# Wideband Tuning of the Tunneling Frequency in a Narrowed Epsilon-Near-Zero Channel

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***Abstract*—We propose a design of foam ENZ waveguide with E- and H-plane step discontinuities at the same waveguide cross-section. This enables using channel dielectric permittivity greater than the permittivity in input waveguides, which is required for foam ENZ waveguide realization. A method for wideband tuning of tunneling frequency by changing the length of two longitudinal slots was also proposed and experimentally verified. Simulated tuning range is 1.06 GHz which is in a good agreement with experimentally achieved 0.99 GHz. This tuning range is accompanied by amplitude decrease of 1.6 dB.**

***Index Terms*—ENZ metamaterial, rectangular waveguide, E- and H-plane step discontinuity, frequency tuning.**

## I. Introduction

ENERGY tunneling through a very narrow channel obtained by reducing the height of a rectangular waveguide has attracted great attention of the scientific community in the past several years. It has been theoretically shown that it is possible to obtain transmission of electromagnetic waves excited in an input waveguide through a very narrow channel despite the significant E-plane discontinuity between these two waveguides [1]. It was further explained [2] that this unusual effect called energy tunneling takes place just below the cut-off frequency in the channel, when its effective permittivity becomes close to zero (epsilon-near-zero, ENZ). In addition to the tunneling frequency, the ENZ channel supports propagation at one more frequency above the cut-off, known as the Fabry-Perot (FP) resonance, which strongly depends on the length of the channel.

Propagation wavelength at the tunneling frequency approaches infinity, so the wave propagation can be considered quasistatic, even at a great distance, which means that it does not depend on the length of the channel. This kind of behavior is characteristic for the left-handed zero-order resonator, so the tunneling frequency can be addressed as a zero-order resonance (ZOR). This is why the ENZ channel can be used to enhance the efficiency of the energy transfer through the waveguide discontinuities, as well as for energy confinement below the diffraction limit [1]. Other applications involve dielectric sensing [3], as well as multiband filtering and tunable high directivity [4] or omnidirectional antennas [5].

ENZ behavior was accomplished either by using the dispersion characteristic of the rectangular waveguide near the cut-off [6], or by adding complementary split-ring resonators inside the channel [7]. If the channel width is equal to the width of input waveguides (which is usually the case), the dielectric permittivity in the ENZ channel ($\varepsilon_{rch}$) should be lower than in the input waveguides ($\varepsilon_{rwg}$) in order to provide tunneling. Here we have proposed a more flexible design of the ENZ waveguide which consists of H-step discontinuity added at the same cross section as E-step to reduce the channel width in respect to input waveguides. This design allows $\varepsilon_{rch}$ to be greater than $\varepsilon_{rwg}$, which is the case in our paper.

A method of tuning the tunneling frequency by changing the capacitance of a varactor diode placed across the channel was recently proposed [8]. However, the achieved frequency shift is followed by considerable decrease of transmission amplitude as well as the $Q$-factor, which are the main drawbacks of this approach. Here we have proposed and experimentally verified a method of tuning the tunneling frequency using two longitudinal slots on the broad side of the narrowed channel. The tuning range of the tunneling frequency is optimized by changing the offset and length of two longitudinal slots. This way of tuning the tunneling frequency has little influence on transmission amplitude, and can be efficiently applied to a very thin channel, which is not the case in the only previously reported method of tuning the tunneling frequency [8]. Frequency tuning can be done continuously by means of sliding backshort, or discretely by using PIN diodes across the slots. Proposed tuning method can be used to widen the frequency range of ENZ sensors for measurements of dielectric permittivity [9], as well as in a design of tunable multiband filters using multilayered structure [10], but with narrowed channels instead of the wire media.

## II. Foam ENZ Waveguide

A novel design of foam ENZ waveguide which consists of a thin microwave substrate Taconic TLY-5A ($\varepsilon_{rch} = 2.17$, $tg\delta_{ch} = 0.001$) which serves as an ENZ channel, and also as a carrier of the input waveguides is shown in Fig. 1. The use of a microwave substrate in the channel is very advantageous since it allows precise control of channel height and roughness (*rms*) of the metal coating (*rms* $= 0.5\mu m$, $\sigma = 58MS/m$), which can significantly increase the losses in the channel if we lower channel height below a certain point, or the roughness is too high [11]. Input waveguides are made of foam dielectric ROHACELL 200 WF ($\varepsilon_{rwg} = 1.22$, $tg\delta_{wg} = 0.0009$) which is easily shaped and cut.

The propagation of the first order mode ($\mathrm{TE}_{10}$) in a narrow channel can be described as a propagation of the TEM mode in a parallel-plate waveguide with effective permittivity, which

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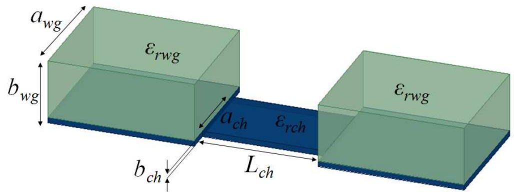


Fig. 1. Foam ENZ waveguide with relevant dimensions: $a_{wg} = 22$ mm, $a_{ch} = 14$ mm, $b_{wg} = 11$ mm, and $b_{ch} = 0.508$ mm. Outer metal coating is removed so the dielectric layers can be seen.

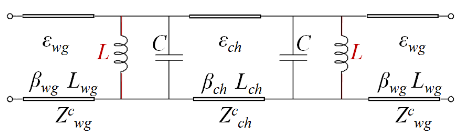


Fig. 2. Equivalent circuit of a narrowed ENZ waveguide.

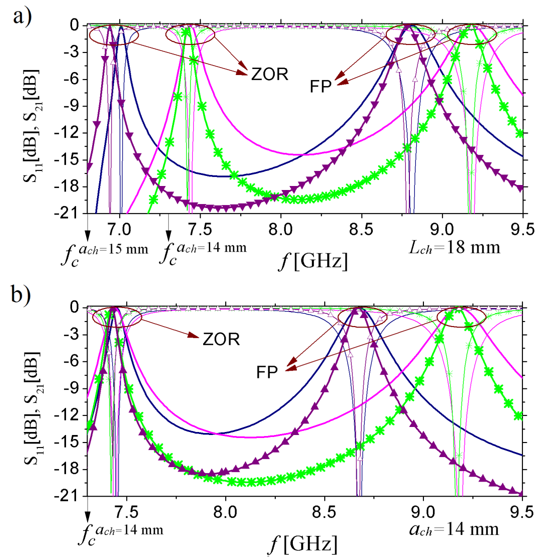


Fig. 3. Comparison of $S_{11}$ (thin lines) and $S_{21}$ (thick lines) parameters obtained by full-wave simulation (with markers) and equivalent circuit (without markers) for: (a) two different channel widths ($a_{ch} = 14$ mm - light lines, and $a_{ch} =15$ mm - dark lines); (b) two different channel lengths ($L_{ch} = 18$ mm - light lines, and $L_{ch} =22$ mm - dark lines). The channel cut-off frequencies are denoted with arrows.

is given as [3]:

$$\varepsilon_{reff} \approx \varepsilon_{rch} - \frac{c^2}{4f^2 a_{ch}^2}. \tag{1}$$

It can be seen that the effective permittivity, $\varepsilon_{reff}$ equals zero at the channel cut-off frequency $f^{ch}_{TE_{10}}$ at which the tunneling of energy appears:

$$f_{tun} \approx f^{ch}_{TE_{10}} = \frac{c}{2a_{ch}\sqrt{\varepsilon_{rch}}}. \tag{2}$$

The frequency range in which energy tunneling may occur is within the passband of the input waveguide, i.e. between cut-off frequency of $\mathrm{TE}_{10}$ mode and the frequency at which the first higher mode appears ($f^{wg}_{TE_{10}} < f_{tun} < f^{wg}_{TE_{20}}$). In order to fit the tunneling frequency in this passband, we can keep the same width of the channel and the input waveguides, but put dielectric with smaller relative permittivity in the channel, or we can narrow the channel in respect to the input waveguides, so its relative permittivity can be equal or even greater than the relative permittivity in the input waveguides. This can be described by defining a condition [12]:

$$\frac{\varepsilon_{rwg}}{4} \leq \left(\frac{a_{ch}}{a_{wg}}\right)^2 \varepsilon_{rch} \leq \varepsilon_{rwg}. \tag{3}$$

The equivalent circuit of the proposed ENZ waveguide is given in Fig. 2. E-step discontinuity between the input waveguide and the channel is modeled by parallel capacitance $C$ [13], while the H-step discontinuity i.e. the change in the channel width is modeled by parallel inductance $L$ [13]. The tunneling frequency, as well as the Fabry-Perot resonance, come from the resonating nature of the obtained $LC$ circuit. The propagation constant in input waveguides filled with microwave substrate and foam dielectric is $\beta_{wg} = 2\pi/\lambda_{gwg}$, where $\lambda_{gwg}$ is given as [13]:

$$\lambda_{gwg} = \frac{\lambda_0}{\sqrt{\frac{\varepsilon_{rch}}{1-((b_{wg}-b_{ch})/b_{wg})(1-\varepsilon_{rch}/\varepsilon_{rwg})} - (\frac{\lambda_0}{2a_{wg}})^2}}. \tag{4}$$

It should be pointed out that ZOR resonance appears above the channel cut-off frequency, which is not the case with ENZ waveguide based only on E-plane step discontinuity. Since the step in E-plane can be represented as capacitance, and the channel characteristic impedance has an inductive nature below the cutoff, the tunneling in this case is forced to occur at the frequency which is just below the channel cut-off in order to provide the necessary inductance for $LC$ resonator.

Using proposed equivalent circuit we are able to predict the frequency of tunneling and Fabry-Perot resonance quite accurately, without any full-wave simulation. This is illustrated in Fig. 3 (a) and (b), which is showing the comparison of $S_{11}$ and $S_{21}$ coefficients obtained using an equivalent circuit and a full-wave analysis for the different widths and lengths of the channel, respectively. The small discrepancy in results obtained using equivalent circuit comes from the fact that the expressions for L and C [13] are valid only for $\mathrm{TE}_{10}$ mode propagation. Since the input waveguides are composed of two layers, that causes a hybrid mode to arise. However, the height of one layer in regard to the other one is very small, so we can approximate it by a $\mathrm{TE}_{10}$ mode.

## III. Methods of Tuning

According to expression (2), the tunneling frequency is fixed for the given channel geometry, since it depends only on channel width and dielectric permittivity in the channel. However, in this article we have shown that it is possible to shift the tunneling frequency down by changing the length of two longitudinal slots placed on the broad face of the ENZ channel (see Fig. 4).

It is well known that a longitudinal slot whose length is nearly equal to the half free-space wavelength is a commonly used radiator in antenna systems. It can be represented by a shunt admittance which mainly depends on the slot offset, i.e. the slot displacement from the channel centerline. For tuning the tunneling frequency we have proposed two very

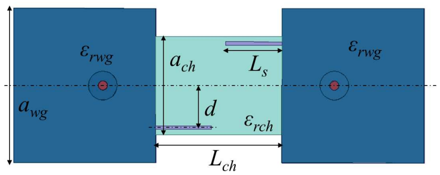


Fig. 4. ENZ channel with coaxial connectors for signal excitation and longitudinal slots with length $L_s$ for frequency tuning. The slot displacement from the channel centerline (offset) is denoted with $d$.

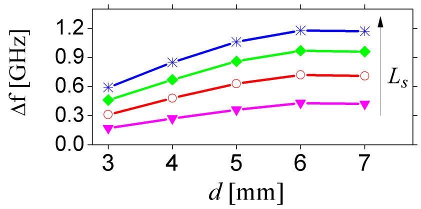


Fig. 5. Simulated tuning range vs. slot offset, for the different slot lengths $L_s$= 4, 6, 7.1 and 8 mm respectively in the direction of the arrow.

short longitudinal slots whose length $L_s$ ranges from $0.1\lambda_0$ to $0.24\lambda_0$, and which are placed far from the channel centerline in order to increase the tuning range. The tuning range of the tunneling frequency depends on the length of the slot, and also on its offset. Changing of the slot length can be done either continuously by sliding planar backshort, or discretely by using PIN diodes across the slots.

Simulated tuning range $\Delta f$ as a function of offset and slot length for continuous tuning is shown in Fig. 5. It can be seen that the slots with different lengths exhibit the maximum tuning range for the same displacement which is slightly apart from the channel ends (offset $d = 6$ mm). The maximum simulated frequency shift of 1.18 GHz is achieved in the range 7.03 to 5.85 GHz (16.8% of the tuning range) using slots with the length $L_s = 8$ mm.

For the discrete tuning we can place a finite number of PIN diodes across the slots, as it is shown in Fig. 6. We have simulated the influence of a PIN diode switch, which is reflected in a way that the OFF state of the diode has no influence on the slot, and the ON state short circuits the slot. This is how the slot effectively becomes divided into two parts, from which the longer one is predominantly responsible for the frequency shift. Therefore, the position of the diode in respect to the E-step can be determined using corresponding slot length from Fig. 5. to achieve desired frequency shift. This has been verified in Fig. 7. (see curves ii-iia and iii-iiia). The minimal frequency shift is obtained when the slot is short circuited in the middle, and as a consequence, we have a discrete tuning range ($\Delta f_D$) which is smaller than the frequency range for continuous tuning ($\Delta f_C$), as it is shown in Fig.7.

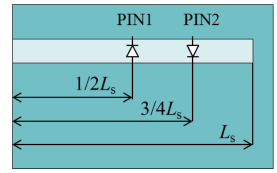


Fig. 6. Tuning slot ($L_s$= 8 mm, $d$=5 mm) with marked positions of PIN diodes.

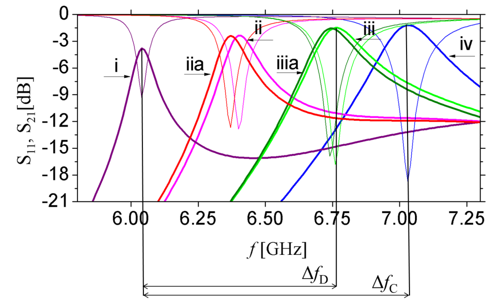


Fig. 7. Simulated $S_{11}$ (thin lines) and $S_{21}$ (thick lines) parameters for discrete tuning using PIN diodes across the slot in the case of: i) both diodes turned OFF; ii) only PIN2 switched ON; iii) only PIN1 switched ON; compared to the results obtained using slots without diodes of the length iia) $L_s$= 6 mm, and iiia) $L_s$= 4 mm. For comparison of discrete ($\Delta f_D$) and continuous ($\Delta f_C$) tuning ranges, the case: iv) without slots, is also given.

## IV. Experimental Verification

We have experimentally verified continuous tuning method using silver conductive paste to shorten the slots. For the experiment the input waveguides were excited by coaxial connectors whose position in respect to the channel and backshorts, as well as the length of pins, has been optimized to obtain the best impedance matching near the tunneling frequency.

Comparison between the measured and simulated $S_{11}$ and $S_{21}$ parameters around the tunneling frequency (ZOR) as a function of the slot length for slot offset $d = 5$ mm is given in Fig. 8. The simulated tuning range is 1.06 GHz, which is in a good agreement with experimentally achieved 0.99 GHz. This tuning range is accompanied by an amplitude decrease of 1.6 dB. However, the maximum achievable tuning range is about an octave, i.e. between cut-off frequencies of $TE_{10}$ mode and the first higher $TE_{20}$ mode of input waveguides. To achieve such a wide tuning range it is necessary to excite ZOR without tuning slots below, but near to the cut-off frequency of $TE_{20}$ mode of the input waveguides.

The ENZ waveguide is a dual band device because it supports two transmission bands: ZOR and FP. By tuning the tunneling frequency by means of longitudinal slots, the FP resonance is shifted down simultaneously, but this shift is smaller than the ZOR one. Experimentally achieved tuning range is 0.78 GHz while theoretical prediction gives 0.86 GHz, as can be seen in Fig. 9. Obtained results show that there is no degradation of the amplitude of FP resonance during the frequency tuning due to better matching of input ports at lower FP frequencies.

The experimental model with coaxial connectors is shown in Fig. 10.

## V. Conclusion

We have proposed a more flexible design of ENZ waveguide which includes both E- and H-step discontinuity at the same cross section of the rectangular waveguide. In this way dielectric permittivity required for energy tunneling can now

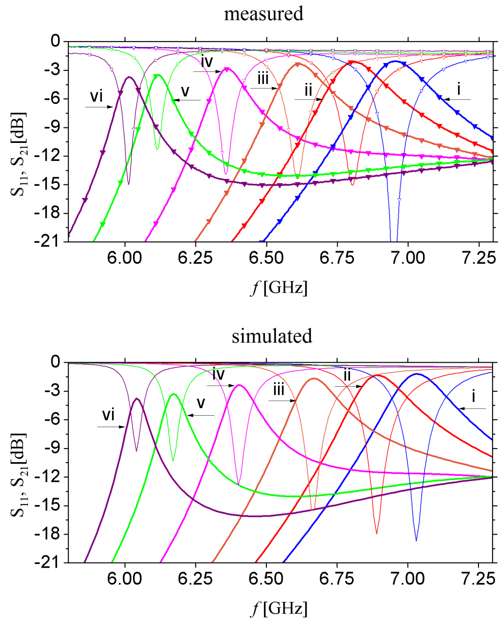


Fig. 8. Comparison of the experimentally obtained $S_{11}$ and $S_{21}$ parameters and the simulated ones for ZOR resonance for different slot lengths: i) without slots, ii) $L_s = 3$ mm, iii) $L_s = 4.6$ mm, iv) $L_s = 6$ mm, v) $L_s = 7.1$ mm and vi) $L_s = 8$ mm. Slot offset is $d = 5$ mm.

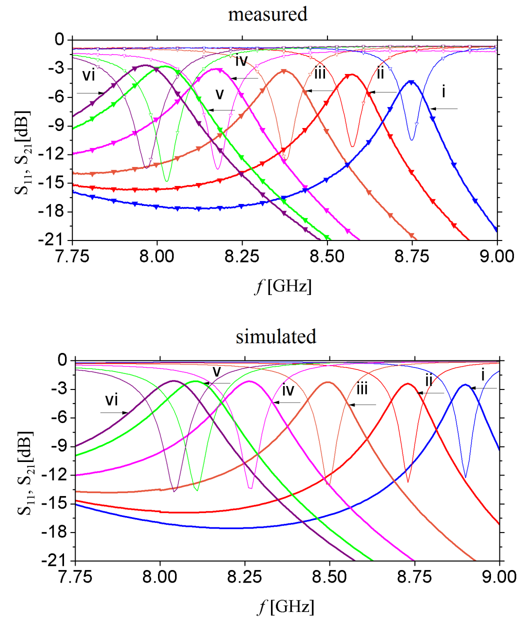


Fig. 9. Comparison of the experimentally obtained $S_{11}$ and $S_{21}$ parameters and the simulated ones for FP resonance for different slot lengths: i) without slots, ii) $L_s = 3$ mm, iii) $L_s = 4.6$ mm, iv) $L_s = 6$ mm, v) $L_s = 7.1$ mm and vi) $L_s = 8$ mm. Slot offset is $d = 5$ mm.

be greater than the dielectric permittivity in input waveguides, which is useful for the design of foam waveguides. We have also proposed the method of tuning the tunneling frequency by means of two longitudinal slots placed along the ENZ channel. The method is experimentally verified for continuous variation of the slot length and the tuning range of approximately 1GHz is obtained. The maximum achievable tuning range can be as wide as an octave, i.e. between cut-off frequencies of $TE_{10}$ and $TE_{20}$ modes of input waveguides. This design and the proposed frequency tuning method using PIN diode switch is suitable for multiband filters with agile, real-time frequency tuning capabilities, while continuous tuning with sliding backshort is more useful for sensors for wideband measurements of dielectric permittivity.

Fig. 10. Side view of fabricated ENZ waveguide with coaxial connectors.

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