Morandotti R., *Phys. Rev. Lett.*, **102** (2009) 203903.
[24] Segev M. and Silberberg Y., *Phys. Rep.*, **463** (2008) 1.
[25] Torner L. and Kartashov Y. V., *Opt. Lett.*, **34** (2009) 1129.
[26] Pertsch T., Zentgraf T., Peschel U., Bräuer A. and Lederer F., *Phys. Rev. Lett.*, **88** (2002) 093901.
[27] Mihalache D., Mazilu D., Lederer F. and Kivshar Yu. S., *Opt. Lett.*, **32** (2007) 2091.
[28] Sakaguchi H., Li B. and Malomed B. A., *Phys. Rev. E*, **89** (2014) 0320920.
[29] Zhang Y. C., Zhou Z. W., Malomed B. A. and Pu H., *Phys. Rev. Lett.*, **115** (2015) 253902.
[30] Kartashov Y. V., Malomed B. A., Konotop V. V., Lobanov V. E. and Torner L., *Opt. Lett.*, **40** (2015) 1045.
[31] Dai C. Q., Wang X. G. and Zhou G. Q., *Phys. Rev. A*, **89** (2014) 013834.
[32] Kivshar Y. S. and Agrawal G. P., *Optical Solitons: From Fibers to Photonic Crystals* (Academic, San Diego) 2003.
[33] Greiner M., Mandel O., Esslinger T., Hansch T. W. and Bloch I., *Nature (London)*, **415** (2002) 6867.
[34] Petrović N., Aleksić N. and Belić M., *Opt. Express*, **23** (2015) 10616.

Nonautonomous vector matter waves in two-component Bose-Einstein condensates with combined time-dependent harmonic-lattice potential

Article *in* Journal of optics · October 2015

DOI: 10.1088/2040-8978/17/10/105605

4 authors, including:

Nikola Z Petrović
Institute of Physics Belgrade

Milivoj R. Belić
Texas A&M University at Qatar

IOP Publishing Journal of Optics
J. Opt. 17 (2015) 105605 (6pp) doi:10.1088/2040-8978/17/10/105605

Nonautonomous vector matter waves in two-component Bose-Einstein condensates with combined time-dependent harmonic-lattice potential

Si-Liu Xu[1,5], Guo-Peng Zhou[2], Nikola Petrović[3] and Milivoj R Belić[4]

[1] The School of Electronic and Information Engineering, HuBei University of Science and Technology, Xianning 437100, People's Republic of China
[2] Institute of Engineering and Technology, HuBei University of Science and Technology, Xianning 437100, People's Republic of China
[3] Institute of Physics, University of Belgrade, PO Box 68, 11001 Belgrade, Serbia
[4] Texas A&M University at Qatar, PO Box 23874 Doha, Qatar

E-mail: xusiliu1968@163.com

Received 14 April 2015, revised 2 June 2015
Accepted for publication 8 June 2015
Published 25 August 2015

Abstract

We construct exact self-similar soliton solutions of three-dimensional coupled Gross–Pitaevskii equations for two-species Bose–Einstein condensates (BECs) in a combined time-dependent harmonic-lattice potential. Based on these solutions, we investigate the control and manipulation of solitary waves for three kinds of BECs with changing diffraction and nonlinearity coefficients; the solutions include Ma breathers and Peregrine and Akhmediev soliton solutions. Our results indicate that matter waves readily propagate in this system. It is shown that diffraction and lattice potential factors play important roles in the beam evolution characteristics, such as the peak, the phase offset, the linear phase, and the chirp.

Keywords: Bose-Einstein condensates, vector solitons, harmonic-lattice potential

1. Introduction

Bose–Einstein condensates (BECs) in weakly interacting atomic gases have offered a practical means of studying nonlinear behavior by using the concept of matter waves. A BEC soliton is a macroscopic localized excitation, which is quite common and has been extensively studied both theoretically and experimentally. Usually, in one-dimensional (1D) systems, a repulsive BEC or an attractive BEC is believed to be stable, bright and dark solitons are expected to exist, respectively [1]. In integrable nonlinear systems, solitons have a variety of applications in fiber optics as well as other fields [2, 3].

In order to get stable BEC solitons, an effective method is to consider the effects of periodic time modulation of the optical-lattice (OL) potential [4, 5]. The time-dependent OL can suppress the leading-order diffraction; hence the formation of sub-diffractive solitons was realized in BECs [6, 7]. It had been demonstrated that the strong, periodically shaken OLs could cause the phase coherence of BECs [8]. Moreover, a remarkable OL is the combined harmonic-lattice potential, in which the periodic time-dependent lattice is accompanied by a harmonic confining potential. Using the Feshbach resonance (FR) technique, harmonic-lattice potential may be realized in BECs by tuning the external magnetic field and the optically controlled interactions [9, 10]. The dynamics of BEC solitons in the harmonic-lattice potentials has become a hot research topic both in theory [11–13] and experiment [14, 15]. Some more recent accounts have been presented in [16–19].

Under a harmonic trapping potential, 1D exact vector solitons and localized nonlinear matter waves in two-component BECs were found in reference [16]. In reference [17] Feijoo *et al* have discussed the possibility of emitting vector

[5] Author to whom any correspondence should be addressed.

solitons from a two-component elongated BEC by manipulating in time the inter- or intra-species scattering lengths with FR tuning. In the case of repulsive and attractive condensates, the 2D vector solutions have been studied in references [20] and [21]. In the experiments, Dalfovo *et al* have realized an attractive two-component BEC by simultaneous magnetic trapping of atoms of ^{7}Li [22]. Moreover, Papp *et al* realized two-component BECs with tunable inter-component interaction [23]. In addition, dynamical creation of fractionalized vortices and vortex lattices was considered in [24], while the dynamics of bright solitons in BECs with time-dependent atomic scattering length in an expulsive parabolic potential was considered in [25].

With the basis of this motivation in mind, we investigate in this paper the explicit novel solutions of the (3 + 1)-dimensional coupled Gross–Pitaevskii equations (GPE) with temporal modulation of the nonlinearities and the harmonic-lattice potential. Three kinds of soliton solutions are presented, and the stability of these solitons is investigated numerically.

The paper is organized as follows. The model describing vector matter waves in two-component Bose–Einstein condensates with combined time-dependent harmonic-lattice potential is introduced in section 2. In section 3, novel Ma breathers and Peregrine and Akhmediev soliton solutions in the combined time-dependent harmonic-lattice potential are demonstrated, and the evolution characteristics of rogue nonlinear matter wave soliton solutions are discussed. Concluding remarks, with a simple summary, are given in section 4.

2. The model

We describe nonlinear interactions in a binary mixture of BECs, consisting of two different spin states for the complex macroscopic wave functions $u_{1,2}$:

$$\mathrm{i}\frac{\partial u_1}{\partial t} + \frac{\beta(t)}{2}\Delta u_1 + \chi(t)\left(c_1\left|u_1\right|^2 + c\left|u_2\right|^2\right)u_1 + V(t, x, y, z)u_1 = \mathrm{i}\gamma(t)u_1, \tag{1a}$$

$$\mathrm{i}\frac{\partial u_2}{\partial t} + \frac{\beta(t)}{2}\Delta u_2 + \chi(t)\left(c_2\left|u_2\right|^2 + c\left|u_1\right|^2\right)u_2 + V(t, x, y, z)u_2 = \mathrm{i}\gamma(t)u_2, \tag{1b}$$

where Δ is the three-dimensional (3D) Laplacian. All coordinates are made dimensionless by the choice of variables. The functions $\beta(t)$ and $\chi(t)$ represent the diffraction coefficient and the nonlinearity coefficient, respectively. In this paper, we choose the time-dependent harmonic-lattice potential as $V(t, x, y, z) = V_1(t) + V_2(t)(x + y + z) + V_3(t)(x^2 + y^2 + z^2)$, where V_j are, as yet, not specified functions. The parameter $\gamma(t)$ is the gain or loss coefficient. The constants c, c_1, and c_2 determine the ratio of the coupling strengths of the cross-phase modulation to the self-phase modulation. For the linearly polarized eigenmodes we have $c_1 = c_2 = 1, c = 2/3$ [26], and the total number of particles of each component is determined by the norm $N_{1,2} = \iiint |u_{1,2}|^2 \mathrm{d}x\mathrm{d}y\mathrm{d}z$.

To obtain exact analytical solutions of equation (1), we introduce a self-similar transformation of the solution [27]:

$$u_j(t, x, y, z) = \rho_0\left(\frac{c - c_{j-1}}{c^2 - c_1c_2}\right)^{1/2} D^3(t)U(T, X) \times \mathrm{e}^{\mathrm{i}\varphi(t,x,y,z)+\Gamma(t)}, \tag{2}$$

where $j = 1,\ 2$, $c_0 = c_2$, and ρ_0 is a constant. The similarity variable T, the effective distance X, and the phase φ are assumed to be real functions. $U = U(T, X)$ is a complex function, $X = X(t, x, y, z)$ is the multivariate self-similar variable, and $\varphi(t, x, y, z)$ is the phase of the matter wave. Substituting equation (2) into equation (1), one obtains the standard nonlinear Schrödinger (NLS) equations with transformed variables

$$\mathrm{i}\frac{\partial U}{\partial T} + \frac{1}{2}\frac{\partial^2 U}{\partial X^2} + |U|^2 U = 0, \tag{3}$$

provided the following two conditions are satisfied:

$$\beta(t)\left(X_x^2 + X_y^2 + X_z^2\right) = T_t, \tag{4a}$$

$$D^6\chi(t) = T_t. \tag{4b}$$

Here, the variable subscripts stand for the corresponding partial derivatives. Considering equation (3), one obtains for D, T, X, and φ the following solutions:

$$D = \exp\left[-\int_0^t \beta(\tau)c_p(\tau)\mathrm{d}\tau\right], \tag{5a}$$

$$T = \int_0^t \left(k^2 + l^2 + m^2\right)\beta(\tau)D^4(\tau)\mathrm{d}\tau, \tag{5b}$$

$$X = (kx + ly + mz)D^2 - (k + l + m)\int_0^t \beta(\tau)D^4(\tau)H(\tau)\mathrm{d}\tau, \tag{5c}$$

$$\varphi = c_p(t)\left(x^2 + y^2 + z^2\right) + H(t)D^2(t)(x + y + z) - \frac{3}{2}\int_0^t \beta(\tau)D^4(\tau)H^2(\tau)\mathrm{d}\tau, \tag{5d}$$

where c_p is the chirp function, $H(t) = H_0 + \int_0^t V_2(\tau)/D^2(\tau)\mathrm{d}\tau$ and $\Gamma(t) = \int_0^t \gamma(\tau)\mathrm{d}\tau$. The subscript '0' denotes the initial values of the corresponding functions at time $t = 0$. The parameters k, l, m, and H_0 are arbitrary constants.

The transformation of equation (1) into an integrable equation (3) imposes the following constraint on $\chi(z)$:

$$\chi(t) = \frac{\left(k^2 + l^2 + m^2\right)}{D^2(t)}\beta(t), \tag{6a}$$

and a constraint on the quadratic term of the lattice potential:

$$V_3(t) = 2c_{p,t} + 4\beta c_p^2. \tag{6b}$$

Therefore, substituting equation (3) into (2), the self-similar solitons of equation (1) can be obtained. The solutions of

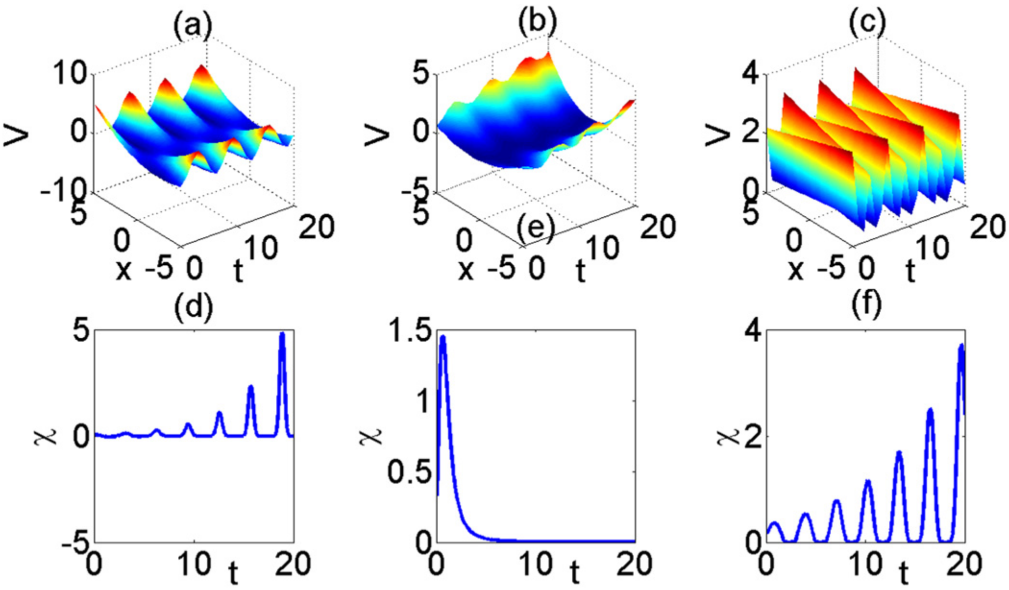


Figure 1. Profiles of the OL potential and the nonlinearity coefficient with $V_1 = 0$ and $V_2 = \cos(2t)$. (a), (d) The first lattice potential $V_3 = 2[1 + 0.05\exp(2t)]$ and $\beta(t) = \cos(2t)$; (b), (e) The second lattice potential $V_3 = 4(1 + t)$, $\beta(t) = 2/(1 + t)^2$; (c), (f) The third lattice potential $V_3 = \exp(\cos(t))$, $\beta(t) = 2\exp[2\sin(2t)/(1 + \sin(2t))]$, respectively; other parameters are $c_{p0} = -0.1$, $l = m = \gamma = 0$, $c_1 = -1$, $c_2 = -8$, $c = 5$, and $H_0 = \rho_0 = 1$.

equation (3) can be considered as seeds to generate various solutions of equation (1) under the conditions (4).

Following reference [28] and using the inverse scattering technique, the breather solution (Ma breather) of the standard NLS equation (3) can be found:

$$U(T, X) = \left[2\frac{\cos\left(2\sqrt{2}T\right) + i\sqrt{2}\sin\left(2\sqrt{2}T\right)}{\sqrt{2}\cosh(2X) - \cos\left(2\sqrt{2}T\right)} - 1\right]e^{iT}. \tag{7a}$$

In the same way, when the period becomes infinite, the rogue wave solution with the following basic structure is obtained [29, 30]:

$$U_n(T, X) = \left[(-1)^n + \frac{G_n(T, X) + iH_n(T, X)}{D_n(T, X)}\right]e^{iT}, \tag{7b}$$

where $n = 1, 2, 3 \ldots$. When $n = 1$, the Peregrine soliton [31] is found, with $G_1 = 4$, $H_1 = 8X$, and $D_1 = 1 + 4T^2 + 4X^2$; if $n = 2$, one finds the Akhmediev soliton solution [32]. Thus, collecting the partial solutions together, we obtain the exact rogue wave solution of equation (1):

$$u_j(t, x, y, z) = \rho_0\left(\frac{c - c_{j-1}}{c^2 - c_1c_2}\right)^{1/2} D^3(t)U(T, X) \times e^{i\varphi(t,x,y,z)+\Gamma(t)}, \tag{8}$$

where U is given by equations (7a) and (7b). The expressions above indicate that the width $W(t)$ of the soliton for two components is related to $D^{-3}(t)$, the amplitude of the solitons is proportional to $\rho_0\left(c - c_{j-1}/c^2 - c_1c_2\right)^{1/2}D^3(t)\exp[\Gamma(t)]$, and the center of the solitons is located at $\beta(t)D^2(t)H(t)$. The linear part of the phase is determined by $H(t)D^2(t)$, and the chirp in the phase c_p is related to the lattice potential V_3 and the diffraction coefficient β. It is easily seen that the diffraction coefficient $\beta(t)$ and the lattice potential V play an important role in determining the evolution characteristics of the solitons.

3. Solitary solutions

In this paper we demonstrate explicit vector matter wave solutions in combined time-dependent magneto-optical potentials (TDMOPs). Using the FR techniques [33, 34], as previously described, the combined TDMOP may be realized in BECs by tuning the external magnetic field and the optically controlled interactions. Their profiles and nonlinearity coefficients are plotted in figure 1.

We study the dynamics of the exact localized nonlinear waves of equation (1). To illustrate the characteristics of the analytic solution (8), we present the corresponding system management scheme with the parameter functions $\beta(t)$, $\chi(t)$, $\gamma(t)$, and $V(x, y, z, t)$, the choice of which leads to the controlled development of rogue waves. Without much loss of generality we assume $\gamma(t) = 0$ and $V_1 = 0$. We also take $V_2 = \cos(2t)$ throughout and make three choices of the V_3 potential.

First, we study the influence of the first lattice potential, with $V_3 = 2[1 + 0.05\exp(2t)]$ in equation (1), and in the diffraction management for $\beta = 2\cos(2t)$. As the condition (6b) is a variable-coefficient differential equation, the analytical solution cannot be found; thus a Runge–Kutta numerical solution is used. By combining equations (5)–(7) with equation (8), the spatiotemporal evolution of the soliton wave

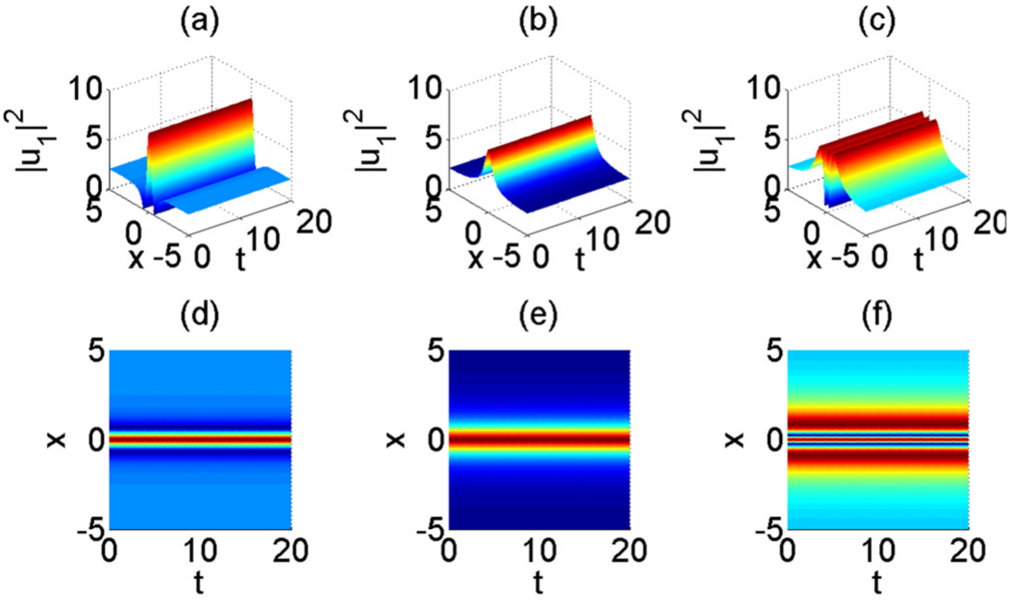


Figure 2. Condensate distributions of the soliton wave $|u_1|^2$ in the first lattice potential. (a) Ma breather; (b) Peregrine first-order wave; (c) Akhmediev second-order wave. Other parameters are as in figure 1.

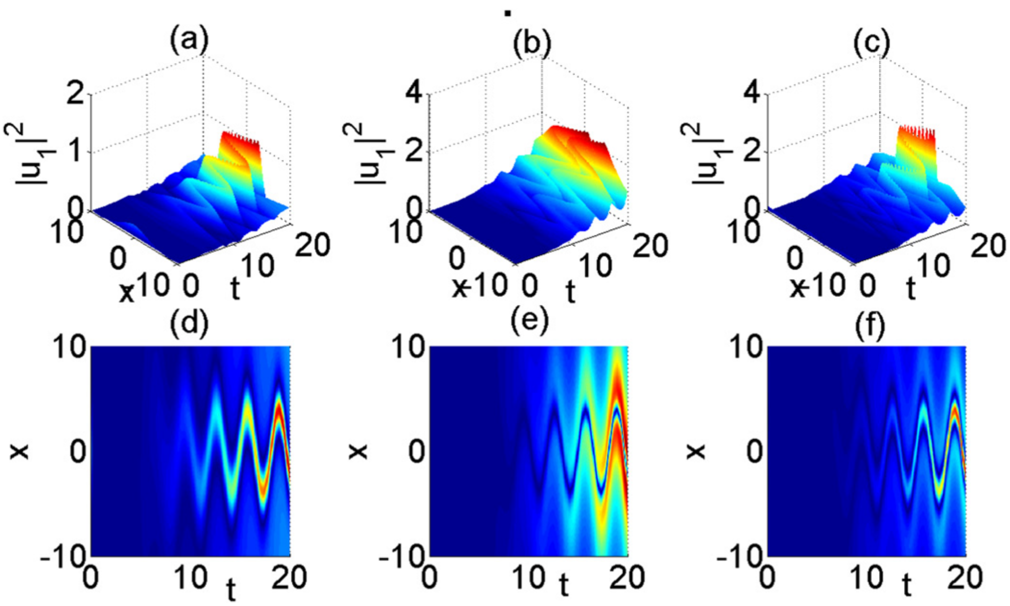


Figure 3. Same as figure 2 but for the second potential.

$|u_1|^2$ is shown in figure 2. This figure displays the intensity profiles of the Ma breather and the Peregrine and the Akhmediev rogue waves of equation (8), represented as functions of $kx + ly + mz$ and t, without loss. One can see that the Ma breather and the Peregrine and Akhmediev waves have one, two, and three symmetrical maxima in the center of the $kx + ly + mz$ direction. It is noteworthy to observe that the three wave peaks keep the same intensity with the change in time; however, their intensity distributions differ.

Figure 3 demonstrates the condensate distributions of the three kinds of solitary waves $\left|u_1\right|^2$ with the parameters $\beta(t) = 2/(1 + t)^2$ and $V_3 = 4(1 + t)$. It is shown that the condensates oscillate in the $kx + ly + mz$ direction as a function of time. When $t < 10$, the intensities increase very slowly. After $t = 20$ one can see that the intensities remain constant. Concerning the Peregrine soliton, the two usual symmetric oscillating distributions are found. However, the usual symmetry of the Akhmediev solutions is changed, and the central peak is much larger than the two side peaks.

Figure 4 demonstrates the condensate distributions of the three kinds of solitary waves $\left|u_1\right|^2$ with the parameters $V_3(t) = \exp[\cos(2t)]$, $\beta(t) = 2\exp[2\sin(2t)/(1 + \sin(2t))]$. Similar to figure 2, the three centrosymmetric solutions in the $kx + ly + mz$ direction are shown. For the Ma breather, the

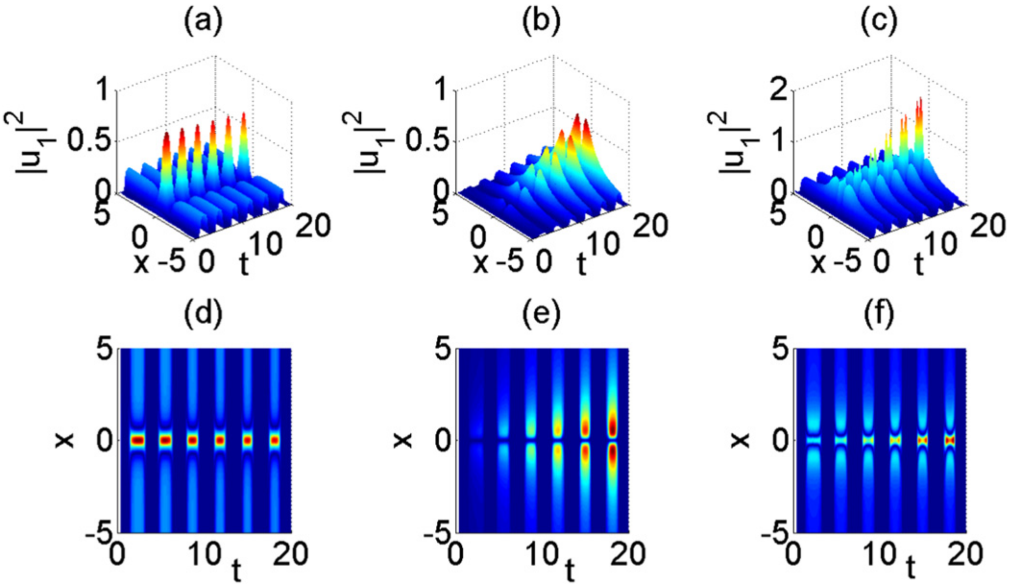


Figure 4. Same as figure 2 but for the third potential.

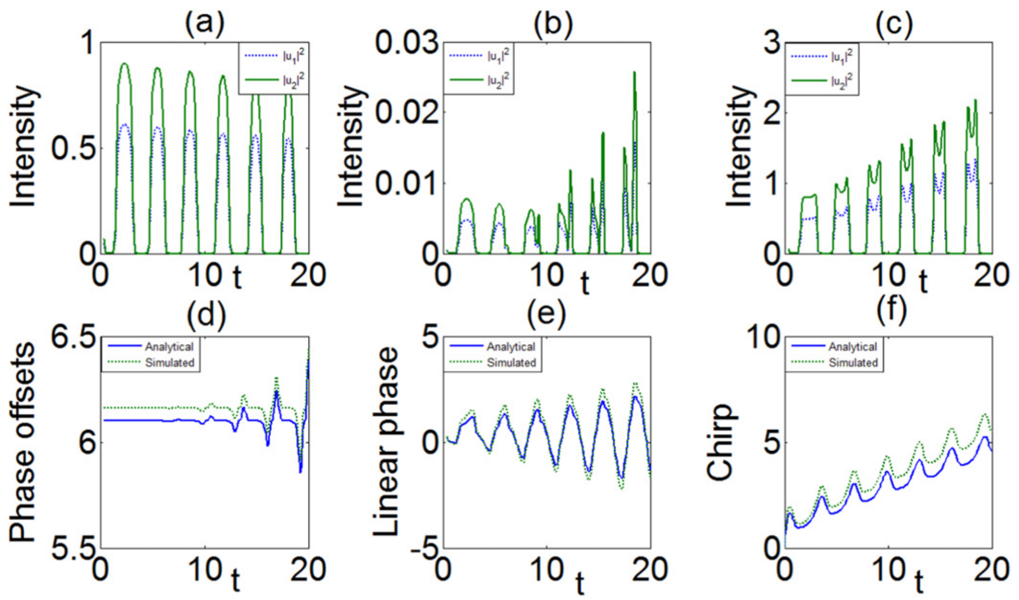


Figure 5. (a)–(c) The condensate distributions of wave packets $|u_1|^2$ and $|u_2|^2$ for $kx + ly + mz = 0$. (d)–(f) Comparison of the analytical solution with the numerical simulation of the phase offsets, the linear phase, and the chirp of wave packets. The parameters are as in figure 4.

difference is that the peak decreases slowly with time. However, for the Peregrine and the Akhmediev, the peaks increase gradually. Also, the usual symmetries of the Peregrine and Akhmediev solutions are now displayed.

Figures 5(a)–(c) depict the comparison of the condensate distributions of wave packets $|u_1|^2$ with $|u_2|^2$ for $kx + ly + mz = 0$. One can see that the central intensities of the $\left|u_1\right|^2$ wave packets are smaller than those of $\left|u_2\right|^2$. These analytical properties have been confirmed by the direct numerical integration of equation (1). We used a 3D split-step fast Fourier technique and considered an initial form of the solution given by equation (8) at $t = 0$. The comparison between the simulations and analytical predictions for this case are shown in figures 5(d)–(f). These figures highlight significant features of the evolution of 3D vector wave packets. The simulated phase offset, the linear phase, and the chirp are in good agreement with the analytical results. These results confirm the fact that the diffraction managements and TDMOPs of the type considered can prolong the life of condensate wave packets.

4. Conclusion

In summary, we have demonstrated condensate solitary waves supported by different TDMOPs in the coupled vector

GPE. The control and manipulation of solitary waves are investigated for the three kinds of changing diffraction, potential, and nonlinearity parameters of GPE, considering Ma breathers and the Peregrine and Akhmediev soliton solutions. Our results indicate that diffraction and lattice-potential factors play important roles in the evolution characteristics, such as the peak, the phase offset, the linear phase, and the chirp of the beam.

Acknowledgments

This work was supported in China by the National Natural Science Foundation of China (61340042), the Natural Science Foundation of Hubei Province (2013CFC011), and the National college students' innovative entrepreneurial training program at local colleges, under Grant 201210927052. In addition, work in Qatar was supported by the NPRP 6-021-1-005 project with the Qatar National Research Fund (a member of the Qatar Foundation) and in China by the Natural Science Foundation of Guangdong Province under Grant No. 1015283001000000. The work in Serbia is supported by the Serbian Ministry of Education and Science under project OI 171006.

References

[1] Ruprecht P A, Holland M J, Burnett K and Edwards M 1995 *Phys. Rev.* A **51** 4704
[2] Sulem C and Sulem P L 1999 *Nonlinear Schrödinger Equations: Self-focusing and Wave Collapse* (New York: Springer)
[3] Carr L D, Clark Charles W and Reinhardt W P 2000 *Phys. Rev.* A **62** 063610
[4] Poletti D, Alexander T J, Ostrovskaya E A, Li B and Kivshar Y S 2008 *Phys. Rev. Lett.* **101** 150403
[5] Xu S-L, Liang J C and Yi L 2010 *J. Opt. Soc. Am.* B **27** 99
[6] Staliunas K, Herrero R and de Valćarcel G J 2006 *Phys. Rev.* E **73** 065603
[7] Staliunas K, Herrero R and de Valćarcel G J 2007 *Phys. Rev.* A **75** 011604
[8] Lignier H, Sias C, Ciampini D, Singh Y, Zenesini A, Morsch O and Arimondo E 2007 *Phys. Rev. Lett.* **99** 220403
[9] Roberts J L, Claussen N R, Burke J P, Greene C H, Cornell E A and Wieman C E 1998 *Phys. Rev. Lett.* **81** 5109
[10] Theis M, Thalhammer G, Winkler K, Hellwig M, Ruff G, Grimm R and Denschlag J H 2004 *Phys. Rev. Lett.* **93** 123001
[11] Schneider P I and Saenz A 2013 *Phys. Rev.* A **87** 052712
[12] Zhang R, Sapiro R E, Mhaskar R R and Raithel G 2008 *Phys. Rev.* A **78** 053607
[13] Ott H, de Mirandes E, Ferlaino F, Roati G, Türck V, Modugno G and Inguscio M 2004 *Phys. Rev. Lett.* **93** 120407
[14] Eiermann B, Anker T, Albiez M, Taglieber M, Treutlein P, Marzlin K P and Oberthaler M K 2004 *Phys. Rev. Lett.* **92** 230401
[15] Blakie P B, Bezett A and Buonsante P J 2007 *Phys. Rev.* A **75** 063609
[16] Zhang X-F, Hu X-H, Liu X-X and Liu W M 2009 *Phys. Rev.* A **79** 033630
[17] Feijoo D, Paredes A and Michinel H 2013 *Phys. Rev.* A **87** 063619
[18] Kanna T, Babu M R, Tsitoura F, Nistazakis H E and Frantzeskakis D J 2013 *J. Phys. A: Math. Theor.* **46** 475201
[19] Goodman Roy H, Kevrekidis P G and Carretero-González R 2015 *SIAM J. Appl. Dyn. Syst.* **14** 699–729
[20] García-Ripoll J J and Pérez-García V M 2000 *Phys. Rev. Lett.* **84** 4264
[21] Musslimani Z H, Segev M, Christodoulides D N and Soljačić M 2000 *Phys. Rev. Lett.* **84** 1164
[22] Dalfovo F, Giorgini S, Pitaevskii L P and Stringari S 1999 *Rev. Mod. Phys.* **71** 463
[23] Papp S B, Pino J M and Wieman C E 2008 *Phys. Rev. Lett.* **101** 040402
[24] Ji A C, Liu W M, Song J L and Zhou F 2008 *Phys. Rev. Lett.* **101** 010402
[25] Liang Z X, Zhang Z D and Liu W M 2005 *Phys. Rev. Lett.* **94** 050402
[26] Cao X D and Meyerhofer D D 1994 *J. Opt. Soc. Am.* B **11** 380–85
[27] Dai C and Zhu H 2013 *J. Opt. Soc. Am.* B **30** 3291–97
[28] Ma Y C 1979 *Stud. Appl. Math.* **60** 43
[29] Osborne A R, Onorato M and Serio M 2000 *Phys. Lett.* A **275** 386
[30] Chabchoub A, Hoffmann N P and Akhmediev N 2011 *Phys. Rev. Lett.* **106** 204502
[31] Peregrine D H 1983 *J. Aust. Math. Soc. Ser. B Appl. Math* **25** 16
[32] Akhmediev N, Ankiewicz A and Taki M 2009 *Phys. Lett.* A **373** 675
[33] Staliunas K and Longhi S 2008 *Phys. Rev.* A **78** 033606
[34] Mayteevarunyoo T and Malomed B A 2009 *Phys. Rev.* A **80** 013827

Opt Quant Electron (2016) 48:268
DOI 10.1007/s11082-016-0522-1

General Jacobi elliptic function expansion method applied to the generalized (3 + 1)-dimensional nonlinear Schrödinger equation

Nikola Z. Petrović[1] · Moiz Bohra[2]

Received: 19 November 2015 / Accepted: 15 March 2016 / Published online: 31 March 2016

Abstract We generalize the Jacobi elliptic function expansion method used to solve the (3 + 1)-dimensional nonlinear Schrödinger equation for the case of an arbitrary inverse of an elliptic integral. Among the obtained solutions are functions based on the Weierstrass elliptic function and the inverses of Carlson's elliptic integrals.

Keywords F-expansion · Jacobi elliptic functions · Nonlinear Schrödinger equation

1 Introduction

The generalized nonlinear Schrödinger equation (NLSE) is a generic model that is very important in NL optics, where it describes the full spatiotemporal optical solitons or light bullets (Akhmediev and Ankiewicz 1997; Kivshar and Agrawal 2003; Hasegawa and Matsumoto 2003; Malomed 2006). For this equation, along with several other related equations such as the Klein–Gordon (KG) equation and the Korteweg de-Vries (KdV) equation, there is a large interest in finding novel exact solutions (Drazin and Johnson 1989). One of the most popular approaches is to use an expansion method which assumes a certain ansatz for the solutions and the solutions are then expanded in terms of one or perhaps more than one function, as was done in Ref. Zhang (2010). Some of the most popular expansion methods are the trigonometric function expansion method (Zhang 2008;

This article is part of the Topical Collection on Advances in the Science of Light.

Guest Edited by Jelena Radovanovic, Milutin Stepić, Mikhail Sumetsky, Mauro Pereira and Dragan Indjin.

✉ Nikola Z. Petrović
nzpetr@ipb.ac.rs

[1] Institute of Physics, University of Belgrade, Pregrevica 118, Belgrade, Serbia

[2] Qatar Science Leadership Program, Qatar Foundation, P.O. Box 5825, Doha, Qatar

Zhang et al. 2011c), including hyperbolic trigonometric functions (Malfliet and Hereman 1996; Zhang et al. 2013a), the exponential function expansion method (He and Wu 2006), the $\left(\frac{G'}{G}\right)$-expansion method (Li and Wang 2009; Miao and Zhang 2011; Zhang et al. 2013b), the bifurcation method (Zhang et al. 2011a, b) and the first integral method (Zhang et al. 2013c). These forms of solutions have in common that they assume a linear form for the phase of the solution. However, Kruglov et al. (2003) have proposed an ansatz for the solution to the NLSE which involves a quadratic term in the phase, commonly known as the chirp (Lai and Cai 2011).

A prominent expansion method that has emerged is the Jacobi elliptic function (JEF) expansion method (Zhang 2012, 2015). The JEF is a natural choice for the NLSE with a Kerr nonlinearity because it satisfies the second order nonlinear differential involving a nonlinear term of the third degree and also because it encompasses both traveling and solitary wave solutions (Olver 2010). Recently, there has been a huge development in obtaining stable spatiotemporal soliton solutions, with and without chirp, for a higher number of transverse dimensions (Malomed 2006; Zhong 2008) using the JEF-expansion method and the principle of harmonic balance. The traveling wave and soliton solutions to the generalized NLSE in (3 + 1) dimensions ((3 + 1)-D) for the cubic nonlinearity were first developed in Belić (2008) for the anomalous dispersion and were generalized in Petrović (2009) for the normal dispersion. However, the important task remains to generalize these solutions to a wider range of functions.

2 Method

In this paper we expand the work done in Belić (2008) to find new spatiotemporal traveling wave solutions to the NLSE. We consider the standard form of the NLSE (Malomed et al. 2005):

$$i\partial_z u + \frac{\beta(z)}{2}\left(\Delta_\perp u + \partial_t^2 u\right) + \chi(z)|u|^2 u = i\gamma(z)u, \tag{1}$$

which describes evolution of a slowly-varying wavepacket envelope $u(x, y, z, t)$ in a diffractive nonlinear Kerr medium with anomalous dispersion, in the paraxial approximation. Here, z is the propagation coordinate, $\Delta_\perp = \partial_x^2 + \partial_y^2$ represents the transverse Laplacian, and t is the reduced time, i.e. time in the frame of reference moving with the wavepacket. All coordinates are made dimensionless by the choice of coefficients. The functions β, χ, and γ stand for the diffraction/dispersion, nonlinearity, and gain coefficients, respectively. As in Belić (2008) we define u in terms of amplitude and phase $u(z, x, y, t) = A(z, x, y, t)\exp(iB(z, x, y, t))$ and assume the following form of the solutions:

$$A = f_1(z) + f_2(z)F(\theta) + f_3(z)F^{-1}(\theta), \tag{2}$$

$$\theta = k(z)x + l(z)y + m(z)t + \omega(z), \tag{3}$$

$$B = a(z)\left(x^2 + y^2 + t^2\right) + b(z)(x + y + t) + e(z), \tag{4}$$

where $f, g, k, l, m, \omega, a, b, e$ are parameter functions to be determined.

The key difference from the previous paper is that we will assume F to be the solution of a more general differential equation:

$$\left(\frac{dF}{d\theta}\right)^2 = c_0 + c_1F + c_2F^2 + c_3F^3 + c_4F^4, \tag{5}$$

whereas for the Jacobi elliptic functions (JEFs) used in Belić (2008) we had $c_1 = c_3 = 0$. This differential equation has been previously used in several papers, namely in Fan (2002), Lan-Fang et al. (2011), Wang et al. (2006) and in Zhang et al. (2013d) for $c_0 = c_1 = 0$. However, the ansatz with the more general equation has never been applied to the (3 + 1)-D NLSE and in no case was the effect of chirp considered. In this paper we will explicitly assume $c_1 \neq 0$ or $c_3 \neq 0$.

Applying the F-expansion method and the principle of harmonic balance we obtain the following system of algebraic and first order differential equations for f_i $(i = 1, 2, 3), a, b, k, l, m$ and ω:

$$\frac{df_j}{dz} + 3a\beta f_j - \gamma f_j = 0, \tag{6}$$

$$\frac{dk}{dz} + 2ka\beta = 0, \quad \frac{dl}{dz} + 2la\beta = 0, \quad \frac{dm}{dz} + 2ma\beta = 0, \tag{7}$$

$$\frac{da}{dz} + 2\beta a^2 = 0, \tag{8}$$

$$\frac{db}{dz} + 2\beta ab = 0, \tag{9}$$

$$\frac{d\omega}{dt} + \beta(k + l + m)b = 0. \tag{10}$$

Instead of two equations for χ as was obtained in Belić (2008) we obtain four equations:

$$f_2\left(\beta\left(k^2 + l^2 + m^2\right)c_4 + \chi f_2^2\right) = 0, \tag{11}$$

$$f_3\left(\beta\left(k^2 + l^2 + m^2\right)c_0 + \chi f_3^2\right) = 0, \tag{12}$$

$$f_2\left(\beta\left(k^2 + l^2 + m^2\right)\frac{c_3}{4} + \chi f_1 f_2\right) = 0, \tag{13}$$

$$f_3\left(\beta\left(k^2 + l^2 + m^2\right)\frac{c_1}{4} + \chi f_1 f_3\right) = 0. \tag{14}$$

Equation (11) (or Eq. (12) if $f_2 = 0$) will give us the formula for $\chi(z)$, given arbitrary values of $\beta(z)$ and $\gamma(z)$, while the remaining three equations, if not automatically satisfied, will impose constraints on c_i $(i = 0, \ldots, 4)$. For the equation for e we obtain two equations instead of one:

$$\frac{de}{dz} - 6\chi f_2 f_3 - \chi f_1^2 + \frac{\beta}{4}\left(6b^2 - \left(k^2 + l^2 + m^2\right)\frac{f_2c_1 + f_3c_3}{f_1}\right) = 0, \tag{15}$$

$$\frac{de}{dz} - 3\chi f_2 f_3 - 3\chi f_1^2 + \frac{\beta}{2}\left(3b^2 - \left(k^2 + l^2 + m^2\right)c_2\right) = 0. \tag{16}$$

We will require an additional matching condition imposed on the coefficients c_i $(i = 0, \ldots, 4)$ for both equations to be satisfied simultaneously.

3 Results

We now proceed to solve the system of ordinary algebraic and differential equations obtained in Sect. 2. Using standard elementary calculus, Eqs. (6)–(10) are solved as in Belić (2008) to obtain:

$$f_i(z) = \alpha^{3/2} f_{i0} \exp\left(\int_0^z \gamma dz\right), \tag{17}$$

$$k(z) = \alpha k_0, \quad l(z) = \alpha l_0, \quad m(z) = \alpha m_0, \tag{18}$$

$$\omega(z) = \omega_0 - \alpha(k_0 + l_0 + m_0) b_0 \int_0^z \beta dz, \tag{19}$$

$$a(z) = \alpha a_0, \tag{20}$$

$$b(z) = \alpha b_0, \tag{21}$$

where:

$$\alpha = \frac{1}{1 + 2a_0 \int_0^z \beta dz} \tag{22}$$

is the chirp function and subscript '0' denoted the value of the respective function at $z = 0$.

From solving Eqs. (11)–(14) we obtain three distinct cases.

3.1 Case I: $f_3 = 0$

Assuming $f_3 = 0$, Eqs. (12) and (14) are automatically satisfied, and from Eqs. (11) and (13) we obtain:

$$f_2 = f_1 \frac{4c_4}{c_3}, \tag{23}$$

$$\chi(z) = -\beta c_4 \left(k_0^2 + l_0^2 + m_0^2\right) f_{10}^{-2} \exp\left(-2\int_0^z \gamma dz\right) \Big/ \alpha. \tag{24}$$

The matching condition is:

$$c_2 = \frac{c_3^2}{4c_4} + \frac{2c_4 c_1}{c_3} \ . \tag{25}$$

The formula for e is:

$$e(z) = e_0 + \frac{\alpha}{2} \int_0^z \beta dz \cdot \left(\left(k_0^2 + l_0^2 + m_0^2\right) \left(\frac{2c_4 c_1}{c_3} - \frac{c_3^2}{8c_4} \right) - 3b_0^2 \right). \tag{26}$$

For $c_1 = 0$, the condition (25) reduces to $c_3 = \pm 2\sqrt{c_2 c_4}$. A range of functions that can be used in this case is given in Refs. Lan-Fang et al. (2011) and Zhang et al. (2013d).

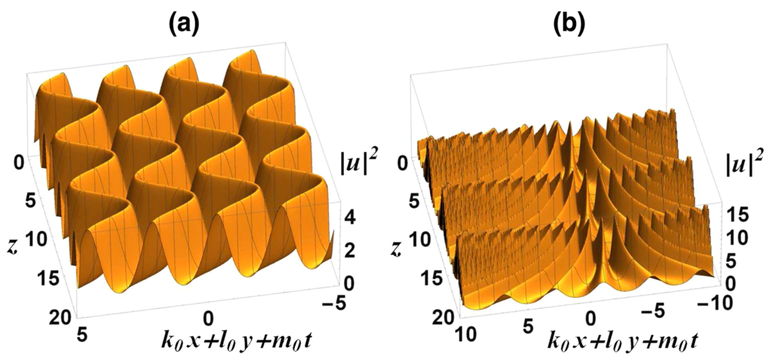


Fig. 1 A solution for Case II using the Weierstrass elliptic function. The parameters are: $F = \wp(x;\sqrt[3]{32},1), f_{10} = b_0 = k_0 = l_0 = m_0 = 1, \omega_0 = 0$ and **a** $a_0 = 0$, **b** $a_0 = 0.3$

3.2 Case II: $f_2 = 0$

Assuming $f_2 = 0$, Eqs. (11) and (13) are automatically satisfied, and from Eqs. (12) and (14) we obtain:

$$f_3 = f_1 \frac{4c_0}{c_1}, \tag{27}$$

$$\chi(z) = -\beta c_4 \left(k_0^2 + l_0^2 + m_0^2\right) f_{20}^{-2} \exp\left(-2\int_0^z \gamma dz\right) \Big/ \alpha. \tag{28}$$

The matching condition is:

$$c_2 = \frac{c_1^2}{4c_0} + \frac{2c_0c_3}{c_1}. \tag{29}$$

The formula for e is:

$$e(z) = e_0 + \frac{\alpha}{2}\int_0^z \beta dz \cdot \left(\left(k_0^2 + l_0^2 + m_0^2\right)\left(\frac{2c_0c_3}{c_1} - \frac{c_1^2}{8c_0}\right) - 3b_0^2\right). \tag{30}$$

For $c_3 = 0$, the condition (29) reduces to $c_1 = \pm 2\sqrt{c_2c_0}$.

For this case, the function of most interest which satisfies the necessary requirement $c_0, c_1 \neq 0$ is the Weierstrass elliptic function (WEF) (Lawden 1989). It is well known that for the WEF $\wp(x; g_2, g_3)$ we have $c_4 = c_2 = 0$, $c_3 = 4$, $c_1 = -g_2$ and $c_0 = -g_3$. Using Eq. (29) we get $g_2^3 = 32g_3^2$. In Fig. 1 we see the solutions plotted with and without chirp for $g_3 = 1$ and $g_2 = \sqrt[3]{32}$. Thus we have shown that the WEF can also be used in the F-expansion method for solving the (3 + 1)-D NLSE.

3.3 Case III: $f_2, f_3 \neq 0$

Assuming both f_2 and f_3 are non-zero, we obtain the following equations:

$$f_2 = f_1 \frac{4c_4}{c_3}, \quad f_3 = f_1 \frac{4c_0}{c_1}, \tag{31}$$

and an additional condition:

$$\frac{c_3^2}{c_4} = \frac{c_1^2}{c_0} \tag{32}$$

needed to make Eqs. (11)–(14) consistent. The formula for $\chi(z)$ is the same as the one in Eqs. (24). The matching condition for Eqs. (15)–(16) is:

$$c_2 = \frac{c_3^2}{4c_4} - \frac{2c_4c_1}{c_3}. \tag{33}$$

The formula for e is:

$$e(z) = e_0 - \frac{\alpha}{2}\int_0^z \beta dz \cdot \left(\left(k_0^2 + l_0^2 + m_0^2\right)\left(\frac{2c_4c_1}{c_3} + \frac{c_3^2}{8c_4}\right) + 3b_0^2 \right). \tag{34}$$

Note that a symmetric formula for (33) is also available using c_0, c_1 and c_3.

For this case we will chose the following values of coefficients, which satisfy conditions (32) and (33): $c_4 = -1$, $c_3 = 2$, $c_2 = 0$, $c_1 = 1$ and $c_0 = -1/4$. The polynomial $p(F) = c_0 + c_1F + c_2F^2 + c_3F^3 + c_4F^4$ has two positive zeros, giving us a range for $\frac{dF}{d\theta}$ that insures bounded solutions (Drazin and Johnson 1989). When the polynomial P has distinct zeros, function F can be considered an inverse of Carlson's elliptic integrals (Carlson 1987), though cases with complex conjugate zeros can also be evaluated (Carlson 1977). A plot is given in Fig. 2. The plots depict roughly a single period of the function. We can see that the combination of two elliptic functions, given that both f_2 and f_3 are non-zero. Also, it is important to note that unlike JEF the maxima and minima of F are not the same width, due to the asymmetry of the elliptic integral. The modulational stability of these solutions is not addressed in this paper, though is likely that the solutions are stable since work done in Petrović et al. (2015) has shown that in most regimes solutions to Eq. (1) obtained in Belić (2008), Petrović (2009) are either unconditionally modulationally stable or stable under the regime of dispersion management.

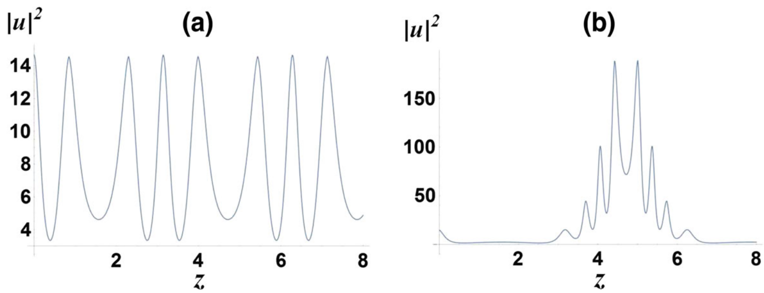


Fig. 2 A solution for Case III where function F satisfied the differential equation $\frac{dF}{d\theta} = -\frac{1}{4} + F + 2F^3 - F^4$ for $x = y = t = 0$. Other parameters are the same as in Fig. 1

4 Conclusion

To sum up, we have generalized the Jacobi elliptic function expansion method for the case of a general fourth order-polynomial for the elliptic integral. We have shown that it is possible to use the Weierstrass elliptic function in the F-expansion method for the (3 + 1)-D NLSE. Finally, we have shown that general inverses of elliptic integrals may be used for solving the (3 + 1)-D NLSE. This opens the door to a far more general class of solutions for the (3 + 1)-D NLSE than what was previously obtained.

Acknowledgments The work in Serbia is supported by the Serbian Ministry of Education and Science under project OI 171006.

References

Akhmediev, N., Ankiewicz, A.: Solitons. Chapman and Hall, London (1997)

Belić, M., et al.: Analytical light bullet solutions to the generalized (3 + 1)-dimensional nonlinear Schrödinger equation. Phys. Rev. Lett. **101**, 123904 (2008)

Carlson, B.: Elliptic integrals of the first kind. SIAM J. Math. Anal. **8**(2), 231–242 (1977)

Carlson, B.: A table of elliptic integrals of the second kind. Math. Comput. **49**(180), 595–606 (1987)

Drazin, P.G., Johnson, R.S.: Solitons: An Introduction, Chapter 2. Cambridge University Press, Cambridge (1989)

Fan, E.: Travelling wave solutions in terms of special functions for nonlinear coupled evolution systems. Phys. Lett. A **300**, 243–249 (2002)

Hasegawa, A., Matsumoto, M.: Optical Solitons in Fibers. Springer, Berlin (2003)

He, J., Wu, X.: Exp-function method for nonlinear wave equations. Chaos Solitons Fractals **30**, 700–708 (2006)

Kivshar, Y., Agrawal, G.: Optical Solitons, from Fibers to Photonic Crystals. Academic, New York (2003)

Kruglov, V., Peacock, C., Harvey, J.: Exact self-similar solutions of the generalized nonlinear Schrödinger equation with distributed coefficients. Phys. Rev. Lett. **90**, 113902 (2003)

Lai, X., Cai, X.: Chirped wave solutions of a generalized (3 + 1)-dimensional nonlinear Schrödinger equation. Z. Naturforsch. **66a**, 392–400 (2011)

Lan-Fang, S., Cai-Sheng, C., Xian-Chun, Z.: The extended auxiliary equation method for the KdV equation with variable coefficients. Chin. Phys. B **20**, 100507 (2011)

Lawden, D.: Elliptic Functions and Applications. Springer, New York (1989)

Li, L., Wang, M.: The $\left(\frac{G'}{G}\right)$-expansion method and travelling wave solutions for a higher-order nonlinear Schrödinger equation. Appl. Math. Comput. **208**, 440–445 (2009)

Malfliet, W., Hereman, W.: The tanh method: I. Exact solutions of nonlinear evolution and wave equations. Phys. Scr. **54**, 563–568 (1996)

Malomed, B.: Soliton Management in Periodic Systems. Springer, Berlin (2006)

Malomed, B.A., Michalache, D., Wise, F., Torner, L.: Spatiotemporal optical solitons. J. Opt. B **7**, R53–R72 (2005)

Miao, X., Zhang, Z.: The modified $\left(\frac{G'}{G}\right)$-expansion method and traveling wave solutions of nonlinear the perturbed nonlinear Schrödingers equation with Kerr law nonlinearity. Commun. Nonlinear Sci. Numer. Simul. **16**, 4259–4267 (2011)

Olver, F.W.: NIST Handbook of Mathematical Functions, Chapter 22. Cambridge University Press, Cambridge (2010)

Petrović, N., et al.: Exact spatiotemporal wave and soliton solutions to the generalized (3 + 1)-dimensional Schrödinger equation for both normal and anomalous dispersion. Opt. Lett. **34**, 1609–1611 (2009)

Petrović, N., Aleksić, N.B., Belić, M.R.: Modulation stability analysis of exact multidimensional solutions to the generalized nonlinear Schrödinger equation and the Gross–Pitaevskii equation using a variational approach. Opt. Exp. **23**, 10616–10630 (2015)

Wang, Q., Chen, Y., Zhang, H.: Generalized algebraic method and new exact traveling wave solutions for (2 + 1)-dimensional dispersive long wave equation. Appl. Math. Comput. **181**, 247–255 (2006)

Zhang, Z.: New exact traveling wave solutions for the nonlinear Klein–Gordon equation. Turk. J. Phys. **32**, 235–240 (2008)

Zhang, Z.: Jacobi elliptic function expansion method for the modified Korteweg–de Vries–Zakharov-Kuznetsov and the Hirota equations. Rom. J. Phys. **60**, 1384–1394 (2015)

Zhang, Z., et al.: New exact solutions to the perturbed nonlinear Schrödinger's equation with Kerr law nonlinearity. Appl. Math. Comput. **216**, 3064–3072 (2010)

Zhang, Z., et al.: Qualitative analysis and traveling wave solutions for the perturbed non-linear Schrödinger equation with Kerr law nonlinearity. Phys. Lett. A **375**, 1275–1280 (2011a)

Zhang, Z., Gan, X., Yu, D.: Bifurcation behaviour of the travelling wave solutions of the perturbed nonlinear Schrödinger equation with Kerr law nonlinearity. Z. Naturforsch **66a**, 721–727 (2011b)

Zhang, Z., et al.: New exact solutions to the perturbed nonlinear Schrödinger's equation with Kerr law nonlinearity via modified trigonometric function series method. Commun. Nonlinear Sci. Numer. Simul. **16**, 3097–3106 (2011c)

Zhang, Z., et al.: A note on exact traveling wave solutions of the perturbed nonlinear Schrödingers equation with Kerr law nonlinearity. Commun. Theor. Phys. **57**, 764–770 (2012)

Zhang, Z., et al.: Abundant exact traveling wave solutions for the Klein–Gordon–Zakharov equations via the tanh–coth expansion method and and Jacobi elliptic function expansion method. Rom. J. Phys. **58**, 749–765 (2013a)

Zhang, Z., et al.: A new method to construct travelling wave solutions for the Klein–Gordon–Zakharov equations. Pramana **80**, 766–777 (2013b)

Zhang, Z., et al.: First integral method and exact solutions to nonlinear partial differential equations arising in mathematical physics. Rom. Rep. Phys. **65**, 1155–1169 (2013c)

Zhang, Z., et al.: Exact traveling wave solutions of the perturbed Klein–Gordon equation with quadratic nonlinearity in (1+1)-dimension, part I: without local inductance and dissipation effect. Turk. J. Phys. **37**, 259–267 (2013d)

Zhong, W., et al.: Exact spatial soliton solutions of the two-dimensional generalized nonlinear Schrdinger equation with distributed coefficients. Phys. Rev. A **78**, 023821 (2008)

6th International Conference on

Photonics

July 31- August 01, 2017 Milan, Italy

Theme: New Research Horizons: Photonics in a Changing World

Nikola Z Petrović (https://optics.physicsmeeting.com/speaker/2017/nikola-z-petrovi--university-of-belgrade-serbia)

University of Belgrade, Serbia

Title: General analytic solutions to the various forms of the nonlinear Schrödinger equation using the Jacobi elliptic function expansion method

Biography

Biography: Nikola Z Petrović (https://optics.physicsmeeting.com/speaker/2017/nikola-z-petrovi--university-of-belgrade-serbia)

Abstract

The advent of meta-materials has made materials with a negative refractive index possible. This has opened up a possibility of finding stable solutions to various nonlinear equations that naturally occur in the field of nonlinear optics through the use of dispersion management. Finding such stable solutions is invaluable for the field of photonics (http://optics.physicsmeeting.com/)and has many potential practical applications. In our work we use the F-expansion method applied to the Jacobi elliptic function, along with the principle of harmonic balance to find novel solutions to various forms of the Nonlinear Schrödinger equation (NLSE). This approach allowed us to assume a quadratic form for the phase with respect to the longitudinal variable and thus find solutions both with and without chirp. Earlier work done on the NLSE with Kerr nonlinearity, with both normal and anomalous dispersion, was generalized to nonlinearities of arbitrary polynomial nonlinearity. Stable solutions were also obtained for the Gross-Pitaevskii equation. These solutions were determined to be modulationally stable, either unconditionally or with dispersion management, depending on the signs of various parameters in the original equation. The method was subsequently generalized for functions satisfying an arbitrary elliptic differential equation, including Weierstrass elliptic functions. A relatively new line of research has been finding solutions to the NLSE in a parity-time (PT) conserving potential, i.e. one for which the real part is an even function and the complex part is an odd function. We found a rich new class of exact solutions where the potential resembles the Scarf II potential.

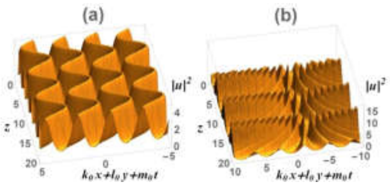


Figure 1: Solution to the NLSE with Kerr nonlinearity using the Weierstrass elliptic function described in: (a) without chirp (b) with chirp

1. S L Xu, Y Zhao, N Z Petrović and M R Belić (2016) Spatiotemporal soliton supported by parity-time symmetric potential with competing nonlinearities. Europhysics Letters. 115: 14006.
2. N Z Petrović and M Bohra (2016) General Jacobi elliptic function expansion method applied to the generalized (3+1)-dimensional nonlinear Schrödinger equation. Optical and Quantum Electronics. 48: 1-8.
3. S L Xu, N Petrović, M R Belić and W Deng (2016) Exact solutions for the quintic nonlinear Schrödinger equation with time and space. Nonlinear Dynamics 84: 251.
4. N Z Petrović, N B Aleksić and M R Belić (2015) Modulational stability analysis of exact multidimensional solutions to the generalized nonlinear Schrödinger equation and the Gross-Pitaevskii equation using a variational approach. Optics Express 23: 10616.
4. W P Zhong, L Chen, M R Belić and N Petrović (2014) Controllable parabolic-cylinder optical rogue wave. Physical Review E. 90: 043201.

General analytic solutions to the various forms of the Nonlinear Schrödinger Equation using the Jacobi elliptic function expansion method

Nikola Petrović[1]

(1) *Institute of Physics, 118 Pregrevica, 11080 Belgrade, Serbia*

Contact: Nikola Petrović (nzpetr@ipb.ac.rs)

Abstract. In recent years there have been great developments towards finding exact solutions to various forms of the Nonlinear Schrödinger equation (NLSE). In particular, the combination of several ideas, most notably the F-expansion method, the principle of harmonic balance and the use of the Jacobi elliptic function (JEF) for the expansion function, has yielded a rich new class of solutions for a wide range of parameters of the NLSE. Thanks to the mathematical properties of JEFs, both solitary wave and traveling wave solutions can be realized and the effect of chirp can be added to all the solutions [1].

The fundamental ansatz for the solution to the basic NLSE with distributed coefficients, anomalous dispersion and Kerr nonlinearity was described in [1]. This ansatz was subsequently modified in order to find the analytical solution to the NLSE with the normal dispersion in [2] and was also modified to account for nonlinearities of an arbitrary polynomial order [3]. Further adaptation has lead to the analytical solutions of the Gross-Pitaevskii equation [4,5]. Here, it turned out to be necesssary to solve the Ricatti differential equation for the chirp function, which gave rise to very complex forms of solutions. The stability analysis performed in [6] revealed that in the majority of solutions obtained we had either unconditional stability or stability achievable through the use of dispersion management. Other systems that were covered in subsequent papers include the NLSE with linear potential, pairs of co- or counter-propatating beams in a Kerr medium and solutions involving the Weierstrass elliptic function. Current topics of interest include the study of vortex clusters in the NLSE and finding analytic solutions for NLSEs with a non-integer degree of nonlinearity.

REFERENCES

[1] M. Belić, N. Z. Petrović, W. P. Zhong, R. H. Xie and G. Chen, Phys. Rev. Lett.101, 0123904 (2008)
[2] N. Z. Petrović, M. Belić, W. P. .Zhong, R. H. Xie and G. Chen, Opt. Lett.34, 1609 (2009).
[3] N. Z. Petrović, M. Belić and W. P .Zhong, Phys. Rev. E 83, 026604 (2011).
[4] N. Z. Petrović, M. Belić and W. P .Zhong, Phys. Rev. E 81, 016610 (2010).
[5] N. Z. Petrović, N. Aleksić, A. Al Bastami and M. Belić, Phys. Rev. E 83, 036609 (2011).
[6] N. Z. Petrović, N. Aleksić and M. Belić, Optics express 23 (8), 10616-10630 (2015).

Конференција **Десета радионица фотонике (2017)**

Зборник апстраката

Копаоник 26.2.-2.3.2017.

Издаје

Институт за физику Универзитета у Београду

За издавача

др Александар Богојевић, директор

Уредник

др Драган Лукић

Тираж

100 примерака

ИСБН 978-86-82441-45-8

Штампа

Развојно-истраживачки центар,
Технолошко-металуршког факултета у Београду

Карнегијева 4, Београд

CIP - Каталогизација у публикацији
Народна библиотека Србије, Београд

CIP

535(048)
681.7(048)
66.017/.018(048)

PHOTONICS Workshop (10 ; 2017 ; Kopaonik)
Book of Abstracts / 10th Photonics Workshop, Kopaonik, 26.2-2.3.2017. = Зборник апстраката / Конференција Десета радионица фотонике (2017), Копаоник, 26.2-2.3.2017. ; [urednik Dragan Lukić]. - Београд : Институт за физику Универзитета, 2017 (Београд : Развојно-истраживачки центар графичког инжењерства ТМФ). - X, 46 str. : ilustr. ; 25 cm

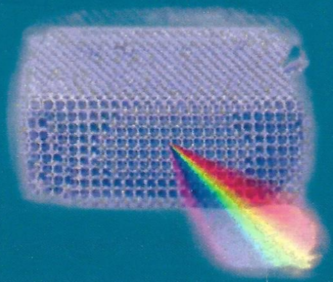
Тираж 100. - Реч уредника: str. VII. - Registar.

ISBN 978-86-82441-45-8

a) Оптика - Апстракти b) Оптоелектроника - Апстракти c) Технички материјали - Апстракти
COBISS.SR-ID 229745420

УНИВЕРЗИТЕТ У БЕОГРАДУ

Институт за физику Београд



Конференција

Десета радионица фотонике (2017)

Зборник апстраката

10th Photonics Workshop
Book of Abstracts
Kopaonik, 26.2-2.3 2017

Копаоник, 26.2–2.3 2017.

WORKSHOP >> >> Send link to a friend

Workshop Program: Monday, February 27, 2017

Final program

The schedule of lectures can be downloaded in pdf format (pdf, ~500 kB).

Monday, February 27, 2017

from	to	activity	carried by	type
16:00	16:30	*REFRESHMENT*		
		chairperson		
16:30	17:00	Building your own Light Sheet Fluorescence Microscope	A. Kranz	1
17:00	17:30	H2020-CARDIALLY – Noninvasive capturing and quantitative analysis of multi-scale multi-channel diagnostic data of the cardiovascular system	Lj. Hadžievski	1
17:30	17:40	*BREAK*		
17:40	18:10	General analytic solutions to the various forms of the Nonlinear Schrödinger Equation using the Jacobi elliptic function expansion method	N. Petrović	1
18:10	18:30	Electro-optic techniques and THz time domain spectroscopy (pdf)	R. Pan	2
18:30	20:00	*DINNER BREAK*		
20:00	20:20	FTIR Spectrometer for spectral characterization of THz undulator at FLASH1 (pdf)	E. Zapolnova	2
20:20	20:40	Raman spectroscopy: a tool for the characterization of antioxidant components of mature tomato fruits	I. Petrović	2
20:40	21:00	Imaging of functional and structural alterations in primary cortical astrocytes isolated from the transgenic rat model of ALS	S. Stamenković	2
21:00	21:10	*BREAK*		
21:10	21:30	Deformation of Fermi Surface for Ultracold Dipolar Fermi Gases	V. Veljić	2
21:30	21:50	Benefits of implementing online Dynamic Bandwidth Allocation algorithm in energy efficient WDM EPON	B. Pajčin	2
21:50	22:10	Performances of BB84 and B92 QKD authentication protocols analyzed by proposed physical model	N. Miljković	2
22:10	22:30	Ejection fraction calculation using multiparametric cardiac measurement system	M. Miletić	2
22:30	22:50	G-Protein Coupled Receptors structure prediction by Bayesian probabilistic approach principle	M. Mudrinić	2

IMPORTANT DATES

1st announcement: Nov 14, 2016
Registration (early): Dec 31, 2016
Registration (late): Feb 26, 2017
Abstract upload: ~~Jan 28, 2017~~
Feb 15, 2017

Workshop start: Feb 26, 2017

SPONSORS

ORGANIZERS

J Opt (April–June 2015) 44(2):172–177
DOI 10.1007/s12596-015-0243-z

RESEARCH ARTICLE

Two-dimensional dark solitons in diffusive nonlocal nonlinear media

Si-Liu Xu [1] · Nikola Petrović [2] · Milivoj R. Belić [3]

Received: 22 April 2014 / Accepted: 30 March 2015 / Published online: 26 April 2015

Abstract Using the balance principle and the F-expansion method, we find dark soliton solutions in a general nonlocal nonlinear optical model with a diffusive type of nonlinearity. These solutions are modeled by numerical simulation, in order to study how they propagate and interact with each other. Our results show that the multidimensional nonlocal solitary waves can be manipulated and controlled by changing the degree of nonlocality and the diffraction coefficient.

OCIS codes (190.0190) Nonlinear optics · (190.6135) Spatial solitons

PACS number(s) 42.65.Tg · 05.45.Yv

Keywords

Introduction

Nonlocality is an inherent feature of many settings in optics, plasmas, and Bose-Einstein condensates. Spatial-domain nonlinear dynamics of light waves in nonlocal optical media with different response functions, characterized by the respective correlation lengths, were studied in detail theoretically and experimentally in Królikowski et al. [1]. In particular, the nonlocal nonlinearity can support solitons in various forms [2, 3]. Two-dimensional (2D) spatial solitons stabilized by the nonlocality were observed in vapors [4] and lead glasses featuring strong thermal nonlinearity [5], as well as in liquid crystals [6]. Further studies included stabilized solitons in photonic lattices [7, 8], vortices [9, 10], spatial solitons in soft matter [11], and multipole vector solitons in nonlocal nonlinear media [12]. In addition, it was shown that long-range cubic nonlinearity induced by long-range interactions between atoms carrying polarized magnetic momenta in effectively 2D Bose-Einstein condensates also leads to the prediction of stable 2D solitons [13]. It has also been demonstrated that the nonlocal nonlinear (NN) response allows for suppression of the modulation instability of plane waves, arrest of the collapse of multidimensional beams, and, generally, fosters the stabilization of solitons in NN media [14, 15].

In principle, it is important to find exact solutions to the of nonlinear (NL) partial differential equations. The goal of this paper is to adapt the homogeneous balance principle and the F-expansion technique, which were used in finding solutions to the general 2D NL paraxial wave equation, to identify traveling and solitary wave solutions of a general NN optical system of equations, with both diffractive and diffusive equations. We study the spatial solitary waves in a general NN optical system with a diffusive type of nonlocality and a Kerr-type of nonlinearity.

The paper is structured as follows. "The model" section introduces the general 2D nonlocal NL optical system and the solution method for the problem. "The solitary solutions" section analyzes different forms of spatial dark solitons. In "Stability analysis" section, the stability is investigated by accurate computation of growth rates for the perturbed eigenmodes. The concluding remarks, with a simple summary, are given in "Conclusion" section.

✉ Si-Liu Xu
xusiliu1968@163.com

[1] The School of Electronic and Information Engineering, HuBei University of Science and Technology, Xianning 437100, China

[2] Institute of Physics, University of Belgrade, P.O. Box 68, 11001 Belgrade, Serbia

[3] Science Program, Texas A&M University at Qatar, P.O. Box 23874, Doha, Qatar

The model

We consider the propagation of a paraxial laser beam along the z axis in a generic NN optical medium. The beam propagation in 2D is described by the following system of equations for the slowly varying field amplitude u and the NL contribution to the change of the refractive index n [16–20]:

$$iu_z+\frac{\beta(z)}{2}\left(u_{xx}+u_{yy}\right)+un=0, \tag{1a}$$

$$d\left(n_{xx}+n_{yy}\right)-n+|u|^2=0, \tag{1b}$$

where x are y are the suitably scaled transverse and longitudinal coordinates, z is the longitudinal coordinate, $\beta(z)$ is transverse diffraction coefficient, and the parameter d stands for the degree of nonlocality of the NL response n. Thus, the system of equations given in (1) consists of the paraxial propagation equation for the field envelope and the diffusion equation for the nonlocal nonlinearity. When $d\to 0$ the system describes a local response and reduces to the simple NL Schrödinger equation in Kerr medium, whereas when $d\to\infty$ it describes a strongly nonlocal response. A generalization of Eq. (1) in (3+1)D was considered by Mihalache et al. [21, 22] as a model for the generation of stable 3D spatiotemporal solitons. The stability of the fundamental solitons was demonstrated through the computation of the corresponding stability eigenvalues in direct simulations. It is clear that the parameter d can be eliminated from the equations by rescaling the coordinates. Nevertheless, we prefer to keep d as an explicit parameter, as it directly controls the system's nonlocality degree.

In general, the system of equations (1) can be applied to optical propagation in generic NN media, for example the propagation of an electromagnetic wave in a nematic liquid crystal [23] or the optical beam propagation in partially ionized plasmas [10]. The degree of nonlocality d can be modulated externally, by changing an applied voltage to the crystal. System (1) possess several conserved quantities, among other the power E and the Hamiltonian

$$E=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}|u|^2\,dxdy, \tag{2}$$

$$H=\frac{\beta}{2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left(\left|\frac{\partial u}{\partial x}\right|^2+\left|\frac{\partial u}{\partial y}\right|^2-n|u|^2\right)dxdy. \tag{3}$$

To determine the amplitude profiles of spatial solitons with $u,n\to 0$ at $x,y\to\infty$, we assume $u(z,x,y)=A(\theta)e^{iB(z,x,y)}$ and $n(z,x,y)=D(\theta)$, where $A(\theta)$ and $D(\theta)$ are real functions of their argument, $B(z,x,y)=kx+ly+\Omega z+B_0$ is the phase of the complex amplitude and $\theta=px+qy+\omega z+\theta_0$ is the traveling wave variable. B_0 and θ_0 represent the initial values of the respective functions at $z=0$. In this manner, the system (1) reduces to the following coupled equations:

$$\frac{\beta\left(k^2+l^2\right)}{2}\frac{\partial^2 A}{\partial^2\theta}+(D-\Omega)A-\frac{\beta(p^2+q^2)}{2}A=0, \tag{4a}$$

$$D-\left(k^2+l^2\right)d\frac{\partial^2 D}{\partial^2\theta}-A^2=0, \tag{4b}$$

with the following dispersion relation having been established: $\omega=-\beta(z)(kp+lq)$. Linear waves cannot be a proper solution of (4) since the second derivatives of A and D are proportional not only to the functions themselves, but also to the product and the square of the functions. Therefore, we seek the traveling nonlinear wave solutions of (4) that satisfy the established dispersion relation. Using the balance principle and the F-expansion technique [24, 25], we assume the solutions to be of the form

$$A(\theta)=a_0+a_1F(\theta)+a_2G(\theta)+a_3F^2(\theta)+a_4F(\theta)G(\theta), \tag{5a}$$

$$D(\theta)=b_0+b_1F(\theta)+b_2G(\theta)+b_3F^2(\theta)+b_4F(\theta)G(\theta), \tag{5b}$$

where a_j, b_j (j=0,1,2,3,4) are the parameters to be determined, and $F(\theta)$ and $G(\theta)$ are Jacobi elliptic functions (JEFs). These functions satisfy the following NL differential equations:

$$\left(\frac{dF}{d\theta}\right)^2=c_0+c_2F^2+c_4F^4, \tag{6a}$$

$$\frac{d^2F}{d\theta^2}=c_2F+2c_4F^3, \tag{6b}$$

$$\left(\frac{dG}{d\theta}\right)^2=e_0+e_2G^2+e_4G^4, \tag{6c}$$

$$\frac{d^2G}{d\theta^2}=e_2G+2e_4G^3, \tag{6d}$$

where c_0,c_2,c_4 and e_0,e_2,e_4 are real constants related to the elliptic modulus m ($0\le m\le 1$) of the JEF [24, 25]. Table 1 lists the dependence of c_0,c_2,c_4 and e_0,e_2,e_4 on m for some of JEFs. As can be seen from the table, the elliptic modulus greatly influences the final form of the solution. When $m\to 0$ the JEFs become the trigonometric functions and the periodic traveling wave solutions become the periodic trigonometric solutions. When $m\to 1$ the JEFs become the hyperbolic functions and the periodic traveling wave solutions become the soliton solutions.

Table 1 Jacobi elliptic functions

Solution	$c_0(e_0)$	$c_2(e_2)$	$c_4(e_4)$	$F(\theta)$
1	1	$-(m^2+1)$	m^2	$sn(\theta)$
2	$1-m^2$	$2m^2-1$	$-m^2$	$cn(\theta)$
3	m^2-1	$2-m^2$	-1	$dn(\theta)$
4	m^2	$-(1+m^2)$	1	$ns(\theta)$
5	$-m^2$	$2m^2-1$	$1-m^2$	$nc(\theta)$
6	-1	$2-m^2$	m^2-1	$nd(\theta)$
7	1	$2-m^2$	$1-m^2$	$sc(\theta)$
8	1	$2m^2-1$	$-m^2(1-m^2)$	$sd(\theta)$
9	$1-m^2$	$2-m^2$	1	$cs(\theta)$
10	$-m^2(1-m^2)$	$2m^2-1$	1	$ds(\theta)$

We now proceed to solve Eqs. (4). Inserting expressions (5) and (6) into (4) and making the coefficient of each power of $F^j G^q$ $(j=q=0,1,2,3,4)$ and $\sqrt{c_0+c_2F^2+c_4F^4}$ $\sqrt{e_0+e_2G^2+e_4G^4}$ equal to zero, we obtain a set of algebraic polynomials for the parameters a_j, b_j, k,l,p,q,Ω, and ω:

$$a_0=\frac{c_2-\sqrt{c_2^2+3c_0c_4}}{2\sqrt{c_2^2+3c_0c_4}}\sqrt{\frac{\beta}{2d}}, \tag{7a}$$

$$a_3=\frac{3c_4}{4\sqrt{c_2^2+3c_0c_4}}\sqrt{\frac{2\beta}{d}}, \tag{7b}$$

$$b_0=\frac{2c_2^2+3c_0c_4-(12dc_0c_4+2c_2)\sqrt{c_2^2+3c_0c_4}}{8d\left(c_2^2+3c_0c_4\right)^2}\beta, \tag{7c}$$

$$b_3=\frac{-3c_4\beta}{4d\sqrt{c_2^2+3c_0c_4}}, \tag{7d}$$

$$k^2+l^2=\frac{1}{4d\sqrt{c_2^2+3c_0c_4}}, \tag{7e}$$

$$p^2+q^2=\frac{-2\Omega+2b_0}{\beta}+\frac{3c_0c_4\left(c_2+3\sqrt{c_2^2-4c_0c_4}\right)}{d\left(3c_2^2-9c_0c_4+3c_2\sqrt{c_2^2-4c_0c_4}\right)\left(9c_0c_4-2c_2^2\right)}. \tag{7f}$$

with the signs of a_0 and a_3 equal to each other and the rest of the a_j and b_j coefficients equal to zero. We will choose the signs so that a_0 and a_3 are positive. Note that G turns out to be proportional to F and thus it does not explicitly figure in the solutions. Thus, the solutions with cross products of different functions F and G seem not to be possible, according to the solution method utilized here.

By solving Eqs. (7) self-consistently, one can find exact periodic and solitary solutions of (1). The solutions of (1) are found in the form:

$$u(z,x,y)=\left[a_0+a_3F^2(\theta)\right]e^{i(kx+ly+\Omega z+B_0)}, \tag{8a}$$

$$n(z,x,y)=\left[b_0+b_3F^2(\theta)\right]. \tag{8b}$$

The traveling wave solution in Eq. (8) is a stationary solution as long as l, p, and Ω are constant. In the most general case, l and p are independent of d. As long as we choose the constants c_0,c_2,c_4 according to the relationships listed in Table 1, and substitute the appropriate a_j, b_j k,l,p,q,Ω, and ω into (8), we obtain exact periodic traveling wave solutions of (1).

We concentrate in this paper on the dark solitary solutions. The soliton propagation characteristics and shape greatly depend on parameters d and β, which are chosen according to some actual physical modeling requirements. In order to study this dependence, we consider F=sn, which for dark solitary waves ($m=1$) reduces to F=tanh.

Substituting the corresponding JEF from Table 1 into (8), with the condition $m=1$, and noting the conditions: $\beta d\geq 0$, $d\neq 0$, and we obtain the required solitary wave solution of (1). They are discussed in the following section.

The solitary solutions

First, we consider the strongly nonlocal case, $d>>1$. For $d=12$, $k=p=\Omega=1$, $\beta=2$, and $\theta_0=B_0=0$, the evolution of the corresponding soliton and the distribution of the refractive index are depicted in Fig. 1a and b. It is seen that the spatial soliton

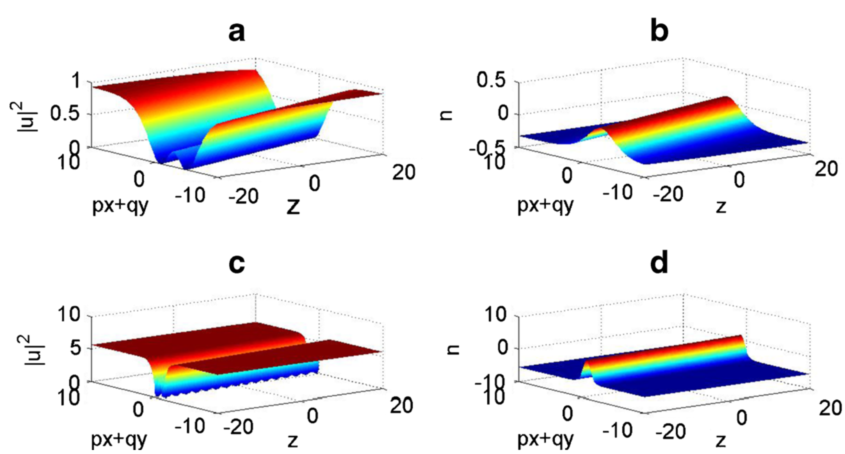


Fig. 1 Exact solitary wave solution and the distribution of the refractive index in the strongly nonlocal case, with $\beta=2$, $d=12$, (**a**, **b**) and the weakly nonlocal case $d=0.7$ (**c**, **d**)

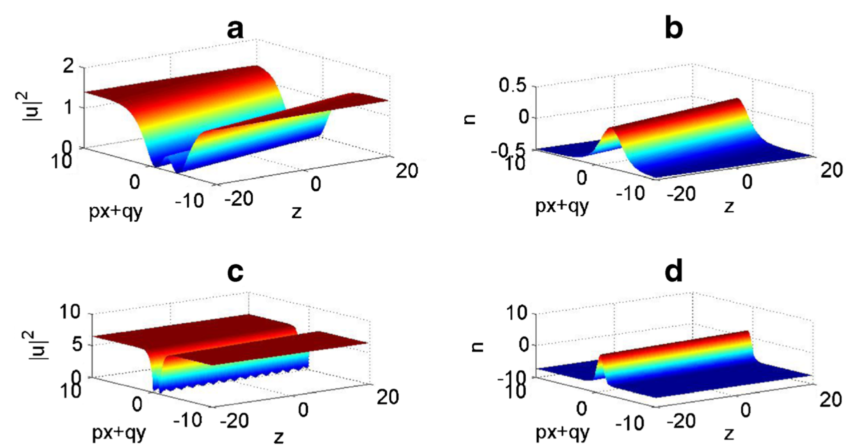


Fig. 2 Exact solitary wave solution and the distribution of the refractive index in the strongly nonlocal case, with d=−12, β=−2 (**a**, **b**) and the weakly nonlocal case and d=−0.1, β=−0.1 (**c**, **d**)

profile and its width remain unchanged with the increasing propagation distance. Next, we consider the Kerr-like medium case in the weakly nonlocal case, d close but not equal to 1/2. For d=0.7 and the other parameters the same as in Fig. 1a and b, the shape of the solitary wave oscillates, as is evident in Fig. 1c and d. The quasi-periodic change in the solitary wave shape during its motion is clearly seen. Such behavior is characteristic of nonlocal solitary waves and quite different from the standard Kerr solitons.

Figure 2 presents the evolution of the exact solution for the self-focusing and nonlocal Kerr medium case with d<0. Similar to Fig. 1, we can discern that for the strong nonlocality, the spatial soliton profile and its width remain unchanged with the increasing propagation distance; in the weak nonlocality case, quasi-periodic oscillations can be seen.

Figure 3 presents soliton amplitude profiles with different degrees of nonlocality (a), (b) and different diffraction coefficients (c), (d). We analyze the impact of different values of the nonlocality parameter d and the diffraction coefficient on the soliton amplitude profile $|u|$. Evidently, an increase in the degree of nonlocality parameter $|d|$ results in a decrease of the soliton width. The nonlocality suppresses the change in the refractive index profile, thereby leading to a narrowing of the beam. This effect is more clearly seen in Fig. 3a and b. We can see that the soliton width changes monotonically with the degree of nonlocality $|d|$. Furthermore, we show the influence of different values of the diffraction coefficient β on the soliton amplitude profile $|u|$ in Fig. 3c and d. In the strongly nonlocal case (d=12 and d=−12), the diffraction coefficient contracts the change in the refractive index profile, thereby leading to a narrowing of the beam, which results in a decrease in the soliton width.

Stability analysis

To study the stability of stationary solitons in a more accurate manner, the full stability of solitons is investigated using the equations for small perturbations linearized around the analytical solution [22, 26]

$$u = \left[u(x,y) + \varepsilon\left(f(x,y,z)e^{\delta z} + g^*(x,y,z)e^{\delta^* z}\right)\right]e^{ibz}, \quad (10a)$$

$$n = \left[n(x,y) + \varepsilon\left(q(x,y,z)e^{\delta z} + q^*(x,y,z)e^{\delta^* z}\right)\right], \quad (10b)$$

where ε is a small parameter, b is the transmission constant and f, g and q are the eigenfunctions of the linearized

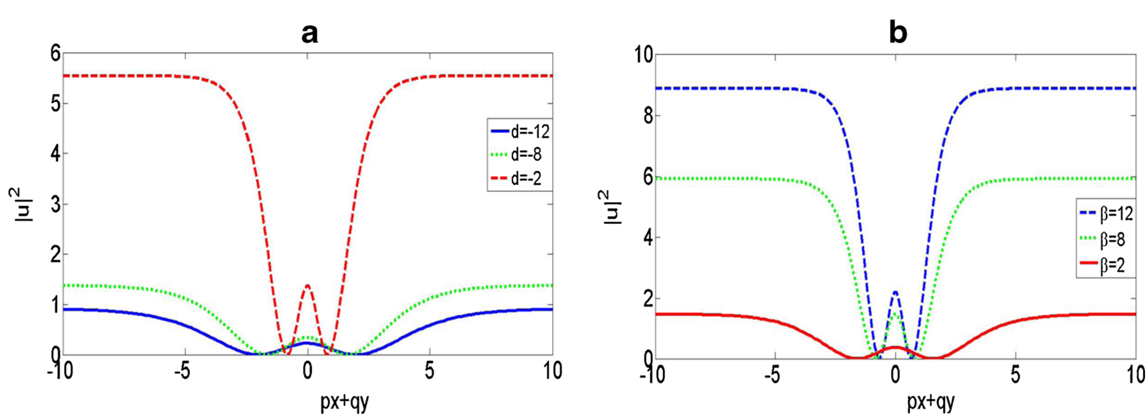


Fig. 3 Soliton intensity profiles with different degrees of nonlocality (**a**, **b**) and different diffraction coefficients (**c**, **d**). The parameters are taken to be z=30, B_0=θ_0=0, β=1.5 (**a**), β=−1.5 (**b**), d=15 (**c**), and d=−15, respectively

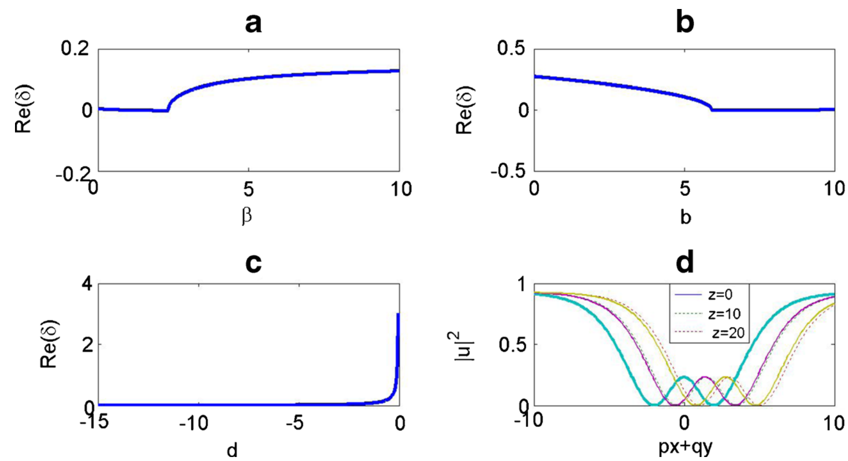


Fig. 4 The largest real part of the perturbation growth rate (eigenvalue) vs **a** diffraction coefficient β, **b** transmission constant b, and the nonlocality parameter d, respectively. **d** Comparison of amplitude profiles for the exact solution (*solid lines*) and the numerical simulation (*dashed lines*) at different propagation distances $z=0,10,20$, from left to right, with $\beta=1.8$ and $d=12$

equations (1). Here, δ is the instability growth rate, which may be complex, and * stands for the complex conjugate. Functions $u(x,y)$ and $n(x,y)$ represent the initial values of u and n, as given in Eq. (9), at $z=0$. The substitution of the perturbed solution in Eq. (1) leads to the linearized equations [26]

$$2i\delta f - 2bf + \beta\left(f_{xx} + f_{yy}\right) + 2(nf + uq) = 0, \tag{11a}$$

$$-2i\delta g - 2bg + \beta\left(g_{xx} + g_{yy}\right) + 2(ng + uq) = 0, \tag{11b}$$

$$q - d\left(q_{xx} + q_{yy}\right) - u(f + g) = 0. \tag{11c}$$

Here, the function q is not to be confused with the parameter q in the first part of the paper. The growth rate δ is found as an eigenvalue at which Eqs. (11) have a non-singular localized solution. The eigenfunctions f and g, and the eigenvalue δ can be found only numerically. Stable solitons are those for which $\mathrm{Re}(\delta)=0$ for different values of β and d. These features are presented in Fig. 4 for $\Omega=1$, $k=0.2$, and $\theta_0=B_0=0$.

Figure 4a and c show the largest real part of the perturbation growth rate (eigenvalue) vs β and d. It can be shown that there exist certain ranges of values for β and d, $-1.5\leq\beta\leq1.5$ and $-2.7\leq d\leq2.7$, in which no stabile soliton solutions in the requested form exist. Without loss of generality, here we only present the case of $\beta\geq0$ in Fig. 4a. Figure 4b represents the perturbation growth rate (eigenvalue) vs the transmission constant b. One can see that when $b<8.2$ the instability appears, i.e. the larger the value of b, the more stable the exact solutions. Finally, to demonstrate the stability of such exact solutions, we compare in Fig. 4d the exact solution with the numerical simulation. As expected, no collapse is seen, and the numerical solutions are in good agreement with the analytical solutions.

Conclusion

We have solved analytically the generic nonlocal NL 2D optical system, using the homogeneous balance principle and the F-expansion technique. A type of new exact dark soliton solutions is demonstrated. The stability is corroborated by an accurate computation of growth rates for the perturbed eigenmodes. Numerical simulations are performed, to ascertain the stability of such soliton solutions. A procedure is presented for controlling NN solitons, in which one may select the nonlocality parameter d and the diffraction coefficient β, to control the propagation behavior of solitons.

Acknowledgments This work is supported in China by the Young teachers corporate action plan project (XD2012358), the National college students' innovative entrepreneurial training program at local colleges, under Grant 201210927052. In addition, work in China is further supported by the Natural Science Foundation of Guangdong Province under Grant No. 1015283001000000 and in Qatar by the NPRP 09-462-1-074 project with the Qatar National Research Fund (a member of the Qatar Foundation). Work in Belgrade, Serbia, is supported under the project OI 171006 of the Ministry of Education, Science and Technological Development.

References

1. W. Królikowski, O. Bang, N.I. Nikolov, D. Neshev, J. Wyller, J.J. Rasmussen, D. Edmundson, J. Opt. B: Quantum Semiclassical Opt. **6**, S288 (2004)
2. A.W. Snyder, D.J. Mitchell, Science **276**, 1538 (1997)
3. C. Conti, M. Peccianti, G. Assanto, Phys. Rev. Lett. **91**, 073901 (2003)
4. D. Suter, T. Blasberg, Phys. Rev. A **48**, 4583 (1993)
5. C. Rotschild, O. Cohen, O. Manela, M. Segev, T. Carmon, Phys. Rev. Lett. **95**, 213904 (2005)
6. C. Conti, M. Peccianti, G. Assanto, Phys. Rev. Lett. **92**, 113902 (2004)
7. Y.V. Kartashov, V.A. Vysloukh, L. Torner, Phys. Rev. Lett. **93**, 153903 (2004)
8. Z. Xu, Y.V. Kartashov, L. Torner, Phys. Rev. Lett. **95**, 113901 (2005)
9. D. Briedis et al., Opt. Express **13**, 435 (2005)
10. A.I. Yakimenko, Y.A. Zaliznyak, Y. Kivshar, Phys. Rev. E **71**, 065603(R) (2005)
11. C. Conti, G. Ruocco, S. Trillo, Phys. Rev. Lett. **95**, 183902 (2005)
12. Y.V. Kartashov, L. Torner, V.A. Vysloukh, D. Mihalache, Opt. Lett. **31**, 1483 (2006)

13. P. Pedri, L. Santos, Phys. Rev. Lett. **95**, 200404 (2005)
14. O. Bang, W. Królikowski, J. Wyller, J.J. Rasmussen, Phys. Rev. E **66**, 046619 (2002)
15. F. Ye, B.A. Malomed, Y. He, B. Hu, Phys. Rev. A **81**, 043816 (2010)
16. W.P. Zhong, M. Belic, T. Huang, Appl. Phys. B **102**, 53–58 (2011)
17. Z. Xu, Y.V. Kartashov, L. Torner, Phys. Rev. E **73**, 055601 (2006)
18. Y.V. Kartashov, L. Torner, Opt. Lett. **32**, 946 (2007)
19. N.I. Nikolov, D. Neshev, Opt. Lett. **29**, 286 (2007)
20. B. Alfassi, C. Rotschild, O. Manela, M. Segev, D.N. Christodoulides, Phys. Rev. Lett. **98**, 213901 (2007)
21. D. Mihalache, D. Mazilu, F. Lederer, B.A. Malomed, Y.V. Kartashov, L.-C. Crasovan, L. Torner, Phys. Rev. E **73**, 025601 (2006)
22. D. Mihalache, Rom. Rep. Phys. **59**(2), 515–522 (2007)
23. M. Peccianti, G. Assanto, A. Dyadyusha, M. Kaczmarek, Opt. Lett. **32**, 271 (2007)
24. W.P. Zhong, R.H. Xie, M. Belic, N. Petrovic, G. Chen, Phys. Rev. A **78**, 023821 (2008)
25. M. Belic, N. Petrovic, W.P. Zhong, R.H. Xie, G. Chen, Phys. Rev. Lett. **101**, 123904 (2008)
26. T.R. Marchant, N.F. Smyth, J. Phys. A Math. Theor. **41**, 365201 (2008)