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Exact spatiotemporal wave and soliton solutions to the generalized (3 + 1)-dimensional nonlinear Schrödinger equation with linear potential

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Abstract

Exact extended traveling wave and spatiotemporal soliton solutions to the generalized (3 + 1)-dimensional Schrödinger equation with distributed coefficients and linear potential are found. We obtain solutions for four combinations of the temporal dependence of the coefficients: constant and sinusoidal diffraction coefficients and constant and sinusoidal external electric fields. A number of stable periodic soliton solutions are obtained whose signal does not decrease with time in the absence of an externally induced loss.

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(Some figures in this article are in colour only in the electronic version.)

1. Introduction

The generalized nonlinear Schrödinger equation (NLSE) is a generic model that is very important in nonlinear optics, where it describes the full *spatiotemporal* optical solitons [1] or *light bullets* [2]. It has also been very instrumental in describing the matter wavefunction in Bose–Einstein condensates (BECs) [3]. The stable exact soliton solutions to the generalized NLSE with the cubic nonlinearity were known to exist in (1 + 1) dimensions ((1 + 1)D) for quite some time [4]. Recently, there has been significant improvement in obtaining stable spatiotemporal soliton solutions for a *higher* number of transverse dimensions [5, 6]. The traveling wave and soliton solutions to the generalized NLSE in (3 + 1)D for a cubic nonlinearity were first developed in [7] for anomalous dispersion and were generalized in [8] for normal dispersion. Exact solutions for varying potential and nonlinearity were found in [9] by similarity transformations.

It should be noted that the stability of localized solutions of the generalized NLSE with time-dependent coefficients is of *lesser* concern than that in the case when the coefficients are constant. It is known that in homogeneous media, multidimensional solitary waves tend to blow up; they can be stabilized by dispersion management techniques [5] developed recently. When the coefficients in NLSE keep changing with time, the solutions to the equation become *transient* at all times. They can diminish, blow up, oscillate or tend to specific profiles. The question of stability of such solutions then is not of great importance.

The NLSE with cubic nonlinearity and a linear potential has also been of great interest to various fields of physics. The topic was first covered in [10], where detailed solutions were offered for the (1 + 1)D case of BECs at constant diffraction and constant linear potential in time. Then in [11, 12], the Hirota method was used to find both one- and two-soliton solutions in (1 + 1)D. Finally, in [13], (1 + 1)D solutions without chirp for the time-dependent linear potential in BECs were developed, using the *F*-expansion technique. Here, we present analytical traveling wave and soliton solutions to the NLSE in (3 + 1)D with cubic nonlinearity and time-dependent linear external potential. We specifically concentrate on the solutions with chirp, as this *internal* parameter function of the solutions offers a wide range of localized and wave solutions to the multidimensional generalized NLSE.

This paper is organized as follows. Section 2 introduces the method of F-expansion and the balance principle. Section 3 summarizes the results obtained for the four different cases considered: constant diffraction and field, constant diffraction and sinusoidal field, sinusoidal diffraction and constant field and sinusoidal diffraction and field. In section 4, we graphically present the solutions obtained and discuss the stability of these solutions. Finally, section 5 presents the conclusion.

2. Method

We consider the NLSE with distributed coefficients in (3+1)D, with a linear potential of the form [1, 4, 6]

$$\mathrm{i}\partial_t u + \frac{\beta(t)}{2}\Delta u + \chi(t)|u|^2 u + \epsilon(t)(x+y+z)u = \mathrm{i}\gamma(t)u. \quad (1)$$

Here t is time, $\Delta = \partial_x^2 + \partial_y^2 + \partial_z^2$ is the 3D Laplacian, $r = \sqrt{x^2+y^2+z^2}$ is the position coordinate and $\epsilon(t)$ stands for the strength of the linear potential as a function of time. The strength of the linear potential is related to the strength of the electric field established in the medium; hence, we will use the term 'electric field' to refer to ϵ. The direction of the electric field is purposely chosen to symmetrize the three coordinates, although other choices are possible. The functions $\beta(t)$, $\chi(t)$ and $\gamma(t)$ stand for the diffraction, nonlinearity and gain/loss coefficients, respectively.

Equation (1) will in principle be solvable by our method for arbitrary β, ϵ and γ, provided an integrability condition is satisfied by the nonlinearity coefficient χ. However, only certain forms of the coefficients will be studied in detail: in particular, the sinusoidal β and ϵ. The reason we take such a form of β and ϵ is that there exists increased interest in systems with periodic dispersion and nonlinearity, as these tend to produce stable solitary and traveling wave solutions [7]. In addition, the chirp function contained in these solutions will also be periodic and therefore the solutions will not decay or collapse, as will become apparent in the four cases of study. The two types of external field that are of most experimental interest are, of course, constant and alternating.

According to the F-expansion and the balance principle techniques [14, 15], we search for the solution of equation (1) in the form [6, 7]

$$u(x,y,z,t) = A(x,y,z,t)\exp[\mathrm{i}B(x,y,z,t)], \quad (2)$$

where

$$A = f(t)F(\theta) + g(t)F^{-1}(\theta), \quad (3)$$

$$\theta = k(t)x + l(t)y + m(t)z + \omega(t), \quad (4)$$

$$B = a(t)r^2 + b(t)(x+y+z) + e(t). \quad (5)$$

Here f, g, k, l, m, ω, a, b and e are parameter functions to be determined, and F is one of the Jacobi elliptic functions (JEFs). When expressions (3)–(5) are plugged into equation (1) and the detailed balance principle [6, 7] is applied to the resulting expressions, the following set of differential equations for the parameter functions are obtained:

$$\frac{\mathrm{d}f}{\mathrm{d}t} + 3a\beta f - \gamma f = 0, \quad (6)$$

$$\frac{\mathrm{d}g}{\mathrm{d}t} + 3a\beta g - \gamma g = 0, \quad (7)$$

$$\frac{\mathrm{d}k}{\mathrm{d}t} + 2ka\beta = 0, \quad (8)$$

$$\frac{\mathrm{d}l}{\mathrm{d}t} + 2la\beta = 0, \quad (9)$$

$$\frac{\mathrm{d}m}{\mathrm{d}t} + 2ma\beta = 0, \quad (10)$$

$$\frac{\mathrm{d}a}{\mathrm{d}t} + 2\beta a^2 = 0, \quad (11)$$

$$\frac{\mathrm{d}b}{\mathrm{d}t} + 2\beta ab - \epsilon = 0, \quad (12)$$

$$\frac{\mathrm{d}\omega}{\mathrm{d}t} + \beta(k+l+m)b = 0, \quad (13)$$

$$\frac{\mathrm{d}e}{\mathrm{d}t} + \frac{\beta}{2}[3b^2 - (k^2+l^2+m^2)c_2] - 3\chi f_1 f_2 = 0. \quad (14)$$

In addition, two algebraic relations involving the functions f and g are obtained:

$$f[\beta(k^2+l^2+m^2)c_4 + \chi f^2] = 0, \quad (15)$$

$$g[\beta(k^2+l^2+m^2)c_0 + \chi g^2] = 0. \quad (16)$$

The coefficients c_0 and c_4, together with the coefficient c_2 appearing in the equation for e, are the coefficients of the Jacobi elliptic differential equation [6]. In [7], we treated light bullets, and in [16], we considered the Gross–Pitaevskii equation by a similar method. The major difference compared to [16] is that equations (11) and (12) are different. These differences will produce a significant change in the form of solutions.

3. Results

Although the problem treated in [7] is different from the problem treated here, equations (6)–(11) and (15)–(16) of this paper are the same as the corresponding equations in [7]; hence the solutions to all the parameter functions except b, ω and e will be the same as those found in [7]:

$$f = (\alpha)^{3/2} f_0 \exp\int_0^z \gamma\,\mathrm{d}z, \quad g = \delta\sqrt{\frac{c_0}{c_4}}f, \quad (17)$$

$$a = \alpha a_0, \quad k = \alpha k_0, \quad l = \alpha l_0, \quad m = \alpha m_0, \quad (18)$$

where $\alpha = (1 + 2a_0\int_0^t \beta\,\mathrm{d}t)^{-1}$ is the normalized *chirp* function, and $\delta = 0, \pm 1$. The subscript 0 denotes the value of the given function at $t = 0$. The integrability condition for $\chi(t)$ is also the same as that in [7]:

$$\chi = -\beta c_4(k_0^2 + l_0^2 + m_0^2) f_0^{-2}\,\mathrm{e}^{-2\int_0^t \gamma\,\mathrm{d}t}/\alpha. \quad (19)$$

In the absence of gain/loss and chirp, i.e. when $a_0 = 0$, χ is of the same form as β. This testifies to the importance of the chirp function: it provides not only a means of producing different solutions, but also a means of producing different coefficients. Equations (17)–(19) hold for arbitrary $\beta(t)$ and $\epsilon(t)$.

We now proceed to solve the remaining equations for four distinct cases: constant ϵ and β; constant ϵ, $\beta = \beta_0\cos\Omega t$; constant β, $\epsilon = \epsilon_0\cos\Omega t$; and $\beta = \beta_0\cos\Omega t$, $\epsilon = \epsilon_0\cos\Omega t$.

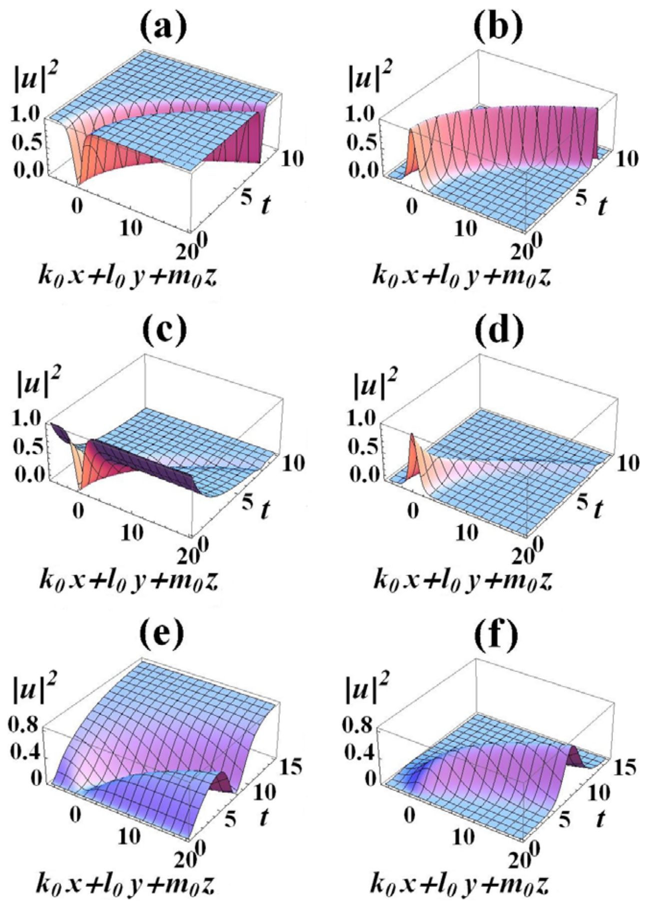

Figure 1. Soliton solutions for constant β and ϵ, as functions of time. Intensity $|u|^2$ is presented as a function of $k_0x+l_0y+m_0z$ and t. ((a), (c), (e)) Dark solitons ($F = \tan h$ for $M = 1$). ((b), (d), (f)) Bright solitons ($F = \sec h$ for $M = 1$). The parameters are $M = 1$, $\beta = 1$, $b_0 = 0$, $e_0 = 0$, $k_0 = l_0 = m_0 = 1$, $\omega_0 = 0$, $\epsilon = 0.2$ and $\delta = 0$. Panels (a) and (b) are with no chirp or gain: $a_0 = 0$, $\gamma(t) = 0$; panels (c) and (d) are with chirp: $a_0 = 0.1$, $\gamma(t) = 0$; and panels (e) and (f) are with chirp and gain: $a_0 = 0.5$, $\gamma(t) = 3/(2t)$.

3.1. Case 1: constant ϵ and β

In the case of constant ϵ and β, we obtain

$$b = \alpha(b_0 + \epsilon t + a_0\beta\epsilon t^2), \tag{20}$$

$$\omega = \omega_0 - \alpha\beta(k_0+l_0+m_0)\left(b_0 t + \frac{\epsilon t^2}{2}\right), \tag{21}$$

$$e = e_0 + \frac{\alpha\beta t}{2}\left[q - \left(3b_0^2 + 3b_0\epsilon t + \epsilon^2 t^2 + \frac{a_0\beta\epsilon^2 t^3}{2}\right)\right], \tag{22}$$

where the normalized chirp function is now $\alpha = (1+2a_0\beta t)^{-1}$ and $q = (k_0^2 + l_0^2 + m_0^2)(c_2 - 6\delta\sqrt{c_0 c_4})$.

3.2. Case 2: constant ϵ and $\beta = \beta_0 \cos \Omega t$

In the case of constant ϵ and $\beta = \beta_0 \cos \Omega t$, we obtain

$$b = \alpha\left[b_0 + \epsilon t + \frac{2a_0\beta_0\epsilon}{\Omega^2}(1 - \cos \Omega t)\right], \tag{23}$$

$$\omega = \omega_0 + \alpha\beta_0(k_0+l_0+m_0)\,[\epsilon(1-\cos\Omega t) - \Omega(b_0+\epsilon t)\sin\Omega t]\,/\,\Omega^2, \tag{24}$$

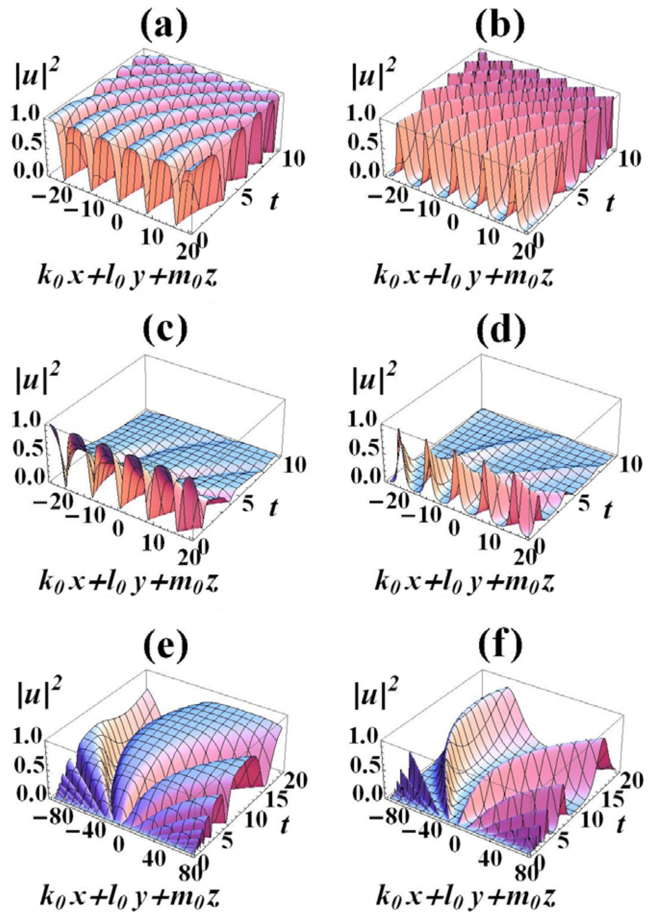

Figure 2. Traveling wave solutions for β and ϵ constant, as functions of time. The setup and parameters are the same as in figure 1, except that $M = 0.995$.

$$\begin{aligned} e = e_0 + \frac{\alpha\beta_0}{2\Omega^4}&([(q-3b_0^2)\Omega^3 + 3\epsilon^2\Omega + 12a_0\beta_0\epsilon^2\Omega t + 6b_0\epsilon\Omega^3 \\ &- 3\epsilon^2\Omega^2 t^2]\sin\Omega t + (6\epsilon^2\Omega^2 t - 12a_0\beta_0\epsilon^2 - 6b_0\epsilon\Omega^2)\cos\Omega t \\ &- 3a_0\beta_0\epsilon^2\cos(2\Omega t) + 15a_0\beta_0\epsilon^2 + 6b_0\epsilon\Omega^2), \end{aligned} \tag{25}$$

where $\alpha = (1 + 2a_0\beta_0 \sin\Omega t/\Omega)^{-1}$ is the normalized chirp function here.

3.3. Case 3: constant β and $\epsilon = \epsilon_0 \cos \Omega t$

In the case of constant β and $\epsilon = \epsilon_0 \cos \Omega t$, we obtain

$$b = \alpha\left(b_0 + \frac{\epsilon_0 \sin\Omega t}{\Omega}(1+2a_0\beta t) + \frac{2a_0\beta\epsilon_0}{\Omega^2}(\cos\Omega t - 1)\right), \tag{26}$$

$$\omega = \omega_0 - \alpha\beta(k_0+l_0+m_0)\left[b_0 t + \epsilon(1-\cos\Omega t)/\Omega^2\right], \tag{27}$$

$$\begin{aligned} e = e_0 + \frac{\alpha\beta_0}{2\Omega^4}&((q-3b_0^2)\Omega^4 t + 36a_0\beta\epsilon_0^2 - 24\beta_0\epsilon_0\Omega^2 - 6\epsilon_0^2\Omega^2 t \\ &- 12a_0\beta\epsilon_0^2\Omega^2 + (24b_0\epsilon_0\Omega^2 - 48a_0\beta\epsilon_0^2)\cos\Omega t \\ &+ 12a_0\beta\epsilon_0^2\cos(2\Omega t) + (3\epsilon_0^2\Omega + 6a_0\beta\epsilon_0^2\Omega t)\sin(2\Omega t)), \end{aligned} \tag{28}$$

where $\alpha = (1+2a_0\beta t)^{-1}$ is the appropriate normalized chirp function.

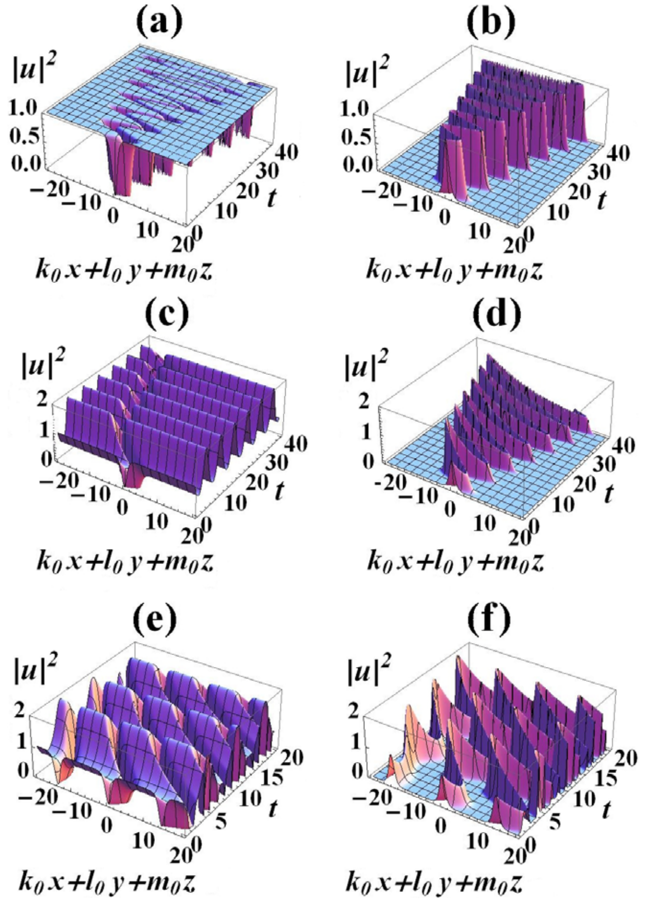

Figure 3. Soliton and traveling wave solutions for $\beta = \beta_0 \cos \Omega t$ and constant ϵ, as functions of time. Intensity $|u|^2$ is presented as a function of $k_0x + l_0y + m_0z$ and t. ((a), (c), (e)) $F = sn$ ($F = \tan h$ for $M = 1$). ((b), (d), (f)) $F = cn$ ($F = \sec h$ for $M = 1$). The parameters are $\beta_0 = 1$, $\gamma(t) = 0$, $b_0 = 1$, $e_0 = 0$, $k_0 = l_0 = m_0 = 1$, $\omega_0 = 0$, $\epsilon = 0.1$, $\Omega = 1$, $\gamma(t) = 0$ and $\delta = 0$. Panels (a) and (b) are solitary waves with no chirp: $a_0 = 0$, $M = 1$; panels (c) and (d) are solitary waves with chirp: $a_0 = 0.1$, $M = 1$; panels (e) and (f) are traveling waves with chirp: $a_0 = 0.1$, $M = 0.999\,99$.

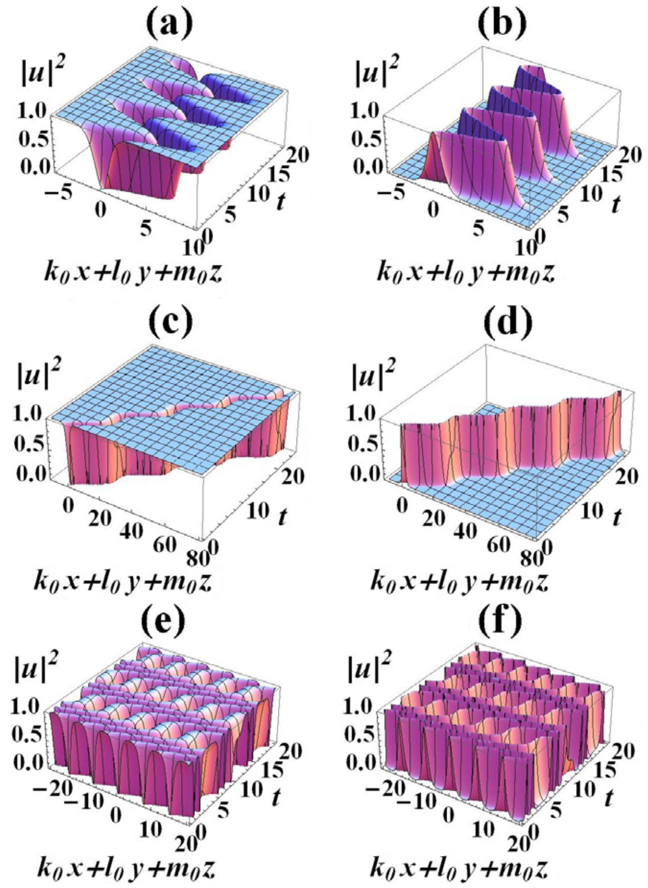

Figure 4. Soliton and traveling wave solutions for constant β and $\epsilon = \epsilon_0 \cos \Omega t$, without chirp and gain. Intensity $|u|^2$ is presented as a function of $k_0x + l_0y + m_0z$ and t. ((a), (c), (e)) $F = sn$ ($F = \tan h$ for $M = 1$). ((b), (d), (f)) $F = cn$ ($F = \sec h$ for $M = 1$). The parameters are $\beta = 1$, $\gamma(t) = 0$, $a_0 = 0$, $e_0 = 0$, $k_0 = l_0 = m_0 = 1$, $\omega_0 = 0$, $\epsilon_0 = 1$, $\Omega = 1$ and $\delta = 0$. Panels (a) and (b) are solitary waves with no b: $M = 1$, $b_0 = 0$; panels (c) and (d) are solitary waves with b: $M = 1$; $b_0 = 1$; panels (e) and (f) are traveling waves with b: $M = 0.99$, $b_0 = 1$.

3.4. Case 4: $\beta = \beta_0 \cos \Omega t$ and $\epsilon = \epsilon_0 \cos \Omega t$

In the case of $\beta = \beta_0 \cos \Omega t$ and $\epsilon = \epsilon_0 \cos \Omega t$, we obtain

$$b = \alpha \left(b_0 + \frac{\epsilon_0}{\Omega} \sin \Omega t + \frac{a_0\beta_0\epsilon_0}{\Omega^2} (\sin^2 \Omega t) \right), \qquad (29)$$

$$\omega = \omega_0 - \alpha\beta_0(k_0 + l_0 + m_0)\frac{\sin \Omega t}{\Omega} \left(b_0 + \frac{\epsilon_0}{2\Omega} \sin \Omega t \right), \qquad (30)$$

$$e = e_0 + \frac{\alpha\beta_0 \sin \Omega t}{16\Omega^4}(8(q - 3b_0^2)\Omega^3 - 4\epsilon_0^2\Omega - 3\epsilon_0(a_0\beta_0\epsilon_0 + 8b_0\Omega^2) \sin \Omega t + a_0\beta_0\epsilon_0^2 \sin(3\Omega t)), \qquad (31)$$

where $\alpha = (1 + 2a_0\beta_0 \sin \Omega t/\Omega)^{-1}$ again is the normalized chirp function. We note that in all four cases the solutions with $\epsilon = 0$ reduce to those found in [7].

4. Discussion

We are now in a position to study the effect of external linear potential on our solutions. We will not include solutions without the field turned on, i.e. $\epsilon = 0$; these can be found in [7]. Still, for $\epsilon = 0$, one should remark that for $b_0 = 0$ the waves travel in a straight line for both constant β and $\beta = \beta_0 \cos \Omega t$. For $\epsilon = 0$, constant β and $b_0 = 1$, the solution is linear as a function of the longitudinal variable, unlike the case $\beta = \beta_0 \cos \Omega t$ depicted in [7], where the solution oscillates. Although no results on the stability of these solutions are provided here, numerical work done in [7] and the study of modulational stability of solutions in [17], obtained using a method similar to that described here, all point to these solutions as being numerically and modulationally stable.

4.1. Case 1: constant ϵ and β

First we discuss the first case: constant ϵ and β. Figure 1 depicts the soliton solutions when the elliptic modulus M of JEFs equals 1. In figures 1(a) and (b), we see the effect of adding the electric field to a solitary wave. Pushed by the external field, the chirpless solution moves away parabolically from the center. The effect of chirp dramatically changes the situation, as can be discerned from figures 1(c) and (d). As a consequence of the chirp, the solution decays. The same will hold true for the third case: constant β and $\epsilon = \epsilon_0 \cos \Omega t$.

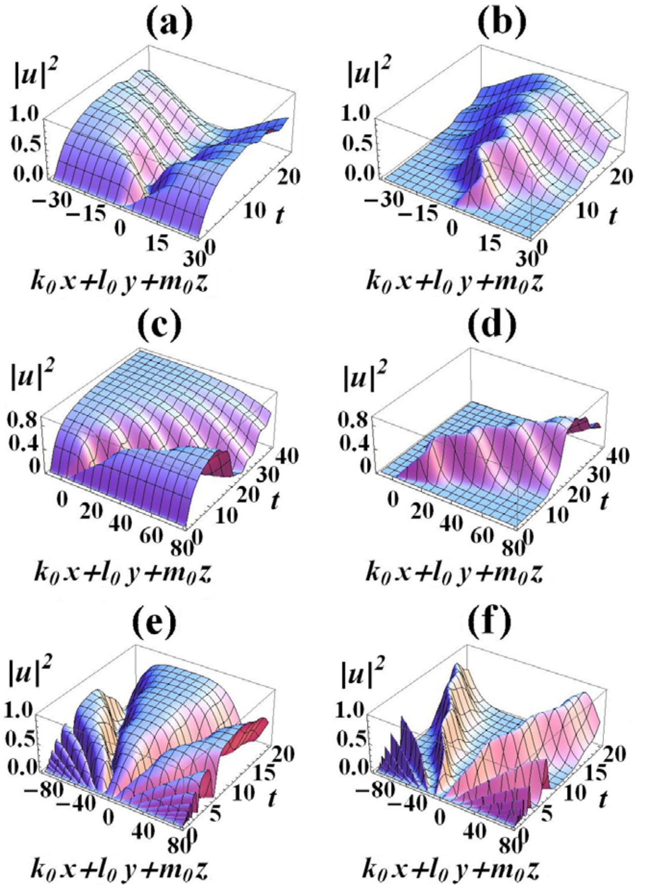

Figure 5. Soliton and traveling wave solutions for constant β and $\epsilon = \epsilon_0 \cos \Omega t$, with chirp and gain. The figure layout is as in figure 4. The parameters are the same as in figure 4, except for $a_0 = 0.5$, $\gamma(t) = 3/(2t)$.

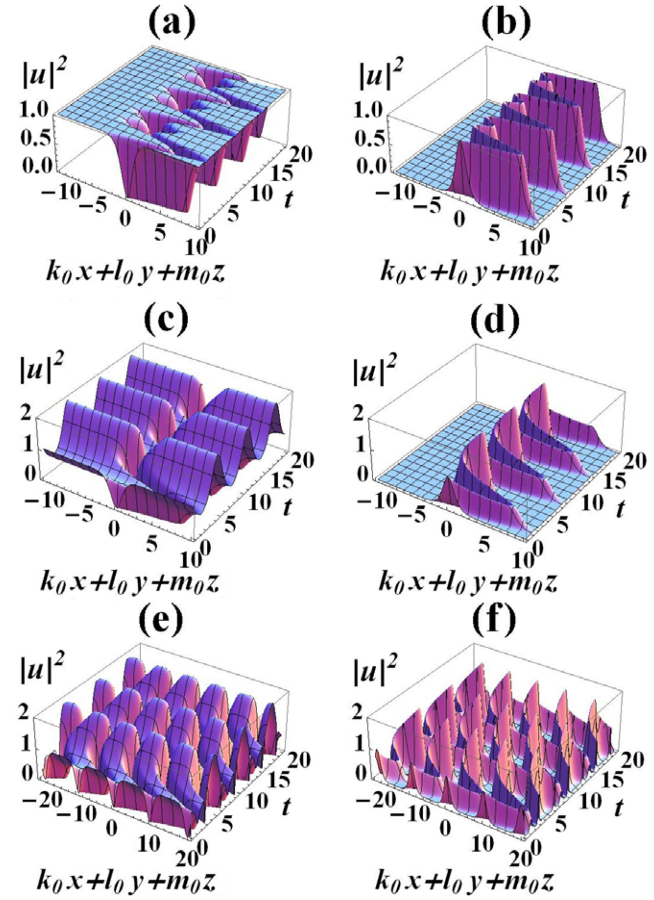

Figure 6. Soliton and traveling wave solutions for $\beta = \beta_0 \cos \Omega t$ and $\epsilon = \epsilon_0 \cos \Omega t$, as functions of time. The figure layout is the same as in figure 3. The parameters are $\beta_0 = 1$, $\gamma(t) = 0$, $a_0 = 0$, $b_0 = 1$, $e_0 = 0$, $k_0 = l_0 = m_0 = 1$, $\omega_0 = 0$, $\epsilon_0 = 3$, $\Omega = 1$ and $\delta = 0$. Panels (a) and (b) are the solitary waves with no chirp: $a_0 = 0$, $M = 1$; panels (c) and (d) are the solitary waves with chirp: $a_0 = 0.1$, $M = 1$; panels (e) and (f) are the traveling waves with chirp: $a_0 = 0.1$, $M = 0.999$.

The effect of chirp can be mitigated to a certain degree by including some gain in the problem. Specifically, when γ is set to be $\gamma(t) = 3/(2t)$, the long-time influence of the chirp will be canceled out by the exponential in the formulae for f and g. This idea was first used in [16]. The value to which the amplitude of the pulse converges is $1/(2a_0\beta)^{(3/2)}$. Dark and bright soliton solutions with chirp and gain are given in figures 1(e) and (f). Naturally, when gain is present, the power will increase with time; the solutions will not blow up, but will become wider.

In figure 2, we see the same solutions as in figure 1, except that the traveling waves are shown; that is, the only parameter different from figure 1 is the elliptic modulus of JEFs, $M = 0.995$. By comparing figures 2(a) and (b) with figures 2(c) and (d), we see that the effect of the chirp is not only to reduce the intensity but also to broaden the waves. In figures 2(e) and (f), once again the effect of adding the gain is seen. Note that the picture is not symmetrical with respect to the transverse variable, and that the central crest curls to the right (towards positive values of the transverse variable), and along with it all other crests.

4.2. Case 2: constant ϵ and $\beta = \beta_0 \cos \Omega t$

We now turn our attention to the second case: $\beta = \beta_0 \cos \Omega t$ and constant ϵ. Here we see the effect of a sinusoidal form of β, analyzed in more detail in [7], in the presence of a constant electric field. In figures 3(a) and (b) we see that the effect of the electric field is to widen the amplitude of oscillation of the soliton. This widening effect happens even in the absence of b_0. For $b_0 = 0$ and without the electric field, the solution obtained is that of a pulse moving in a straight line.

In figures 3(c) and (d) one sees the effect of adding chirp, which roughly corresponds to what was obtained in [7]. Since β has the form of a sine function, the chirp does not cause the pulse to decay (given sufficiently low a_0 to avoid singularities), but rather the amplitude of the pulse oscillates. As in [7], there is the effect of stretching along the transverse variable; that is, the solution is no longer periodic with respect to the transverse variable. In figures 3(e) and (f), we see the corresponding effects on traveling waves.

4.3. Case 3: constant β and $\epsilon = \epsilon_0 \cos \Omega t$

We now analyze the third case: constant β and $\epsilon = \epsilon_0 \cos \Omega t$. In the absence of b_0 the solutions oscillate as a sine wave, as in figures 4(a) and (b). However, if $b_0 \neq 0$, then the solution veers towards positive values of the transverse variable, as in figures 4(c) and (d). In figures 4(e) and (f), we see the traveling wave solutions for $b_0 \neq 0$. The presence of the chirp causes the solutions to decay, as in the first case. However, the same trick can be applied as in the first case: adding the right amount of gain to counteract the loss of the signal due to chirp. We see the effect in figure 5.

4.4. Case 4: $\beta = \beta_0 \cos \Omega t$ and $\epsilon = \epsilon_0 \cos \Omega t$

Finally, for the fourth case we have $\beta = \beta_0 \cos \Omega t$ and $\epsilon = \epsilon_0 \cos \Omega t$. In this case the oscillations of β and ϵ combine, as shown in figure 6. In figures 6(c) and (d) we see the effect of adding chirp. As can be seen, the chirp warps the amplitude of the signal in surprising ways. Finally, in figures 6(e) and (f) we see the same results as in figures 6(c) and (d) but for the traveling waves.

5. Conclusion

We have obtained exact solutions for the NLSE with a cubic, time-dependent nonlinearity, in a linear time-dependent potential. We have analyzed in detail the dependence of the form of the solutions on the time dependence of the linear potential and the diffraction coefficient, and obtained exact solutions for four cases that are of particular interest.

Acknowledgments

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References

[1] Akhmediev N N and Ankiewicz A A 1997 *Solitons* (London: Chapman and Hall)
Kivshar Y S and Agrawal G P 2003 *Optical Solitons: From Fibers to Photonic Crystals* (New York: Academic)
Hasegava A and Matsumoto M 2003 *Optical Solitons in Fibers* (Berlin: Springer)
[2] Malomed B A, Mihalache D, Wise F and Torner L 2005 Spatiotemporal optical solitons *J. Opt. B: Quantum Semiclass. Opt.* **7** R53
[3] Kevrekidis P G, Frantzeskakis J and Carretero-Gonzales R (ed) 2008 *Emergent Nonlinear Phenomena in Bose–Einstein Condensates: Theory and Experiment* (Berlin: Springer)
[4] Kruglov V I, Peacock A C and Harvey J D 2003 Exact self-similar solutions of the generalized nonlinear Schrödinger equation with distributed coefficients *Phys. Rev. Lett.* **90** 113902
Kruglov V I, Peacock A C and Harvey J D 2005 Exact solutions of the generalized nonlinear Schrödinger equation with distributed coefficients *Phys. Rev.* E **71** 056619
[5] Malomed B A 2006 *Soliton Management in Periodic Systems* (New York: Springer)
[6] Zhong W P *et al* 2008 Exact spatial soliton solutions of the two-dimensional generalized nonlinear Schrödinger equation with distributed coefficients *Phys. Rev.* A **78** 023821
[7] Belić M *et al* 2008 Analytical light bullet solutions to the generalized (3 + 1)-dimensional nonlinear Schrödinger equation *Phys. Rev. Lett.* **101** 0123904
[8] Petrović N, Belić M, Zhong W P, Xie R H and Chen G 2009 Exact spatiotemporal wave and soliton solutions to the generalized (3 + 1)-dimensional nonlinear Schrödinger equation for both normal and anomalous dispersion *Opt. Lett.* **34** 1609
[9] Yan Z and Konotop V V 2009 Exact solutions to three-dimensional generalized nonlinear Schrödinger equations with varying potential and nonlinearities *Phys. Rev.* E **80** 036607
[10] Chen H-H and Liu C-S 1976 Solitons in nonuniform media *Phys. Rev. Lett.* **37** 693
[11] Li Z-D *et al* 2007 Hirota method for the nonlinear Schrödinger equation with an arbitrary linear time-dependent potential *Ann. Phys.* **322** 2545
[12] Li Q-Y *et al* 2009 Nonautonomous solitons of Bose–Einstein condensation in a linear potential with an arbitrary time-dependence *Opt. Commun.* **282** 1676
[13] Yang Q and Zhang J-f 2006 Bose–Einstein solitons in time-dependent linear potential *Opt. Commun.* **258** 35
[14] Zhou Y B, Wang M L and Wang Y M 2003 Periodic wave solutions to a coupled KdV equations with variable coefficients *Phys. Lett.* A **308** 31
[15] Zhou Y B, Wang M L and Miao T D 2004 The periodic wave solutions and solitary wave solutions for a class of nonlinear partial differential equations *Phys. Lett.* A **323** 77
[16] Petrović N Z, Belić M and Zhong W-P 2010 Exact spatiotemporal wave and soliton solutions to the generalized (3 + 1)-dimensional Gross–Pitaevskii equation *Phys. Rev.* E **81** 016610
[17] Petrović N Z, Aleksić N R, Bastami A A and Belić M R 2011 Analytical traveling-wave and solitary solutions to the generalized Gross–Pitaevskii equation with sinusoidal time-varying diffraction and potential *Phys. Rev.* E **83** 036609