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# DIFFERENTIATING BETWEEN Δ- AND Y-STRING CONFINEMENT: CAN ONE SEE THE DIFFERENCE IN BARYON SPECTRA?*

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We use $O(4) \simeq O(3) \times O(3)$ algebraic methods to calculate the energy-splitting pattern of the $K = 2$, 3 excited states of the Y-string in two dimensions. To this purpose we use the dynamical O(2) symmetry of the Y-string in the shape space of triangles and compare our results with known results in three dimensions and find qualitative agreement.

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## 1. Introduction

QCD seems to demand a genuine three-quark confining potential: the so-called Y-junction string three-quark potential, defined by

$$V_{\rm Y} = \sigma \min_{\boldsymbol{x}_0} \sum_{i=1}^{3} |\boldsymbol{x}_i - \boldsymbol{x}_0| \, , \tag{1}$$

or, explicitly

$$V_{\rm string} \; = \; V_{\rm Y} = \sigma \sqrt{\tfrac{3}{2}\left(\boldsymbol{\rho}^2 + \boldsymbol{\lambda}^2 + 2|\boldsymbol{\rho} \times \boldsymbol{\lambda}|\right)} \, . \tag{2}$$

The complete Y-string potential contains "additional" two-body terms that are valid only in certain parts of the three-particle configuration space, and which we shall ignore here. The $|\boldsymbol{\rho} \times \boldsymbol{\lambda}|$ term is proportional to the area of the triangle subtended by the three quarks. The Y-string potential was proposed as early as 1975, see Refs. [1, 2] and the first schematic calculation (using perturbation theory) of the baryon spectrum for $K \leq 2$ followed soon

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thereafter, Ref. [3]. References [4–6] elaborated on this. The first non-perturbative calculations (variational approximation) of the $K = 3$ band with the Y-string potential were published in the early 1990s, Ref. [7] and extended to the $K = 4$ band later in that decade, Ref. [8]. Yet, some of the most basic properties, such as the ordering of the low-lying states in the spectrum of this potential, without the "QCD hyperfine interaction" and/or relativistic kinematics, remain unknown.

The first systematic attempt to solve the Y-string spectrum, albeit only for the $K \leq 2$ states, can be found in Ref. [9]. That paper used the hyper-spherical harmonics formalism, where the Y-string potential can be written as a function of hyper-angles

$$V_{\rm Y} \; = \; \sigma \sqrt{\tfrac{3}{2} R^2 \left(1 + \sin 2\chi |\sin\theta|\right)} \, . \tag{3}$$

This led to the discovery, see Ref. [10], of a new dynamical O(2) symmetry in the Y-string potential, with the permutation group $S_3 \subset O(2)$ as the subgroup of the dynamical O(2) symmetry. That symmetry was further elaborated in Ref. [11]. The present report is a continuation of that line of work.

The three-body sum of two-body potentials has only the three-body permutation group $S_3$ as its symmetry. When one changes variables from the hyper-angles $(\chi, \theta)$ to $z^{'} = z = \cos 2\chi$ (vertical axis), and $x^{'} = x\sqrt{1 - z^2} = \cos\theta \sin 2\chi$, one can see the full $S_3$ symmetry, Fig. 1. The area of the triangle $\frac{\sqrt{3}}{2} |\boldsymbol{\rho} \times \boldsymbol{\lambda}|$ and the hyper-radius $R$ are related to the Smith–Iwai variables $\alpha$, $\phi$ as follows

$$(\cos\alpha)^2 \; = \; \left(\frac{2\boldsymbol{\rho} \times \boldsymbol{\lambda}}{R^2}\right)^2 , \tag{4}$$

$$\tan\phi \; = \; \left(\frac{2\boldsymbol{\rho} \cdot \boldsymbol{\lambda}}{\boldsymbol{\rho}^2 - \boldsymbol{\lambda}^2}\right) . \tag{5}$$

The Y-string potential becomes

$$V_{\rm Y} \; = \; \sigma \sqrt{\tfrac{3}{2} R^2 \left(1 + |\cos\alpha|\right)} \, . \tag{6}$$

Independence of the potential on the variable $\phi$ is equivalent to its invariance under (infinitesimal) "kinematic rotation" O(2) transformations $\delta x^{'} = 2\varepsilon z^{'}, \delta z^{'} = -2\varepsilon x^{'}$ or, in terms of the original Jacobi variables, $\delta\boldsymbol{\rho} = \varepsilon\boldsymbol{\lambda}, \delta\boldsymbol{\lambda} = -\varepsilon\boldsymbol{\rho}$, in the six-dimensional hyper-space. This invariance leads to the new integral of motion $G_3 = \frac{1}{2}\left(\boldsymbol{p}_\rho \cdot \boldsymbol{\lambda} - \boldsymbol{p}_\lambda \cdot \boldsymbol{\rho}\right)$, References [10, 11], associated with the dynamical symmetry (Lie) group $O(2)$ that is a subgroup of the (full hyper-spherical) $O(6)$ Lie group.

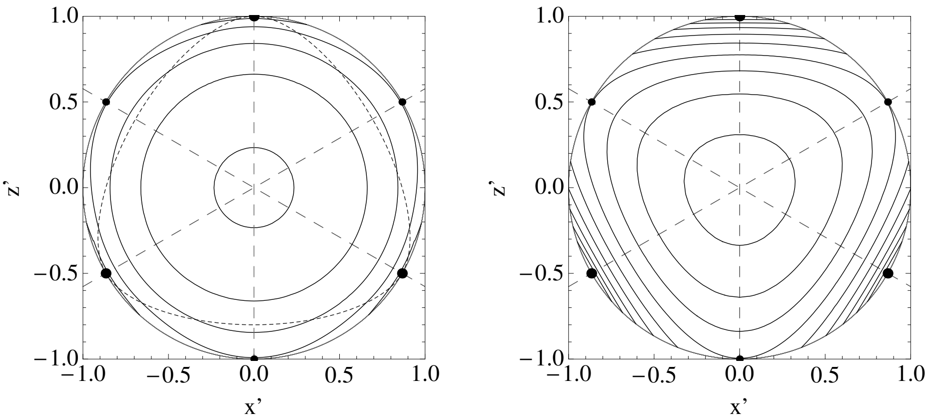


Fig. 1. Left: The equipotential contours for the central Y-string potential (black solid), and the boundary between the central Y-string and two-string potentials (blue dashes). Right: The equipotential contour plot of the Δ-string potential as functions of $z^{'} = z = \cos 2\chi$ (vertical axis), and $x^{'} = x\sqrt{1 - z^2} = \cos\theta \sin 2\chi$ (horizontal axis). The three straight lines (red long dashes) of reflection symmetry correspond to the three binary permutations, or "transpositions" $S_2$ subgroups of $S_3$. The rotations through $\phi = \pm\frac{2\pi}{3}$ correspond to two cyclic three-body permutations. The rotation symmetry of the Y-string potential (left panel) about the axis pointing out of the plane of the figure should be visible to the naked eye.

Of course, the sums of two-body potentials, such as the Δ-string potential, are invariant only under the finite rotations through $\phi = \pm\frac{2\pi}{3}$, that correspond to cyclic permutations, as well as under reflections about the three symmetry axes. In that case, this generalized hyper-angular momentum $G_3$ is not an exact integral of motion, but an approximate one. The precise consequences in the energy spectra of systems with such a broken (approximate) symmetry will be shown below.

## 2. The O(4) algebraic method

The existence of an additional dynamical symmetry strongly suggests an algebraic approach, such as those used in Refs. [12–15]. A careful perusal of Ref. [12, 13] shows, however, that an O(2) group had been used as an enveloping structure for the (discrete) permutation group $S_3 \subset O(2)$, but was not interpreted as a (possible) dynamical symmetry. References [14, 15] did not use this symmetry, however. For the sake of technical simplicity, we confine ourselves to the two spatial dimensions here. In two dimensions (2D), the non-relativistic three-body kinetic energy is a quadratic form of the two Jacobi two-vector velocities, $\dot{\boldsymbol{\rho}}, \dot{\boldsymbol{\lambda}}$, so its "hyper-spherical symmetry" is O(4), and the residual dynamical symmetry of the Y-string potential is $O(2) \otimes O_L(2) \subset O(4)$, where $O_L(2)$ is the (orbital) angular momentum. As the $O(4)$ Lie group can be "factored" in two mutually commuting O(3) Lie groups: $O(4) \simeq O(3) \otimes O(3)$, one may use for our purposes many of the O(3) group results, such as the Clebsch–Gordan coefficients. The 3D case is more complicated than the 2D one; for reasons of simplicity, we limit ourselves to the two-dimensional case in this report.

We (re)formulate the problem in terms of $O(4)$ symmetric variables and then bring the potential into a form that can be (exactly) solved, *i.e.* we expand it in O(4) hyperspherical harmonics $\mathcal{Y}^{JJ}_{LM}$. The energy spectrum is a function of the O(4) hyperspherical expansion coefficients for the potential, and of the O(4) Clebsch–Gordan coefficients, that are products of the ordinary O(3) Clebsch–Gordan coefficients. As the potential is $O_L(2)$ "rotation-symmetric", we have an additional constraint on the allowed hyperspherical harmonics and one finds that for values of $K \leq 3$ one needs only three terms: (1) the "hyper-spherical average", *i.e.* the $\mathcal{Y}^{00}_{00}$ matrix element, (2) the area-term containing the O(4) hyperspherical harmonic $\mathcal{Y}^{22}_{00}$ (which is related to the O(3) spherical harmonic $Y_{20}(\alpha, \phi)$ of the shape space (hyper)spherical angles $(\alpha, \phi)$, *i.e.*, the $V_4$ term in the notation of Richard and Taxil [17]) that is present in both the two-body and the Y-string potentials; and (3) the O(2) symmetry-breaking term containing $\mathcal{Y}^{33}_{0\pm3} \simeq Y_{3\pm3}(\alpha, \phi)$, *i.e.*, the $V_6$ term in the notation of Richard and Taxil [17], that is important in the two-body potentials, and not at all in the Y-string potential Eq. (2).

## 3. Results

We have evaluated the $K = 2, 3$ bands' splittings in 2D, Ref. [16] and compare them with the 3D case, Ref. [17]:

(1) The only difference between the 2D and 3D $K = 2$ states' splittings is that the $[70, 0^+]$ and $[56, 2^+]$ states are degenerate in 2D, whereas in 3D they are split by one half of the energy difference between $[70, 2^+]$ and $[70, 0^+]$. This shows that the 2D case does relate fairly closely to the 3D one.

(2) We compare our 2D Y-string potential $K = 3$ results with the 3D $K = 3$ two-body potential results of Ref. [17] and find certain similarities, and a few distinctions. There are six SU(6) multiplets in the $K = 3$ sector (other than the hyper-radial excitation $[70, 1^-]''$ of the $K = 1$ state): $[20, 1^-]$, $[56, 1^-]$, $[70, 3^-]$, $[56, 3^-]$, $[70, 2^-]$, $[20, 3^-]$ in 3D. The main difference between the 2D and 3D is that the $[70, 2^-]$ state disappears in 2D.

In 3D two-body potential the energy splittings can be divided in two parts in Ref. [17]: (a) those due to the $V_4$ perturbation; and (b) due to the $V_6$ perturbation. This corresponds to our $Y_{20}$ and $Y_{3\pm3}$ terms, respectively.

(a) In the $V_4 \neq 0$, $V_6 \to 0$ limit, the states can be (roughly) divided in two groups: the $[20,1^-]$, $[56,1^-]$, $[70,3^-]$ which are pushed down, and the $[56,3^-]$, $[70,2^-]$, $[20,3^-]$ which are pushed up by the $V_4$ perturbation. Two pairs of states are left degenerate: ($[20,1^-]$, $[56,1^-]$) in the lower set and ($[56,3^-]$, $[20,3^-]$) in the upper set. In this limit, in 2D we find complete degeneracy of all three members of the lower- ($[20,1^-]$, $[56,1^-]$, $[70,3^-]$) and upper levels ($[56,3^-]$, $[70,2^-]$, $[20,3^-]$), Fig. 2 (b).

(b) In the $V_4 \neq 0$, $V_6 \neq 0$ case, the remaining degeneracy of states is removed in 3D: the $[20,1^-]$ and the $[56,1^-]$ are split in the "lower set" and the $[56,3^-]$ and the $[20,3^-]$ in the "upper set". In 2D, we find the same pattern of splitting, and a similar ratio of strengths, Fig. 2 (b).

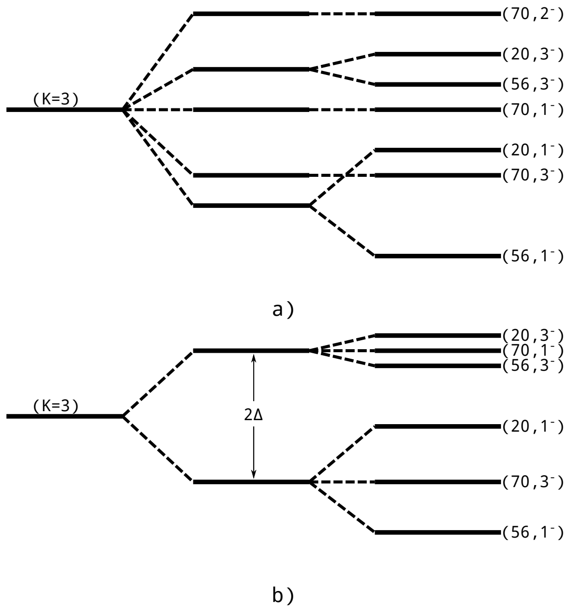


Fig. 2. Schematic representation of the $K = 3$ band in the energy spectrum of the Δ-string potential in (a) three dimensions, following Ref. [17]; and (b) two dimensions (present calculation). The sizes of the two splittings (the $v^{\Delta}_{20}$-induced $\Delta$ and the subsequent $v^{\Delta}_{3\pm3}$-induced splitting) are not on the same scale, the latter having been increased, so as to be clearly visible. The $\Delta$ here is the same as the $\Delta$ in the $K = 2$ band.

So, in the $K = 2, 3$ bands, one sees similarities of dynamical symmetry-breaking patterns in 2D and 3D. This lends credence to the belief that this similarity may persist at higher values of $K$, where there are not known 3D results, at present.

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