A LETTERS JOURNAL EXPLORING THE FRONTIERS OF PHYSICS

OFFPRINT

Modeling elastic momentum transfer cross-sections from mobility data

Ž. D. Nikitović, V. D. Stojanović and Z. M. Raspopović

EPL, **114** (2016) 25001

OVER
560,000
full text downloads in 2013

24 DAYS
average accept to online publication in 2013

10,755
citations in 2013

"We greatly appreciate the efficient, professional and rapid processing of our paper by your team."

Cong Lin
Shanghai University

Six good reasons to publish with EPL

We want to work with you to gain recognition for your research through worldwide visibility and high citations. As an EPL author, you will benefit from:

1. **Quality** – The 50+ Co-editors, who are experts in their field, oversee the entire peer-review process, from selection of the referees to making all final acceptance decisions.
2. **Convenience** – Easy to access compilations of recent articles in specific narrow fields available on the website.
3. **Speed of processing** – We aim to provide you with a quick and efficient service; the median time from submission to online publication is under 100 days.
4. **High visibility** – Strong promotion and visibility through material available at over 300 events annually, distributed via e-mail, and targeted mailshot newsletters.
5. **International reach** – Over 2600 institutions have access to EPL, enabling your work to be read by your peers in 90 countries.
6. **Open access** – Articles are offered open access for a one-off author payment; green open access on all others with a 12-month embargo.

Details on preparing, submitting and tracking the progress of your manuscript from submission to acceptance are available on the EPL submission website **www.epletters.net**.

If you would like further information about our author service or EPL in general, please visit **www.epljournal.org** or e-mail us at **info@epljournal.org**.

EPL is published in partnership with:

European Physical Society | Società Italiana di Fisica | EDP Sciences | IOP Publishing

epl A Letters Journal Exploring the Frontiers of Physics

April 2016

EPL, **114** (2016) 25001
doi: 10.1209/0295-5075/114/25001

www.epljournal.org

Modeling elastic momentum transfer cross-sections from mobility data

Ž. D. Nikitović, V. D. Stojanović and Z. M. Raspopović

Institute of Physics, University of Belgrade - Pregrevica 118, Belgrade, Serbia

received 3 February 2016; accepted in final form 27 April 2016
published online 10 May 2016

PACS 51.10.+y – Kinetic and transport theory of gases
PACS 52.20.Hv – Atomic, molecular, ion, and heavy-particle collisions
PACS 52.65.Pp – Monte Carlo methods

Abstract – In this letter we present a new method to simply obtain the elastic momentum transfer cross-section which predicts a maximum of reduced mobility and its sensitivity to the temperature variation at low energies. We first determined the transport cross-section which resembles mobility data for similar closed-shell systems by using the Monte Carlo method. Second, we selected the most probable reactive processes and compiled cross-sections from experimental and theoretical data. At the end, an elastic momentum transfer cross-section is obtained by subtracting the compiled cross-sections from the momentum transfer cross-section, taking into account the effects of the angular scattering distributions. Finally, the cross-section set determined in such a way is used as an input in a final Monte Carlo code run, to calculate the flux and bulk reduced mobility for $Ne^+ + CF_4$ which were discussed as functions of the reduced electric field E/N (N is the gas density) for the temperature $T = 300\,\mathrm{K}$.

Introduction. – Cold plasmas are often exploited in new technologies where they favorably offer non-intrusive production or modification of various substances [1]. The main characteristics of these plasmas are their high electron temperature and low gas temperature where the non-equilibrium behavior of a large number of species takes place [2]. The present computer resources allow studies of complex global models [3,4] describing the behavior of such plasmas by taking into account a very large number of particles. In such cases the knowledge of the ion-neutral reaction and transport parameters for ions resurges as interesting mostly in cases in which reactive processes take place [5–7]. In spite of great efforts and numerous results [8–15], the knowledge about ion-molecule reactions holding a prominent place in the descriptions of these plasmas, is far from satisfactory. The main reason for that are the poorly known ion scattering properties in various gases that must be guessed in order to obtain transport properties during modelling [12,16–18].

Quantum-mechanical calculation of the ion-neutral scattering cross-sections requires the knowledge of accurate potential energy surfaces which are known only to within limits of appropriate (usually very complex) theories [19], while experiments at low energies are sensitive to small stray fields and many other technical problems. Thus, simple and effective methods based on known principles are highly valuable.

Calculation of the cross-section. – Due to the lack of experimental and theoretical mobility data Gatland *et al.* [20] found that all the experimental mobility curves can be unified into a single mobility curve by using a model interaction potential. Their interaction potential predicts a peak in reduced mobility (Ko) and suggests generalized mobility curves for alkali ions in rare gases (closed-shell systems). The generalized mobility curves are defined as Ko normalized by a polarization limit (PL) value K_{PL} [6,20] as a function of the effective temperature normalized to the well depth of the potential minimum. Takebe [21] made a further step in the unification of mobility data. Apart from similar conclusions as in [20], *e.g.* that the ratio $K\mathrm{omax}/K_{\mathrm{PL}}$ dominantly depends on the observed ion, he showed that transport of ions proceeds in the presence of clustering collisions and found out that the activation energy for the reversible process accounts for half of the potential well depth. Takebe's reduced mobilities in the $T \to 0$ limit were slightly above the PL values. Takebe's approach is justified only for closed-shell systems but presents a good base for upgrading for more complex systems. By following a vast amount of data in [10] one

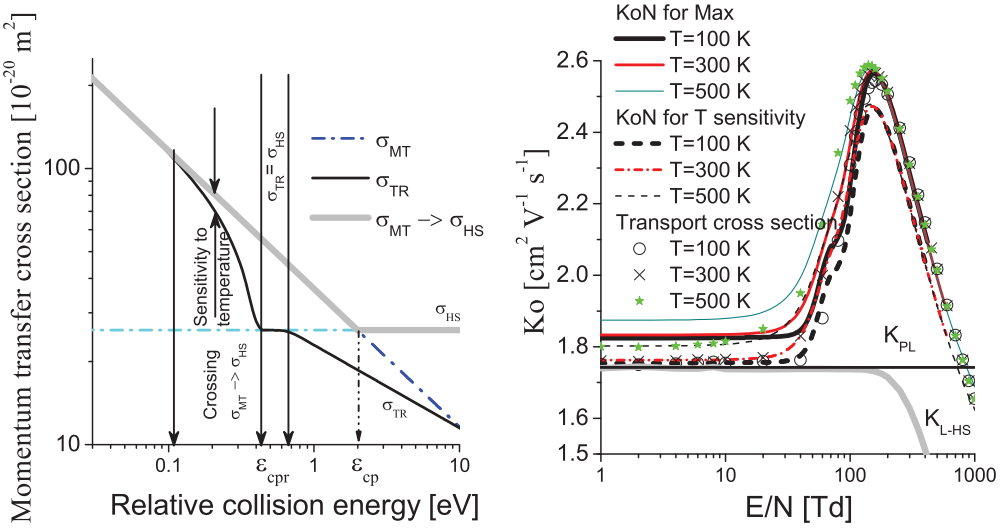

Fig. 1: (Colour online) (a) Transport cross-sections as a function of relative collision energy and (b) reduced mobility as a function of E/N where temperature T is used as a parameter.

may conclude that for a broad range of systems mobility behaves similarly to closed-shell systems, *i.e.*, where, apart from the association reaction, other reactions are negligible and the dominant effect on transport comes from the elastic momentum transfer cross-section. Since at low pressures the association cross-section is few percent of the elastic momentum transfer cross-section and is dominant at low collision energies, at $T \to 0$, $E \to 0$ affects the reduced mobility which goes to values slightly higher than the polarization limit. For the same reasons the association will not significantly affect the mobility peak. We than exploited a hard-sphere cross-section in order to define the maximum cross-section for reactive collisions.

Modelling the transport cross-section by mobility curve. In order to model the transport cross-section one may exploit either numerical solutions of Boltzmann's kinetic equation or the stochastic Monte Carlo (MC) method [22, 23]. In this work we used the well-tested MC code [23].

Central information about the interaction is drawn from the differential cross-section with all the possible quantum-mechanical effects stored within, not exclusively from interaction potentials. Thus, in the MC modeling the momentum of the transfer cross-section σ_{MT} [22,24–26] with the assumption of isotropic scattering in the center-of-mass frame is very often used. The idea behind is the need to properly account for ion momentum losses which also appear in balance equations [12] and are desired in modeling many technologically interesting cases. Ion mobility is exactly described with σ_{MT}, at low energies where the average distance at which the ion and the molecule collide is within the range of the ion-induced dipole potential $\sim r^{-4}$. The same potential for the reaction coefficient gives a constant value calculated from the known gas polarizability and reduced mass of ion and gas.

Average polarizabilities for most gaseous atoms/molecules are generally known [27] so simple approximations based on them are useful for a wide range of systems. For r^{-4} the potential momentum transfer cross-section $\sigma_{\mathrm{MT}} = \sigma_{\mathrm{DL}} = 1.105 \cdot \sigma_{\mathrm{L}}$ has the same energy dependence ($\sim \varepsilon^{-0.5}$) as Langevin's cross-section σ_{L} [17] and a constant mobility (see solid line denoted by K_{PL} in fig. 1(b)) is obtained analytically by the so-called mean-free time theories such as, for example, the theory in [22]. In this case the obtained mobility is not a function of temperature and pressure [6,26] and is taken as a limiting case for the polarization attraction between ion and neutral [17]. If for the same potential, Langevin's cross-section is used either partially or fully to describe the loss of particles, for example, the association reaction [5] reduced mobility increases. Generally inelastic collisions cause the decrease of mobility [28] while reactive collisions increase the mobility [6] which in general can be limited by the possibility of inverse reactions. Both reduced mobility and reaction rate coefficients are easily calculated by the MC code [24], where σ_{MT} is used as previously explained. The calculated values are precise (reproducible) to a few significant digits depending on the computing resources [4,24].

A hard-sphere (HS) cross-section σ_{HS} [29,30] is often used to represent ion transport [24,30] at high collision energies (see fig. 1(a)) where actually the repulsive nature of collisions becomes dominant. Scattering in this case, described by a potential $\sim r^{-n}$ ($n \to \infty$), is isotropic in the center-of-mass frame [24] and is exactly represented by $\sigma_{\mathrm{MT}} = \sigma_{\mathrm{HS}}$ in Monte Carlo codes. At the same time σ_{HS} is used as a starting point in theories of ion reactions, for example as a good approximation in cases in which processes of charge transfer dominate [29,31] so it can be a potentially good approximation for more complex cases. If one crudely joins the above-mentioned approximations simply by using $\sigma_{\mathrm{MT}} = 1.105 \cdot \sigma_{\mathrm{L}}$ below $\varepsilon_{\mathrm{cp}}$ and $\sigma_{\mathrm{MT}} = \sigma_{\mathrm{HS}}$ above $\varepsilon_{\mathrm{cp}}$, trying to describe ion mobility in a wider energy range (see thick solid line in fig. 1(a)) than the MC

Table 1: Characteristic values for mobility peaks with respect to polarization limit values in percent [10,33] and appropriate E/N values in Td ($1\,\mathrm{Td} = 10^{-17}\,\mathrm{Vcm}^2$).

		^{4}He	^{20}Ne	^{40}Ar	^{84}Kr	^{131}Xe	^{222}Rn
$^7Li^+$	$K\mathrm{omax}/K\mathrm{o_{PL}}(\%)$	71	59	62	66	65	–
	E/N (Td)	48	50	117	112	144	–
$^{11}Be^+$	$K\mathrm{omax}/K\mathrm{o_{PL}}(\%)$	47	47	68	61	52	52
	E/N (Td)	30	40	165	190	265	263
$^{23}Na^+$	$K\mathrm{omax}/K\mathrm{o_{PL}}(\%)$	57	42	46	47	50	56
	E/N (Td)	34	53	151	156	156	174
$^{27}Al^+$	$K\mathrm{omax}/K\mathrm{o_{PL}}(\%)$	37	35	37	43	40	41
	E/N (Td)	20	38	129	173	213	223
$^{39}K^+$	$K\mathrm{omax}/K\mathrm{o_{PL}}(\%)$	36	27	33	34	37	43
	E/N (Td)	27	44	131	155	185	171
$^{40}Ca^+$	$K\mathrm{omax}/K\mathrm{o_{PL}}(\%)$	36	27	33	34	37	43
	E/N (Td)	27	44	131	155	185	171

code [22,23] which properly takes into account thermal collisions [10], gives a reduced mobility in fig. 1(b) (solid line labelled by $K_{\text{L-HS}}$). $K_{\text{L-HS}}$ at $T = 300\,\mathrm{K}$ overlaps the PL curve and is a uniformly decreasing function at high average energies.

It is commonly accepted that the mobility peak represents the average energy where attractive intermolecular forces balance repulsive forces [24,32]. The height of the peak is best represented with respect to the polarization limit value, while the E/N's scale can be well normalized by the well depth of the interaction potential [20,21].

In table 1 are shown the mobility peak values obtained from the newest theoretical data for the reduced ion mobility for closed-shell systems [10] which are the best fit to all the existing experimental data, all normalized at the polarization limit value ($K\mathrm{omax}/K_{\mathrm{PL}}$). It is evident that $K\mathrm{omax}$ mainly depends on ion species in closed-shell systems, so one may pull a more general conclusion that $K\mathrm{omax}/K_{\mathrm{PL}}$ decreases with the increase of the ion atomic number in inert gases, and even in minor cases a small discrepancy exists from such average trend. With the increase of the atomic number $K\mathrm{omax}/K_{\mathrm{PL}}$ are at higher E/N's since less collision energy is transferred from the laboratory system with lighter ions to the gas for the similar effect.

For our test case ($^{20}Ne^+ + {}^{88}CF_4$) we chose $^{23}Na^+ + {}^{84}Kr$ as nearest closed-shell system due to its similarity with the atomic number and polarizability value.

From table 1, one may see that the mobility peak for the Na^+ ion is 46% relatively to the polarization limit data and is characteristic not only for Kr [21] but also for all closed-shell atoms. Thus, by analogy, the mobility peak for $Ne^+ + CF_4$ should be very similar to the peak for $Na^+ + Kr$ if one neglects the different reactivity of these two systems. For $Na^+ + Kr$ both interaction potential and bulk of the experimental measurements for the transport coefficients are available [33–36], so in the following we will refer to these data in order to model the mobility of Ne^+ in CF_4. The theoretical data for $K\mathrm{o}$ from [36] (see also [10]) give an excellent fit of the data in [33,34] at $T = 300\,\mathrm{K}$ so as to be used also for the description of the temperature variation of the reduced mobility. We selected data for temperatures $T = 100\,\mathrm{K}$, $300\,\mathrm{K}$ and $500\,\mathrm{K}$ and normalized them at the $Ne^+ + CF_4$ polarization limit (solid lines denoted as "KoN for Max" in fig. 1(b)). Ne^+ has slightly lower atomic number than Na^+ and so slightly higher $K\mathrm{omax}/K_{\mathrm{PL}}$ for which we used 49% at $E/N = 155\,\mathrm{Td}$.

Most probable processes in $Ne^+ + CF_4$ scattering. It is known [29,37] that the scattering probability for Ne^+ on CF_4 is largest for charge transfer that has been measured with a guided ion beam apparatus [29] (CF_3^+, CF_2^+ and CF^+) for energies below 50 eV and where also the absence of $NeCF_4^+$ and CF_4^+ was notified. The cross-section extrapolation up to 1000 eV was done according to the emission measurements of Motohashi *et al.* [37] and data in [31]. The line spectra of excited atoms obtained in spectrometric measurements in CF_4 indicate that the charge transfer reaction is by far the most dominant process in collisions with inert-gas ions, so other processes such as Ne excitation can be safely neglected. Having in mind measurements of the charge transfer cross-sections for F^+ and C^+ production at high energies [31] we also deduced these cross-sections by using similarities with data for other radical ions [29,31], from $He^+ + CF_4$ scattering data [29], and for the emission cross-sections in [37]. Thresholds for the production of all radical ions were selected from the data for ionization energies for CF_4 [29].

Since it is not known whether dissociative processes proceed from excited CF_4 or a $NeCF_4^+$ complex, we calculated the dissociation cross-sections by using basic gas phase enthalpies of formation for CF_4 and products [29]. With the same input data we calculated thermodynamic thresholds and used them as cross-section thresholds.

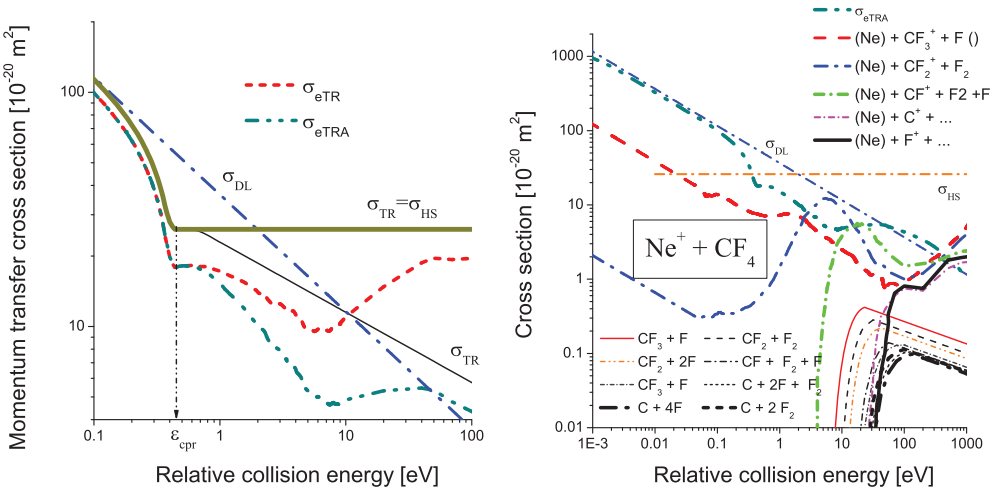

Fig. 2: (Colour online) (a) Momentum transfer cross-section and (b) cross-section set as a function of collision energy for $Ne^+ + CF_4$.

Modelling the elastic momentum transfer cross-section. Since reactive collisions reduce the total number of trajectories, then σ_{TR} can be obtained by deducing all non-continuing trajectories represented by reactive cross-sections. The same is easily implemented in Monte Carlo codes by simply removing the particle followed from the simulation. This introduces non-conservativity in transport coefficients dividing them into flux and bulk coefficients [38]. Due to the uncertainties related to the reactive cross-sections it is possible that at high collision energies the sum of the reactive cross-sections be higher than σ_{TR}. With that in mind the momentum transfer cross-section is approximated by $\sigma_{\mathrm{TR}} = \sigma_{\mathrm{HS}}$ for collision energies $\varepsilon > \varepsilon_{\mathrm{cp}}$ (see fig. 2(a)) in order to account for the possible reactions and if combined to a soft-sphere cross-section which is the one actually determined from the mobility of the closed-shell system to take into account scattering anisotropy. If now reactive cross-sections are subtracted, then the resulting cross-section σ_{eTR} represents all the elastic and inelastic losses. Now we may correct σ_{eTR} for more realistic angular dependences at high collision energies by applying the function $A(\varepsilon)$ to obtain the cross-section $\sigma_{\mathrm{eTRA}} = A\sigma_{\mathrm{eTR}}$ in fig. 2(a). If inelastic collisions can be neglected σ_{eTRA} represents the elastic momentum transfer cross-section $\sigma_{\mathrm{elm}} \sim \sigma_{\mathrm{eTRA}}$ which can be easily treated in Monte Carlo codes. If inelastic scattering is included in σ_{eTRA} the momentum balance will still be holding providing a precise calculation of the ion drift velocity and consequently of the flux. For the case of the previously derived σ_{eTRA} and reactive processes selected for $Ne^+ + CF_4$, one may claim that the cross-section set for $Ne^+ + CF_4$ is derived, which recovers the transport properties described by the given transport cross-section for a similar closed-shell system and set of reactive cross-sections (fig. 2(b)).

The mobility of high recombination energy ions such as Ne^+ ions in CF_4 is not measured up to now although the small reactivity at low E/N does not present a significant difficulty.

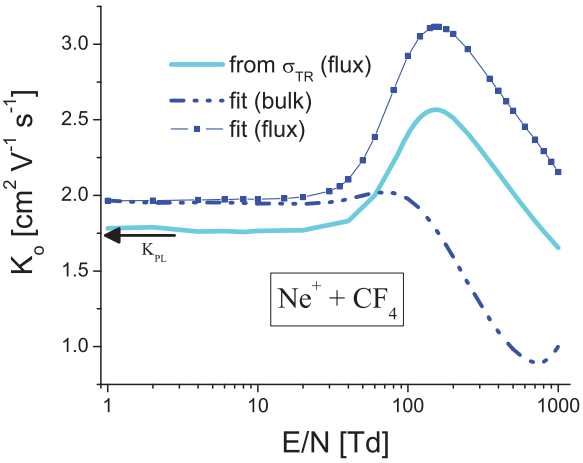

Fig. 3: (Colour online) Reduced mobility as a function E/N for Ne^+ ions in CF_4 gas for temperature $T = 300\,\mathrm{K}$.

Transport parameters. – The reduced mobility for Ne^+ ions in CF_4 as a function of E/N (E is the electric-field strength, N the gas number density) compared with bulk and flux values is shown in fig. 3. The bulk drift velocity ($W = \mathrm{d}\langle x\rangle/\mathrm{d}t$) is the reaction corrected flux drift velocity ($w = \langle v\rangle$): $W = w + S$, where S is the term representing a measure of the effect of the reactions on the drift velocity. The difference between bulk and flux reduced mobility is a consequence of the energy-dependent reactions.

Very different values of flux and bulk reduced mobility are obtained (above about 20 Td), both with peaks of different height, as a consequence of reactive collisions. A large mobility peak appears for the flux component of the reduced mobility, while a much smaller peak appears in the bulk component of the reduced mobility. The flux

reduced mobility peaks at the same E/N values where the reduced mobility peak is obtained from the momentum transfer cross-section corresponding to the closed-shell system.

Conclusion. – We presented a simple and effective method to obtain an elastic momentum transfer cross-section for a modeled system from mobility data for a similar closed-shell system. The method can be described in a step-by-step manner as follows. A similar closed-shell system is used as a reference system with the absence of reactivity. For that system the elastic momentum transfer cross-section (σ_{TR}) as a function of energy is deduced by using Monte Carlo simulations where possible clustering reactions and temperature-dependent mobility data are taken into account. The momentum transfer cross-section of the modeled system is then constructed from σ_{TR} by using its polarization limit and atomic mass. At the same time at high energies a hard-sphere cross-section is used in order to recover the reactivity of the modeled system by subtracting from it the reactive cross-sections. By taking into account the angular dependence of the reference system one finally arrives at an elastic momentum transfer cross-section as a function of the collision energy and at the assessment of the cross-section set.

The method is applied for the case of Ne^+ scattering on CF_4 for which the cross-section set is determined. By using the Monte Carlo technique in the final run we calculated transport parameters and discussed flux and bulk mobility as a function of E/N, which were not available up to now.

* * *

The results of this paper were obtained in the Laboratory of Gaseous Electronics, Institute of Physics, University of Belgrade, under the auspices of the Ministry of Education, Science and Technology, Projects Nos. 171037 and 410011.

REFERENCES

[1] Makabe T. and Petrović Z., *Plasma Electronics: Applications in Microelectronic Device Fabrication* (Taylor and Francis, CRC Press, New York) 2006; Petrović Z. Lj., Maguire P., Radmilović-Radjenović M., Radetić M., Puač N., Marić D., Mahony C. and Malović G., in *Nanotechnology for Electronics, Photonics and Renewable Energy*, edited by Korkin A. *et al.* (Springer) 2010, p. 85.

[2] Robson R. E., White R. D. and Petrović Z. Lj., *Rev. Mod. Phys.*, **77** (2005) 1303.

[3] Murakami T., Niemi K., Gans T., O'Connell D. and Graham W. G., *Plasma Sources Sci. Technol.*, **22** (2013) 015003; Van Gaens W. and Bogaerts A., *J. Phys. D: Appl. Phys.*, **46** (2013) 275201.

[4] Samukawa S., Hori M., Rauf S., Tachibana K., Bruggeman P., Kroesen G., Whitehead J. C., Murphy A. B., Gutsol A. F., Starikovskaia S., Kortshagen U., Boeuf J.-P., Sommerer T. J., Kushner M. J., Czarnetzki U. and Mason N., *J. Phys. D: Appl. Phys.*, **45** (2012) 253001.

[5] Nikitović Ž., Raspopović Z., Stojanović V. and Jovanović J., *EPL*, **108** (2014) 35004.

[6] Raspopović Z., Stojanović V. and Nikitović Ž., *EPL*, **111** (2015) 45001.

[7] Stojanović V., Raspopović Z., Jovanović J. V., de Urquijo J. and Petrović Z. Lj., *J. Phys.: Conf. Ser.*, **514** (2014) 012059.

[8] Nedner-Schatteburg G. and Bondybey V. E., *Chem. Rev.*, **100** (2000) 4059.

[9] Jalili A. H., Behnejad H., Afsahi G., Gharibi M. and Viehland L. A., *Chem. Phys. Lett.*, **584** (2013) 49.

[10] Viehland database, `www.lxcat.net`, retrieved on May 4, 2016.

[11] Armentrout P. B., Ervin K. M. and Rodgers M. T., *J. Phys. Chem.*, **112** (2008) 10071.

[12] Phelps A. V., *Phys. Rev. E*, **79** (2009) 066401.

[13] Troe J., *J. Phys. Chem. A*, **110** (2006) 2831.

[14] Petrie S. and Bohme D. K., *Top. Curr. Chem.*, **225** (2003) 37; Bohme D. K. and Schwarz H., *Angew. Chem., Int. Ed.*, **44** (2005) 2336; Bohme D. K., *Can. J. Chem.*, **86** (2008) 177.

[15] Geppert W. D. and Larsson M., *Chem. Rev.*, **113** (2013) 8872.

[16] Phelps A. V., *J. Appl. Phys.*, **76** (1994) 747.

[17] Krstić P. S. and Schultz D. R., *J. Phys. B: At. Mol. Opt. Phys.*, **42** (2009) 065207.

[18] Petrović Z. Lj. and Stojanović V. D., *J. Vac. Sci. Technol. A*, **16** (1998) 329.

[19] Falcetta M. F., Dorko M. J. and Siska P. E., *J. Chem. Phys.*, **113** (2000) 11044.

[20] Gatland I. R., Viehland L. A. and Mason E. A., *J. Chem. Phys.*, **66** (1977) 537.

[21] Takebe M., *J. Chem. Phys.*, **78** (1983) 7223.

[22] Skullerud H., *J. Phys. B: At. Mol. Phys.*, **6** (1973) 728.

[23] Ristivojević Z. and Petrović Z. Lj., *Plasma Sources Sci. Technol.*, **21** (2012) 035001.

[24] Mason E. A. and McDaniel E. W., *Transport Properties of Ions in Gases* (John Wiley and Sons, New York) 1988, p. 560.

[25] Denpoh K. and Nanbu K., *J. Vac. Sci. Technol. A*, **16** (1998) 1201.

[26] Petrović Z. Lj., Raspopović Z. M., Stojanović V. D., Jovanović J. V., Malović G., Makabe T. and de Urquijo J., *J. Appl. Surf. Sci.*, **253** (2007) 6619.

[27] Haynes W. M. (Editor), *CRC Handbook of Chemistry and Physics*, 96th edition (CRC Press) 2015.

[28] Ohtsuki K., Hananoe M. and Matsuzawa M., *Phys. Rev. Lett.*, **95** (2005) 213201.

[29] Fisher E. R., Weber M. E. and Armentrout P. B., *J. Chem. Phys.*, **92** (1990) 2296.

[30] Mc Daniel E. W., Čermak V., Dalgarno A., Ferguson E. E. and Friedman L., *Ion-Molecule Reactions* (Wiley Interscience) 1970.

[31] Parker E. and El-Ashhab F. S. M., *Int. J. Mass Spectrom. Ion Phys.*, **47** (1983) 159.

[32] Jovanović J. and Petrović Z. Lj., *J. Phys.: Conf. Ser.*, **162** (2009) 012004.

[33] Ellis H. W., Thackston M. G., Mc Daniel E. W. and Mason E. A., *At. Data Nucl. Data Tables*, **31** (1984) 113.

[34] Thackston M. G., Sanchez M. S., Neely G. N., Pope W. M., Eisele F. L., Gatland I. R. and McDaniel E. W., *J. Chem. Phys.*, **73** (1980) 2012.
[35] Koutselos A. D., Mason E. A. and Viehland L. A., *J. Chem. Phys.*, **93** (1990) 7125.
[36] Viehland L. A., Lozeille J., Soldan P., Lee E. P. F. and Wright T. G., *J. Chem. Phys.*, **119** (2003) 3729.
[37] Motohashi K., Takahashi T., Takahashi N. and Tsurubuchi S., *J. Phys. B: At. Mol. Opt. Phys.*, **38** (2005) 3339.
[38] Robson R. E., *Aust. J. Phys.*, **44** (1991) 685.